Please check the examination details be	w before entering your ca	ndidate information
Candidate surname	Other name	es
Centre Number Candidate N	mber	
Pearson Edexcel Inter	national Adv	vanced Level
Time 1 hour 30 minutes	Paper reference W	M02/01
Mathematics		0 0
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You must have:	l Tables (Vallaus) sala	Total Marks
Mathematical Formulae and Statistic	i Tables (Yellow), calc	ulator

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## **Instructions**

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  - there may be more space than you need.
- You should show sufficient working to make your methods clear.
   Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
  - use this as a guide as to how much time to spend on each question.

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶







1. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Express the complex number

$$-4 - 4\sqrt{3}i$$

in the form  $r(\cos \theta + i \sin \theta)$ , where r > 0 and  $-\pi < \theta \leqslant \pi$ 

**(3)** 

(b) Solve the equation

$$z^3 + 4 + 4\sqrt{3}i = 0$$

giving your answers in the form  $re^{i\theta}$ , where r > 0 and  $-\pi < \theta \leqslant \pi$ 

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**3.** 

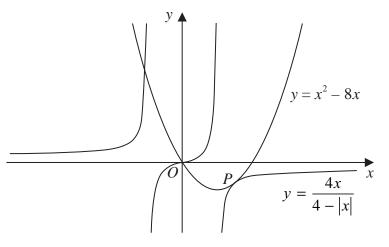


Figure 1

Figure 1 shows a sketch of the curve  $C_1$  with equation

$$y = \frac{4x}{4 - |x|}$$

and the curve  $C_2$  with equation

$$y = x^2 - 8x$$

For x > 0,  $C_1$  has equation  $y = \frac{4x}{4 - x}$ 

- (a) Use algebra to show that  $C_1$  touches  $C_2$  at a point P, stating the coordinates of P (5)
- (b) Hence or otherwise, using algebra, solve the inequality

$$x^2 - 8x > \frac{4x}{4 - |x|} \tag{6}$$

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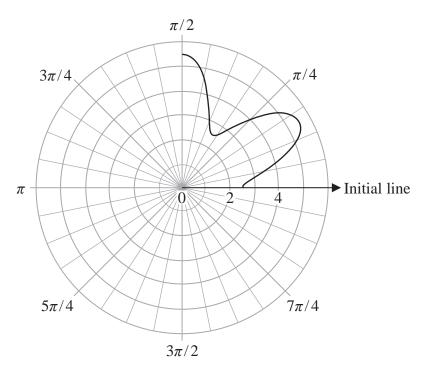


Figure 2

Figure 2 shows part of the curve with polar equation

$$r = 4 - \frac{3}{2}\cos 6\theta \qquad 0 \leqslant \theta < 2\pi$$

- (a) Sketch, on the polar grid in Figure 2,
  - (i) the rest of the curve with equation  $r = 4 \frac{3}{2}\cos 6\theta$   $0 \le \theta < 2\pi$
  - (ii) the polar curve with equation

$$r = 1$$

$$0 \leqslant \theta < 2\pi$$

A spare copy of the grid is given on page 15.

(3)

**(7)** 

In part (b) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(b) Determine the exact area enclosed between the two curves defined in part (a).

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5.

$$y = \sqrt{4 + \ln x} \qquad x > \frac{1}{2}$$

(a) Show that

$$\frac{d^2y}{dx^2} = -\frac{9 + 2\ln x}{4x^2(4 + \ln x)^{\frac{3}{2}}}$$

(b) Hence, or otherwise, determine the Taylor series expansion about x = 1 for y, in ascending powers of (x - 1), up to and including the term in  $(x - 1)^2$ , giving each

coefficient in simplest form. (3)




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**6.** Given that A > B > 0, by letting  $x = \arctan A$  and  $y = \arctan B$ 

(a) prove that

$$\arctan A - \arctan B = \arctan\left(\frac{A - B}{1 + AB}\right)$$

(b) Show that when A = r + 2 and B = r

$$\frac{A-B}{1+AB} = \frac{2}{(1+r)^2} \tag{2}$$

(c) Hence, using the method of differences, show that

$$\sum_{r=1}^{n} \arctan\left(\frac{2}{(1+r)^2}\right) = \arctan(n+p) + \arctan(n+q) - \arctan 2 - \frac{2}{4}$$

where p and q are integers to be determined.

**(4)** 

**(3)** 

(d) Hence, making your reasoning clear, determine

$$\sum_{r=1}^{\infty} \arctan\left(\frac{2}{(1+r)^2}\right)$$

giving the answer in the form  $k\pi$  – arctan 2, where k is a constant.

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7. A transformation from the z-plane to the w-plane is given by

$$w = \frac{(1+i)z + 2(1-i)}{z-i} \qquad z \neq i$$

The transformation maps points on the imaginary axis in the z-plane onto a line in the w-plane.

(a) Find an equation for this line.

**(2)** 

The transformation maps points on the real axis in the *z*-plane onto a circle in the *w*-plane.

(b) Find the centre a	ınd radius	of this	circle
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**(6)** 


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**8.** (a) Show that the transformation v = y - 2x transforms the differential equation

$$\frac{dy}{dx} + 2yx(y - 4x) = 2 - 8x^3$$
 (I)

into the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}x} = -2xv^2 \tag{II}$$

- (b) Solve the differential equation (II) to determine v as a function of x (4)
- (c) Hence obtain the general solution of the differential equation (I). (1)
- (d) Sketch the solution curve that passes through the point (-1, -1).

On your sketch show clearly the equation of any horizontal or vertical asymptotes.

You do **not** need to find the coordinates of any intercepts with the coordinate axes or the coordinates of any stationary points.

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