Task 1:

FD1: {A} → {B,C} FD2: {C} → {A,D} FD3: {D,E} → {F}

a) Decomposition of FD1 : A → B , A → C
   Decomposition of FD2 : C → A , C → D
   Transitivity of C → A and A → B, then C → B

b) Decomposition of FD1 : A → B , A → C
   Decomposition of FD2 : C → A , C → D
   Transitivity of C → D and DE → F, then CE → F
   Pseudo-Transitivity of A → C and CE → F, then AE → F

Task 2:

a) X = { A }
   X^+ = { A, B, C, D, F }

b) X = { C, E }
   X^+ = { A, B, C, D, E, F }

Task 3:

FD1: {A,B} → {C,D,E,F} FD2: {E} → {F} FD3: {D} → {B}

a) {A,B} because {A,B} → {A,B,C,D,E,F}, which means every attribute in R.
   {A,D} because {D} → {B} and {A,B} → {A,B,C,D,E,F}.

b) FD2, and FD3.
   FD2: IF X = {E}
   X^+ = { E, F }
   FD3: IF X = {D}
   X^+ = { B, C, D, E, F }

c) FD2 => R1{A,B,C,D,E} with FD3 and FD4 AB → CDE => CK = {AB} and {AD} using transitivity of D → B and AB → CDE,
   R2{E,F} with FD2 => CK = {E}.
   R1 is still not in BCNF because of FD3, while R2 is in BCNF.

FD3 => R1A{A,C,D,E} with FD {A,D} → {C,D,E} using transitivity and decomposition of D → B and {A,B} → {C,D,E,F}, => CK = {A} and {D},
R1B{D,B} with FD3 => CK = {D}.
All of them are in BCNF now.

Task 4:

FD1: {A,B,C} → {D,E} FD2: {B,C,D} → {A,E} FD3: {C} → {D}

a) Decomposition of FD1 : A →D, A→E, B→D, B→E, C→D, C→E
Decomposition of FD2 : B →A, B→E, C→A, C→E, D→A, D→E
X = {A,B,C} => X^+ = {A,B,C,D,E} => super key
X = {B,C,D} => X^+ = {A,B,C,D,E} => super key
X = C => X^+ = { A, D, E } => Not super key => Violates BCNF condition.

b) FD3 => R1{C,D} with FD3 => CK = {C},
c) R2{A,B,C,E} with FD : {A,B,C} → {E} using decomposition of {A,B,C} → {D,E} => CK = {B,C}
Both are in BCNF.