

Mark Scheme (Results) January 2009

GCE

GCE Mathematics (6684/01)



January 2009 6684 Statistics S2 Mark Scheme

Scheme	Mark	(S
The random variable X is the number of daisies in a square. Poisson(3)	B1	
$1 - P(X \le 2) = 1 - 0.4232 \qquad 1 - e^{-3}(1 + 3 + \frac{3^2}{2!})$ $= 0.5768$	M1 A1	(3)
	M1	
$\mu = 3.69$	B1	(2)
$Var(X) = \frac{1386}{80} - \left(\frac{295}{80}\right)^2$ = 3.73/3.72/3.71 accept s ² = 3.77	M1 A1	
For a Poisson model, Mean = Variance; For these data 3.69≈3.73 ⇒ Poisson model	B1	(3)
$\frac{e^{-3.6875}3.6875^4}{4!} = 0.193$ allow their mean or var	M1	(1)
Awrt 0.193 or 0.194	A1 ft	(2)
	The random variable X is the number of daisies in a square. Poisson(3) $1 - P(X \le 2) = 1 - 0.4232 \qquad 1 - e^{-3}(1 + 3 + \frac{3^2}{2!})$ $= 0.5768$ $P(X \le 6) - P(X \le 4) = 0.9665 - 0.8153 \qquad e^{-3} \left(\frac{3^5}{5!} + \frac{3^6}{6!}\right)$ $= 0.1512$ $\mu = 3.69$ $Var(X) = \frac{1386}{80} - \left(\frac{295}{80}\right)^2$ $= 3.73/3.72/3.71 \qquad accept s^2 = 3.77$ For a Poisson model, Mean = Variance; For these data $3.69 \approx 3.73$ $\Rightarrow Poisson model$ $\frac{e^{-3.6875}3.6875^4}{4!} = 0.193 \qquad allow their mean or var$	The random variable X is the number of daisies in a square. Poisson(3) $1 - P(X \le 2) = 1 - 0.4232 \qquad 1 - e^{-3}(1+3+\frac{3^2}{2!}) \qquad \qquad M1$ $= 0.5768 \qquad \qquad A1$ $P(X \le 6) - P(X \le 4) = 0.9665 - 0.8153 \qquad e^{-3} \left(\frac{3^5}{5!} + \frac{3^6}{6!}\right) \qquad \qquad M1$ $= 0.1512 \qquad \qquad A1$ $\mu = 3.69 \qquad \qquad B1$ $Var(X) = \frac{1386}{80} - \left(\frac{295}{80}\right)^2 \qquad \qquad M1$ $= 3.73/3.72/3.71 \qquad \text{accept s}^2 = 3.77 \qquad A1$ For a Poisson model , Mean = Variance ; For these data $3.69 \approx 3.73$ $\Rightarrow \text{Poisson model} \qquad B1$ $\frac{e^{-3.6875}3.6875^4}{4!} = 0.193 \qquad \text{allow their mean or var}$

Question Number	Scheme	Marks	
2 (a)	$f(x) = \begin{cases} \frac{1}{9} & -2 \le x \le 7\\ 0 & otherwise \end{cases}$	B1 B1 (2	2)
(b)	1/9	B1 B1	
(c)	$E(X) = 2.5 \text{Var } (X) = \frac{1}{12} (7+2)^2 \text{ or } \underline{6.75} $ both	(2 B1	2)
	$E(X^2) = Var(X) + E(X)^2$	M1	
	$= 6.75 + 2.5^{2}$ = 13 alternative	A1 (3	3)
	$\int_{-2}^{7} x^2 f(x) dx = \left[\frac{x^3}{27} \right]_{-2}^{7}$ attempt to integrate and use limits of -2 and 7 $= 13$	B1 M1	
(d)	$P(-0.2 < X < 0.6) = \frac{1}{9} \times 0.8$	M1	
	$=\frac{4}{45}$ or 0.0889 Or equiv awrt 0.089	A1	
		(2	2)

	estion mber	Scheme	Mark	(S
3	(a)	$X \sim B(20, 0.3)$	M1	
		$P(X \le 2) = 0.0355$		
		$P(X \ge 11) = 1 - 0.9829 = 0.0171$		
		Critical region is $(X \le 2) \cup (X \ge 11)$	A1 A1	(3)
	(b)	Significance level = $0.0355 + 0.0171$, = 0.0526 or 5.26%	M1 A1	(2)
	(c)	Insufficient evidence to reject H_0 Or sufficient evidence to accept H_0 /not significant	B1 ft	
		x = 3 (or the value) is not in the critical region or 0.1071> 0.025	B1 ft	(2)
		Do not allow inconsistent comments		

Question Number	Scheme	Marks
Number 4 (a) (b)	Scheme $\int_{0}^{10} kt dt = 1 \qquad \text{or Area of triangle} = 1$ $\left[\frac{kt^{2}}{2}\right]_{0}^{10} = 1 \qquad \text{or } 10 \times 0.5 \times 10 \text{k} = 1 \text{ or linear equation in k}$ $50k = 1 \qquad \text{cso}$ $\int_{6}^{10} kt dt = \left[\frac{kt^{2}}{2}\right]_{0}^{10} = \frac{16}{25}$ $E(T) = \int_{0}^{10} kt^{2} dt = \left[\frac{kt^{3}}{3}\right]_{0}^{10} = 6\frac{2}{3}$ $Var(T) = \int_{0}^{10} kt^{3} dt - \left(6\frac{2}{3}\right)^{2} = \left[\frac{kt^{4}}{4}\right]_{0}^{10}; -\left(6\frac{2}{3}\right)^{2}$ $= 50 - \left(6\frac{2}{3}\right)^{2}$ $= 5\frac{5}{9}$ 10	Marks M1 M1 A1 (3) M1 A1 (2) M1 A1 M1;M1dep A1 (5) B1 (1) B1 (1)

Question Number	Scheme	Mark	S
5 (a)	X represents the number of defective components.		
	$P(X=1) = (0.99)^9 (0.01) \times 10 = 0.0914$	M1A1	4-5
(b)	$P(X \ge 2) = 1 - P(X \le 1)$ $= 1 - (p)^{10} - (a)$ $= 0.0043$	M1 A1√ A1	(3)
(c)	$X \sim \text{Po}(2.5)$	B1B1	
	$P(1 \le X \le 4) = P(X \le 4) - P(X = 0)$ $= 0.8912 - 0.0821$	M1	
	= 0.809	A1	
			(4)
	Normal distribution used. B1for mean only		
	Special case for parts a and b If they use 0.1 do not treat as misread as it makes it easier. (a) M1 A0 if they have 0.3874 (b) M1 A1ft A0 they will get 0.2639 (c) Could get B1 B0 M1 A0 For any other values of p which are in the table do not use misread. Check using the tables. They could get (a) M1 A0 (b) M1 A1ft A0 (c) B1 B0 M1 A0		

Question Number	Scheme	Marks
6 (a)(i)	$H_0: \lambda = 7$ $H_1: \lambda > 7$	B1
	$X =$ number of visits. $X \sim Po(7)$	B1
	$P(X \ge 10) = 1 - P(X \le 9) = 0.1695 1 - P(X \le 10) = 0.0985 1 - P(X \le 9) = 0.1695$	M1
	$\begin{array}{c} -0.1093 & 1-1(X \le 9) - 0.1093 \\ \text{CR } X \ge 11 \end{array}$	A1
	$0.1695 > 0.10$, $CR X \ge 11$ Not significant or it is not in the critical region or do not reject H_0 The rate of visits on a Saturday is not greater/ is unchanged	M1 A1 no ft
(ii)	X = 11	B1 (7)
(b)	(The visits occur) randomly/ independently or singly or constant rate	(7) B1 (1)
(c)	$[H_0: \lambda = 7$ $H_1: \lambda > 7$ (or $H_0: \lambda = 14$ $H_1: \lambda > 14)]$	
	X~N;(14,14)	B1;B1
	$P(X \ge 20) = P\left(z \ge \frac{19.5 - 14}{\sqrt{14}}\right) + /-0.5, \text{ stand}$ $= P(z \ge 1.47)$ $= 0.0708 \text{or } z = 1.2816$	M1 M1 A1dep both
	0.0708 < 0.10 therefore significant. The rate of visits is greater on a Saturday	A1dep 2 nd M (6)

Question Number	Scheme	Mark	S
7 (a)	$F(x_0) = \int_1^x -\frac{2}{9}x + \frac{8}{9} dx = \left[-\frac{1}{9}x^2 + \frac{8}{9}x \right]_1^x$	M1A1	
	$= \left[-\frac{1}{9}x^2 + \frac{8}{9}x \right] - \left[-\frac{1}{9} + \frac{8}{9} \right]$ $= -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9}$	A1	(3)
(b)	$F(x) = \begin{cases} 0 & x < 1 \\ -\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} & 1 \le x \le 4 \\ 1 & x > 4 \end{cases}$	B1B1√	
	F(x) = 0.75; or F(2.5) = $-\frac{1}{9} \times 2.5^2 + \frac{8}{9} \times 2.5 - \frac{7}{9}$	M1;	(2)
	$-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.75$		
	$4x^{2} - 32^{x} + 55 = 0$ $-x^{2} + 8x - 13.75 = 0$ $x = 2.5$ $= 0.75$ cso	A1	
	and $F(x) = 0.25$ $-\frac{1}{9}x^2 + \frac{8}{9}x - \frac{7}{9} = 0.25$	M1	
	$-x^{2} + 8x - 7 = 2.25$ $-x^{2} + 8x - 9.25 = 0$ $x = \frac{-8 \pm \sqrt{8^{2} - 4 \times -1 \times -9.25}}{2 \times -1}$ quadratic 3 terms =0	M1 dep M1 dep	
(4)	x = 1.40	M1	(6)
(d)	$Q_3 - Q_2 > Q_2 - Q_1$ Or mode = 1 and mode < median Or mean = 2 and median < mode Sketch of pdf here or be referred to if in a different part of the question Box plot with Q_1 , Q_2 , Q_3 values marked on	IVII	
	Positive skew	A1	(2)