Surname	Other n	ames
Pearson Edexcel GCE	Centre Number	Candidate Number
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Mathema Advanced/Advan	atics FP3	
Mathem	atics FP3 ced Subsidiary Afternoon	Paper Reference 6669/01

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets
 use this as a quide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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2. The ellipse E has equation

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

The line *l* is the normal to *E* at the point *P* (6 cos θ , 5 sin θ), where $0 < \theta < \frac{\pi}{2}$

(a) Use calculus to show that an equation of l is

$$6x\sin\theta - 5y\cos\theta = 11\sin\theta\cos\theta$$

(5)

The line l meets the x-axis at the point Q.

The point R is the foot of the perpendicular from P to the x-axis.

(b) Show that $\frac{OQ}{OR} = e^2$, where e is the eccentricity of the ellipse E.

(4)

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3.	(a) Using the definition for $\cosh x$ in terms of exponentials, show that	
	$\cosh 2x \equiv 2\cosh^2 x - 1$	
		(3)
	(b) Find the exact values of x for which	
	$29\cosh x - 3\cosh 2x = 38$	
	giving your answers in terms of natural logarithms.	(6)

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Use the substitution $x + 2 = u^2$, where u > 0, to show that

$$\int_{-1}^{7} \frac{(x+2)^{\frac{1}{2}}}{x+5} \, \mathrm{d}x = a + b\pi\sqrt{3}$$

where a and b are rational numbers to be found.	(9)



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5.	The plane Π_1 has equation $x - 2y - 3z = 5$ and the plane Π_2 has equation $6x + y - 4z = 7$
	(a) Find, to the nearest degree, the acute angle between Π_1 and Π_2
	(3)
	The point <i>P</i> has coordinates $(2, 3, -1)$. The line <i>l</i> is perpendicular to Π_1 and passes through the point <i>P</i> . The line <i>l</i> intersects Π_2 at the point <i>Q</i> .
	(b) Find the coordinates of Q .
	(4)
	The plane Π_3 passes through the point Q and is perpendicular to Π_1 and Π_2
	(c) Find an equation of the plane Π_3 in the form $\mathbf{r} \cdot \mathbf{n} = p$ (4)



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6. The matrix \mathbf{M} is given by

$$\mathbf{M} = \begin{pmatrix} 1 & k & 0 \\ 2 & -2 & 1 \\ -4 & 1 & -1 \end{pmatrix}, k \in \mathbb{R}, k \neq \frac{1}{2}$$

(a) Show that det $\mathbf{M} = 1 - 2k$.

(2)

(b) Find \mathbf{M}^{-1} in terms of k.

(4)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix

$$\begin{pmatrix}
1 & 0 & 0 \\
2 & -2 & 1 \\
-4 & 1 & -1
\end{pmatrix}$$

Given that l_2 has cartesian equation

$$\frac{x-1}{5} = \frac{y+2}{2} = \frac{z-3}{1}$$

(c) find a cartesian equation of the line l_1

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7.

$$I_n = \int_0^{\ln 2} \cosh^n x \, dx, \quad n \geqslant 0$$

(a) Show that, for $n \ge 2$,

$$I_n = \frac{3a^{n-1}}{nb^n} + \frac{n-1}{n}I_{n-2}$$

where a and b are integers to be found.

(6)

(b) Hence, or otherwise, find the exact value of

$$\int_0^{\ln 2} \cosh^4 x \, \mathrm{d}x$$

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8. The curve C has equation

$$y = \ln\left(\frac{e^x + 1}{e^x - 1}\right), \quad \ln 2 \leqslant x \leqslant \ln 3$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2\mathrm{e}^x}{\mathrm{e}^{2x} - 1}$$

(b) Find the length of the curve C, giving your answer in the form $\ln a$, where a is a rational number.

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