# Optimization of Regional GDP During the CoVID-19 Pandemic using Full Factorial Design of Experiments

McMaster University
Department of Engineering
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# 1 Overview

The corona virus pandemic has affected countries worldwide and is evident through business closures, limited transportation, increasing mortality rates, increasing unemployment, and declining economies. As a result, the need to implement regulatory restrictions within local communities has been increasing. To avoid an economic crisis, it is important to investigate how different factors such as regional population density and level of policy control affect the economy.

## 1.1 Objective

The objective of this experiment is to maximize USA's economy (GDP) while varying the different factors in place during this COVID-19 pandemic.

# 2 Parameter

## 2.1 Response variable

Gross Domestic Product (GDP) will be the response variable for this experiment. It is measured in a similar way to real life. There is no constraint for this variable, economy growth is represented as a positive change in GDP while economy collapse is represented as a negative change in GDP.

Gross Domestic Product (GDP) is measured in units of US dollars-$. The GDP can be measured for this experiment by keeping track of unit price & quantity produced of locally produced goods/services and calculating total GDP at each time interval. GDP = (unit price of item 1 x quantity of item 1 produced) + (unit price of item 2 x quantity of item 2 produced) + ……+ (unit price of item n x quantity of item n produced) [1]. The GDP calculation is reproducible; however, in the simulation, the exact value of the outcome (economy) is not reproducible.

## 2.2 Factors

*Table 1: Identification of Important Factors*

| Factors: | How Factors are Measured: | Range:* | Level of Control |
|---|---|---|---|
| Policy Control | Lockdown level, indoor and outdoor event capacity limit. | None (0.1) to Restricted (0.9) | Varied |
| Population Density | Local census in each area at each time interval | 461 to 2000 people/km$^2$ | Varied |
| Time Period | Timer | 2200 hours (constant) | Controlled |
| Initial Diseased Population | Public statistics of number of active corona virus cases | 3 ppl (constant) | Controlled |

**Please refer to Appendix A for detailed information.*

*Table 2: Investigated Factors and Predetermined Set Factor Levels*

| | **Levels*** | | | |
|---|---|---|---|---|
| | **Policy Control** | | **Population Density (ppl/km$^2$)** | |
| **Model** | - | + | - | + |
| 1 | 0.1 | 0.4 | 461 | 615 |
| 2 | 0.7 | 0.9 | 461 | 615 |
| 3 | 0.7 | 0.9 | 923 | 1352 |
| 4 | 0.7 | 0.9 | 1352 | 2000 |

**Please refer to Appendix A for detailed information.*

## 2.3 Expected Effect of Factors on GDP

The economy is directly affected by the immune, diseased, and deceased populations due to the spread of the virus. For every death, the GDP will drop by $1700 and for every diseased individual, the GDP will drop by $4500. For every individual that gains immunity, the GDP will increase by $220. (Refer to *Appendix A* for more detail)

Increasing policy levels will increase the response variable. Policy levels directly affect the amount of interaction between individuals within a population. Regardless of the population density, the virus will spread at a faster rate with increased interaction.

Additionally, increasing the population density will increase the response variable. By increasing the population density, the number of people who can be exposed to the virus increases. However, the extent to which population density affects the economy will be less than policy levels.

## 2.4 Nuisance Factors

Considering the time period of the experiment, holidays and festivals would affect GDP by increasing it, as additional exclusive services/products such as festive decorations, would be produced. It can be accounted for by using trends from previous years and interpreting accordingly. Further detailed experiments can be carried out to narrow down the exact cause.

Season is another factor that could affect GDP since the services/products and quantity produced could change depending on the weather. However, it can be accounted for by using trends from previous years and adjusting accordingly.

Age could be another factor that affects GDP but not investigated in the experiment. When younger people that are working gets infected, the goods/services produced could decrease and thereby have a much larger effect on GDP compared to when older/retired people are diseased. However, in the experiment, age is not taken into account when assessing the effect of diseased population on GDP.

The socioeconomic status of the individuals partaking in this experiment can affect the spread of the virus, which in turn, affects the GDP. Individuals of lower socioeconomic status are more susceptible to contracting the disease through factors such as higher reliance on public transport, decreased sanitation of important establishments, lower access to healthcare and overcrowding in public spaces. These factors are not considered through this experiment.

# 3 Experiment Program

## 3.1 Design Choices to Account for Nuisance Factors

The potential nuisance factors known to us are included in the sections above, which are: Holiday and festival, seasons, population age structure, socioeconomic status, duration of the experiment, and initial disease population. Therefore, if this experiment was to be performed in real life, these factors would need to be blocked to ensure the result is not affected by those nuisances. However, for this simulation, the nuisance factors mentioned above are controlled and are held constant for all experiments ran; the experiment is appropriately designed.

## 3.2 Experiment Design

This design is a **full factorial design** - the two factors, policy & population density, are investigated at two set levels per model. Thus, leading to a 2 by 2 factorial DOE design per model. As depicted in *Table 2*, there will be 4 models to investigate different set factor levels. The experiment was repeated 20 times for each model to improve precision. All combinations of the factors, policy, population density & time period, (see *Table 1)* were possible. Time period was set at a constant value to improve the simplicity of the design. The only factor that cannot be controlled is initial diseased population as that is a fixed variable within the simulated model.

Since the experiment is simulated, blocking will not be used and there will be no confounding in this experiment. Due to the simulated nature of the experiment, additional randomization techniques will not be incorporated. The order in which the experiments are conducted are randomized.

## 3.3 Centre points

*Table 3: Centre Points of Relevant Factors for Individual Models*

| Model | Policy | Population Density (ppl/km$^2$) |
|---|---|---|
| I | 0.25 | 538 |
| II | 0.8 | 538 |
| III | 0.8 | 1138 |
| IV | 0.8 | 1677 |

# 4 Analysis of Data

Since the occurrence of events are randomized, multiple experiments need to be performed to get data that reflects the GDP of a certain policy level. Therefore, twenty datasets were collected for each model. The datasets are averaged and used for analysis. The standard tables for each model are found in *Appendix C*.

## 4.1 Accuracy of the Model Parameters

Based on the summary of linear regression model *(Appendix B: Fig7, 8, 9, 10(e)),* the p-values for all parameters are larger than 0.1, indicating that the parameters for each model are not statistically significant. This may be due to high variance in the data collected.

## 4.2 Visualization of Raw Data.

**Please refer to Appendix B for visualization of Raw Data (Fig1-6).*

## 4.3 Possible Issues with Data Collection

Since the collision and the event (lockdown/restriction) happens to each character in this simulation randomly, some extreme events may occur and cause the results to have huge variance. To avoid this situation, experiments are repeated 20 times for each model and the averages of the datasets are used to do the analysis.

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## 4.4 Derived Prediction Models & Sensitivity Analysis

Least-squares modelling was used to describe the system. Overall, the formula for each model is:

**Model I** 127921 - 10552(Policy) + 19557(Population Density) + 15808(Policy*Density)

**Model II** 170682.5 +23088.1 (Policy) – 13868.7(Population Density) -496.2(Policy*Density)

**Model III** 26243 + 2308(Policy) + 20545(Population Density) + 20526(Policy*Density)

**Model IV** 186608 - 81507(Policy) + 43697(Population Density) + 2626(Policy*Density)

**Please view Appendix B for the linear regression summary & Pareto plot of each model.*

**Model I** In Model I, policy level has a negative but the lowest effect on GDP. While population density as well as the interaction of policy level and population density has a positive and higher effect. This can be seen in the Pareto plot in Appendix B, Fig 7b. It can be interpreted as, when the policy level increases, GDP decreases but the amount the GDP decreases by is less than the increase in GDP due to increase in population density as well as due to interaction between population density and policy level.

**Model II** In Model II, population density and the interaction effect of policy level and population density has a negative effect on GDP, where the interaction effect of policy level and population density has the lowest effect. While policy level has a positive and the highest effect. This can be seen in the Pareto plot in Appendix B, Fig 8b. It can be interpreted as, when the population density increases, GDP decreases but the amount the GDP decreases by is less than the increase in GDP due to increase in policy level.

**Model III** In Model III, all policy level, population density and interaction between policy level and population density has positive effect on GDP, in which policy level has the lowest effect while population density has the highest effect. This can be seen in the Pareto plot in Appendix B, Fig 9b. It can be interpreted as, when the population density, or policy level, or both increases, GDP increases.

**Model IV** In Model IV, population density and interaction between population density and policy level has positive effect on GDP, while policy level has a negative and the highest effect on GDP. This can be seen in the Pareto plot in Appendix B, Fig 10b. It can be interpreted as, when the policy level increases, GDP decreases where the amount the GDP decreases by is greater than the increase in GDP due to increase in population density.

## 4.5 Optimization

**Please view Appendix B (Fig 11) for Visual Representation and refer to Table 1 for model details.*

Based on Model I (*Fig 7a-e*), it can be seen that the GDP increases as the population density and policy control increase. Therefore, in Model II (*Fig 8a-e*), policy control level is increased from (0.25 (center point)) to (0.8), but the population density is kept the same. From the result of Model II, it is found that the population density needs to be increased to further increase the GDP. Therefore, in Model III (*Fig 9a-e*), population density is increased from 538 ppl/km$^2$ to 1138 ppl/km$^2$ and it is expected that the GDP will increase further when population density is increased. So, in Model IV (Fig 10a-e), population density is increased from 1138 ppl/km$^2$ to 1677 ppl/km$^2$. However, inspection of the contour plot and the interaction plot, reveals that the GDP starts to decrease when the population density is further increased. The maximum GDP appears to be when population density is 1352 ppl/km$^2$ and when the policy control level is 0.9.

# 5 Conclusion

In this experiment, the relationship between regional GDP and the factors; population density & policy level, is investigated. Based on the result, when the regional population density is lower than 1353 ppl/km$^2$, the GDP will increase as the population density increases and the GDP will increase as the policy level increases. Followed by this tendency, the maximum GDP is found when population density equals to 1353 ppl/km$^2$ and policy level is restricted to 0.9. In following experiments, additional factors should be considered to provide a more realistic model and improve the generalizability of the results. For example, socioeconomic status could be incorporated into model to test how all three factors affect the regional GDP.

# 6 References

1. "Measuring the Economy 1," *Sparknotes.com*. [Online]. Available: https://www.sparknotes.com/economics/macro/measuring1/section1/. [Accessed: 04-Aug-2021].

# 7 Appendices

## 7.1 Appendix A: Simulation Mechanism

Assumptions

1. The simulation is isolated, indicating the people inside the simulation can't go inside or outside. The total number of people will keep the same.
2. The time for simulation and initial disease population will be the same.
3. The *age structure effect* is ignored. Every individual will have the same probability for experiencing each event.

Simulation Link: https://drive.google.com/file/d/1VNkNrQMMC6xAjO1PzYG2fyHRkN5SHbOT/view?usp=sharing
GitHub Code Link: https://github.com/RayLyu-Mac/COVID-19-SImulation

Each character has a certain number of health points (50). With each collision with a person who carries the disease, the character will lose their health points. Once there are no more health points remaining, the character may turn into one of the following options: person carrying the disease, party people or quarantine people. Both party and disease populations will carry the disease, but the party population will move faster than the disease population. Quarantine population will remain in a corner and will not carry disease. After a certain period, the following events may occur: death, immunity or remain in the same condition,

*Probabilities for Events Based on Population*

| Death | Immunity | Remain the Same Condition |
|---|---|---|
| Quarantine < Disease < Party | Quarantine > Disease > Party | Quarantine < Disease = Party |

*GDP Equation:*
GDP = Population (People without disease (exclude quarantine and party population)*X+ Disease*Y

*X & Y Values for Different Events:*

| Event | X | Y |
|---|---|---|
| Restrict | 1.85 | 0.8 |
| No Control | 3.3 | 1.4 |
| Lockdown | 0.35 | 0.05 |
| Prevent | 2.5 | 0.6 |

The number of events will be monitored during the simulation. Once five events have occurred, there will be a steady time for GDP to grow. Based on different policy level, different events may happen in random order.

*Changes in GDP Based on Events that Happen to Different Populations:*

| Dead | Disease | Immune |
|---|---|---|
| GDP – 4500 / person | GDP -1700 / person | GDP + 220 / person |

When the healthy population drops under 55%, the society will collapse, to simulate the possible scenario where the mortality rate is too high.

*Properties Dependent on Events:*

| **Moving speed for character** | **Economy Growth** |
|---|---|
| No control > Prevent > Restricted > Lockdown | No control > Prevent > Restricted > Lockdown |
| **Population Behaviours** | |
| **Party** | **Quarantine** |
| No control > Prevent > Restricted > Lockdown | No control < Prevent < Restricted < Lockdown |

*Probabilities for Events to Occur at Different Policy Levels:*

| **Lock Down** | **No Control** | **Restricted** |
|---|---|---|
| Restricted > Lev II >Lev I >No control (0.9>0.7>0.4>0.1) | Restricted<Lev II<Lev I<No control (0<0.2<0.5<0.8) | Restricted = Lev II = Lev I = No control = 0.1 |

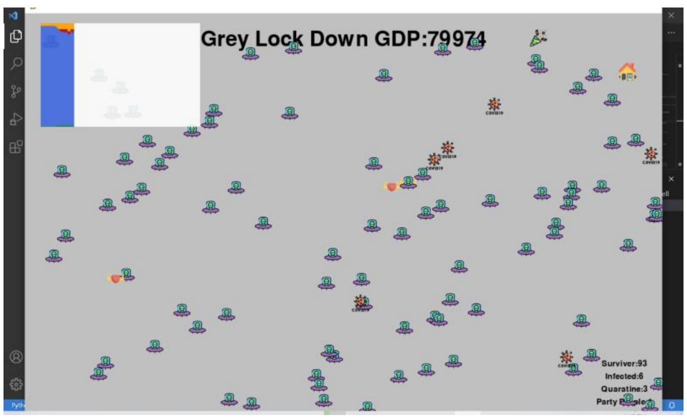


*Fig 1*: Grey Lockdown


*Fig 2*: Green Prevent


*Fig 3*: White No Control

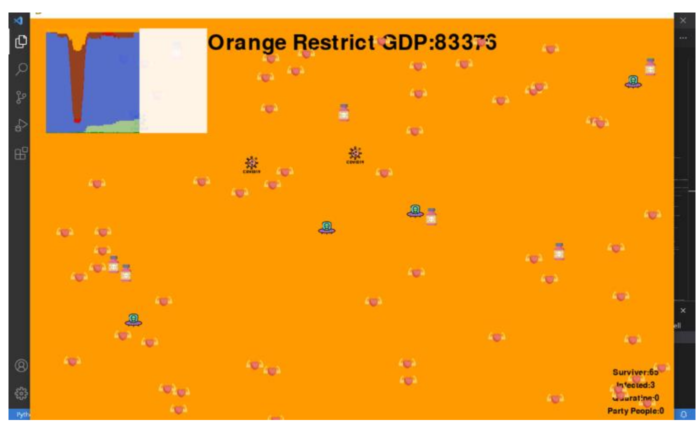


*Fig 4*: Orange Restricted.

## 7.2 Appendix B: Plots

### *7.2.1 Raw Data Visualization*

*Visualization of GDP for Specific Population Density Levels*

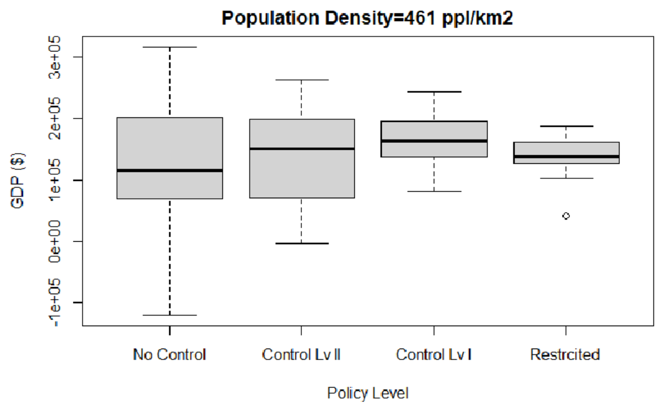


*Fig 1*: Population Density of 461ppl/km$^2$

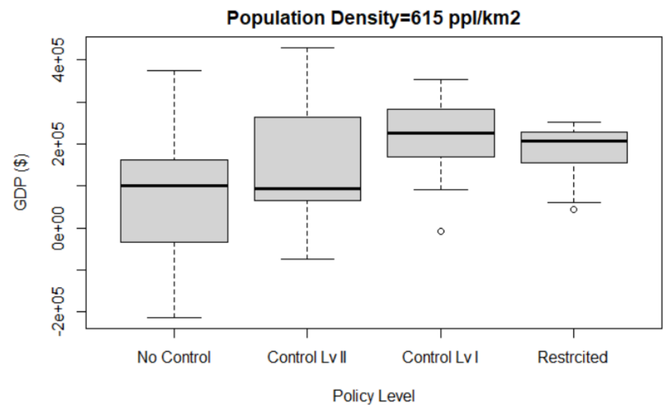


*Fig 2*: Population Density of 615 ppl/km$^2$

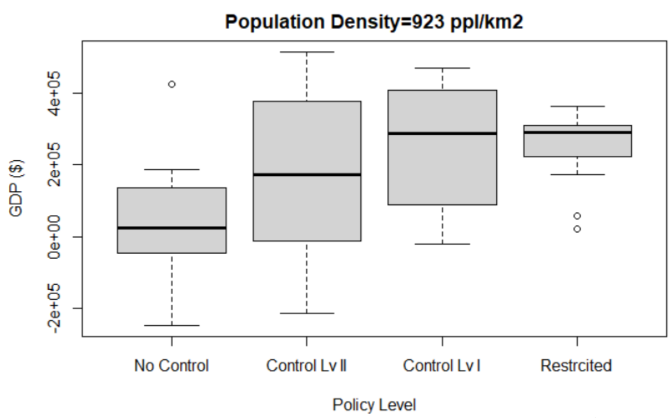


*Fig 3*: Population Density of 923 ppl/km$^2$

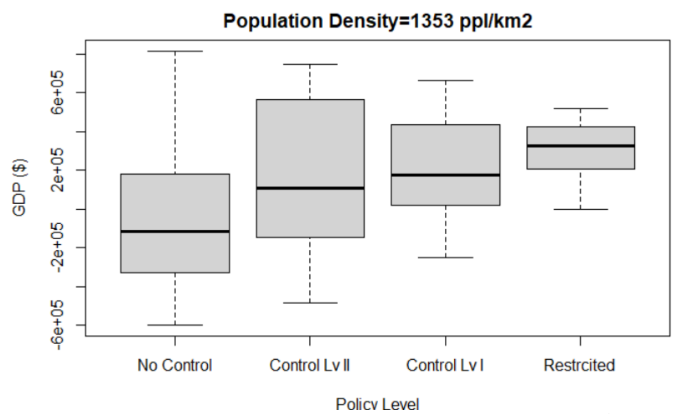


*Fig 4*: population density of 1353 ppl/km$^2$

*Visualization of GDP for Specific Policy Levels*

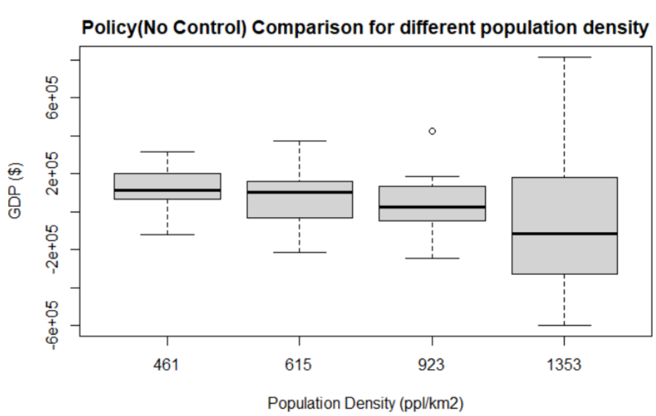


*Fig 5*: Policy of No control

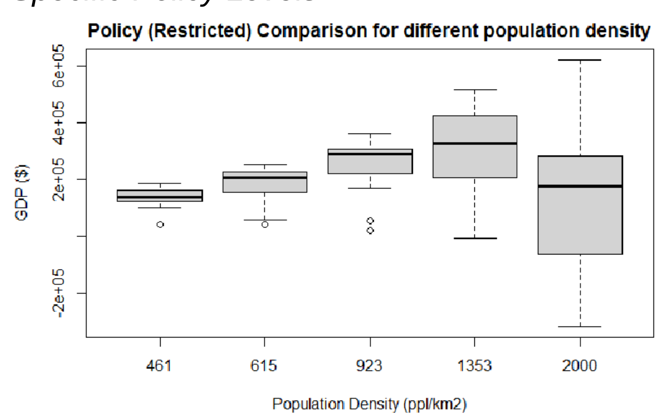


*Fig 6*: Policy of Restricted

### *7.2.2 Plots for Model 1 & Linear Regression Summary*

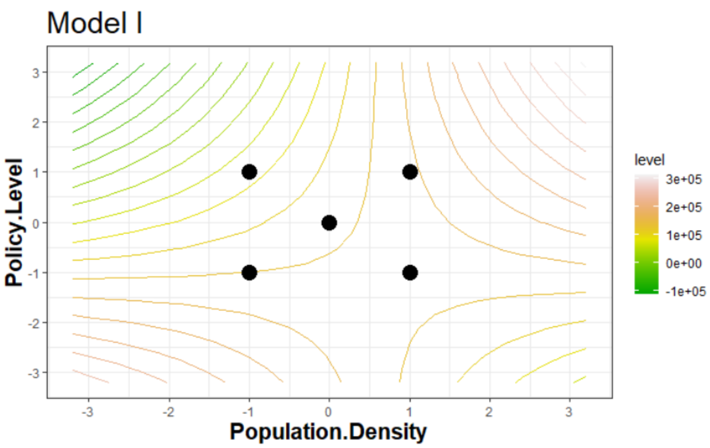


*Fig 7a*: Contour Plot for Model I

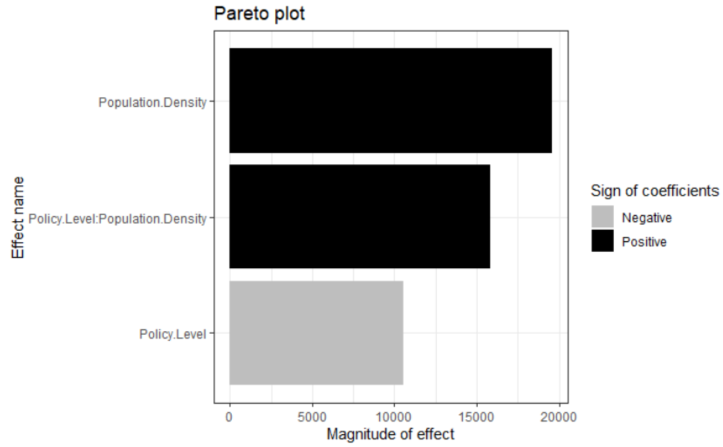


*Fig 7b*: Pareto Plot for Model I

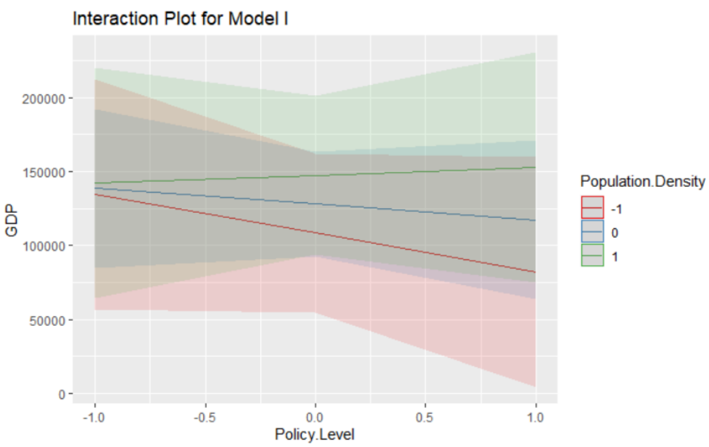


*Fig 7c*: Interaction plot for Policy Level

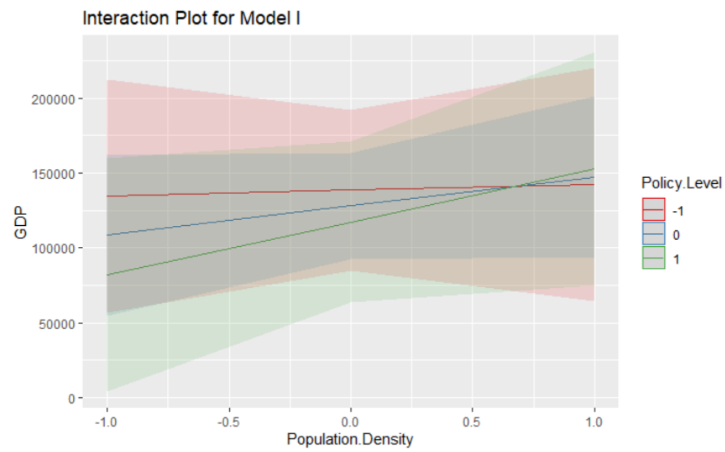


*Fig 7d*: Interaction plot for Population Density

```
Call:
lm(formula = GDP ~ Policy.Level * Population.Density)

Residuals:
    1      2      3      4      5
-9105 -9105 -9105 -9105 36420

Coefficients:
                                Estimate Std. Error t value Pr(>|t|)
(Intercept)                       127921      18210   7.025    0.090 .
Policy.Level                      -10552      20359  -0.518    0.696
Population.Density                 19557      20359   0.961    0.513
Policy.Level:Population.Density    15808      20359   0.776    0.580
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 40720 on 1 degrees of freedom
Multiple R-squared:  0.6421,    Adjusted R-squared:  -0.4315
F-statistic: 0.5981 on 3 and 1 DF,  p-value: 0.7134
```

*Fig 7e*: Linear Regression Summary

### *7.2.3 Plots for Model 2 & Linear Regression Summary*

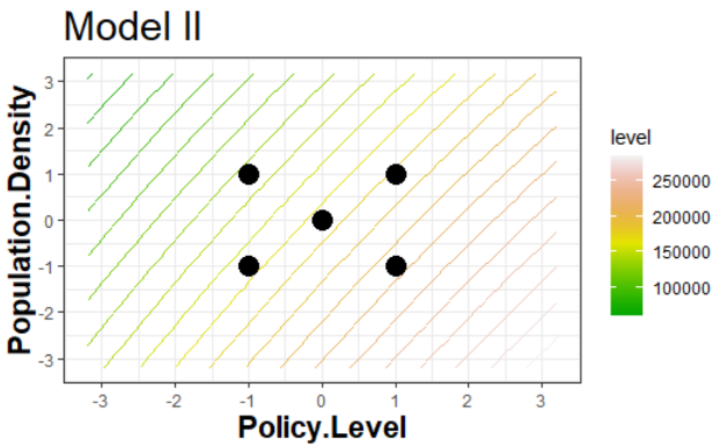


*Fig 8a*: Contour Plot for Model II

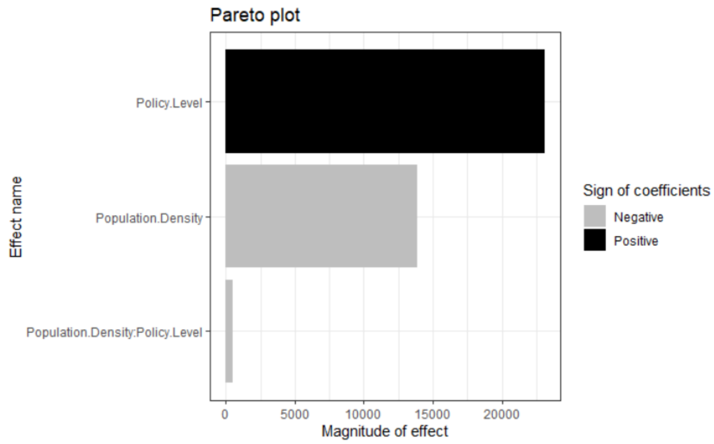


*Fig 8b*: Pareto Plot for Model II

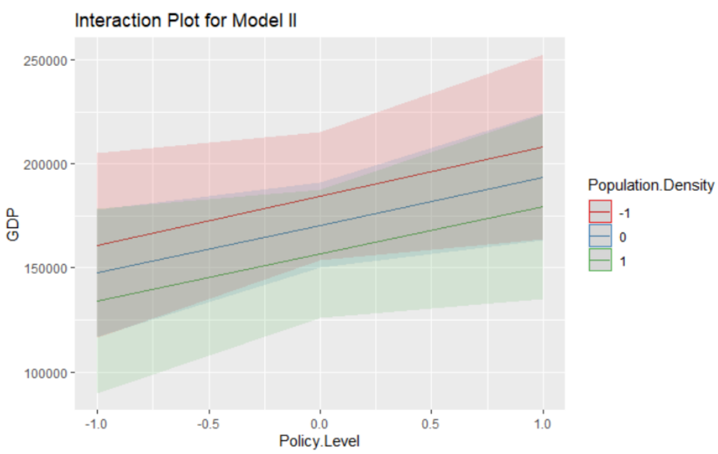


*Fig 8c*: Interaction Plot for Policy Level

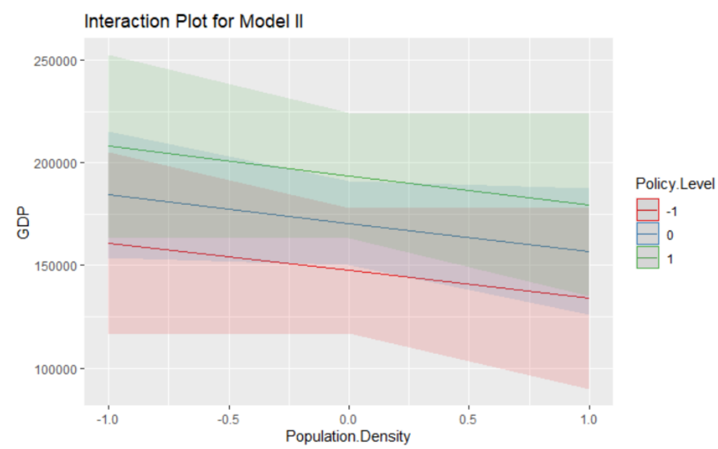


*Fig 8d*: Interaction Plot for Population Density

```
Call:
lm(formula = GDP ~ Population.Density * Policy.Level)

Residuals:
     1      2      3      4      5 
  5205   5205   5205   5205 -20821 

Coefficients:
                                Estimate Std. Error t value Pr(>|t|)  
(Intercept)                     170682.5    10410.5  16.395   0.0388 *
Population.Density              -13868.7    11639.3  -1.192   0.4445  
Policy.Level                     23088.1    11639.3   1.984   0.2973  
Population.Density:Policy.Level   -496.2    11639.3  -0.043   0.9729  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 23280 on 1 degrees of freedom
Multiple R-squared:  0.8427,	Adjusted R-squared:  0.3707 
F-statistic: 1.785 on 3 and 1 DF,  p-value: 0.4914
```

*Fig 8e*: Linear Regression Summary

### *7.2.4 Plots for Model 3 & Linear Regression Summary*

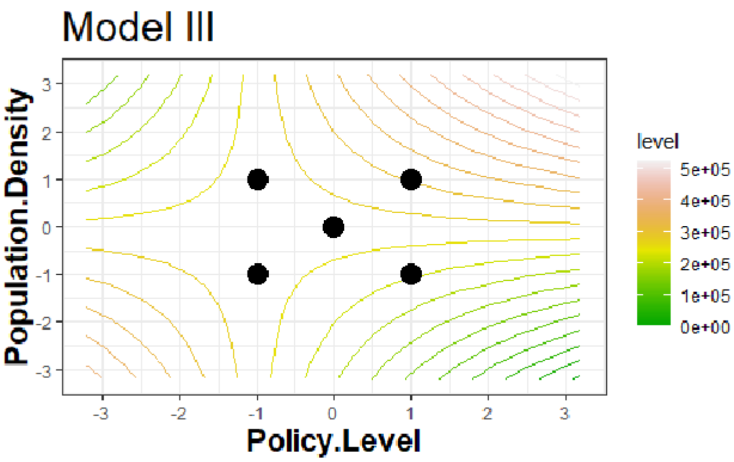


*Fig 9a*: Contour Plot for Model III

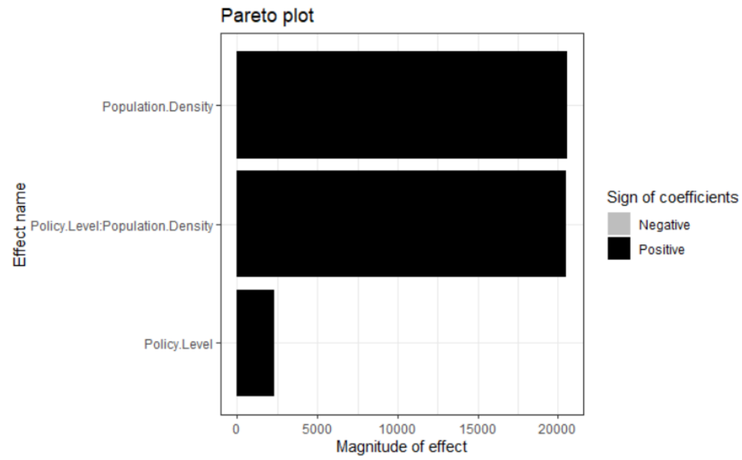


*Fig 9b*: Pareto Plot for Model III

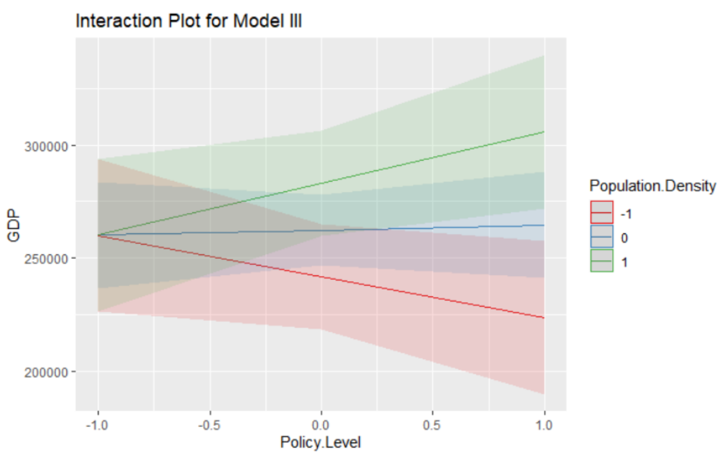


*Fig 9c*: Interaction Plot for Policy Level

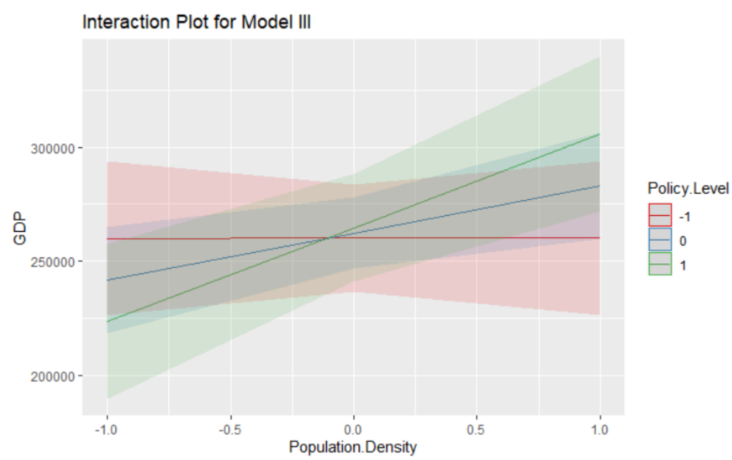


*Fig 9d*: Interaction Plot for Population Density

```
Call:
lm(formula = GDP ~ Policy.Level * Population.Density)

Residuals:
    1     2     3     4     5 
-3961 -3961 -3961 -3961 15844 

Coefficients:
                                Estimate Std. Error t value Pr(>|t|)  
(Intercept)                       262423       7922  33.125   0.0192 *
Policy.Level                        2308       8857   0.261   0.8377  
Population.Density                 20545       8857   2.320   0.2591  
Policy.Level:Population.Density    20526       8857   2.317   0.2593  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 17710 on 1 degrees of freedom
Multiple R-squared:  0.9154,	Adjusted R-squared:  0.6616 
F-statistic: 3.606 on 3 and 1 DF,  p-value: 0.3651
```

*Fig 9e*: Linear Regression Summary

### *7.2.5 Plots for Model 4 & Linear Regression Summary*

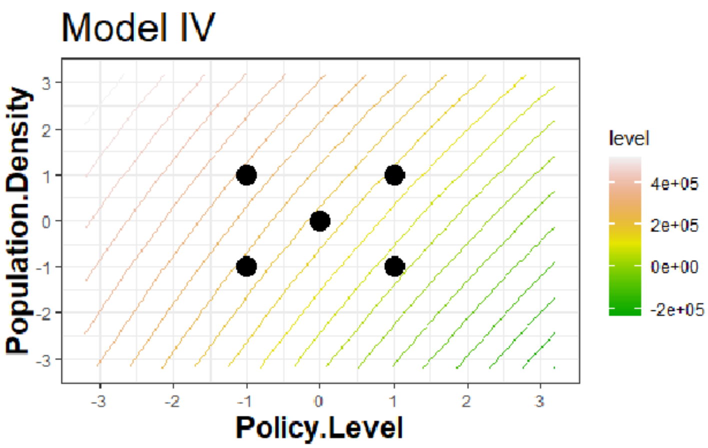


*Fig 10a*: Contour Plot for Model IV

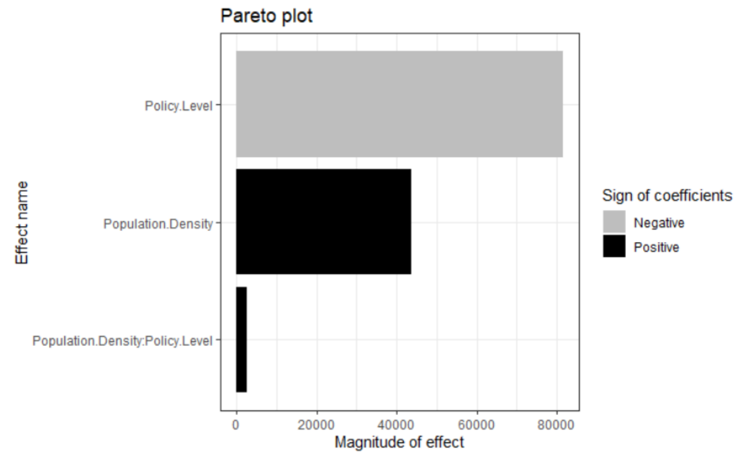


*Fig 10b*: Pareto Plot for Model IV

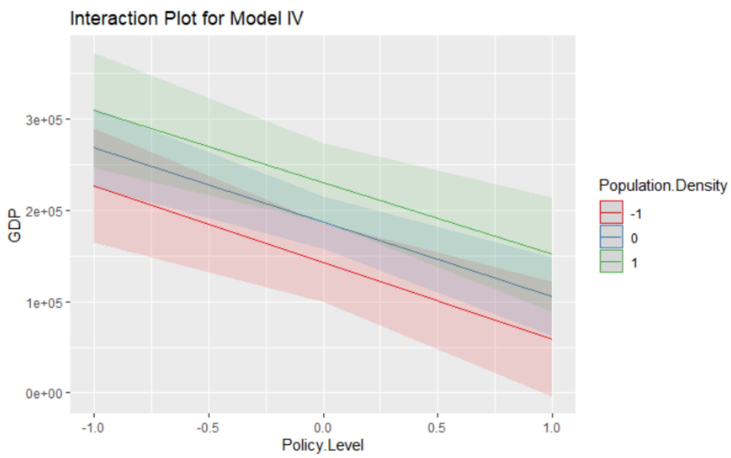


*Fig 10c*: Interaction Plot for Policy Level

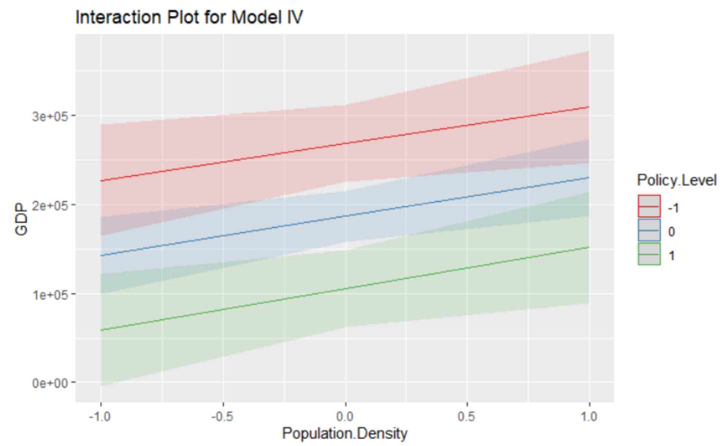


*Fig 10d*: Interaction Plot for Population Density

```
Call:
lm(formula = GDP ~ Population.Density * Policy.Level)

Residuals:
    1      2      3      4      5
-7345  -7345  -7345  -7345  29380

Coefficients:
                                Estimate Std. Error t value Pr(>|t|)
(Intercept)                       186608      14690  12.703    0.050 .
Population.Density                 43697      16424   2.661    0.229
Policy.Level                      -81507      16424  -4.963    0.127
Population.Density:Policy.Level     2626      16424   0.160    0.899
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 32850 on 1 degrees of freedom
Multiple R-squared:  0.9694,    Adjusted R-squared:  0.8778
F-statistic: 10.58 on 3 and 1 DF,  p-value: 0.2214
```

*Fig 10e*: Linear Regression Summary

### 7.2.6 Overall Experiment Visual with 4 models

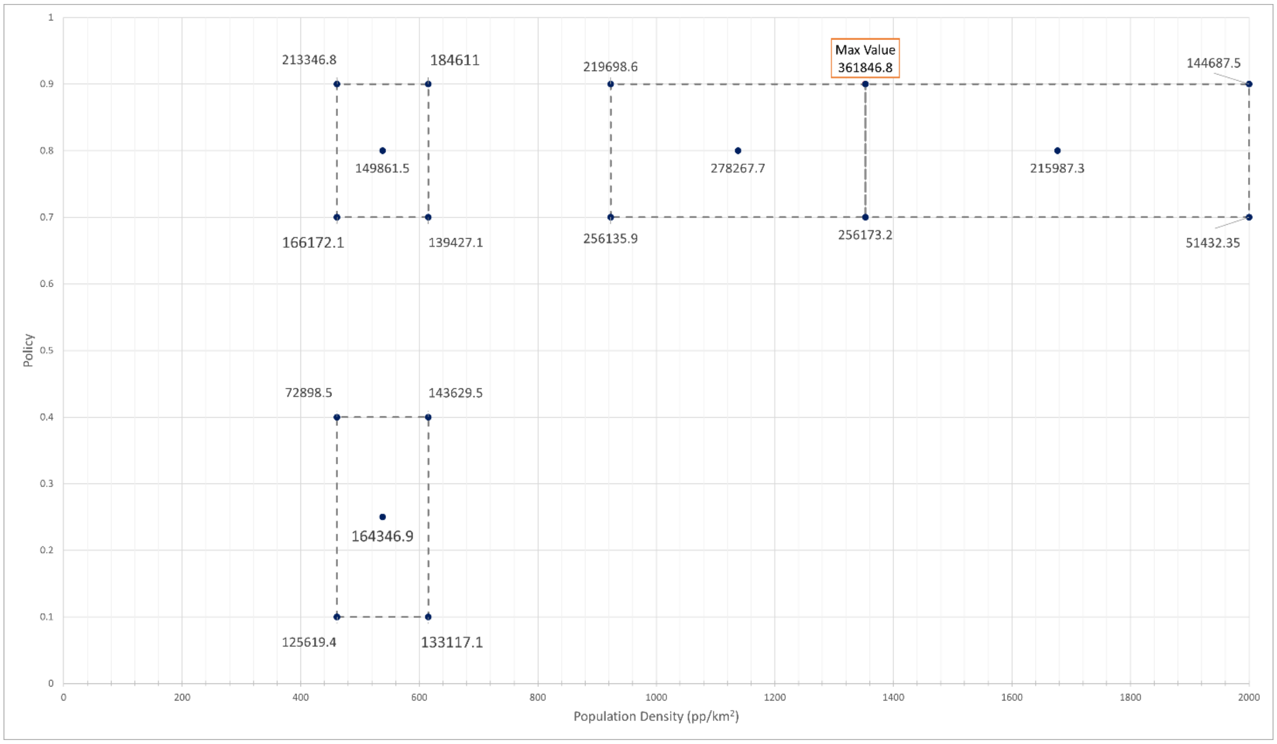


*Fig 11*: Overall Visualization of Cube Plots

## 7.3 Appendix C: Standard Tables

| **Model I:** *Policy (-: 0.1, +0.4), Population Density (-:461, +615)* | | | |
|---|---|---|---|
| Experiment | Policy | Population density | GDP ($) |
| 1 | - | - | 125619.4 |
| 2 | - | + | 133117.1 |
| 3 | + | - | 72898.5 |
| 4 | + | + | 143629.5 |
| 5 | 0 | 0 | 164340.9 |

| **Model II:** *Policy (-: 0.7, +0.9), Population Density (-:461, +615)* | | | |
|---|---|---|---|
| Experiment | Policy | Population density | GDP ($) |
| 1 | - | - | 166172.1 |
| 2 | - | + | 139427.1 |
| 3 | + | - | 213340.8 |
| 4 | + | + | 184611 |
| 5 | 0 | 0 | 149861.5 |

| **Model III:** *Policy (-: 0.7, +0.9), Population Density (-:923, +1352)* | | | |
|---|---|---|---|
| Experiment | Policy | Population density | GDP ($) |
| 1 | - | - | 256135.9 |
| 2 | - | + | 256173.2 |
| 3 | + | - | 219698.6 |
| 4 | + | + | 301840.8 |
| 5 | 0 | 0 | 278267.7 |

| **Model IV:** *Policy (-: 0.7, +0.9), Population Density (-:1352, +2000)* | | | |
|---|---|---|---|
| Experiment | Policy | Population density | GDP ($) |
| 1 | - | - | 219698.6 |
| 2 | - | + | 51432.35 |
| 3 | + | - | 301840.8 |
| 4 | + | + | 144078.5 |
| 5 | 0 | 0 | 215987.3 |

## 7.4 Appendix D: R Code

```
d=read.csv('DOE.csv')
policies<-c("No Control","Control Lv II","Control Lv I","Restrcited")
boxplot(d$GDP.70[1:20],d$GDP.70[21:40],d$GDP.70[41:60],d$GDP.70[61:80],names = policies,main="Population Den
sity=461 ppl/kzZm2",ylab="GDP ($)",xlab="Policy Level")
```

```
mean(d$GDP.70[1:20])

## [1] 125619.4

mean(d$GDP.70[21:40])

## [1] 133117.1

mean(d$GDP.70[41:60])

## [1] 166172.1

mean(d$GDP.70[61:80])

## [1] 139427.1

boxplot(d$GDP.100[1:20],d$GDP.100[21:40],d$GDP.100[41:60],d$GDP.100[61:80],names = policies,main="Population
 Density=615 ppl/km2",ylab="GDP ($)",xlab="Policy Level")
```

```
mean(d$GDP.100[1:20])

## [1] 72898.5

mean(d$GDP.100[21:40])

## [1] 143629.5

mean(d$GDP.100[41:60])

## [1] 213340.8

mean(d$GDP.100[61:80])

## [1] 184611

boxplot(d$GDP.150[1:20],d$GDP.150[21:40],d$GDP.150[41:60],d$GDP.150[61:80],names = policies,main="Population
 Density=923 ppl/kAm2",ylab="GDP ($)",xlab="Policy Level")
```

```
mean(d$GDP.150[1:20])

## [1] 43526.65

mean(d$GDP.150[21:40])

## [1] 170857.8

mean(d$GDP.150[41:60])

## [1] 256135.9

mean(d$GDP.150[61:80])

## [1] 256173.2
```

```
boxplot(d$GDP.220[1:20],d$GDP.220[21:40],d$GDP.220[41:60],d$GDP.220[61:80],names = policies,main="Population
 Density=1353 ppl/km2",ylab="GDP ($)",xlab="Policy Level")
```

```
mean(d$GDP.220[1:20])

## [1] -27852.4

mean(d$GDP.220[21:40])

## [1] 147426.9

mean(d$GDP.220[41:60])

## [1] 219698.6

mean(d$GDP.220[61:80])

## [1] 301840.8

pd<-c(461,615,923,1353)
boxplot(d$GDP.70[1:20],d$GDP.100[1:20],d$GDP.150[1:20],d$GDP.220[1:20],names = pd,main="Policy(No Control) C
omparison for different population density",ylab="GDP ($)",xlab="Population Density (ppl/km2)")
```

```
pd<-c(461,615,923,1353)
boxplot(d$GDP.70[61:80],d$GDP.100[61:80],d$GDP.150[61:80],d$GDP.220[61:80],names = pd,main="Policy (Restrict
ed) Comparison for different population density",ylab="GDP ($)",xlab="Population Density (ppl/km2)")
```

```
g185<-c(299305,213842, 329788, 187368, 363760, 271328, 82727, 366634, 417668, 241524, 403128, 330026, 411482
,420842,353057, 467985, 374109, 128923, -15253, -82889)
mean(g185)

## [1] 278267.7

GDP<-c(125619.4,133117.1,72898.5,143629.5,164340.9)

L<-c(-1,-1,1,1,0)
O<-c(-1,1,-1,1,0)
Population.Density<-O
Policy.Level<-L
DModel=lm(GDP~Policy.Level*Population.Density)
summary(DModel)

##
## Call:
## lm(formula = GDP ~ Policy.Level * Population.Density)
##
## Residuals:
##     1     2     3     4     5
## -9105 -9105 -9105 -9105 36420
##
## Coefficients:
##                                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)                       127921      18210   7.025    0.090 .
## Policy.Level                      -10552      20359  -0.518    0.696
## Population.Density                 19557      20359   0.961    0.513
## Policy.Level:Population.Density    15808      20359   0.776    0.580
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 40720 on 1 degrees of freedom
## Multiple R-squared:  0.6421, Adjusted R-squared:  -0.4315
## F-statistic: 0.5981 on 3 and 1 DF,  p-value: 0.7134
```

```
contourPlot(DModel,"Population.Density","Policy.Level", main = "Model I")
```

```
paretoPlot(DModel)
```

```
plot_model(DModel, type = "pred", terms = c("Policy.Level", "Population.Density"),title = "Interaction Plot
for Model I")
```

```
plot_model(DModel, type = "pred", terms = c("Population.Density","Policy.Level"),title = "Interaction Plot f
or Model I")
```

```
GDP<-c(256135.9,256173.2,219698.6,301840.8,278267.7)

L<-c(-1,-1,1,1,0)
O<-c(-1,1,-1,1,0)

Population.Density<-O
Policy.Level<-L
DModel=lm(GDP~Policy.Level*Population.Density)
summary(DModel)

##
## Call:
## lm(formula = GDP ~ Policy.Level * Population.Density)
##
## Residuals:
##     1     2     3     4     5
## -3961 -3961 -3961 -3961 15844
##
## Coefficients:
##                                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)                       262423       7922  33.125   0.0192 *
## Policy.Level                        2308       8857   0.261   0.8377
## Population.Density                 20545       8857   2.320   0.2591
## Policy.Level:Population.Density    20526       8857   2.317   0.2593
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 17710 on 1 degrees of freedom
## Multiple R-squared:  0.9154, Adjusted R-squared:  0.6616
## F-statistic: 3.606 on 3 and 1 DF,  p-value: 0.3651

contourPlot(DModel,"Policy.Level","Population.Density",main = "Model III")
```

```
paretoPlot(DModel)
```

```
plot_model(DModel, type = "pred", terms = c("Policy.Level", "Population.Density"),title = "Interaction Plot
for Model III")
```

```
plot_model(DModel, type = "pred", terms = c("Population.Density","Policy.Level"),title = "Interaction Plot f
or Model III")
```

```
gdp85<-c(124934, 105422, 144103, 350026, 39361, 136083, 252514, 99996, 156002, 54433, 148591, 252008, -220,
341352, 114839, 89923, 275037, 304257, 109621, 188536)
```

```
gdp325<-c(-310439, 201489, 220120, 293823, 243064, -24801, 464426, 344555, -140184, 86044, 204043, 75721, 40
244, -155597, 226992, -819044, 318994, 170727, -295451, -116079, 622679, 389695, 170355, 327421, 86298, -114
61, -41837, 364887, -82615, 539170, 241249, 135684, -108892, 215231, -133013, 241791, 222122, -314499, 18103
4, -163729)
gdp273<-c(-175090, 190470, 194727, 161382, 275124, 284229, 182657, 58468, 44725, 515284, -41662, -118056, 45
7220,
120644, 186032, 558274, 364987, 169261, 383579, 507491)

boxplot(gdp85,g185, gdp325[1:20],gdp325[21:40],gdp273,names = c("P=85, Policy=0.25","P=185, Policy=0.8","P=3
25, Policy=0.7","P=325, Policy=0.9","P=273, Policy=0.8"),main="Center Point Box Plot")
```

```
mean(gdp85)

## [1] 164340.9

mean(gdp325[1:20])

## [1] 51432.35

mean(gdp325[21:40])

## [1] 144078.5

mean(gdp273)

## [1] 215987.3

GDP<-c(219698.6,301840.8,51432.35,144078.5,215987.3)

L<-c(-1,-1,1,1,0)
O<-c(-1,1,-1,1,0)

Population.Density<-O
Policy.Level<-L
DModel=lm(GDP~Population.Density*Policy.Level)
summary(DModel)

##
## Call:
## lm(formula = GDP ~ Population.Density * Policy.Level)
##
## Residuals:
##     1     2     3     4     5
## -7345 -7345 -7345 -7345 29380
##
## Coefficients:
##                                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)                       186608      14690  12.703    0.050 .
## Population.Density                 43697      16424   2.661    0.229
## Policy.Level                      -81507      16424  -4.963    0.127
## Population.Density:Policy.Level     2626      16424   0.160    0.899
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 32850 on 1 degrees of freedom
## Multiple R-squared:  0.9694, Adjusted R-squared:  0.8778
## F-statistic: 10.58 on 3 and 1 DF,  p-value: 0.2214

contourPlot(DModel,"Policy.Level","Population.Density",main = "Model IV")
```

```
paretoPlot(DModel)
```

```
plot_model(DModel, type = "pred", terms = c("Policy.Level", "Population.Density"),title = "Interaction Plot
for Model IV")
```

```
plot_model(DModel, type = "pred", terms = c("Population.Density","Policy.Level"),title = "Interaction Plot f
or Model IV")
```

```
G<-c(111250, 178990, 165044, 188042, 176328, 69560, -9837, 184123, 184094, 75735, 215564, 200884, 69887, 224
369, 209162, 20190, 221408, 227362, 35333, 249741)
GDP<-c(166172.1,139427.1,213340.8,184611,mean(G))
mean(G)

## [1] 149861.5

L<-c(-1,-1,1,1,0)
O<-c(-1,1,-1,1,0)

Population.Density<-O
Policy.Level<-L
DModel=lm(GDP~Population.Density*Policy.Level)
summary(DModel)

##
## Call:
## lm(formula = GDP ~ Population.Density * Policy.Level)
##
## Residuals:
##      1      2      3      4      5
##   5205   5205   5205   5205 -20821
##
## Coefficients:
##                                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)                     170682.5    10410.5  16.395   0.0388 *
## Population.Density              -13868.7    11639.3  -1.192   0.4445
## Policy.Level                     23088.1    11639.3   1.984   0.2973
## Population.Density:Policy.Level   -496.2    11639.3  -0.043   0.9729
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 23280 on 1 degrees of freedom
## Multiple R-squared:  0.8427, Adjusted R-squared:  0.3707
## F-statistic: 1.785 on 3 and 1 DF,  p-value: 0.4914

contourPlot(DModel,"Policy.Level","Population.Density",main = "Model II")
```

```
paretoPlot(DModel)
```

```
plot_model(DModel, type = "pred", terms = c("Policy.Level", "Population.Density"),title = "Interaction Plot
for Model II")
```

```
plot_model(DModel, type = "pred", terms = c("Population.Density","Policy.Level"),title = "Interaction Plot f
or Model II")
```