

Let,

**Player\_1** moves are  $(0,0), (2,0), (2,1), (0,2)$  and  $(1,2)$ .

**Player\_2** moves are  $(0,1), (1,0), (1,2)$  and  $(2,2)$ .

$$\text{Then, Centroid } (g,f) = \left[ \frac{0+2+2+0+1}{5}, \frac{0+0+1+2+2}{5} \right]$$

$$\therefore G(g,f) = (1, 1)$$

Again,

Reference Vector from  $G$  to each Point,

$$\vec{V}_0 = (0,0) - (1,1) = (0-1, 1-1) = (-1, -1)$$

$$\vec{V}_1 = (2,0) - (1,1) = (2-1, 0-1) = (1, -1)$$

$$\vec{V}_2 = (2,1) - (1,1) = (2-1, 1-1) = (1, 0)$$

$$\vec{V}_3 = (0,2) - (1,1) = (0-1, 2-1) = (-1, 1)$$

$$\vec{V}_4 = (1,2) - (1,1) = (1-1, 2-1) = (0, 1)$$

Then,

Norms of all of these Norms Vector,

$$|\vec{V}_0| = |\vec{V}_1| = |\vec{V}_3| = \sqrt{2} = 1.414$$

$$|\vec{V}_2| = |\vec{V}_4| = \sqrt{1} = 1$$

Now,

All the possible Pairs of all the Reference Vectors are ,

$\vec{V}_0 \vec{V}_1, \vec{V}_0 \vec{V}_2, \vec{V}_0 \vec{V}_3, \vec{V}_0 \vec{V}_4, \vec{V}_1 \vec{V}_2, \vec{V}_1 \vec{V}_3, \vec{V}_1 \vec{V}_4, \vec{V}_2 \vec{V}_3, \vec{V}_2 \vec{V}_4$  and  $\vec{V}_3 \vec{V}_4$  .

Cosines between  $\vec{V}_0$  and  $\vec{V}_1$  =  $\vec{V}_0 \cdot \vec{V}_1 / (|\vec{V}_0| |\vec{V}_1|) = 1(-1) + (-1)(-1) / (2) = 0 = 90$

Cosines between  $\vec{V}_0$  and  $\vec{V}_2$  =  $\vec{V}_0 \cdot \vec{V}_2 / (|\vec{V}_0| |\vec{V}_2|) = 1(-1) + (0)(-1) / (1.414) = -0.7071 = 135$

Cosines between  $\vec{V}_0$  and  $\vec{V}_3$  =  $\vec{V}_0 \cdot \vec{V}_3 / (|\vec{V}_0| |\vec{V}_3|) = (-1)(-1) + 1(-1) / (2) = 0 = 90$

Cosines between  $\vec{V}_0$  and  $\vec{V}_4$  =  $\vec{V}_0 \cdot \vec{V}_4 / (|\vec{V}_0| |\vec{V}_4|) = 0(-1) + 1(-1) / (1.414) = -0.7071 = 135$

Cosines between  $\vec{V}_1$  and  $\vec{V}_2$  =  $\vec{V}_1 \cdot \vec{V}_2 / (|\vec{V}_1| |\vec{V}_2|) = 1(1) + (0)(-1) / (1.414) = 0.7071 = 45$

Cosines between  $\vec{V}_1$  and  $\vec{V}_3$  =  $\vec{V}_1 \cdot \vec{V}_3 / (|\vec{V}_1| |\vec{V}_3|) = 1(-1) + 1(-1) / (2) = -1 = 180 (\text{Ignore})$

Cosines between  $\vec{V}_1$  and  $\vec{V}_4$  =  $\vec{V}_1 \cdot \vec{V}_4 / (|\vec{V}_1| |\vec{V}_4|) = 1(0) + 1(-1) / (1.414) = -0.7071 = 135$

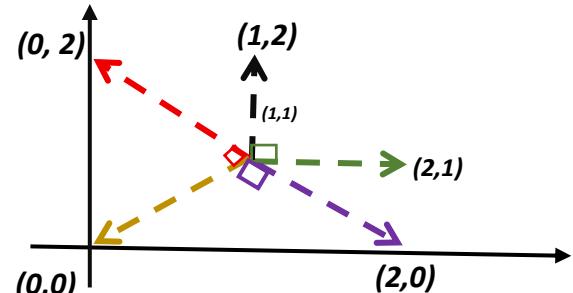
Cosines between  $\vec{V}_2$  and  $\vec{V}_3$  =  $\vec{V}_2 \cdot \vec{V}_3 / (|\vec{V}_2| |\vec{V}_3|) = 1(-1) + 0(1) / (1.414) = -0.7071 = 135$

Cosines between  $\vec{V}_2$  and  $\vec{V}_4$  =  $\vec{V}_2 \cdot \vec{V}_4 / (|\vec{V}_2| |\vec{V}_4|) = 1(0) + 1(0) / (1.414) = 0 = 90$

Cosines between  $\vec{V}_3$  and  $\vec{V}_4$  =  $\vec{V}_3 \cdot \vec{V}_4 / (|\vec{V}_3| |\vec{V}_4|) = 0(-1) + 1(1) / (1.414) = 0.7071 = 45$

X	X	O
O	O	X
X	O	X

**Fig(i) :** **Player\_1** = X , **Player\_2** = O



**Fig(ii) :** **Player\_1** moves plotted in Graph

Now ,

Score ( Avg Cosines ) of **Player\_1** =  $(3 * 90 + 4 * 135 + 2 * 45) / 9 = 100$

Similarly, For Player 2 :

Score ( Avg Cosines ) of **Player\_2** =  $(76 + 104 + 157 + 53 + 127) / 5 = 116$

Since, Avg Cosines Score of ( **Player\_1** < **Player\_2** ) :

**∴ Player\_1 is the Winner .** // Higher the Avg Cosines Angle, more Player moves are Scattered and less of Straight lines (Win Chance).