The MIU System and the Impossibility of MIII

Goal

The goal is to determine whether it is possible to derive `MIII` starting from `MI` using the rules of the MIU system.

Rules

Rule I: If you possess a string whose last letter is `I`, you can add a `U` at the end.

Rule II: Suppose you have `Mx`. Then you may add `Mxx` to your collection.

Rule III: If `III` occurs in one of the strings in your collection, you may make a new string with `U` in place of `III`.

Rule IV: If `UU` occurs inside one of your strings, you can drop it.

Attempts

```
                        v----------------------v
MI - MIU - MIUIU - MIUUIUU - MII - MIIU - MIIUU - MII *
                              MIIII - MUI - MUI - MUIIU
*THOUGHT* If possible, try to get MIII
```

For example: starting with one `I`, Rule II can double it, giving two total `I`'s with a `U` in between. Because the only way for the number of `I`'s to grow is doubling, every time Rule III might apply, there will always be one leftover. This means it is never possible to reach exactly three `I`'s.

Conclusion

None of the rules leads to the number of `I`'s being three.

- Rule I has no effect on the number of `I`'s.

- Rule II can only double the `I`'s, which means they are always even after the first step. Getting exactly three `I`'s is therefore impossible.

- Rule III is the important rule, because if we could get three `I`'s, this is how we would turn it into `MU`. But since we can't reach three `I`'s, the rule never helps.

- Rule IV has no effect on the number of `I`'s.

Therefore, it is impossible to derive `MIII` from `MI`.