| Please check the examination details bel              | low before ente    | ring your candidate in | nformation  |
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| Pearson Edexcel Inter                                 | nation             | al Advanc              | ed Level    |
| <b>Time</b> 1 hour 30 minutes                         | Paper<br>reference | WFM                    | 03/01       |
| Mathematics   |                    |                        | •           |
| International Advanced S                              | uhsidiar           | v/Δdvanced             | Level       |
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| Further Pure Mathematics                              | 5 F3               |                        |             |
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|   |                    |                        |             |
| You must have:  Mathematical Formulae and Statistical | al Tables (Ye      | llow), calculator      | Total Marks |
|   |                    |                        |             |

Candidates may use any calculator permitted by Pearson regulations.

Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## **Instructions**

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
   there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  - use this as a guide as to how much time to spend on each guestion.

## **Advice**

- Read each guestion carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶







1. (a) Use the definitions of  $\sinh x$  and  $\cosh x$  in terms of exponentials to show that

$$\cosh A \cosh B + \sinh A \sinh B \equiv \cosh(A + B)$$
(2)

(b) Hence find the value of x for which

$$\cosh(x + \ln 2) = 5\sinh x$$

giving your answer in the form  $\frac{1}{2} \ln k$ , where k is a rational number to be determined. (5)

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2. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(i) Determine

$$\int \frac{1}{\sqrt{5+4x-x^2}} \, \mathrm{d}x \tag{3}$$

(ii) Use the substitution  $x = 3 \sec \theta$  to determine the exact value of

$$\int_{2\sqrt{3}}^{6} \frac{18}{(x^2 - 9)^{\frac{3}{2}}} \, \mathrm{d}x$$

Give your answer in the form  $A + B\sqrt{3}$  where A and B are constants to be found.

**(6)** 

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$$\mathbf{M} = \begin{pmatrix} -2 & 5 & 0 \\ 5 & 1 & -3 \\ 0 & -3 & 6 \end{pmatrix}$$

Given that i + j + k is an eigenvector of M,

(a) determine the corresponding eigenvalue.

**(1)** 

Given that 8 is an eigenvalue of M,

(b) determine a corresponding eigenvector.

**(2)** 

(c) Determine a diagonal matrix  $\boldsymbol{D}$  and an orthogonal matrix  $\boldsymbol{P}$  such that

$$\mathbf{D} = \mathbf{P}^{\mathrm{T}} \mathbf{M} \mathbf{P}$$

**(5)** 



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4.

$$y = \operatorname{artanh}\left(\frac{\cos x + a}{\cos x - a}\right)$$

where a is a non-zero constant.

Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = k \tan x$$

where k is a constant to be determined.

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5. A curve has parametric equations

$$x = 4e^{\frac{1}{2}t} \qquad y = e^t - t \qquad 0 \leqslant t \leqslant 4$$

The curve is rotated through  $2\pi$  radians about the *x*-axis.

Show that the area of the curved surface generated is

$$\pi(e^8 + Ae^4 + B)$$

where A and B are constants to be determined.

**(7)** 

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**6.** 
$$\mathbf{A} = \begin{pmatrix} x & 1 & 3 \\ 2 & 4 & x \\ -4 & -2 & -1 \end{pmatrix}$$

| (a) | Show that A is non-singular for an real values of $x$ . |            |
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| (b) Determine, in terms of $x$ , $\mathbf{A}^{-1}$ | (4) |
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| Question 6 continued              |
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| (Total for Question 6 is 8 marks) |
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$$I_n = \int \frac{x^n}{\sqrt{10 - x^2}} \, \mathrm{d}x \qquad n \in \mathbb{N} \qquad |x| < \sqrt{10}$$

(a) Show that

$$nI_n = 10(n-1)I_{n-2} - x^{n-1}(10 - x^2)^{\frac{1}{2}}$$
  $n \ge 2$  (6)

(b) Hence find the exact value of

$$\int_0^1 \frac{x^5}{\sqrt{10 - x^2}} \, \mathrm{d}x$$

giving your answer in the form  $\frac{1}{15}(p\sqrt{10}+q)$  where p and q are integers to be determined.

**(4)** 



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| (Total for Question 7 is 10 marks) |



**8.** The plane  $\Pi$  has equation

$$3x + 4y - z = 17$$

The line  $l_1$  is perpendicular to  $\Pi$  and passes through the point P(-4, -5, 3)

The line  $l_1$  intersects  $\Pi$  at the point Q

(a) Determine the coordinates of Q

**(4)** 

Given that the point R(-1, 6, 4) lies on  $\Pi$ 

(b) determine a Cartesian equation of the plane containing PQR

**(4)** 

The line  $l_2$  passes through P and R

The line  $l_3$  is the reflection of  $l_2$  in  $\Pi$ (c) Determine a vector equation for  $l_3$ 

**(4)** 



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**9.** The ellipse E has equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

The line *l* has equation y = kx - 3, where *k* is a constant.

Given that E and l meet at 2 distinct points P and Q

(a) show that the x coordinates of P and Q are solutions of the equation

$$(9k^2 + 4)x^2 - 54kx + 45 = 0$$
(2)

The point M is the midpoint of PQ

(b) Determine, in simplest form in terms of k, the coordinates of M

**(3)** 

(c) Hence show that, as k varies, M lies on the curve with equation

$$x^2 + py^2 = qy$$

where p and q are constants to be determined.

**(5)** 



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