AI1110 Assignment 1

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EE22BTECH11215

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Question: 10.13.2.12 Sushma tosses a coin 3 times and gets tail each time. Do you think that the outcome of next toss will be a tail? Give reasons.
Solution: As the coin is tossed 3 times and gets a tail each time but it is not necessary that 4th time will be a tail. It may be either tail or head in any further toss.

Let X be the random variable for the occurrence of tail.

(i) In this binomial distribution, $n = 3$.

$$\Pr(X = r) = {}^{n}C_r p^r q^{n-r} \tag{1}$$

where,

$$0 \le X \le 3. \tag{2}$$

$p = q = \frac{1}{2}$

$$\Pr(X = 3) = {}^{3}C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0 = \frac{1}{8} \tag{3}$$

(ii) In this binomial distribution, $n = 4$.

$$\Pr(X = r) = {}^{n}C_r p^r q^{n-r} \tag{4}$$

where,

$$0 \le X \le 4 \tag{5}$$

$p = q = \frac{1}{2}$

$$\Pr(X = 4) = {}^{4}C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0 = \frac{1}{16} \tag{6}$$

Let Y be a Bernoulli random variable for coin on fourth toss with heads as success.

$\Pr(Y = 1) = p = \frac{1}{2}$

$\Pr(Y = 0) = 1 - p = \frac{1}{2}$

Comparing from the cases,

$$\Pr(X = 4) = \Pr(X = 3) \times \Pr(Y = 0) = \frac{1}{16} \tag{7}$$

$$\Pr(X = 3) \times \Pr(Y = 1) = \frac{1}{8} \times \frac{1}{2} = \frac{1}{16} \tag{8}$$

Clearly,

$$\Pr(X = 3) \times \Pr(Y = 0) = \Pr(X = 3) \times \Pr(Y = 1) = \frac{1}{16} \tag{9}$$

Probability of Head = Tail = $\frac{1}{2}$ in every single case. Hence, the given statement is false.