Candidate surname	Other names
Pearson Edexcel International Advanced Level	tre Number Candidate Number
Friday 15 May 2	2020
Afternoon (Time: 1 hour 30 minutes)	Paper Reference WFM02/01
Mathematics	
International Advanced Su Further Pure Mathematics	•

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 there may be more space than you need.
- You should show sufficient working to make your methods clear.
 Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
 - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶







1.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 3x \, \frac{\mathrm{d}y}{\mathrm{d}x} = 2\cos x$$

(a) Express $\frac{d^3y}{dx^3}$ in terms of x, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

(3)

At
$$x = 0$$
, $y = 2$ and $\frac{dy}{dx} = 5$

(b) Determine the value of $\frac{d^3y}{dx^3}$ at x = 0

(1)

(c) Express y as a series in ascending powers of x, up to and including the term in x^3

(3)



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2. (a) Write $\frac{3r+1}{r(r-1)(r+1)}$ in partial fractions.

(2)

(b) Hence find

$$\sum_{r=2}^{n} \frac{3r+1}{r(r-1)(r+1)} \qquad n \geqslant 2$$

giving your answer in the form

$$\frac{an^2 + bn + c}{2n(n+1)}$$

where a, b and c are integers to be determined.

(5)

(c) Hence determine the exact value of

$$\sum_{r=15}^{20} \frac{3r+1}{r(r-1)(r+1)}$$

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4. (a) Express the complex number $18\sqrt{3} - 18i$ in the form	
$r(\cos\theta + i\sin\theta) - \pi < \theta \leqslant \pi$	(2)
	(3)
(b) Solve the equation	
4 10 5 10:	
$z^4 = 18\sqrt{3} - 18i$	
giving your answers in the form $re^{i\theta}$ where $-\pi < \theta \leqslant \pi$	
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5. The transformation T from the z-plane to the w-plane is given by

$$w = \frac{z - 3i}{z + 2i} \qquad z \neq -2i$$

The circle with equation |z| = 1 in the z-plane is mapped by T onto the circle C in the w-plane.

Determine

- (i) the centre of C,
- (ii) the radius of C.

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$$x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + (x\cot x + 2)xy = 4\sin x \qquad 0 < x < \pi$$

Give your	answer	in the	form	<i>y</i> =	f(x)
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7.

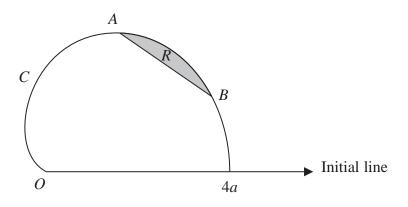


Figure 1

The curve C, shown in Figure 1, has polar equation

$$r = 2a(1 + \cos \theta)$$
 $0 \le \theta \le \pi$

where a is a positive constant.

The tangent to C at the point A is parallel to the initial line.

(a) Determine the polar coordinates of A.

(6)

The point *B* on the curve has polar coordinates $\left(a\left(2+\sqrt{3}\right),\frac{\pi}{6}\right)$

The finite region R, shown shaded in Figure 1, is bounded by the curve C and the line AB.

(b) Use calculus to determine the exact area of the shaded region R.

Give your answer in the form

$$\frac{a^2}{4} \Big(d\pi - e + f\sqrt{3} \Big)$$

where d, e and f are integers.

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8. (a) Show that the transformation $x = e^u$ transforms the differential equation

$$x^{2} \frac{d^{2}y}{dx^{2}} + 3x \frac{dy}{dx} - 8y = 4 \ln x \qquad x > 0$$
 (I)

into the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}u^2} + 2\frac{\mathrm{d}y}{\mathrm{d}u} - 8y = 4u \tag{II}$$

(b) Determine the general solution of differential equation (II), expressing y as a function of u.

(7)

(c) Hence obtain the general solution of differential equation (I). (1)



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