Econometrics HW5

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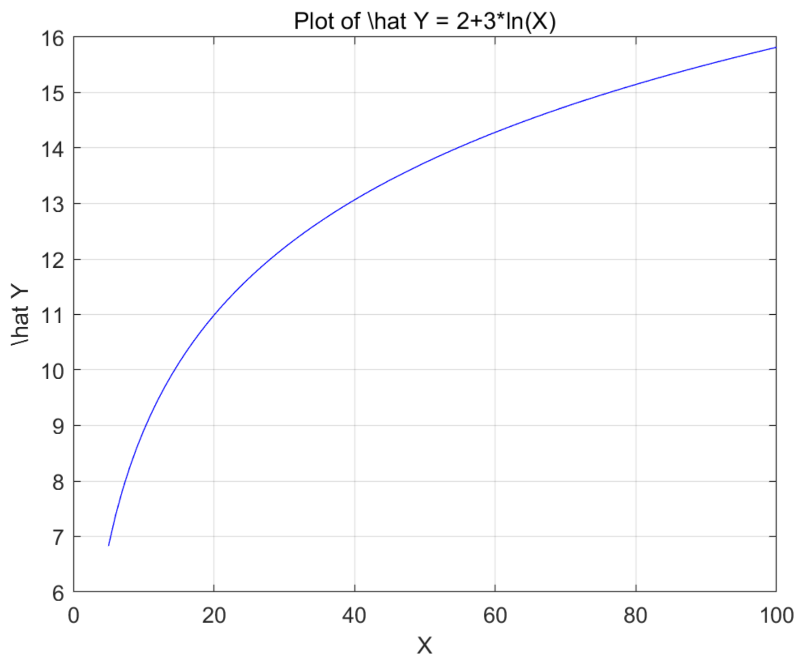
1 EX 8.3

(a) The regression functions, which are in alignment with the educator' s statement, are characterized by two regression coefficients:$\beta_1 > 0$and$\beta_2 < 0$ When the test score is plotted against the student-teacher ratio (STR), the regression will exhibit three horizontal segments. The first segment corresponds to $STR < 20$, the second segment corresponds to $20 \leq STR \leq 25$, and the final segment corresponds to $STR > 25$. The first segment is higher than the second, and the second segment is higher than the third.

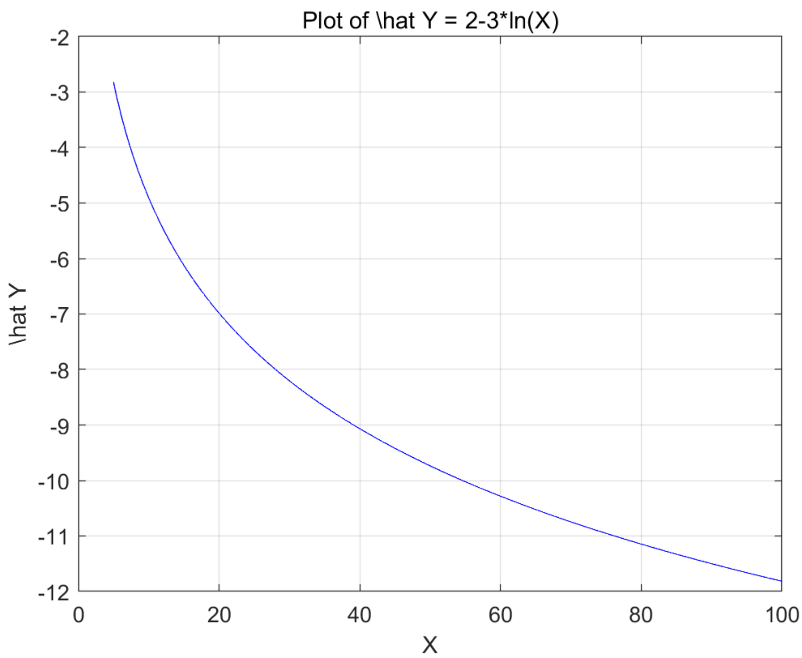
(b) The occurrence of these segments is due to **perfect multicollinearity**. When all three binary variables representing class size are included in the regression, it becomes impossible to compute the Ordinary Least Squares (OLS) estimates. This is because the intercept becomes a perfect linear function of the three class size regressors. This situation highlights the challenges of dealing with multicollinearity in regression analysis.

2 EX 8.8

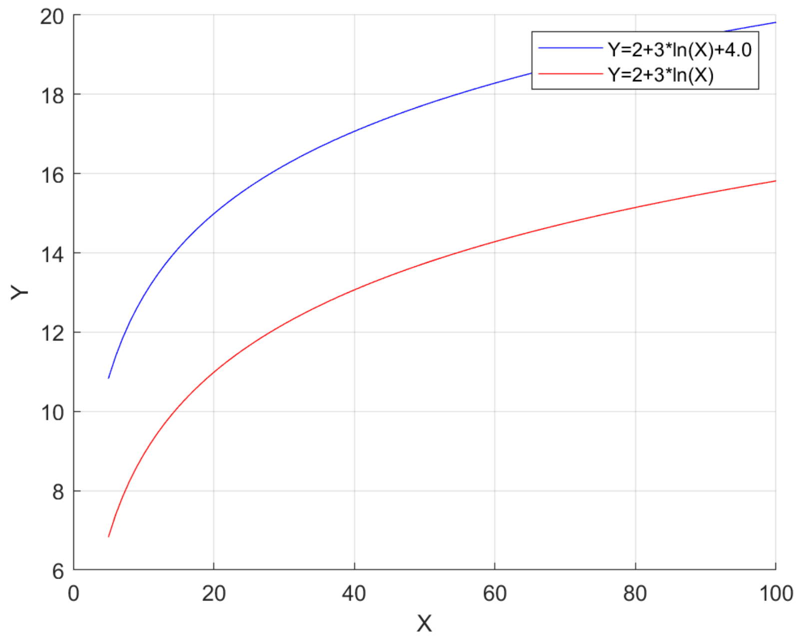
(a)



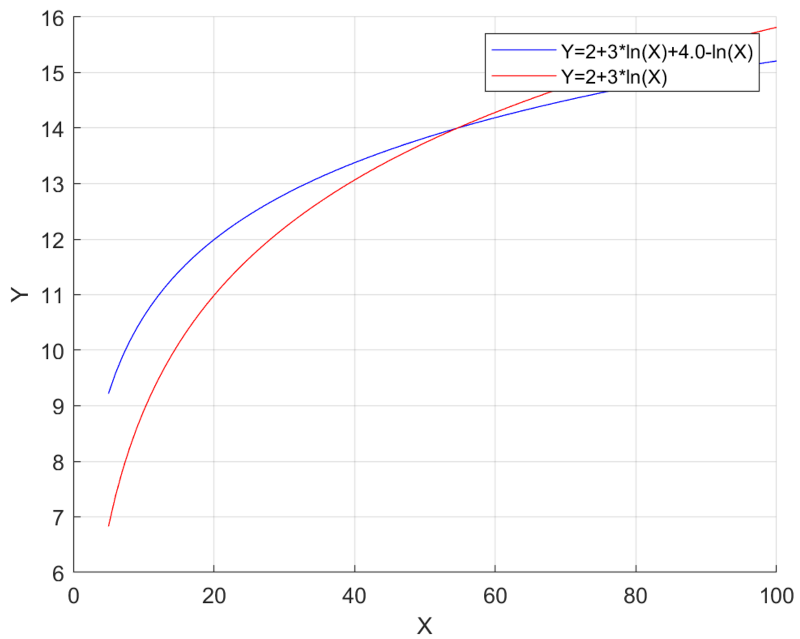
(b)


Plot of \hat Y = 2-3*ln(X)
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
\hat Y
0
20
40
60
80
100
X

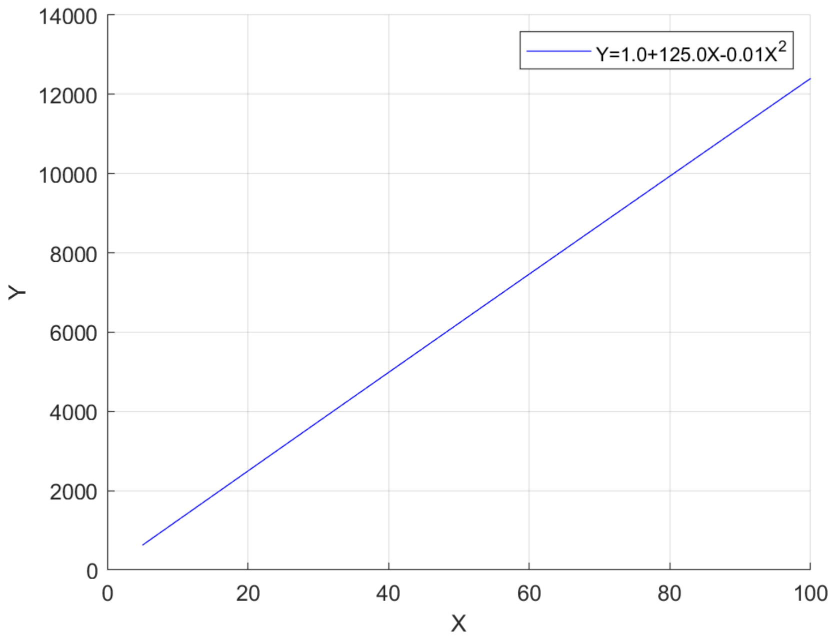
(c)


Y=2+3*ln(X)+4.0
Y=2+3*ln(X)
20
18
16
14
12
10
8
6
Y
0
20
40
60
80
100
X

(d)



(e)



3 EX 8.10

(a) **proof** we have:

$$\triangle Y = f(X_1 + \triangle X_1, X_2) - f(X_1, X_2) = \beta_1 \triangle X_1 + \beta_3 \triangle X_1 \times X_2$$

therefore,

$$\frac{\triangle Y}{\triangle X_1} = \beta_1 + \beta_3 X_2$$

(b) **proof** we have:

$$\triangle Y = f(X_1, X_2 + \triangle X_2) - f(X_1, X_2) = \beta_2 \triangle X_2 + \beta_3 X_1 \times + \triangle X_2$$

therefore,

$$\frac{\triangle Y}{\triangle X_2} = \beta_2 + \beta_3 X_1$$

(c) **proof** We have:

$$\begin{aligned}\triangle Y &= f(X_1 + \triangle X_1, X_2 + \triangle X_2) f(X_1, X_2)\\ &= \beta_0 + \beta_1(X_1 + \triangle X_1) + \beta_2(X_2 + \triangle X_2) + \beta_3(X_1 + \triangle X_1)(X_2 + \triangle X_2) - (\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2)\\ &= (\beta_1 + \beta_3 X_2) \triangle X_1 + (\beta_2 + \beta_3 X_1) \triangle X_2 + \beta_3 \triangle X_1 \triangle X_2\end{aligned}$$

4 EX 8.12

(a) Because of random assignment within the group of returning students $E(X_{1i}|u_i) = 0$ in "$\gamma - regression$", therefore, $\hat{\gamma}_1$ is unbiased for γ_1.

(b) Because of random assignment within the group of returning students $E(X_{1i}|u_i) = 0$ in "$\delta - regression$", so that $\hat{\delta}_1$ is an unbiased estimator of δ_1.

We write $E(u_i|X_{1i}, X_{2i}) = 0 = E(u_i|X_{2i} = \lambda_0 + \lambda_1 X_{2i}$ where linearity is assumed for the conditional expected value. Therefore,

$$\begin{aligned}E(Y|X_1, X_2) &= \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + E(u_i|X_{i1}, X_{i2})\\ &= (\beta_0 + \lambda_0) + \beta_1 X_1 + (\beta_2 + \lambda_1) X_2 + \beta_3 X_1 X_2\end{aligned}$$

By our discussion, $E(Y|X_1 = 1, X_2 = 0) - E(Y|X_1 = 0, X_2 = 0) = \beta_1$ which is equivalent to γ_1 from (a). Similarly, $E(Y|X_1 = 1, X_2 = 1) - E(Y|X_1 = 0, X_2 = 1) = \beta_1 + \beta_3$ which is equivalent to δ_1 from (b). Therefore, we can conclude that $\beta_3 + \delta_1 = \gamma_1$.

(d) We can define that $\epsilon_i = u_i - E(u_i|X_{1i}, X_{2i}) = u_i - E(u_i|X_{2i}$, then we have:

$$Y_i = (\beta_0 + \lambda_0) + \beta_1 X_1 + (\beta_2 + \lambda_1) X_2 + \beta_3 X_1 X_2 + \epsilon_i$$

where $E(\epsilon_i|X_{1i}, X_{2i}) = 0$, Thus, applying OLS to the equation will yield a biased estimate of the constant term $[E(\hat{\beta}_0) = \beta_0 + \lambda_0]$, an unbiased estimate of $\beta_1[E(\hat{\beta}_1 = \beta_1)]$, a biased estimate of $\beta_2[E(\hat{\beta}_2) = \beta_2 + \lambda_1]$, and an unbiased estimate of $\beta_3[E(\hat{\beta}_3 = \beta_3)]$.