

## Mark Scheme (Results) January 2008

**GCE** 

GCE Mathematics (6678/01)



## January 2008 6678 Mechanics M2 Mark Scheme

Question Number	Scheme	Marks
1.	(a) KE lost is $\frac{1}{2} \times 2.5 \times 8^2 = 80$ (J)	M1 A1 (2)
	(b) Work energy $80 = R \times 20$ ft their (a) $R = 4$	M1 A1 ft A1 (3) [5]
	Alternative to (b) $0^2 = 8^2 - 2 \times a \times 20 \implies a = (-)1.6$	
	N2L $R = 2.5 \times 1.6$ ft their $a = 4$	M1 A1ft A1 (3)
2.	(a) $\dot{\mathbf{p}} = (6t - 6)\mathbf{i} + (9t^2 - 4)\mathbf{j} \qquad (ms^{-1})$	M1 A1 (2)
	(b) $9t^2 - 4 = 0 \\ t = \frac{2}{3}$	M1 DM1 A1 (3)
	(c) $t = 1 \implies \dot{\mathbf{p}} = 5\mathbf{j}$ ft their $\dot{p}$ $(+/-)$ $2\mathbf{i} - 6\mathbf{j} = 0.5(\mathbf{v} - 5\mathbf{j})$	B1ft M1
	$\mathbf{v} = 4\mathbf{i} - 7\mathbf{j}  \left(\mathbf{m}\mathbf{s}^{-1}\right)$	M1 A1 (4) [9]

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3.	(a) $20000 = 16F  (F = 1250)$ $F = 550 + 1000 \times 9.8 \sin \theta$ ft their $F$ Leading to $\sin \theta = \frac{1}{14}$ $\star$ cso	M1 A1 M1 A1ft A1 (5)
	(b) N2L $7$ 550 + 1000 × 9.8 × sin $\theta$ = 1000 $a$ (550 + 1000 × 9.8 × $\frac{1}{14}$ = 1000 $a$ ) or 1250 = 1000 $a$ ( $a$ = ( $-$ )1.25)	M1 A1
	$v^{2} = u^{2} + 2as \implies 16^{2} = 2 \times 1.25 \times y$ $y \approx 102$ accept 102.4, 100	M1 A1 (4) [9]
	Alternative to (b) Work-Energy $\frac{1}{2} \times 1000 \times 16^2 - 1000 \times 9.8 \times \frac{1}{14} y = 550 y$ $y \approx 102$ accept 102.4, 100	M1 M1 A1 A1 (4)
4.	(a) Triangle Circle $S$ Mass ratio 126 $9\pi$ 126 $-9\pi$ (28.3) (97.7)	B1 B1ft
	$\frac{\overline{x}}{\overline{y}}$ 7 5 $\frac{\overline{x}}{\overline{y}}$ 4, 7 seen	B1
	$126 \times 7 = 9\pi \times 5 + (126 - 9\pi) \times \overline{x}  \text{ft their table values}$ $\overline{x} \approx 7.58 \ (\frac{882 - 45\pi}{126 - 9\pi}) \qquad \text{awrt 7.6}$	M1 A1ft A1
	$126 \times 4 = 9\pi \times 5 + (126 - 9\pi) \times \overline{y}  \text{ft their table values}$ $\overline{y} \approx 3.71  (\frac{504 - 45\pi}{126 - 9\pi})  \text{awrt } 3.7$	M1 A1ft A1 (9)
	(b) $\tan \theta = \frac{\overline{y}}{21 - \overline{x}}$ ft their $\overline{x}$ , $\overline{y}$ $\theta \approx 15^{\circ}$	M1 A1ft A1 (3) [12]

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5.	(a) $ \begin{array}{c} N & B \\ 2a \\ 30^{\circ} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	
	$M(A) \qquad N \times 4a \cos 30^{\circ} = 3mg \times a \sin 30^{\circ} + mg \times 2a \sin 30^{\circ}$ $N = \frac{5}{4} mg \tan 30^{\circ} \ (= \frac{5}{4\sqrt{3}} mg = 7.07m)$ $\rightarrow F_r = N  ,  \uparrow  R = 4mg$ $U \sin g  F_r = \mu R$ $\frac{5}{4\sqrt{3}} mg = \mu R  \text{for their } R$	M1 A2(1,0)  DM1 A1  B1, B1  B1
	$\mu = \frac{5}{16\sqrt{3}}$ awrt 0.18	A1 (10)
	Alternative method: M(B): $mg \times 2a \sin 30 + 3mg \times 3a \sin 30 + F \times 4a \cos 30 = R \times 4a \sin 30$ $11mga \sin 30 + F \times 4a \cos 30 = R \times 4a \sin 30$ $\frac{11mg}{2} + F \frac{4\sqrt{3}}{2} = 2R$ $\uparrow R = 4mg,$ Using $F_r = \mu R$ $8\mu\sqrt{3} = \frac{5}{2}, \qquad \mu = \frac{5}{16\sqrt{3}}$	[10] M1A3(2,1,0) DM1A1 B1 B1 H1

6.	(a) $\rightarrow 30 = 2ut$ $\uparrow -47.5 = 5ut - 4.9t^2$ $-47.5 = 75 - 4.9t^2$ eliminating $u$ or $t$ $t^2 = \frac{75 + 47.5}{4.9} (= 25)$ $t = 5 *$ cso	B1 M1 A1 DM1 DM1 A1 (6)
	(b) $30 = 2ut \implies 30 = 10u \implies u = 3$	M1 A1 (2)
	(c) $ \uparrow \qquad \dot{y} = 5u - 9.8t = -34 \qquad \text{M1 requires both} $ $ \rightarrow \qquad \dot{x} = 2u = 6 \qquad \qquad \dot{x} \text{ and } \dot{y} $ $ v^2 = 6^2 + (-34)^2 \qquad \qquad v \approx 34.5  (\text{m s}^{-1}) \qquad \text{accept } 35 $	M1 A1 A1 DM1 A1 (5)
	Alternative to (c) $\frac{1}{2}mv_{B}^{2} - \frac{1}{2}mv_{A}^{2} = m \times g \times 47.5 \text{ with } v_{A}^{2} = 6^{2} + 15^{2} = 261$ $v_{B}^{2} = 261 + 2 \times 9.8 \times 47.5 \text{ (= 1192)}$ $v_{B} \approx 34.5 \text{ (m s}^{-1}\text{)} \text{ accept 35}$ BEWARE: Watch out for incorrect use of $v^{2} = u^{2} + 2as$	[13] M1 A(2,1,0)  DM1 A1 (5)

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7.	$ \begin{array}{cccc} 2u & u \\ \hline 2m & 3m \\ \hline x & y \end{array} $	
	LM $4mu + 3mu = 2mx + 3my$ NEL $y - x = \frac{1}{2}u$ Solving to $y = \frac{8}{5}u$ * cso	M1 A1 B1 M1 A1 (5)
	(b) $x = \frac{11}{10}u \qquad \text{or equivalent}$ Energy loss $\frac{1}{2} \times 2m \left( \left( 2u \right)^2 - \left( \frac{11}{10}u \right)^2 \right) + \frac{1}{2} \times 3m \left( u^2 - \left( \frac{8}{5}u \right)^2 \right)$ $= \frac{9}{20}mu^2$	B1 M1 A(2,1,0) A1 (5)
	(c) $\frac{\frac{8}{5}u}{3m}$ $\frac{m}{t}$ $LM \qquad \frac{24}{5}mu = 3ms + mt$ $NEL \qquad t - s = \frac{8}{5}eu$ $Solving to  s = \frac{2}{5}u(3 - e)$	M1 A1 B1 M1 A1
	For a further collision $\frac{11}{10}u > \frac{2}{5}u(3-e)$ $e > \frac{1}{4}$ ignore $e \le 1$	M1 A1 (7) [17]