DSIP Assignment (first and second part)

What we expect from you Turn in your solution to all of the exercises keeping in mind that this is a good part of your exam. You are expected to work on the exercises after you studied the theory covered in class and the labs session we went through together. If you experience any difficulty go back to the material we covered, check the notes, and find out where you need to study more. You can ask questions anytime you want on what was covered in class and labs but you are not going to receive help on the exercises you find below.

Deadline No deadline in place. Turn in your solution when you feel ready for the exam.

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Fourier series: Hand in a Python notebook which computes the Fourier expansion of a real valued function f where

- a is sampled uniformly in the interval $[-2,-1]$
- b is sampled uniformly in the interval $[1,2]$
- and the function $f:[a,b]\to\mathbb{R}$ is defined as

$$f(t)=\begin{cases} pe^{qt}\sin(\ln(1+rt^2)) & t\in[a,0) \\ s & t\in[0,b] \end{cases}$$

with p,q,r and s are sampled uniformly in the interval $[0,1]$.

Comment the results you obtain, Gibbs phenomenon included, when using the first 20, 40, and 60 harmonics.

Fourier transform: Hand in a Python notebook which computes the Fourier transform of

$$f(t)=\alpha\cos(\omega_0 t)+\beta\cos(\omega_1 t)$$

where

- α and β are sampled uniformly in the interval $[-1,1]$
- ω_0 is sampled uniformly in the interval $[1,5]$ and
- ω_1 is sampled uniformly in the interval $[10,20]$

Comment the results you obtain in terms of δ functions.

Aliasing: Consider the function $p_T(t)$ with T sampled uniformly in the interval $[1,4]$. Starting from the train of impulses for two different sampling rates and the corresponding train of impulses in frequency in the two cases, derive the temporal sampling rate such that the distance between two consecutive impulses in frequency correspond to a decay of 10% and 1% of the Fourier Transform $F(\omega)$ with respect to the maximum value. Comment the type of aliasing you can expect in the two cases. Explain the reason for which with functions like $p_T(t)$ aliasing can be reduced but not completely eliminated.

Linear Time-Invariant Systems: Compute the convolution product between the functions f and g where $f,g:[a,b]\to\mathbb{R}$ with

$$f(t)=pe^{qt}\sin(\ln(1+rt^2)) \quad \text{and} \quad g(t)=p_T(t)$$

and p,q,r and T sampled uniformly in the interval $[0,2]$. Here again sample a uniformly in the interval $[-2,-1]$ and b uniformly in the interval $[1,2]$.

Verify that you obtain the same result in both the temporal and the frequency domain.

Kalman filtering: Hand in a Python notebook which implements a Kalman filter for the estimation of a function f and its first derivative f' in the interval $[0, 10]$ from 1000 equally spaced noisy samples. Define $f : [0, 1] \to \mathbb{R}$ as

$$f(t) = \sin(pt)e^{qt}$$

with p and q sampled uniformly in the interval $[2, 4]$. For $i = 0, 1, \ldots, 999$ let $t_i = i/1000$ and

$$f_i = f(t_i) + \epsilon_i$$

with ϵ_i sampled from a Gaussian distribution with $\mu = 0$ and $\sigma = 0.1$ and $\sigma = 0.5$. Plot the estimates against the values for f and f' computed analytically in the two cases. Study how the filtered estimates change for increasing values of the noise process covariance.

Wavelets: Compute the Haar wavelet transform of two 32-pixel image, $\mathcal{I}_{32}$ and $\mathcal{J}_{32}$ using the appropriate analysis filters. Sample the 32 values forming $\mathcal{I}_{32}$ uniformly from the set $\{0, 1, \ldots, 255\}$. Sample the first 16 values forming $\mathcal{J}_{32}$ uniformly from the set $\{24, 25, 26, 27\}$ and the second 16 values from the set $\{201, 202, 203, 204\}$. In both cases compute the fraction of the details coefficients larger than 1/100 and comment on the obtained results.