Conditional Probability

Problem 05: A box contains 10 green and 8 red balls. Two balls are drawn successively at random from the box. Find the probability that both balls will be red, considering both cases with and without replacement the balls during drawing.

Solution: A box contains 10 green and 8 red balls. The total number of balls is 10+8=18. If two balls are drawn successively, two cases are raised.

Case I (with replacement): The probability of the first ball is to be red is $\frac{8}{18}$. If the ball is replaced to the box, the number of red ball and the number of total balls will remain same. Thus, the probability will be same for the second ball drawn $\frac{8}{18}$. Therefore, the probability of the both balls to be red is $\frac{8}{18} \times \frac{8}{18} = \frac{16}{81}$.

Case II (without replacement): The probability of the first ball is to be red is $\frac{8}{18}$. If the ball is not replaced to the box, the number of red balls will be 8-1=7 and the number of total balls will be 18-1=17. Thus, the probability of drawing the second ball to be red is $\frac{7}{17}$. Therefore, the probability of the both balls to be red is $\frac{8}{18} \times \frac{7}{17} = \frac{56}{306}$.

Problem 06: A box contains 2 red and 3 blue marbles. Find the probability that if two marbles are drawn at random successively (without replacement), (a) both are blue, (b) both are red, (c) first ball is red and second is blue. (HW)

Problem 07: Find the probability of drawing 3 aces at random, successively, from a deck of 52 ordinary cards if the cards are (a) replaced, (b) not replaced. (HW)

Problem 08: Find the probability of getting a 7 or 11 total on either of two tosses of a pair of fair dice.

Solution: If two dice are tossed at random, the sample space is

$$
\begin{aligned}
S = \{&(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),\\
&(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),\\
&(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),\\
&(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),\\
&(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
\end{aligned}
$$

$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

Let A be the probability of getting a total 7 and B be the probability of getting a total 11.

$A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$ and $B = \{(5,6), (6,5)\}$.

We have to find $P(A \cup B)$. As the event A and B are mutually exclusive (i.e., $A \cap B = \{\}$)

Then the law of probability is $P(A \cup B) = P(A) + P(B)$

or, $P(A \cup B) = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)}$

or, $P(A \cup B) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$.

Problem 9: Find the probability of getting a 5 or greater than 10 totals on either of two tosses of a pair of fair dice. (HW)