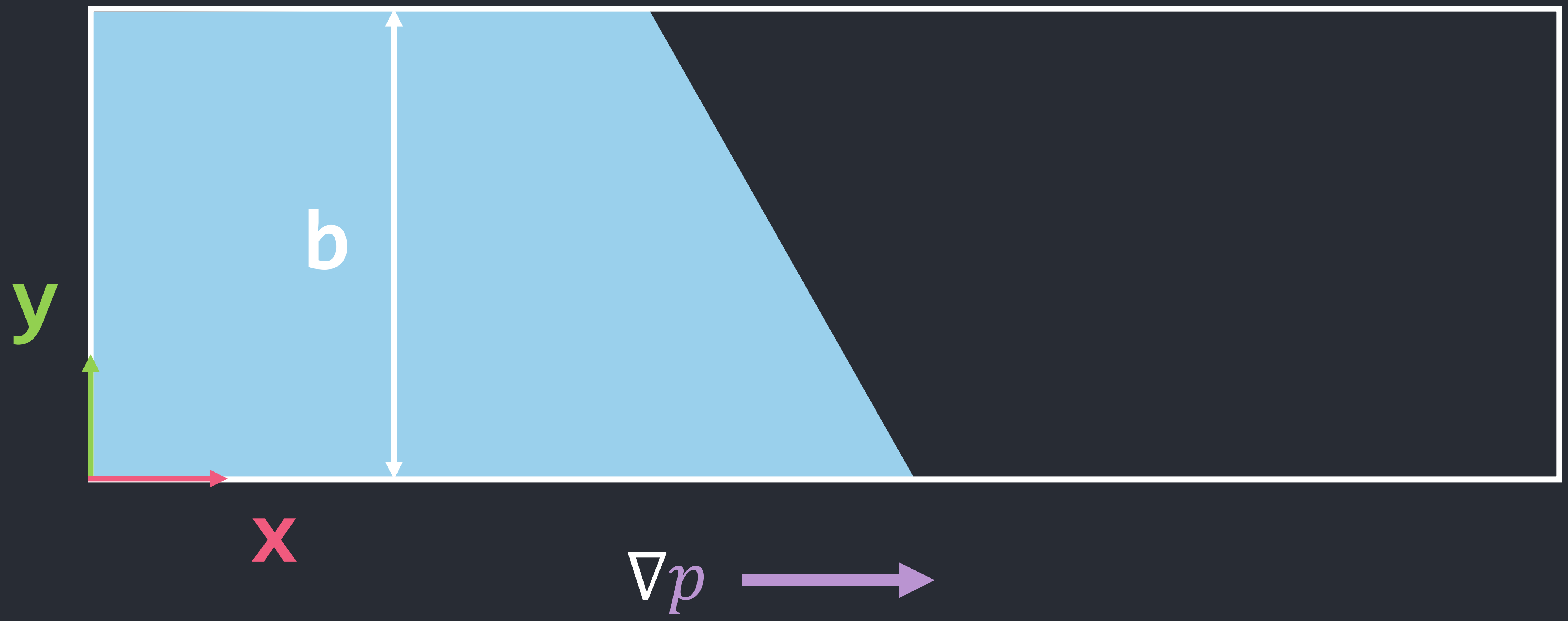
$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = -\nabla p + \mu\nabla^2\boldsymbol{u} + \mathbf{F}$$

Navier-Stokes Equation

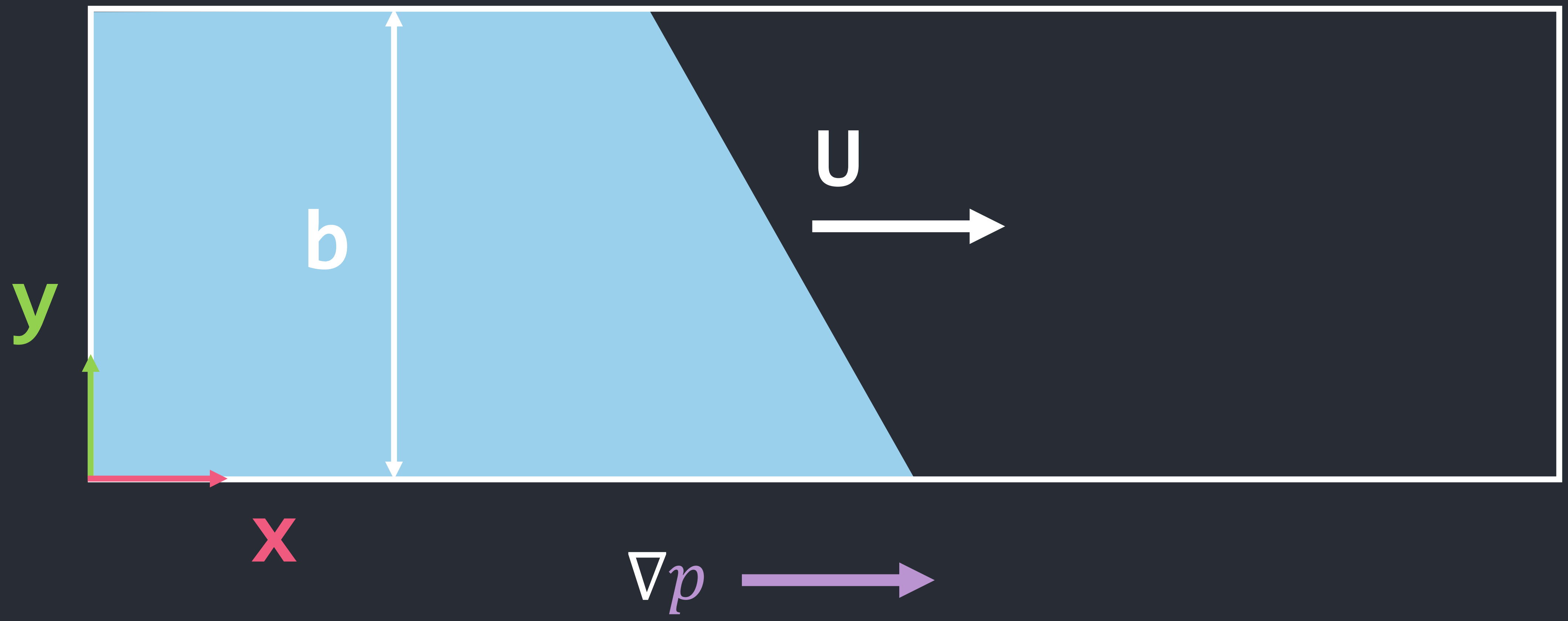
$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}\right) = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \boldsymbol{F}$$


y
x

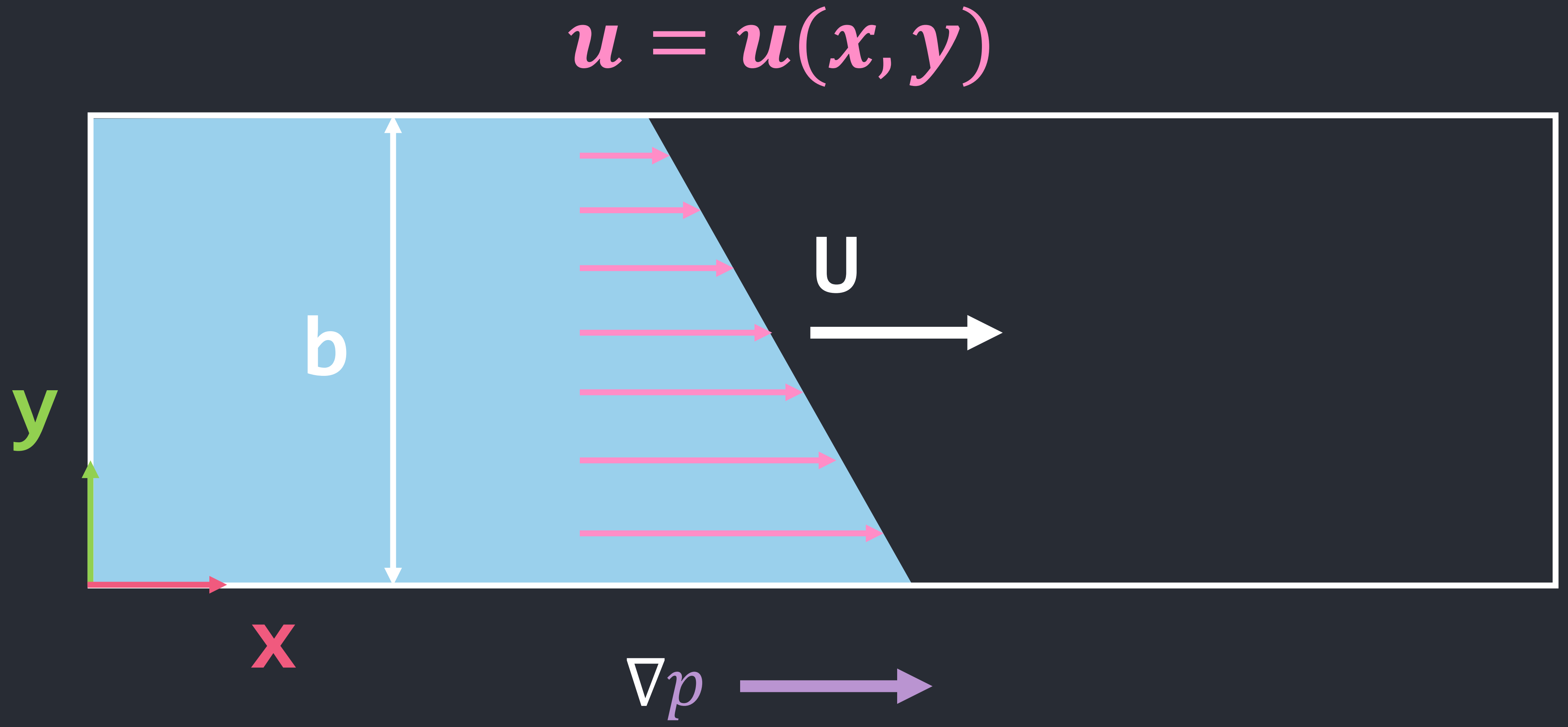
b
y
x



b
y
x
∇p

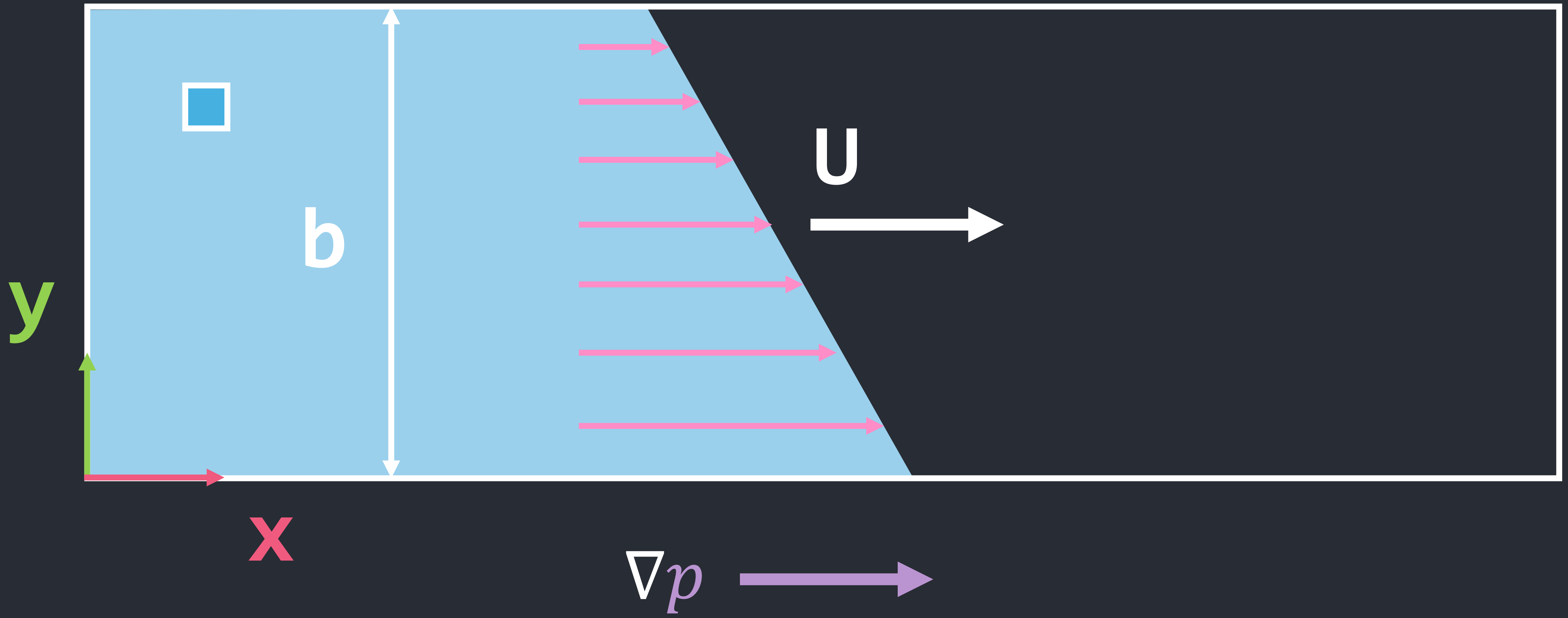


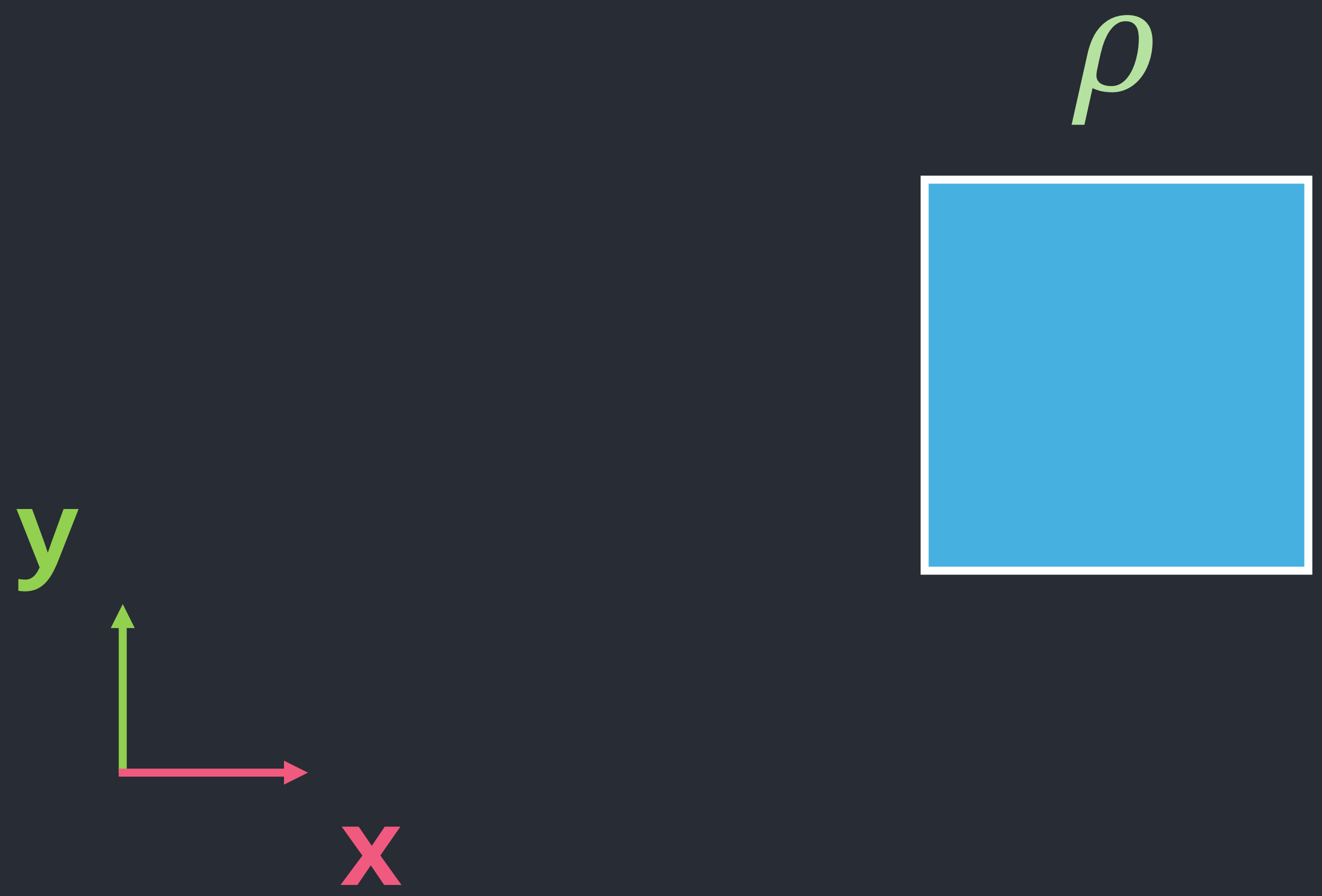
U
b
y
x
∇p



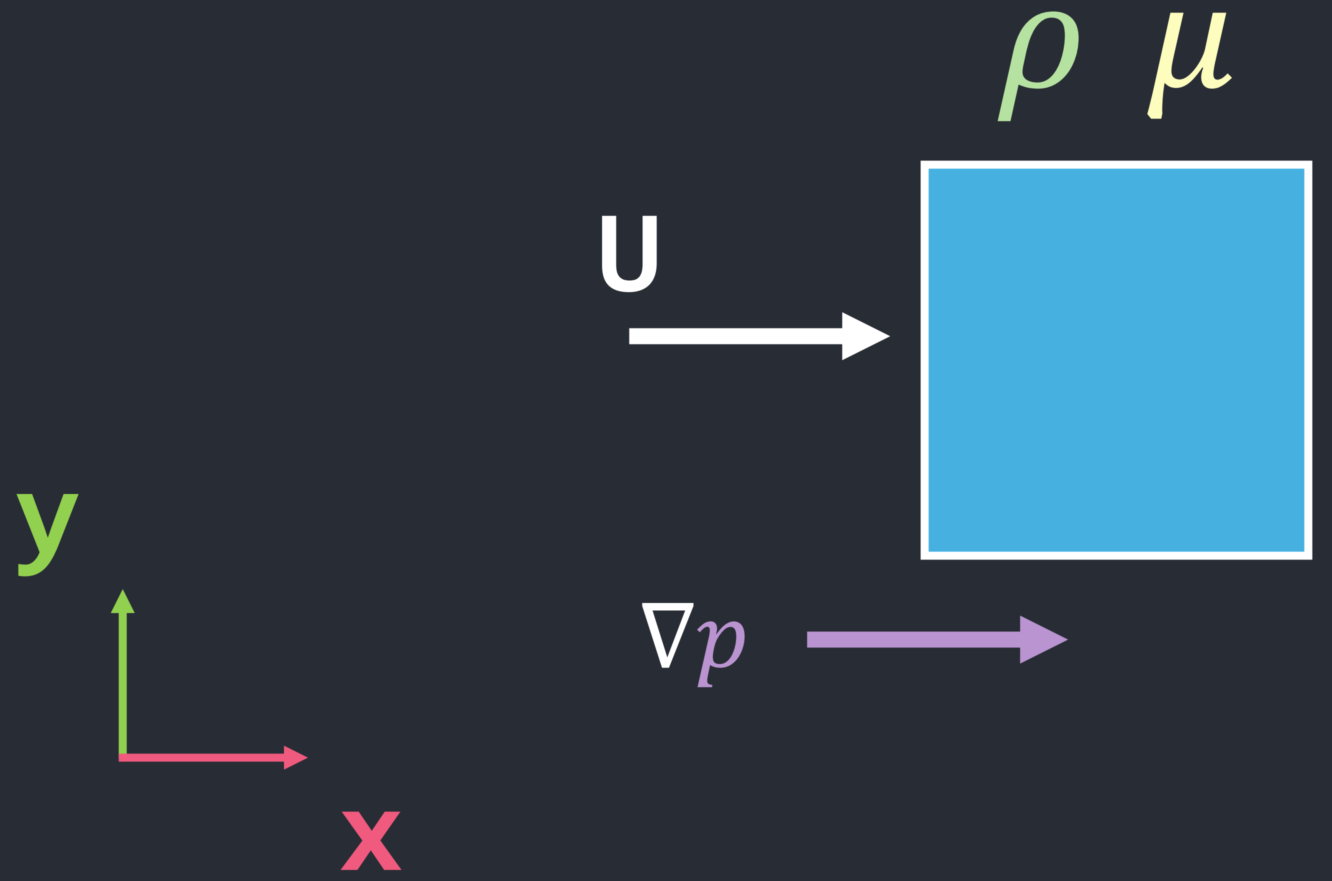
$u = u(x, y)$
U
b
y
x
∇p

$$u = u(x, y)$$

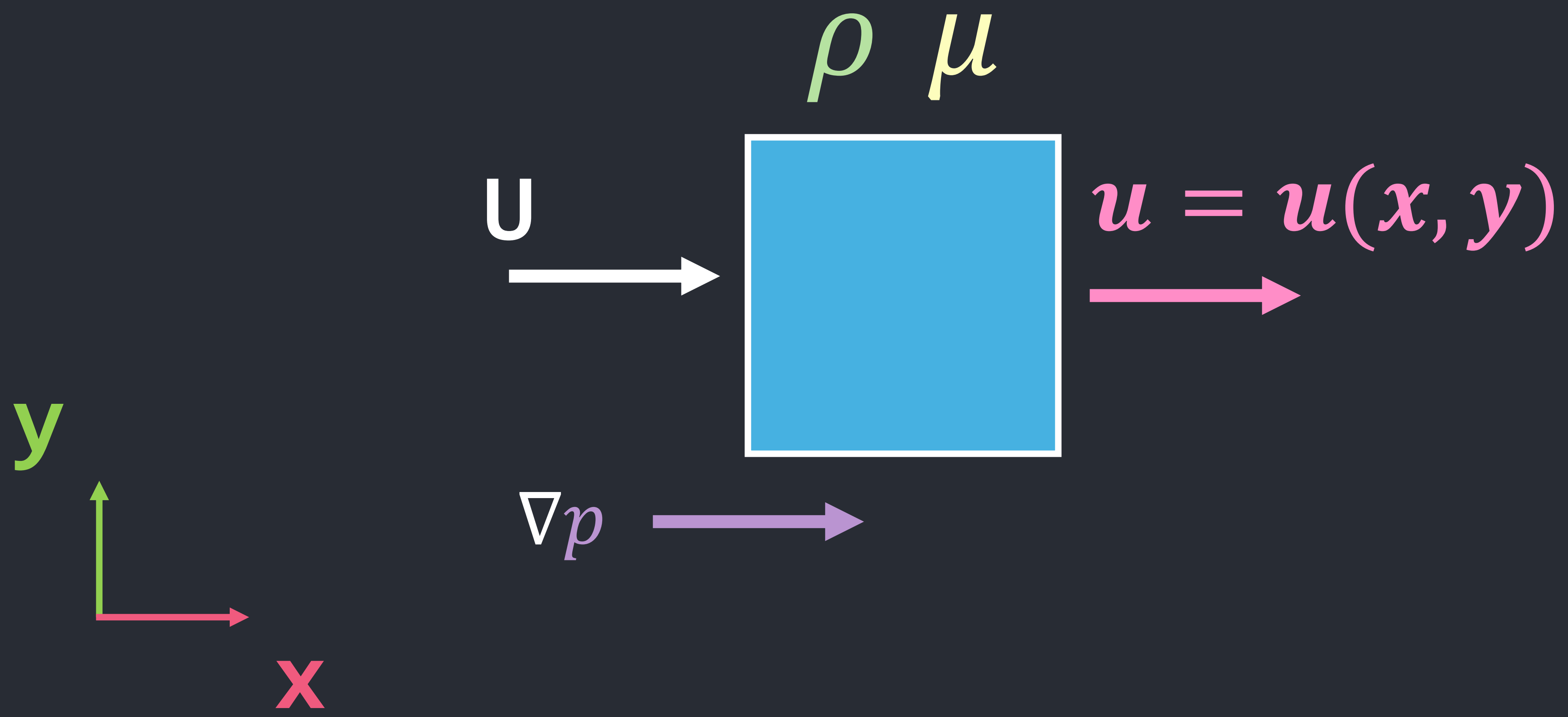




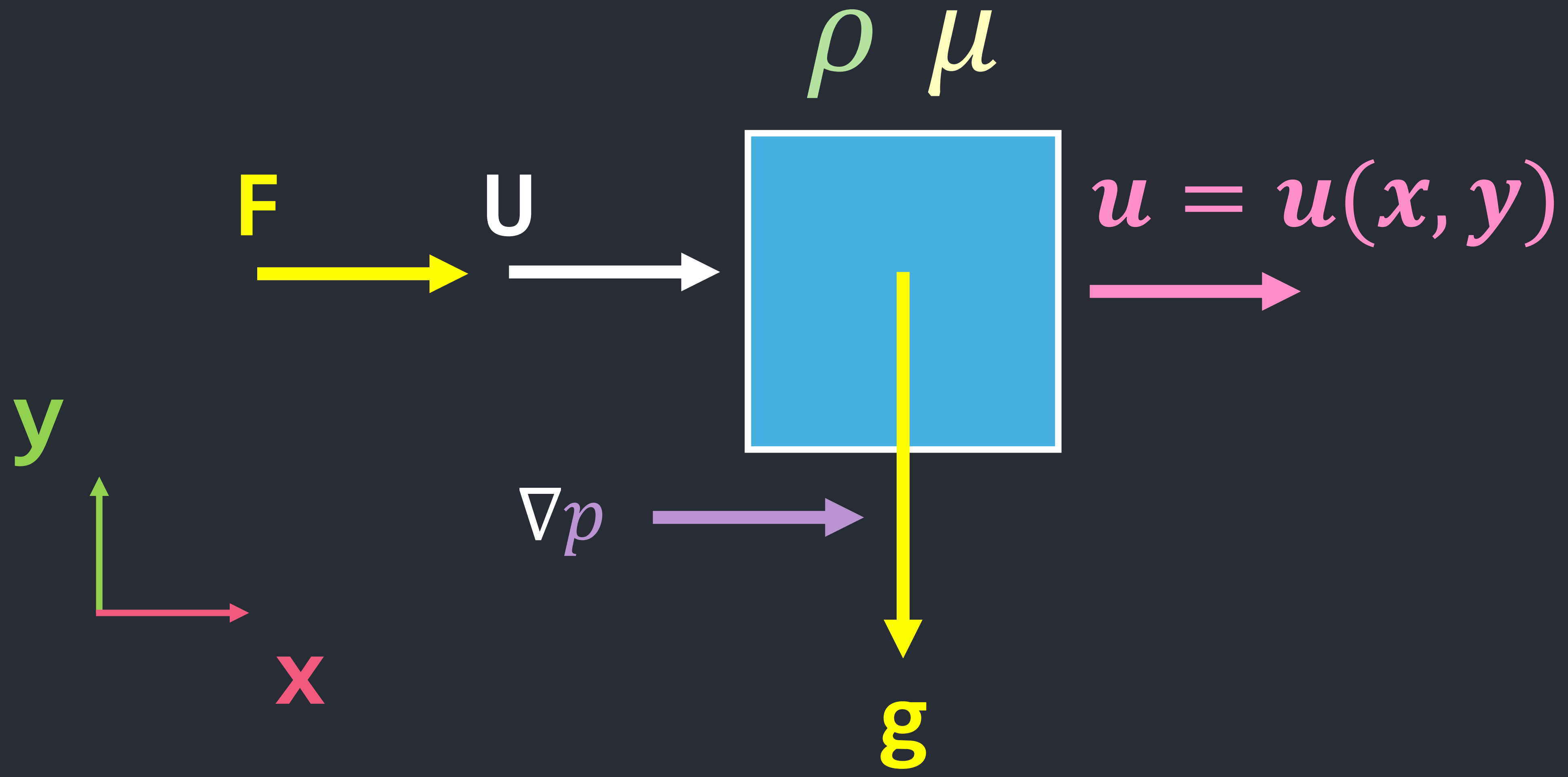
ρ
y
x



ρ μ
U
∇p
y
x



ρ μ
U
$\boldsymbol{u} = \boldsymbol{u}(x, y)$
∇p
y
x



ρ μ
F
U
$\boldsymbol{u} = \boldsymbol{u}(x, y)$
y
x
∇p
g

Navier-Stokes Equation

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}\right) = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \boldsymbol{F}$$

$$\boldsymbol{u} \rightarrow Speed:\ m/s$$

$$\rho \rightarrow Fluid\ Density:\ kg/m^2$$

$$p \rightarrow Pressure:\ Pa$$

$$\mu \rightarrow Dynamic\ Viscousity:\ Pa$$

$$\boldsymbol{F} \rightarrow External\ Force: N$$

$$\boldsymbol{t} \rightarrow Time: s$$

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}\right) = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \boldsymbol{F}$$

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}\right) = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \boldsymbol{F}$$

Newton's 2nd Law

$$ma = \sum F$$

$$\underbrace{\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}\right)}_{\dfrac{\boldsymbol{ma}}{\boldsymbol{V}}} = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \boldsymbol{F}$$

$$\underbrace{\rho\left(\frac{\partial \boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\nabla\boldsymbol{u}\right)}_{\frac{\boldsymbol{ma}}{\boldsymbol{V}}}=\underbrace{-\nabla p+\mu\nabla^2\boldsymbol{u}+\boldsymbol{F}}_{\frac{\boldsymbol{F}}{\boldsymbol{V}}}$$

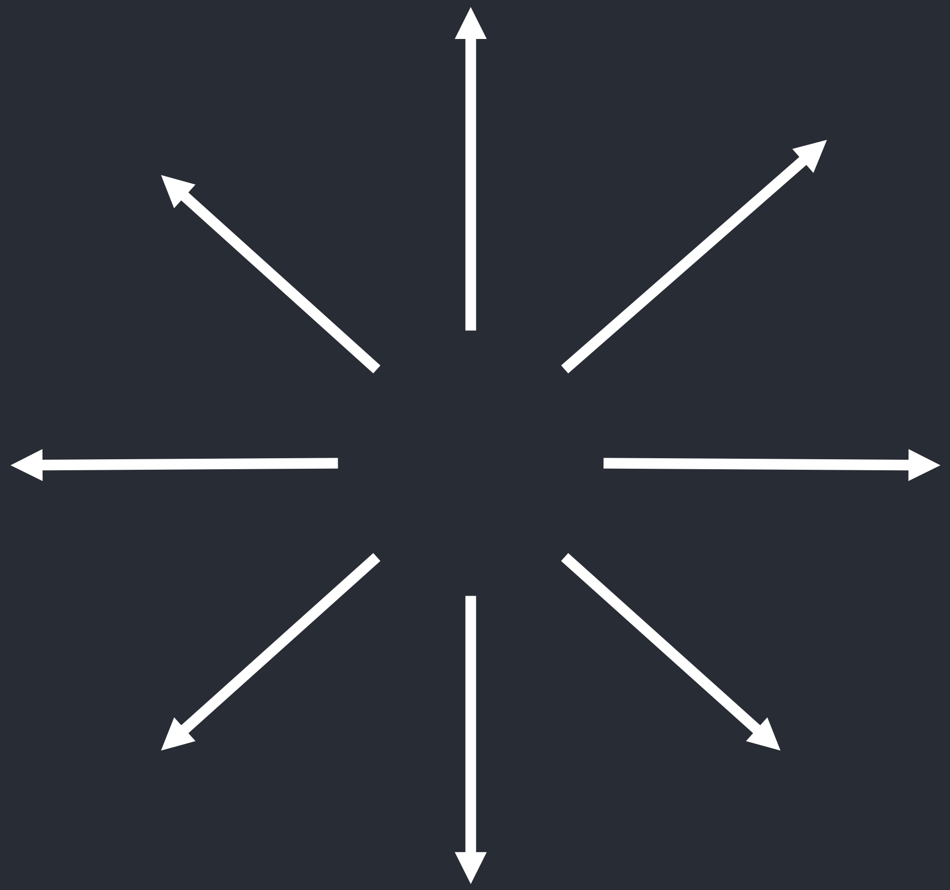
$$\boldsymbol{Divergence}$$

$$\nabla \cdot \boldsymbol{u}$$

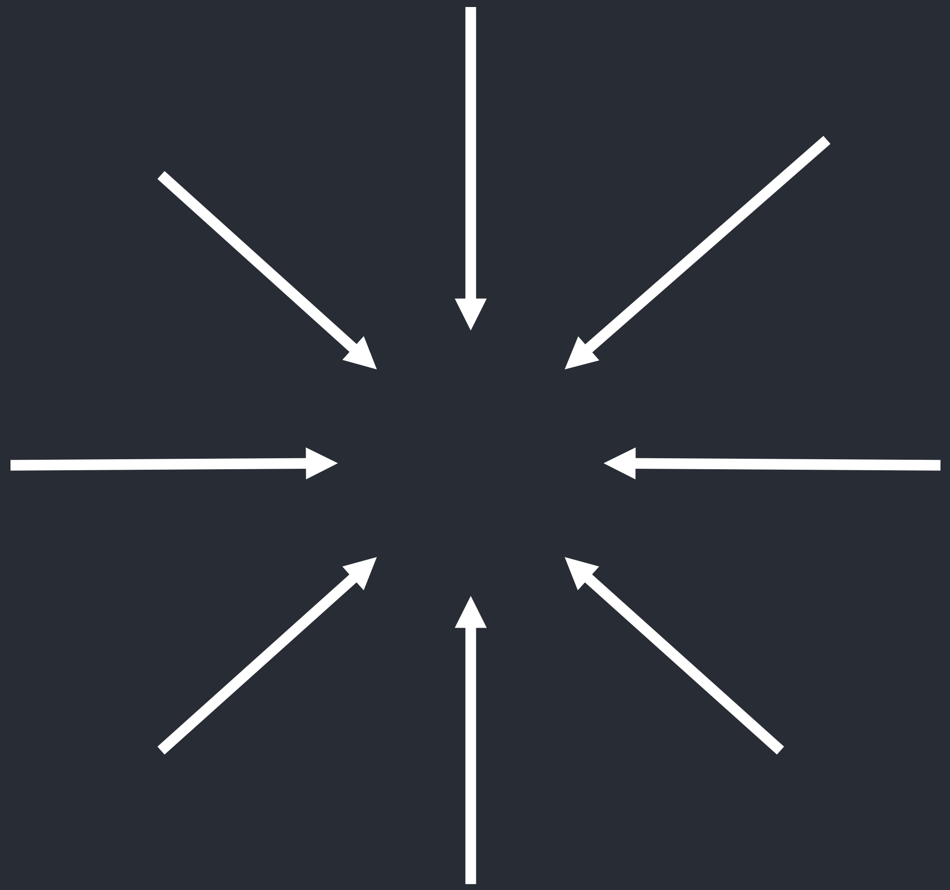
$$\boldsymbol{u} = u(x, y)$$

$$\nabla \cdot \boldsymbol{u} = \boldsymbol{div}\,\boldsymbol{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}$$

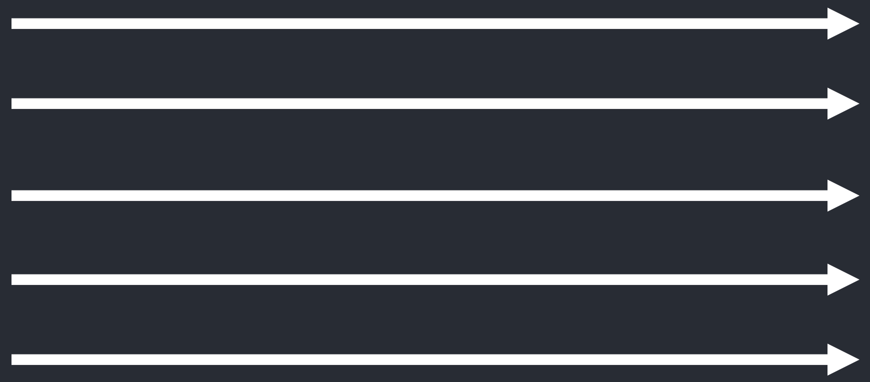
$$\nabla \boldsymbol{u} > \mathbf{0}$$



$$\nabla \boldsymbol{u} < 0$$



$$\nabla \boldsymbol{u} = \mathbf{0}$$

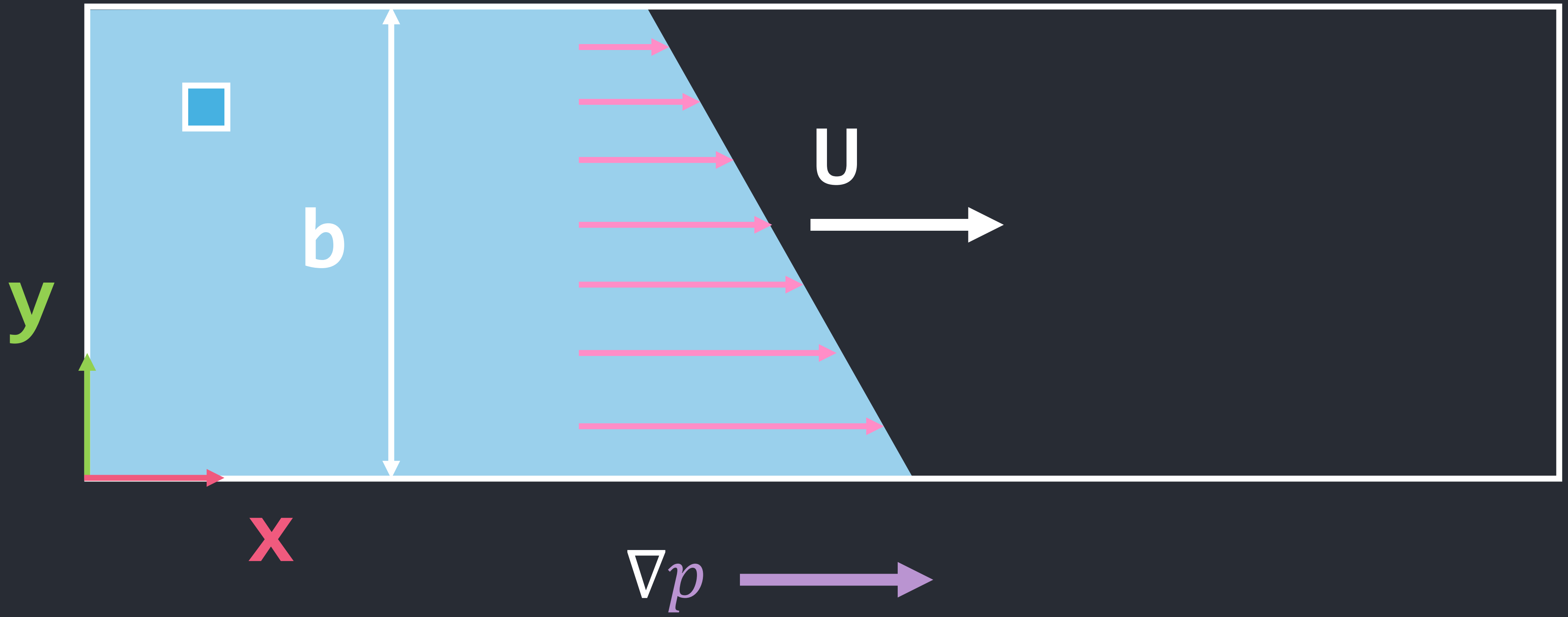


$$\boldsymbol{Incompressible\ Flow}$$

$$\boldsymbol{(AKA\text{: } Continuity\ Equation)}$$

$$\nabla \cdot \boldsymbol{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = \boldsymbol{0}$$

$$u = u(x, y)$$



Conditions

- $No\ flow\ in\ vertical\ direction \rightarrow u_y = 0\ \frac{m}{s}$
- $Flow\ driven\ by\ pressure\ gradient \rightarrow \frac{\partial p}{\partial x} \neq 0\ \frac{Pa}{m}$
- $No\ gravity\ effects\ \rightarrow g = 0\ \frac{m}{s^2}$
- $Boundary\ of\ the\ pipe\ \rightarrow y = [0, b]$
- $Incompressible\ Flow \rightarrow \nabla \boldsymbol{u} = \boldsymbol{0}$

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}\right) = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \boldsymbol{F}$$

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{t}} + \left(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y}\right)\right) = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \rho g$$

Incompressible Flow

(Continuity Equation)

$$\nabla \cdot \boldsymbol{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = \mathbf{0}$$

Fully Developed Flow

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = \mathbf{0}$$

Fully Developed Flow

$$\frac{\partial u_x}{\partial x} + \xcancel{\frac{\partial u_y}{\partial y}} = 0$$

0 ***No flow in y***

Fully Developed Flow

$$\frac{\partial u_x}{\partial x} = \mathbf{0}$$

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{t}}+\left(u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}\right)\right)=-\nabla p+\mu\nabla^2\boldsymbol{u}+\rho g$$

Steady Flow

No flow in y

No g effects

$$\rho\left(\cancelto{0}{\frac{\partial \boldsymbol{u}}{\partial t}} + \left(u_x \cancelto{0}{\frac{\partial u_x}{\partial x}} + \cancelto{0}{u_y} \frac{\partial u_x}{\partial y}\right)\right) = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \cancelto{0}{\rho g}$$

Fully Developed

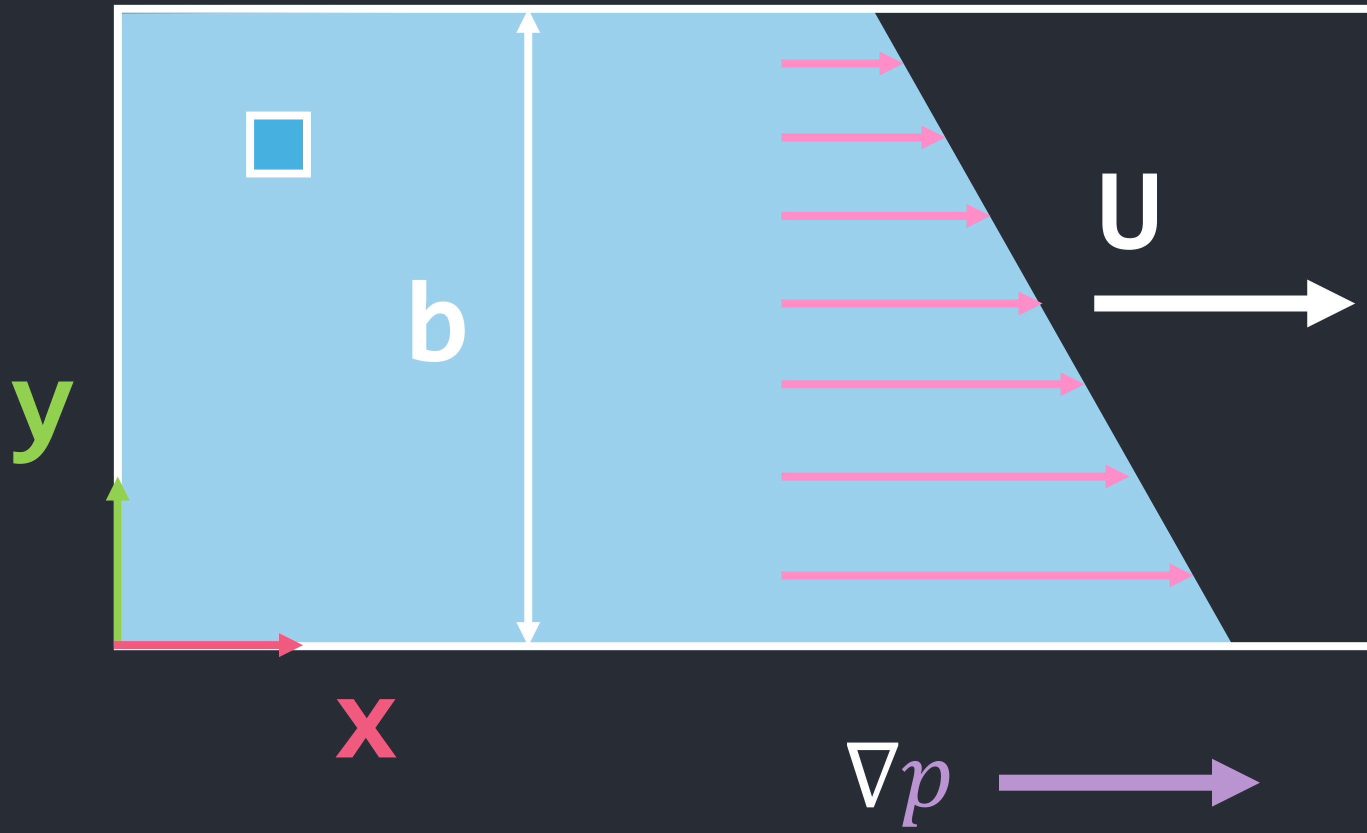
$$0 = -\nabla p + \mu \nabla^2 \boldsymbol{u}$$

Fully Developed

$$0 = -\nabla p + \mu\left(\cancelto{0}{\frac{\partial^2 u_x}{\partial x^2}} + \frac{\partial^2 u_x}{\partial y^2}\right)$$

$$0 = -\nabla p + \mu \left(\frac{\partial^2 u_x}{\partial y^2} \right)$$

$$u = u(x, y)$$



Flow Profile

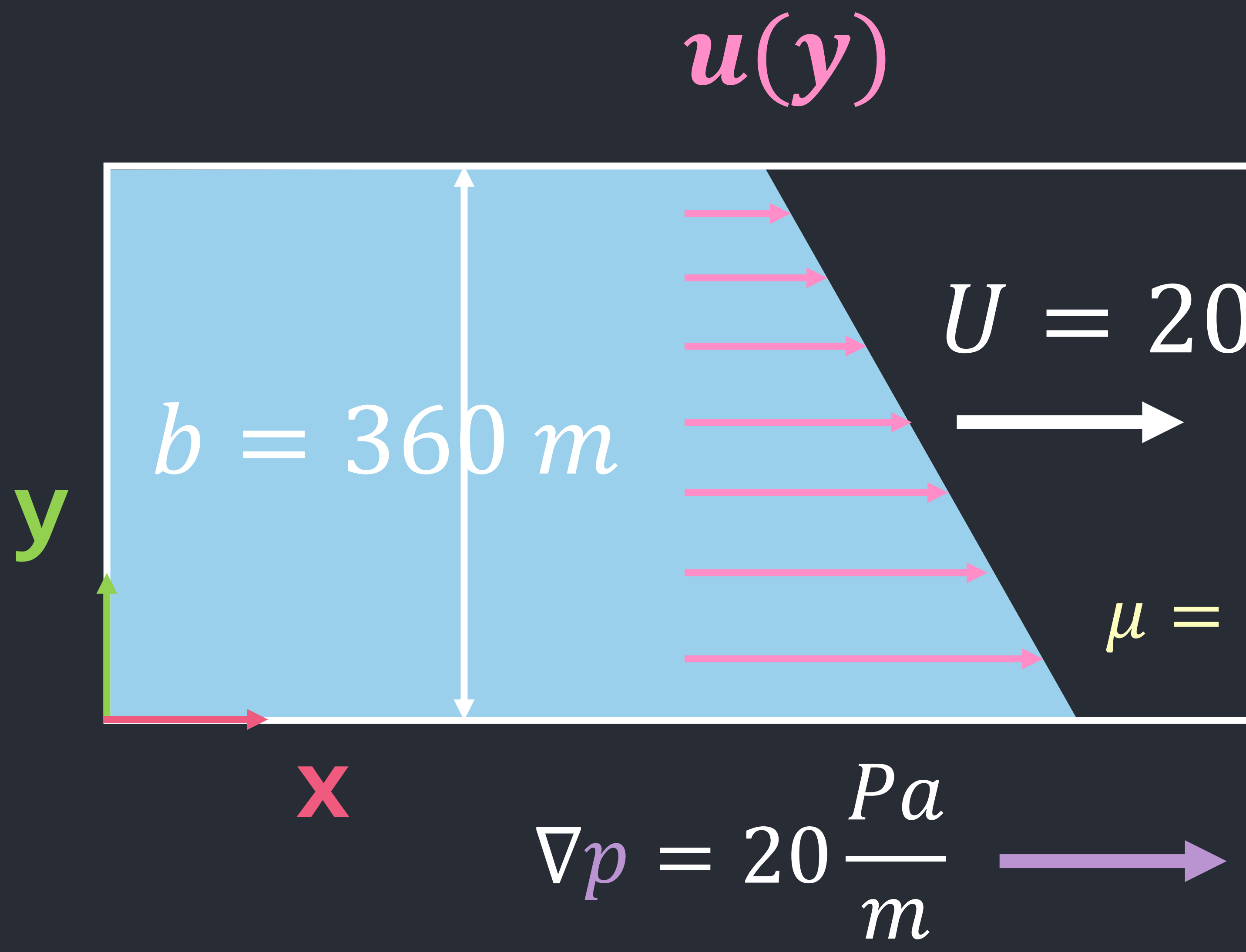
$$\boldsymbol{u}(\boldsymbol{y}) = ???$$

$$0 = -\nabla p + \mu \left(\frac{\partial^2 u_x}{\partial y^2} \right)$$

$$\frac{\partial^2 u_x}{\partial y^2} = \frac{1}{\mu} \nabla p = const$$

Flow Profile

$$u(y) = \frac{1}{2\mu} \nabla p (b^2 - by) + U\left(1 - \frac{y}{b}\right)$$



u(y)
b = 360 m
U = 20 m/s
μ = 1001600.0 Pa · s
y
x
∇p = 20 Pa/m

Rust + Bevy