

Econometrics HW 3 陈子睿

EX 5.1

(a) The 95% confidence interval of β_1 is

$$[-5.82 - 1.96 \times 2.21, -5.82 + 1.96 \times 2.21] \\ = [-10.152, -1.4884].$$

(b) We want to test

$$H_0: \beta_1 \geq 0, H_1: \beta_1 < 0.$$

The test statistic is:

$$t = \frac{\hat{\beta}_1 - 0}{SE(\hat{\beta}_1)} = -2.6335.$$

The p-value for the hypothesis test is:

$$p\text{-value} = 2\Phi(-|t|) = 2\Phi(-2.6335) = 0.00845 < 0.05.$$

Therefore we reject H_0 at the significance level $\alpha = 5\%$.

(c) The t-statistic is

$$t = \frac{\hat{\beta}_1 - (-5.6)}{SE(\hat{\beta}_1)} = 0.09955.$$

The p-value is:

$$p\text{-value} = 2\Phi(-|t|) = 2\Phi(-0.09955) = 0.9207 > 0.05$$

Therefore we fail reject H_0 at the significance level $\alpha = 0.05$, i.e. -5.6 is contained in the 95% confidence interval for β_1 .(d) The 99% confidence interval for β_0 is:

$$[520.4 \pm 2.58 \times 20.4] = [467.7, 573.0]$$

EX 5.2

(a) The estimated gender gap is

$$\beta_1 \times (\text{Male} - \text{Female}) = 2.12$$

(b) The t-statistic is

$$t = \frac{\hat{\beta}_1 - 0}{SE(\hat{\beta}_1)} = 5.8889$$

The p-value is

$$p\text{-value} = 2\Phi(-5.8889) \approx 0 < 0.05$$

Therefore we reject the H_0 at $\alpha = 0.05$

(c) The 95% confidence interval for the gender gap is:

$$[2.12 \pm 1.96 \times 0.31] = [1.4144, 2.8256]$$

(d) Mean Wage of male = $12.52 + 2.12 = 14.64$ — of female = 12.52

(e) The new regression is

$$\widehat{\text{Wage}} = 14.64 - 2.12 \times \text{Female}, R^2 = 0.06, SER = 4.2$$

As R^2 and SER are independent of the choice of explanatory variables.

EX 5.9.

(a) Since

$$\bar{y} = \frac{\bar{Y}}{\bar{x}} = \frac{1}{n\bar{x}} \cdot (Y_1 + Y_2 + \dots + Y_n)$$

therefore \bar{y} is a linear function of Y_1, Y_2, \dots, Y_n .(b). Since $E[Y_i | X_1, X_2, \dots, X_n] = \beta_1 X_i$, therefore

$$E[\bar{y} | X_1, X_2, \dots, X_n] = E\left[\frac{1}{n\bar{x}} (Y_1 + \dots + Y_n) | X_1, \dots, X_n\right]$$

$$= \frac{1}{n\bar{x}} \cdot \beta_1 \cdot (X_1 + \dots + X_n)$$

$$= \beta_1$$

EX 5.13.

(a). True

(b) True

(c) Obviously unchanged.

(d) (a) is true, and (b) will be no longer true since the errors are not conditionally homoskedasticity.