

Introduction to Cryptography, Spring 2024

Homework 2

Due: 3/19/2024 (Tuesday)

Notes:

- (1) For Part A, submit a “hardcopy” right after the class on the due day.
- (2) For any question about the online judge system Formosa OJ, consult TA’s.
- (3) TAs will run plagiarism check on your submitted programs. Write your own code and do not copy from others or anywhere.

Part A: Written exercises

1. Decrypt the following ciphertext that is generated by the Vigenere autokey cipher with keyword=apple:

cgnaxqcmvr wd wtrmjmfmmek

2. Consider the one-time pad cipher with a skewed key distribution. Assume that the plaintext M is 2-bit long with distribution $\Pr[M=00]=0.2$, $\Pr[M=01]=0.25$, $\Pr[M=10]=0.4$ and $\Pr[M=11]=0.15$ and the key is picked with distribution $\Pr[K=00]=0.2$, $\Pr[K=01]=0.35$, $\Pr[K=10]=0.15$ and $\Pr[K=11]=0.3$.
 - a) What is the distribution of the ciphertext $C=M\oplus K$?
 - b) What is the deduced plaintext distribution after a ciphertext $C=10$ is observed? That is, to compute $\Pr[M=b_1b_2|C=10]$ for $b_1, b_2 \in \{0,1\}$
 - c) If you intercept a ciphertext $C=11$, what would you guess about the plaintext M ? Explain the reason.

Part B: Programming

1. This homework is to implement DES, which encrypts a 64-bit plaintext block to a 64-bit ciphertext block with a key of 64 bits (with parity bits). Do not call crypto library directly since you need to modify the code during the on-site test.
 - a. Input format: an ordered pair of key and plaintext in characters, such as “12345678 Pachinko”. Each character is interpreted as its 8 bit-ASCII code, e.g., ‘A’ = 41 (Hex)
 - b. Output format: 16 hex characters, such as “C45077C10E08B3D0” which is the ciphertext of the above key and plaintext.
 - c. Use C++ programming language in order to use the Formosa Online Judge system.
2. Submission:
 - a. Submit before 9:00am, 3/19 (Tuesday). The submission system will close on time.

- b. Submit a file DES.cpp to Formosa OJ (<https://formosa.oj.cs.nycu.edu.tw/>) with your own account.
- c. **Your code needs to read the input from `stdin`**, which contains 5 ordered pairs of key and plaintext, one in each line, such as, "12345678 Pachinko".
- d. Output: print 5 lines of ciphertexts (in Hex) for the test data **that are read from `stdin`**.
- e. Formosa OJ will compile your code and judge it on the test data **from `stdin`**.
3. On-site test
 - a. Test time: 5:30-9:00pm, 3/22 (Friday).
 - b. Test site: Computer rooms (EC315、EC316、EC324)
 - c. It is your responsibility to reserve sufficient time for completing the test. The system will close at 9 pm on time.
 - d. You will be asked to modify your DES implementation, which is your submitted C++ file on Formosa OJ, according to the given specification.
 - e. **Your code needs to read the input from `stdin`, which has the same format as the submitted version. The output format is the same also.**
4. Grade evaluation
 - a. 50%: the submitted programs and test results
 - b. 50%: correctness of the on-site test

Appendix: Join the course group on Formosa OJ

1. Please find the course "515611 密碼學概論" in the group list (<https://formosa.oj.cs.nycu.edu.tw/groups/>), and press the "Join" button.
2. **Important:** Login Formosa OJ by NYCUC OAuth2. If you don't login by NYCUC OAuth2, your username will not be the student ID and you won't have any grade on this homework.

$$\begin{aligned}
 (2)(a) \Pr[C=10] &= \Pr[M=00, K=00] + \Pr[M=01, K=01] + \Pr[M=10, K=10] + \Pr[M=11, K=11] \\
 &= 0.2 \times 0.3 + 0.3 \times 0.3 + 0.4 \times 0.15 + 0.15 \times 0.2
 \end{aligned}$$

$$\begin{aligned}
 (2)(a) \Pr[C=10] &= \Pr[M=00, K=00] + \Pr[M=01, K=01] + \Pr[M=10, K=10] + \Pr[M=11, K=11] \\
 &= 0.2 \times 0.2 + 0.25 \times 0.75 + 0.4 \times 0.15 + 0.15 \times 0.3 \\
 &= 0.2325 \\
 \Pr[C=01] &= 0.2625 \\
 \Pr[C=10] &= 0.2375 \\
 \Pr[C=11] &= 0.2675
 \end{aligned}$$

$$\begin{aligned}
 (b) \Pr[M=00 | C=10] &= \Pr[M=00, C=10] / \Pr[C=10] \\
 &= \Pr[M=00, K=10] / \Pr[C=10] = \frac{0.2 \times 0.15}{0.2375} = 0.1263
 \end{aligned}$$

$$\Pr[M=01 | C=10] = 0.1158$$

$$\Pr[M=10 | C=10] = 0.3368$$

$$\Pr[M=11 | C=10] = 0.2211$$

$$(c) \Pr[M=00 | C=11] = 0.2243$$

$$\Pr[M=01 | C=11] = 0.1402$$

$$\Pr[M=10 | C=11] = 0.5233$$

$$\Pr[M=11 | C=11] = 0.1121$$

Since $M=10$ is most likely among all possible $b_1 b_2$,
we would guess $M=10$ on observing $C=11$.