3

This experiment consists of two steps. First step has four possible outcome a_1, a_2, a_3 and a_4 and the second step has three possible outcomes b_1, b_2 and b_3 .

According to the multiplication rule, the total number of outcome of the experiment is $4 \times 3 = 12$ and each of the outcome consists of two elements one from each step.

For example, the outcome (a_3, b_2) means that third route from Benghazi to Tripoli and second route from Tripoli to Paris.

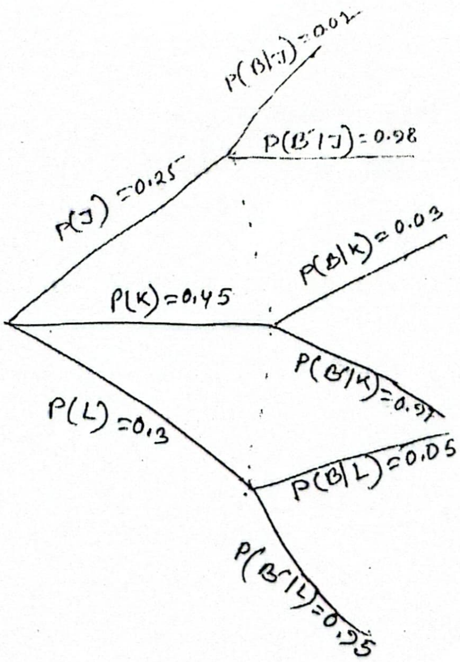
Problem: In a factory, three machines, *J*, *K* and *L* are used to make biscuits. Machine *J* makes 25% of the biscuits. Machine *K* makes 45% of the biscuits. The rest of the biscuits are made by machine *L*.

It is known that 2% of the biscuits made by machine *J* are broken, 3% of the biscuits made by machine *K* are broken and 5% of the biscuits made by machine *L* are broken.

a. Draw a tree diagram to illustrate all the possible outcomes and associated probabilities. A biscuit is selected at random.
b. Calculate the probability that the biscuit is made by machine *J* and is not broken.
c. Calculate the probability that the biscuit is broken.
d. Given that the biscuit is broken, find the probability that it was not made by machine *K*.

Solution:

a.



b. $P(J \cap B') = P(J)\,P(B'|J)$.

$= 0.25\ (0.98)$

$= 0.245$

c. $P(B) = P(J \cap B) + P(K \cap B) + P(L \cap B)$

$= P(J)\,P(B|J) + P(K)\,P(B|K) + P(L)\,P(B|L)$

$= 0.25(0.02) + 0.45(0.03) + 0.3(0.05)$

$=0.0335$

d. $P(K'|B) = \dfrac{P(K' \cap B)}{P(B)} = \dfrac{P(J \cap B) + P(L \cap B)}{P(B)}$

$= \dfrac{0.25\,(0.02) + 0.3\,(0.05)}{0.0335}$

$=0.5970$

$=0.597$

Problem: A disease is known to be present in 2% of a population. A test is developed to help determine whether or not someone has the disease.

Given that a person has the disease, the test is positive with probability 0.95.

Given that a person does not have the disease, the test is positive with probability 0.03.

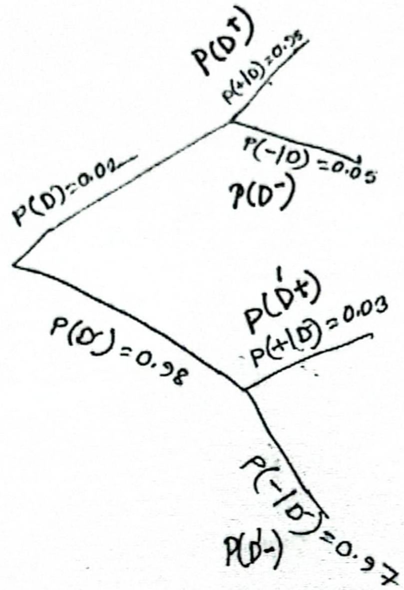
a. Draw a tree diagram to represent this information
   A person in selected at random from the population and tested for this disease.
b. Find the probability that the test is positive.

A doctor randomly selects a person from the population and tests him for the disease. Given that the test is positive,

c. Find the probability that he does not ~~have~~ known have the disease.
d. Comment on the usefulness of this test.

Solution:

a.



b. $P(+) = P(D \cap +) + P(D' \cap +)$

$= P(D)P(+|D) + P(D')P(+|D')$

$= 0.02\ (0.95) + 0.98\ (0.03)$

$= 0.0484$

c. $P(D'|+) = \dfrac{P(D' \cap +)}{P(+)} = \dfrac{0.98(0.03)}{0.0484} = 0.607$

d. $P(+)$=0.0484 and $P(D'|+)$=0.6074

Not useful since: $P(+)$ is very small when we would expect a high probability.

$P(D'|+)$ is high when we would expect a small probability.

Problem:

A manufacturer carried out a survey of the defects in their soft toys. It is found that the probability of a toy having poor stitching is 0.03 and that a toy with poor stitching has a probability of 0.7 of splitting open. A toy without poor stitching has a probability of 0.02 of splitting open.

(a) Draw a tree diagram to represent this information.

(3)

(b) Find the probability that a randomly chosen soft toy has exactly one of the two defects, poor stitching or splitting open.

(3)

The manufacturer also finds that soft toys can become faded with probability 0.05 and that this defect is independent of poor stitching or splitting open. A soft toy is chosen at random.

(c) Find the probability that the soft toy has none of these 3 defects.

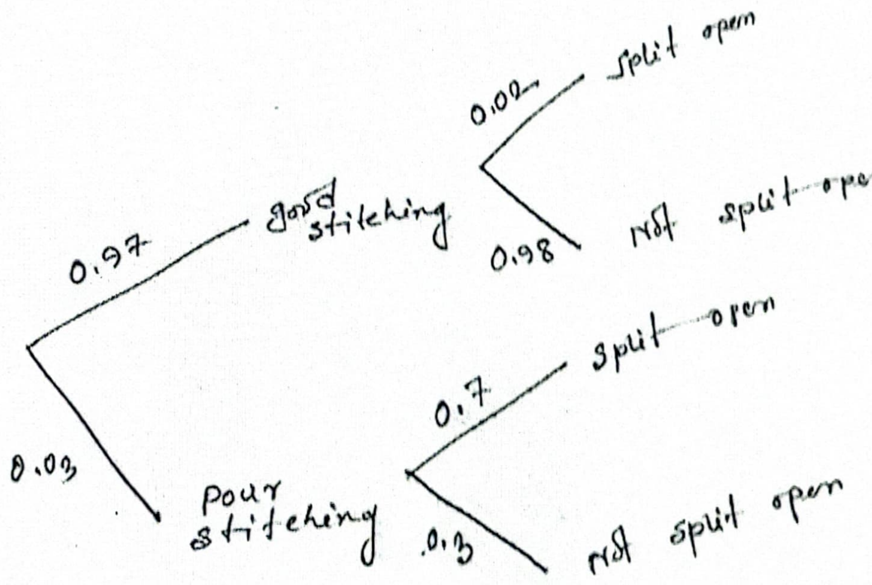
(2)

(d) Find the probability that the soft toy has exactly one of these 3 defects.

(4)

Solution:

a.



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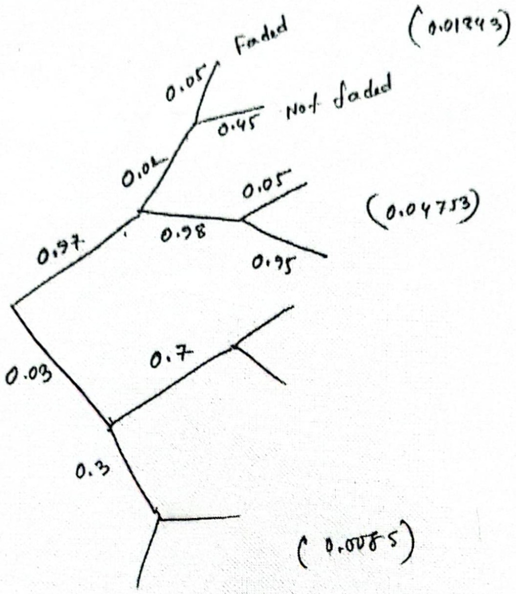
b. P (only one defect) =0.97 (0.02) + 0.03 (0.3) = 0.0284
c. P (no defects) = 0.97 (0.98) (0.95) = 0.90307 =0.903
d.



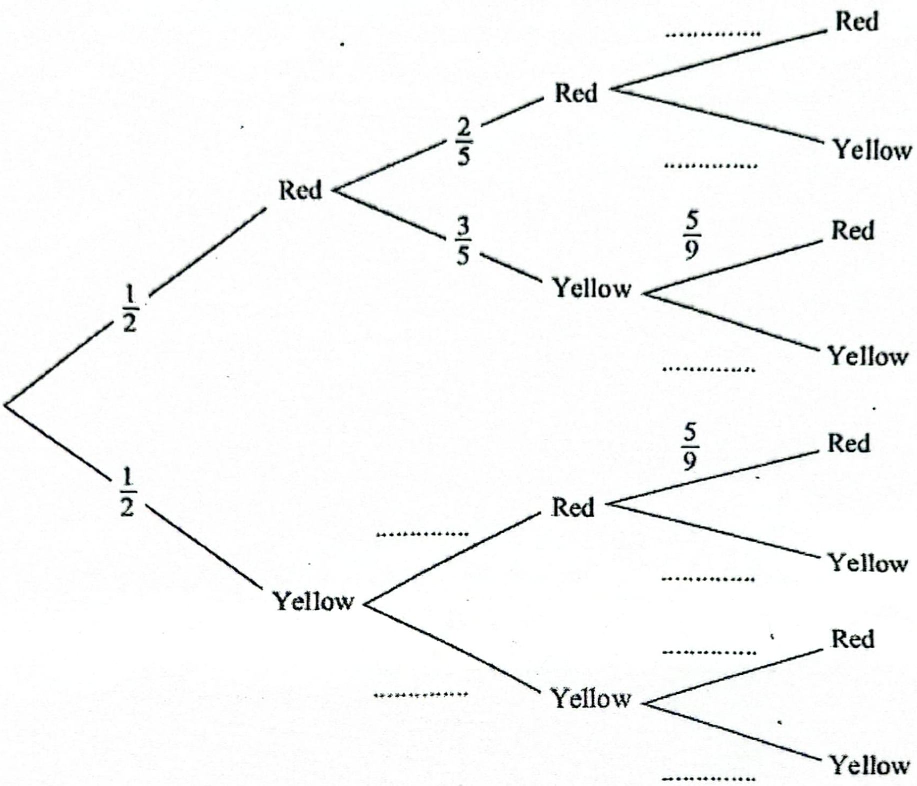
P (1 defect) = 0.07450 =0.0745

B Farjana

Problem: The bag P contains 6 balls of which 3 are red and 3 are yellow.
The bag Q contains 7 balls of which 4 are red and 3 are yellow.
A ball is drawn at random from bag P and placed in bag Q. A second ball is drawn at random from bag P and placed in bag Q.
A third ball is then drawn at random from the 9 balls in bag Q.

The event A occurs when 2 balls drawn from bag P are of the same colour.
The event B occurs when the ball drawn from bag Q is red.

(a) Complete the tree diagram shown below.



(b) Find P(A)

(c) Show that $P(B) = \frac{5}{9}$

(d) Show that $P(A \cap B) = \frac{2}{9}$

(e) Hence find $P(A \cup B)$

(f) Given that all three balls drawn are the same colour, find the probability that the all red.

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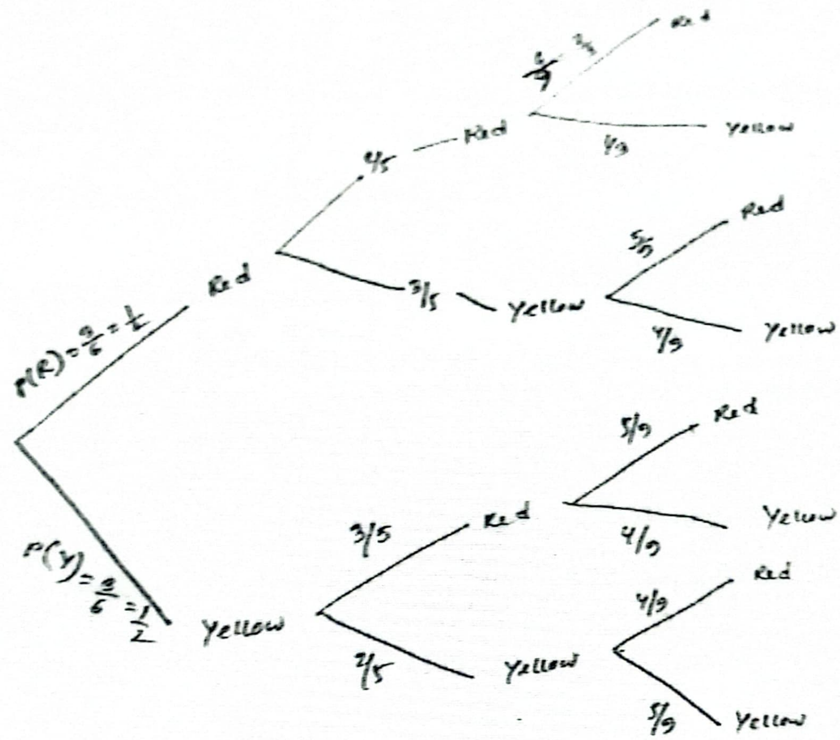
Solution:

a.



b. $P(A)=P(R,R)+P(Y,Y)=\frac{1}{2}\left(\frac{2}{5}\right)+\frac{1}{2}\left(\frac{2}{5}\right)=\frac{2}{5}$

c. $P(B)=P(R,R,R)+P(R,Y,R)+P(Y,R,R)+P(Y,Y,R)$

$=\frac{1}{2}\left(\frac{2}{5}\right)\left(\frac{2}{3}\right)+\frac{1}{2}\left(\frac{3}{5}\right)\left(\frac{5}{9}\right)+\frac{1}{2}\left(\frac{3}{5}\right)\left(\frac{5}{9}\right)+\frac{1}{2}\left(\frac{2}{5}\right)\left(\frac{4}{9}\right)$

$=\frac{5}{9}$

16, 18, 20

21, 22, 24, 26

H.W d. $P(A\cap B)=P(R,R,R)+P(Y,Y,R)=\frac{1}{2}\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)+\frac{1}{2}\left(\frac{2}{5}\right)\left(\frac{4}{9}\right)=\frac{2}{9}$

e. $P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{2}{5}+\frac{5}{9}-\frac{2}{9}=\frac{11}{15}$

f. $$P(all\, R \mid all 3\, same color) = \frac{P(all\, R)}{P(all 3\, same\ color)}$$
$$= \frac{P(R,R,R)}{P(R,R,R)+P(Y,Y,Y)}$$
$$= \frac{\frac{1}{2}\left(\frac{2}{5}\right)\left(\frac{2}{3}\right)}{\frac{1}{2}\left(\frac{2}{5}\right)\left(\frac{2}{3}\right)+\frac{1}{2}\left(\frac{2}{5}\right)\left(\frac{5}{9}\right)}$$

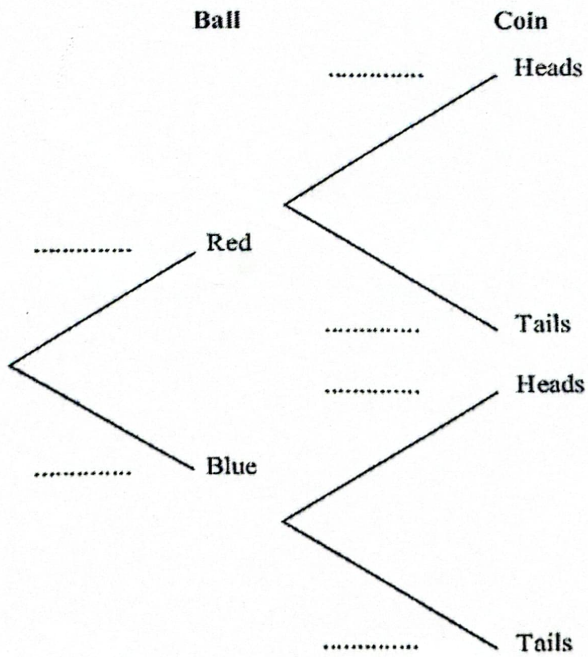
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{6}{11}.$$

Problem: An experiment consists of selecting a ball from a bag and spinning a coin. The bag contains 5 red balls and 7 blue balls. A ball is selected at random from the bag, its color is noted and then the ball is returned to the bag.

When a red ball is selected, a biased coin with probability 2/3 of landing heads is spun.

When the blue ball is selected a fair coin is spun

(a) Complete the tree diagram below to show the possible outcomes and associated probabilities.



Shivani selects a ball and spins the appropriate coin.

(b) Find the probability that she obtains a head.
(2)

Given that Tom selected a ball at random and obtained a head when he spun the appropriate coin,

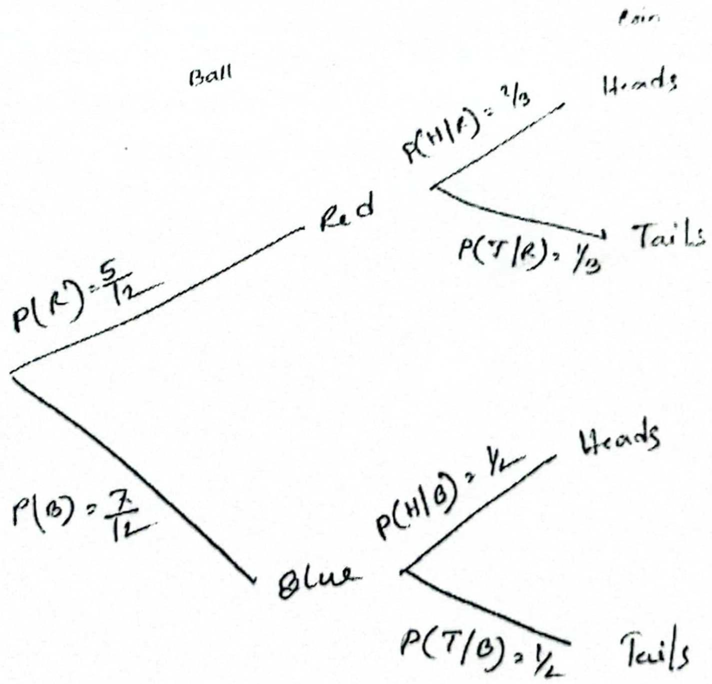
(c) find the probability that Tom selected a red ball.
(3)

Shivani and Tom each repeat this experiment.

(d) Find the probability that the colour of the ball Shivani selects is the same as the colour of the ball Tom selects.
(3)

Solution:

a.



b. $P(H) = P(R \cap H) + P(B \cap H)$

$$= P(R)P(H|R) + P(B)P(H|B)$$

$$= \frac{5}{12}\left(\frac{2}{3}\right) + \frac{7}{12}\left(\frac{1}{2}\right) = \frac{41}{72}$$

c. $P(R|H) = \dfrac{P(R \cap H)}{P(H)} = \dfrac{\frac{5}{12} \times \frac{2}{3}}{\frac{41}{72}} = \dfrac{20}{41}$ e

d. $P(\text{both balls the same}) = P(R)P(R) + P(B)P(B) = \frac{5}{12} \times \frac{5}{12} + \frac{7}{12} \times \frac{7}{12} = \frac{37}{72}$