## Statistical Modeling: A Fresh Approach

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# Preface to this electronic version

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Example: Applying to Law School

Example: Nitrogen Fixing

**Example: Sex Discrimination** 

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- 1.2 Observation and Knowledge
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Reading Questions

# Data: Cases, Variables, Samples

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