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# GCE

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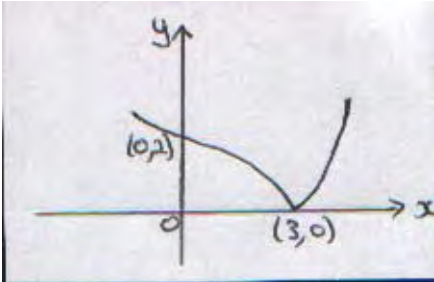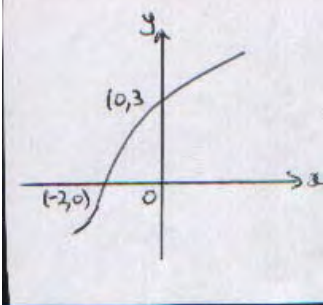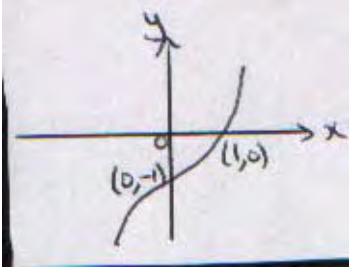
Mathematics

Core Mathematics C3 (6665)

June 2006

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Mark Scheme (Results)

Edexcel GCE

# Mathematics

**EDEXCEL CORE MATHEMATICS C3 (6665) – JUNE 2006**     **MARK SCHEME**

| Question Number | | Scheme | Marks |
|---|---|---|---|
| **1.** | (*a*) | $\dfrac{(3x+2)(x-1)}{(x+1)(x-1)}, = \dfrac{3x+2}{x+1}$ | M1B1, A1  (3) |
| | | **Notes**<br> M1 attempt to factorise numerator, *usual rules*<br> B1 factorising denominator seen anywhere in (a),<br> A1 given answer<br> If factorisation of denom. not seen, correct answer implies B1 | |
| | (*b*) | Expressing over common denominator<br><br>$\dfrac{3x+2}{x+1} - \dfrac{1}{x(x+1)} = \dfrac{x(3x+2)-1}{x(x+1)}$ | M1 |
| | | [Or "Otherwise" : $\dfrac{(3x^2-x-2)x-(x-1)}{x(x^2-1)}$ ]<br>Multiplying out numerator and attempt to factorise | M1 |
| | | $\left[3x^2+2x-1 \equiv (3x-1)(x+1)\right]$ | |
| | | Answer:  $\dfrac{3x-1}{x}$ | A1       (3) |
| | | | **(6 marks)** |
| **2.** | (*a*) | $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3\,\mathrm{e}^{3x} + \dfrac{1}{x}$ | B1M1A1(3) |
| | | **Notes**<br><br>B1 $3e^{3x}$<br><br>M1 : $\dfrac{a}{bx}$          A1: $3\,\mathrm{e}^{3x} + \dfrac{1}{x}$ | |
| | (b) | $\left(5+x^2\right)^{\frac{1}{2}}$ | B1 |
| | | $\dfrac{3}{2}\left(5+x^2\right)^{\frac{1}{2}} \cdot 2x \qquad = 3x\left(5+x^2\right)^{\frac{1}{2}}$          M1 for $kx(5+x^2)^m$ | M1 A1     (3) |
| | | | **(6 marks)** |

**EDEXCEL CORE MATHEMATICS C3 (6665) – JUNE 2006**     **MARK SCHEME**

| Question Number | | Scheme | Marks |
|---|---|---|---|
| **3.** | (a) |  Mod graph, reflect for $y < 0$ | M1 |
| | | (0, 2), (3, 0) or marked on axes | A1 |
| | | Correct shape, including cusp | A1     (3) |
| | (b) |  Attempt at reflection in $y = x$ | M1 |
| | | Curvature correct | A1 |
| | | (–2, 0), (0, 3) or equiv. | B1     (3) |
| | (c) |  Attempt at 'stretches' | M1 |
| | | (0, –1) or equiv. | B1 |
| | | (1, 0) | B1     (3) |
| | | | **(9 marks)** |
| **4.** | (a) | 425 ºC | B1     (1) |
| | (b) | $300 = 400\,e^{-0.05t} + 25 \quad \Rightarrow 400\,e^{-0.05t} = 275$ | M1 |
| | | sub. $T = 300$ and attempt to rearrange to $e^{-0.05t} = a$, where $a \in \mathbb{Q}$ | |
| | | $e^{-0.05t} = \dfrac{275}{400}$ | A1 |
| | | M1 correct application of logs | M1 |
| | | $t = 7.49$ | A1     (4) |
| | (c) | $\dfrac{dT}{dt} = -20\ e^{-0.05\,t}$          ( M1 for $k\,e^{-0.05\,t}$ ) | M1 A1 |
| | | At $t = 50$,   rate of decrease = $(\pm)\ 1.64$ ºC/min | A1     (3) |
| | (d) | $T > 25$,   (since $e^{-0.05\,t} \to 0$ as $t \to \infty$ ) | B1     (1) |
| | | | **(9 marks)** |

**EDEXCEL CORE MATHEMATICS C3 (6665) – JUNE 2006**          **MARK SCHEME**

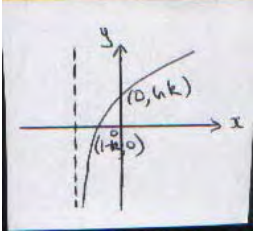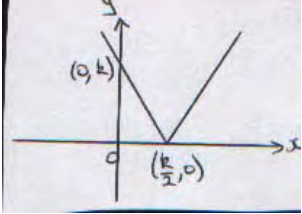| Question Number | | Scheme | Marks |
|---|---|---|---|
| **5.** | (*a*) | Using product rule: $\dfrac{dy}{dx} = 2\tan 2x + 2(2x-1)\sec^2 2x$ | M1 A1 A1 |
| | | Use of "$\tan 2x = \dfrac{\sin 2x}{\cos 2x}$" and "$\sec 2x = \dfrac{1}{\cos 2x}$" | M1 |
| | | $[= 2\dfrac{\sin 2x}{\cos 2x} + 2(2x-1)\dfrac{1}{\cos^2 2x}\ ]$ | |
| | | Setting $\dfrac{dy}{dx} = 0$ and multiplying through to eliminate fractions | M1 |
| | | $[\Rightarrow\ 2\sin 2x\cos 2x + 2(2x-1) = 0]$ | |
| | | Completion: producing $4k + \sin 4k - 2 = 0$ with no wrong working seen and at least previous line seen.          AG | A1*          (6) |
| | (*b*) | $x_1 = 0.2670,\quad x_2 = 0.2809,\quad x_3 = 0.2746,\quad x_4 = 0.2774,$ | M1 A1 A1  (3) |
| | | **Note:**   M1  for first correct application, first A1 for two correct, second A1 for all four correct <br> Max –1 deduction, if ALL correct to > 4 d.p.   M1 A0 A1 <br> SC: degree mode:  M1  $x_1 = 0.4948$ , A1 for $x_2 = 0.4914$, then A0;  max 2 | |
| | (*c*) | Choose suitable interval for *k*: e.g.  [0.2765, 0.2775]  and evaluate f(*x*) at these values | M1 |
| | | Show that  $4k + \sin 4k - 2$  changes sign and deduction | A1          (2) |
| | | [f(0.2765) = –0.000087.., f(0.2775) = +0.0057] | |
| | | **Note:** <br> Continued iteration: (no marks in degree mode) <br> Some evidence of further iterations leading to 0.2765 or better  M1; <br> Deduction A1 | |
| | | | **(11 marks)** |

**EDEXCEL CORE MATHEMATICS C3 (6665) – JUNE 2006**　　　　**MARK SCHEME**

| Question Number | | Scheme | Marks |
|---|---|---|---|
| **6.** | (*a*) | Dividing $\sin^2\theta + \cos^2\theta \equiv 1$ by $\sin^2\theta$ to give $$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} \equiv \frac{1}{\sin^2\theta}$$ | M1 |
| | | Completion: $1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta \Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$　　AG | A1*　　　(2) |
| | (*b*) | $\operatorname{cosec}^4\theta - \cot^4\theta \equiv \left(\operatorname{cosec}^2\theta - \cot^2\theta\right)\left(\operatorname{cosec}^2\theta + \cot^2\theta\right)$ | M1 |
| | | $\equiv \left(\operatorname{cosec}^2\theta + \cot^2\theta\right)$　　using (a)　　　　　AG | A1*　　　(2) |
| | | **Notes:** (i) Using LHS = $(1 + \cot^2\theta)^2 - \cot^4\theta$, using (a) & elim. $\cot^4\theta$　M1, conclusion {using (a) again}　　A1* (ii) Conversion to sines and cosines: needs $\dfrac{(1-\cos^2\theta)(1+\cos^2\theta)}{\sin^4\theta}$　for M1 | |
| | (*c*) | Using (b) to form　　　$\operatorname{cosec}^2\theta + \cot^2\theta \equiv 2 - \cot\theta$ | M1 |
| | | Forming quadratic in $\cot\theta$ $\Rightarrow 1 + \cot^2\theta + \cot^2\theta \equiv 2 - \cot\theta$　　　　　{using (a)} | M1 |
| | | $2\cot^2\theta + \cot\theta - 1 = 0$ | A1 |
| | | Solving:　$(2\cot\theta - 1)(\cot\theta + 1) = 0$　　　　to $\cot\theta =$ | M1 |
| | | $\left(\cot\theta = \dfrac{1}{2}\right)$　or　　$\cot\theta = -1$ | A1 |
| | | $\theta = 135°$　　　　(or correct value(s) for candidate dep. on 3Ms) | A1√　　(6) |
| | | **Note:** Ignore solutions outside range Extra "solutions" in range loses A1√, but candidate may possibly have more than one "correct" solution. | |
| | | | **(10 marks)** |

**EDEXCEL CORE MATHEMATICS C3 (6665) – JUNE 2006**          **MARK SCHEME**

| Question Number | Scheme | Marks |
|---|---|---|
| **7.** (*a*) |  Log graph: Shape | B1 |
| | Intersection with –ve *x*-axis | dB1 |
| | $(0, \ln k)$, $(1 - k, 0)$ | B1 |
| |  Mod graph :V shape, vertex on +ve *x*-axis | B1 |
| | $(0, k)$ and $\left(\dfrac{k}{2}, 0\right)$ | B1   (5) |
| (*b*) | $f(x) \in R$ ,   $-\infty < f(x) < \infty$ ,   $-\infty < y < \infty$ | B1   (1) |
| (*c*) | $fg\left(\dfrac{k}{4}\right) = \ln\{k + \mid\dfrac{2k}{4} - k\mid\}$   or   $f\left(\mid-\dfrac{k}{2}\mid\right)$ | M1 |
| | $= \ln\left(\dfrac{3k}{2}\right)$ | A1   (2) |
| (*d*) | $\dfrac{dy}{dx} = \dfrac{1}{x + k}$ | B1 |
| | Equating (with $x = 3$) to grad. of line;     $\dfrac{1}{3 + k} = \dfrac{2}{9}$ | M1; A1 |
| | $k = 1\frac{1}{2}$ | A1√   (4) |
| | | **(12 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **8.** (*a*) | Method for finding $\sin A$ | M1 |
| | $\sin A = -\dfrac{\sqrt{7}}{4}$ | A1 A1 |
| | **Note:**    First A1 for $\dfrac{\sqrt{7}}{4}$,    exact. | |
| | Second A1 for sign (even if dec. answer given) | |
| | **Use** of $\sin 2A \equiv 2\sin A\cos A$ | M1 |
| | $\sin 2A = -\dfrac{3\sqrt{7}}{8}$  or equivalent exact | A1√    (5) |
| | **Note:**   $\pm$ f.t.    Requires exact value, dependent on 2nd M | |
| (*b*)(i) | $\cos\left(2x+\dfrac{\pi}{3}\right)+\cos\left(2x-\dfrac{\pi}{3}\right) \equiv \cos 2x\cos\dfrac{\pi}{3} - \sin 2x\sin\dfrac{\pi}{3}\; + \cos 2x\cos\dfrac{\pi}{3} + \sin 2x\sin\dfrac{\pi}{3}$ | M1 |
| | $\equiv 2\cos 2x\cos\dfrac{\pi}{3}$ | A1 |
| | [This can be just written down (using factor formulae) for M1A1] | |
| | $\equiv \cos 2x$        AG | A1*    (3) |
| | **Note:** | |
| | M1A1 earned, if $\equiv 2\cos 2x\cos\dfrac{\pi}{3}$  just written down, using factor theorem | |
| | Final A1*   requires some working after first result. | |
| (*b*)(ii) | $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 6\sin x\cos x - 2\sin 2x$ | B1 B1 |
| | or    $6\sin x\cos x - 2\sin\left(2x+\dfrac{\pi}{3}\right) - 2\sin\left(2x-\dfrac{\pi}{3}\right)$ | |
| | $= 3\sin 2x - 2\sin 2x$ | M1 |
| | $= \sin 2x$     AG | A1*    (4) |
| | **Note:**  First B1 for $6\sin x\cos x$ ; second B1 for remaining term(s) | |
| | | **(12 marks)** |