Given a planning instance I_δ from Sheila's algorithm, we construct an NFA $M = (Q, A, \Delta, q_0, F)$ as follows:

1. Q is the set of state objects $S_0, \ldots, S_n$ mentioned in I_δ, plus S_f;
2. A is the set of operator names in I_δ, plus *end*;
3. Δ is the transition between state objects defined by operator description, plus $\Delta(S_n, end) = \{S_f\}$;
4. q_0 is S_0;
5. F is $\{S_f\}$.

As above, we know that A contains primitive actions and some auxiliary actions like *noop*, *test*, $free$ and *end*. Here we mark NT the subset of A, as it contains all the *noop* and *test* actions.

From the given transition $\Delta(s, a) = s'$ for states s and s' and action a, we can define the transition for states set R and action sequence $\vec{c}$ as classical automaton as follows:

1. $\Delta(R, a) = \bigcup_{s \in R} \Delta(s, a)$;
2. $\Delta(R, \vec{c} \cdot a) = \Delta(\Delta(R, \vec{c}), a)$.

So we say an action sequence $\vec{c}$ is accepted by M, if $S_f \in \Delta(S_0, \vec{c})$.

Now we construct an NFA $M' = (Q', A, \Delta', q_0, F)$ from $M = (Q, A, \Delta, q_0, F)$ constructed form I_δ, as follows:

- $Q' = \{S_0\} \cup \bigcup_{s \in Q, a \in A - NT} \Delta(s, a)$;
-

$$\Delta'(s, \vec{c}) = \begin{cases} \Delta(s, \vec{c}) & \text{if } s \in Q' \text{ and } \vec{c} = \vec{nt} \cdot a, \text{ where } \vec{nt} \in NT^* \text{ and } a \in A - NT \\ \emptyset & \text{else} \end{cases}$$

We want to demonstrate the equivalence of the two NFA M and M': an action sequence $\vec{c}$ is accepted by M', iff $\vec{c}$ is accepted by M.

To prove the equivalence, we firstly prove that: for arbitrary $s \in Q'$ and $\vec{c} = \vec{nt}_1 \cdot a_1 \cdots \vec{nt}_m \cdot a_m$, where $\vec{nt}_i \in NT^*$ and $a_i \in A - NT$ for $1 \le i \le m$, $\Delta'(s, \vec{c}) = \Delta(s, \vec{c})$.

Proof: we use structure induction to prove.

(1) Base: consider $\vec{c} = \vec{nt} \cdot a$, then $\Delta'(s, \vec{c}) = \Delta(s, \vec{c})$ from the definition.

(2) Induction step: consider $\vec{c} = \vec{nt}_1 \cdot a_1 \cdots \vec{nt}_m \cdot a_m$, where $m > 1$:

$\Delta'(s, \vec{nt}_1 \cdot a_1 \cdots \vec{nt}_m \cdot a_m)$
$= \Delta'(s, (\vec{nt}_1 \cdot a_1 \cdots \vec{nt}_{m-1} \cdot a_{m-1}) \cdot \vec{nt}_m \cdot a_m)$
$= \Delta'(\Delta'(s, \vec{nt}_1 \cdot a_1 \cdots \vec{nt}_{m-1} \cdot a_{m-1}), \vec{nt}_m \cdot a_m)$

We mark $\Delta(s, \vec{nt}_1 \cdot a_1 \cdots \vec{nt}_{m-1} \cdot a_{m-1})$ as P for short, and from inductive assumption, we have $\Delta'(s, \vec{nt}_1 \cdot a_1 \cdots \vec{nt}_{m-1} \cdot a_{m-1}) = P$. Then we get:

$\Delta'(s, \vec{nt}_1 \cdot a_1 \cdots \vec{nt}_m \cdot a_m)$
$= \bigcup_{s' \in P} \Delta'(s', \vec{nt}_m \cdot a_m)$
$= \bigcup_{s' \in P} \Delta(s', \vec{nt}_m \cdot a_m)$[from definition]
$= \Delta(s, \vec{nt}_1 \cdot a_1 \cdots \vec{nt}_m \cdot a_m)$

Therefore, the proof is done.$\blacksquare$

And now the equivalence of M and M' is easy to demonstrate. As *end* is always the final and only action to transit to S_f, every action sequence $\vec{c}$ accepted by M, i.e. $S_f \in \Delta(S_0, \vec{c})$, should be the form $\vec{nt}_1 \cdot a_1 \cdots \vec{nt}_m \cdot end$. Hence we know $\vec{c}$ is accepted by M', as $\Delta'(S_0, \vec{c}) = \Delta(S_0, \vec{c})$ contains S_f too. Vice verse.

Our improvement is to combine the action sequence $\vec{nt} \cdot a$ into an action a', of which the name is as the same as a's; and precondition is the conjunctions of the preconditions of $nt_i (1 \leq i \leq k)$ and a, with state function restrict updated from M'; and effects are the union of the effects of nt_i and a, also with state function updated from M'; and lastly the parameters should be the union of the ones of nt_i and a. And we mark the new planning instance transformed from I_δ as I'_δ.

As we know that NFAs M and M' are equivalence, we conclude that the plans for I_δ and I'_δ, filtering the actions in NT, would be exact the same.