Sets, Sample space & Event

Some important theorem on probability:

i) $P(A_2 - A_1) = P(A_2) - P(A_1)$, where $P(A_2) \geq P(A_1)$.

ii) For every event A, $0 \leq P(A) \leq 1$. i.e., probability lies between 0 and 1.

iii) $P(\Phi) = 0$. i.e., the probability of an impossible event is 0 (zero).

iv) If A' is the complement of A, $P(A') = 1 - P(A)$.

v) If $A = A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n$ where $A_1, A_2, A_3, \ldots, A_n$ are mutually exclusive event then $P(A) = P(A_1) + P(A_2) + P(A_3) + \ldots P(A_n)$

vi) For two events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

vii) If an event A must result in the occurrence of one of the mutually exclusive events $A_1, A_2, A_3, \ldots., A_n$, then $P(A) = P(A \cap A_1) + P(A \cap A_2) + \ldots + P(A \cap A_n)$

Problem 1: If a coin is tossed what is the probability to get head.

Solution: If a coin is being tossed, the sample space $S = \{H, T\}$. Suppose A is the event of getting head. Thus $A = \{H\}$.

Now $P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$.

Problem 2: If 10 of 500 randomly selected cars manufactured at a certain factory are found to be the default. What is the probability of a randomly chosen car will be default?

Solution: Suppose the total number of cars, $n(S) = 500$ and the total number of default cars $n(A) = 10$.

Thus $P(A) = \frac{n(A)}{n(S)} = \frac{10}{500} = \frac{1}{50}$.

Problem 3: A single die is tossed once. Find the probability of a 2 or 5 turning up.

Solution: If a die is tossed, the sample space $S = \{1, 2, 3, 4, 5, 6\}$. Suppose *A='turning up 2'* and *B='turning up 5'.*

Now, it is known to us the addition law of probability

Problem 04: Two coins are tossed "*A*" is the event of "getting two heads" and "*B*" is the event of "getting the second coin shows head". Evaluate $P(A \cup B)$.

Solution: If two coins are tossed ones, the sample space could be written as $S = \{HH, HT, TH, TT\}$.

Let A=the event of "getting two heads"$= \{HH\}$ and

B=the event of "getting the second coin shows head" $= \{HH, TH\}$.

$A \cap B = \{HH\} \cap \{HH, TH\} = \{HH\} \neq \{\}$, implies A and B is not mutually exclusive.

It is known to us for not mutually exclusive event

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\text{or, } P(A \cup B) = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A\cap B)}{n(S)}$$

$$\text{or, } P(A \cup B) = \frac{1}{4} + \frac{2}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}.$$

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}.$$

Problem 05:

A ball is drawn at random from a box containing 6 red balls, 4 white balls, and 5 blue balls. Determine the probability that it is (a) red, (b) white, (c) blue, (d) not red, (e) red or white.

(a) **Method 1**

Let R, W, and B denote the events of drawing a red ball, white ball, and blue ball, respectively. Then

$$P(R) = \frac{\text{ways of choosing a red ball}}{\text{total ways of choosing a ball}} = \frac{6}{6+4+5} = \frac{6}{15} = \frac{2}{5}$$

Method 2

Our sample space consists of $6 + 4 + 5 = 15$ sample points. Then if we assign equal probabilities $1/15$ to each sample point, we see that $P(R) = 6/15 = 2/5$, since there are 6 sample points corresponding to "red ball."

(b) $P(W) = \dfrac{4}{6+4+5} = \dfrac{4}{15}$

(c) $P(B) = \dfrac{5}{6+4+5} = \dfrac{5}{15} = \dfrac{1}{3}$

(d) $P(\text{not red}) = P(R') = 1 - P(R) = 1 - \dfrac{2}{5} = \dfrac{3}{5}$ by part (a).

(e) **Method 1**

$$P(\text{red or white}) = P(R \cup W) = \frac{\text{ways of choosing a red or white ball}}{\text{total ways of choosing a ball}}$$

$$= \frac{6+4}{6+4+5} = \frac{10}{15} = \frac{2}{3}$$

This can also be worked using the sample space as in part (a).

Method 2

$$P(R \cup W) = P(B') = 1 - P(B) = 1 - \frac{1}{3} = \frac{2}{3} \text{ by part (c).}$$

Method 3

Since events R and W are mutually exclusive, it follows from (4), page 5, that

$$P(R \cup W) = P(R) + P(W) = \frac{2}{5} + \frac{4}{15} = \frac{2}{3}$$

Problem 05: Two coins are tossed "A" is the event of "getting two tails" and "B" is the event of "getting the second coin shows head". Evaluate $P(A \cup B)$. **(HW)**

Problem 06: A dice is tossed once. Assume "A" is the event of getting "odd numbers" and "B" is the event of getting "the number divisible by 3". Calculate $P(A \cup B)$. **(HW)**