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PS5

CS542

Cover

To achieve the best result; I decide to use hand to write down the written assignment part. (Typing in math symbols and equations in LaTex really interrupts my logic)

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1.(a) Bishop 8.3

we have 3 binaries a,b,c, we want to show $P(a,b) \neq P(a)P(b)$, but when c coming in, $P(a,b|c) = P(a|c)P(b|c)$ for $c=0$ and $c=1$.

For first part we follow the question and show by direct evaluation

b	a	$P(a,b)$	$P(a)P(b)$
1	0	264	244800
0	1	256	236800
0	0	336	356600
1	1	144	162800

$\Rightarrow$ we can see directly from these that a,b are marginally dependent which $P(a,b) \neq P(a)P(b)$

For second part, now we have a condition c, we know that

$$P(a,b|c) = \frac{P(a,b,c)}{\sum_{a\in(0,1)}\sum_{b\in(0,1)} P(a,b,c)}$$

we can get $P(a|c)$ and $P(b|c)$ the same way

Thus, we can have below:

$$\underbrace{\frac{P(a,b,c)}{\sum_{a\in(0,1)}\sum_{b\in(0,1)}P(a,b,c)}}_{P(a,b|c)} = \underbrace{\frac{\sum_{b\in(0,1)}P(a,b,c)}{\sum_{a\in(0,1)}\sum_{b\in(0,1)}P(a,b,c)}}_{P(a|c)} \underbrace{\frac{\sum_{a\in(0,1)}P(a,b,c)}{\sum_{a\in(0,1)}\sum_{b\in(0,1)}P(a,b,c)}}_{P(b|c)}$$

$\Rightarrow$ following the question

According to table 2, we can tell that when $c=0, c=1$, above equation is valid so that we successfully show when c involved, $P(a,b|c) = P(a|c)P(b|c)$ for $c=0,1$.

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1.(b) Bishop 8.4

In this question, we are asked to use Table 8.2 again, we can still show by direct evaluation.

$p(a)=\sum_{b\in\{0,1\}}\sum_{c\in\{0,1\}} p(a,b,c)$ $\quad p(b|c)=\frac{\sum_{a\in\{0,1\}} p(a,b,c)}{\sum_{a\in\{0,1\}}\sum_{b\in\{0,1\}} p(a,b,c)}$ $\quad p(c|a)=\frac{\sum_{b\in\{0,1\}} p(a,b,c)}{\sum_{b\in\{0,1\}}\sum_{c\in\{0,1\}} p(a,b,c)}$

$$p(a,b,c) = p(a)\,p(c|a)\,p(b|c)$$

a	p(a)
1	0.6
0	0.4

a	c	p(c|a)
0	0	0.4
1	1	0.4
0	1	0.6
1	0	0.6

b	c	p(b|c)
0	0	0.8
1	1	0.6
1	0	0.2
0	1	0.4

Now Let's calculate then compare to information from Table 8.2

a	b	c	p(a,b,c)	our calculated p(a,b,c)
0	0	0	0.192	0.192
0	0	1	0.144	0.144
0	1	0	0.048	0.048
0	1	1	0.216	0.216
1	0	0	0.192	0.192
1	0	1	0.064	0.064
1	1	0	0.048	0.048
1	1	1	0.096	0.096

$\Rightarrow$ Thus, we prove that $p(a,b,c)=p(a)p(c|a)p(b|c)$

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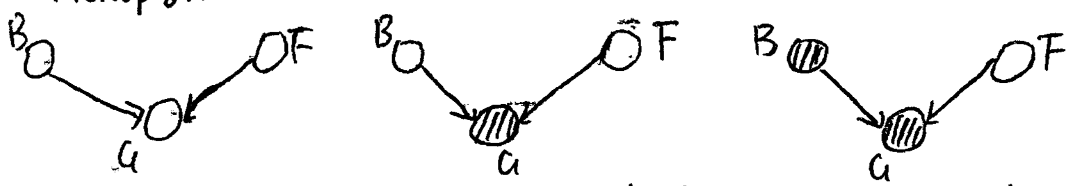
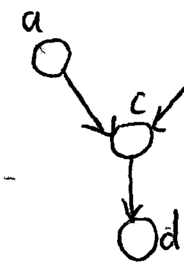
1. (c) Bishop 8.11



This is Figure 8.21 system, instead of observing G directly, we have a D here, so the situation should be like 8.54



First, we need to evaluate the Prob of tank is empty when $D=0$. Simply use Bayes' Theorem.

$$\text{Prob}(\text{Empty tank} \mid D=0) = \frac{P(D=0 \mid \text{Empty tank})\, P(\text{empty tank})}{P(D=0)}$$

$P(D=0) \rightarrow \sum_{F,G,B} P(D=0|G)P(B)P(F)P(G|B,F) = 0.352$

$P(D=0 \mid \text{Empty tank}) = \sum_{B,G} P(D=0|G)P(B)P(G|B, \text{Empty tank}) = 0.748$

$\therefore$ overall $\text{Prob}(\text{Empty tank} \mid D=0) = \underline{\underline{0.21}}$

Then we want to calculate when battery is flat which is $B=0$, we still use Bayes' Theorem and can get $\text{Prob}(\text{Empty tank} \mid B=0, D=0) = \underline{\underline{0.11}}$

The <u>intuition behind the results is below</u>:

Firstly 0.21 is less than 0.257 which is from 8.32, this shows that driver is not really dependable, then 0.11 is also less than 0.111 which is from 8.33. Just like what explained in the textbook: "This accords with our intuition that finding out that the battery is flat explains away the observation that the fuel gauge reads empty."

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1. (d) Bishop 8.14

We have a particular case based on 8.42 $E(X,y)=h\sum_i x_i-\beta\sum_{\{i,j\}}x_ix_j-\eta\sum_i x_iy_i$ which now $\beta=h=0$. We want to show that the most probable configuration of the latent variables is given by $x_i=y_i$ for all i.

It is pretty clear to show so for this particular case. We know most probable configuration means lowest energy configuration. In 8.42 we know η is a positive constant and we have the special case $\beta=h=0$, then $x_iy_i\in\{-1,+1\}$ it is obtained when $x_i=y_i$ where $i=1,\dots,D$. Thus, we successfully prove that the most probable configuration of the latent variables is given by $x_i=y_i$ for all i