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# GCSE

## Edexcel GCE

## Core Mathematics C4 (6666)

Summer 2005

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Mark Scheme (Results)

**Final Version**

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| Question Number | Scheme | Marks |
|---|---|---|
| **1.** | $(4-9x)^{\frac{1}{2}} = 2\left(1 - \frac{9x}{4}\right)^{\frac{1}{2}}$ | B1 |
| | $= 2\left(1 + \frac{\frac{1}{2}}{1}\left(-\frac{9x}{4}\right) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}\left(-\frac{9x}{4}\right)^2 + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}\left(-\frac{9x}{4}\right)^3 + \ldots\right)$ | M1 |
| | $= 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \ldots\right)$ | |
| | $= 2 - \frac{9}{4}x, -\frac{81}{64}x^2, -\frac{729}{512}x^3 + \ldots$ | A1, A1, A1 |
| | | **[5]** |

*Note* The M1 is gained for $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}(\ \ldots\ )^2$ or $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}(\ \ldots\ )^3$

*Special Case*

If the candidate reaches $= 2\left(1 - \frac{9}{8}x - \frac{81}{128}x^2 - \frac{729}{1024}x^3 + \ldots\right)$ and goes no further

allow A1 A0 A0

| Question Number | Scheme | Marks |
|---|---|---|
| **2.** | $$2x + \left(2x\frac{dy}{dx} + 2y\right) - 6y\frac{dy}{dx} = 0$$ $$\frac{dy}{dx} = 0 \quad \Rightarrow \quad x + y = 0 \qquad \text{or equivalent}$$ Eliminating either variable and solving for at least one value of $x$ or $y$. $$y^2 - 2y^2 - 3y^2 + 16 = 0 \qquad \text{or the same equation in } x$$ $$y = \pm 2 \qquad \qquad \text{or } x = \pm 2$$ $$(2, -2), (-2, 2)$$ Note: $\dfrac{dy}{dx} = \dfrac{x+y}{3y-x}$ *Alternative* $$3y^2 - 2xy - \left(x^2 + 16\right) = 0$$ $$y = \frac{2x \pm \sqrt{\left(16x^2 + 192\right)}}{6}$$ $$\frac{dy}{dx} = \frac{1}{3} \pm \frac{1}{3}.\frac{8x}{\sqrt{\left(16x^2 + 192\right)}}$$ $$\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{8x}{\sqrt{\left(16x^2 + 192\right)}} = \pm 1$$ $$64x^2 = 16x^2 + 192$$ $$x = \pm 2$$ $$(2, -2), (-2, 2)$$ | M1 (A1) A1 M1 M1 A1 A1 **[7]** M1 A1± A1 M1 M1 A1 A1 **[7]** |

6666 Core C4
June 2005 Advanced Subsidiary/Advanced Level in GCE Mathematics

| Question Number | Scheme | Marks |
|---|---|---|
| **3.** | (a) $\dfrac{5x+3}{(2x-3)(x+2)} = \dfrac{A}{2x-3} + \dfrac{B}{x+2}$ $$5x+3 = A(x+2) + B(2x-3)$$ Substituting $x = -2$ or $x = \frac{3}{2}$ and obtaining $A$ or $B$; or equating coefficients and solving a pair of simultaneous equations to obtain $A$ or $B$. $$A = 3, B = 1$$ If the cover-up rule is used, give M1 A1 for the first of $A$ or $B$ found, A1 for the second. (b) $\displaystyle\int \dfrac{5x+3}{(2x-3)(x+2)}\,\mathrm{d}x = \dfrac{3}{2}\ln(2x-3) + \ln(x+2)$ $$\Big[\ \dots\ \Big]_2^6 = \dfrac{3}{2}\ln 9 + \ln 2$$ $$= \ln 54 \qquad\qquad\qquad \text{cao}$$ | M1 A1, A1 **(3)** M1 A1ft M1 A1 A1 **(5)** **[8]** |

| Question Number | Scheme | Marks |
|---|---|---|
| **4.** | $\displaystyle\int \frac{1}{\left(1-x^2\right)^{\frac{1}{2}}}\, dx = \int \frac{1}{\left(1-\sin^2\theta\right)^{\frac{3}{2}}}\cos\theta\, d\theta$    Use of $x=\sin\theta$ and $\dfrac{dx}{d\theta}=\cos\theta$ | M1 |
| | $\displaystyle = \int \frac{1}{\cos^2\theta}\, d\theta$ | M1 A1 |
| | $\displaystyle = \int \sec^2\theta\, d\theta = \tan\theta$ | M1 A1 |
| | Using the limits 0 and $\frac{\pi}{6}$ to evaluate integral | M1 |
| | $\displaystyle \left[\tan\theta\right]_0^{\frac{\pi}{6}} = \frac{1}{\sqrt{3}} \quad \left(=\frac{\sqrt{3}}{3}\right)$    cao | A1 |
| | | **[7]** |
| | *Alternative for final* M1 A1 | |
| | Returning to the variable $x$ and using the limits 0 and $\frac{1}{2}$ to evaluate integral | M1 |
| | $\displaystyle \left[\frac{x}{\sqrt{\left(1-x^2\right)}}\right]_0^{\frac{1}{2}} = \frac{1}{\sqrt{3}} \quad \left(=\frac{\sqrt{3}}{3}\right)$    cao | A1 |

| Question Number | Scheme | Marks |
|---|---|---|
| **5.** | (a) $\quad \int x\mathrm{e}^{2x}\mathrm{d}x = \dfrac{1}{2}x\mathrm{e}^{2x} - \dfrac{1}{2}\int \mathrm{e}^{2x}\,\mathrm{d}x$   Attempting parts in the right direction | M1 A1 |
| | $\qquad\qquad = \dfrac{1}{2}x\mathrm{e}^{2x} - \dfrac{1}{4}\mathrm{e}^{2x}$ | A1 |
| | $\qquad \left[\dfrac{1}{2}x\mathrm{e}^{2x} - \dfrac{1}{4}\mathrm{e}^{2x}\right]_{0}^{1} = \dfrac{1}{4} + \dfrac{1}{4}\mathrm{e}^{2}$ | M1 A1 |
| | | **(5)** |
| | (b) $\qquad\qquad\qquad\quad x = 0.4 \;\Rightarrow\; y \approx 0.890\,22$ <br> $\qquad\qquad\qquad\quad x = 0.8 \;\Rightarrow\; y \approx 3.962\,43$    Both are required to 5 d.p . | B1 |
| | | **(1)** |
| | (c) $\quad I \approx \dfrac{1}{2} \times 0.2 \times \big[\,\ldots\,\big]$ | B1 |
| | $\quad \approx \; \ldots \; \times\big[0 + 7.389\,06 + 2(0.298\,36 + .890\,22 + 1.992\,07 + 3.962\,43)\big]$ <br> $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ft their answers to (b) | M1 A1ft |
| | $\quad \approx 0.1 \times 21.675\,22$ <br> $\quad \approx 2.168$                         cao | A1    **(4)** <br> **[10]** |
| | $Note \;\; \dfrac{1}{4} + \dfrac{1}{4}\mathrm{e}^{2} \approx 2.097 \;\ldots$ | |

| Question Number | Scheme | Marks |
|---|---|---|
| **6.** (a) | $\dfrac{\mathrm{d}x}{\mathrm{d}t} = -2\operatorname{cosec}^2 t,\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = 4\sin t\cos t$      both | M1 A1 |
| | $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-2\sin t\cos t}{\operatorname{cosec}^2 t}\ \left(=-2\sin^3 t\cos t\right)$ | M1 A1 |
| | | **(4)** |
| (b) | At $t=\frac{\pi}{4}$, $x=2$, $y=1$      both $x$ and $y$ | B1 |
| | Substitutes $t=\frac{\pi}{4}$ into an attempt at $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ to obtain gradient $\left(-\dfrac{1}{2}\right)$ | M1 |
| | Equation of tangent is $y-1=-\dfrac{1}{2}\left(x-2\right)$ | M1 A1 |
| | Accept $x+2y=4$ or any correct equivalent | **(4)** |
| (c) | Uses $1+\cot^2 t = \operatorname{cosec}^2 t$, or equivalent, to eliminate $t$ | M1 |
| | $1+\left(\dfrac{x}{2}\right)^2 = \dfrac{2}{y}$      correctly eliminates $t$ | A1 |
| | $y=\dfrac{8}{4+x^2}$      cao | A1 |
| | The domain is $x\dots 0$ | B1    **(4)** |
| | | **[12]** |
| | *An alternative in* (c) | |
| | $\sin t = \left(\dfrac{y}{2}\right)^{\frac{1}{2}};\ \cos t = \dfrac{x}{2}\sin t = \dfrac{x}{2}\left(\dfrac{y}{2}\right)^{\frac{1}{2}}$ | |
| | $\sin^2 t + \cos^2 t = 1 \ \Rightarrow\ \dfrac{y}{2}+\dfrac{x^2}{4}\times\dfrac{y}{2}=1$ | M1 A1 |
| | Leading to $y=\dfrac{8}{4+x^2}$ | A1 |

| Question Number | Scheme | Marks |
|---|---|---|
| **7.** | (a) **k** component $\quad 2+4\lambda=-2 \;\Rightarrow\; \lambda=-1$ | M1 A1 |
| | $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ *Note* $\mu=2$ | |
| | $\qquad$ Substituting their $\lambda$ (or $\mu$) into equation of line and obtaining $B$ | M1 |
| | $\qquad\qquad\qquad B: \left(2,2,-2\right) \qquad\qquad$ Accept vector forms | A1 |
| | | **(4)** |
| | (b) $\qquad\qquad \left|\begin{pmatrix}1\\-1\\4\end{pmatrix}\right|=\sqrt{18}; \quad \left|\begin{pmatrix}1\\-1\\0\end{pmatrix}\right|=\sqrt{2} \qquad$ both | B1 |
| | $\qquad\qquad \begin{pmatrix}1\\-1\\4\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\0\end{pmatrix}=1+1+0\left(=2\right)$ | B1 |
| | $\qquad\qquad\qquad \cos\theta=\dfrac{2}{\sqrt{18}\sqrt{2}}=\dfrac{1}{3} \qquad\qquad$ cao | M1 A1 |
| | | **(4)** |
| | (c) $\quad \overrightarrow{AB}=-\mathbf{i}+\mathbf{j}-4\mathbf{k}\Rightarrow\left|\overrightarrow{AB}\right|^{2}=18 \;$ or $\;\left|\overrightarrow{AB}\right|=\sqrt{18} \qquad$ ignore direction of vector | M1 |
| | $\qquad \overrightarrow{BC}=3\mathbf{i}-3\mathbf{j}\Rightarrow\left|\overrightarrow{BC}\right|^{2}=18 \;$ or $\;\left|\overrightarrow{BC}\right|=\sqrt{18} \qquad$ ignore direction of vector | M1 |
| | $\qquad\qquad\qquad$ Hence $\left|\overrightarrow{AB}\right|=\left|\overrightarrow{BC}\right| \quad *$ | A1 $\qquad$ **(3)** |
| | (d) $\quad \overrightarrow{OD}=6\mathbf{i}-2\mathbf{j}+2\mathbf{k} \qquad\qquad$ Allow first B1 for any two correct | B1 B1 |
| | $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ Accept column form or coordinates | **(2)** |
| | | **[13]** |

| Question Number | Scheme | Marks |
|---|---|---|
| **8.** | (a)  $\dfrac{dV}{dt}$ is the rate of increase of volume (with respect to time) | B1 |
| | $-kV$ : $k$ is constant of proportionality and the negative shows decrease (or loss) giving $\dfrac{dV}{dt} = 20 - kV$  ✱      These Bs are to be awarded independently | B1 |
| | | **(2)** |
| | (b)      $\displaystyle\int \dfrac{1}{20 - kV}\, dV = \int 1\, dt$          separating variables | M1 |
| | $-\dfrac{1}{k}\ln(20 - kV) = t \quad (+C)$ | M1 A1 |
| | Using $V = 0, t = 0$ to evaluate the constant of integration | M1 |
| | $c = -\dfrac{1}{k}\ln 20$ | |
| | $t = \dfrac{1}{k}\ln\left(\dfrac{20}{20 - kV}\right)$ | |
| | Obtaining answer in the form $V = A + B\,e^{-kt}$ | M1 |
| | $V = \dfrac{20}{k} - \dfrac{20}{k}e^{-kt}$          Accept $\dfrac{20}{k}\left(1 - e^{-kt}\right)$ | A1  **(6)** |
| | (c)          $\dfrac{dV}{dt} = 20\,e^{-kt}$          Can be implied | M1 |
| | $\dfrac{dV}{dt} = 10, t = 5 \;\Rightarrow\; 10 = 20\,e^{-kt} \;\Rightarrow\; k = \dfrac{1}{5}\ln 2 \approx 0.139$ | M1 A1 |
| | At $t = 10, \quad V = \dfrac{75}{\ln 2}$          awrt 108 | M1 A1  **(5)** |
| | | **[13]** |
| | *Alternative to* (b) | |
| | Using printed answer and differentiating  $\dfrac{dV}{dt} = -kB\,e^{-kt}$ | M1 |
| | Substituting into differential equation $-kB\,e^{-kt} = 20 - kA - kB\,e^{-kt}$ | M1 |
| | $A = \dfrac{20}{k}$ | M1 A1 |
| | Using $V = 0, t = 0$ in printed answer to obtain $A + B = 0$ | M1 |
| | $B = -\dfrac{20}{k}$ | A1  **(6)** |