

1. (1)

$$\begin{aligned}
 p(\lambda|x) &\propto p(\lambda)p(x|\lambda) \\
 &\propto B|\lambda|^{\frac{1}{2}(V_0-D-1+N)} \exp\left(-\frac{1}{2}\text{Tr}(W_0^{-1}\lambda)\right) \cdot |\lambda|^{\frac{N}{2}} \exp\left\{-\frac{1}{2}\sum_{n=1}^N (x_n - \mu)^T \lambda (x_n - \mu)\right\} \\
 &\propto B|\lambda|^{\frac{1}{2}(V_0-D-1+N)} \exp\left\{-\frac{1}{2}\text{Tr}(W_0^{-1}\lambda) - \frac{1}{2}\text{Tr}(S\lambda)\right\} \\
 &\propto B|\lambda|^{\frac{1}{2}(V_0+N-D-1)} \exp\left\{-\frac{1}{2}\text{Tr}(W_0^{-1} + S)\lambda\right\} \\
 &\sim W(\lambda | (W_0^{-1} + S)^{-1}, V_0 + N), \quad S = \sum_n (x_n - \mu)(x_n - \mu)^T \\
 &\sim W(\lambda | W_N, V_N) \\
 \\
 \frac{\partial \ln p(\lambda|x)}{\partial \lambda} &= 0 \Rightarrow \frac{\partial}{\partial \lambda} \left[\ln B|\lambda|^{\frac{1}{2}(V_0+N-D-1)} - \frac{1}{2}\text{Tr}(W_0^{-1} + S)\lambda \right] = 0 \\
 \text{令微分} &= 0 \Rightarrow \frac{1}{2}[V_0+N-D-1](\lambda^{-1})^T + (-\frac{1}{2})(W_0^{-1} + S)^T = 0 \\
 \Rightarrow \lambda_{\text{map}} &= \left(\frac{S + W_0^{-1}}{V_0 + N - D + 1} \right)^{-1} \\
 S &= \sum_n (x_n - \mu)(x_n - \mu)^T
 \end{aligned}$$

(2)

pre1 =

3.8308 -2.0267
-2.0267 1.9616

pre2 =

8.8510 -3.6687
-3.6687 2.6131

pre3 =

7.6965 -3.1764
-3.1764 2.4727

2. (1)

```

mn_10: [0.65816839, 6.56900994, 4.47161766, -5.19232775, 0.30683732, -2.70731155, -12.93480385]
sn_10: [[ 11.74844606 -45.13183578 57.51651542 -53.28263165 47.81732525
-23.09680649 4.98836982]
 [ -45.13183578 186.11942006 -243.6042185 226.73082952 -203.6058234
 98.3513931 -21.24177114]
 [ 57.51651542 -243.6042185 327.17144482 -313.8674939 283.55142864
-137.0512211 29.60342635]
 [ -53.28263165 226.73082952 -313.8674939 318.58097503 -293.07005739
142.21832712 -30.76334672]
 [ 47.81732525 -203.6058234 283.55142864 -293.07005739 276.29194886
-137.93198009 30.41415771]
 [ -23.09680649 98.3513931 -137.0512211 142.21832712 -137.93198009
74.77288329 -19.93938959]
 [ 4.98836982 -21.24177114 29.60342635 -30.76334672 30.41415771
-19.93938959 9.07926812]]

```

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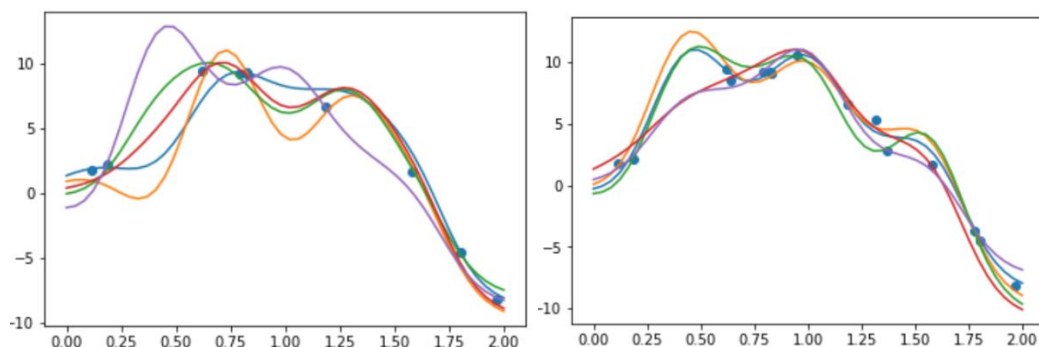
mn_15: [-1.29839592, 14.9439402, -7.78968709, 8.31956943, -12.02884643, 2.83821224, -13.79152896]
sn_15: [[ 3.9101095 -11.80732662 11.79648808 -6.61631585 3.74523425
 -1.45634936 0.52849964]
 [ -11.80732662 44.42869426 -49.04149853 27.91609081 -15.84926623
 6.16666844 -2.23835515]
 [ 11.79648808 -49.04149853 57.71564428 -35.45778281 20.707921
 -8.11304334 2.95345492]
 [ -6.61631585 27.91609081 -35.45778281 26.84820193 -17.85259394
 7.40666631 -2.7755787 ]
 [ 3.74523425 -15.84926623 20.707921 -17.85259394 14.85384119
 -8.77275241 3.96100555]
 [ -1.45634936 6.16666844 -8.11304334 7.40666631 -8.77275241
 10.05835841 -6.80399544]
 [ 0.52849964 -2.23835515 2.95345492 -2.7755787 3.96100555
 -6.80399544 6.25063527]]

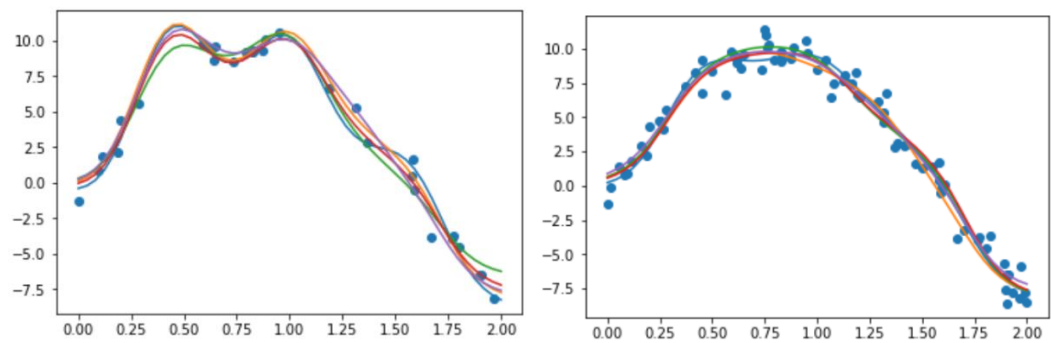
mn_30: [-1.91468609, 17.1267577, -9.55822961, 8.01656775, -10.11101665, -1.71138079, -9.89206398]
sn_30: [[ 1.36509916 -3.59710179 3.41142343 -2.0383279 1.16791206
 -0.39413822 0.10306912]
 [ -3.59710179 11.94420157 -12.90391636 7.90485805 -4.55451624
 1.53907999 -0.40269729]
 [ 3.41142343 -12.90391636 15.58050315 -10.90817398 6.58613523
 -2.25617425 0.59380843]
 [ -2.0383279 7.90485805 -10.90817398 10.31921394 -7.44251917
 2.79842845 -0.77036676]
 [ 1.16791206 -4.55451624 6.58613523 -7.44251917 7.48596272
 -4.46505472 1.51298983]
 [ -0.39413822 1.53907999 -2.25617425 2.79842845 -4.46505472
 4.73375351 -2.5399153 ]
 [ 0.10306912 -0.40269729 0.59380843 -0.77036676 1.51298983
 -2.5399153 2.26539449]]

mn_80: [0.25681676, 9.39255749, 0.07712854, 0.4227124, -4.6364583, -4.20883399, -9.33299811]
sn_80: [[ 0.42259863 -0.77573232 0.52491237 -0.26550415 0.1400047 -0.06579142
 0.02294464]
 [-0.77573232 1.86915539 -1.69530005 0.94343168 -0.51158981 0.24185308
 -0.08445695]
 [ 0.52491237 -1.69530005 2.19167309 -1.71239151 1.04855707 -0.50972738
 0.17912946]
 [-0.26550415 0.94343168 -1.71239151 2.1769828 -1.81675643 0.9714415
 -0.35005766]
 [ 0.1400047 -0.51158981 1.04855707 -1.81675643 2.19383496 -1.59317568
 0.63895177]
 [-0.06579142 0.24185308 -0.50972738 0.9714415 -1.59317568 1.76175332
 -0.98212236]
 [ 0.02294464 -0.08445695 0.17912946 -0.35005766 0.63895177 -0.98212236
 0.80201586]]

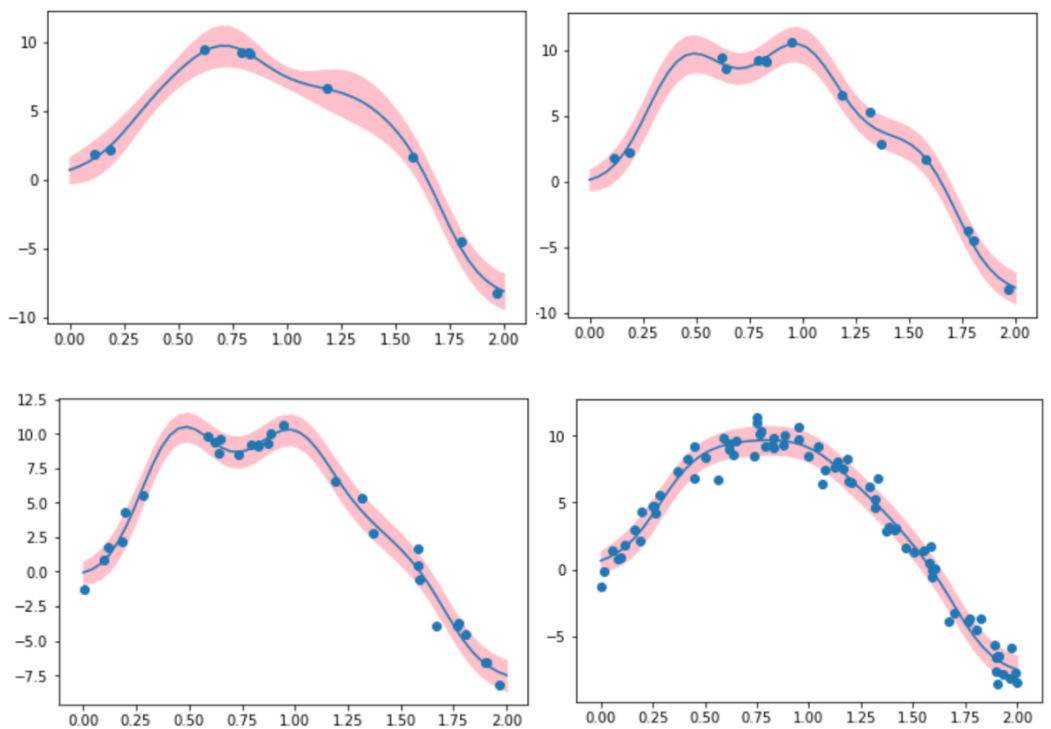
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(2)



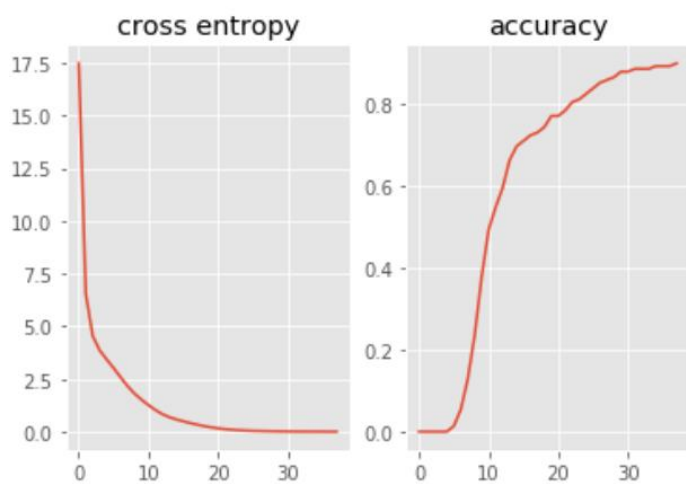


(3)



3.

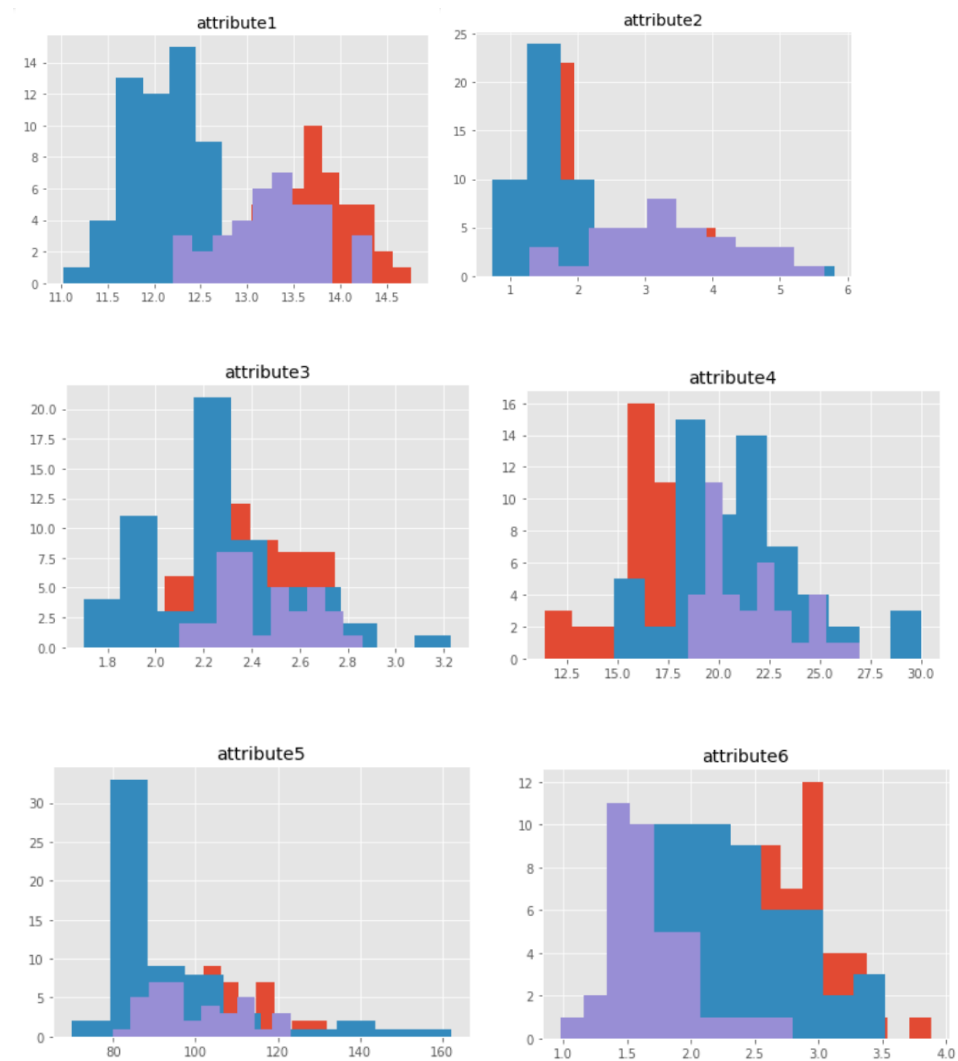
(1)

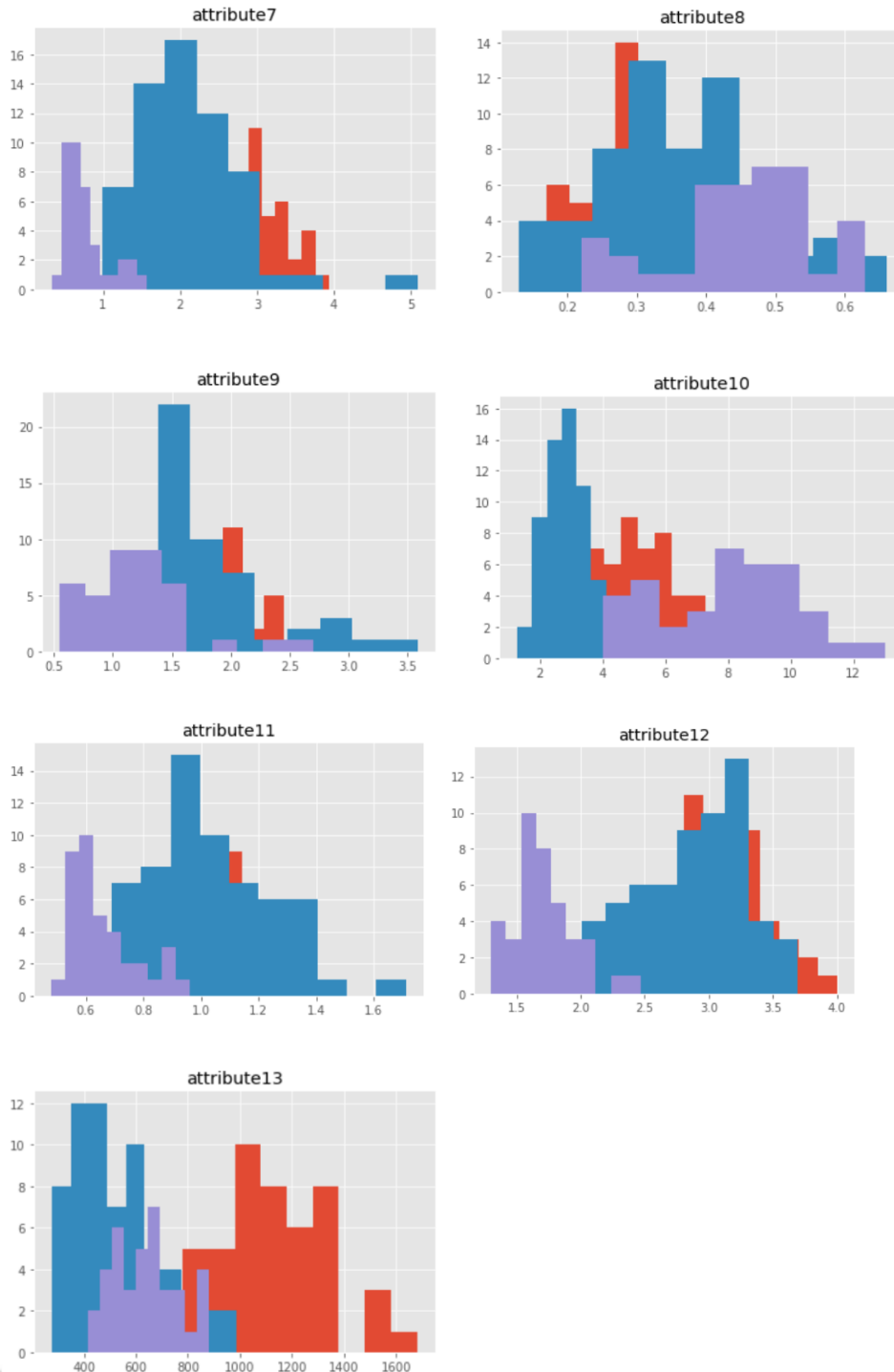


(2)

classification		classification		classification	
0	class3	10	class2	20	class1
1	class3	11	class2	21	class1
2	class3	12	class2	22	class1
3	class3	13	class2	23	class1
4	class3	14	class2	24	class1
5	class3	15	class2	25	class1
6	class3	16	class2	26	class1
7	class3	17	class2	27	class1
8	class3	18	class2	28	class1
9	class3	19	class3	29	class1

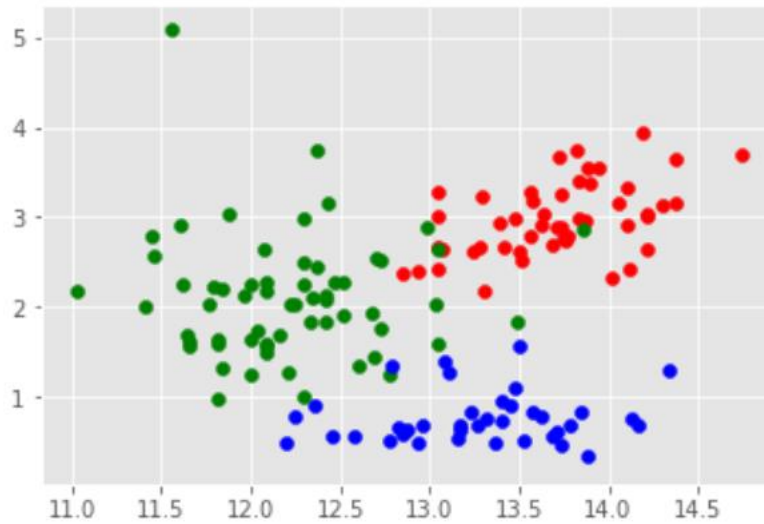
(3)





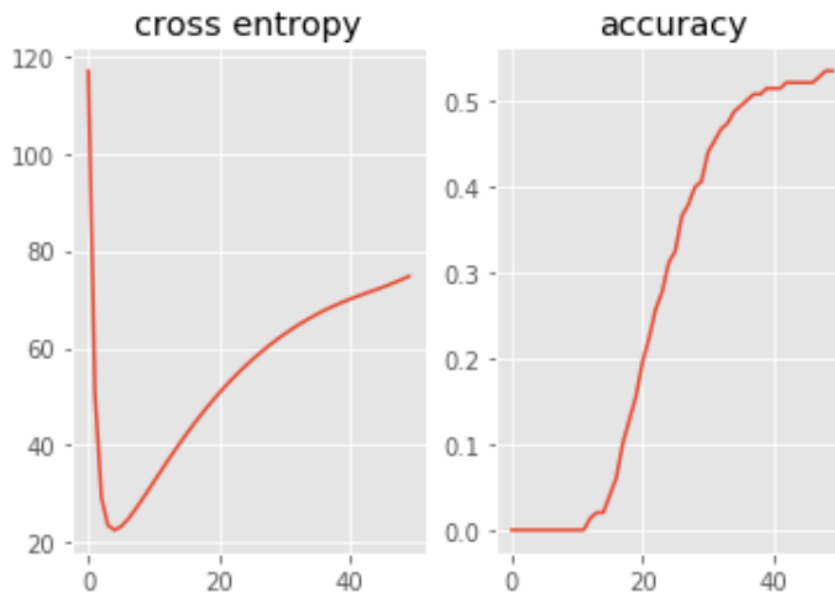
(4)並不是所有的Newton-Raphson algorithm都有global minimum，因為更新W的function為一個convex function，所以會有global minimum

(5)attribute1 and attribute7的散佈圖



(6)

For training data



For testing data

classification		classification		classification	
0	class3	10	class1	20	class1
1	class3	11	class2	21	class1
2	class3	12	class2	22	class1
3	class3	13	class1	23	class1
4	class3	14	class2	24	class1
5	class3	15	class2	25	class1
6	class3	16	class2	26	class1
7	class3	17	class3	27	class1
8	class3	18	class2	28	class1
9	class3	19	class1	29	class1