PCA:

1. $Scale\ the\ given\ matrix\ \boldsymbol{X}\ with\ repect\ to\ its\ mean\ i.e\ X - \bar{X}$

2. $Find\ covariance\ matrix\ of\ given\ matrix\ \boldsymbol{X}$

$$\boldsymbol{covariance\ matrix}\ (\Sigma) = \frac{\boldsymbol{XX^T}}{\boldsymbol{n-1}}$$

3. $Find\ the\ eigen\ values\ of\ the\ system$

$$\Sigma\boldsymbol{\alpha_i} = \boldsymbol{\lambda_i \alpha_i}$$

$where\ \boldsymbol{\alpha_i}\ is\ an\ eigen\ vector\ corresponding\ to\ the\ eigen\ value\ \boldsymbol{\lambda_i}$

$These\ \boldsymbol{\alpha_i}\ are\ called\ principal\ components.$

4. $The\ components\ corresponding\ to\ larger\ eigen\ values\ explain\ most\ of\ the\ variance\ in\ the\ data\ and\ are\ hence\ considered\ important\ where\ as\ the\ eigen\ vectores\ corresponding\ to\ lower\ eigen\ values\ can\ be\ discarded.$

KPCA:

1. $Project\ the\ data\ onto\ a\ higher\ dimensional\ space\ that\ makes\ it\ linearly\ separable\ (similar\ to\ SVM).this\ is\ done\ by\ adding\ a\ new\ dimension\ which\ is\ a\ function\ of\ existing\ dimensions.$

$The\ high\ dimensional\ mapping\ is\ represented\ as\ \varphi.$

$$\boldsymbol{ex}:\ (\boldsymbol{x_1}, \boldsymbol{x_2}) \to (\boldsymbol{x_1},\ \boldsymbol{x_2},\ \boldsymbol{x_1^2} + \boldsymbol{x_2^2}). \qquad \boldsymbol{\varphi}: \boldsymbol{R^2} \to \boldsymbol{R^3}.$$

2. $The\ covariance\ matrix\ in\ the\ higher\ dimension\ is\ represented\ as$

$$\Sigma = \varphi(X)\varphi(X)^T.$$

3. $This\ can\ be\ easily\ calculated\ as\ dotproduct\ of\ feature\ vectors\ in\ the\ high\ dimensional\ space\ without\ actually\ calculating\ the\ high\ dimensional\ representation.$

4. $Typically\ done\ via\ kernel\ method\ to\ obtain\ a\ matrix\ as\ follows.$

$$\mathrm{K} = \begin{bmatrix} k(\mathbf{x}_1, \mathbf{x}_1) & k(\mathbf{x}_1, \mathbf{x}_2) & \dots & k(\mathbf{x}_1, \mathbf{x}_d) \\ k(\mathbf{x}_2, \mathbf{x}_1) & k(\mathbf{x}_2, \mathbf{x}_2) & \dots & k(\mathbf{x}_2, \mathbf{x}_d) \\ \vdots & \vdots & & \vdots \\ k(\mathbf{x}_d, \mathbf{x}_1) & k(\mathbf{x}_d, \mathbf{x}_2) & \dots & k(\mathbf{x}_d, \mathbf{x}_d) \end{bmatrix}$$

$k(x1, x2)\ is\ called\ the\ kernel\ function.Typically\ Gaussian\ function\ used.$

$$k(x, y) = \exp\left(-\frac{||x-}{2\sigma}\right.$$

$most\ used\ ones\ are\ linear, polynomial\ and\ simoid\ kernel\ functions.$

1. $pick\ a\ kernel\ function\ k(x_i, x_j).$
2. $Calculate\ the\ kernel\ matrix\ K.$
3. $Center\ the\ Kernel\ matrix.$
4. $solve\ the\ eigen\ system\ \widetilde{K}\alpha_i = \lambda_i\alpha_i.$

$same\ as\ PCA\ .$