Polytech Nice

Asian Options Princing

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Sommaire

1 Introduction

In this project, we aim to model the pricing of an Asian option, one of the most well-known and widely used exotic financial instruments in the financial markets. These options are unique in that their value depends not only on the final value of the underlying asset but also on its average over a specific period. This distinctive feature makes them particularly useful for reducing volatility and mitigating the risks of market manipulation.

The primary objective of this work is to develop a rigorous approach to calculate the price of an Asian option. To achieve this, we have structured this report into several sections, outlining the steps taken from theoretical calculations to the implementation of the necessary numerical methods. We began by establishing the mathematical foundations of the problem, focusing specifically on the differential equation associated with pricing. Subsequently, we explored and implemented various numerical techniques to solve this equation, validating our results by comparing the two methods used.

This project has been highly educational as it allowed us to apply and deepen the knowledge acquired during courses on numerical methods and option pricing, taught by Mr. Auroux. Furthermore, it provided an opportunity to gain a deeper understanding of the practical challenges related to handling exotic instruments in a realistic financial framework.

This report reflects our efforts to combine theoretical rigor with practical relevance in solving a complex problem while highlighting the many subtleties inherent to Asian options.

2 Vanillla Options

2.1 Vanilla Options :

The option market is a forward market that can be divided into two subcategories : the over-the-counter (OTC) market and the organized market.

An **organized market**, also called a regulated or listed market, is a trading venue where negotiations follow a set of rules enforced by a regulator. Contracts are standardized and created by market authorities such as the CME Group (Chicago Mercantile Exchange), Eurex in Europe, and Euronext in France.

An OTC market directly connects the buyer and the seller, either on their initiative or through a bank or broker. The main advantage of the OTC market is the ability to trade customized products. However, it has a significant disadvantage : counterparty risk, i.e., the risk that one of the parties does not fulfill its obligations.

The purpose of the option market is to facilitate the establishment of purchase or sale options for a financial product. The word *option* comes from the Latin *option*, meaning *right* but not *obligation*. An option is, therefore, a contract that grants the right, but not the obligation, to exercise a previously acquired right. There are two types of options :

— **Call options (purchase options)** : A call option gives the right to buy an asset at a predetermined price (called the *strike price*) at a fixed date (called the *maturity date*). The payoff function of a call is given by the following formula :

$$\text{Payoff} = \max(S - K, 0)$$

where S represents the price of the underlying asset at the option's expiration date, and K the exercise price.

— **Put options (sale options)** : A put option gives the right to sell an asset at a predetermined price (K) at a fixed date. The payoff function (or intrinsic value) of a put is given by the following formula :

$$\text{Payoff} = \max(K - S, 0)$$

where S represents the price of the underlying asset at the option's expiration date, and K the exercise price.

3 Exotic Options

Exotic options are a specialized class of derivatives that go beyond the simplicity of vanilla options by incorporating innovative and complex features. These options are designed to cater to the unique demands of sophisticated investors or to implement highly specific market strategies. Their bespoke nature makes them ideal for addressing non-standard investment scenarios or managing intricate risk exposures.

What sets exotic options apart is their integration of advanced mechanisms such as price barriers, average price calculations, or conditions tied to multiple market events. For instance, Asian options base their payout on the average price of the underlying asset over a given period, while barrier options activate or deactivate depending on whether the underlying asset crosses a predetermined price threshold.

Exotic options are predominantly traded in OTC markets, where flexibility and customization are paramount. Popular types include Asian options, single-barrier options, and double-barrier options, all of which serve as valuable tools for achieving targeted investment objectives and managing nuanced risks in dynamic market environments.

3.1 Asian Options

An Asian option is a specific type of option contract. For Asian options, the payoff is determined by the average price of the underlying asset over a predefined period. This differs from the standard European and American options, where the option's payoff depends on the underlying asset's price at the time of exercise. Asian options are thus one of the fundamental forms of exotic options

— **Asian Call Option** : An Asian call option gives the right to buy an asset at a predetermined price (called the *strike price*) at a fixed date (called the *maturity date*). The payoff function of an Asian call option is given by the following formula :

$$\text{Payoff} = \max(A - K, 0)$$

where $\bar{S}$ represents the average price of the underlying asset over the predefined observation period, and K is the exercise price.

— **Asian Put Option** : An Asian put option gives the right to sell an asset at a predetermined price (K) at a fixed date. The payoff function (or intrinsic value) of an Asian put option is given by the following formula :

$$\text{Payoff} = \max(K - A, 0)$$

where $\bar{S}$ represents the average price of the underlying asset over the predefined observation period, and K is the exercise price.

— To calculate the average price of the asset, two methods can be used :

1. **Arithmetic Average** : The average price is calculated as the arithmetic mean of the observed prices of the underlying asset.
2. **Geometric Average** : The average price is calculated as the geometric mean of the observed prices of the underlying asset.

4 Asian option pricing using Black and Shcoles Method

4.1 Find the new B.S differential equation for an Asian Option

For a call option, the value V is defined as follows :

$$V(A, S, T) = \max(A - K, 0)$$

- A_t, the average of the underlying asset up to time t,
- S_t, the price of the underlying asset at time t,
- T, the maturity of the option,
- K, the strike price.

In a continuous framework, the average of the underlying asset up to time t is given by :

$$A_t = \frac{1}{t}\int_0^t S_u \, \mathrm{d}u$$

Differentiating this expression with respect to time t, we obtain :

$$\frac{\mathrm{d}A_t}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{t}\int_0^t S_u \, \mathrm{d}u\right)$$

Applying the differentiation rule for a product :

$$\frac{\mathrm{d}A_t}{\mathrm{d}t} = -\frac{1}{t^2}\int_0^t S_u \, \mathrm{d}u + \frac{1}{t}S_t$$

This can be rewritten as :

$$\frac{\mathrm{d}A_t}{\mathrm{d}t} = -\frac{1}{t}A_t + \frac{1}{t}S_t$$

Thus, we obtain :

$$\mathrm{d}A_t = \frac{S_t - A_t}{t}\,\mathrm{d}t$$

The payoff of the call depends on two stochastic processes, dS_t and dA_t. The process dS_t is known because we assume that the returns of the underlying asset are Gaussian. This implies that the value of the asset follows a geometric Brownian motion, which is the solution to the following stochastic differential equation :

$$\mathrm{d}S_t = rS_t \, \mathrm{d}t + \sigma S_t \, \mathrm{d}W_t$$

where :

- r is the risk-free interest rate,
- σ is the volatility of the underlying asset,
- W_t is a standard Brownian motion.

The dynamics of S_t are described by the equation above, while A_t is obtained through the time average of S_t. The equation $\mathrm{d}A_t = \frac{S_t - A_t}{t}\,\mathrm{d}t$ links these two processes to model and compute the payoff of the Asian option.

Considering $V(t, x_t, y_t)$, a function that depends on two stochastic processes x_t and y_t, Itô's Lemma can be applied. This gives the following equation for $\mathrm{d}V$:

$$\mathrm{d}V = \frac{\partial V}{\partial t}\,\mathrm{d}t + \frac{\partial V}{\partial x}\,\mathrm{d}x + \frac{\partial V}{\partial y}\,\mathrm{d}y + \frac{1}{2}\frac{\partial^2 V}{\partial x^2}\,(\mathrm{d}x)^2 + \frac{1}{2}\frac{\partial^2 V}{\partial y^2}\,(\mathrm{d}y)^2 + \frac{\partial^2 V}{\partial x \partial y}\,\mathrm{d}x\,\mathrm{d}y$$

where :

- $\frac{\partial V}{\partial t}$ represents the partial derivative of V with respect to time t,
- $\frac{\partial V}{\partial x}$ and $\frac{\partial V}{\partial y}$ are the partial derivatives of V with respect to x_t and y_t,
- $\frac{\partial^2 V}{\partial x^2}$ and $\frac{\partial^2 V}{\partial y^2}$ are the second-order partial derivatives of V,

— $\frac{\partial^2 V}{\partial x \partial y}$ represents the mixed partial derivative of V.

In our case, the process S_t is stochastic, while A_t is deterministic. Consequently, for each term in Itô's Lemma, we have :

— $\frac{\partial V}{\partial t}\,\mathrm{d}t$: This term remains unchanged and corresponds to the partial derivative of V with respect to time.

— $\frac{\partial V}{\partial S_t}\,\mathrm{d}S_t$: By substituting $\mathrm{d}S_t$ with its dynamics given by the geometric Brownian motion equation :

$$\mathrm{d}S_t = rS_t\,\mathrm{d}t + \sigma S_t\,\mathrm{d}W_t,$$

we obtain :

$$\frac{\partial V}{\partial S_t}\,\mathrm{d}S_t = \frac{\partial V}{\partial S_t}\left(rS_t\,\mathrm{d}t + \sigma S_t\,\mathrm{d}W_t\right).$$

— $\frac{\partial V}{\partial A_t}\,\mathrm{d}A_t$: By substituting $\mathrm{d}A_t$ with its deterministic expression :

$$\mathrm{d}A_t = \frac{S_t - A_t}{t}\,\mathrm{d}t,$$

this term becomes :

$$\frac{\partial V}{\partial A_t}\,\mathrm{d}A_t = \frac{\partial V}{\partial A_t}\left(\frac{S_t - A_t}{t}\right)\mathrm{d}t.$$

— $(\mathrm{d}S_t)^2$: Using the diffusion equation for S_t, we have :

$$(\mathrm{d}S_t)^2 = \left(rS_t\,\mathrm{d}t + \sigma S_t\,\mathrm{d}W_t\right)^2.$$

Neglecting higher-order terms and retaining only those where $(\mathrm{d}W_t)^2 = \mathrm{d}t$, this simplifies to :

$$(\mathrm{d}S_t)^2 \simeq \sigma^2 S_t^2\,\mathrm{d}t.$$

Since A_t is deterministic, we neglect terms like $(\mathrm{d}A_t)^2$ and $\mathrm{d}S_t \cdot \mathrm{d}A_t$.

Finally, assuming risk neutrality, and grouping terms associated with $\mathrm{d}t$ and $\mathrm{d}W_t$, the PDE describing the value of the Asian option V under the risk-neutral measure is :

$$\frac{\partial V}{\partial t} + rS_t\frac{\partial V}{\partial S_t} + \frac{S_t - A_t}{t}\frac{\partial V}{\partial A_t} + \frac{1}{2}\sigma^2 S_t^2\frac{\partial^2 V}{\partial S_t^2} - rV = 0$$

4.2 First Attempt to discretize the new B.S

By discretizing the terms of the previous equation, we obtain :

$$\frac{V_{i+1,j,k} - V_{i,j,k}}{\Delta t} + rS_j\frac{V_{i,j+1,k} - V_{i,j,k}}{\Delta S} + \frac{V_j - A_k}{t}\frac{V_{i,j,k+1} - V_{i,j,k}}{\Delta A} + \frac{1}{2}\sigma^2 S_j^2\frac{V_{i,j+1,k} - 2V_{i,j,k} + V_{i,j-1,k}}{\Delta S^2} = 0$$

By grouping the terms and reorganizing the equation, we obtain :

$$V_{i+1,j,k}\frac{1}{\Delta t} + V_{i,j,k}\left(-\frac{1}{\Delta t} - \frac{rS_j}{\Delta S} - \frac{\sigma^2 S_j^2}{\Delta S^2} - \frac{S_j - A_k}{t\Delta A} - r\right) + V_{i,j+1,k}\left(\frac{rS_j}{\Delta S} + \frac{1}{2}\frac{\sigma^2 S_j^2}{\Delta S^2}\right)$$
$$+V_{i,j-1,k}\frac{1}{2}\frac{\sigma^2 S_j^2}{\Delta S^2} + V_{i,j,k+1}\frac{S_j - A_k}{t\Delta A} = 0$$

However, due to the complexity associated with the dimensionality of this scheme, its implementation is particularly challenging. To simplify the problem and reduce dimensionality, we will perform a variable change.

4.3 Variable Change

It is prudent to perform a variable change to reduce the dimensionality of the problem. Thus, we define :

$$x = \frac{K - tA/T}{S}$$

We aim to express the value of the option $V(T, S, A)$ in the form :

$$V(T, S, A) = Sf(t, x),$$

where $f(t, x)$ is a function we need to determine.

— K is the strike price of the option,

— t is the current time,

— A is the arithmetic mean of the underlying asset up to t,

— T is the maturity of the option,

— S is the current price of the underlying asset.

4.3.1 Derivative of V with respect to t

Applying the chain rule, we have :

$$\frac{\partial V}{\partial t} = S\left(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{\partial x}{\partial t}\right).$$

Let us calculate $\frac{\partial x}{\partial t}$. By definition :

$$x = \frac{K - \frac{tA}{T}}{S},$$

$$\frac{\partial x}{\partial t} = \frac{\partial}{\partial t}\left(\frac{K - \frac{tA}{T}}{S}\right) = -\frac{A}{TS}.$$

Thus, the derivative becomes :

$$\frac{\partial V}{\partial t} = S\frac{\partial f}{\partial t} - \frac{A}{T}\frac{\partial f}{\partial x}.$$

4.3.2 Derivative of V with respect to S

Applying the chain rule, we have :

$$\frac{\partial V}{\partial S} = f(t, x) + S\frac{\partial f}{\partial x}\frac{\partial x}{\partial S}.$$

Let us calculate $\frac{\partial x}{\partial S}$. By definition :

$$x = \frac{K - \frac{tA}{T}}{S},$$

$$\frac{\partial x}{\partial S} = -\frac{K - \frac{tA}{T}}{S^2} = -\frac{x}{S}.$$

Thus, the derivative becomes :

$$\frac{\partial V}{\partial S} = f(t, x) - x\frac{\partial f}{\partial x}.$$

4.3.3 Second Derivative of V with respect to S

Applying the chain rule, we have :

$$\frac{\partial^2 V}{\partial S^2} = \frac{\partial}{\partial S}\left(f(t,x) - x\frac{\partial f}{\partial x}\right).$$

Let us expand the terms separately :

$$\frac{\partial}{\partial S}\left(f(t,x)\right) = \frac{\partial f}{\partial x}\frac{\partial x}{\partial S} = -\frac{x}{S}\frac{\partial f}{\partial x},$$

$$\frac{\partial}{\partial S}\left(x\frac{\partial f}{\partial x}\right) = \frac{\partial x}{\partial S}\frac{\partial f}{\partial x} + x\frac{\partial}{\partial S}\left(\frac{\partial f}{\partial x}\right).$$

This gives :

$$\frac{\partial^2 V}{\partial S^2} = -\frac{x}{S}\frac{\partial f}{\partial x} + \frac{x}{S}\frac{\partial f}{\partial x} + \frac{x^2}{S^2}\frac{\partial^2 f}{\partial x^2}.$$

Simplifying :

$$\frac{\partial^2 V}{\partial S^2} = \frac{x^2}{S^2}\frac{\partial^2 f}{\partial x^2}.$$

4.3.4 Derivative of V with respect to A

Applying the chain rule, we have :

$$\frac{\partial V}{\partial A} = S\frac{\partial f}{\partial x}\frac{\partial x}{\partial A}.$$

Let us calculate $\frac{\partial x}{\partial A}$. By definition :

$$x = \frac{K - \frac{tA}{T}}{S},$$

$$\frac{\partial x}{\partial A} = \frac{\partial}{\partial A}\left(\frac{K - \frac{tA}{T}}{S}\right) = -\frac{t}{TS}.$$

Thus, the derivative becomes :

$$\frac{\partial V}{\partial A} = -\frac{t}{T}\frac{\partial f}{\partial x}.$$

4.3.5 Substitution into the Black-Scholes Equation

The Black-Scholes equation is given by :

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2} + \frac{S-A}{t}\frac{\partial V}{\partial A} - rV = 0.$$

Substitute the calculated derivatives :

$$\left(S\frac{\partial f}{\partial t} - \frac{A}{T}\frac{\partial f}{\partial x}\right) + rS\left(f(t,x) - x\frac{\partial f}{\partial x}\right) + \frac{1}{2}\sigma^2 S^2\left(\frac{x^2}{S^2}\frac{\partial^2 f}{\partial x^2}\right) - \frac{S-A}{t}\frac{t}{T}\frac{\partial f}{\partial x} - rSf(t,x) = 0.$$

Simplify :

$$\frac{\partial f}{\partial t} - \frac{A}{TS}\frac{\partial f}{\partial x} + \frac{1}{2}\sigma^2 x^2\frac{\partial^2 f}{\partial x^2} - rx\frac{\partial f}{\partial x} - \frac{S-A}{TS}\frac{\partial f}{\partial x} = 0.$$

Which gives :

$$\frac{\partial f}{\partial t} - \left(\frac{1}{T} + rx\right)\frac{\partial f}{\partial x} + \frac{1}{2}\sigma^2 x^2\frac{\partial^2 f}{\partial x^2} = 0.$$

4.4 Finite Difference Schemes

4.4.1 Implicit Euler Method

We now consider the following equation :

$$\frac{\partial f}{\partial t} - \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 f}{\partial x^2} + \left(\frac{1}{T} + rx\right)\frac{\partial f}{\partial x} = 0, \quad t > 0.$$

Subject to the following conditions :

— $f(T, x) = \max(-x, 0)$

— $f(t, x) = 0, \quad x \to -\infty$

— $f(t, x) = 0, \quad x \to +\infty$

We define :

$$x = \frac{K - At/T}{S}, \quad \text{with } S = \frac{K}{x}.$$

From this, we deduce :

$$S = \frac{K}{K - At/T}.$$

The finite difference discretization of the equation gives :

$$-\frac{f_{i,j} - f_{i+1,j}}{\Delta t} - \frac{1}{2}\sigma^2 x_j^2 \frac{f_{i+1,j+1} - 2f_{i+1,j} + f_{i+1,j-1}}{\Delta x^2} + \left(\frac{1}{T} + rx_j\right)\frac{f_{i+1,j+1} - f_{i+1,j-1}}{2\Delta x} = 0$$

Reorganizing to isolate $f_{i,j}$:

$$f_{i+1,j+1} \cdot \left(\frac{\sigma^2 x_j^2}{2\Delta x^2} - \frac{\frac{1}{T} + rx_j}{2\Delta x}\right) + f_{i+1,j} \cdot \left(-\frac{1}{\Delta t} - \frac{\sigma^2 x_j^2}{\Delta x^2}\right) + f_{i+1,j-1} \cdot \left(\frac{\sigma^2 x_j^2}{2\Delta x^2} + \frac{\frac{1}{T} + rx_j}{2\Delta x}\right) = -\frac{f_{i,j}}{\Delta t}.$$

Care must be taken to respect the spatial boundary conditions such as :

$$X_{min} \leq 0,$$

$$X_{max} \geq \frac{K}{S}.$$

Additionally, it should be noted that the scheme is unconditionally stable, which justifies its use.

4.5 Results and Coherence

An Asian call option depends on the average price of the underlying asset over a specific period. The payoff is given by :

$$\text{Payoff} = \max(\bar{A} - K, 0),$$

where :

— $\bar{A}$: The average price of the underlying asset over the observation period.

— K : The strike price of the option.

The behavior of the Asian call option varies depending on the relationship between S_0 (the initial price of the underlying asset) and K (the strike price) :

1. When S_0 is close to K

— The average price $\bar{A}$ is likely to stay close to K, especially if the volatility is low.

— The payoff will be close to zero if $\bar{A} < K$, but it will increase as $\bar{A}$ exceeds K.

— The sensitivity of the option price to changes in S_0 is moderate, as the averaging effect reduces the impact of short-term fluctuations.

2. When $S_0 \gg K$ (Initial price much higher than strike)

- The average price $\bar{A}$ is likely to remain above K, leading to a positive payoff $(\bar{A} - K)$.
- The option price will be relatively high because the likelihood of a positive payoff is significant.
- The averaging effect ensures that the option is less sensitive to extreme upward movements in S_t during the observation period.

3. When $S_0 \ll K$ (Initial price much lower than strike)

- The average price $\bar{A}$ is likely to remain below K, resulting in a zero payoff $(\bar{A} - K = 0)$.
- The option price will be low since the probability of $\bar{A}$ exceeding K is minimal.
- Even if S_t increases significantly during the observation period, the averaging effect reduces the impact on $\bar{A}$, making the option less responsive to large upward movements.

```python
import numpy as np
import matplotlib.pyplot as plt
import time

# PARAMETERS
M = 100                  # Number of time steps
T = 1.0                  # Maturity
K = 100                  # Strike price of the option
S0 = 95                  # Initial price of the underlying asset
r = 0.05                 # Risk-free interest rate
sigma = 0.2              # Volatility
N = 1000                 # Number of price grid points
dt = T / M               # Time step
Xmax = 10.0               # Maximum for the spatial grid (x) must be > K/S
Xmin = -10.0              # Minimum for the spatial grid (x) must be < 0.0
dx = (Xmax - Xmin) / N  # Spatial step

# Initializing matrices and vectors
C = np.zeros((M + 1, N + 1))  # Option price matrix
X = np.linspace(Xmin, Xmax, N + 1)  # Spatial grid
t = np.linspace(0, T, M + 1)        # Time grid

# Boundary conditions
C[M, :] = np.maximum(-X, 0)  # At maturity f(T, x) = max(-x, 0)
C[:, 0] = 0.0                # For x -> - ou  (1/r*T) * (1 - np.exp(-r*(T-t))) ?
C[:, N] = 0.0                # For x ->


# Matrix A for the implicit scheme
main_diag = -1 / dt - (sigma**2 * X[1:N]**2) / dx**2
upper_diag = 0.5 * (sigma**2 * X[2:N]**2 / dx**2) - ((1 / T + r * X[2:N]) / (2 * dx))
lower_diag = 0.5 * (sigma**2 * X[1:N-1]**2 / dx**2) + ((1 / T + r * X[1:N-1]) / (2 * dx))

A = np.diag(main_diag) + np.diag(upper_diag, 1) + np.diag(lower_diag, -1)
```

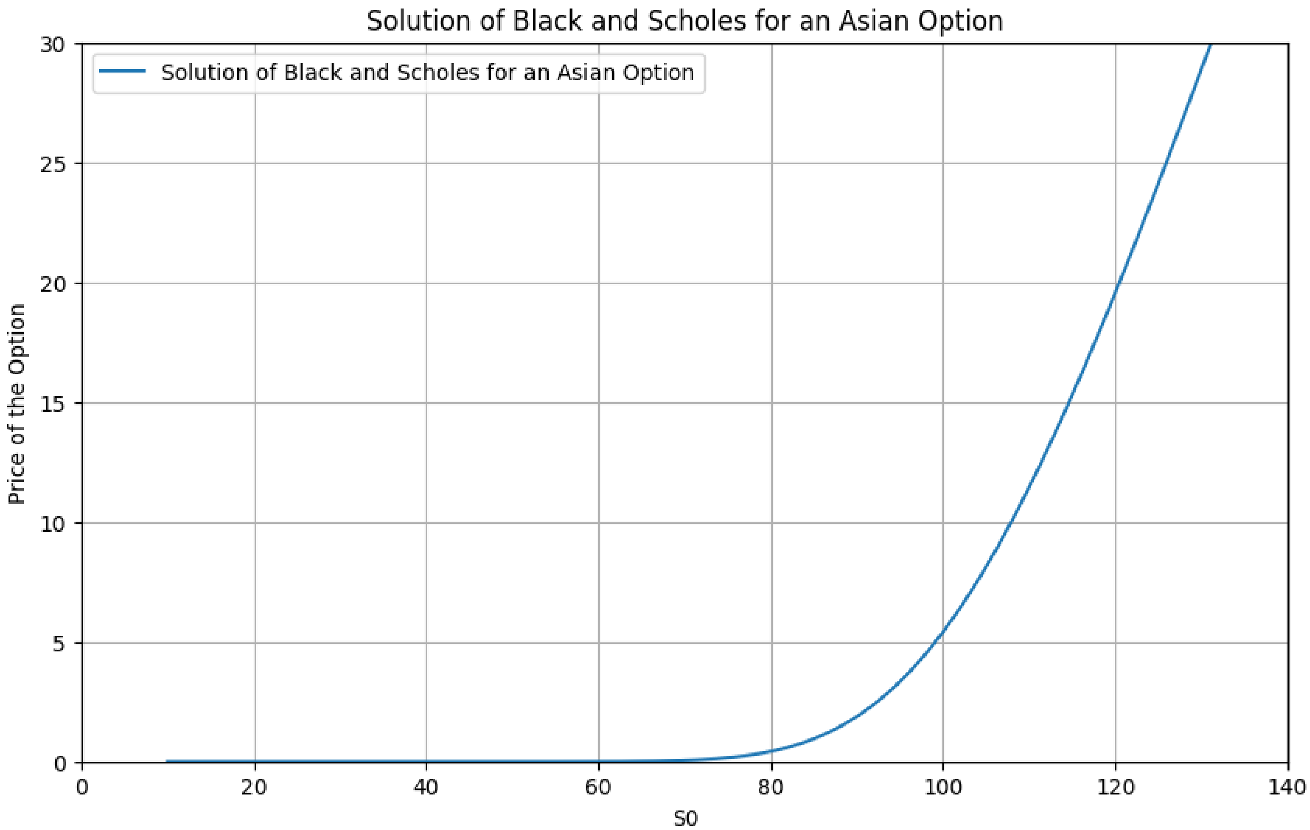

FIGURE 1 – Solution of B.S using Euler Implicit Method

Pricing of the option

$$S0 = 95 \text{ et } K = 100 \rightarrow V = 3.10$$

5 Asian option pricing using Monte-Carlo Method

5.1 Definition of the Asian Option Payoff

An Asian option depends on the average of the prices of the underlying asset $S(t)$ over a given period. The payoff is given by :

— For an **Asian call** :

$$\text{Payoff} = \max\left(\frac{1}{N}\sum_{i=1}^{N} S(t_i) - K, 0\right),$$

— For an **Asian put** :

$$\text{Payoff} = \max\left(K - \frac{1}{N}\sum_{i=1}^{N} S(t_i), 0\right),$$

where :

— K is the strike price,

— $S(t_i)$ is the price of the underlying asset at time t_i,

— N is the number of time points.

5.2 Underlying Asset Dynamics

The dynamics of the price of the underlying asset $S(t)$ follow the Black-Scholes model, defined by the following stochastic differential equation :

$$dS(t) = rS(t)dt + \sigma S(t)dW(t),$$

where :

- r is the risk-free rate,
- σ is the volatility,
- $W(t)$ is a standard Brownian motion.

5.3 Discretization of the Dynamics

To simulate the trajectories of $S(t)$, we discretize the previous equation using the explicit Euler scheme :

$$S_{i+1} = S_i \exp\left((r - \frac{\sigma^2}{2})\Delta t + \sigma\sqrt{\Delta t}Z_i\right),$$

where :

- $\Delta t = \frac{T}{N}$ is the time step,
- $Z_i \sim \mathcal{N}(0,1)$ are independent standard normal random variables.

5.4 Monte Carlo Simulation

The Monte Carlo method consists of the following steps :

1. Generate M simulated trajectories of $S(t)$ for different time steps t_i following the discretized dynamics.
2. Compute the arithmetic average of the simulated prices for each trajectory :

$$\bar{A} = \frac{1}{N}\sum_{i=1}^{N} S(t_i).$$

3. Calculate the payoff for each trajectory :

$$\text{Payoff} = \max\left(\bar{A} - K, 0\right) \quad \text{(Asian call)}.$$

5.5 Price of the Asian Option

The price of the Asian option is given by the present value of the expected payoff under the risk-neutral measure. With Monte Carlo, this translates to :

$$V_0 = e^{-rT} \cdot \frac{1}{M}\sum_{j=1}^{M} \text{Payoff}_j,$$

where :

- e^{-rT} is the discount factor,
- Payoff_j is the simulated payoff for the j-th trajectory.

5.6 Results and Analysis

The price obtained using Monte Carlo converges to the true value as the number of simulations M becomes very large. The use of variance reduction techniques, such as control variates, can improve accuracy and reduce error. This is a method we did not implement in this project. Additionally, the volatility is constant in this Monte Carlo model. Below are the parameters used for the following simulation :

```python
# PARAMETERS
np.random.seed(42) # the number of life :)
M = 1000           # number of simulation by the M.C method
T = 1              # maturity
K = 205            # strike of the option
S0 = 200           # initial price
r = 0.035          # rate
sigma = 0.3        # volatility
n = 100000         # number of values to get to compute the mean of S
dt = T / n         # time step for edp resolution
```

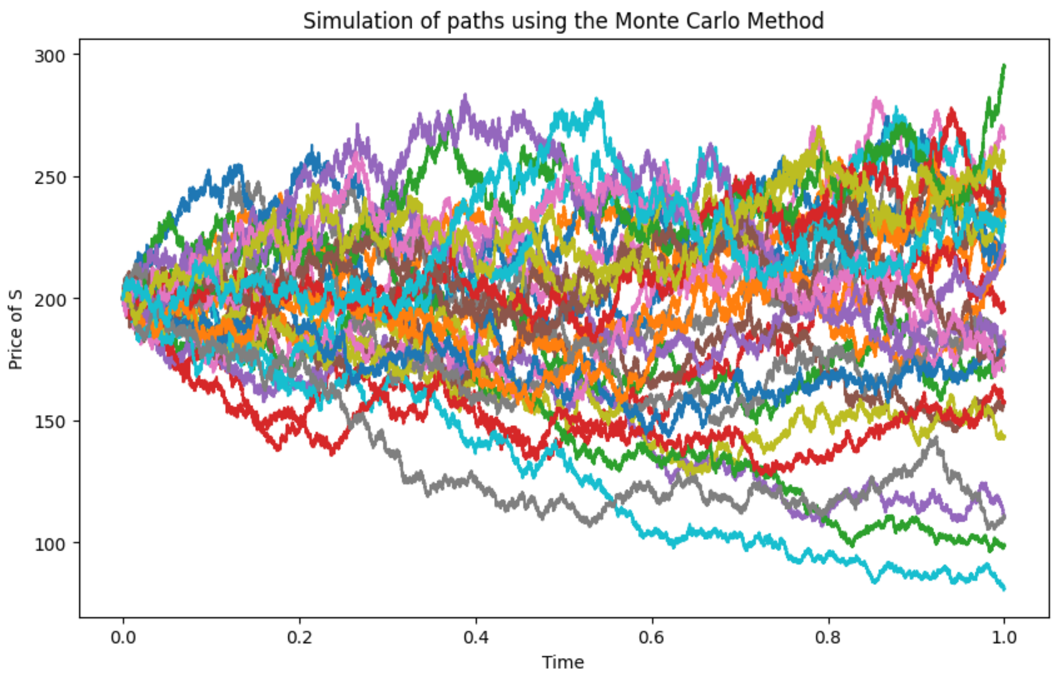

FIGURE 2 – Simulation and Display of 30 paths using Monte Carlo Method

Pricing of the option

$$S0 = 95 \text{ et } K = 100 \rightarrow V = 3.17$$

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