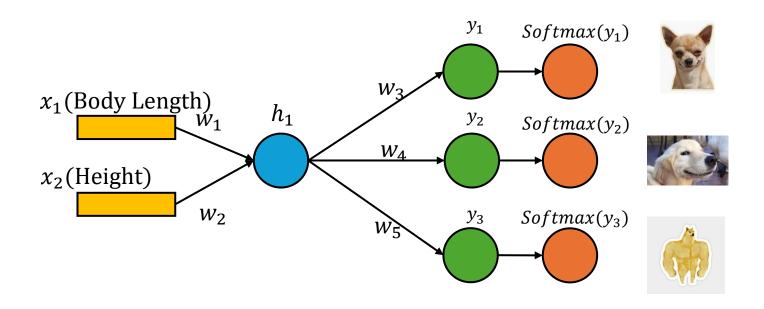
## Softmax Cross Entropy



$$CE = -\sum_{i=1}^{n} Observed \cdot \log(P_i)$$

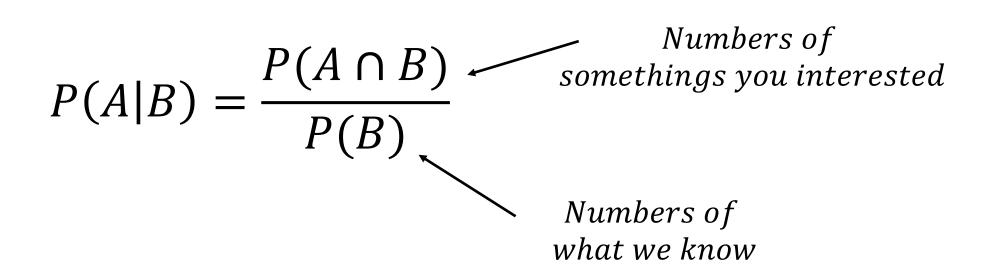
#### Classification Neural Network





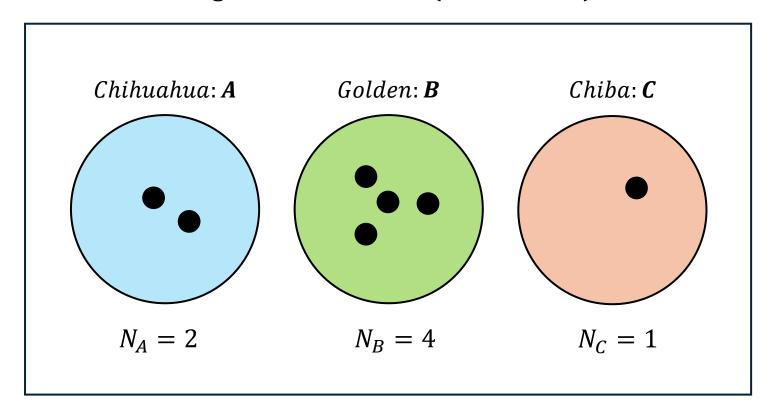
### Why Softmax + Cross Entropy is works with classification???

#### **Conditional Probabilities**



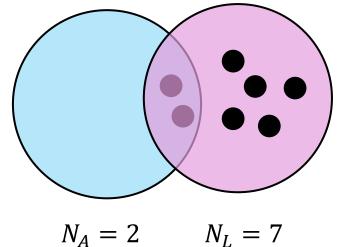
#### **Conditional Probabilities**

Dog Lovers Land. (Short in **L**)



Total Populations =  $N_A + N_B + N_C = 7$ 

Chihuahua: A Dog Lovers Land: L



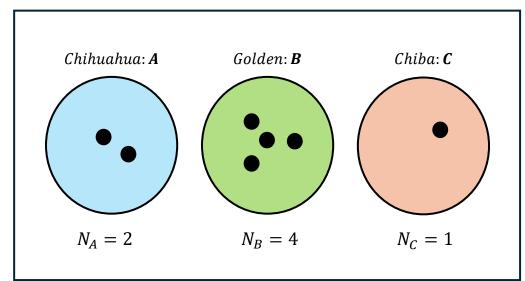
$$P(A|L) = \frac{P(A \cap L)}{P(L)}$$

$$P(A|L) = \frac{Number\ of\ A\cap L}{Number\ of\ L}$$

$$P(A|L) = \frac{2}{7} = 0.29$$

Range of probability: [0, 1]

#### Dog Lovers Land. (Short in L)



$$Total\ Populations = N_A + N_B + N_C = 7$$

Chihuahua 
$$\rightarrow P(A|L) = \frac{2}{7} = 0.29$$

$$Golden \rightarrow P(B|L) = \frac{4}{7} = 0.57$$

Chiba 
$$\to P(C|L) = \frac{1}{7} = 0.14$$

$$P(A|L) + P(B|L) + P(C|L) = 0.29 + 0.14 + 0.57$$

$$P(A|L) + P(B|L) + P(C|L) = 1$$

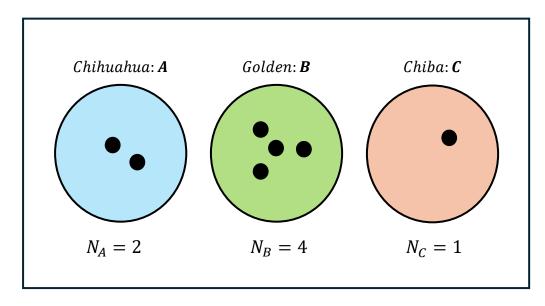
#### Why we need Softmax???

#### Softmax

$$Softmax(x_i) = \frac{e^{x_i}}{\sum_{j=1}^{n} e^{x_j}}$$

It can normalize outputs to scale within the range of [0, 1].

#### Dog Lovers Land. (Short in L)



 $Total\ Populations = N_A + N_B + N_C = 7$ 

$$Softmax(x_i) = \frac{e^{x_i}}{\sum_{j=1}^{n} e^{x_j}}$$

$$Softmax(x_A) = \frac{e^{x_A}}{e^{x_A} + e^{x_B} + e^{x_C}}$$

$$Softmax(x_B) = \frac{e^{x_B}}{e^{x_A} + e^{x_B} + e^{x_C}}$$

$$Softmax(x_C) = \frac{e^{x_C}}{e^{x_A} + e^{x_B} + e^{x_C}}$$

$$Softmax(x_A) = \frac{e^{x_A}}{e^{x_A} + e^{x_B} + e^{x_C}} = \frac{e^{0.29}}{e^{0.29} + e^{0.57} + e^{0.14}} = 0.31$$

$$Softmax(x_B) = \frac{e^{x_B}}{e^{x_A} + e^{x_B} + e^{x_C}} = \frac{e^{0.57}}{e^{0.29} + e^{0.57} + e^{0.14}} = 0.42$$

$$Softmax(x_C) = \frac{e^{x_C}}{e^{x_A} + e^{x_B} + e^{x_C}} = \frac{e^{0.14}}{e^{0.29} + e^{0.57} + e^{0.14}} = 0.27$$

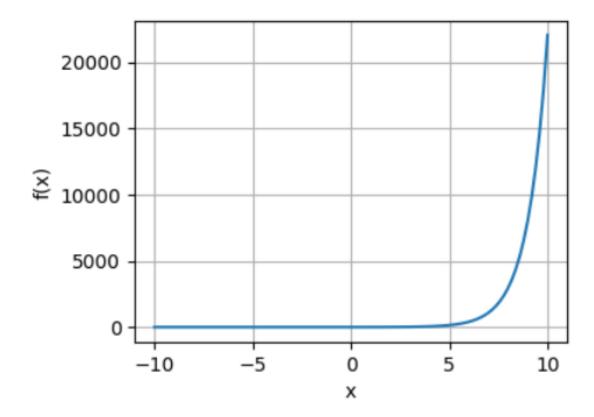
$$Softmax(x_A) + Softmax(x_B) + Softmax(x_C) = 1.0$$

### If you just need to scale it into range of [0,1]

Why not we just use normalization??

# The output from sum of node in neural network maybe negative number

$$f(x) = e^x$$



*Range of output always be*:  $(0, +\infty)$ 

Furthermore, the **Softmax** can boost the value of output to closer of the expectation.

#### **Example**

$$Output = [10, 20]$$

**Normalization** = 
$$\left[\frac{10}{10+20}, \frac{20}{10+20}\right] = [0.333..., 0.666...]$$

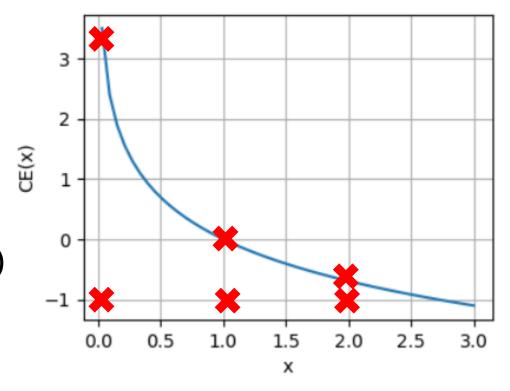
$$Softmax = \left[\frac{e^{10}}{e^{10} + e^{20}}, \frac{e^{20}}{e^{10} + e^{20}}\right] = [0.00 \dots, 0.999 \dots]$$

## Next, Why we always use Cross Entropy with Softmax ???

#### **Cross Entropy**

$$CE = -\sum_{i=1}^{n} Observed \cdot \log(P_i)$$

$$CE = -\sum_{i=1}^{n} Observed \cdot \log(Softmax_i)$$



Therefore, If we try to minimize the Cross Entropy
The error is going to less.

