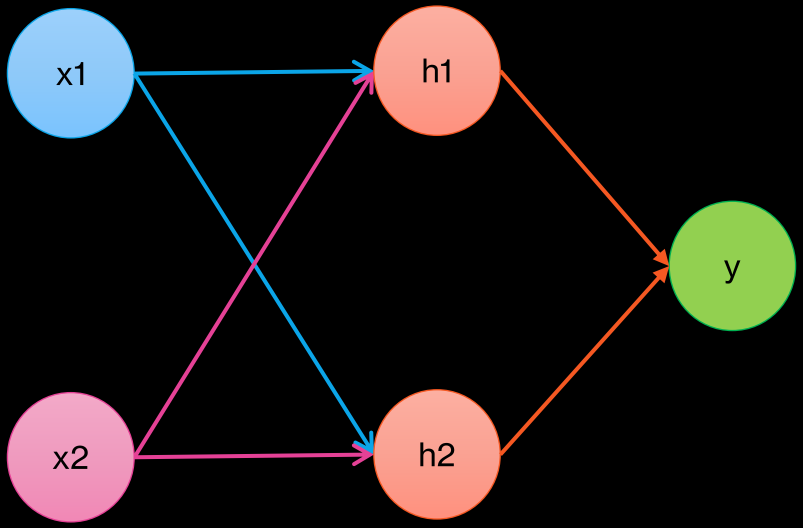
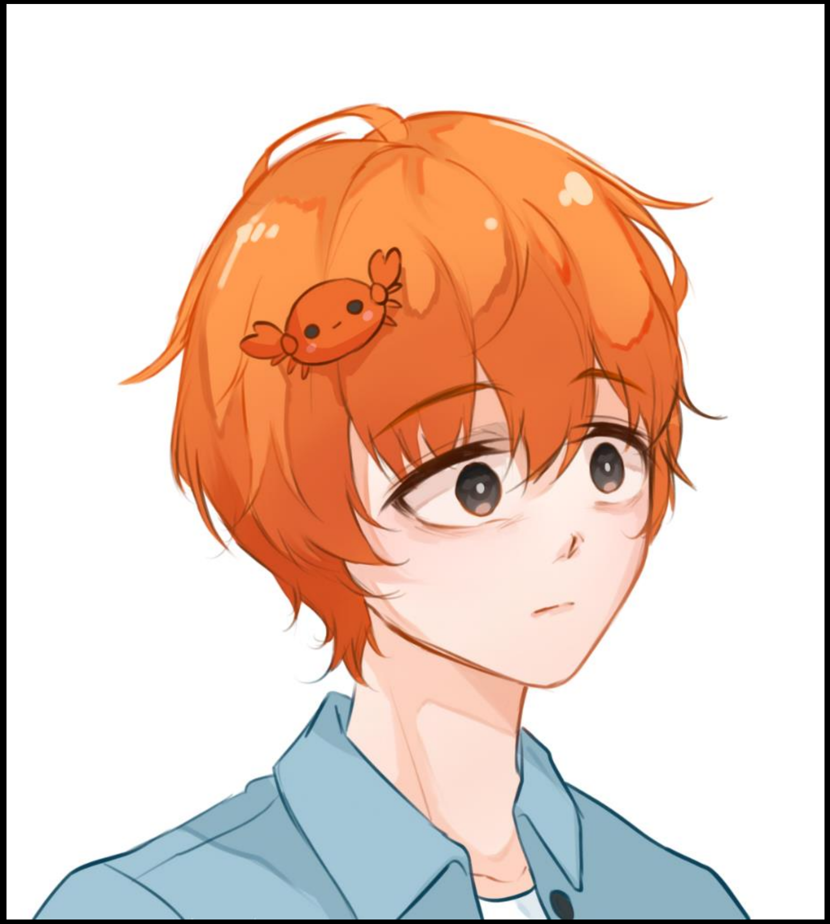
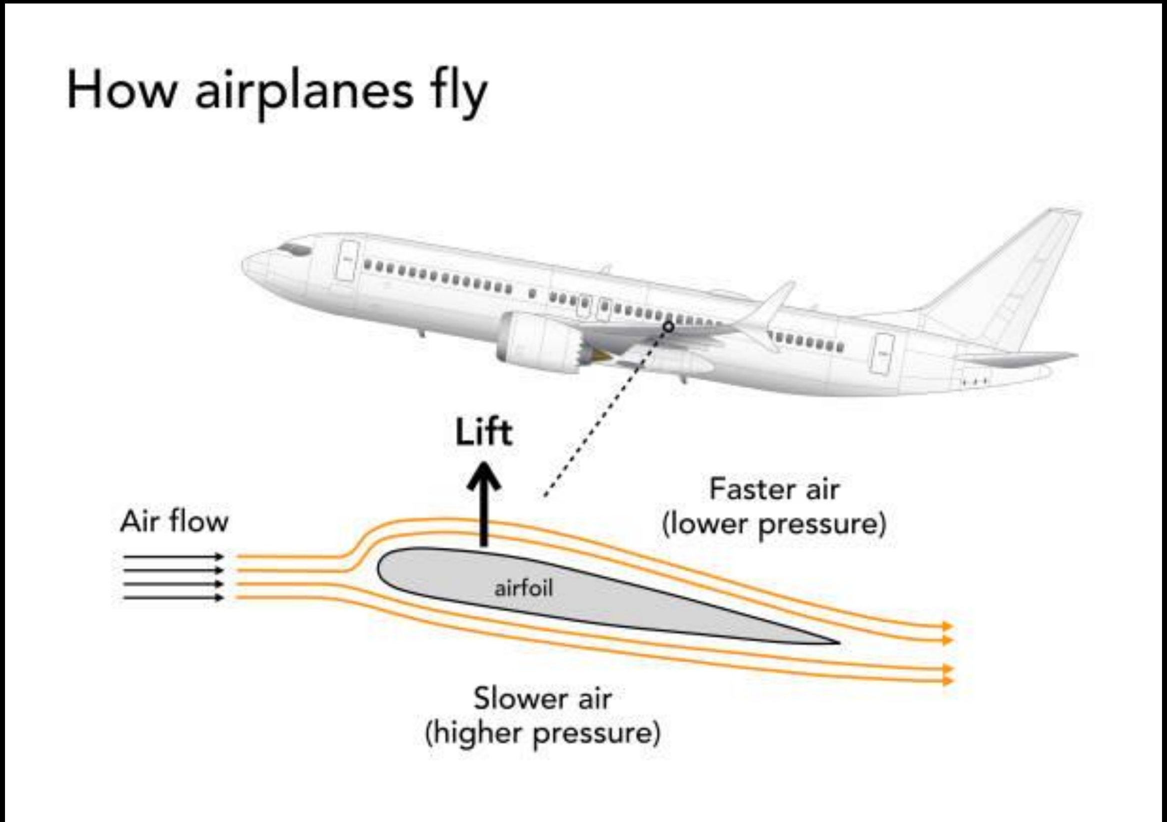
Neural Network

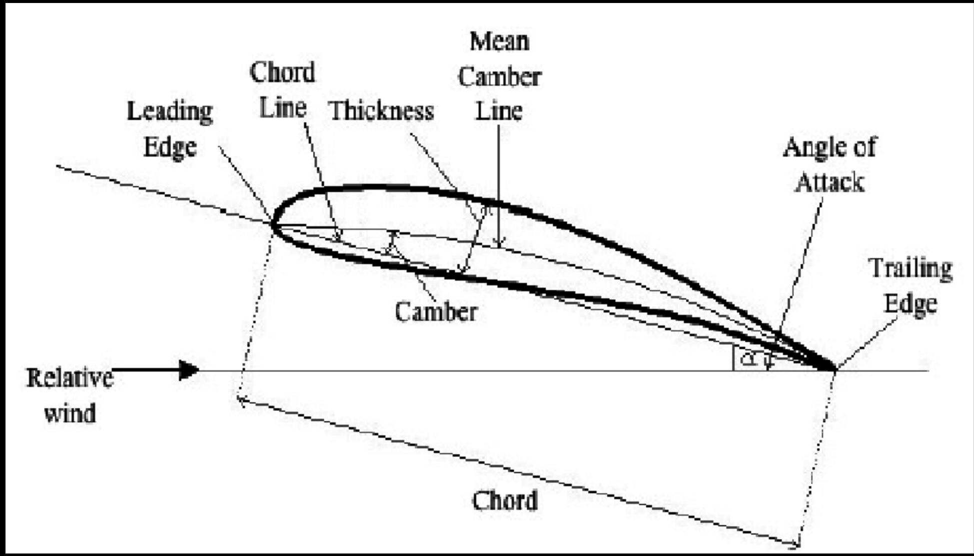




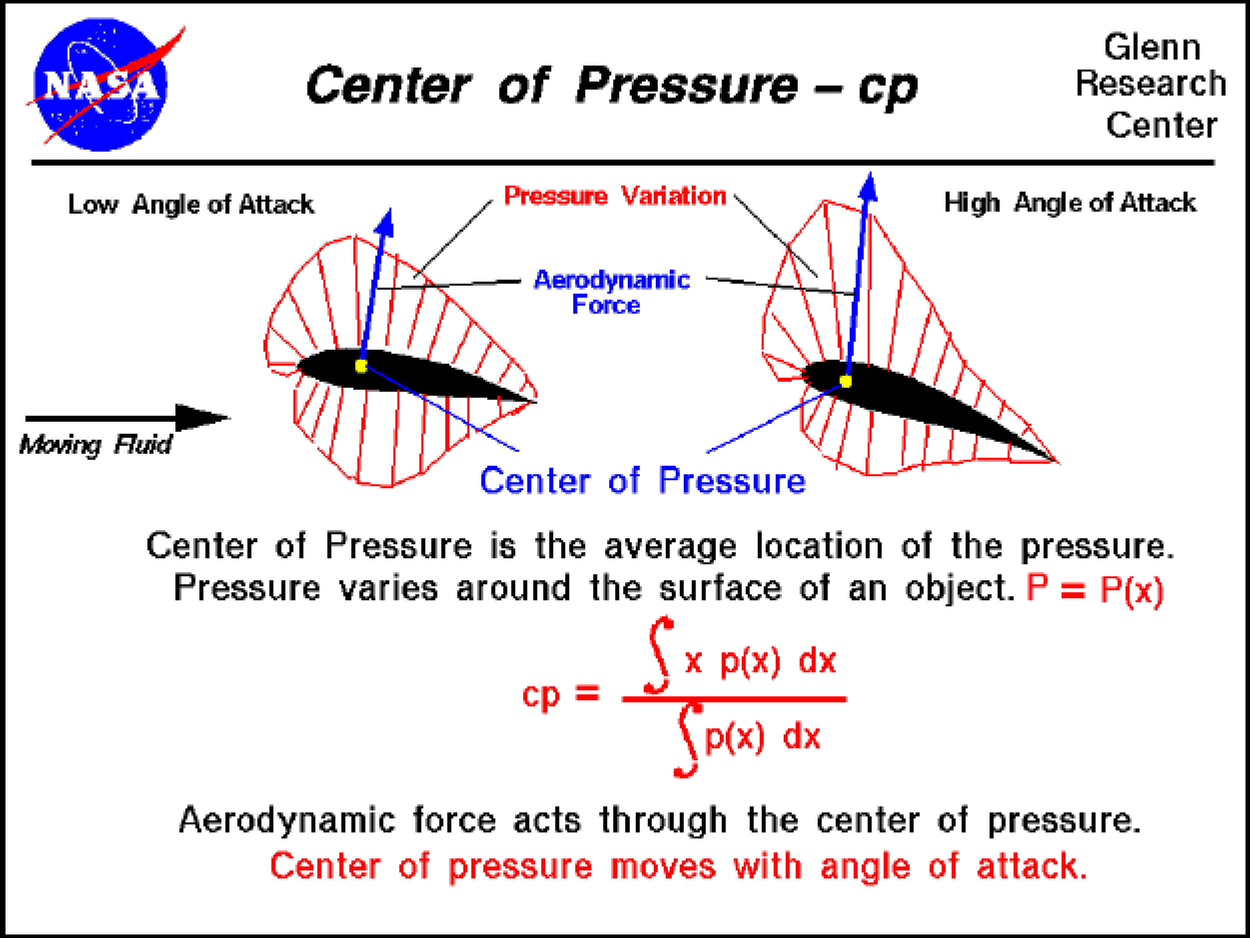
Dancing with My Code

Usecase


How airplanes fly
Lift
Faster air
(lower pressure)
Air flow
airfoil
Slower air
(higher pressure)


Mean
Camber
Line
Chord
Line
Thickness
Leading
Edge
Angle of
Attack
Trailing
Edge
Camber
Relative
wind
Chord

Airfoil pressure distribution



Navier-Stokes Equations

Navier–Stokes Equations
3 – dimensional – unsteady

Glenn Research Center

Coordinates: (x,y,z)

Velocity Components: (u,v,w)

Time : t

Density: ρ

Total Energy: Et

Pressure: p

Stress: τ

Heat Flux: q

Reynolds Number: Re

Prandtl Number: Pr

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X – Momentum:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right]$$

Y – Momentum:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right]$$

Z – Momentum

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right]$$

Energy:

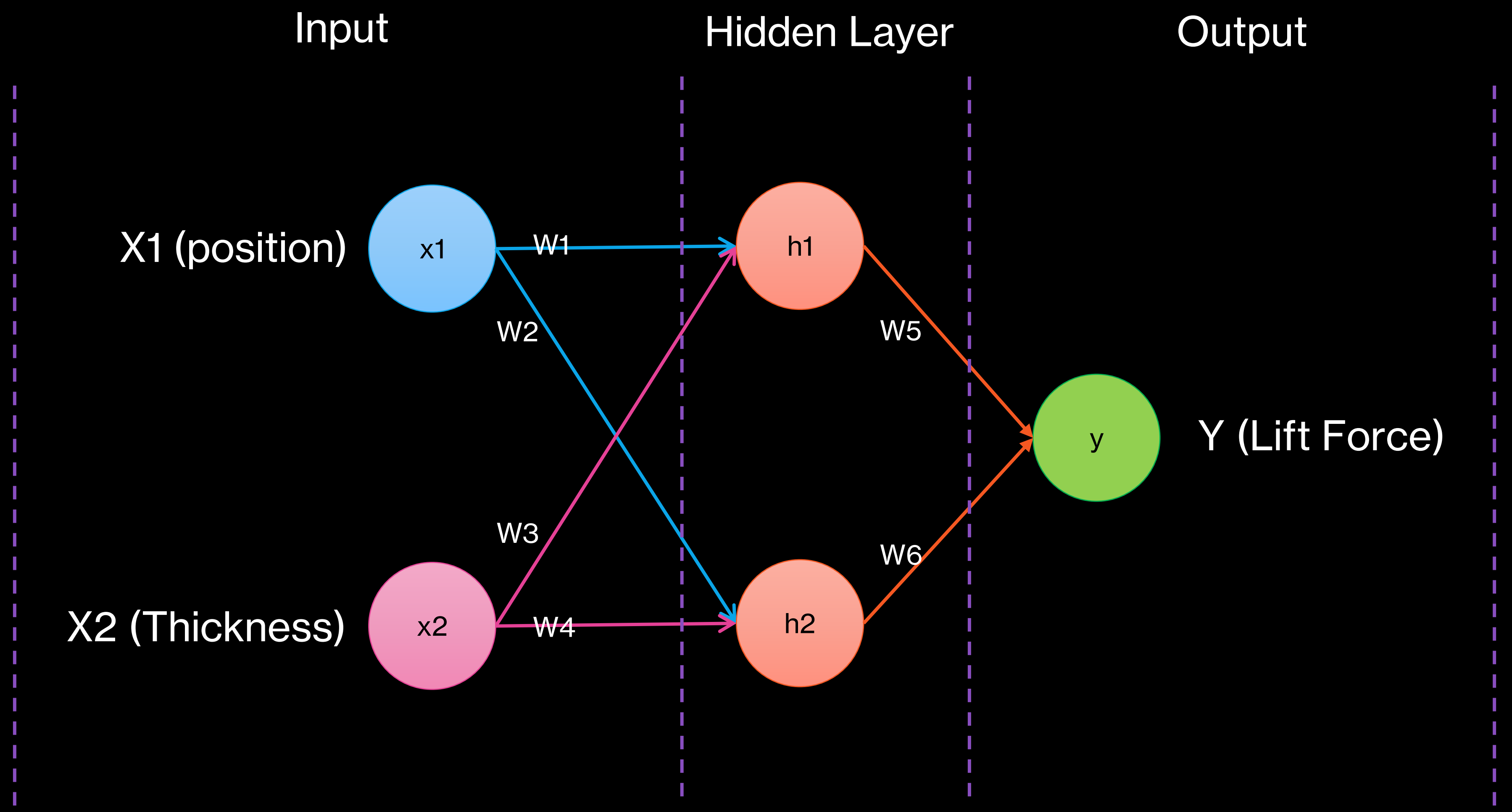
$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z} - \frac{1}{Re_r Pr_r}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$$

$$+ \frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u\,\tau_{xx} + v\,\tau_{xy} + w\,\tau_{xz}) + \frac{\partial}{\partial y}(u\,\tau_{xy} + v\,\tau_{yy} + w\,\tau_{yz}) + \frac{\partial}{\partial z}(u\,\tau_{xz} + v\,\tau_{yz} + w\,\tau_{zz})\right]$$

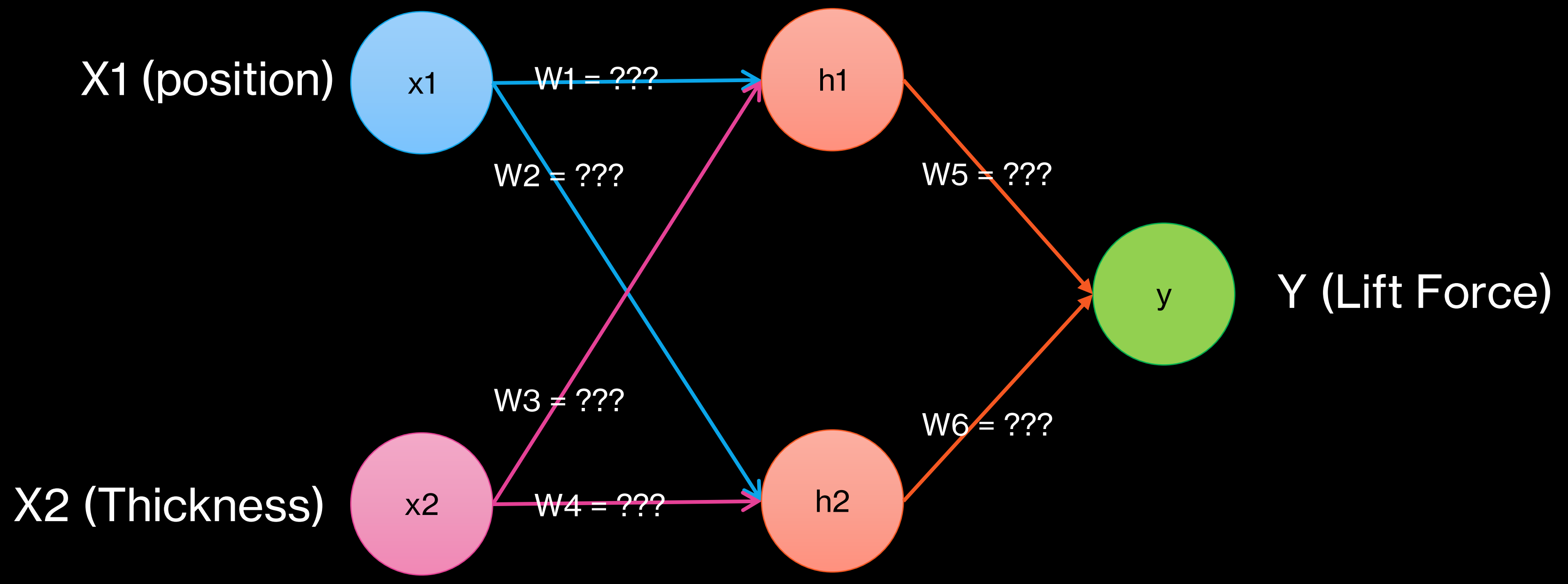
Datasets

X1 (position in unit)	X2 (max thickness)	Y (lift in unit)
0.6	0.2	1.0
0.4	0.1	0.8
0.5	0.4	0.6
0.3	0.3	0.7
0.3	0.05	0.5

Neural Network

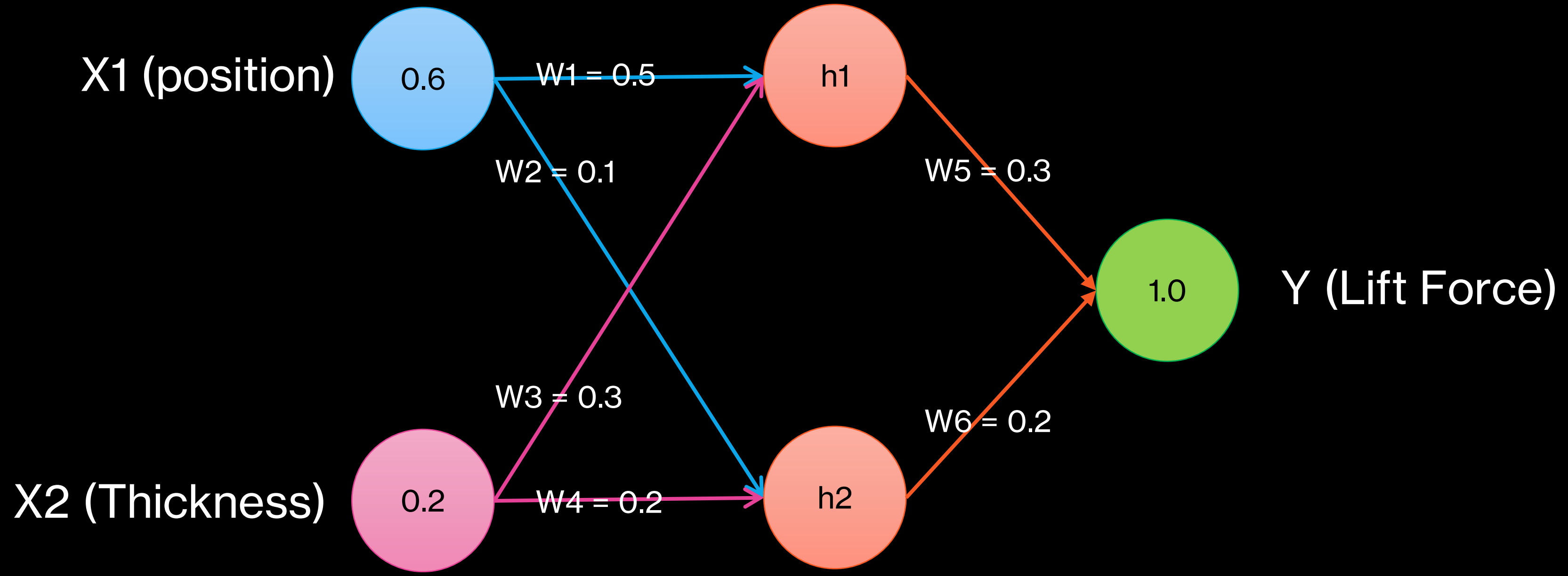


Forward Propagation

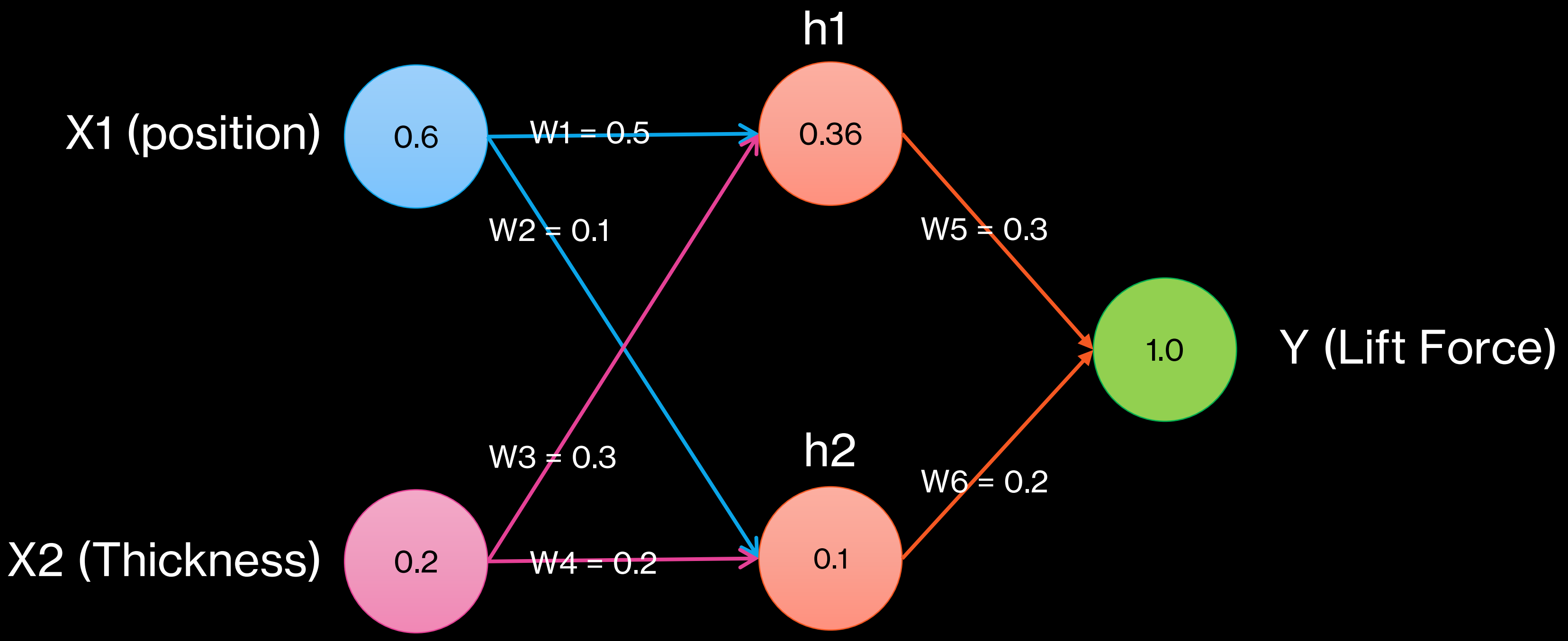


Forward Propagation

Random all weights



Forward Propagation

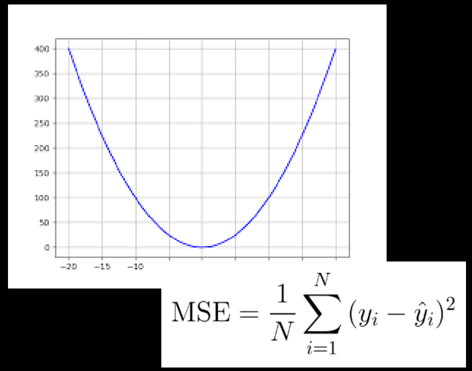


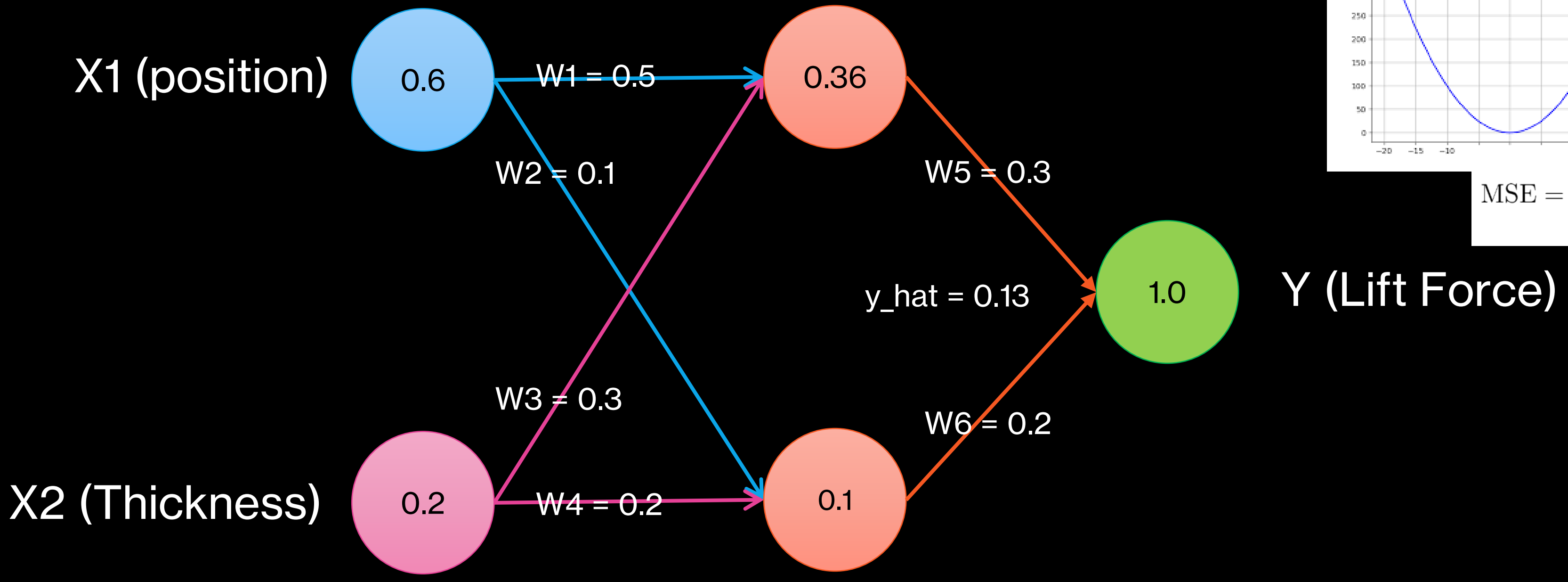
$$h_1 = (x_1 w_1 + x_2 w_3) = (0.6)(0.5) + (0.2)(0.3) = 0.36$$

$$h_2 = (x_1 w_2 + x_2 w_4) = (0.6)(0.1) + (0.2)(0.2) = 0.1$$

Forward Propagation

MSE loss function



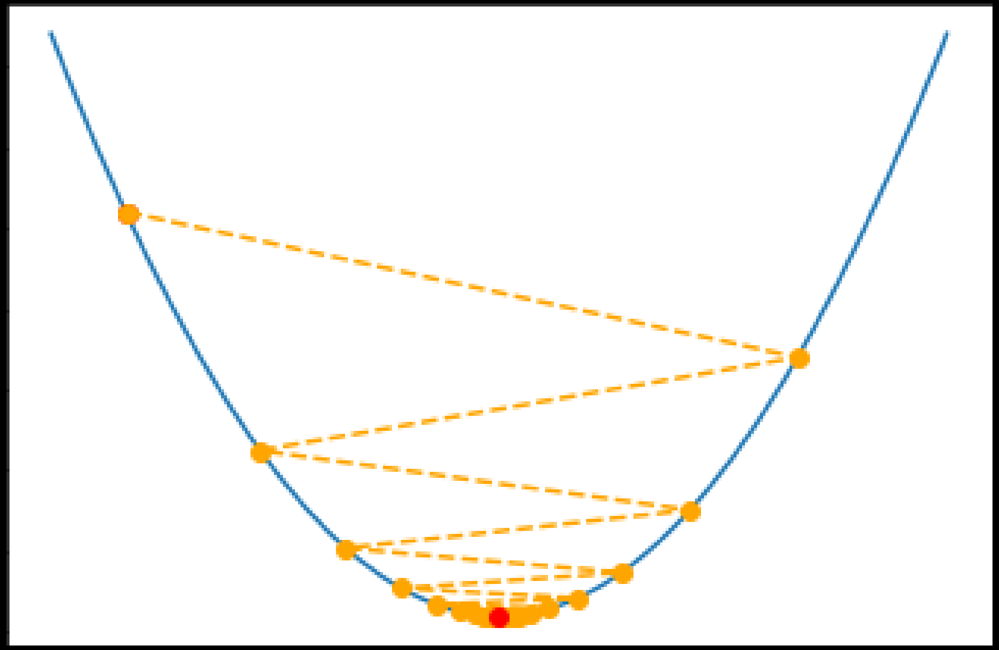


$$y = (h_1 w_5 + h_2 w_6) = ((0.36)(0.3) + (0.1)(0.2)) = 0.13$$

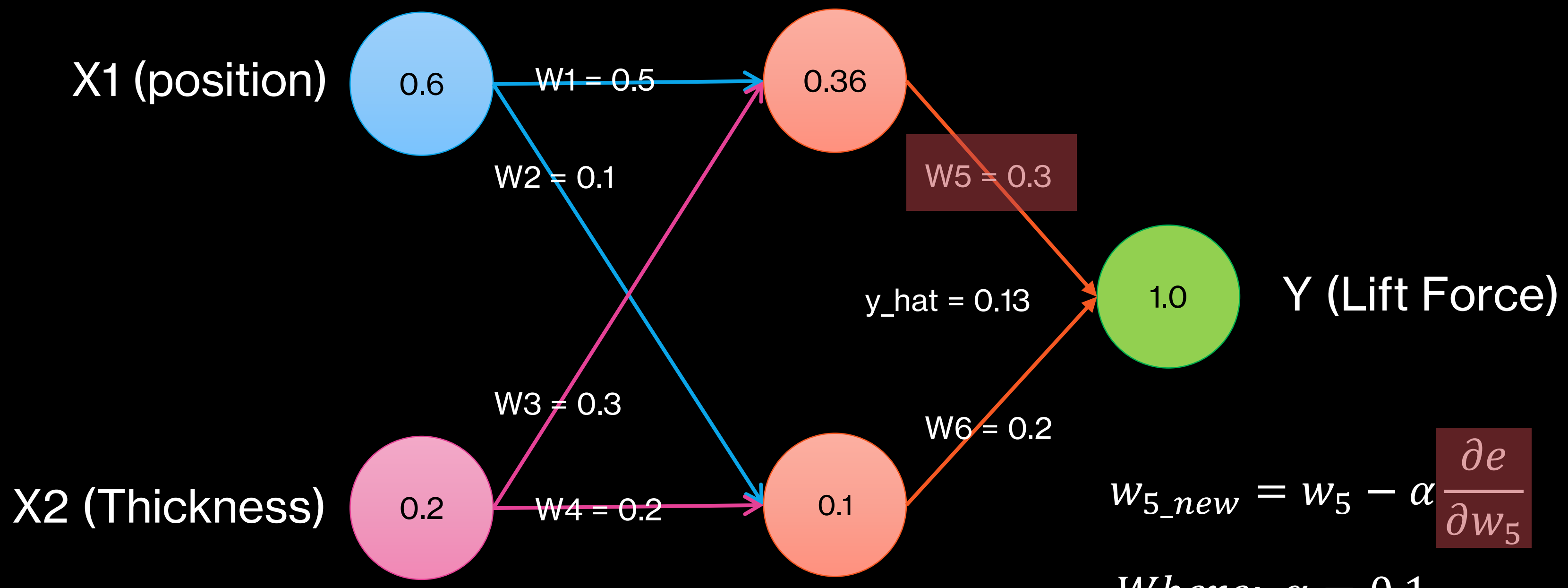
$$error = \frac{1}{n}\sum_{i}^{n}(y - \hat{y})^2 = \frac{1}{1}\sum_{i}^{n}(1 - 0.13)^2 = 0.76$$

Gradient Descent Optimization

$$w_{next} = w_{current} - \alpha \frac{\partial e}{\partial w}$$



Back Propagation



$$w_{5\_new} = w_5 - \alpha \frac{\partial e}{\partial w_5}$$

$$Where: \alpha = 0.1$$

Back Propagation

Let's apply the chain rule.

$$\frac{\partial e}{\partial w_5} = ???$$

$$= \frac{\partial e}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_5}$$

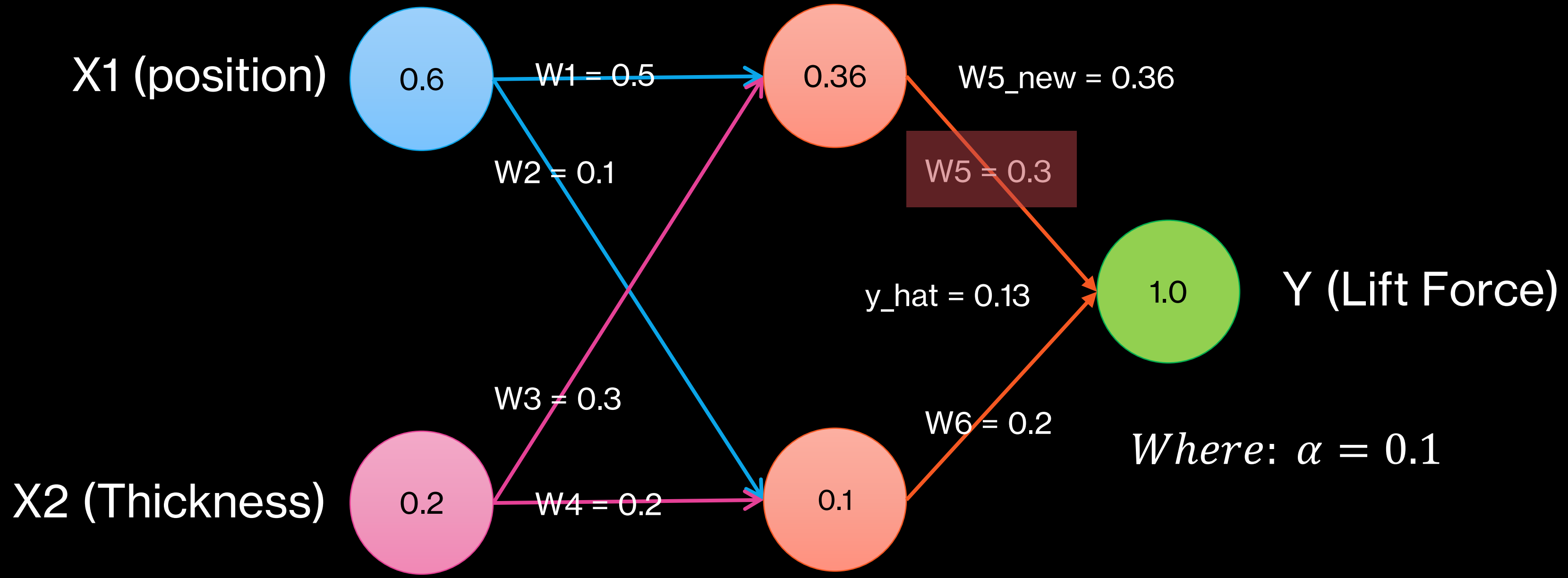
$$= \frac{\partial (y - \hat{y})^2}{\partial \hat{y}} \cdot \frac{\partial (h_1 w_5 + h_2 w_6)}{\partial w_5}$$

$$= -2(y - \hat{y}) \cdot h_1$$

$$= -2(1 - 0.13)(0.36)$$

$$= -0.63$$

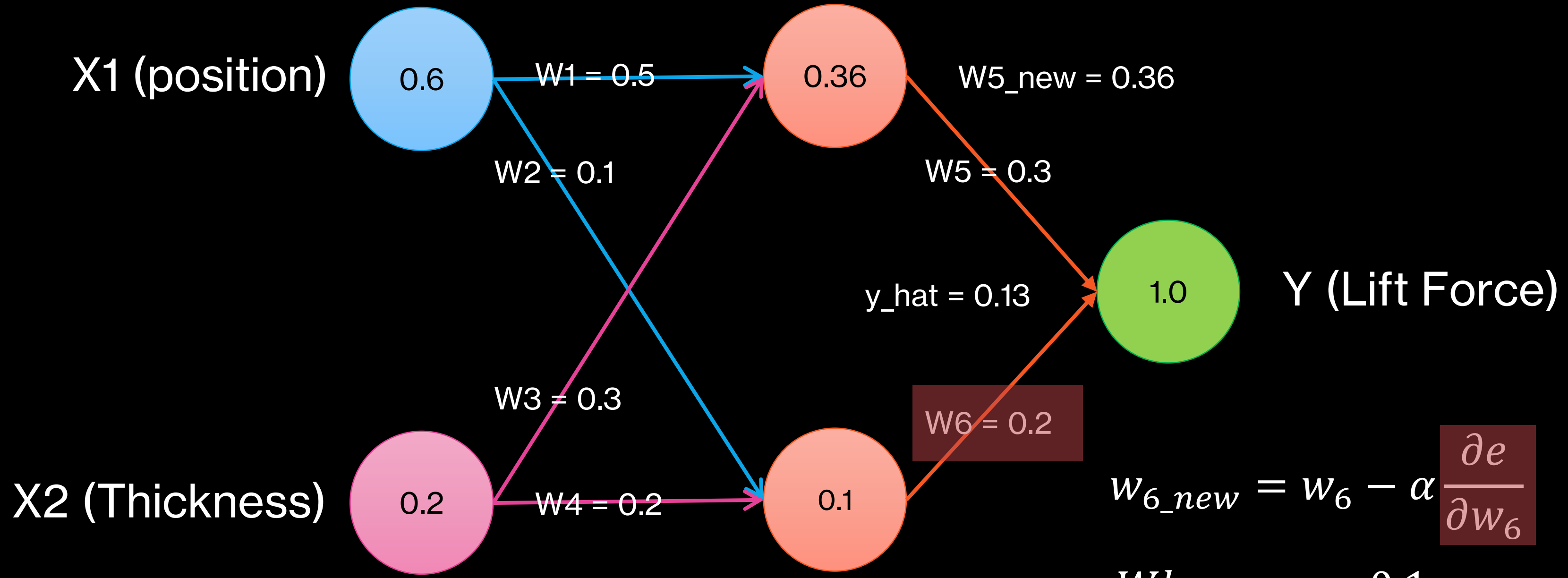
Back Propagation



$Where: \alpha = 0.1$

$$w_{5\_new} = w_5 - \alpha \frac{\partial e}{\partial w_5} = 0.3 - (0.1)(-0.63) = 0.36$$

Back Propagation



$$w_{6\_new} = w_6 - \alpha \frac{\partial e}{\partial w_6}$$

$$Where:\ \alpha = 0.1$$

Back Propagation

Let's apply the chain rule.

$$\frac{\partial e}{\partial w_6} = ???$$

$$= \frac{\partial e}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial w_6}$$

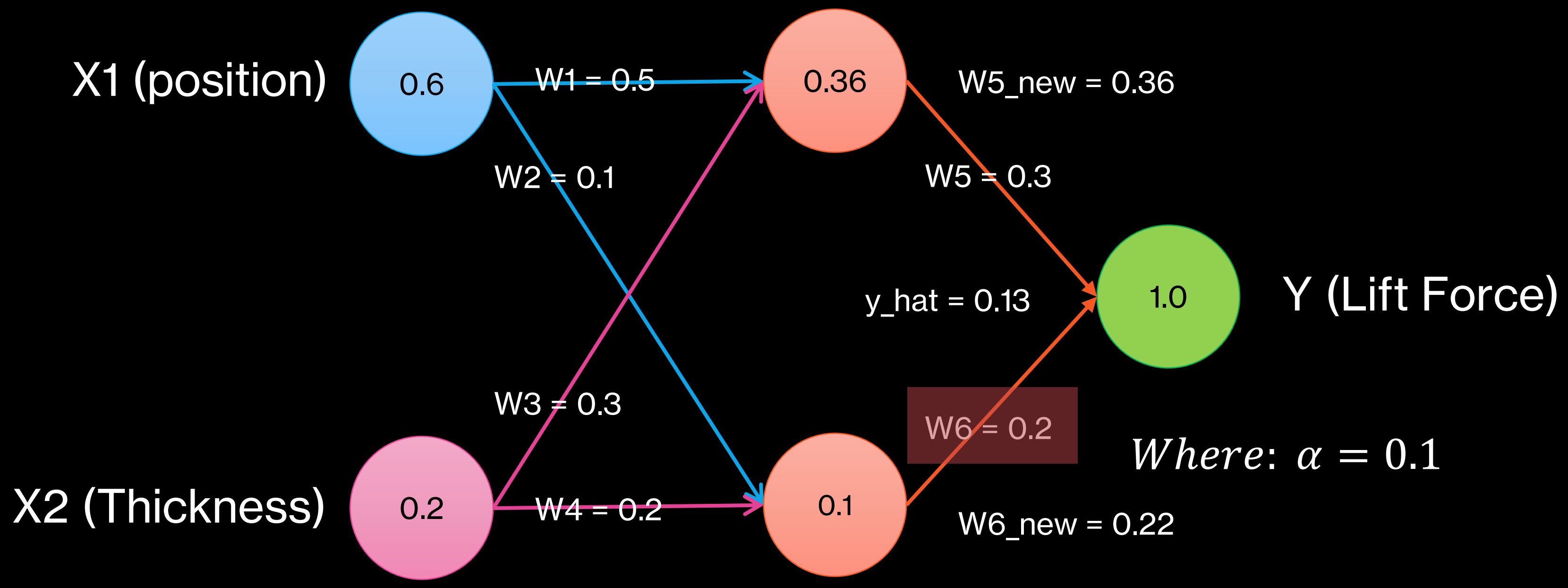
$$= \frac{\partial (y - \hat{y})^2}{\partial \hat{y}} \cdot \frac{\partial (h_1 w_5 + h_2 w_6)}{\partial w_6}$$

$$= -2(y - \hat{y}) \cdot h_2$$

$$= -2(1 - 0.13)(0.1)$$

$$= -0.17$$

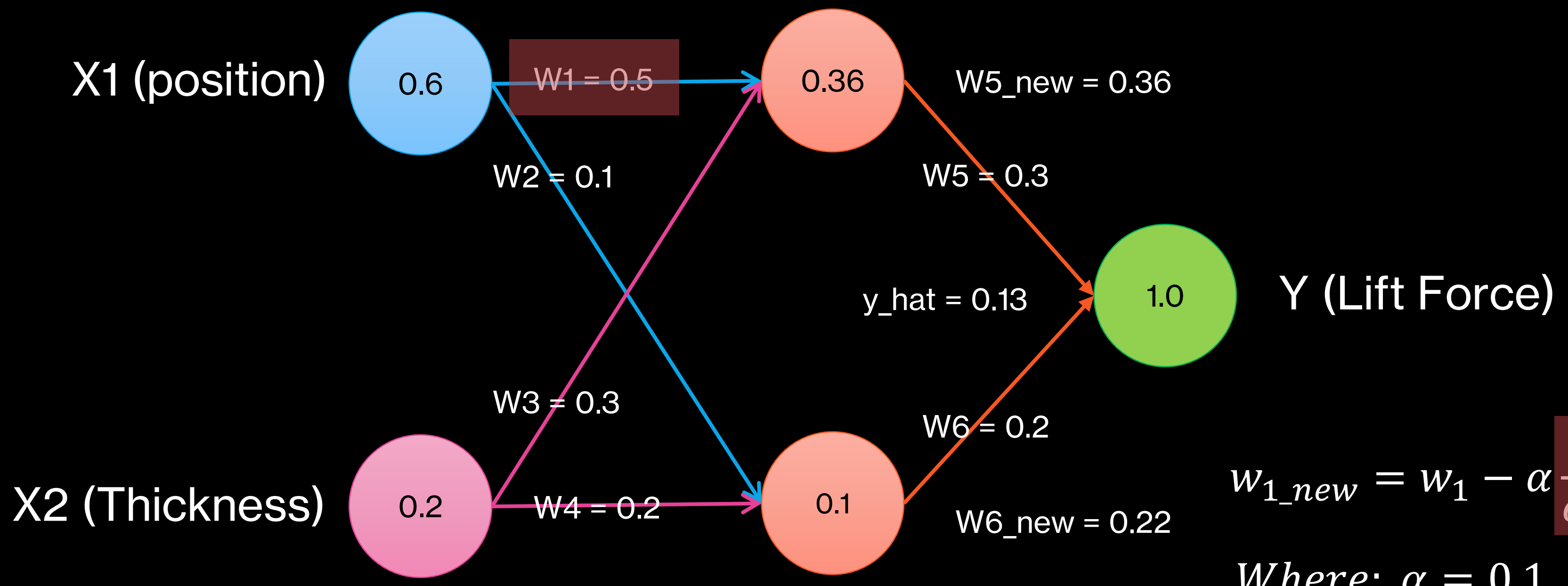
Back Propagation



$Where\colon\ \alpha = 0.1$

$$w_{6\_new} = w_6 - \alpha \frac{\partial e}{\partial w_6} = 0.2 - (0.1)(-0.17) = 0.22$$

Back Propagation



$$w_{1\_new} = w_1 - \alpha \frac{\partial e}{\partial w_1}$$

$$Where:\ \alpha = 0.1$$

Back Propagation

Let's apply the chain rule.

$$\frac{\partial e}{\partial w_1} = ???$$

$$= \frac{\partial e}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h_1} \cdot \frac{\partial h_1}{\partial w_1}$$

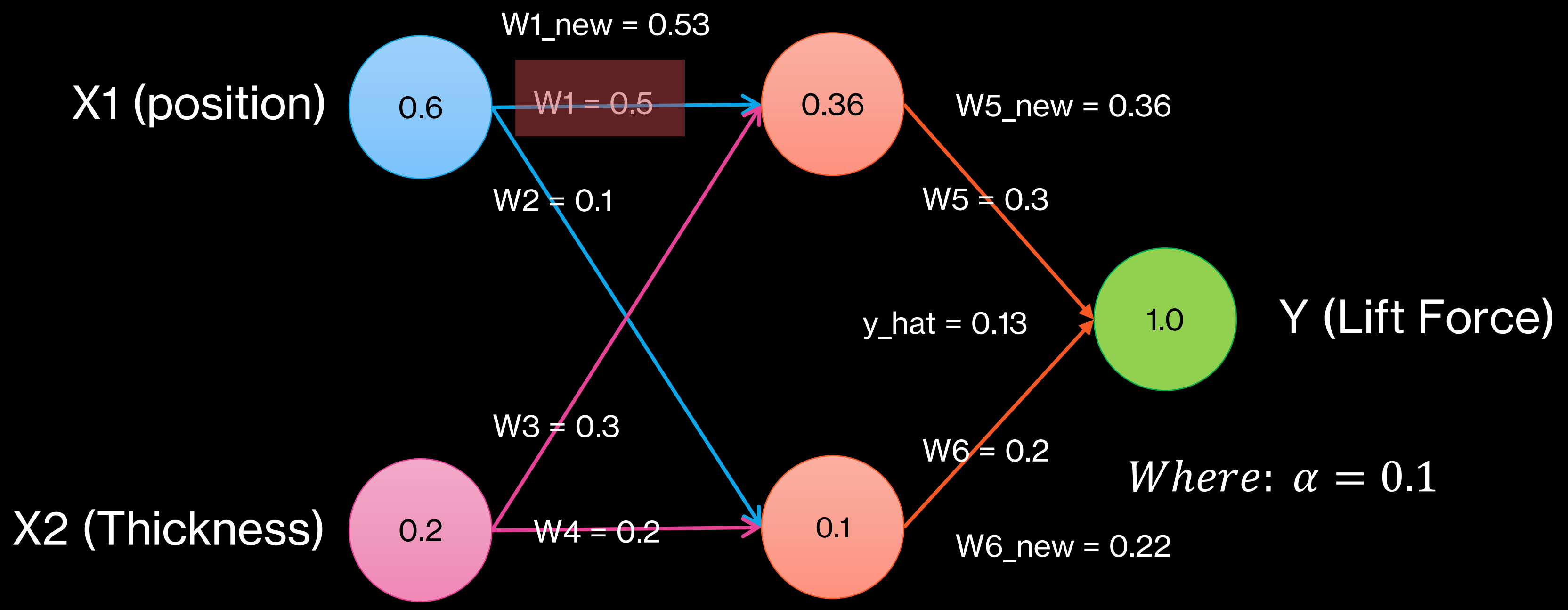
$$= \frac{\partial (y - \hat{y})^2}{\partial \hat{y}} \cdot \frac{\partial (h_1 w_5 + h_2 w_6)}{\partial h_1} \cdot \frac{\partial (x_1 w_1 + x_2 w_3)}{\partial w_1}$$

$$= -2(y - \hat{y}) \cdot w_5 \cdot x_1$$

$$= -2(1 - 0.13)(0.3)(0.6)$$

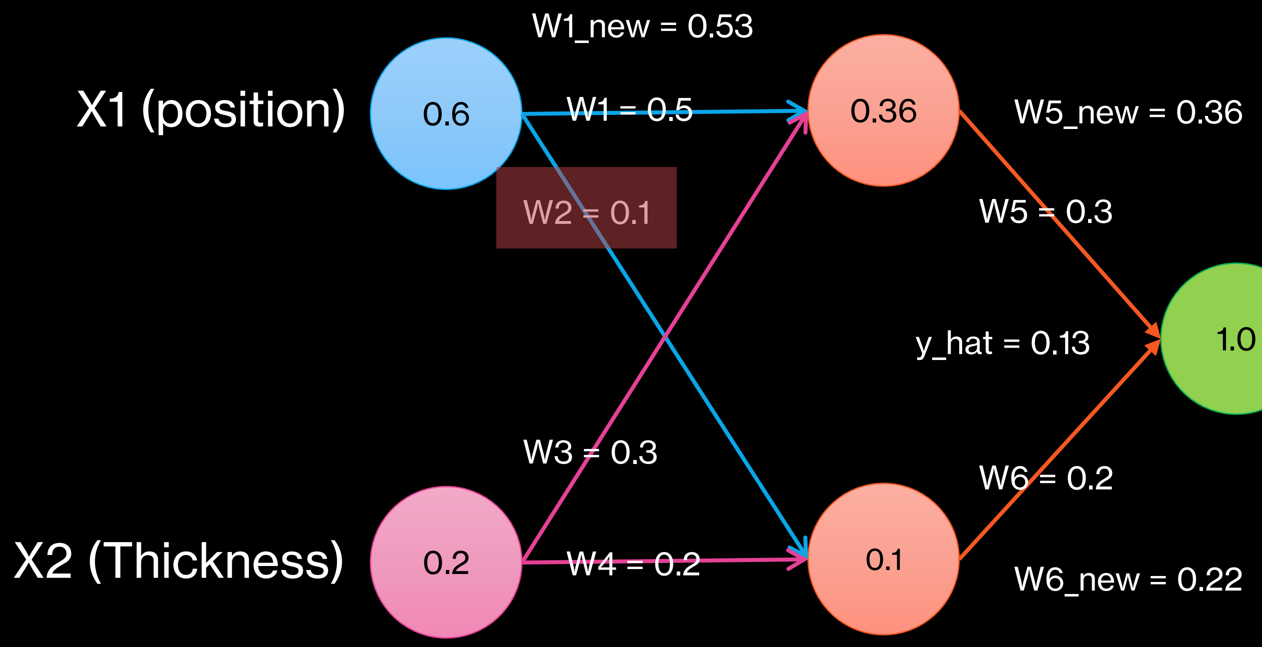
$$= -0.31$$

Back Propagation



$$w_{1\_new} = w_1 - \alpha \frac{\partial e}{\partial w_1} = 0.5 - (0.1)(-0.31) = 0.53$$

Back Propagation



$$w_{2\_new} = w_2 - \alpha \frac{\partial e}{\partial w_2}$$

$$Where\colon\ \alpha = 0.1$$

Back Propagation

Let's apply the chain rule.

$$\frac{\partial e}{\partial w_2} = ???$$

$$= \frac{\partial e}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h_2} \cdot \frac{\partial h_2}{\partial w_2}$$

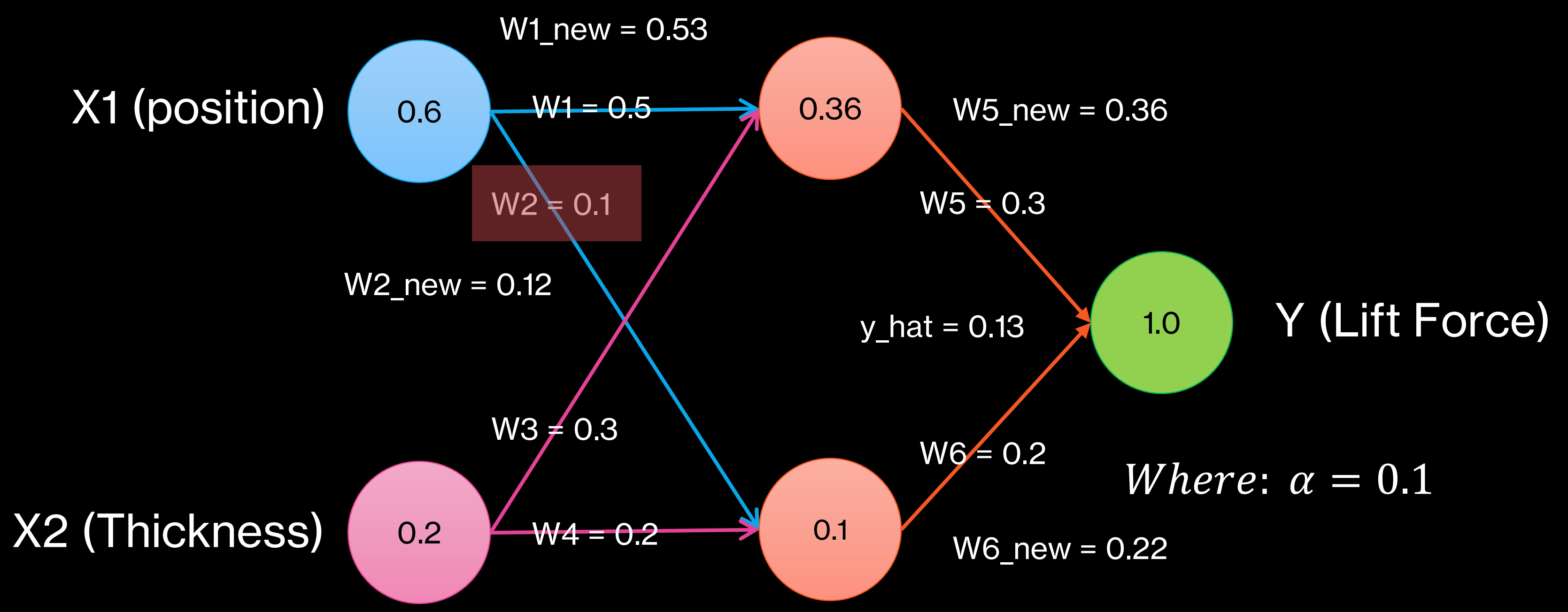
$$= \frac{\partial (y - \hat{y})^2}{\partial \hat{y}} \cdot \frac{\partial (h_1 w_5 + h_2 w_6)}{\partial h_2} \cdot \frac{\partial (x_1 w_2 + x_2 w_4)}{\partial w_2}$$

$$= -2(y - \hat{y}) \cdot w_6 \cdot x_1$$

$$= -2(1 - 0.13)(0.2)(0.6)$$

$$= -0.21$$

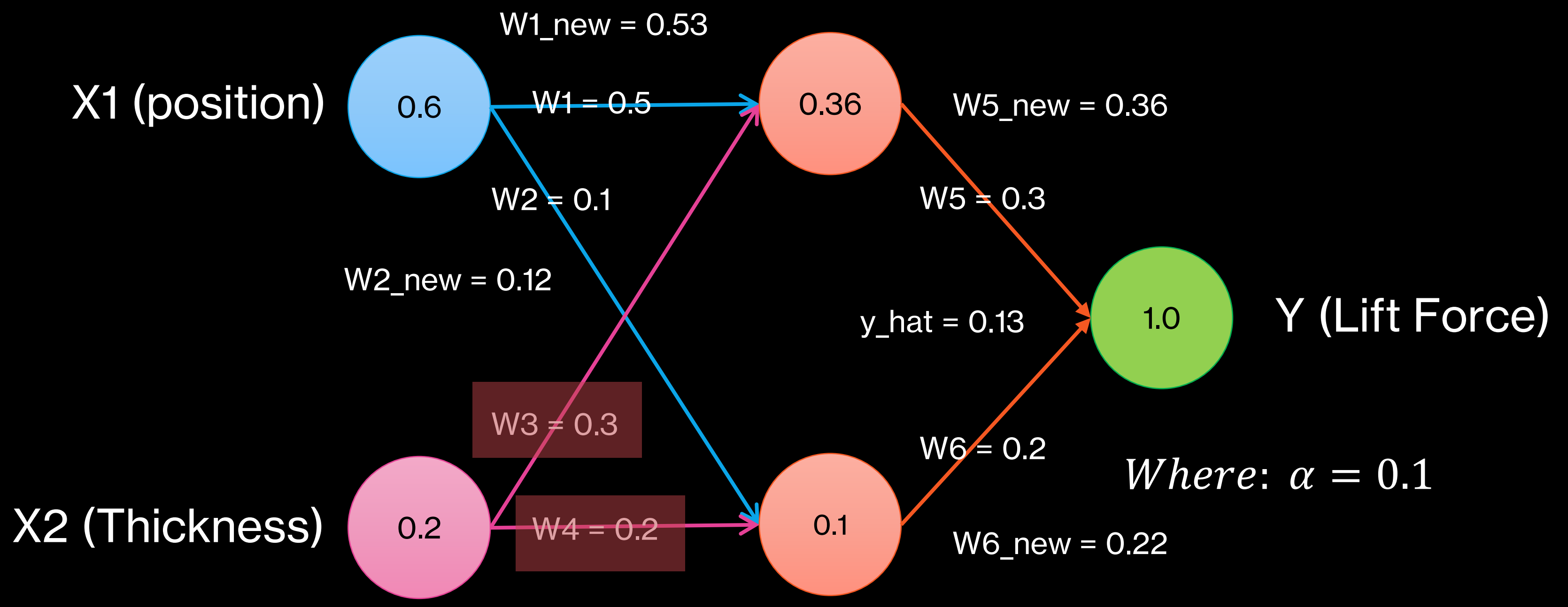
Back Propagation



$$Where: \alpha = 0.1$$

$$w_{2\_new} = w_2 - \alpha \frac{\partial e}{\partial w_2} = 0.1 - (0.1)(-0.21) = 0.12$$

Back Propagation



$Where: \alpha = 0.1$

$$w_{3\_new} = w_3 - \alpha \frac{\partial e}{\partial w_3} \qquad w_{4\_new} = w_4 - \alpha \frac{\partial e}{\partial w_4}$$

Back Propagation

Let's apply the chain rule.

$$\frac{\partial e}{\partial w_3} = ???$$

$$= \frac{\partial e}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h_1} \cdot \frac{\partial h_1}{\partial w_3}$$

$$= \frac{\partial (y-\hat{y})^2}{\partial \hat{y}} \cdot \frac{\partial (h_1 w_5 + h_2 w_6)}{\partial h_1} \cdot \frac{\partial (x_1 w_1 + x_2 w_3)}{\partial w_3}$$

$$= -2(y - \hat{y}) \cdot w_5 \cdot x_2$$

$$= -2(1 - 0.13)(0.3)(0.2)$$

$$= -0.1$$

$$\frac{\partial e}{\partial w_4} = ???$$

$$= \frac{\partial e}{\partial y} \cdot \frac{\partial y}{\partial h_2} \cdot \frac{\partial h_2}{\partial w_4}$$

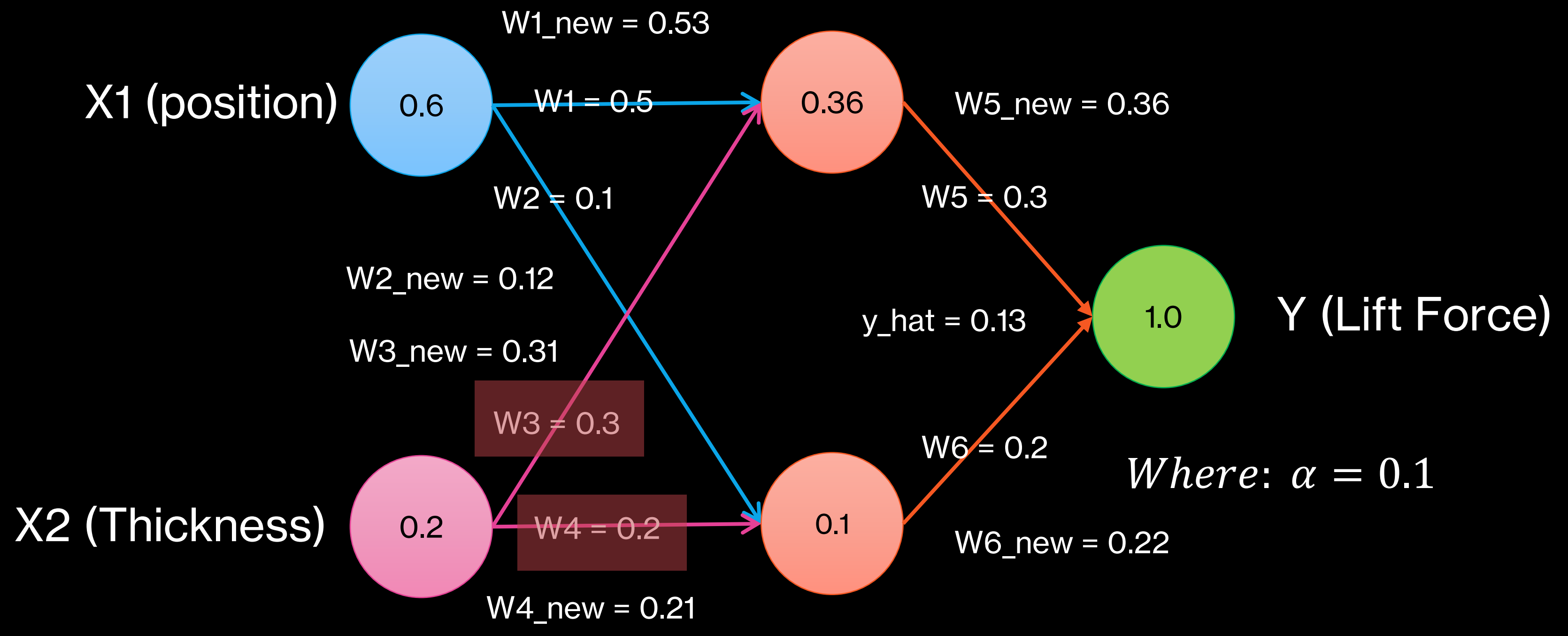
$$= \frac{\partial (y-\hat{y})^2}{\partial \hat{y}} \cdot \frac{\partial (h_1 w_5 + h_2 w_6)}{\partial h_2} \cdot \frac{\partial (x_1 w_2 + x_2 w_4)}{\partial w_4}$$

$$= -2(y - \hat{y}) \cdot w_6 \cdot x_2$$

$$= -2(1 - 0.13)(0.2)(0.2)$$

$$= -0.07$$

Back Propagation

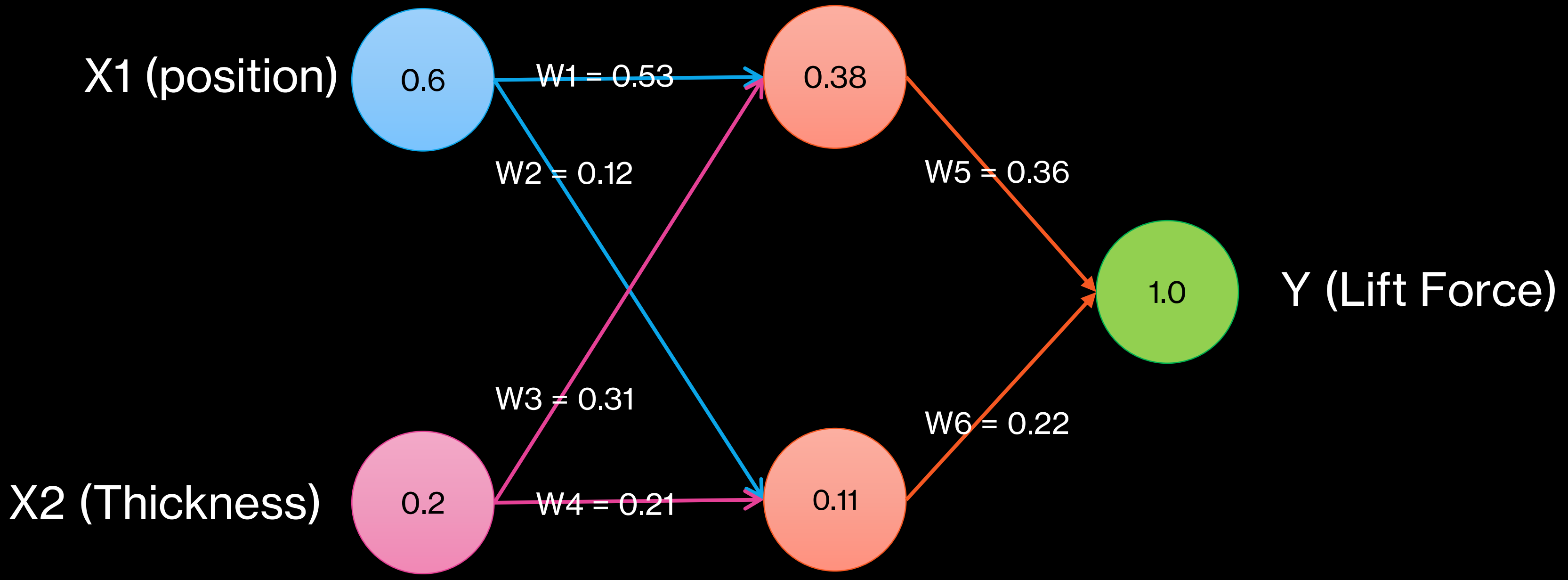


$$Where:\ \alpha = 0.1$$

$$w_{3\_new} = w_3 - \alpha \frac{\partial e}{\partial w_3} = 0.3 - (0.1)(-0.1) = 0.31$$

$$w_{4\_new} = w_4 - \alpha \frac{\partial e}{\partial w_4} = 0.2 - (0.1)(-0.07) = 0.21$$

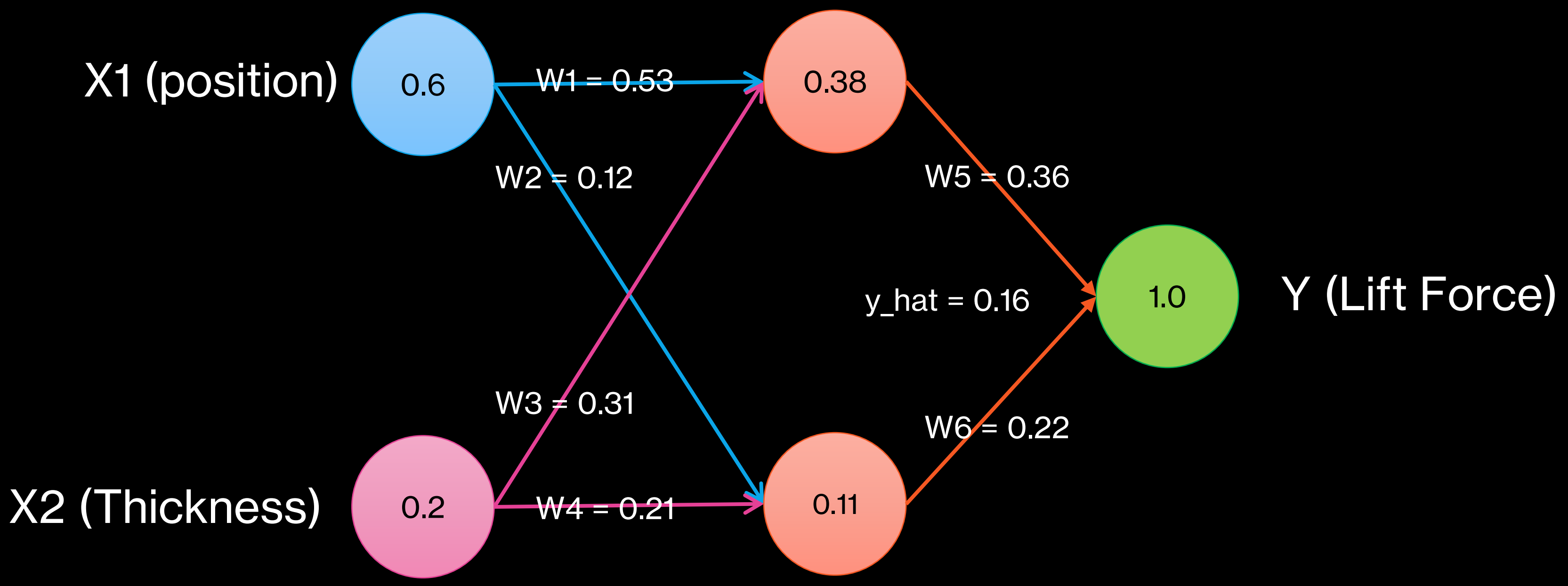
Forward Propagation



$$h_1 = (x_1 w_1 + x_2 w_3) = (0.6)(0.53) + (0.2)(0.31) = 0.38$$

$$h_2 = (x_1 w_2 + x_2 w_4) = (0.6)(0.12) + (0.2)(0.21) = 0.11$$

Forward Propagation



$$y = (h_1 w_5 + h_2 w_6) = ((0.38)(0.36) + (0.11)(0.22)) = 0.16$$

$$error = \frac{1}{n}\sum_{i}^{n}(y - \hat{y})^2 = \frac{1}{1}\sum_{i}^{n}(1 - 0.16)^2 = 0.71$$