AI1110 Assignment 1

Indian Institute of Technology, Hyderabad

EE22BTECH11215

Rayi Giri Varshini

Question: 10.13.2.12 Sushma tosses a coin 3 times and gets tail each time. Do you think that the outcome of next toss will be a tail? Give reasons.
Solution: As the coin is tossed 3 times and gets a tail each time but it is not necessary that 4th time will be a tail. It may be either tail or head in any further toss.

Let X be the random variable for the occarance of tail.

(i) In this binomial distribution, $n = 3$.

$$\Pr(X = r) = {}^{n}C_r p^r q^{n-r} \tag{1}$$

where,

$$X \in \{0, 1, 2, 3\} \tag{2}$$

$p = q = \frac{1}{2}$

$$\Pr(X = 0) = {}^{3}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^3 = \frac{1}{8} \tag{3}$$

$$\Pr(X = 1) = {}^{3}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2 = \frac{3}{8} \tag{4}$$

$$\Pr(X = 2) = {}^{3}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 = \frac{3}{8} \tag{5}$$

$$\Pr(X = 3) = {}^{3}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0 = \frac{1}{8} \tag{6}$$

(ii) In this binomial distribution, $n = 4$.

$$\Pr(X = r) = {}^{n}C_r p^r q^{n-r} \tag{7}$$

where,

$$X \in \{0, 1, 2, 3, 4\} \tag{8}$$

$p = q = \frac{1}{2}$

$$\Pr(X = 0) = {}^{4}C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 = \frac{1}{16} \tag{9}$$

$$\Pr(X = 1) = {}^{4}C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 = \frac{1}{4} \tag{10}$$

$$\Pr(X = 2) = {}^{4}C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 = \frac{3}{8} \tag{11}$$

$$\Pr(X = 3) = {}^{4}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^1 = \frac{1}{4} \tag{12}$$

$$\Pr(X = 4) = {}^{4}C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 = \frac{1}{16} \tag{13}$$

Comparing from the both cases, Probability of fourth tail on fourth toss = Probability of three tails till third toss$\times\frac{1}{2} = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$. Probability of head on fourth toss for first time = Probability of three tails till third toss$\times\frac{1}{2} = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$. Clearly, Probability of head and tail on fourth toss for first three tosses being tails is the same = $\frac{1}{16}$. Probability of Head = Tail = $\frac{1}{2}$ in every single case. Hence, the given statement is false.