

## homework HW1 Math6373 due date thursday feb 10th at midnight

Data set:

use four years of daily market data; download 7 daily closing prices of **Gold, Platinum, Silver, DowJones, Euro, Yen, Renminbi**,

Note: Renminbi = Yuan

On day "t":  $V(t)$  = line vector of 7 prices =  $[V_1(t) \dots V_7(t)]$

the four years data set contains N actual days

replace calendar dates by index  $t=1,2,3 \dots N$

$X(t)$  = feature vector has dimension  $5 \times 7 = 35$

$X(t)$  = long line vector  $[V(t), V(t-1), V(t-2), V(t-3), V(t-4)]$

case # t is INITIALLY described by feature vector  $X_t$

Goal: construct an MLP to predict (on day t) the future gold price  $Z(t) = V_1(t+1)$

data set =  $\{X(1), X(2), \dots, X(N)\}$  cases observed over 4 years

true value  $Z(t)$  is known on the data set

Q0

for each  $j = 1 \dots 7$  compute  $M_j$  = mean over all t of the values  $V_j(t)$

for  $j=1 \dots 6$  construct the graph displaying both  $V_j(t)/M_j$  and  $V_7(t)/M_7(t)$

Visual interpretation?

Q1

replace each price  $V_j(t)$  by rate of return  $rV_j(t) = [V_j(t) - V_j(t-1)] / V_j(t-1)$

replace  $Z(t)$  by  $rZ(t) = [Z(t) - Z(t-1)] / [Z(t-1)] = [V_1(t+1) - V_1(t)] / V_1(t)$

replace  $X_t$  by  $rX(t) = [rV(t), rV(t-1), rV(t-2), rV(t-3), rV(t-4)]$

for case # t, the new feature vector is  $rV(t)$ , the true target variable to be predicted is  $rZ(t)$

compute mean  $rZ$  = average of the N values  $|rZ(t)|$

display separately the two curves

Q2

define the first attempted architecture of your MLP with 3 layers as follows

Input layer  $L_1 \rightarrow$  hidden layer  $L_2 \rightarrow$  Output layer  $L_3$

size  $L_1 = 35$  ; size  $L_2 = h$  ; size  $L_3 = 1$ ;

The integer h will be finalized below

denote  $\text{param}(h)$  the total # of weights and thresholds in this MLP

give a formula for  $\text{param}(h)$

Q3

randomly select 80% of all cases as your training set; display  $\text{TRN}$  = size of training set

the remaining 20% of cases will be the test set

apply the parsimony principle : impose  $\text{param}(h) < \#$  informations brought by the training set

compute the maximum value  $h^*$  of h , derived from this parsimony principle

Q4

fix 2 possible values for  $h$  namely  $h_1 = h^*$  and  $h_2 = 3 h^*$

note that  $h_2$  does not verify the parsimony principle

for each such value of  $h$ , launch the automatic learning of your MLP

you will need to select (and report your choices)

the type of response function( RELU is suggested)

the type of initialization of the weights and thresholds (default random choices in tensorflow)

the type of gradient descent optimizer ( Adams is a good generic choice)

the Batch Size BATS ( try 4 possible BATS values :  $TRN/40$ ,  $TRN/20$ ,  $TRN/10$ ,  $TRN/2$  )

the type of loss function (MSE)

the criterion used to stop the automatic learning (explain the basic choices in tensorflow)

for each of the 8 choices of the pair ( $h$  , BATS)

display the computing time necessary for automatic learning

display the total number numBATS of batches

display the terminal value trainMSE of MSE on the whole training set

give a comparative interpretation of these results

Q5

Monitoring of EACH one of the eight automatic learning

for  $k=1,2, \dots$  after **each epoch #  $m$**

compute trainMSE( $m$ ) on the whole training set and testMSE( $m$ ) on the whole test set

compute and display the curve  $||\text{grad MSE}(m)|| / \sqrt{\text{param}(h)}$

Q6

compute the two normalized accuracy curves

$\text{trainAcc}(k) = \sqrt{\text{trainMSE}(k)} / \text{mean.rZ}$

$\text{testAcc}(k) = \sqrt{\text{testMSE}(k)} / \text{mean.rZ}$

on the same graph display the two curves { trainAcc( $k$ ) versus  $k$  } and {testAcc( $k$ ) versus  $k$ }

interpret these results for each automatic learning ;

check if and when there is overfit;

comment the behaviour of the  $||\text{gradMSE}(k)||$

for each learning, determine an optimal stopping time  $k_{\text{opt}}$  and trainAcc , testAcc for  $k=k_{\text{opt}}$

Q7

use your preceding analyzis to determine the best pair ( $h$  , BATS), and the corresponding best weights  $W_{ij}$  + thresholds  $B_i$  reached at optimal stopping time

display the histogram of all  $|weights| = |W_{ij}|$  linking neuron  $j$  of L1 to neuron  $i$  of L2

identify the 10 smallest and the 10 largest  $|W_{ij}|$

display the histogram of all  $|weights| = |m(i,1)|$  linking neuron  $i$  of L2 to neuron 1 of L3

rank the  $|m(k,1)|$  in increasing order and display this increasing curve

Q8

Most Influential Hidden Neurons

identify the neuron  $i^*$  in L2 such that  $|m(i^*,1)| > \text{all } |m(k,1)|$

this neuron is strongly influential on the output

Q9

most influential explanatory variables

the neuron  $i^*$  is connected to 35 inputs by weights  $W(i^*,1) \dots W(i^*, 35)$

for each neuron  $j$  in L1, compute average impact of  $\text{input}(j)$  on neuron  $i^*$  by

$\text{impact}(j \text{ on } i^*) = |W(i^*,j)| \times \text{mean}|\text{input}(j)|$  where

$\text{mean}|\text{input}(j)| = \text{average value of } |\text{input}(j)| \text{ over all cases}$

rank the impacts( $j, i^*$ ) in increasing order and display these 35 ordered values

identify the 2 explanatory variables which have the highest influence on neuron  $i^*$

identify the 2 explanatory variables which have the lowest influence on neuron  $i^*$

conclusions?

Q10

your suggestions to improve the architecture of the MLP ?