Econometrics HW9

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EX 1

Solution (a) Based on the consequence in column (1), we are inferred that

$$z_{Matthew} = 0.712 + 0.031 \times 10 = 1.022.$$

And by the result from the standard normal distribution, we have that

$$\phi(1.022) = 0.847.$$

Similarly, we have

$$z_{Christopher} = 0.712 + 0.031 \times 0 = 0.712,$$

with

$$\phi(0.712) = 0.762.$$

(b) Based on column (2), the t-statistic for coefficient on *Experience* is $t = \frac{0.040}{0.016} = 2.5$, which is significant at the 5% level. Therefore,

$$Prob_{Matthew} = \frac{1}{1 + exp\{-(1.059 + 0.040 \times 10)\}} = \frac{1}{1 + exp\{-1.459\}} \approx 0.811$$

$$Prob_{Christopher} = \frac{1}{1 + exp\{-(1.059 + 0.040 \times 0)\}} = \frac{1}{1 + exp\{-1.059\}} \approx 0.742.$$

(c) Based on column (3), the t-statistic for coefficient on *Experience* is $t = \frac{0.006}{0.002} = 3$, which is significant at the 1% level. Therefore,

$$Prob_{Matthew} = 0.774 + 0.006 \times 10 = 0.834$$

$$Prob_{Christopher} = 0.774 + 0.006 \times 0 = 0.774.$$

EX 2

Solution Based on the result in column (7), we have that

(a) $\phi(0.806 + 0.041 \times 10 \times -0.074 \times 1 - 0.015 \times 1 \times 10) = 0.839$.

(b) $\phi(0.806 + 0.041 \times 2 - 0.074 \times 0 - 0.015 \times 0 \times 2) = 0.813$.

(c) The t-statistic on the interaction term is $\frac{-0.015}{0.019} = -0.79$, which is NOT significant at the 10% level.

EX 3

Solution Under the assumption that $\{X_i, Y_i\}_{i=1}^n$ are i.i.d, the joint probability distribution of $Y_1, ..., Y_n$ conditioning on X is given by

$$\begin{aligned} Prob\{Y_1 = y_1, ..., Y_n = y_n | X_1, ..., X_n\} &= \prod_{i=n}^{n} Prob(Y_i = y_i | X_i) \\ &= \prod_{i=n}^{n} p_i^{y_i}(1-p_i)^{1-y_i} \\ &= \prod_{i=n}^{n} (\beta_0 + \beta_1 X_1)^{y_i}[1-(\beta_0+\beta_1 X_1)]^{1-y_i}. \end{aligned}$$

EX 3

Solution First, we consider the conditional pdf of $Y_t|X_t$,

$$f_{Y_t|X_t}(y|x,\beta) = \frac{1}{\sqrt{2\pi}} exp\{-\frac{(y-x'\beta)^2}{2}\},$$

Then we have

$$\sum_{t=1}^{n} \ln f_{Y_t|\Psi_t}(Y_t|\Psi_t,\beta) = -\frac{n}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1}^{n}(Y_t - X_t'\beta)^2.$$

Solving the FOCs,

$$\frac{\partial \sum_{t=1}^{n} \ln f_{Y_t|\Psi_t}(Y_t|\Psi_t,\hat{\beta})}{\partial \beta} = \frac{1}{2}\sum_{t=1}^{n}(Y_t - X_t'\hat{\beta})^2,$$

We obtain

$$\hat{\beta} = (X'X)^{-1}X'Y.$$

Which indicates that the MLE estimators are the same as the OLS estimator.