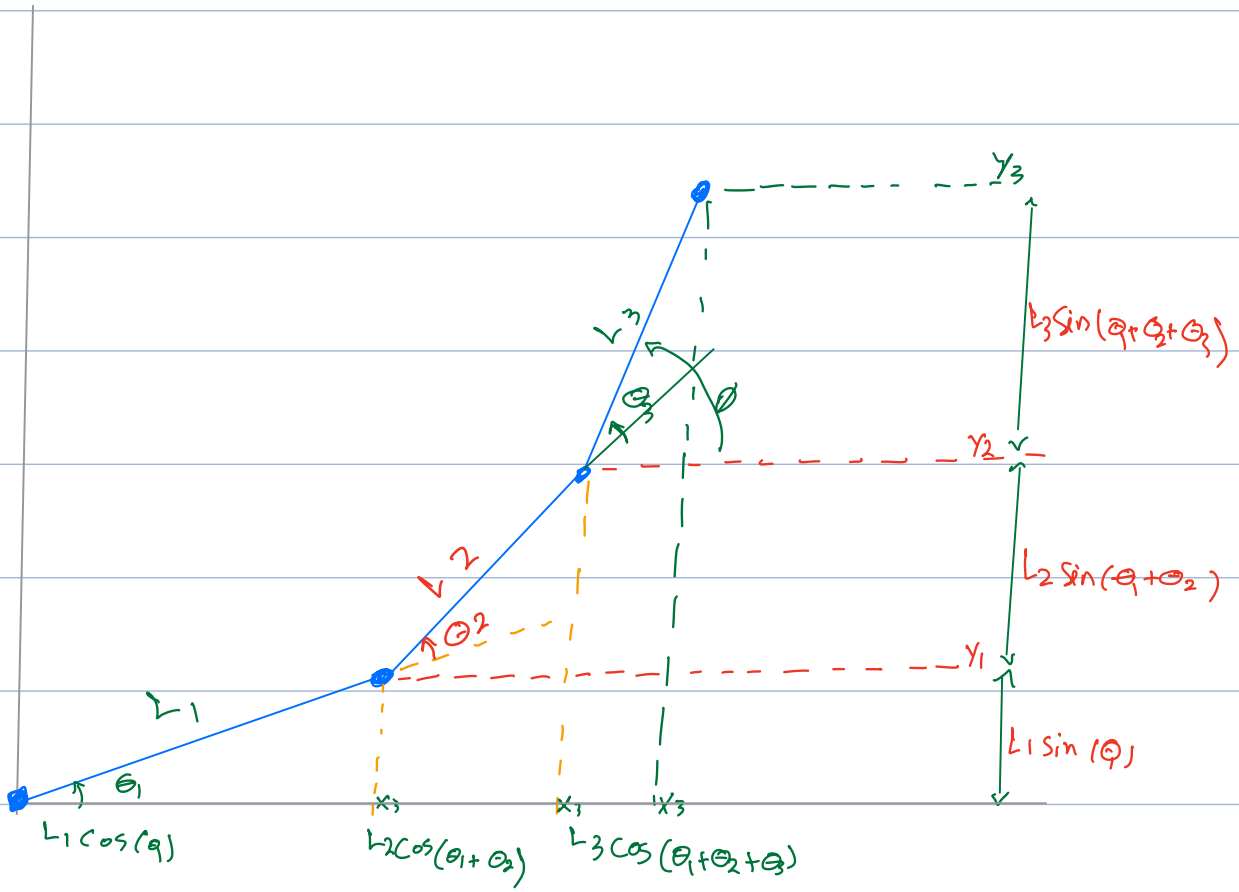

Forward given $\{\theta_1, \theta_2, \theta_3\}$

$$X_1 = L_1 \cos(\theta_1)$$

$$X_2 = X_1 + L_2 \cos(\theta_1+\theta_2) = L_1 \cos(\theta_1) + L_2 \cos(\theta_1+\theta_2)$$

$$X_3 = X_1 + X_2 + L_3 \cos(\theta_1+\theta_2+\theta_3) = L_1 \cos(\theta_1) + L_2 \cos(\theta_1+\theta_2) + L_3 \cos(\theta_1+\theta_2+\theta_3)$$

$$Y_1 = L_1 \sin(\theta_1)$$

$$Y_2 = Y_1 + L_2 \sin(\theta_1 + \theta_2) = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2)$$

$$Y_3 = Y_1 + Y_2 + L_3 \sin(\theta_1 + \theta_2 + \theta_3) = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

Inverse $[x_3, y_3, \phi]$ given

$$X_2 = X_3 - L_3 \cos(\theta_1 + \theta_2 + \theta_3)$$

$$Y_2 = Y_3 - L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$

$$\cos(\theta_2) = \frac{X_2^2 + Y_2^2 - (L_1^2 + L_2^2)}{2 L_1 L_2}$$

$$\cos(\theta_1) = \frac{X_2 [L_1 + L_2 \cos(\theta_2)]}{X_2^2 + Y_2^2}$$

$$\theta_3 = \phi - (\theta_1 + \theta_2)$$