MA201,
End Semester Examination, Autumn 2020-21
Remote Session-2

Instructions:
(i) This is an open book/notes/internet examination. **Solve all problems**. The duration of the examination is **60 minutes**. (ii) Clearly state all the assumptions, if any, while answering the problem. (iii) **Write your answer neatly and submit on time**. (iv) AnswerScript Filename: **Remote2_MA201_EndSem_YourRollNo_YourName.pdf**

Qus-(1) One direct application of Markov Chains (one memory model) is its utilization in genetics. According to one statistics, the probability of a child of left-handed person to be left-handed is 81/90, whereas the probability of a child of right-handed person to be left-handed is 1/90. Use the above information:

(a) Write the state transition matrix and draw the state transition diagram.

(b) Find the probability that the grandchild of a left-handed person will be left-handed.

(c) Find the steady state distribution of left-handed and right-handed people.

[3+4+4=11 Marks]

Qus-(2) A sample of n observations is collected from a continuous distribution with density $f(x) = \lambda^2 \, x \, e^{-\lambda x}$ for $x > 0$.

(a) Find the estimate of λ using the maximum likelihood method.

(b) Name the distribution $f(x)$ and estimate λ using the moment method.

(c) Calculate the value of the estimate of λ found in (a) and (b) of the sample $X = [6, 97, 45, 5, 7]$ generated by $f(x)$. Are both the estimates, evaluated in (a), and in (b), the same?

[4+4+3=11 Marks]

Qus-(3) A manager evaluates effectiveness of major hardware upgrade by running a certain process 5 times before the upgrade (sample $X = [12, 8, 6, 29, 57]$) and 5 times after it (sample $Y = [5, 3, 15, 74, 2]$). Based on these data, the average running time is 22.4 minutes before the upgrade, 19.8 minutes after it. Historically, the standard deviation of this Gaussian data has been 2.0975 minutes, and presumably it has not changed. Given $z_{0.025} = 1.96$ (i.e. $\Pr(Z \geq 1.96) = 0.025$; Z is standard normal r.v.),

(a) Construct a two-sided 95% confidence interval for population mean of the sample X and as well as for Y .

(b) Find the size of the sample X or Y to have the margin below 0.3.

(c) Construct a one sided 97.5% confidence interval for difference of population mean of the samples X and Y . Why the margin for confidence interval in (c) should be more than that in (a)?

(d) Looking at (c), was the hardware up-gradation successful with 97.5% confidence (or 5% level of significance)?

[3+3+3+3=12 Marks]

Qus-(4) Twenty people were attacked by a disease and only 18 survived. Design the right-sided proportion test, which is "the survival rate of a person, if attacked by this disease, is 85%"

(a) Assuming the large sample test (i.e. Z test), Find the value of Z.

(b) Will you reject the null hypothesis that the survival rate, if attacked by this disease, is 85% in favour of the alternate hypothesis that it is more that %85, at 5% level. Given $z_{0.05} = 1.645$ (i.e. $\Pr(Z \geq 1.645) = 0.05$; Z is standard normal r.v.),

(c) Find the P-Value and comment on the rejection of the null hypothesis. Given $z_{0.2364} = 0.633$ (i.e. $\Pr(Z \geq 0.633) = 0.2364$; Z is standard normal r.v.),

[4+4+3=11 Marks]