

AI1110 Assignment 1

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EE22BTECH11215

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Question: 10.13.2.12 Sushma tosses a coin 3 times and gets tail each time. Do you think that the outcome of next toss will be a tail? Give reasons.

Solution: As the coin is tossed 3 times and gets a tail each time but it is not necessary that 4th time will be a tail. It may be either tail or head in any further toss.

Let X be the random variable for the occurrence of tail.

(i) In this binomial distribution, $n = 3$.

$$\Pr(X = r) = {}^nC_r p^r q^{n-r} \quad (1)$$

where,

$$0 \leq X \leq 3. \quad (2)$$

$$p = q = \frac{1}{2}$$

$$\Pr(X = 3) = {}^3C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0 = \frac{1}{8} \quad (3)$$

(ii) In this binomial distribution, $n = 4$.

$$\Pr(X = r) = {}^nC_r p^r q^{n-r} \quad (4)$$

where,

$$0 \leq X \leq 4 \quad (5)$$

$$p = q = \frac{1}{2}$$

$$\Pr(X = 4) = {}^4C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 = \frac{1}{16} \quad (6)$$

Let Y be a Bernoulli random variable for coin on fourth toss with heads as success.

$$\Pr(Y = 1) = p = \frac{1}{2}$$

$$\Pr(Y = 0) = 1 - p = \frac{1}{2}$$

Comparing from the cases,

$$\Pr(X = 4) = \Pr(X = 3) \times \Pr(Y = 0) = \frac{1}{16} \quad (7)$$

$$\Pr(X = 3) \times \Pr(Y = 1) = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16} \quad (8)$$

Clearly,

$$\Pr(X = 3) \times \Pr(Y = 0) = \Pr(X = 3) \times \Pr(Y = 1) = \frac{1}{16} \quad (9)$$

Code

```
from scipy.stats import binom
n,p = 4, 0.5
print(f'The PMF of the binomial distribution for X
=4 is: {binom.pmf(4,n,p)}')
```

Hence,

$$\Pr(X = 4) = 0.0625 \quad (10)$$

Let Y be the the event of getting tails in first 3 tosses, H be the event of getting head in fourth toss and T be the event of getting tail in fourth toss. Events Y and H are independent. Similarly, events Y and T are independent. $P(H) = 1 - P(T)$

$$P(T|Y) = \frac{P(YT)}{P(Y)} = \frac{P(Y) \times P(T)}{P(Y)} = \frac{\frac{1}{8} \times \frac{1}{2}}{\frac{1}{8}} = \frac{1}{2} \quad (11)$$

Also,

$$P(H|Y) = \frac{P(YH)}{P(Y)} = \frac{P(Y) \times P(H)}{P(Y)} = \frac{\frac{1}{8} \times \frac{1}{2}}{\frac{1}{8}} = \frac{1}{2} \quad (12)$$

So, Probability of getting head or tail in fourth toss given that tail occurred in first three tosses is the same.

$$P(T|Y) = P(H|Y)$$

Probability of Head = Tail = $\frac{1}{2}$ in every single case. Hence, the given statement is false.