The Quantum Hall Effect

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The Classic: Hall Effect

- The Drude model: $m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -e\mathbf{E} - e\mathbf{v} \times \mathbf{B} - \frac{m\mathbf{v}}{\tau}$.
  Equilibrium:
  $$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = 0 \;\Rightarrow\; \mathbf{v} + \frac{e\tau}{m}\mathbf{v} \times \mathbf{B} = -\frac{e\tau}{m}\mathbf{E}. \tag{1}$$
- Setup: Electrons restricted to move in the 2D (x,y)-plane while a constant magnetic field points in the z-direction.
- Angular frequency unit: $\omega_B = \frac{eB}{m}$. Recall: $\mathbf{J} = -ne\mathbf{v}$.
  Plugging in Eq.(1) $\Rightarrow \begin{pmatrix} 1 & \omega_B\tau \\ -\omega_B\tau & 1 \end{pmatrix} \mathbf{J} = \sigma_{DC}\mathbf{E}$, where $\sigma_{DC} = \frac{ne^2\tau}{m}$ is the resistivity without magnetic field.
- From above, we have $\mathbf{J} = \sigma\mathbf{E}, \rho\mathbf{J} = \mathbf{E}$.
- $\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}$, ρ similar. (Cross terms because of $\mathbf{B}$.)

The Classic: Hall Effect

- $\frac{1}{\sigma_{DC}} \begin{pmatrix} 1 & \omega_B \tau \\ -\omega_B \tau & 1 \end{pmatrix} = \rho \Rightarrow$
  longitudinal resistivity $\rho_{xx} = \frac{1}{\sigma_{DC}} = \frac{m}{ne^2 \tau}$,
  Hall resistivity $\rho_{xy} = \frac{\omega_B \tau}{\sigma_{DC}} = \frac{B}{ne} := R_H B$.
- R_H and ρ_{xy} is irrelevant of $\tau \Rightarrow$ intrinsic property.
  ρ_{xx} depends on τ. $\tau \to \infty$, $\rho_{xx} \to 0$. (No scattering.)

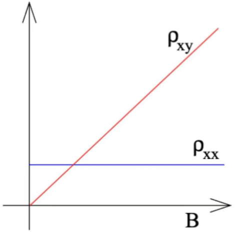

Figure: Expected experimental result from Hall Effect

Credit: *D. Tong, The Quantum Hall Effect – TIFR Infosys Lectures. (2016)*

The Quantum Hall Effect

More fun at low temperature ($\sim 4K$) and strong magnetic field.

- Our setup is similar as the classic Hall effect.
- Integer QHE (1981): $\rho_{xy} = \frac{2\pi\hbar}{e^2}\frac{1}{v}$, $v \in \mathbb{Z}$.
- Fractional QHE (1982): $\rho_{xy} = \frac{2\pi\hbar}{e^2}\frac{1}{v}$, $v \in \mathbb{Q}$.

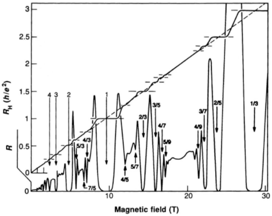

Figure: Hall coefficient – B relations for fractional QHE

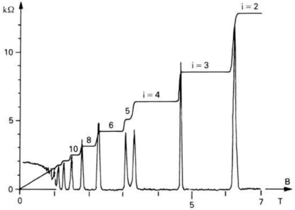

Figure: Hall coefficient – B relations for integer QHE

Credit: *D. Tong, The Quantum Hall Effect – TIFR Infosys Lectures. (2016).*

Materials where Quantum Hall Happens

- Initial discovery of integer QHE: Si MOSFET ("metal-oxide-semiconductor field-effect transistor"), a metal-insulator- semiconductor sandwich.
- Initial discovery of fractional QHE: GaAs (Gallium Arsenide).
- Recent: integer and fractional QHE in graphene [1] [2].

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[1] K. S. Novoselov et al., Room-Temperature Quantum Hall Effect in Graphene. Science 315,1379-1379 (2007).

[2] Bolotin, K., Ghahari, F., Shulman, M. et al. Observation of the fractional quantum Hall effect in graphene . Nature 462, 196–199 (2009).

Topology and Hall Conductivity

TKNN formula:

$$\sigma_{xy} = -\frac{e^2}{2\pi\hbar} C, \tag{2}$$

where the first Chern number $C \in \mathbb{Z}$.

Autonomous Hall

- AHE: $\rho_{xy} = R_0 B + R_s M$, where M is the magnetization (net magnetic moment per volume).

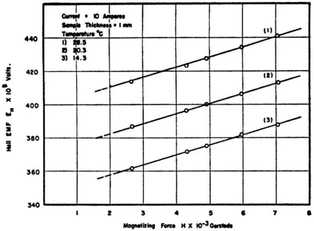

Fig. 1. Hall electromotive force as a function of the applied magnetic field in a Ni sample which has reached its saturation magnetisation. The slope of the lines gives the ordinary Hall coefficient while the *y*-intercepts give the anomalous Hall coefficients. From (Pugh, et al., 1950).

Credit: *Emerson M. Pugh and Norman Rostoker, Hall Effect in Ferromagnetic Materials. Rev. Mod. Phys. 25, 151 (1953).*

- Quantum Autonomous Hall – a popular field

Summary: The Hall Effects

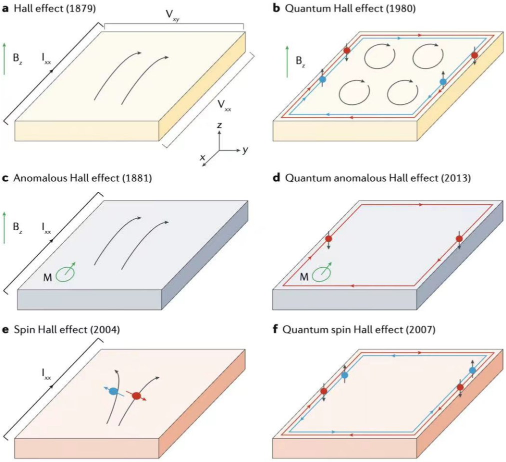

Figure: All the Hall Effects, some of which not covered above.

Credit: *von Klitzing, K., Chakraborty, T., Kim, P. et al. 40 years of the quantum Hall effect. Nat Rev Phys 2, 397–401 (2020).*