	ils bel	ow before ente	ring your candidate information
Candidate surname			Other names
Pearson Edexcel International Advanced Level	Cen	tre Number	Candidate Number
Time 1 hours 30 minutes		Paper reference	WFM03/01
Mathematics International Advance		-	//Advanced Level
Further Pure Mathema	TICS	: F3	

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

Turn over ▶







1. (a) Using the definition	tions of hyperbolic functions in terms of exponentials, show that $1 - \tanh^2 x \equiv \operatorname{sech}^2 x$	(3)
(b) Solve the equation	on	(-)
	$2 \operatorname{sech}^2 x + 3 \tanh x = 3$	
giving your ans	wer as an exact logarithm.	(3)



Overtion 1 continued	
Question 1 continued	

2.	A	curve	has	equation

$$y = \sqrt{9 - x^2} \qquad 0 \leqslant x \leqslant 3$$

(a) Using calculus, show that the length of the curve is $\frac{3\pi}{2}$

(4)

The curve is rotated through 2π radians about the *x*-axis.

(b) Using calculus, find the exact area of the surface generated.

(3)

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		(3	1	p	
3.	$\mathbf{M} =$	1	1	2	where p is a real constant
		-1	p	2)	

(a) Find the exact values of p for which \mathbf{M} has no inverse.

(4)

Given that M does have an inverse,

(b) find \mathbf{M}^{-1} in terms of p.

(5)

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4. (i)	$f(x) = x \arccos x$ $-1 \le x \le 1$	
Find the exact value or	f f (0.5).	(3)
(ii)	$g(x) = \arctan(e^{2x})$	` /
	g(x) = arctan(c)	
Show that		
	$g''(x) = k \operatorname{sech}(2x) \tanh(2x)$	
where k is a constant t	o be found.	
		(5)

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 $I_n = \int \sec^n x \, \mathrm{d}x \qquad n \geqslant 0$

(a) Prove that for $n \ge 2$

$$(n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$$
(6)

(b) Hence, showing each step of your working, find the exact value of

$$\int_0^{\frac{\pi}{4}} \sec^6 x \, \mathrm{d}x$$

(4)

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6. The line l_1 has equation

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k})$$

and the line l_2 has equation

$$r = 2\mathbf{i} + s\mathbf{j} + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

where s is a constant and λ and μ are scalar parameters.

Given that $l_{\scriptscriptstyle 1}$ and $l_{\scriptscriptstyle 2}$ both lie in a common plane $\Pi_{\scriptscriptstyle 1}$

(a) show that an equation for Π_1 is 3x + y - z = 3

(4)

(b) find the value of s.

(1)

The plane Π_2 has equation $\mathbf{r} \cdot (\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = 3$

(c) Find an equation for the line of intersection of Π_1 and Π_2

(4)

(d) Find the acute angle between Π_1 and Π_2 giving your answer in degrees to 3 significant figures.

(4)



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Question 6 continued	



Question 6 continued	

7. Using calculus, find the exact values of

(i)
$$\int_{1}^{2} \frac{1}{x^2 - 4x + 5} \, \mathrm{d}x$$

(3)

(ii)
$$\int_{\sqrt{3}}^{3} \frac{\sqrt{x^2 - 3}}{x^2} \, \mathrm{d}x$$

(5)



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The hyperbola *H* has equation

$$4x^2 - y^2 = 4$$

(a) Write down the equations of the asymptotes of H.

(1)

(b) Find the coordinates of the foci of H.

(2)

The point $P(\sec \theta, 2\tan \theta)$ lies on H.

(c) Using calculus, show that the equation of the tangent to H at the point P is

$$y \tan \theta = 2x \sec \theta - 2$$

(4)

The point V(-1, 0) and the point W(1, 0) both lie on H.

The point $Q(\sec \theta, -2\tan \theta)$ also lies on H.

Given that P, Q, V and W are distinct points on H and that the lines VP and WQ intersect at the point S,

(d) show that, as θ varies, S lies on an ellipse with equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where a and b are integers to be found.

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