

CPSC-354 Report

Ray Hettleman
Chapman University
rhettleman@chapman.edu

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Abstract

This report collects my weekly notes, homework, and reflections for CPSC-354.

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1 Introduction

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2 Week by Week

2.1 Week 1

2.1.1 Notes and Exploration

We studied the MIU system from Hofstadter's *Gödel, Escher, Bach*. The task was to decide whether the string MIII can be derived from MI using the system's rules.

2.1.2 Homework

Rules.

- I. If a string ends with I, you may append U.
- II. From Mx you may infer Mxx.
- III. Replace any occurrence of III by U.
- IV. Delete any occurrence of UU.

Reasoning. We begin with MI. Rule I allows MIU. Rule II doubles the sequence after M: $MI \Rightarrow MII$, then MIIII, etc. Rule III can only replace consecutive III with a U. But because doubling produces powers of two I's (1, 2, 4, 8, ...), we never get exactly three I's. Thus, no sequence of rules produces MIII.

Conclusion. It is impossible to derive MIII from MI. The parity of the number of I's (always even after the first doubling) prevents reaching 3.

2.1.3 Questions

What is a rule that could be implemented that, while still requiring many steps, makes the MU-Puzzle solvable?


2.2 Week 2


2.2.1 Notes and Exploration

We explored Abstract Reduction Systems (ARS), focusing on termination, confluence, and unique normal forms (UNFs). Each ARS was represented as a graph, and we determined its key properties.

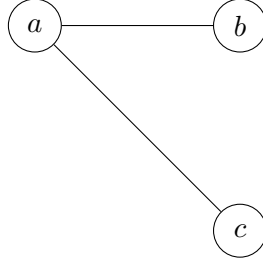
2.2.2 Homework

1. $A = \emptyset$ No nodes or edges. Terminating: True. Confluent: True. UNFs: True.

2. $A = \{a\}$, $R = \emptyset$ 
Normal forms: a . Terminating: True. Confluent: True. UNFs: True.


3. $A = \{a\}, R = \{(a, a)\}$ 

Infinite sequence $a \rightarrow a \rightarrow \dots$. Terminating: False. Confluent: True. UNFs: False.

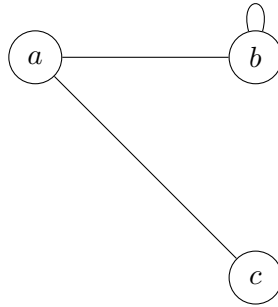


4. $A = \{a, b, c\}, R = \{(a, b), (a, c)\}$

b and c are normal forms, so from a two distinct endpoints are reachable. Terminating: True. Confluent: False. UNFs: False.

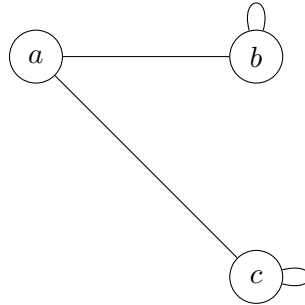
5. $A = \{a, b\}, R = \{(a, a), (a, b)\}$ 

b is normal; all paths from a can reach b . Terminating: False. Confluent: True. UNFs: True.



6. $A = \{a, b, c\}, R = \{(a, b), (b, b), (a, c)\}$

Non-terminating through $b \rightarrow b$; c is normal but unreachable from b . Terminating: False. Confluent: False. UNFs: False.



7. $A = \{a, b, c\}, R = \{(a, b), (b, b), (a, c), (c, c)\}$

Both b and c loop indefinitely. Terminating: False. Confluent: False. UNFs: False.

Summary Table:

#	(A, R)	confluent	terminating	unique NFs
1	(\emptyset, \emptyset)	True	True	True
2	$(\{a\}, \emptyset)$	True	True	True
3	$(\{a\}, \{(a, a)\})$	True	False	False
4	$(\{a, b, c\}, \{(a, b), (a, c)\})$	False	True	False
5	$(\{a, b\}, \{(a, a), (a, b)\})$	True	False	True
6	$(\{a, b, c\}, \{(a, b), (b, b), (a, c)\})$	False	False	False
7	$(\{a, b, c\}, \{(a, b), (b, b), (a, c), (c, c)\})$	False	False	False

All 8 Combinations:

confluent	terminating	unique NFs	example
True	True	True	ARS 2 (or 1)
True	True	False	<i>Impossible</i>
True	False	True	ARS 5
True	False	False	ARS 3
False	True	True	<i>Impossible</i>
False	True	False	ARS 4
False	False	True	<i>Impossible</i>
False	False	False	ARS 6 (or 7)

2.2.3 Questions

Is there an easy way to tell from the graph if an ARS will terminate, or do you always have to trace every path?

2.3 Week 3

2.3.1 Notes and Exploration

Exercises 5 and 5b extended ARS analysis with additional looping rules.

2.3.2 Homework

Exercise 5. $A = \{a, b\}$ with $a \rightarrow a$, $a \rightarrow b$. Loop $a \rightarrow a$ means not terminating, but since all paths eventually lead to b , the system is confluent with a unique NF.

Exercise 5b. Add c with $a \rightarrow c$, $c \rightarrow c$. Now one branch terminates (b) and the other loops, breaking confluence.

2.3.3 Questions

Could we make a version of 5b that still loops but somehow keeps unique normal forms?

2.4 Week 4

2.4.1 Notes and Exploration

We introduced termination proofs and measure functions.

2.4.2 Homework

HW 4.1 (Euclidean Algorithm).

```
while b != 0:
    temp = b
    b = a mod b
    a = temp
return a
```

This algorithm terminates when $b > 0$ because b decreases with each iteration. The measure function $m(a, b) = b$ is always non-negative and strictly decreases.

HW 4.2 (Merge Sort).

```
function merge_sort(arr, left, right):
    if left >= right: return
    mid = (left + right) / 2
    merge_sort(arr, left, mid)
    merge_sort(arr, mid+1, right)
    merge(arr, left, mid, right)
```

The measure $\varphi(left, right) = right - left + 1$ decreases with every recursive call. When it reaches 1, the recursion stops, proving termination.

2.4.3 Questions

What would it look like to design a sorting algorithm that checks to see if an input is terminating?

2.5 Week 5

2.5.1 Notes and Exploration

We practiced λ -calculus reduction and substitution.

2.5.2 Homework

Evaluate:

$$(\lambda f. \lambda x. f(f(x))) (\lambda f. \lambda x. f(f(f(x)))).$$

Step 1. Parentheses group left:

$$((\lambda f. \lambda x. f(f(x))) (\lambda f. \lambda x. f(f(f(x)))))$$

Step 2. Apply β -reduction:

$$\mapsto \lambda x. (\lambda f. \lambda x. f(f(f(x)))) ((\lambda f. \lambda x. f(f(f(x)))) x).$$

Step 3. Rename inner $x \rightarrow y$ and reduce:

$$(\lambda f. \lambda y. f(f(f(y)))) x \mapsto \lambda y. x(x(x(y))).$$

Step 4. Apply again:

$$\lambda x. \lambda y. x^9(y).$$

Conclusion. The function composes f twice and f^3 thrice, resulting in a ninefold application.

2.5.3 Questions

What would happen if we swapped the two functions in the workout?

2.6 Week 6

2.6.1 Notes and Exploration

We computed factorial using the fixed-point combinator `fix`.

2.6.2 Homework

Compute:

```
let rec fact = \n. if n=0 then 1 else n * fact (n-1) in 3
```

Using the rules:

$$\text{fix } F \mapsto F (\text{fix } F), \quad \text{let rec } f = e_1 \text{ in } e_2 \mapsto \text{let } f = (\text{fix } (\lambda f. e_1)) \text{ in } e_2.$$

We unfold recursively:

$$\text{fact } 3 \mapsto 3 * (\text{fix } F) \ 2 \mapsto 3 * 2 * 1 = 6.$$

Conclusion. Following only the allowed computation rules, `fact 3` reduces to `6`.

2.6.3 Questions

Would using α -conversion anywhere in the factorial example change the result, or just make it cleaner?

2.7 Week 7

2.7.1 Notes and Exploration

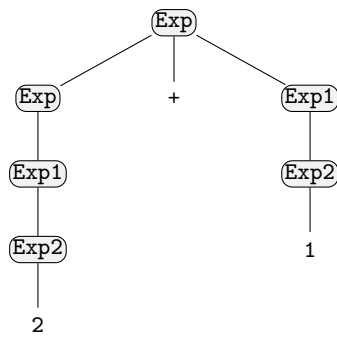
Parsing and context-free grammars. We use the grammar (nonterminals `Exp`, `Exp1`, `Exp2`):

```
Exp  -> Exp '+' Exp1
Exp1 -> Exp1 '*' Exp2
Exp2 -> Integer
Exp2 -> '(' Exp ') '
Exp  -> Exp1
Exp1 -> Exp2
```

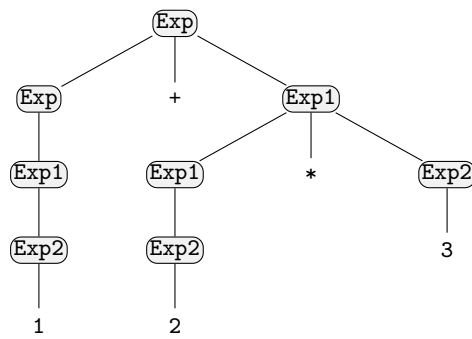
2.7.2 Homework: Parse Trees

Below are derivation trees (concrete syntax trees) for the required expressions. Nonterminals are boxed; terminals are leaves.

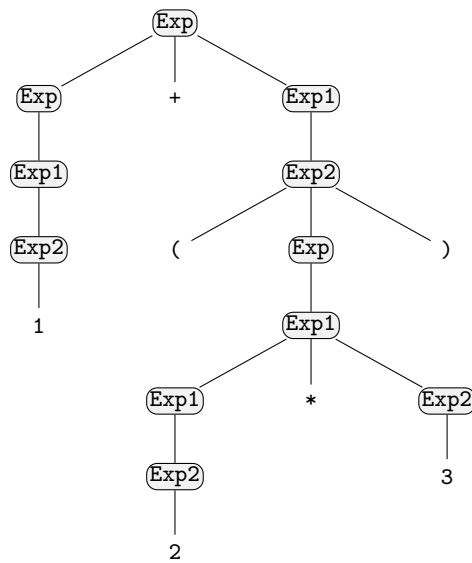
(a) $2+1$



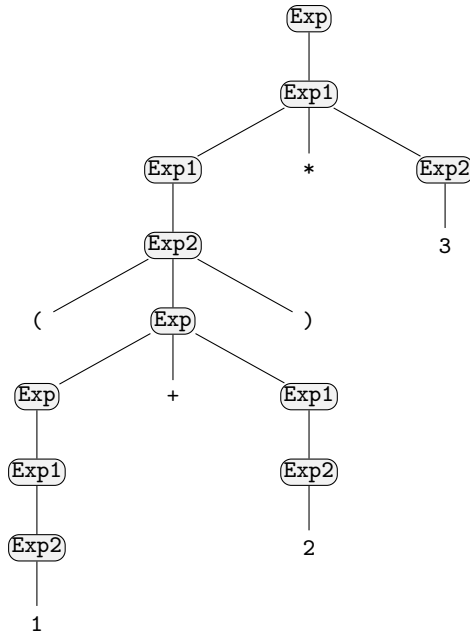
(b) $1+2*3$



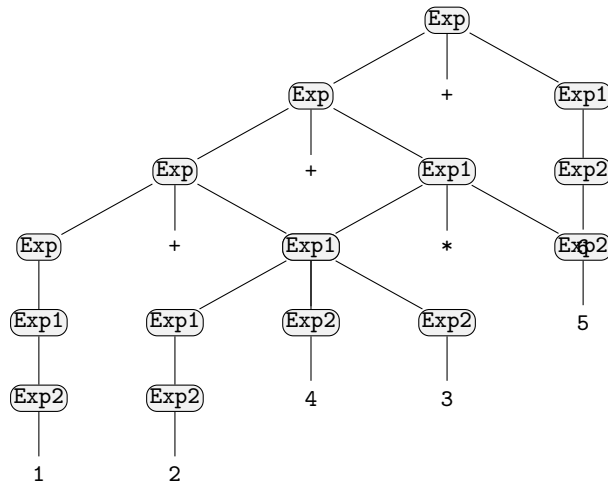
(c) $1+(2*3)$



(d) $(1+2)*3$



(e) $1+2*3+4*5+6$



2.7.3 Questions

Could this grammar still work if “+” and “*” had the same priority?

2.8 Week 8

2.8.1 Notes and Exploration

I completed Levels 5–8 of the *Natural Number Game* (NNG). The key tools were:

- **Rewriting** with `rw [lemma]` to substitute equals for equals.
- `rf1` to close a goal when both sides are definitionally identical.
- Peano-style equations for addition and numerals, including `add_zero`, `add_succ`, `add_one`, `one_eq_succ_zero`, and `two_eq_succ_one`.

2.8.2 Homework (NNG Levels 5–8)

5/8: Adding zero. *Goal:* $a + (b + 0) + (c + 0) = a + b + c$.

Lean steps: `rw [add_zero] (on b + 0), rw [add_zero] (on c + 0), rfl`.

6/8: Precision rewriting. *Goal:* $a + (b + 0) + (c + 0) = a + b + c$.

Lean steps: `rw [add_zero] (on b + 0), rw [add_zero] (on c + 0), rfl`.

7/8: add_succ. *Goal:* $\text{succ } n = n + 1$.

Lean steps: `rw [one_eq_succ_zero]` to get $n + 1 = n + \text{succ } 0$; `rw [add_succ]` to obtain $\text{succ}(n + 0)$; `rw [add_zero]; rfl`.

8/8: $2 + 2 = 4$. *Outline:* Rewrite 2 and 4 as successors (`two_eq_succ_one`, etc.), use `add_succ` to pull `succ` out of addition stepwise, and finish by `rfl` once both sides match.

2.8.3 Natural-Language Proof (English math proof)

Claim. $2 + 2 = 4$.

Assumptions/Definitions. Let 0 be the base natural number and $\text{succ}(n)$ the successor of n . Define $1 = \text{succ}(0)$, $2 = \text{succ}(1)$, and $4 = \text{succ}(\text{succ}(\text{succ}(\text{succ}(0))))$. Addition is defined by the Peano axioms:

$$\forall a, a + 0 = a \quad \text{and} \quad \forall a, b, a + \text{succ}(b) = \text{succ}(a + b).$$

Proof. We compute $2 + 2$ using the addition axioms. First rewrite $2 = \text{succ}(1)$, so

$$2 + 2 = 2 + \text{succ}(1) = \text{succ}(2 + 1) \quad (\text{by the second axiom}).$$

Again rewrite $1 = \text{succ}(0)$ to get

$$2 + 1 = 2 + \text{succ}(0) = \text{succ}(2 + 0) = \text{succ}(2) \quad (\text{using both axioms}).$$

Therefore

$$2 + 2 = \text{succ}(2 + 1) = \text{succ}(\text{succ}(2)).$$

Since $2 = \text{succ}(1)$, we have

$$\text{succ}(\text{succ}(2)) = \text{succ}(\text{succ}(\text{succ}(1))) = \text{succ}(\text{succ}(\text{succ}(\text{succ}(0)))) = 4.$$

Hence $2 + 2 = 4$. \square

2.8.4 Discord Question

When using `rw [add_zero]`, how does Lean know which part of the equation to rewrite first?

3 Essay (Synthesis)

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4 Evidence of Participation

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5 Conclusion

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References