Surname	Other na	imes
Pearson Edexcel International Advanced Level	Centre Number	Candidate Number
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Advanced/Advance		
	d Subsidiary	Paper Reference WFM01/01

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided - there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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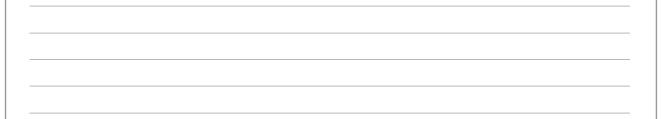


(4)

1. Use the standard results for $\sum_{r=1}^{n} r$ and for $\sum_{r=1}^{n} r^3$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r^2 - 3) = \frac{n}{4}(n+a)(n+b)(n+c)$$

where a , b	and	c are	integers	to	be	found.
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	Q1
(Total 4 marks)	



2.	A parabola P has cartesian equation $y^2 = 28x$. The point S is the focus of the parabola P.
	(a) Write down the coordinates of the point <i>S</i> . (1)
	Points A and B lie on the parabola P . The line AB is parallel to the directrix of P and cuts the x -axis at the midpoint of OS , where O is the origin.
	(b) Find the exact area of triangle <i>ABS</i> . (4)
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3.

$$f(x) = x^2 + \frac{3}{x} - 1, \quad x < 0$$

The only real root, α , of the equation f(x) = 0 lies in the interval [-2, -1].

(a) Taking -1.5 as a first approximation to α , apply the Newton-Raphson procedure once to f(x) to find a second approximation to α , giving your answer to 2 decimal places.

(5)

(b) Show that your answer to part (a) gives α correct to 2 decimal places.

(2)

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4. Given that

$$\mathbf{A} = \begin{pmatrix} k & 3 \\ -1 & k+2 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

(a) show that $det(\mathbf{A}) > 0$ for all real values of k,

(3)

(b) find A^{-1} in terms of k.

(2)

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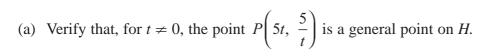
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6. The rectangular hyperbola H has equation xy = 25



(1)

The point *A* on *H* has parameter $t = \frac{1}{2}$

(b) Show that the normal to H at the point A has equation

$$8y - 2x - 75 = 0$$

(5)

This normal at A meets H again at the point B.

(c) Find the coordinates of B.

(4)

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7.

$$\mathbf{P} = \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation U represented by the matrix P.

The transformation V, represented by the 2×2 matrix \mathbf{Q} , is a reflection in the line with equation y = x

(b) Write down the matrix \mathbf{Q} .

(1)

Given that the transformation V followed by the transformation U is the transformation T, which is represented by the matrix \mathbf{R} ,

(c) find the matrix \mathbf{R} .

(2)

(d) Show that there is a value of k for which the transformation T maps each point on the straight line y = kx onto itself, and state the value of k.

(4)

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8.

$$f(z) = z^4 + 6z^3 + 76z^2 + az + b$$

where a and b are real constants.

Given that -3 + 8i is a complex root of the equation f(z) = 0

(a) write down another complex root of this equation.

- (1)
- (b) Hence, or otherwise, find the other roots of the equation f(z) = 0
- **(6)**
- (c) Show on a single Argand diagram all four roots of the equation f(z) = 0
- **(2)**



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9. The quadratic equation

$$2x^2 + 4x - 3 = 0$$

has roots α and β .

Without solving the quadratic equation,

- (a) find the exact value of
 - (i) $\alpha^2 + \beta^2$
 - (ii) $\alpha^3 + \beta^3$

(5)

(b) Find a quadratic equation which has roots $(\alpha^2 + \beta)$ and $(\beta^2 + \alpha)$, giving your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers.

(4)



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(6)

10. (i) A sequence of positive numbers is defined by

$$u_1 = 5$$

 $u_{n+1} = 3u_n + 2, \quad n \ge 1$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = 2 \times (3)^n - 1$$
 (5)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} \frac{4r}{3^r} = 3 - \frac{(3+2n)}{3^n}$$

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