

# CPSC 354 — Report

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## 1 Assignment 1: MIU System — Can we derive MIII from MI?

### Problem (restated)

Starting from MI, and using the MIU rules:

- I. If a string ends with I, you may append U.
- II. From Mx you may infer Mxx.
- III. Replace any occurrence of III by U.
- IV. Delete any occurrence of UU.

Determine whether MIII is derivable. Explain your reasoning.

### Solution

**Idea.** Track the count of I's. Rule I and Rule IV do not change that count. Rule II doubles it; Rule III removes 3 but can only apply if 3 consecutive I's already exist.

Starting from MI (one I), the only growth is doubling. So after the first step the number of I's is always even. Removing triples never gives exactly three. So MIII cannot appear.

### Conclusion

It is **impossible** to derive MIII from MI. Reason: the number of I's is never equal to three under these rules.


## 2 Assignment 2: ARS Pictures & Properties


### Task (restated)

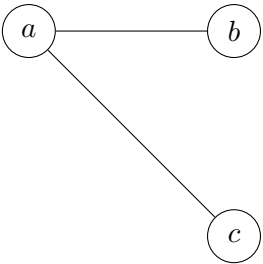
For each given abstract reduction system (ARS)  $(A, R)$ : draw the graph, decide whether it is terminating, confluent, and whether it has unique normal forms (UNFs).


## Instances

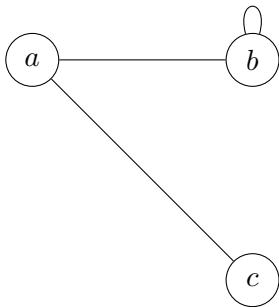
1.  $A = \emptyset$  No nodes/edges. *Terminating*: True. *Confluent*: True. *UNFs*: True.

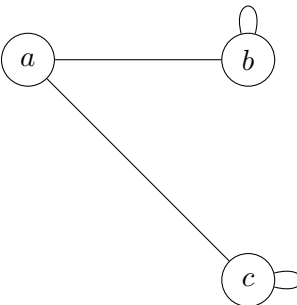
2.  $A = \{a\}$ ,  $R = \emptyset$    
Normal forms:  $a$ . *Terminating*: True. *Confluent*: True. *UNFs*: True.

3.  $A = \{a\}$ ,  $R = \{(a, a)\}$    
Infinite  $a \rightarrow a \rightarrow \dots$ : *Terminating*: **False**. *Confluent*: **True**. *UNFs*: **False**.

4.  $A = \{a, b, c\}$ ,  $R = \{(a, b), (a, c)\}$   
 $b, c$  are normal; from  $a$  two distinct NFs. *Terminating*: **True**. *Confluent*: **False**. *UNFs*: **False**.  


5.  $A = \{a, b\}$ ,  $R = \{(a, a), (a, b)\}$    
 $b$  is normal; every element has the (unique) NF  $b$ . *Terminating*: **False**. *Confluent*: **True**. *UNFs*: **True**.

6.  $A = \{a, b, c\}$ ,  $R = \{(a, b), (b, b), (a, c)\}$   
Non-terminating via  $b \rightarrow b$ ;  $c$  is normal,  $b$  has no NF. *Confluent*: **False**. *Terminating*: **False**. *UNFs*: **False**.  


7.  $A = \{a, b, c\}$ ,  $R = \{(a, b), (b, b), (a, c), (c, c)\}$   


Both  $b$  and  $c$  loop; no NFs reachable from  $a$ . Terminating: **False**. Confluent: **False**. UNFs: **False**.

### Summary Table

#	$(A, R)$	confluent	terminating	unique normal forms
1	$(\emptyset, \emptyset)$	True	True	True
2	$(\{a\}, \emptyset)$	True	True	True
3	$(\{a\}, \{(a, a)\})$	True	False	False
4	$(\{a, b, c\}, \{(a, b), (a, c)\})$	False	True	False
5	$(\{a, b\}, \{(a, a), (a, b)\})$	True	False	True
6	$(\{a, b, c\}, \{(a, b), (b, b), (a, c)\})$	False	False	False
7	$(\{a, b, c\}, \{(a, b), (b, b), (a, c), (c, c)\})$	False	False	False

### All 8 combinations

confluent	terminating	unique NFs	example
True	True	True	e.g. ARS 2 (or 1)
True	True	False	<i>Impossible</i>
True	False	True	ARS 5
True	False	False	ARS 3
False	True	True	<i>Impossible</i>
False	True	False	ARS 4
False	False	True	<i>Impossible</i>
False	False	False	ARS 6 (or 7)

## 3 Assignment 3: Exercises 5 and 5b

### Problem (restated)

Exercise 5 asks us to analyse a given ARS and check for termination, confluence, and unique normal forms. Exercise 5b asks us to consider a small variation and explain the difference.

### Solution to Exercise 5

For the ARS with  $A = \{a, b\}$  and rules  $a \rightarrow a$ ,  $a \rightarrow b$ :

- There is a loop  $a \rightarrow a$ , so the system is **not terminating**.
- From  $a$  we can still reach  $b$ , and  $b$  is normal. Every path from  $a$  eventually has the option to reach  $b$ , and once at  $b$  no rules apply.
- Thus the ARS is **confluent**: all reductions can be joined at  $b$ .
- Normal forms are unique: everything reduces to  $b$ .

## Solution to Exercise 5b

Now suppose we add  $c$  with  $a \rightarrow c$  and  $c \rightarrow c$ :

- Again,  $a$  can reduce to both  $b$  and  $c$ .
- But  $c$  loops forever and never reaches  $b$ .
- That means the system is no longer confluent: starting from  $a$  you can end in either the normal form  $b$  or the non-terminating loop on  $c$ .
- Unique normal forms are therefore lost.

## Conclusion

Exercise 5 shows a non-terminating but confluent system (everything has the unique NF  $b$ ). Exercise 5b shows that adding another looping branch breaks confluence, since from  $a$  different outcomes are possible.