Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | Candidate Number

Monday 8 June 2020

Morning (Time: 1 hour 30 minutes) | Paper Reference **WFM03/01**

Mathematics

**International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3**

You must have:
Mathematical Formulae and Statistical Tables (Blue), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations.
Calculators must not have the facility for symbolic algebra manipulation,
differentiation and integration, or have retrievable mathematical
formulae stored in them.**

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name,
  centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are
  clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers
  without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise
  stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer
  and any working underneath.

Turn over ▶

Pearson

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1. (a) Use the definition of sinh x in terms of exponentials to show that

$$\sinh 3x \equiv 4\sinh^3 x + 3\sinh x$$

(2)

(b) Hence determine the exact coordinates of the points of intersection of the curve with equation $y = \sinh 3x$ and the curve with equation $y = 19\sinh x$, giving your answers as simplified logarithms where necessary.

(5)

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Question 1 continued

Q1

(Total 7 marks)

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2. Determine

(i) $\displaystyle\int \frac{1}{3x^2 + 12x + 24}\, \mathrm{d}x$

(4)

(ii) $\displaystyle\int \frac{1}{\sqrt{27 - 6x - x^2}}\, \mathrm{d}x$

(4)

Question 2 continued

Question 2 continued

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Question 2 continued

Q2

(Total 8 marks)

7

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3.

$$\mathbf{M} = \begin{pmatrix} 3 & -4 & k \\ 1 & -2 & k \\ 1 & -5 & 5 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

Given that 3 is an eigenvalue of $\mathbf{M}$,

(a) find the value of k.

(3)

(b) Hence find the other two eigenvalues of $\mathbf{M}$.

(3)

(c) Find a normalised eigenvector corresponding to the eigenvalue 3

(3)

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

(Total 9 marks)

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4.
$$I_n = \int x^n \cos x \, dx$$

(a) Show that, for $n \geqslant 2$

$$I_n = x^n \sin x + nx^{n-1} \cos x - n(n-1)I_{n-2}$$

(4)

(b) Hence find the functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ such that

$$\int x^4 \cos x \, dx = \mathrm{f}(x)\sin x + \mathrm{g}(x)\cos x + c$$

where c is an arbitrary constant.

(5)

Question 4 continued

Question 4 continued

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Question 4 continued

(Total 9 marks)

Q4

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5. The hyperbola H has equation $\dfrac{x^2}{25} - \dfrac{y^2}{4} = 1$

The line l has equation $y = mx + c$, where m and c are constants.

Given that l is a tangent to H,

(a) show that $25m^2 = 4 + c^2$

(4)

(b) Hence find the equations of the tangents to H that pass through the point (1, 2).

(5)

(c) Find the coordinates of the point of contact each of these tangents makes with H.

(3)

Question 5 continued

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Question 5 continued

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Question 5 continued

(Total 12 marks)

Q5

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6.

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & a \end{pmatrix} \quad a \neq 1$$

(a) Find $\mathbf{A}^{-1}$ in terms of a.

(4)

$$\mathbf{B} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}$$

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix $\mathbf{B}$.

The equation of l_2 is

$$\left(\mathbf{r} - (12\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})\right) \times (-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = \mathbf{0}$$

(b) Find a vector equation for the line l_1

(4)

Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total 8 marks)

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7. The curve C has parametric equations

$$x = \cosh t + t, \quad y = \cosh t - t \quad 0 \leqslant t \leqslant \ln 3$$

(a) Show that

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 2\cosh^2 t$$

(3)

The curve C is rotated through 2π radians about the x-axis. The area of the curved surface generated is given by S.

(b) Show that

$$S = 2\sqrt{2} \int_0^{\ln 3} \left(\cosh^2 t - t\cosh t\right) dt$$

(2)

(c) Hence find the value of S, giving your answer in the form

$$\frac{\pi\sqrt{2}}{9}\left(a + b\ln 3\right)$$

where a and b are constants to be determined.

(7)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 12 marks)

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8. The plane Π_1 has equation

$$x - 5y + 3z = 11$$

The plane Π_2 has equation

$$3x - 2y + 2z = 7$$

The planes Π_1 and Π_2 intersect in the line l.

(a) Find a vector equation for l, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors and λ is a scalar parameter.

(5)

The point $P(2, 0, 3)$ lies on Π_1

The line m, which passes through P, is parallel to l.

The point $Q(3, 2, 1)$ lies on Π_2

The line n, which passes through Q, is also parallel to l.

(b) Find, in exact simplified form, the shortest distance between m and n.

(5)

Question 8 continued

Question 8 continued

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Question 8 continued

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Question 8 continued

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Q8

(Total 10 marks)

TOTAL FOR PAPER: 75 MARKS

END

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