Task 1:

FD1:
$$\{A\} \rightarrow \{B,C\}$$
 FD2: $\{C\} \rightarrow \{A,D\}$ FD3: $\{D,E\} \rightarrow \{F\}$

- a) Decomposition of FD1: A → B, A → C
 Decomposition of FD2: C → A, C → D
 Transitivity of C → A and A → B, then C → B
- b) Decomposition of FD1: A → B, A → C
 Decomposition of FD2: C → A, C → D
 Transitivity of A → C and C → D, then AE → F

Task 2:

a)
$$X = \{A\}$$

 $X^+ = \{A, B, C, D, F\}$

b)
$$X = \{ C, E \}$$

 $X^+ = \{ A, B, C, D, E, F \}$

Task 3:

FD1:
$$\{A,B\} \rightarrow \{C,D,E,F\}$$
 FD2: $\{E\} \rightarrow \{F\}$ FD3: $\{D\} \rightarrow \{B\}$

- a) $\{A,B\}$ because $\{A,B\} \rightarrow \{A,B,C,D,E,F\}$, which means every attribute in R.
- b) FD2, and FD3.
- c) FD2 => R1{A,B,C,D,E} with FD1 and FD3,R2{E,F} with FD2 R1 is still not in BCNF because of FD3, while R2 is in BCNF.

 $FD3 \Rightarrow R1A\{A,C,D,E\}$ with FD1, $R1B\{D,B\}$ with FD3, $R2\{E,F\}$ with FD2

All of them are in BCNF now.

Task 4:

FD1: $\{A,B,C\} \to \{D,E\} \text{ FD2: } \{B,C,D\} \to \{A,E\} \text{ FD3: } \{C\} \to \{D\}$

- a) Decomposition of FD1: $A \rightarrow D$, $A \rightarrow E$, $B \rightarrow D$, $B \rightarrow E$, $C \rightarrow D$, $C \rightarrow E$ Decomposition of FD2: $B \rightarrow A$, $B \rightarrow E$, $C \rightarrow A$, $C \rightarrow E$, $D \rightarrow A$, $D \rightarrow E$ $X = \{A,B,C\} => X^+ = \{A,B,C,D,E\} => \text{super key}$ $X = \{B,C,D\} => X^+ = \{A,B,C,D,E\} => \text{super key}$ $X = C => X^+ = \{A,D,E\} => \text{Not super key} => \text{Violates BCNF condition.}$
- b) FD3 => R1{C,D} with FD3, R2{A,B,C,E} with FD1 and FD2. Both are in BCNF.