Formulation of LPP

for word problem.

Q1) A firm can produce 3 types of cloth, A, B, C. 3 kinds of wool are reqd - red, green & blue

1 unit length of type A needs 2 meters of red wool & 3 meters of blue wool.

1 unit length of type B needs 3 meters of red wool, 2 meters green & 2 meters of blue wool.

1 unit of type C requires 5 meters of green wool & 4 meters of blue wool.

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The firm has stocks of 8 meters of red wool, 10 meters of green wool & 15 meters of blue wool.

The income obtained from 1 unit length of type A cloth is Rs 3, type B cloth is Rs 5 & type C cloth is Rs 4.

Determine how the firm should use the available material, to maximize income.

→ Maximization problem.

Step 1: Formulate table.

Wool	Products A	B	C	Availability
Red	2	3	0	8 mtrs
green	0	2	5	10 mtrs
blue	3	2	4	15 mtrs
Income	3rs	5rs	4rs	

Step 2: Assumptions.

Let x_1, x_2, x_3 be quantity (in meters) of type A, B, C cloth to be produced.

Maximize $Z = 3(x_1) + 5(x_2) + 4(x_3)$

Step 4: Define Constraints

1 row $2(x_1) + 3(x_2) \leq 8$

2nd row $2(x_2) + 5(x_3) \leq 10$

3rd row: $3(x_1) + 2(x_2) + 4(x_3) \leq 15$

$x_1, x_2 \text{ \& } x_3 >= 0$ ie non-negative Constraints

Linear prog widely used applied mathematical technique that helps manager in decision making & planning for optimal allocation of limited resources.

- Deals with optimization of obj fn that are subject of linear eqn's.
- To increase prod profit rate / decrease cost of prod.
- limited resources, capacity of m/c to do work.

Eg: Max $Z = 200x_1 + 175x_2$ ← obj fn

Subject to
$$\left.\begin{aligned} 10x_1 + 22x_2 &\leq 2640 \\ 15x_1 + 18x_2 &\leq 2640 \\ x_1 &\leq 150 \end{aligned}\right\} \text{const eqns}$$

where $x_1, x_2 \geq 0$ ← non negative constr

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A company makes 2 products A & B. Both require processing on 2 m/cs. product A takes 10 & 15 minutes on 2 m/c per unit & prod B takes 22 & 18 minutes per unit on 2 m/cs. Both the m/c's are available for 2640 minutes per week. The products are sold for Rs 200 & Rs 175 respectively per unit. Formulate a LP to maximize revenue? The market can take a max of 150 units of prod A.

→

	Processing Time (minutes)		Selling price (Rs)
	M/C 1	M/C 2	
prod A	10	15	200
prod B	22	18	175
Avail of m/c	2640 min per week	2640 min per week	

Formulation: Obj → To max revenue.
on which parameter? → A & B no of prod.

Let x_1 be no of units of prod A to be made

" x_2 " prod B "

(maximize) $Z = 200x_1 + 175x_2$ ← obj fn

Constraints :-
$10x_1 + 22x_2 \le 2640$
$15x_1 + 18x_2 \le 2640.$
$x_1 \le 150$ (max prod A can be made)

$x_1, x_2 \geq 0$ (non negativity Constraint)

Problems 8th may

Eg. 2.4 Kalavathy Pg no 9

→ Decision variables :

Let x_1 & x_2 be the no of units of two grades of paper of x and Y.

Obj fn :- ∵ profit of x & Y given

$$\text{Max } Z = 200x_1 + 500x_2$$

Constraints :- There are 2 constraints, one referring to raw material & the other to the production hours.

$$Z = 200x_1 + 500x_2$$

$$x_1 \leq 400$$

$$x_2 \leq 300$$

$$0.2x_1 + 0.4x_2 \leq 160$$

Non - negative restriction to $x_1, x_2 \geq 0$.

Eg. 2.7 Kalavathy Pg no 11

→ Let x_1, x_2, x_3 be the area (in acre) to his farm to grow tomatoes, lettuce & radishes respectively.

The farmer produces $2000x_1$ Kg of tomatoes, $3000x_2$ heaps of lettuce

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& $1000x_3$ kg of radishes.

$\therefore$ The total sales of farmer will be

$= ₹ (1.00 \times 2000x_1 + 0.75 \times 3000x_2 + 2 \times 1000x_3)$

$\therefore$ Fertilizer expenditure will be $0.5 \times [100(x_1 + x_2) + 50x_3]$

Labour cost = Rs $20 \times (5x_1 + 6x_2 + 5x_3)$

$\therefore$ Farmer profit will be

$Z =$ Sale (in Rs) − Total expenditure (in Rs)

$= 2000x_1 + 0.75 \times 3000x_2 + 2 \times 1000x_3)$

$- 0.5 \times [100(x_1 + x_2) + 50x_3]$ Fertilizer.

$- 20 \times (5x_1 + 6x_2 + 5x_3)$ (labour)

$Z = 1850x_1 + 2080x_2 + 1875x_3$

$\therefore$ Total area of farm is restricted to 100 acres.

$x_1 + x_2 + x_3 \leq 100$

Also, total man-days labour is restricted to 400 man days.

$5x_1 + 6x_2 + 5x_3 \leq 400$

Hence, the farmers allocation problem can be finally put in the form

Find the value of x_1, x_2 & x_3 so as to

maximize
$Z = 1850x_1 + 2080x_2 + 1275x_3$

Subject to, $x_1 + x_2 + x_3 \le 100$
$5x_1 + 6x_2 + 5x_3 \le 400$
$x_1, x_2, x_3 \ge 0$

Eg 2.6-42 DS Hira pg no 83.

→ Formulation of L.P model.

Key decision is to determine the no of advertisements in each of the media.

Let x_1, x_2 & x_3 denote the no of advertisements in magazine, radio & television respectively.

Obj fn is to maximize the total expected effective exposure. For this, 1st effectiveness coefficient for each advertising media is calculated as follows;

Media	effective coeff
Magazine	0.70 (0.20) + 0.60 (0.40) + 0.45 (0.40) = 0.56
Radio	0.60 (0.20) + 0.50 (0.40) + 0.40 (0.40) = 0.48
Television	0.50 (0.20) + 0.40 (0.40) + 0.30 (0.40) = 0.38

Now effective exposure of each media =

effectiveness coeff × Audience size
Thus effective exposure of
magazine = 0.56 × 800000 = 448000
radio = 0.48 × 1000000 = 480000
television = 0.38 × 1500000 = 570000

∴ The objective function can be expressed as

Maximize $Z = 4,48000\,x_1 + 480000\,x_2 + 570000\,x_3$

Constraints can be formulated as follows:

a) budget const : $10000x_1 + 30000x_2 + 100000x_3 \le 1000000$
or $x_1 + 3x_2 + 10x_3 \le 100$.

b) min no. of adv allowed Const : $x_1 \ge 15$, $x_2 \ge 10$, $x_3 \ge 10$.

c) max no. of adv allowed Constraints $x_1 \le 25$, $x_2 \le 15$, $x_3 \le 15$.

where $x_1, x_2, x_3 \ge 0$.

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Solving LPP by Graphical method

① Eg: 3.1 pg. 20. Kalavathy
Eg 3.2. pg. 21 ''

MODULE 2

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Simplex method

- Developed in 1947, by G.B Dantzig, an American mathematician
- Starts with certain solⁿ, which is feasible, satisfies constraints, as well as non-negativity conditions.
- Improve upon this solⁿ, at consecutive stages, after certain stages, arrive at optimal solⁿ.
- Algorithm consists of moving from one vertex of region of feasible solⁿ to another in such a manner that value of objective fⁿ at succeeding vertex is less in minimization problem (or max) than at preceding vertex.
- procedure is repeated
- ∴ no of vertices is finite, method leads to optimal vertex in finite no of steps.

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2 Condⁿˢ:

i) Feasibility Condⁿ :- It ensures that if starting solⁿ is basic feasible, only basic feasible solⁿ will be obtained during computation.

ii) Optimality Condⁿ :- It guarantees that only better solⁿˢ will be encountered.

3 points :-

1. If there are m equality constraints & $m+n$ no of variables $(m \le n)$ a start for optimal solⁿ is made by putting n unknowns equal to zero, & then solving for m eqⁿˢ.

The n zero variables are called non basic variables & remaining m variables are called basic variables, which form basic solⁿ.

- If solⁿ yields all non-negative basic variables, it is called basic feasible solⁿ. else infeasible.

This step reduces no of alternatives for optimal solⁿ from infinite to finite no, whose max limit can be

$$^{m+n}C_m = \frac{(m+n)!}{m!\,n!}$$

2. WKT in LPP, all variables must be non-negative. (≥ 0)

$\therefore$ basic soln selected by cond 1 above are not necessarily non-negative. no of alternatives can be further reduced by eliminating all infeasible basic soln.

Basic variable set equal to zero is called leaving variable, while the new one is called entering var.

3. Entering variable can be so selected that it improves value of objective fn, so that new soln is better than older one.

achieved by optimality condn

process repeated, final soln is then called optimal basic feasible soln

* It is a iterative procedure for solving LPP in finite no of steps.

* alg moves from one vertex of region of feasible soln to another in such a manner that value of obj fn at succeeding vertex is less/more

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Some Defn :-

min cost / max profit

optimize $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

inequalities $\geq, \leq, =$

Subject to Constraints $a_{11}x_1 + a_{12}x_2 + \dots + a_{mn}x_n \leq, =, \geq b_1$

$\vdots$

b_m

Here $C = \{c_1, c_2, \dots c_n\}$ = price vector

$$a_i = \begin{bmatrix} a_{11} \\ a_2 \\ \vdots \\ \vdots \\ a_n \end{bmatrix} = \text{Activity vector}$$

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} = \text{Requirement vector.}$$

non-negative vectors.

slack variable

Eg 1. Solve the foll LPP

$\max Z = 5x_1 + 3x_2$

Subject to, $3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$

and $x_1, x_2 \ge 0$

→ If want to convert to min multiple by (-)

Convert inequalities into eqns

$\max Z = 5x_1 + 3x_2$

S.T $3x_1 + 5x_2 + x_3 = 15$

$5x_1 + 2x_2 + x_4 = 10$

and $x_1, x_2, x_3, x_4 \ge 0$

$Z = 5x_1 + 3x_2 + 0.x_3 + 0.x_4$

Matrix

$$A = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{bmatrix} 3 & 5 & 1 & 0 \\ 5 & 2 & 0 & 1 \end{bmatrix} \end{array}$$

Coefficients

identity

1st Simplex table

		C_j	5	3	0	0	
C_B	X_B	b_i	x_1	x_2	x_3	x_4	$\frac{b_i}{x_i}$
0	x_3	15	3	5	1	0	$\frac{15}{3} = 5$
0	x_4 exit	10	5 key	2	0	1	$\frac{10}{5} = 2$
	$Z_j - C_j$		-5	-3	0	0	

Entering

find min

$Z_j = \Sigma C_B x_B$

Eliminate x_3, x_4 & bring in x_1 & x_2

$Z_j = \Sigma C_B x_B$

$Z_1 = 0 \times 3 + 0 \times 5 = 0 + 0 = 0$

$Z_j - C_j = 0 - 5 = \underline{-5}$

$Z_2 = 0 \times 5 + 0 \times 2 = 0$

$Z_j - C_j = 0 - 3 = \underline{3}$

$Z_3 = 0 \times 1 + 0 \times 0 = 0$

$Z_j - C_j = 0 - 0 = 0$

$Z_4 = 0 \times 0 + 0 \times 1 = 0$

$Z_j - C_j = 0 - 0 = 0$

Find key element, entry & exit element. until $\forall\ Z_j - C_j \ge 0$.

stop (when no negative found)

Don't count zero, find -ve only.

Look into table i.e (-5) (less)

$\therefore$ entry element $= x_1$

$\min\left(\frac{b_i}{x_i}\right) = (5, 2)$

2 is outgoing

Divide entire row of Table 1 by 5 & result fill in table 2 x_1 row.

2nd simplex table.

		C_j	5	3	0	0	
C_B	x_B	b_i	x_1	x_2	x_3	x_4	b_i/x_i
0	x_3 exit	9	0	19/5	1	-3/5	45/19
5	x_1	10/5 = 2	1	2/5	0	1/5	5
$z_j - c_j$			0	-1	0	+1	

entry

Key element = 5

old line - 3 × new line

$15 - 2 \times 3 = 9$
$3 - 1 \times 3 = 0$
$5 - 2/5 \times 3 = 19/5$
$1 - 0 \times 3 = 1$
$0 - 1/5 \times 3 = -3/5$

fill in x_3 row

Repeat $z_j - c_j$

$z_1 = 0 \times 0 + 5 \times 1 = 5$
$z_j - c_j = 5 - 5 = 0$

$z_2 = 0 \times \frac{19}{5} + 5 \times \frac{2}{5}$
$z_j - c_j = 2 - 3 = -1$

$z_3 = 0 \times 1 + 5 \times 0 = 0$
$z_j - c_j = 0 - 0 = 0$

$z_4 = 0 \times -3/5 + 5 \times 1/5 = 1$
$z_j - c_j = 0 - 1 = -1$

Entering element = -1 => x_2

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$\frac{b_i}{x_i} = \frac{9}{19/5} = \frac{45}{19}$

$\frac{2}{2/5} = \frac{10}{2} = 5$

$\min\left(\frac{45}{19}, 5\right) = 45/19$, exit point

$\boxed{\text{Key element} = 19/5}$

3rd simplex table

		C_j	5	3	0	0
C_B	x_B	b_i	x_1	x_2	x_3	x_4
3	x_2	45/19	0	1	5/19	-3/19
5	x_1	20/19	1	0	-2/19	5/19
$z_j - c_j$			0	0	5/19	16/19

All are +ve & stop.

$\frac{9}{19/5} = \frac{45}{19}$

$\frac{5}{19}$ multiple to 1st row
To make 2/5 to zero.

old line - new line × 2/5

$2 - 45/19 \times 2/5 = \frac{20}{19}$
$1 - 0 \times 2/5 = 1$
$2/5 - 1 \times 2/5 = 0$
$0 - 5/19 \times 2/5 = -\frac{2}{19}$
$1/5 - (-3/19) \times 2/5 = 5/19$

$z_1 = 3 \times 0 + 5 \times 1 - 5 = 0$
$z_2 = 3 \times 1 + 5 \times 0 - 3 = 0$
$z_3 = 3 \times \frac{5}{19} + 5 \times \frac{-2}{19} = 5/19$ $\quad \forall\, z_j - c_j \geq 0$.
$z_4 = \frac{16}{19}$

$x_1 = \frac{20}{19}$ $\quad x_2 = \frac{45}{19}$ $\quad x_3 = x_4 = 0$

$$Z = 5 \times \frac{20}{19} + 3 \times \frac{45}{19}$$

$$\boxed{Z = 235/19}$$ final result

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Dual Simplex Method

Eg: Solve by dual simplex method

min $z = 5x_1 + 6x_2$

Subject to, $x_1 + x_2 \geq 2$

$4x_1 + x_2 \geq 4$

$x_1, x_2 \geq 0$

→ Step 1. formulate the problem
Step 2. Find initial basic sol^n
Step 3: Test for optimality

i) If Mathematical obj fn is min type then change it to maximization type.
ii) all ≥ constraints to ≤ constraints by multiplying −1
iii) Transform every ≤ constr into an = constraints by adding a slack var to every constraint & assign a 0 cost co-eff in obj fn.

i) find initial basic sol^n by setting zero value to the decision variables.

i) if all values of $x_B \geq 0$ & $z_j - c_j \geq 0$ → then current sol^n is optimal sol^n terminate the process

Step 1: Given LPP is

Max $z = -5x_1 - 6x_2$

Subject to $-x_1 - x_2 \leq -2$

$-4x_1 - x_2 \leq -4$ inequality

and $x_1, x_2 \geq 0$

Max $z = -5x_1 - 6x_2 + 0s_1 + 0s_2$ cost

Subject to $-x_1 - x_2 + s_1 = -2$

$-4x_1 - x_2 + s_2 = -4$ eq^n

Step 2: An initial basic feasible sol^n is given by.

$x_1 = x_2 = 0$; $\boxed{s_1 = -2 \quad s_2 = -4}$

decision var.

Step 3: Initial table,

		c_j	−5	−6	0	0	
C_B	B	x_B	x_1	x_2	s_1	s_2	Max ratio
R_1 0	s_1	−2	−1	−1	1	0	
R_2 0	s_2	−4	−4	−1	0	1	1
$z_j - c_j$			5	6	0	0	

key element

$z_j - c_j = C_B x_j - c_j$

$z_1 - c_1 = C_B x_1 - c_1 = 0 - (-5) = 5$

$z_2 - c_2 = 6$

$z_3 - c_3 = 0 \qquad z_4 - c_4 = 0.$

ii) if any $x_B < 0$ then select the most negative x_B & this row is called key row.

(Simplex key col)

Judge on x_B

-4 is most negative.
$\therefore$ S_2 is outgoing. entire row is Key row

iii) Find max ratio $= \dfrac{z_j - c_j}{\text{Key row}}$ & Key row < 0 & this col is called Key col.

$$\text{Max ratio} = \frac{z_j - c_j}{S_2}, \quad S_2 < 0$$

$$\text{Max} = \left[\frac{5}{-4}, \frac{6}{-1}\right] = [-1.25, -6]$$

$$= \left(\frac{5}{-4}\right)$$ incoming vector S_1 Key col

iv) find new sol^n table.

C_B	B	x_B	x_1	x_2	S_1	S_2
c_j			-5	-6	0	0
0	S_1	-1	0	$-3/4$	1	$-1/4$
-5	x_1	$+1$	$+1$	$1/4$	0	$-1/4$
$z_j - c_j$			0	$19/4$	0	$5/4$

R_1', R_2' ; incoming vector ; Key col

$$R_2' \longrightarrow \frac{R_2}{-4}, \quad R_1' = R_1 + R_2'$$

$$z_j - c_j = C_B x_j - c_j$$

Judge on x_B, S_1 is $-ve$ (Key row)

To find key col, find max ratio.

$$\text{Max} = \frac{z_j - c_j}{\text{Key row}} = \left[\frac{z_j - c_j}{S_1}\right] \quad S_1 < 0$$

$$\text{Max} = \left[-, \frac{19/4}{-3/4}, -, \frac{5/4}{-1/4}\right]$$

$$\text{Max} = \left[\frac{-19}{3}, -5\right]$$

$= -5$ is Max value.

Find new table

C_B	B	x_B	x_1	x_2	S_1	S_2
c_j						
0	S_2	4	0	3	-4	1
-5	x_1	2	1	1	-1	0
$z_j - c_j$			0	1	5	0

R_1'', R_2''

$$R_1'' \longrightarrow R_1' \times (-4)$$

$$R_2'' \longrightarrow R_2' + \frac{1}{4} R_1''$$

$R_2' \rightarrow 1 \quad 1 \quad 1/4 \quad 0 \quad -1/4$

$\dfrac{R_1''}{4} \rightarrow 1 \quad 0 \quad 3/4 \quad -1 \quad 1/4 \quad +$

$2 \quad 1 \quad 1 \quad -1 \quad 0$

$z_j - c_j = c_B x_j - c_j$

Judge x_B, all are ≥ 0

$\because$ $z_j - c_j$ all are ≥ 0.

Current soln is optimal.

Terminate process.

The optimal soln is given by

$\boxed{x_1 = 2 \quad x_2 = 0}$

$$\text{Max } Z = \overset{c_B x_B}{0 \times 4 + (-5) \times 2}$$

$$\text{Max } Z = -10$$

$$\text{Min } Z = -\text{Max } Z$$

$$= -(-10)$$

$$\boxed{\text{Min } Z = 10}$$

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Analytical method
(Trial and Error method)

- when more than 2 variables present then instead of graphical method solve by using above method.

Maximization $Z = 3x_1 + 2x_2 + 3x_3 + x_4$

Subject to

$$x_1 + x_2 + x_3 + 4x_4 = 4$$

$$2x_1 + x_2 + x_3 + 5x_4 = 5$$

$$x_1, x_2, x_3, x_4 \geq 0$$

no of variable $= 4 = m$

no of eqn $= 2 = n$

At most soln $= {}^mC_n = {}^4C_2 = \frac{4!}{2!\,2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1}$ $(4-2)$

$= 6$ soln

Take $x_1, x_2 = 0$ = (Non basic variable)

remaining x_3, x_4 = basic variable

$$\text{IIIly} \begin{bmatrix} x_1 x_3 = 0, & x_1 x_4 = 0, & x_2 x_3 = 0, & x_2 x_4 = 0 \\ x_1 x_2 = 0, & x_1 x_3 = 0. & 6 \text{ possibilities} & \end{bmatrix}$$

Let $x_1 = x_2 = 0$, x_3 & x_4 = +ve.

then feasible soln

Follow for all, put in Z, max value obtained is optimal

Eg. Solve foll LPP by Analytical method

minimize $Z = 2x_1 + 3x_2 - x_3$

Subject to $x_1 + x_2 - x_3 = 5$

$2x_1 - x_2 + 3x_3 = 4$

$m = 3$

$n = 2$

and $x_1, x_2, x_3 \geq 0$

$\Rightarrow$ Since there is two linear equation in the constraints involving three variable a basic soln can be obtained by setting any one variable to zero

The total no of basic soln is

$${}^3C_2 = \frac{3!}{2!\,(3-2)!} = 3$$

${}^mC_n$

No. of basic sol m	Non basic var	Basic var	Eqn's of basic var	value of basic var	Value of obj fn $z = 2x_1 + 3x_2 - x_3$	Basic fess sol	Is soln non-degenerate	Optimal soln
1	$x_1 = 0$	x_2, x_3	$x_2 - x_3 = 5$ $-x_2 + 3x_3 = 4$	$x_2 = 19/2$ $x_3 = 9/2$	24	yes	yes	NO
2	$x_2 = 0$	x_1, x_3	$x_1 - x_3 = 5$ $2x_1 + 3x_3 = 4$	$x_1 = \frac{19}{5}$ $x_3 = \frac{-6}{5}$	—	NO	NO	—
3	$x_3 = 0$	x_1, x_2	$x_1 + x_2 = 5$ $2x_1 - x_2 = 4$	$x_1 = 3$ $x_2 = 2$	12	yes	yes	yes

$\therefore$ Min value is 12

Optimal sol^n $z = 12$.

(i) The basic feasible sol^{ns} are $\left(0, \frac{19}{2}, \frac{9}{2}\right)$ & $(3, 2, 0)$ which are also non-degenerate basic sol^n.

ii) The optimal sol^n is $(3, 2, 0)$ which is non-degenerate & feasible also &

$\boxed{Z_{min} = 12}$

Case 1:- let $x_1 = 0$

$$x_2 - x_3 = 5 \quad \&$$
$$-x_2 + 3x_3 = 4$$

Solving these $x_2 = \frac{19}{2}$ $x_3 = \frac{9}{2}$

Thus sol^n is $\left(0, \frac{19}{2}, \frac{9}{2}\right)$

clearly $x_1, x_2, x_3 \geq 0$.

This sol^n is a basic feasible sol^n.

Put in $z = 2(0) + 3\left(\frac{19}{2}\right) - \frac{9}{2}$

$= \frac{57}{2} - \frac{9}{2} = \frac{48}{2} = 24.$

Case 2: let $x_2 = 0$

$$x_1 - x_3 = 5$$
$$2x_1 + 3x_3 = 4$$

Solve this $x_1 = \frac{19}{5}$ $x_3 = \frac{-6}{5}$

Thus sol^n is $\left(\frac{19}{5}, 0, \frac{-6}{5}\right)$

which doesnot satisfy $x_1, x_2 \geq 0$, $x_3 < 0$

This sol^n is infeasible sol^n.

Case 3: let $x_3 = 0$

$$x_1 + x_2 = 5$$
$$2x_1 - x_2 = 4$$

Solve this $x_1 = 3$ $x_2 = 2$

Thus sol^n is $(3, 2, 0)$

which satisfy con $x_i \geq 0$.
This sol^n is basic feasible sol^n.

$$z = 2(3) + 3(2) - 0$$
$$= 6 + 6$$
$$= 12$$

Eg 2: Find all basic feasible sol^n of

$$2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14$$

$m = 3$
$n = 2$
thus $m - n = 1$
need to be 0

S.N	Non basic Var	Basic Var	eqn for B.S	Basic sol^n (if exists)	Is it BFS
1	$x_1 = 0$	x_2, x_3	$x_2 + 4x_3 = 11$ $x_2 + 5x_3 = 14$	$x_1 = 0$, $x_2 = -1$ $x_3 = 3$ -ve value	NO
2	$x_2 = 0$	x_1, x_3	$2x_1 + 4x_3 = 11$ $3x_1 + 5x_3 = 14$	$x_1 = 1/2$ $x_2 = 0$ $x_3 = 5/2$	Yes
3	$x_3 = 0$	x_2, x_1	$2x_1 + x_2 = 11$ $3x_1 + x_2 = 14$	$x_1 = 3$ $x_2 = 5$ $x_3 = 0$	Yes

possible cases $= {}^3C_2 = 3$

Solving all equi

$$x_2 + 4x_3 = 11$$
$$x_2 + 5x_3 = 14$$
$$-x_3 = -3$$
$$\boxed{x_3 = 3}$$

$$x_2 + 4(3) = 11$$
$$\boxed{x_2 = -1}$$

$x_1 = 0$.

Elimination method

$$2x_1 + 4x_3 = 11 \quad \text{— ①} \times 3$$
$$3x_1 + 5x_3 = 14 \quad \text{— ②} \times 2$$

~~x_1 & x_3~~

$$6x_1 + 12x_3 = 33$$
$$6x_1 + 10x_3 = 28$$

$$2x_3 = 5 \Rightarrow \boxed{x_3 = 5/2}$$

$$2x_1 + 4(5/2) = 11$$
$$2x_1 + 10 = 11 \Rightarrow \boxed{x_1 = 1/2} \quad x_2 = 0.$$

$$2x_1 + x_2 = 11$$
$$3x_1 + x_2 = 14$$
$$-x_1 = -3$$
$$\boxed{x_1 = 3}$$

$$2(3) + x_2 = 11$$
$$\boxed{x_2 = 5}$$
$$\boxed{x_3 = 0}$$

Basic feasible sol^n are: $x_1 = \frac{1}{2}, x_2 = 0, x_3 = \frac{5}{2}$

$x_1 = 3,\ x_2 = 5,\ x_3 = 0$

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Artificial Variables Technique

- We used slack variable to make inequalities into proper eqn.

(i) **Big-M method** (Penalty Method)

Max $z = 2x_1 + 4x_2$

subject to, $2x_1 + x_2 \le 18$

$3x_1 + 2x_2 \ge 30$

$x_1 + 2x_2 = 26$

$x_1, x_2 \ge 0$

We don't hv basic f.s

⇒ on using slack & surplus variables.

Max $z = 2x_1 + 4x_2 + 0s_1 + 0s_2 - MA_1 - MA_2$

Auxiliary eqns

$2x_1 + x_2 + s_1 + 0s_2 + 0A_1 + 0A_2 = 18$

$3x_1 + 2x_2 + 0s_1 - s_2 + A_1 + 0A_2 = 30$

$x_1 + 2x_2 + 0s_1 + 0s_2 + 0A_1 + A_2 = 26$

$x_1, x_2, s_1, s_2 \ge 0$

If $x_1, x_2 = 0$, $s_1 = 18$ (or) $s_2 = -30$
basic fs will get but not optimal

A_1, A_2 used to get identity matrix
Assume $-M$ (very large value)
loss, & out of soln

$-M$ = penalty

First simplex table

$C_j \rightarrow$ 2, 4, 0, 0, $-M$, $-M$ (entering col: x_2) $x_j \ge 0$

C_B	B.V	x_B	x_1	x_2	s_1	s_2	A_1	A_2	min ratio x_B/x_2
0	s_1	18	2	1	1	0	0	0	18
$-M$	A_1	30	3	2	0	-1	1	0	15
$-M$	A_2 (outgoing)	26	1	2	0	0	0	1	13 (min ratio)
$Z_j - C_j$			$-4m-2$	$-4m-4$ (most −ve)	0	M	0	0	

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When Artificial var are there in table only they will get eliminated 1st.

A_2 replaced by x_2. (delete A_2 col in next table)

Key ele — most −ve

$C_j \rightarrow$ 2 (enter), 4, 0, 0, $-M$ $x_j \ge 0$

C_B	B.V	x_B	x_1	x_2	s_1	s_2	A_1	Min ratio x_B/x_1
0	s_1	5	3/2	0	1	0	0	10/3
$-M$	A_1	4	2	0	0	-1	1	2 (min)
4	x_2	13	1/2	1	0	0	0	26
$Z_j - C_j$			$-2M$ (−ve)	0	0	M	0	

Still $z_j - c_j$ has −ve

$A_1 \rightarrow$ outgoing $x_1 \rightarrow$ incoming

$s_1 - \frac{3}{4}A_1$ $\quad 5 - \frac{3}{4} \times 4$

C_j: 2, 4, 0, 0

C_B	BV	x_B	x_1	x_2	s_1	s_2
0	s_1	2	0	0	1	3/4
2	x_1	2	1	0	0	$-1/2$
4	x_2	12	0	1	0	1/4
$Z_j - C_j$			0	0	0	0

Divide entire row by 2

$\therefore x_1 = 2$

$x_2 = 12$

$\therefore Z = 2x_1 + 4x_2$
$= 2(2) + 4(12)$
$= 4 + 48$

$\boxed{Z = 52}$

is optimal basic F.S.

(ii) Two phase method

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Max $Z = 2x_1 + 4x_2$
Subject to, $2x_1 + x_2 \le 18$
$3x_1 + 2x_2 \ge 30$
$x_1 + 2x_2 = 26$
$x_1, x_2 \ge 0$

→ Here we take AV cost −1, other var cost = 0 till A.V is in table. (phase 1)
phase 2 – when AV eliminated use actual cost

On using slack & surplus variables we get

Max $Z = 2x_1 + 4x_2 + 0S_1 + 0S_2 + A_1 + A_2$

Sub to $2x_1 + x_2 + S_1 + 0S_2 + 0A_1 + 0A_2 = 18$

$3x_1 + 2x_2 + 0 \cdot S_1 - S_2 + A_1 + 0A_2 = 30$

$x_1 + 2x_2 + 0S_1 + 0S_2 + 0A_1 + A_2 = 26$

$x_1, x_2, S_1, S_2, A_1, A_2 \ge 0$

phase – I

Keep all cost zero here except $A_1, A_2 = -1$

$C_j \rightarrow$			0	0	0	0	−1	−1	
C_B	BV	X_B	x_1	x_2 (enter)	S_1	S_2	A_1	A_2	min ratio X_B/x_2, $x_2 \ge 0$
0	S_1	18	2	1	1	0	0	0	18
−1	A_1	30	3	2	0	−1	1	0	15
−1	A_2 (out going)	26	1	2	0	0	0	1	13 min
$Z_j - C_j$			−4	−4	0	0	0	0	

most −ve → If same value consider any one col

C_j			0	0	0	0	−1	
C_B	BV	X_B	x_1 (enter)	x_2	S_1	S_2	A_1	min ratio $\frac{X_B}{x_1}$, $x_j \ge 0$
0	S_1	5	3/2	0	1	0	0	10/3
−1	A_1 (out)	4	2	0	0	−1	1	2 min
0	x_2	13	1/2	1	0	0	0	26
$Z_j - C_j$			−2	0	0	1	0	

most −ve

phase 2; B/c all AV are out, now use actual cost of other var.

C_j			2	4	0	0
C_B	BV	X_B	x_1	x_2	S_1	S_2
0	S_1	2	0	0	1	3/4
2	x_1	2	1	0	0	−1/2
4	x_2	12	0	1	0	1/4
$Z_j - C_j$			0	0	0	0

$x_1 = 2$
$x_2 = 12$

$\boxed{Z = 52}$

$Z = 2x_1 + 4x_2$
$= 2(2) + 4(12)$
$= 4 + 48$
$= 52$

Absence of Art. in table & exist
all $Z_j - C_j$ are +ve, (optimum)
then this cond^n is No F.S.

— if any col, (key col) become -ve
then it is unbounded soln.

(Whole column calculate)

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Simultaneous equations by Simplex method

Ques Use simplex method to solve the following system of LPP

$$\begin{cases} x_1 - x_3 - 4x_4 = 3 \\ 2x_1 - x_2 = 3 \\ 3x_1 - 2x_2 - x_4 = 1 \end{cases}$$

linear eqn

No slack & surplus vari

and $x_1, x_2, x_3, x_4 \geq 0$

→ Introducing artificial variable in above eqn or constraint eqn.

obj fn not given?

Sign is '=' only add Arti variable.

$$x_1 - x_3 - 4x_4 + 0x_2 + A_1 + 0A_2 + 0A_3 = 3$$
$$2x_1 - x_2 + 0x_3 + 0x_4 + 0A_1 + A_2 + 0A_3 = 3$$
$$3x_1 - 2x_2 + 0x_3 - x_4 + 0A_1 + 0A_2 + 1A_3 = 1$$

Now write obj fun, here not prescribed, so here dummy obj func will be.

$$\max\ z = 0x_1 + 0x_2 + 0x_3 + 0x_4 - 1A_1 - A_2 - A_3$$

where $A_1, A_2, A_3 \geq 0$.

Now Apply simplex method.

$z = C_B x_B = -3 + (-3) + (-1) = -7$

$\Delta_j = C_B x_j - C_j$

		C_j	0	0	0	0	-1	-1	-1	
C_B	BV	x_B	x_1 ↓	x_2	x_3	x_4	A_1	A_2	A_3	x_B/x_1
-1	A_1	3	1	0	-1	-4	1	0	0	3
-1	A_2	3	2	-1	0	0	0	1	0	3/2
-1 ←	A_3	1	$\boxed{3}$	-2	0	-1	0	0	1	1/3
	$z_j = -7$		-6	3	1	5	0	0	0	

	C_B	BV	x_B	x_1	x_2	x_3	x_4 ↓	A_1	A_2	A_3	x_B/x_4
$R_1 - R_3$	-1	a_1	8/3	0	2/3	-1	$\boxed{13/3}$	1	0	×	8/13
$R_2 - 2R_3$	-1	← a_2	7/3	0	1/3	0	2/3	0	1	×	7/12
Divide by 3	0	x_1	1/3	1	-2/3	0	-1/3	0	0	×	—
		$z = -5$		0	-1	1	-5	0	0		

C_B	BV	x_B	x_1	x_2 ↓	x_3	x_4	A_1	A_2	A_3	
0	← x_4	8/13	0	2/13	-3/13	1	×	0	×	8/2
-1	a_2	25/13	0	3/13	2/13	0	×	1	×	25/3
0	x_1	7/13	1	-8/13	1/13	0	×	0	×	
		$z = \frac{25}{19}$	0	$\frac{-3}{13}$	-2/13	0	×	0	×	

$-\frac{8}{3} - \frac{7}{3} = -\frac{15}{3} = -5$

C_B	BV	x_B	x_1	x_2	x_3	x_4	A_1	A_2	A_3	x_B/x_3
0	x_2	4	0	1	$-3/2$	$13/2$	x	0	x	—
-1	a_2	1	0	0	$1/2$	$3/2$	x	1	x	$1/(1/2)$
0	x_1	3	1	0	-1	4	x	0	x	—
	$Z=-1$		0	0	$-1/2$	$3/2$	x	x	x	

0	x_2	7	0	1	0	2	x	x	x
0	x_3	2	0	0	1	-3	x	x	x
0	x_1	5	1	0	0	1	x	x	x
	$Z=0$		0	0	0	0			

All values +ve (zero)
This is optimal soln is attained
ie $Z=0$

$x_1 = 5 \quad x_2 = 7 \quad x_3 = 2$

$x_4 = 0$

eg 2: $4x_1 + 3x_2 = 25$ No obj fn
$2x_1 + x_2 = 11$

$\rightarrow$ $4x_1 + 3x_2 + A_1 + 0A_2 = 25$
$2x_1 + x_2 + 0A_1 + A_2 = 11$

Min $Z = A_1 + A_2$ (virtual)
$\& \; x_1, x_2, A_1, A_2 \geq 0$

Max $z = 0x_1 + 0x_2 - A_2 - A_2$
$4x_1 + 3x_2 + A_1 = 25$
$2x_1 + x_2 + A_2 = 11$
$\& \; x_1, x_2, A_1, A_2 \geq 0$

		0	0	-1	-1		
BV	C_B	x_1	x_2	A_1	A_2	RHS	RHS/ck elem
A_1	-1	4	3	1	0	25	$25/4$
A_2	-1	$\boxed{2}$	1	0	1	11	$11/2$ min
$C_j - Z_j$		6	4	0	0		

		$R_1 - 4R_1$				
A_1	-1	0	1	1	-2	3
x_1	0	1	$1/2$	0	$1/2$	$11/2$
		$R_2/2$				
$C_j - Z_j$		0	4	0	-3	—

Computational Efficiency of Simplex Technique

- Simplex method is iterative in nature. Its Computational Efficiency has 2 factors

(i) Computational effort at each iteration
(ii) The no of iterations.

- Eg: Full tableau implementation needs $O(mn)$ arithmetic ops per iteration

- Same true for revised Simplex method in worst case.

- We will discuss no of iterations

(1) Worst case

- No of extreme points can increase exponentially with no of vars & const
- Typically it takes $O(m)$ pivots to optimal soln.
- Not true for every LP problem.