Analyzing Radioactive Decay

Phsx 815- Project 1

Rakshak Adhikari

February 2021

1 Introduction

This project involves analyzing counts over a fixed time from a radioactive substance, which will be distributed like a Poisson distribution, and testing whether or not it can be distinguished from a Poisson distribution with a different parameter (lambda).

While the code is testing to identify Radioactive substance, the code can be used to test any Poisson Distribution for given lambda values.

2 Code

Simulating the Decay: DecaySimulator.py simulates the radioactive decay using the python standard poisson distribution generator. It takes λ and the number of measurements as input. For the run, the following values were used:

$$\lambda_0 = 50$$
$$\lambda_1 = 70$$
$$\text{no of experiments} = 10^6$$
$$\text{no of measurements per experiment} = 1$$

Poisson distribution is characterized by the following equation.

$$P(x|\lambda) = \frac{exp(-\lambda)\lambda^x}{x!} \tag{1}$$

In the DecaySimulator, λ_0 and $\lambda_1$are used for null and alternative hypothesis respectively. The code outputs the results in textfile.

Analyzing the Output: DecayAnalysis.py reads the output textfile produced by DecaySimulator.py and calculates the Likelihood ratio for each of the parameter values and also plots the histogram.
Λ_{crit} is calculated by sorting the array of the distribution and finding the x value for which (1-α) percent of the values lied to the left of the x value. Beta value can be calculated similarly.
For the sorting needs, python's built in sorting algorithm was utilized.

3 Result

The distribution of the results from the DecaySimulator is plotted in figure 1.

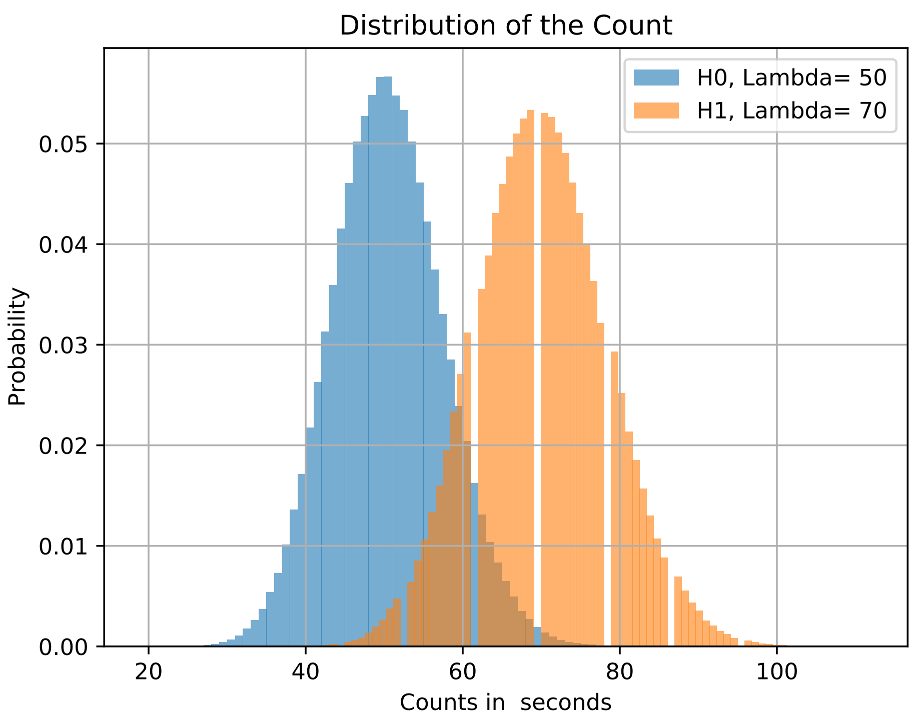
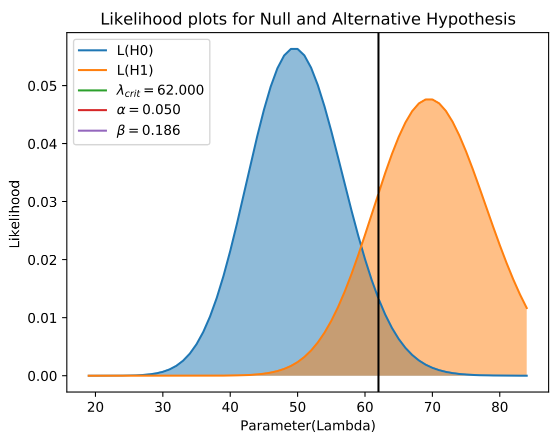


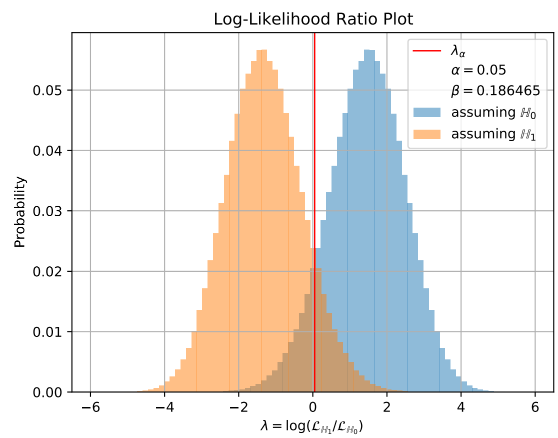
Figure 1: Poisson Distribution Generated using the two Lambda values

Since we are only taking one measurement per experiment, we can use

Likelihood curves for each of the lambda values as shown below instead of the Log-Likelihood plot. We chose our alpha to be 0.05.



(a) Likelihood Plot



(b) Log-Likelihood Ratio Plot

As seen from the plot, for alpha of 0.05, we get Lambda critical of 62 and beta of 0.186.

4 Conclusion

We can conclude from our analysis that if $\Lambda < 62 = \Lambda_{critical}$, we can reject our Alternative Hypothesis in favour of the Null Hypothesis with a confidence level of 95%.
We have sucessfully identified the radioactive substance.
Similar tests can be done to determine whether a set of values follow a Poisson distribution with a known parameter value.

References

[1] `https://github.com/Raxxak/Project-1/blob/main/Python/DecaySimulator.py`

[2] `https://github.com/Raxxak/Project-1/blob/main/Python/DecayAnalysis_LLR.py`

[3] https://github.com/Raxxak/Project-1/blob/main/Python/DecayAnalysis_Likelihood.py