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Homework 2

CSCI-545
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Problem 1

1.a.
No a robot action does not always increase uncertainty,it depends on the action taken.The prediction step always increases uncertainty while the correction or measurement step decreases uncertainty.

1.b.
If at any point in Bayesian filtering the probability of a state assignment becomes 1 then all the subsequent belief will also become 1. This can be avoided by changing the normalizing factor η.

1.c.

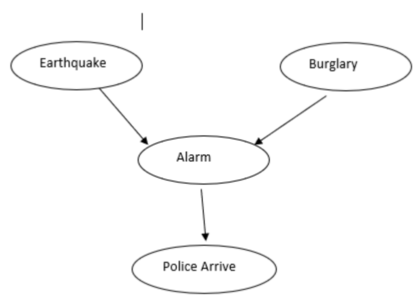

Figure 1:

1.d.

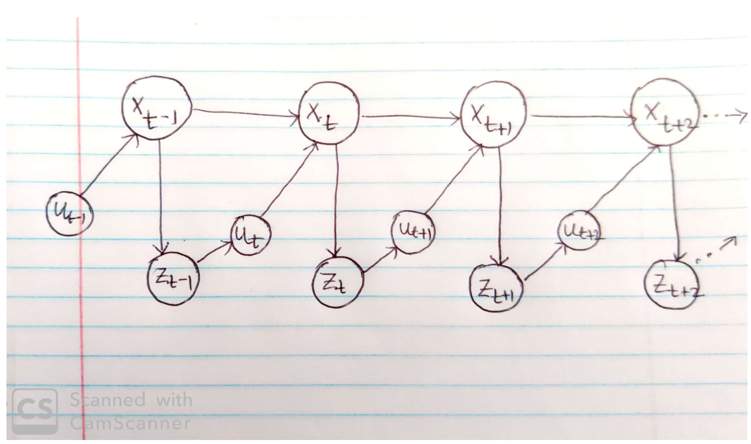

Figure 2:

1.e.

The extended Kalman Filter(EKF) considers non linear dynamics functions g and h for the state and observation respectively which are non Gaussian.Hence the posterior belief calculated by EKF is an approximation given by the mean. The linearization of g and h happens around the mean.

But for multiple hypotheses, eg a robot moving in a maze or a robot at a crossing of multiple paths, the belief at next timestamp may not be at the arithmetic mean of the hypotheses, hence the EKF fails for multi modal hypotheses.

Problem 3

After appending the state with alpha we get the state vector
$X = [x \quad \alpha]^T$
Given the state function f and measurement function h

$f1 = \alpha x + w(t)$
$f2 = \alpha$
$f(X) = \{f1, f2\}$

$h = \sqrt{x^2+1} + v(t)$

$F = \partial f/dX = \begin{bmatrix} \alpha & 0 \\ 0 & 1 \end{bmatrix}$

$H = \partial h/dX = \begin{bmatrix} x/\sqrt{x^2+1} & 0 \end{bmatrix}$

Here F is the state function Jacobian and H is the Measurement function Jacobian.

Writing the equations of Kalman filter for the predict and update state we get

$\overline{\mu} = f(X)$
$\overline{\sum} = F \sum F + R$
$K = \overline{\sum} H^T (H \overline{\sum} H^T + Q)^{-}1$
$\mu = \overline{\mu} + K(Z - h(\overline{\mu}))$
$\sum = (I - KH)\overline{\sum}$

where symbols have usual meanings
We get value of α from $\mu[1]$