

## Mark Scheme (Results) Summer 2009

**GCE** 

GCE Mathematics (6679/01)





## June 2009 6679 Mechanics M3 Mark Scheme

Question Number		Scheme		Marks	
Q1	(a)	6 6	Resolving vertically: $2T \cos \theta = W$		M1A2,1,0
		7.5 T	Hooke's Law:	$T = \frac{80 \times 3.5}{4}$ $W = 84$ N	M1A1 A1
	(b)	EPE = $2 \times \frac{80 \times 3.5^2}{2 \times 4}$ , = 245 (or awrt 245) (alternative $\frac{80 \times 7^2}{16}$ = 245)			M1A1ft,A1
Q2	(a)	Object Mass c of m above base Cone $m$ $2h+3h$ Base $3m$ $h$ Marker $4m$ $d$			B1(ratio masses) B1(distances)
	(b)	d=2h	$\frac{r}{d} = \frac{1}{12}$ $6r = h$		A1 M1A1ft
		r			[8]



Question Number	Scheme	Marks
Q3 (a)	$R \sin \theta = mx\omega^{2}$ $R \times \frac{x}{r} = mx \times \frac{3g}{2r}$ $R = \frac{3mg}{2}$ $\frac{3mg}{2} \times \frac{d}{r} = mg$ $d = \frac{2}{3}r$	M1 A1 M1 A1 M1 A1 M1 A1 [8]
Q4 (a)	Volume = $\int_{\frac{1}{4}}^{1} \pi y^{2} dx = \int_{\frac{1}{4}}^{1} \pi \frac{1}{x^{4}} dx$ = $\left[\pi \times \frac{-1}{3x^{3}}\right]_{\frac{1}{4}}^{1}$ = $\pi(\frac{-1}{3} + \frac{64}{3}) = 21\pi$ $21\pi\rho\bar{x} = \rho\int\pi y^{2}x dx = \rho\int\pi \frac{1}{x^{4}}x dx$ $21\pi\bar{x} = \pi\left[\frac{-1}{2x^{2}}\right]_{\frac{1}{4}}^{1}$ $\bar{x} = \frac{1}{21}(\frac{-1}{2} + \frac{16}{2}) = \frac{5}{14}$ or awrt 0.36 $\bar{y} = 0$ by symmetry	M1A1 A1ft A1 M1A1 A1ft A1ft B1 [9]



Question Number	Sch	eme	Marks
Q5 (a)	cos <sup>1</sup> 4 60°	Energy: $(\frac{1}{2}mu^{2} +)mgl(\cos\theta - \frac{1}{4}) = \frac{1}{2}mv^{2}$ Resolving: $T - mg\cos\theta = \frac{mv^{2}}{l}$ Eliminate $v^{2}$ : $T = mg\cos\theta + \frac{1}{l}(2mgl(\cos\theta - \frac{1}{4}))$	M1A1 M1A1 M1
		$T = 3mg\cos\theta - \frac{mg}{2} $	A1
(b)	A d d 3/	$\theta = 60^{\circ} \Rightarrow mv^2 = 2mgl(\frac{1}{2} - \frac{1}{4})$ $\Rightarrow v^2 = \frac{gl}{2}$ vertical motion under gravity:	M1
	16	$0 = (v\cos 30^\circ)^2 - 2gs$ $0 = \frac{gl}{2} \times \frac{3}{4} - 2gs \Rightarrow s = \frac{3l}{16}$	M1 A1
		Distance below A = $\frac{l}{2} - \frac{3l}{16} = \frac{5l}{16}$	M1A1 [11]
Alternative for end of (b) using energy	60° / v cos 60	$\frac{1}{2}mv^{2} - mgl\cos 60 = \frac{1}{2}m(v\cos 60)^{2} - mgd$ $\frac{gl}{4} - \frac{gl}{2} = \frac{gl}{4} \times \frac{1}{4} - gd$ $d = \frac{1 - 4 + 8}{16}l = \frac{5l}{16}$	M1A1 M1 A1



Question Number	Scheme	Marks
Q6 (a)	At max v, driving force = resistance $Driving force = \frac{80}{v}$	B1
	$\Rightarrow \frac{80}{20} = k \times 20^2 \Rightarrow k = \frac{1}{100}$	M1A1
	$F = \text{ma} \implies 100a = \frac{80}{v} - kv^2  (= \frac{8000 - v^3}{100v})$	M1
4.	$\Rightarrow v \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{8000 - v^3}{10000v}$	A1
(b)	$\int_{4}^{8} \frac{10000v^{2}}{8000 - v^{3}} dv = \int_{0}^{D} 1 dx$	M1A1
	$D = \left[ -\frac{10000}{3} \ln \left  8000 - v^3 \right  \right]_4^8$	A1
	$= \left(-\frac{10000}{3} \ln \frac{7488}{7936}\right) = 193.7 \approx 194 \text{m}  \text{(accept 190)}$	M1 A1
(c)	$\frac{dv}{dt} = \frac{8000 - v^3}{10000v} \Rightarrow \int_0^T 1 dt = \int_4^8 \frac{10000v}{8000 - v^3} dv$	M1A1
	$\Rightarrow T \approx \frac{1}{2} \times 2 \times 10000 \times \left\{ \frac{4}{7936} + \frac{2 \times 6}{7784} + \frac{8}{7488} \right\}$ $\Rightarrow T (= 31.1409) \approx 31$	M1 A1
	→ I (- J1.1407) ≈ J1	[14]



	Scheme	Marks
Q7 (a)	mod=16 a=2  mod=12 a=1  A  5m  5m  d 5-d	
(b)	Hooke's law: Equilibrium $\Rightarrow \frac{16(d-2)}{2} = \frac{12(4-d)}{1}$ $\Rightarrow d = 3.2$ so extensions are 1.2m and 0.8m. If the particle is displaced distance $x$ towards $B$ then $m\ddot{x} = \frac{16(1.2+x)}{12(0.8-x)} = \frac{20x}{1}$	M1A1A1 A1 A1 M1A1ft A1ft
(c)	$\Rightarrow \ddot{x} = -40x \text{ or } \ddot{x} = -\frac{20}{m} (\Rightarrow \text{SHM})$ $T = \frac{2\pi}{\sqrt{40}}$ $a = \frac{\sqrt{10}}{\text{their } \omega}$ $x = a \sin \omega t \text{ their } a, \text{ their } \omega$	A1 B1ft B1ft M1
	$\frac{1}{4} = \frac{1}{2}\sin\sqrt{40}t$ $\sqrt{40}t = \frac{\pi}{6} (\Rightarrow t = \frac{\pi}{6\sqrt{40}})$ Proportion $\frac{4t}{T} = \frac{4\pi}{6\sqrt{40}} \times \frac{\sqrt{40}}{2\pi} = \frac{1}{3}$	A1 M1 M1A1 [16]