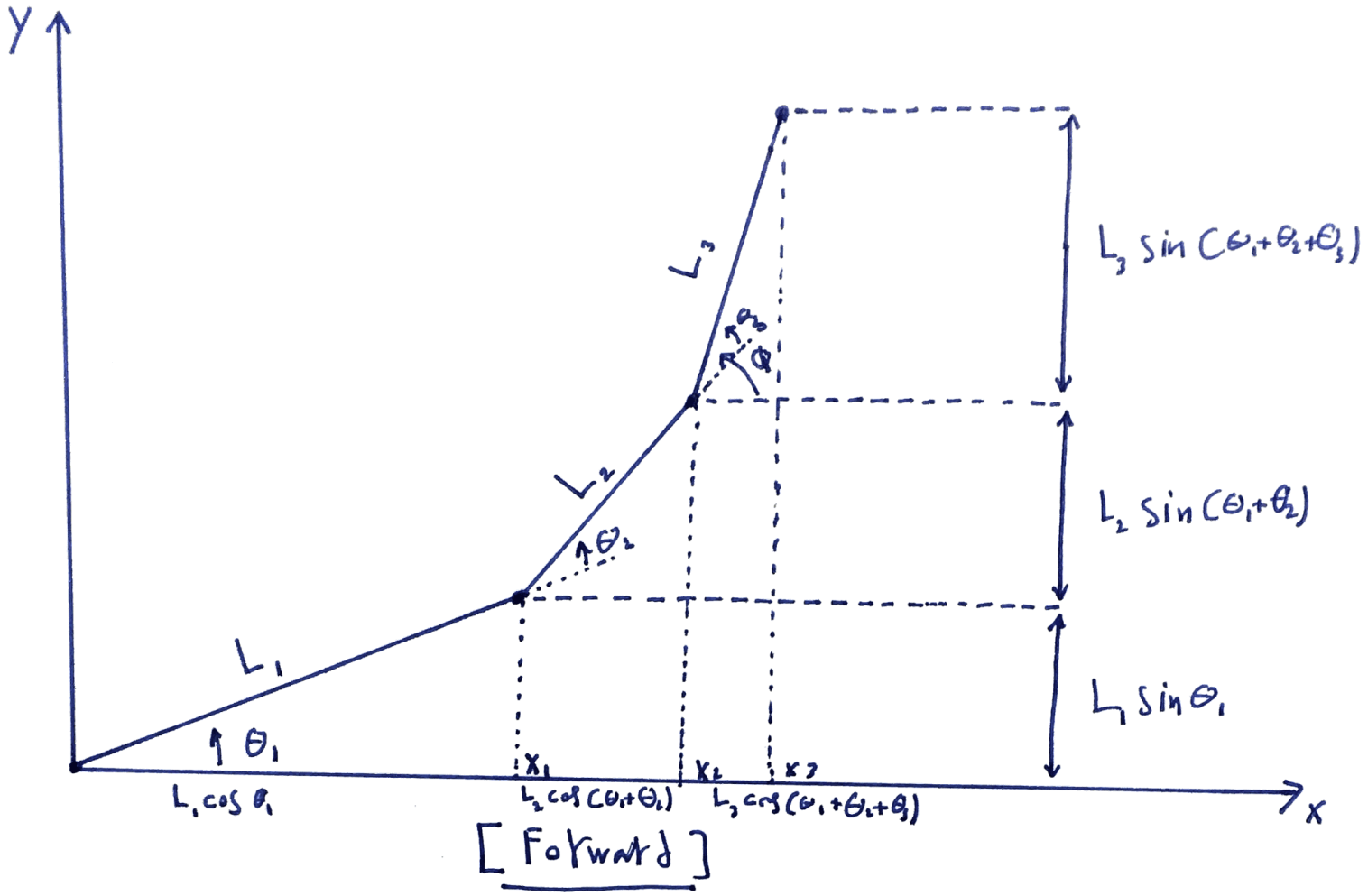
3-DoF Forward and Inverse Kinematics



[Forward]

$$x_1 = L_1\cos\theta_1$$

$$x_2 = x_1 + L_2\cos(\theta_1+\theta_2) = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)$$

$$x_3 = x_1 + x_2 + L_3\cos(\theta_1+\theta_2+\theta_3) = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) + L_3\cos(\theta_1+\theta_2+\theta_3)$$

$$y_1 = L_1\sin\theta_1$$

$$y_2 = y_1 + L_2\sin(\theta_1+\theta_2) = L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2)$$

$$y_3 = y_1 + y_2 + L_3\sin(\theta_1+\theta_2+\theta_3) = L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) + L_3\sin(\theta_1+\theta_2+\theta_3)$$

[Inverse]

$$x_2 = x_3 - L_3\cos(\theta_1+\theta_2+\theta_3)$$

$$y_2 = y_3 - L_3\sin(\theta_1+\theta_2+\theta_3)$$

$$\cos\theta_2 = \frac{x_2^2 + y_2^2 - (L_1^2+L_2^2)}{2L_1L_2}, \quad \cos\theta_1 = \frac{x_2[L_1+L_2\cos\theta_2] + y_2[L_2\sin\theta_2]}{x_1^2+y_2^2}$$

$$\theta_3 = \phi - (\theta_1+\theta_2)$$