Description

The problems are from textbook. **Due Feb 6th 11:59 pm. Please submit the HW via gradescope.**

Part 1: Section 1.1, Exercises 2, 6, 19, 20, 22, 36.

Part 2:Section 1.2, Exercise 4, 10, 20

Part 3:Section 1.2, Exercise 48. Section 1.3, Exercises 20, 22.

Part 1 Section 1.1

2) Find all solutions of the linear system using elimination

$$2. \quad \begin{vmatrix} 4x + 3y = 2 \\ 7x + 5y = 3 \end{vmatrix}$$

① eliminate x. find least common muliple

4 8 12 16 20 24 28 32 36

7 14 21 28 35 42 49 56

② Scale to get x = 28, x = −28

$$\begin{aligned} 7\,(4x + 3y &= 2) \\ -4\,(7x + 5y &= 3) \end{aligned}$$

$$\begin{aligned} 28x + 21y &= 14 \\ -\;28x + -20y &= 12 \end{aligned}$$

$y = 2$

③ Plug in for y

$4x + 3(2) = 2$

$4x = 2 - 6$

$4x = -4$

$x = -1$

④ check $x = -1$ and $y = 2$

$4(-1) + 3(2) == 2$

$-4 + 6 \overset{\checkmark}{=} 2$

$7(-1) + 5(2) == 3$

$-7 + 10 \overset{\checkmark}{=} 3$

$$6. \quad \begin{vmatrix} x + 2y + 3z = 8 \\ x + 3y + 3z = 10 \\ x + 2y + 4z = 9 \end{vmatrix}$$

① Subtract (1) − (2)

$$\begin{aligned} x + 2y + 3z &= 8 \\ -\;x + 3y + 3z &= 10 \end{aligned}$$

$y = 2$

② Subtract to get rid of equation 3

$$\begin{aligned} x + 2y + 3z &= 8 \\ -\;x + 2y + 4z &= 9 \end{aligned}$$

$z = 1$

③ Plug in to get x

a) $x + 2(2) + 3(1) = 8$

$x + 4 + 3 = 8$

$x + 7 = 8 - 7$

$x = 1$

b) $x + 3(2) + 3(1) = 10$

$x + 6 + 3 = 10 - 9$

$x = 1$

c) $x + 2(2) + 4(1) = 9$

$x + 4 + 4 = 9$

$x = 1$

$x = 1$

$y = 2$

$z = 1$

19. Consider the linear system

$$\left| \begin{aligned} x + y - z &= -2 \\ 3x - 5y + 13z &= 18 \\ x - 2y + 5z &= k \end{aligned} \right|,$$

where k is an arbitrary number.

a. For which value(s) of k does this system have one or infinitely many solutions?

b. For each value of k you found in part a, how many solutions does the system have?

c. Find all solutions for each value of k.

One Solution: Consistent Indepent $x=4 \quad y=3$

Many Solution: Consistent, dependent $0=0, 5=5, x=x$ → itself many #'s = ∞ Solutions

No Solution: Inconsistent, Independent $2 \neq 5$

① $\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 3 & -5 & 13 & 18 \\ 1 & -2 & 5 & k \end{array}\right]$

② $r_2 = -3(r_1) + r_2$

$$\begin{array}{cccc} -3 & -3 & 3 & 6 \\ 3 & -5 & 13 & 18 \\ \hline 0 & -8 & 16 & 24 \div -8 \end{array}$$

$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & -2 & -3 \\ 1 & -2 & 5 & k \end{array}\right]$

$r_3 = r_3 + -r(_1)$

$$\begin{array}{cccc} -1 & -1 & 1 & 2 \\ 1 & -2 & 5 & k \\ \hline 0 & -3 & 6 & k+2 \end{array}$$

$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & 1 & -2 & -3 \\ 0 & -3 & 6 & k+2 \end{array}\right]$

$r_1 = r_1 - r_2$ $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & -3 \\ 0 & -3 & 6 & k+2 \end{array}\right]$

$r_3 = r_3 + 3(r_2)$

$$\begin{array}{cccc} 0 & 3 & -6 & -9 \\ 0 & -3 & 6 & k+2 \\ \hline 0 & 0 & 0 & k-7 \end{array}$$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & k-7 \end{array}\right]$

a) One or ∞ Solution: $k = 7$

b/c $k-7$ if $k=7$ then

its $7-7=0$ [0 0 0 0]

b) $k=7$ its ∞ Solution

c) when $k=7$

$$\begin{array}{cccc} x & y & z & b \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$k=7$
$7-7=0$
free parameter

$x + z = 1$

$y - 2z = -3$

$x = 1 - z$

$y = -3 - 2z$

$z = t$ free spot

$x = 1 - t$

$y = -3 - 2t$

$z = t$

20. Consider the linear system

$$\begin{vmatrix} x + y - z = 2 \\ x + 2y + z = 3 \\ x + y + (k^2 - 5)z = k \end{vmatrix},$$

where k is an arbitrary constant. For which value(s) of k does this system have a unique solution? For which value(s) of k does the system have infinitely many solutions? For which value(s) of k is the system inconsistent?

① $\begin{bmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & k^2-5 & k \end{bmatrix}$

$R_2 = R_2 - R_1$ $\begin{bmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & -2 & 1 \\ 1 & 1 & k^2-5 & k \end{bmatrix}$

$R_3 = R_3 - R_1$

$\begin{matrix} 1 & 1 & -1 & 2 \\ 1 & 1 & k^2-5 & k \end{matrix}$ $\begin{bmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & k^2-4 & k-2 \end{bmatrix}$

$\hookrightarrow$ $k^2-4 = 1$ unique

$k^2-4=0$ and $k-2=0$ ∞ solution

$k^2-4=0$ $k-2 \neq 0$ inconsistent

Case 1: $k^2 - 4 \neq 0$ means pivot, non-zero value

$(k^2-4)z = k-2$

$z = \dfrac{k-2}{k^2-4}$ unique solution

Case 2: $k^2 - 4 = 0$, No pivot

$\underbrace{k^2-4}$ = $\underbrace{k-2}$ this to be 0 means $k=2$

thus $2-2=0$

If $k^2-4=0$

$(k-2)(k+2)=0$

If $k=2$ or $k=-2$ means no pivot, ∞ solution, inconsistent, independent

$\downarrow$

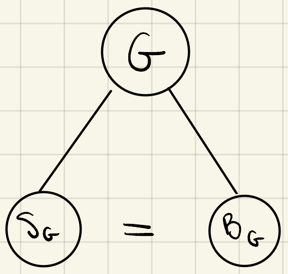
$2^2-4=0$ $\quad$ $2-2=0$ $\quad$ $[\,0 \quad 0\,]$ ∞

22. Emile and Gertrude are brother and sister. Emile has twice as many sisters as brothers, and Gertrude has just as many brothers as sisters. How many children are there in this family? ↳ $2b = S$

① Emile : Male

Gertrude: Female

Gertrude Perspective



Emile sister = $S_G + 1$ (E has as many sister as G +1)

Emile brother = $B_G - 1$ (E has as many brother as G −1)

② Emile formula

$2b = S$

③ Plug in Emile formula to Gertrude formula

$2(B_G - 1) = S_G + 1$

– Since $B_G = S_G$, subsitute B_G for S_G

$2(B_G - 1) = B_G + 1$

$2B_G - 2 = B_G + 1$

$2B_G = B_G + 3$

$-B_G \quad -B_G$

$B_G = 3$

→ Gertrude has 3 brothers therefore 3 Sisters

→ Emile has 2 brothers and 4 sisters

Count:

Gertrude: 6 sibling +1

Emile : 6 sibling +1

$\boxed{= 7 \text{ children}}$

3 sons

4 Daughters

⑤ Check Condition

a) Gertrude $S_G = S_B$
$3 \overset{\checkmark}{=} 3$

ⓑ Emile $2S_E = 1B_E$
$4S \overset{\checkmark}{=} 2B$

36. Find all the polynomials $f(t)$ of degree ≤ 2 [of the form $f(t) = a + bt + ct^2$] whose graphs run through the points (1, 1) and (3, 3), such that $f'(2) = 3$.

NOT sure

$f(1) = 1$

1) $1 = a + b(1) + c(1)^2$

$\boxed{1 = a + b + c}$

$f(3) = 3$

2) $3 = a + b(3) + c(3)^2$

$\boxed{3 = a + 3b + 9c}$

3) $f'(2) = 3$

$b + 2c(2) = 3$

$\boxed{b + 4c = 3}$

4) contradiction use equation 1

$f(x) = a + bt + ct^2$

$a + (3 - 4c) + c = 1$

$\overset{-1}{1} = a + \overset{-1}{3} - 3c$

$\boxed{a = 2 - 3c}$

Part 2 Section 1.2:

Solve using Gauss-Jordan Elimination

4. $\begin{vmatrix} x + y = 1 \\ 2x - y = 5 \\ 3x + 4y = 2 \end{vmatrix}$

1) Augmented Matrix

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & -1 & 5 \\ 3 & 4 & 2 \end{array}\right]$$

$$\boxed{x = 2 \quad y = -1}$$

2) In RREF

a) $-2(r_1) + 2r = 2r$

$$\begin{array}{ccc} -2 & -2 & -2 \\ 2 & -1 & 5 \\ \hline 0 & -3 & 3 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -3 & 3 \\ 3 & 4 & 2 \end{array}\right]$$

b) $-3(r_1) + r_3 = r_3$

$$\begin{array}{cccc} & -3 & -3 & -3 \\ + & 3 & 4 & 2 \\ \hline & 0 & 1 & -1 \end{array}$$

$$\left[\begin{array}{ccc} 1 & 1 & 1 \\ 0 & -3 & 3 \\ 0 & 1 & -1 \end{array}\right]$$

c) Swape $r_2 \leftrightarrow r_1$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & -3 & 3 \end{array}\right]$$

d) $r_3 = r_3 + 3(r_2)$

$$\begin{array}{ccc} 0 & 3 & -3 \\ 0 & -3 & 3 \\ \hline 0 & 0 & 0 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

e) $r_1 = r_1 + -(r_2)$

$$\begin{array}{ccc} 1 & 1 & 1 \\ 0 & -1 & 1 \\ \hline 1 & 0 & 2 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

3) Plug in

Equation 1) $2 + -1 \stackrel{\checkmark}{=} 1$

Equation 2) $2(2) - -1 \stackrel{\checkmark}{=} 5$

equation 3) $3(2) + 4(-1) = 2$

$6 - 4 \quad \checkmark \quad 2$

Solve using Gauss-Jordan Elimination

10. $\left|\begin{aligned} 4x_1 + 3x_2 + 2x_3 - x_4 &= 4 \\ 5x_1 + 4x_2 + 3x_3 - x_4 &= 4 \\ -2x_1 - 2x_2 - x_3 + 2x_4 &= -3 \\ 11x_1 + 6x_2 + 4x_3 + x_4 &= 11 \end{aligned}\right|$

$$\left[\begin{array}{cccc|c} 4 & 3 & 2 & -1 & 4 \\ 5 & 4 & 3 & -1 & 4 \\ -2 & -2 & -1 & 2 & -3 \\ 11 & 6 & 4 & 1 & 11 \end{array}\right]$$

① $R_1 = R_1/4$

$$\left[\begin{array}{cccc|c} 1 & 3/4 & 1/2 & -1/4 & 1 \\ 5 & 4 & 3 & -1 & 4 \\ -2 & -2 & -1 & 2 & -3 \\ 11 & 6 & 4 & 1 & 11 \end{array}\right]$$

$R_2 = R_2 + -5(R_1)$

$$\left[\begin{array}{cccc|c} 1 & 3/4 & 1/2 & -1/4 & 1 \\ 0 & 1/4 & 1/2 & 1/4 & -1 \\ -2 & -2 & -1 & 2 & -3 \\ 11 & 6 & 4 & 1 & 11 \end{array}\right]$$

$R_3 = R_3 + 2(R_1)$

$$\left[\begin{array}{cccc|c} 1 & 3/4 & 1/2 & -1/4 & 1 \\ 0 & 1/4 & 1/2 & 1/4 & -1 \\ 0 & -1/2 & 0 & 3/2 & -1 \\ 11 & 6 & 4 & 1 & 11 \end{array}\right]$$

$R_4 = R_4 + -11(R_1)$

$$\left[\begin{array}{ccc|c} 1 & 3/4 & 1/2 & -1/4 \\ 0 & 1/4 & 1/2 \;\; 1/4 & -1 \\ 0 & -1/2 & 0 \;\; 3/2 & -1 \\ 0 & -9/4 & -3/2 \;\; 13/4 & 0 \end{array}\right]$$

$R_2 = 4(R_2)$

$$\left[\begin{array}{ccc|c} 1 & 3/4 & 1/2 & -1/4 \\ 0 & 1 & 2 \;\; 1 & -4 \\ 0 & -1/2 & 0 \;\; 3/2 & -1 \\ 0 & -9/4 & -3/2 \;\; 13/4 & 0 \end{array}\right]$$

$R_1 = R_1 + -3/4(R_2)$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 4 \\ 0 & 1 & 2 & 1 & -4 \\ 0 & -1/2 & 0 & 3/2 & -1 \\ 0 & -9/4 & -3/2 & 13/4 & 0 \end{array}\right]$$

$R_3 = R_3 + 1/2(R_2)$,

$R_4 = R_4 + 9/4(R_2)$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 4 \\ 0 & 1 & 2 & 1 & -4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 3 & 6 & -9 \end{array}\right]$$

$R_4 = R_4 / 3$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 4 \\ 0 & 1 & 2 & 1 & -4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & 2 & -3 \end{array}\right]$$

$R_5 = R_5 + R_3(-1)$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 4 \\ 0 & 1 & 2 & 1 & -4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & \\ \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 & 2 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = 1$

$1x_2 - 3x_4 = 2 \rightarrow \quad x_2 = 2 + 3x_4$

$x_3 + 2x_4 = -3 \rightarrow \quad x_3 = -3 - 2x_4$

$$\boxed{\begin{aligned} x_1 &= 1 - x_4 \\ x_2 &= 2 + 3x_4 \\ x_3 &= -3 - 2x_4 \\ x_4 &= \text{free} \end{aligned}}$$

20. For which values of a, b, c, d, and e is the following matrix in reduced row-echelon form?

$$A = \begin{bmatrix} 0 & a & 2 & 1 & b \\ 0 & 0 & 0 & c & d \\ 0 & 0 & e & 0 & 0 \end{bmatrix}$$

→ I'm not sure exactly what the question is?

A =

$c = 0$

$a \neq 0$

$e = 1$

b, d = any value

Pivot

$$\begin{bmatrix} 0 & 1 & 2 & 1 & k \\ 0 & 0 & 0 & 0 & k \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Pivot

48. Consider the equations

$$\left| \begin{array}{l} 0 \quad y + 2kz = 0 \\ x + 2y + 6z = 2 \\ kx \quad 0 \quad + 2z = 1 \end{array} \right|,$$

where k is an arbitrary constant.

a. For which values of the constant k does this system have a unique solution?

b. When is there no solution?

c. When are there infinitely many solutions?

$$\left[\begin{array}{ccc|c} 0 & 1 & 2k & 0 \\ 1 & 2 & 6 & 2 \\ k & 0 & 2 & 1 \end{array}\right] \quad r_1 \leftrightarrow r_2 \quad \left[\begin{array}{ccc|c} 1 & 2 & 6 & 2 \\ 0 & 1 & 2k & 0 \\ k & 0 & 2 & 1 \end{array}\right]$$

$R_3 = R_3 - k \cdot R_1$

$$\begin{array}{cccc} -\cancel{k} & -2k & -6k & -2k \\ \cancel{k} & 0 & 2 & 1 \end{array} \qquad \left[\begin{array}{cccc} 1 & 2 & 6 & 2 \\ 0 & 1 & 2k & 0 \\ 0 & -2k & -6k-2 & -2k+1 \end{array}\right]$$

$R_3 = R_3 + 2k(r_2)$

$$\begin{array}{cccc} 0 & \cancel{2}k & 4k^2 & 0 \\ 0 & -\cancel{2}k & -6k-2 & -2k+1 \end{array} \qquad \left[\begin{array}{ccccc} 1 & 2 & 6 & 2 & 2 \\ 0 & 1 & 2k & 0 & 0 \\ 0 & 0 & 4k^2-6k-2 & & -2k+1 \end{array}\right]$$

Unique: $4k^2 - 6k - 2 \neq 0$ $[\,0 \quad 0 \quad 0 \quad -2k+1\,]$

No Solution (inconsistent) $4k^2 - 6k - 2 = 0$ however $-2k+1 = 0$ $[\,0 \quad 0 \quad 0 \quad -2k+1\,]$

Infinite Solution: $4k^2 - 6k - 2 = 0$ And $-2k+1 = 0$ $[\,0 \quad 0 \quad 0 \quad 0\,]$

Part 3 Section 1.3:

$$\begin{matrix} 3\times 2 & & 3\times 2 \\ \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix} & + & \begin{bmatrix} 7 & 5 \\ 3 & 1 \\ 0 & -1 \end{bmatrix} \end{matrix}$$

20. a. Find $\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 7 & 5 \\ 3 & 1 \\ 0 & -1 \end{bmatrix}$.

$$\begin{bmatrix} 2+7 & 3+5 \\ 4+3 & 5+1 \\ 6+0 & 7-1 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ 7 & 6 \\ 6 & 6 \end{bmatrix}$$

22. Consider a linear system of three equations with three unknowns. We are told that the system has a unique solution. What does the reduced row-echelon form of the coefficient matrix of this system look like? Explain your answer.

3 unkown: x_1 x_2 x_3

Unique: number of pivots = # of columns

Ex= $M = \begin{array}{cccc} x_1 & x_2 & x_3 & b \\ \end{array}$

$$M = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 8 \end{array}\right]$$