In this Figure,

$\Delta AGD = \delta$, $\Delta AGB = \alpha$, $\Delta BGC = \beta$, $\Delta CGD = \Theta$,

$\Delta AGC = \alpha + \beta$, $\Delta BGD = \Theta + \beta$

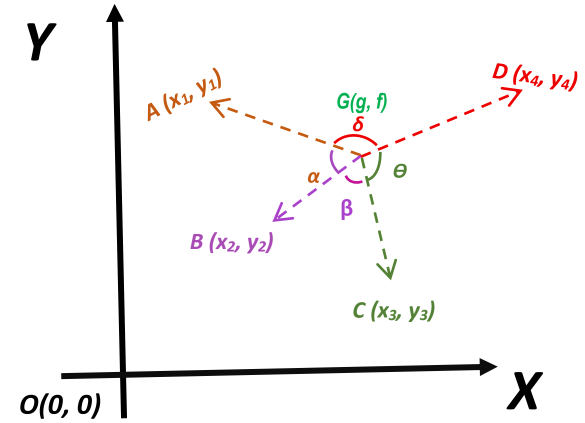

Fig (ii) : Intersection angle between the vectors

Now ,

$$\text{Centroid point} = \left(\frac{x_1 + x_2 + x_3 + \dots + x_n}{n} , \frac{y_1 + y_2 + y_3 + \dots + y_n}{n} \right)$$

$$\therefore G(g, f) = \left(\frac{\sum_1^n x}{n} , \frac{\sum_1^n y}{n} \right) \quad \dots\dots\dots\dots\dots\dots \text{Eq (i)}$$

Then,

Relative vector to G, *from position vector of each vectors ,*

$$\overrightarrow{GA} = (x_1 - g, y_1 - f)$$
$$\overrightarrow{GB} = (x_2 - g, y_2 - f)$$
$$\overrightarrow{GC} = (x_3 - g, y_3 - f)$$
$$\overrightarrow{GD} = (x_4 - g, y_4 - f)$$

$$\overrightarrow{XY} = (X_2 - X_1 , Y_2 - Y_1)$$

Now,

Dot Product of each Vector Pairs ,

$$\overrightarrow{GA} . \overrightarrow{GB} = (x_1 - g)(x_2 - g) + (y_1 - f) (y_2 - f)$$
$$\overrightarrow{GA} . \overrightarrow{GC} = (x_1 - g)(x_3 - g) + (y_1 - f) (y_3 - f)$$
$$\overrightarrow{GA} . \overrightarrow{GD} = (x_1 - g) (x_4 - g) + (y_1 - f) (y_4 - f)$$
$$\overrightarrow{GB} . \overrightarrow{GC} = (x_2 - g)(x_3 - g) + (y_2 - f)(y_3 - f)$$
$$\overrightarrow{GB} . \overrightarrow{GD} = (x_2 - g)(x_4 - g) + (y_2 - f)(y_4 - f)$$
$$\overrightarrow{GC} . \overrightarrow{GD} = (x_3 - g)(x_4 - g) + (y_3 - f)(y_4 - f)$$

$$\overrightarrow{XY} = X_2 . X_1 + Y_2 . Y_1$$

Then,

Cosines between all the Vector Pairs :

Cosines between $\overrightarrow{GA}$ and $\overrightarrow{GB}$ $= (\overrightarrow{GA} . \overrightarrow{GB}) / (|\overrightarrow{GA}| . |\overrightarrow{GB}|) = \alpha$

Cosines between $\overrightarrow{GB}$ and $\overrightarrow{GC}$ $= (\overrightarrow{GB} . \overrightarrow{GC}) / (|\overrightarrow{GB}| . |\overrightarrow{GC}|) = \beta$

Cosines between $\overrightarrow{GC}$ and $\overrightarrow{GD}$ $= (\overrightarrow{GC} . \overrightarrow{GD}) / (|\overrightarrow{GC}| . |\overrightarrow{GD}|) = \Theta$

Cosines between $\overrightarrow{AG}$ and $\overrightarrow{GD}$ $= (\overrightarrow{AG} . \overrightarrow{GD}) / (|\overrightarrow{AG}| . |\overrightarrow{GD}|) = \delta$

Cosines between $\overrightarrow{GA}$ and $\overrightarrow{GC}$ $= (\overrightarrow{GA} . \overrightarrow{GC}) / (|\overrightarrow{GA}| . |\overrightarrow{GC}|) = \alpha + \beta$

Cosines between $\overrightarrow{GB}$ and $\overrightarrow{GD}$ $= (\overrightarrow{GB} . \overrightarrow{GD}) / (|\overrightarrow{GB}| . |\overrightarrow{GD}|) = \Theta + \beta$

$$\Theta = Cos^{-1} \left(\frac{\vec{a} . \vec{b}}{|\vec{a}| |\vec{b}|} \right)$$

Again,

Average Cosines of all Vector Pairs,

$$\therefore \text{Avg} = \frac{(\overrightarrow{GA}. \overrightarrow{GB}) + (\overrightarrow{GB}. \overrightarrow{GC}) + (\overrightarrow{GC} . \overrightarrow{GD}) + (\overrightarrow{AG} . \overrightarrow{GD}) + (\overrightarrow{GA} . \overrightarrow{GC}) + (\overrightarrow{GB} . \overrightarrow{GD})}{n}$$

$$= (\alpha + \beta + \Theta + \delta + \alpha + \beta + \Theta + \beta) / 6$$

$$\left(n = \frac{4!}{2! (4-2)!} = 6 \right)$$

$$\therefore \text{Avg of Cosines} = (2\alpha + 2\Theta + 3\beta + \delta) / 6 \quad \dots\dots\dots\dots\dots\dots\dots\dots \text{Eq (ii)}$$