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Pearson Edexcel nternational Advanced Level	Centre	Number		Candidate Number
Friday 22 Janu	uai	y 20	021	
Afternoon (Time: 1 hour 30 minute	es)	Paper Re	eference V	VFM03/01
Mathematics				
Mathematics International Advanced Further Pure Mathemat		•	y/Adva	nced Level

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
 use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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2.

$$y = \ln\left(\tanh 2x\right) \qquad x > 0$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = p \operatorname{cosech} 4x$$

where p is a constant to be determined.

(4)

(b) Hence determine, in simplest form, the exact value of x for which $\frac{dy}{dx} = 1$

(2)

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3.

$$\mathbf{A} = \begin{pmatrix} 2 & k & 2 \\ 2 & 2 & k \\ 1 & 2 & 2 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Determine the values of k for which **A** is singular.

(2)

Given that A is non-singular,

(b) find A^{-1} , giving your answer in terms of k.

(4)

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(6)

4. Using the substitution $x = 4\cosh\theta$ show that

$$\int \frac{1}{(x^2 - 16)^{\frac{3}{2}}} dx = \frac{ax}{\sqrt{x^2 - 16}} + c \qquad |x| > 4$$

where a is a constant to be determined and c is an arbitrary constant	nt.
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5.

$$\mathbf{M} = \begin{pmatrix} 6 & -2 & -1 \\ -2 & 6 & -1 \\ -1 & -1 & 5 \end{pmatrix}$$

Given that 8 is an eigenvalue of M

(a) determine an eigenvector corresponding to the eigenvalue 8

(2)

(b) Determine the other two eigenvalues of M.

(3)

(c) Hence find an orthogonal matrix \mathbf{P} and a diagonal matrix \mathbf{D} such that $\mathbf{P}^{\mathsf{T}}\mathbf{M}\mathbf{P} = \mathbf{D}$ **(4)**

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6.

$$I_n = \int \frac{x^n}{\sqrt{x^2 + 3}} \, \mathrm{d}x \qquad n \in \mathbb{N}$$

(a) Show that

$$I_{n} = \frac{x^{n-1}}{n} (x^{2} + 3)^{\frac{1}{2}} - \frac{3(n-1)}{n} I_{n-2} \qquad n \geqslant 3$$

(6)

(b) Hence show that

$$\int \frac{x^5}{\sqrt{x^2+3}} dx = \frac{1}{5} (x^2+3)^{\frac{1}{2}} (x^4+px^2+q) + k$$

where p and q are integers to be determined and k is an arbitrary constant.

(4)

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7. The point P has coordinates (1, 2, 1)

The line *l* has Cartesian equation

$$\frac{x-3}{5} = \frac{y+1}{3} = \frac{z+5}{-8}$$

The plane Π_1 contains the point P and the line l.

(a) Show that a Cartesian equation for Π_1 is

$$6x - 2y + 3z = 5 ag{5}$$

The point Q has coordinates (2, k, -7), where k is a constant.

(b) Show that the shortest distance between Π_1 and Q is

$$\frac{2}{7}|k+7|$$

(2)

The plane Π_2 has Cartesian equation 8x - 4y + z = -3

Given that the shortest distance between Π_1 and Q is the same as the shortest distance between Π_2 and Q,

(c) determine the possible values of k.

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8. The curve C has equation

$$y = 2 + \ln(1 - x^2)$$
 $\frac{1}{2} \le x \le \frac{3}{4}$

(a) Show that the length of the curve C is given by

$$\int_{\frac{1}{2}}^{\frac{3}{4}} \left(\frac{1+x^2}{1-x^2} \right) \mathrm{d}x$$

(4)

(b)	Hence,	using	algebraic	integration,	show	that	the	length	of	the	curve	C is	p	+	$\ln q$
	where p	p and q	are ration	nal numbers	to be	deter	min	ed.							

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9. The ellipse E has equation

$$\frac{x^2}{25} + \frac{y^2}{16} = 1$$

The point P lies on the ellipse and has coordinates $(5\cos\theta, 4\sin\theta)$ where $0 < \theta < \frac{\pi}{2}$

The line l is the normal to the ellipse at the point P.

(a) Show that an equation for l is

$$5x\sin\theta - 4y\cos\theta = 9\sin\theta\cos\theta$$

(5)

The point F is the focus of E that lies on the positive x-axis.

(b) Determine the coordinates of F.

(2)

The line l crosses the x-axis at the point Q.

(c) Show that

$$\frac{|QF|}{|PF|} = e$$

where e is the eccentricity of E.

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