Variations of KLT(Baker et al, IJCV 2004) (Cont'd)

- $\frac{\partial}{\partial \Delta \mathbf{p}} \sum_{\mathbf{x}} \left[I\left(W(\mathbf{x}; \mathbf{p}_0)\right) - T(W(\mathbf{x};\ \Delta\mathbf{p}))\right]^2$

$$\approx \frac{\partial}{\partial \Delta \mathbf{p}} \sum_{\mathbf{x}} \left[I\left(W(\mathbf{x}; \mathbf{p}_0)\right) - T(W(\mathbf{x}; 0) - \Delta T \frac{\partial W(x;0)}{\partial p} \Delta\mathbf{p})\right]^2$$

From Chain rule derivate of the above term w.r.t $\Delta\mathbf{p}$

- We equate it to zero

$$2\sum_{\mathbf{x}} \left[\nabla T \frac{\partial W(\mathbf{x};0)}{\partial \mathrm{p}} \right]^{\mathrm{T}} \left[I(W(\mathbf{x}; \mathbf{p}_0)) - T(\mathbf{x}) - \nabla T \frac{\partial W(\mathbf{x};0)}{\partial \mathbf{p}} \Delta\mathbf{p} \right] = 0$$

$$\Delta\mathbf{p} = H^{-1} \sum_{\mathbf{x}} \left[\nabla T \frac{\partial W(\mathbf{x};0)}{\partial \mathrm{p}} \right]^{\mathrm{T}} \left[I(W(\mathbf{x}; \mathbf{p}_0)) - T(\mathbf{x}) \right]$$

- Where $H = \sum_{\mathbf{x}} \left[\nabla I \frac{\partial W(\mathbf{x};0)}{\partial \mathrm{p}} \right]^{\mathrm{T}} \left[\nabla I \frac{\partial W(\mathbf{x};0)}{\partial \mathrm{p}} \right]$

Modified KLT (Baker et al, IJCV 2004)

$$\Delta \mathbf{p} = H^{-1} \sum_{\mathbf{x}} \left[\nabla T \frac{\partial W(\mathbf{x};0)}{\partial \mathrm{p}} \right]^{\mathrm{T}} [I(W(\mathbf{x};\mathbf{p}_0)) - T(\mathbf{x})]$$

1. Warp I with $W(\mathbf{x};\mathbf{p}))$
2. Subtract T from I $[I(W(\mathbf{x};\mathbf{p}_0)) - T(\mathbf{x})]$
3. Compute Gradient ∇T (Only do once)
4. Evaluate the Jacobian $\frac{\partial W}{\partial \mathrm{p}}$ at (x;0) (Only do once)
5. Compute steepest decent $\nabla I \frac{\partial W}{\partial \mathrm{p}}$ (Only do once)
6. Compute Inverse Hessian H^{-1} (Only do once)
7. Multiply steepest descend with error $\sum_{\mathbf{x}} \left[\nabla T \frac{\partial W(\mathbf{x};0)}{\partial \mathrm{p}} \right]^{\mathrm{T}} [I(W(\mathbf{x};\mathbf{p}_0)) - T(\mathbf{x})]$
8. Compute $\Delta \mathbf{p}$ $\Delta \mathbf{p} = H^{-1} \sum_{\mathbf{x}} \left[\nabla T \frac{\partial W(\mathbf{x};0)}{\partial \mathrm{p}} \right]^{\mathrm{T}} [I(W(\mathbf{x};\mathbf{p}_0)) - T(\mathbf{x})]$
9. Update Parameters $\mathbf{p} \rightarrow \mathbf{p} + \Delta \mathbf{p}$