MA201,
End Semester Examination, Autumn 2020-21
Remote Session-1

Instructions:

(i) This is an open book/notes/internet examination. **Solve all problems**. The duration of the examination is **60 minutes**. (ii) Clearly state all the assumptions, if any, while answering the problem. (iii) **Write your answer neatly and submit on time**. (iv) AnswerScript Filename: **Remote1_MA201_EndSem_YourRollNo_YourName.pdf**

Qus-(1) (a) If the sum of n independent exponentially distributed random variables with mean value $\frac{1}{\lambda}$ is gamma distributed, Show that the mean value of gamma distribution is $\frac{n}{\lambda}$.

(b) Prove the memoryless property of geometric distribution.

(c) Show that a geometric sum of independent exponentially distributed random variables with mean value $\frac{1}{\lambda}$ is itself exponentially distributed. what is the mean value of the resultant exponential distribution if the probability of success for the geometric distribution is p?.

(d) Intuitively justify that the mean value of the resultant exponential distribution in (b), will increase or decrease with increasing p.

[2+3+4+2=11 Marks]

Qus-(2) In a certain industrial facility, accidents occur infrequently. It is known that the probability of an accident on any given day is 0.015 and accidents happen independent of each other. Use Binomial-Poisson approximation to find the probability that

(a) in any given period of 600 days, there will be at least 4 accidents.

(b) in any given period of 200 days, there will be one accident on someday.

(c) the the 4th accident will not arrive in 600 days.

(d) Recalculate (c) i.e. $\Pr(X \leq 3)$ using Binomial-Normal approximation. You can use $\phi(2) = \Pr(Z \leq 2) = 0.9772$ for the approximation.

[3+2+2+4=11 Marks]

Qus-(3) It has been observed that the time bank customers spend with a teller can be modeled as a mixed Erlang distribution whose probability density function is $f_x(t) = \alpha\mu e^{-\mu t} + (1-\alpha)\mu^2 t e^{-\mu t}$, $t \geq 0$.

(a) Draw the phase diagram of this phased distribution.

(b) If the sevice time has an average value of four minutes, what is the minimum and maximum value of the standard deviation?

(c) If the sevice time has an average value of four minutes and standard deviation of three minutes, what values should be assigned to the parameters α and μ?

[3+4+4=11 Marks]

Qus-(4) Let U be a Standard Uniform random variable. Write the main step/steps required to generate

(a) an Exponential random variable with the parameter $\lambda = 2.5$;

(b) a Bernoulli random variable with the probability of success 0.77;

(c) a Binomial random variable with parameters $n = 15$ and $p = 0.4$;

(d) a discrete random variable with the distribution $p(x)$, where $p(0) = 0.2$, $p(2) = 0.4$, $p(4) = 0.3$, $p(6) = 0.1$;

(e) a continuous random variable with the density $f(x) = 3x^2$, $0 < x < 1$;

(f) If a computer generates U and the result is $U = 0.3972$, compute the variables generated in (a)-(e).

[1+1+1+1+2+6=12 Marks]