

## Image Compressions

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### Abstract

The concepts acquired from linear algebra are by no means merely theoretical – linear algebra has a plethora of applications in a variety of different fields in our everyday lives. One of these incredibly valuable applications is image compression, which is also used in myriad fields and applications in and of itself. The authors of this paper decided to choose a very intriguing field in which image compression is used – the use of image compression in space missions. This paper will look into the history of image compression implementation in space missions, then explore what can be done to mimic such results with the Egyptian Space Agency (ESA), delving into the mathematical basis of such technology.

### Introduction

The concepts acquired from linear algebra are by no means merely theoretical – linear algebra has a plethora of applications in a variety of different fields in our everyday lives. As modern technology develops and the boldness of mankind advances, we find ourselves surrounded by technological applications and solutions to our plights that are predicated upon linear algebra and its intricate concepts. One of these incredibly valuable applications is image compression, which is also used in myriad fields and applications in and of itself.

However, out of this extreme diversity in which image compression implementation is characterized by, the authors of this paper decided to choose a very intriguing field in which image compression is used – the use of image compression in space missions. This application of image compression is one that is harshly overlooked by mathematicians and researchers alike, but is of crucial significance, especially considering humanity's futuristic fantasies of extraterrestrial endeavors and frontiers that, with the emergence of this new decade, are slowly morphing into a reality. Even though this technology has already been used extensively by NASA in its space flights and missions, the researchers of this paper believe that this study can fill a gap in the approach of Egyptian space technology, thereby enabling Egypt to ascend into the space scene and be a legitimate contender in future space endeavors beside other world superpowers. This paper will look into the history of image compression implementation in space missions, then explore what can be done to mimic such results with the Egyptian Space Agency (ESA), delving into the mathematical basis of such technology and analyzing the results of the researchers' experimentation.

### Methodology

The mathematical idea behind using linear algebra in image compression is quite simple. We can represent any image through its pixels by a matrix. If the pixels of an image are represented by matrix A whose dimensions are m x n, this matrix can be decomposed into three elements: U matrix, which holds the dimensions of the original matrix A; Sigma matrix, which is the result of square rooting the Eigen value obtained from the third equation (larger values are placed first in this matrix in order to ensure minimal loss); V transpose matrix, which is the transpose of the matrix holding the Eigen vector values.

This is the main equation used for reducing the size of the original matrix (aka image compression):

$$X = UV^T$$

Equation 1

This is the equation representing the Eigen pair (Lambda: Eigen value, x: Eigen vector), where matrix A is the product of X and XT:

$$A = \lambda x$$

Equation 2

This is the equation from which we obtain the values of each of the Eigen value, vector in the form of matrices. Utilizing the determinant of A results in a polynomial equation that concludes with the acquisition of the values of the Eigen value (basically solving for lambda). The square root of lambda is what is used as the values of the sigma matrix in the first equation.

$$(A - \lambda I)x = 0$$

Equation 3

Now with A, the Eigen value lambda, and the identity matrix, we can substitute and solve for x, which yields us the values of the Eigen vector. These values are used in the VT matrix in the first equation.

#### Pseudocode:

- 1- Read image  $\rightarrow X$
- 2-  $\text{np.svd}(X) \rightarrow U \sum V^t$
- 3-  $U [ :, i] \sum [ :, i] V^t [i, :]$
- 4- Save Image

```
in [3]: import os
import numpy as np
from PIL import Image
import matplotlib.pyplot as plt
%matplotlib inline

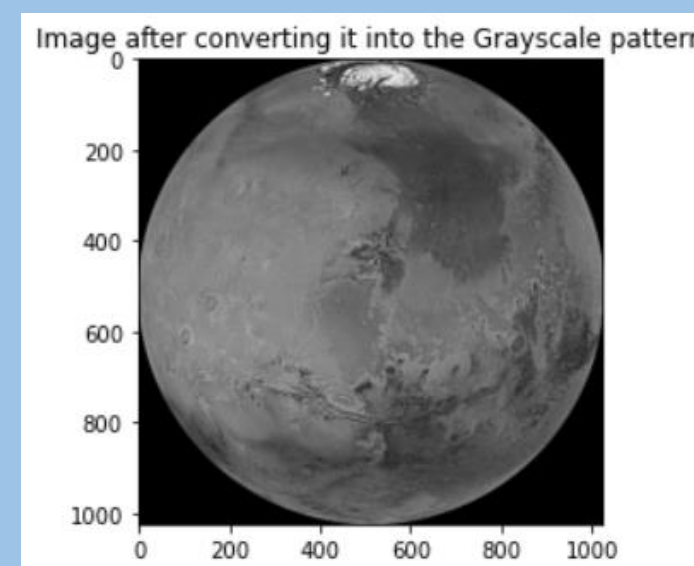
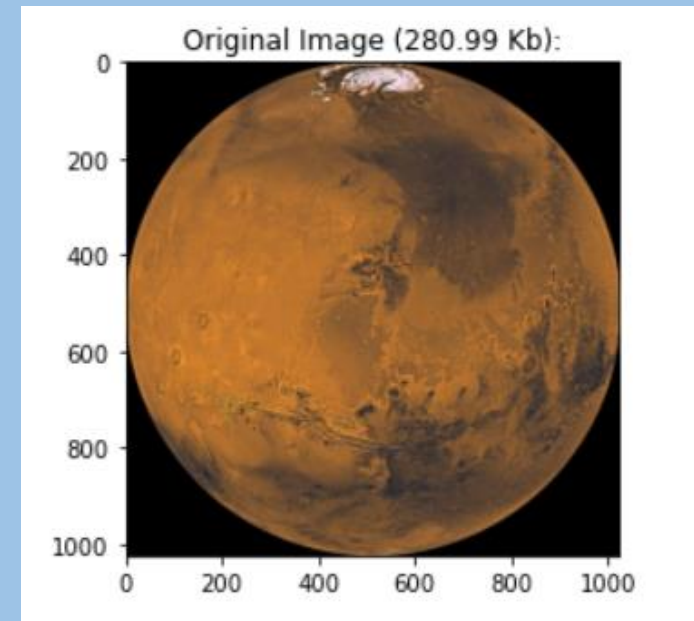
path = 'LB88HCTD78uAAxycy0Ac.jpg'
img = Image.open(path)
s = float(os.path.getsize(path))/1000
print("Size(dimension): ",img.size)
plt.title("Original Image (20.2f Kb): " %s)
plt.imshow(img)

imggray = img.convert('LA')
imgmat = np.array(list(imggray.getdata(band = 0)), float)
imgmat.shape = (imggray.size[1], imggray.size[0])
imgmat = np.matrix(imgmat)
plt.figure()
plt.imshow(imgmat, cmap = 'gray')
plt.title("Image after converting it into the Grayscale pattern")
plt.show()
```

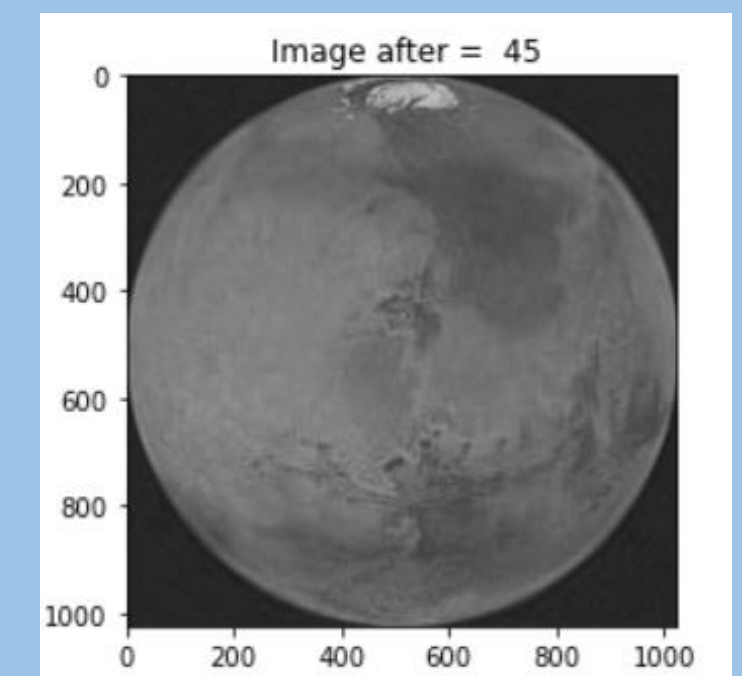
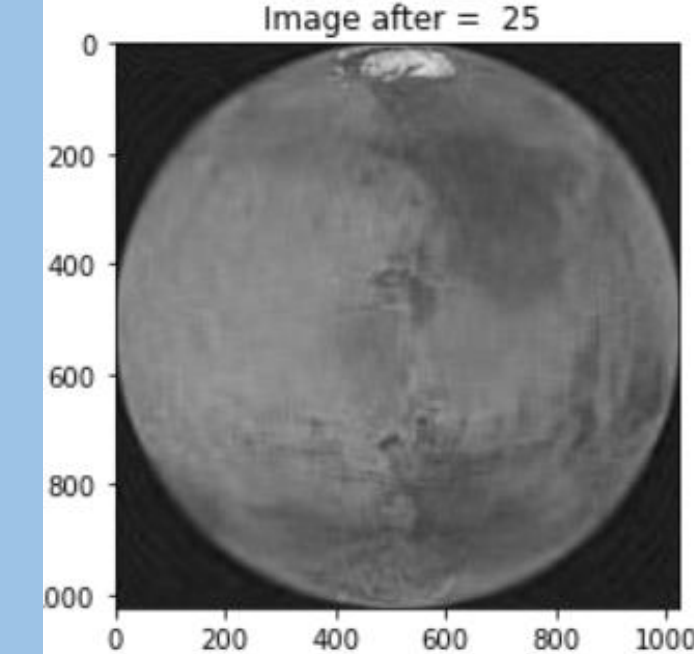
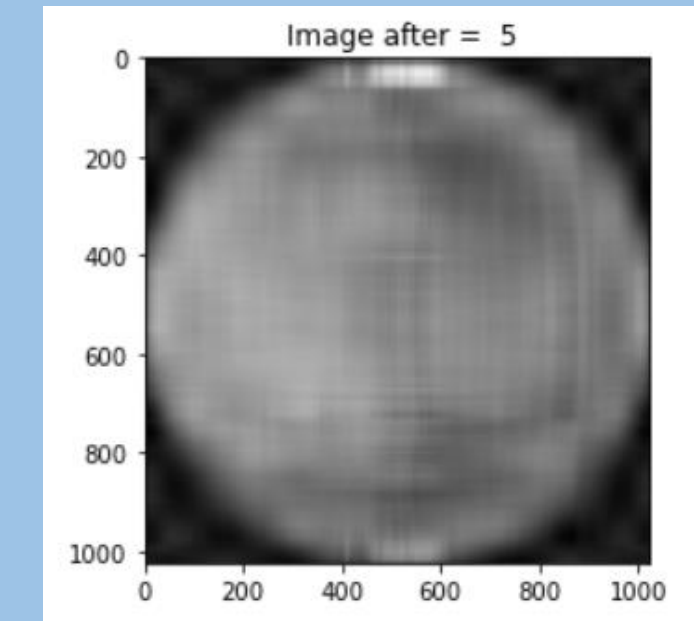
```
print("After compression: ")
U, S, Vt = np.linalg.svd(imgmat) #single value decomposition
for i in range(5, 21, 20):
    cmpimg = np.matrix(0[:, :i]) * np.diag(S[:i]) * np.matrix(Vt[:i,:])
    plt.imshow(cmpimg, cmap = 'gray')
    title = " Image after = %s" %i
    plt.title(title)
    plt.show()
    result = Image.fromarray((cmpimg ).astype(np.uint8))
result.save('OSIRIS_Mars_true_color(compressed).jpg')
```

### Results

#### Before Compression:



#### After Compression:



The image size was  
Reduced by 50% at 45

### References and Literature Review

#### References:

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- 3- Recommendation for Space Data System Standards. IMAGE DATA COMPRESSION. CCSDS 122.0-B-2. Blue Book. September 2017. Retrieved from: <https://public.ccsds.org/Pubs/122x0b2.pdf>
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#### Literature Review:

Data compression in general has been used in space missions of considerable depth since the latter part of the 1960's. Beser details history of the methods used in NASA and international space missions, pointing out that NASA proposed a standard in 1994 to address payload requirements for lossless compression. The report also includes the details surrounding the various data types that have been compressed onboard satellites – images have been used in iconic missions such as Voyager, Mars Observer, Russian Mars 96, Russian Mars 98, Cassini, Mars Global Surveyor, Clementine, Midcourse Space Experiment (MSX), and Galileo. In addition, characteristics are provided such as that pixels range from 8-12 bits, and that data rates range from record playback to real time [1].

A paper by Yeh et al. details a compression algorithm adopted in a recommendation by the CCSDS (Consultative Committee for Space Data Systems) for image data compression, as well as the requirements that determined the selection of this algorithm specifically. Results are then compared with other compressors on a test batch of space images, which proves the algorithm to be suitable for "both frame-based image data and scan-based sensor data, and has applications for near-Earth and deep-space missions." [2]

A Recommendation for Space Data System Standards by CCSDS then attempted to identify a particular payload image data compression algorithm that has widespread applicability to many types of instruments. It compares between the Recommended Standard and many substitute compression methods, such as JPEG2000, which targets the spacecraft's high-rate instruments in particular. It also contextualizes the term "pixel" to be the "coefficient of a pre-processed multispectral or hyperspectral image" [3]. Yu et al. provide a new architecture for on-board image compression systems in order to mitigate and monitor future satellite disasters during missions. [4]

### Conclusion

In a decade where the pursuit of space-related achievements is inevitable, it seems almost necessary to nurture and fast-track the growth of national space programs. This research has aimed to provide for the Egyptian Space Agency a comprehensive report of suitable approaches to compress images during future space exploration and deep space missions. The implementation of this technology will hopefully bolster Egypt's extraterrestrial intel by facilitating transmission of high-definition images of cosmic bodies to command centers, thereby substantially advancing Egypt's position in global space contention and contributing to its technological development.

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