Please check the examination details be	elow before entering your candidate information
Candidate surname	Other names
Centre Number Candidate N	lumber
Pearson Edexcel Inter	rnational Advanced Level
Time 1 hour 30 minutes	Paper reference WFM03/01
Mathematics	
International Advanced S	ubsidiary/Advanced Level
	,
Further Pure Mathematics	s F3
You must have:	Total Marks
Mathematical Formulae and Statistic	- 11
	` "

Candidates may use any calculator permitted by Pearson regulations.

Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for each question are shown in brackets
 - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶







1. The curve C has equation

$$y = \frac{1}{2} \operatorname{arcosh}(2x) \qquad \qquad \frac{7}{2} \leqslant x \leqslant 13$$

Using calculus, determine the exact length of the curve C.

Give your answer in the form $p\sqrt{q}$, where p and q are constants to be found.

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2. Given that

$$\cosh y = x \quad \text{and} \quad y < 0$$

use the definition of $\cosh y$ in terms of exponential functions to prove that

$$y = \ln\left(x - \sqrt{x^2 - 1}\right)$$

 $y = \ln\left(x - \sqrt{x^2 - 1}\right) \tag{6}$

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3. The ellipse E has equation

$$\frac{x^2}{64} + \frac{y^2}{36} = 1$$

The line l is the normal to E at the point $P(8\cos\theta, 6\sin\theta)$.

(a) Using calculus, show that an equation for l is

$$4x\sin\theta - 3y\cos\theta = 14\sin\theta\cos\theta$$

(4)

The line l meets the x-axis at the point A and meets the y-axis at the point B.

The point M is the midpoint of AB.

(b) Determine a Cartesian equation for the locus of M as θ varies, giving your answer in the form $ax^2 + by^2 = c$ where a, b and c are integers.

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4. The matrix M is given by

$$\begin{pmatrix} 2 & 0 & -1 \\ k & 3 & 2 \\ -2 & 1 & k \end{pmatrix}$$

(a) Show that $\det \mathbf{M} = 5k - 10$

(2)

Given that $k \neq 2$

(b) find \mathbf{M}^{-1} in terms of k.

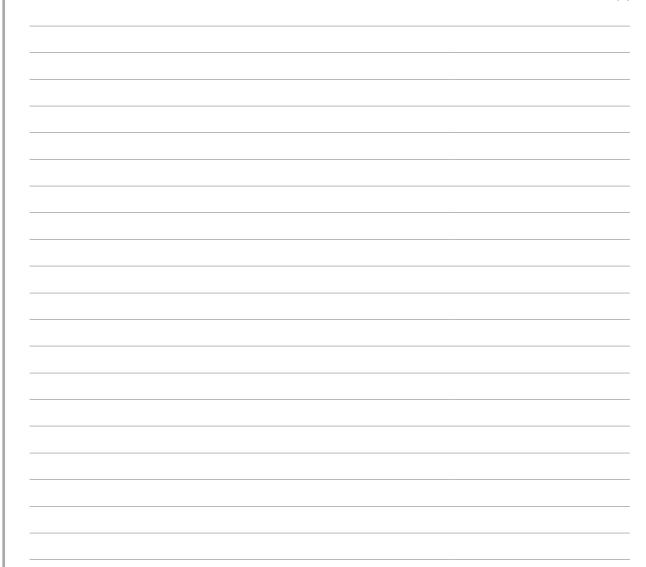
(4)

The points O(0, 0, 0), A(4, -8, 3), B(-2, 5, -4) and C(4, -6, 8) are the vertices of a tetrahedron T.

The transformation represented by matrix M transforms T to a tetrahedron with volume 50

(c) Determine the possible values of k.

(5)





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5. The skew lines  $l_1$  and  $l_2$  have equations

$$l_1: \mathbf{r} = (\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}) + \lambda(5\mathbf{i} + \mathbf{j})$$

and

$$l_2$$
:  $\mathbf{r} = (2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}) + \mu(8\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ 

where  $\lambda$  and  $\mu$  are scalar parameters.

(a) Determine a vector that is perpendicular to both  $\boldsymbol{l_1}$  and  $\boldsymbol{l_2}$ 

(2)

- (b) Determine an equation of the plane parallel to  $\boldsymbol{l_1}$  that contains  $\boldsymbol{l_2}$ 
  - (i) in the form  $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

**(1)** 

(ii) in the form  $\mathbf{r}.\mathbf{n} = p$ 

**(2)** 

(c) Determine the shortest distance between  $l_1$  and  $l_2$  Give your answer in simplest form.

**(5)** 

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**6.** 

$$I_n = \int_0^{\sqrt{\frac{\pi}{2}}} x^n \cos(x^2) dx \qquad n \geqslant 1$$

(a) Prove that, for  $n \ge 5$ 

$$I_n = \frac{1}{2} \left(\frac{1}{2}\right)^{\frac{n-1}{2}} - \frac{1}{4} (n-1)(n-3) I_{n-4}$$

**(6)** 

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**7.** A hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{25} = 1$$

where a is a positive constant.

The eccentricity of H is e.

(a) Determine an expression for  $e^2$  in terms of a.

(1)

The line l is the directrix of H for which x > 0

The points A and A' are the points of intersection of l with the asymptotes of H.

(b) Determine, in terms of e, the length of the line segment AA'.

**(3)** 

The point *F* is the focus of *H* for which x < 0

Given that the area of triangle AFA' is  $\frac{164}{3}$ 

(c) show that a is a solution of the equation

$$30a^3 - 164a^2 + 375a - 4100 = 0$$
 (4)

(d) Hence, using algebra and making your reasoning clear, show that the only possible value of a is  $\frac{20}{3}$ 





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8.

$$y = \arccos(2\sqrt{x})$$

(a) Determine $\frac{dy}{dx}$

(3)

(b) Show that

$$\int y \, dx = x \arccos\left(2\sqrt{x}\right) + \int \frac{\sqrt{x}}{\sqrt{1-4x}} \, dx$$

(2)

(c) Use the substitution $\sqrt{x} = \frac{1}{2}\cos\theta$ to show that

$$\int_0^{\frac{1}{8}} \frac{\sqrt{x}}{\sqrt{1-4x}} \, \mathrm{d}x = \frac{1}{4} \int_a^b \cos^2 \theta \, \mathrm{d}\theta$$

where a and b are limits to be determined.

(4)

(d) Hence, determine the exact value of

$$\int_0^{\frac{1}{8}} \arccos\left(2\sqrt{x}\right) \mathrm{d}x$$

(4)



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