Quick Review of High School Mathematics

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January 12, 2026

Contents

1 Book 1

1.1 Distance of a Point From a Line

$$\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

1.2 Distance Between Two Parallel Lines

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

1.3 Equation of a Circle Given The Endpoints of a Diameter

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$$

1.4 Equations of a Tangent Line or a Chord of Contact on A Circle

$$(x-h)(x_0-h)+(y-k)(y_0-k)=r^2$$

$$x_0x+y_0y+d\frac{x+x_0}{2}+e\frac{y+y_0}{2}+f=0$$

1.5 Lagrange Interpolation

$$\sum_{j=0}^{n} f(x_j)\left(\prod_{\substack{i=0\\ i\neq j}}^{n}\frac{x-x_i}{x_j-x_i}\right)$$

2 Book 2

2.1 Series

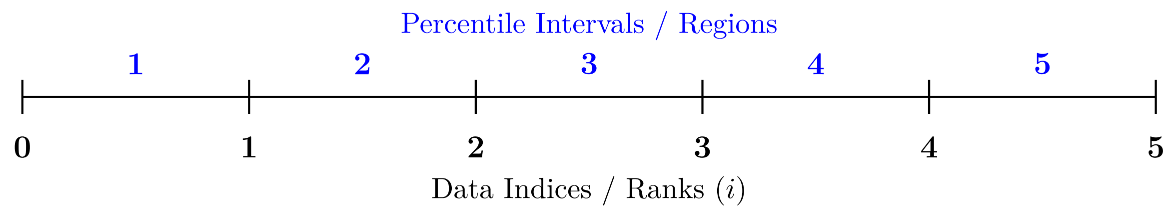
$$\sum_{k=1}^{n} ar^{k-1}=\frac{a(1-r^n)}{1-r}$$

$$\sum_{k=1}^{n} k=\frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2=\frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3=\left[\frac{n(n+1)}{2}\right]^2$$

2.2 Percentiles



2.3 Variance and Standard Deviation

$$\sigma^2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2=\frac{1}{n}\sum_{i=1}^{n}x_i^2-\mu^2$$

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2 - \mu^2}$$

2.4 Correlation Coefficient

$$r = \frac{1}{n}\sum_{i=1}^{n} x_i' y_i' = \frac{S_{xy}}{\sqrt{S_{xx}}\sqrt{S_{xy}}}$$

Note: The numerator is proportional to n, and the denominator is proportional to $\sqrt{n} \times \sqrt{n} = n$, so no extra multiplication or division by n is required.

2.5 Line of Best Fit

$$y - \mu_y = r \times \frac{\sigma_y}{\sigma_x}(x - \mu_x) = \frac{S_{xy}}{S_{xx}}(x - \mu_x)$$

2.6 Pascal's Principle

$$C_k^n = C_{k-1}^{n-1} + C_k^{n-1}$$

2.7 Combination With Repetition

$$H_k^n = C_{n-1}^{n+k-1}$$

2.8 Derangement

$$D_n = n!\sum_{k=0}^{n}\frac{(-1)^k}{k!}$$

2.9 Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

2.10 Cosine Rule

$$a^2 = b^2 + c^2 - 2ab\cos A$$

3 Book 3

3.1 Projection Vector And Its Length

Projection Vector:

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\cdot\frac{\vec{b}}{|\vec{b}|}$$

Length of Projection Vector:

$$\frac{|\vec{a}\cdot\vec{b}|}{|\vec{b}|}$$

3.2 Triangle Inequality

$$|x+y| \le |x| + |y|$$

$$||x| - |y|| \le |x-y|$$

3.3 Cauchy-Schwarz Inequality

$$(a_1^2 + a_2^2)(b_1^2 + b_2^2) \ge (a_1b_1 + a_2b_2)^2$$

The equality holds if and only if $a_1b_2 = a_2b_1$

3.4 Two-Dimensional Determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = -\begin{vmatrix} b & a \\ d & c \end{vmatrix}; \quad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = -\begin{vmatrix} c & d \\ a & b \end{vmatrix}$$

$$\begin{vmatrix} ra & a \\ rc & c \end{vmatrix} = 0; \quad \begin{vmatrix} ra & rb \\ a & b \end{vmatrix} = 0$$

$$\begin{vmatrix} ra & b \\ rc & d \end{vmatrix} = r\begin{vmatrix} a & b \\ c & d \end{vmatrix}; \quad \begin{vmatrix} ra & rb \\ c & d \end{vmatrix} = r\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\begin{vmatrix} a & ka+b \\ c & kc+d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}; \quad \begin{vmatrix} a & b \\ ka+c & kb+d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\begin{vmatrix} a+x & b \\ c+y & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} x & b \\ y & d \end{vmatrix}; \quad \begin{vmatrix} a+x & b+y \\ c & d \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} x & y \\ c & d \end{vmatrix}$$

3.5 Cramer's Rule (2×2)

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

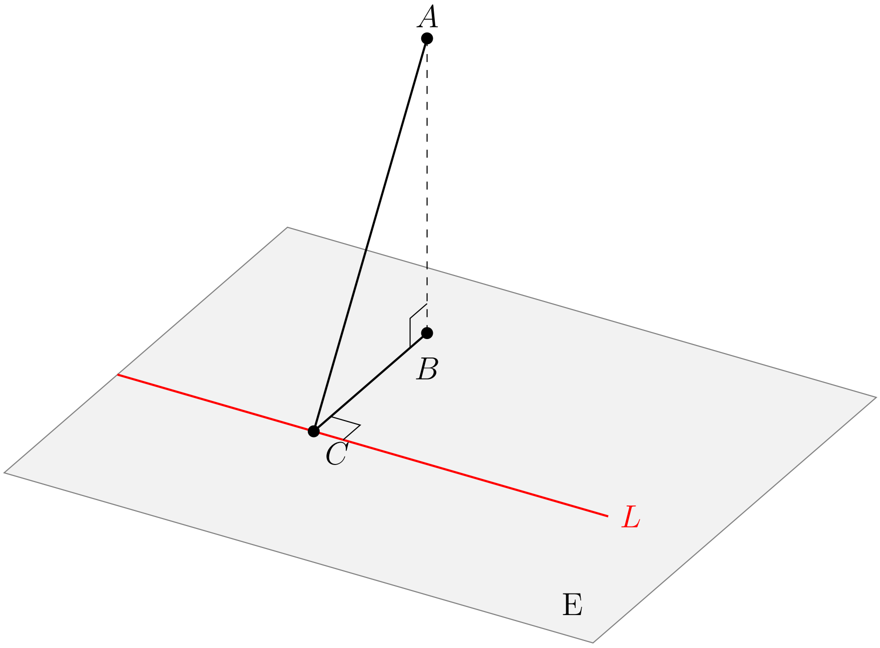
$$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, \quad \Delta_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, \quad \Delta_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

Condition	Solution	Geometric Meaning
$\Delta \neq 0$	Unique ($x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}$)	Intersecting lines
$\Delta = 0$, $(\Delta_x \neq 0 \vee \Delta_y \neq 0)$	None	Parallel distinct lines
$\Delta = \Delta_x = \Delta_y = 0$	Infinite	Coincident lines

4 Book 4

4.1 Theorem of Three Perpendiculars

Let line AB be perpendicular to plane E at point B. If on plane E the line BC is perpendicular to line L at C, then line AC is perpendicular to line L at point C.



4.2 Cross Product

$$(a_1, a_2, a_3) \times (b_1, b_2, b_3) = \left(\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}, \begin{vmatrix} a_3 & a_1 \\ b_3 & b_1 \end{vmatrix}, \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \right)$$

4.3 Volume of Parallelepipeds

$$|\vec{a} \cdot \vec{b} \times \vec{c}| = | \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} |$$

4.4 Laplace Expansion

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

4.5 Point-Normal Form

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

4.6 Distance of a Point From a Plane

$$\frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

4.7 Distance Between Two Parallel Planes

$$\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

4.8 Lines in Space

$$\begin{cases} x = x_0 + at \\ y = y_0 + bt \\ z = z_0 + ct \end{cases}$$

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

4.9 Conditional Probability

$$P(B|A) = \frac{n(A \cap B)}{n(A)} = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(B)P(A|B) = P(A)P(A|B)$$

$$P(B) = P(A' \cap B) + P(A \cap B)$$

4.10 Independent Events

$$P(A \cap B) = P(A)P(B)$$

If the events A, B are independent, then A', B are independent; A, B' are independent; and A', B' are independent.

4.11 Bayes' Theorem

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)} = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(A')P(B|A')}$$

4.12 Operational Properties of Matrices

$$(AB)C = A(BC)$$

$$A(B + C) = AB + AC$$

$$(A + B)C = AC + BC$$

4.13 Inverse Matrix

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

4.14 Scaling Matrix

$$\begin{bmatrix} h & 0 \\ 0 & k \end{bmatrix}$$

4.15 Shear Matrix

$$\begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$$

4.16 Rotation Matrix

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

4.17 Reflection Matrix

$$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

4.18 Transition Matrix

$$\begin{bmatrix} a & b \\ 1-a & 1-b \end{bmatrix}$$

where $0 \leq a, b \leq 1$

5 Additional Tricks

5.1 Centroids of Triangles

$$\overrightarrow{DG} = \frac{1}{3}\overrightarrow{DA} + \frac{1}{3}\overrightarrow{DB} + \frac{1}{3}\overrightarrow{DC}$$

5.2 Circumcenters of Triangles

$$\overrightarrow{AB} \cdot \overrightarrow{AO} = \frac{1}{2}\overline{AB}^2$$

5.3 Incenters of Triangles

$$\overrightarrow{DI} = \frac{\overline{BC}}{\overline{AB}+\overline{BC}+\overline{CA}}\overrightarrow{DA} + \frac{\overline{CA}}{\overline{AB}+\overline{BC}+\overline{CA}}\overrightarrow{DB} + \frac{\overline{AB}}{\overline{AB}+\overline{BC}+\overline{CA}}\overrightarrow{DC}$$

5.4 Orthocenters of Triangles

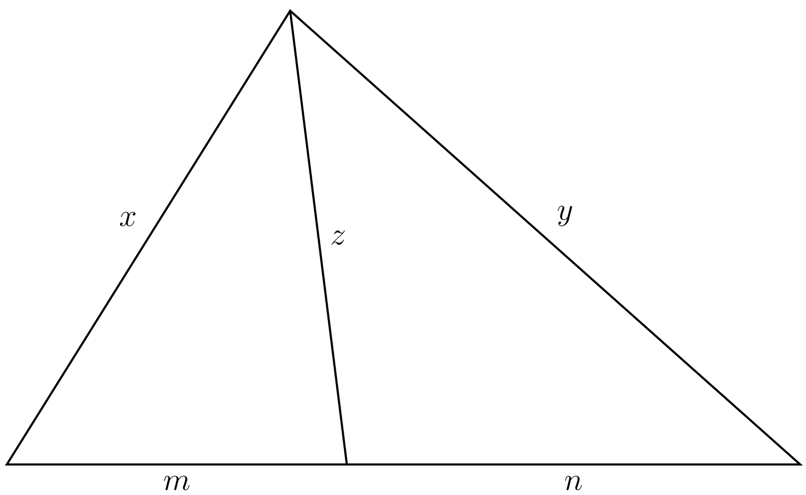
$$\overrightarrow{AB} \cdot \overrightarrow{AC} = \overrightarrow{AB} \cdot \overrightarrow{AH} = \overrightarrow{AC} \cdot \overrightarrow{AH}$$

5.5 The Pairing Method

While dividing some labeled items into groups of two, pick the partners for the unpaired items. For example, the numbers of ways to pair 5 people and one people left is $C_1^5 \times 3 \times 1 = 15$.

5.6 Stewart's Theorem

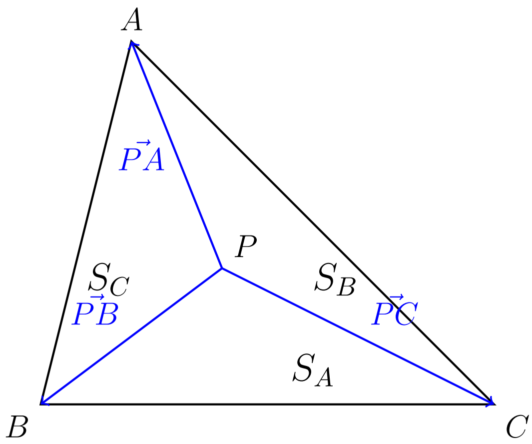
$$z^2 = \frac{n}{m+n}\,x^2 + \frac{m}{m+n}\,y^2 - mn$$



When the dividing line is a bisector, $z^2 = xy - mn$.
When it is a median, $2(z^2 + n^2) = x^2 + y^2$.

5.7 Vectors And Areas about a Point inside a Triangle

$$S_A\vec{PA} + S_B\vec{PB} + S_C\vec{PC} = \vec{0}$$



5.8 Areas of Triangles in Terms of Circumradius

$$Area = \frac{abc}{4R}$$

5.9 Shoelace Formula

$$Area = \frac{1}{2} \left| \begin{vmatrix} x_1 & x_2 & \dots & x_k & x_1 \\ y_1 & y_2 & \dots & y_k & y_1 \end{vmatrix} \right|$$