Sets, Sample space & Event

Set is composed of elements or members, where each elements is a possible outcome. A set is denoted by capital letter.

$$A = \{a, b, c, d\} \qquad a \in A,\ a \text{ belongs to } A$$

$$B = \{e, f, g, h\} \qquad a \notin B,\ \ a, b \in A$$

A Set can be defined in two ways

1. Listing all the elements $A = \{a, b, c, d\}$.
2. Describing the properties held by the members and / or nonmembers.

   $A = \{\text{first } 4 \text{ letters of the alphabet}\}$

Experiment: Experiment is a process leading to one or more possible observation.

EXAMPLE 1.1 If we toss a coin, the result of the experiment is that it will either come up "tails," symbolized by T (or 0), or "heads," symbolized by H (or 1), i.e., one of the elements of the set $\{H, T\}$ (or $\{0, 1\}$).

EXAMPLE 1.2 If we toss a die, the result of the experiment is that it will come up with one of the numbers in the set $\{1, 2, 3, 4, 5, 6\}$.

EXAMPLE 1.3 If we toss a coin twice, there are four results possible, as indicated by $\{HH, HT, TH, TT\}$, i.e., both heads, heads on first and tails on second, etc.

EXAMPLE 1.4 If we are making bolts with a machine, the result of the experiment is that some may be defective. Thus when a bolt is made, it will be a member of the set {defective, nondefective}.

EXAMPLE 1.5 If an experiment consists of measuring "lifetimes" of electric light bulbs produced by a company, then the result of the experiment is a time t in hours that lies in some interval—say, $0 \leq t \leq 4000$—where we assume that no bulb lasts more than 4000 hours.

Sample Space: A set S that consists of all possible outcomes of a random experiment is called a sample space, and each outcome is called a sample point.

Examples: If we toss two coins at a time, the sample space is given by

S= {HH, HT, TH, TT}.

Event: An event is a subset A of the sample space S, i.e., it is a set of possible outcomes. If the outcome of an experiment is an element of A, we say that the event A has occurred. An event consisting of a single point of S is often called a simple or elementary event

Example:

Referring to the experiment of tossing a coin twice, let A be the event "at least one head occurs" and B the event "the second toss results in a tail." Then A = {HT, TH, HH}, B = {HT, TT}, and so we have

Sample space S= {HH, HT, TH, TT}

$$A \cup B = \{HH, HT, TH, TT\}$$

$$A \cap B = \{HT\}$$

$$A - B = \{HH, TH\}$$

$$A' = S - A = \{TT\}$$

Example:

Sample space S={1, 2, 3, 4, 5, 6} toss a DIE

Event = Tossing an even number = {2, 4, 6}

P (Event) = $\frac{3}{6} = \frac{1}{2}$.

Independent event: If $P(B \backslash A) = P(B)$, i.e., the probability of B occurring is not affected by the occurrence or non-occurrence of A, then we say that A and B are independent events. This is equivalent to $P(A \cap B) = P(A)P(B)$.

Mutually Exclusive events: Two events are said to be mutually exclusive event if the occurrence of one of them excludes the occurrence of the other.

If A and B are two mutually exclusive events then $P(A \cap B) = 0$.

Axioms of probability:

i) For every event A, $P(A) \geq 0$.

ii) For sure or certain event S, $P(S) = 1$.

iii) For a number of mutually exclusive events $A_1, A_2, A_3,$ $P(A_1 \cup A_2 \cup A_3 \cup \dots) = P(A_1) + P(A_2) + P(A_3) + \dots$

In particular, for two mutually exclusive events $P(A_1 \cup A_2) = P(A_1) + P(A_2)$.

Bayes' theorem: Suppose that $A_1, A_2, \ldots, A_n$ are mutually exclusive events with $P(A_i) \neq 0$ whose union is the sample space S. Then for an event A that occurs when the experiment is performed, such that $P(A) > 0$, we have

$$P(A_i/A) = \frac{P(A_i)P(A/A_i)}{\sum_{i=1}^{n} P(A_i)P(A/A_i)}.$$

Problems related to Bayes' theorem:

Problem 1: There are three identical boxes containing, 4 white and 3 red balls, 3 white and 7 red balls, 5 white and 4 red balls, respectively. A box is chosen at random and a ball is drawn from it. If the ball is white, find the probability that is of the first box.