A conceptual study of the combined effect of flutter and friction

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September 18, 2022

Outline

Introduction

- Flutter is an aeroelastic instability
- Flutter onset is a linear effect (due to small displacements)
- The amplitude of the blades grow exponentially because of this instability
- Non-linear friction at the fir-tree saturates the instability. The amplitude of vibration is bounded
- There are two time scales in this problem
  1. Fast elastic oscillations of the blade
  2. Amplitude modulation by the effect of flutter and friction

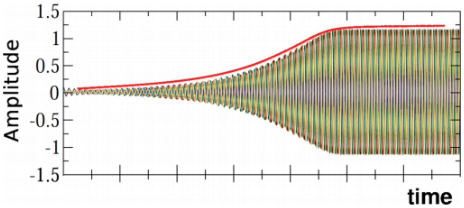


Figure 1: Evolution of the blade vibration amplitude over time (Martel, Corral, Rahul (2015))

Bladed-disk Problem

- A realistic FEM model has around 5,000,000 DOF
- The system of equations of the bladed-disk is

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}+\{F_{nl}\}+\{F_a\}=0, \quad (1)$$

- This model is computationally expensive
- An asymptotic ROM is convenient to study the qualitative behaviour

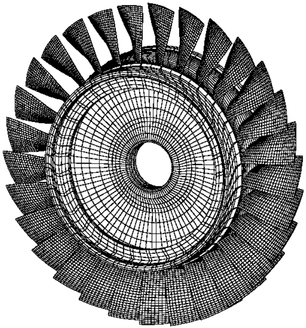


Figure 2: Bladed-disk FEM model

Asymptotic Reduced Order Model

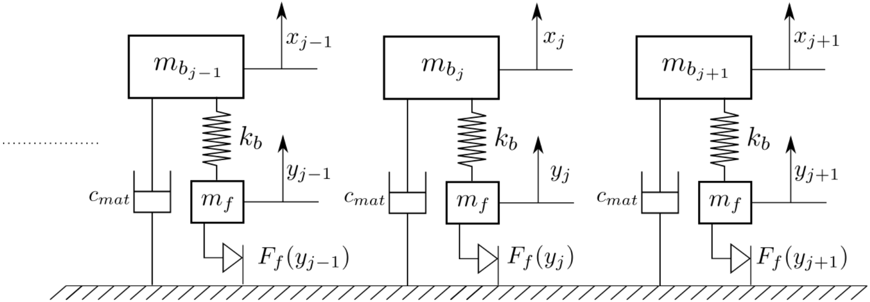


Figure 3: Mass-spring model

$$(m_b[I] + [m]_D)\{\ddot{x}\} + k_b(\{x\} - \{y\}) + [\mathrm{K_c}]\{x\} + c_{mat}\{\dot{x}\} + \{F_{aero}\} = \{F_l\}, \tag{2}$$

$$m_f\{\ddot{y}\} + k_b(\{y\} - \{x\}) + \{F_f(\{y\})\} = \{0\}. \tag{3}$$

Asymptotic Reduced Order Model

- A multiple scales asymptotic technique is used to filter the fast elastic oscillations
- Only solve for the envelope of the displacements
- Remove numerical stiffness from the problem
- The structure and aero matrices are circulant. Therefore the system is diagonalizable by performing the DFT (except the friction matrix)
- The change to between displacements and TW (Traveling Wave) amplitudes is $X = [E]A$. The dimensionless equations for the asymptotic ROM are

$$
\begin{aligned}
2i\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix}\vdots\\ A_j\\ \vdots\end{pmatrix} &= [E]^H\begin{bmatrix}Q(|X_1|) & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & Q(|X_N|)\end{bmatrix}[E]\begin{pmatrix}\vdots\\ A_j\\ \vdots\end{pmatrix}\\
&-2\left(\begin{bmatrix}\frac{\omega_1+\eta_a^1}{\theta} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \frac{\omega_N+\eta_a^N}{\theta}\end{bmatrix}+i\begin{bmatrix}\frac{\xi_{mat}+\xi_a^1}{\theta} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \frac{\xi_{mat}+\xi_a^N}{\theta}\end{bmatrix}\right)\begin{pmatrix}\vdots\\ A_j\\ \vdots\end{pmatrix}.
\end{aligned}
\tag{4}
$$

Asymptotic Reduced Order Model

- For a system with 24 blades

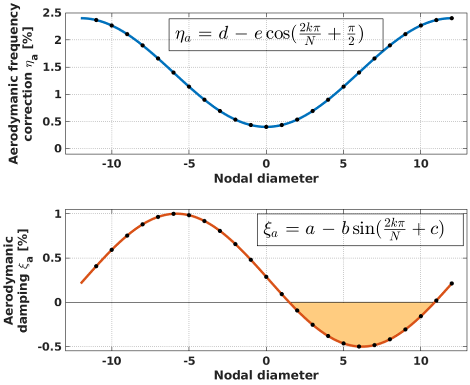


Figure 4: Aerodynamic coefficients

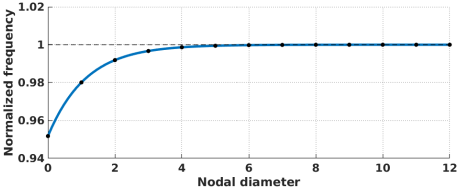


Figure 5: Frequency distribution of the first modal family

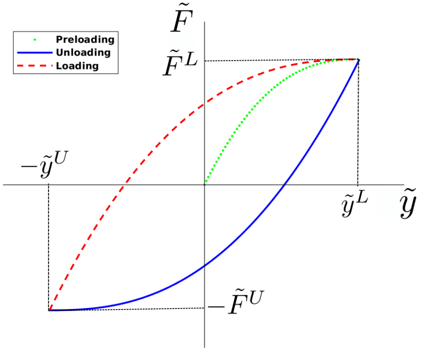


Figure 6: Friction force hysteresis loop. Microslip regime (Olofsson (1996))

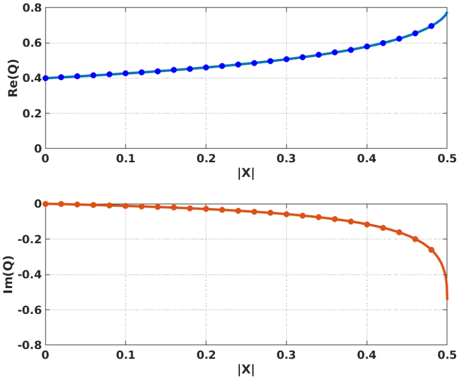


Figure 7: Complex friction coefficient

Asymptotic Reduced Order Model

- The solution of the system with initial condition $A_j = 0.1$ for $j = 1, \ldots, N$ is

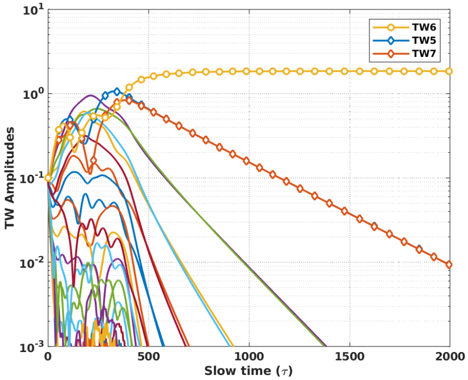


Figure 8: TW amplitude evolution (log scale)

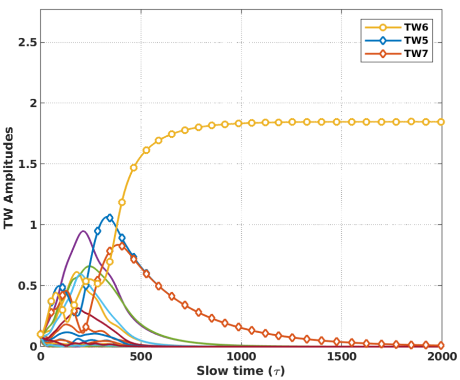
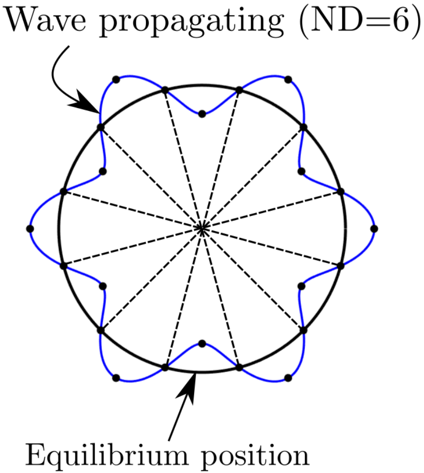


Figure 9: TW amplitude evolution (linear scale)

Asymptotic Reduced Order Model



- The linear system around the trivial solution $A = 0$ is

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix} \vdots \\ a_j \\ \vdots \end{pmatrix} = \begin{bmatrix} -\frac{\xi_{mat}+\xi_a^1}{\theta} + i\left(\frac{\omega_1+\eta_a^1}{\theta} - \frac{\Re[Q(0)]}{2}\right) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\frac{\xi_{mat}+\xi_a^N}{\theta} + i\left(\frac{\omega_N+\eta_a^N}{\theta} - \frac{\Re[Q(0)]}{2}\right) \end{bmatrix} \begin{pmatrix} \vdots \\ a_j \\ \vdots \end{pmatrix}. \tag{5}$$

- The eigenvalues are

$$\lambda_j = -\frac{\xi_{mat} + \xi_a^j}{\theta} + i\left(\frac{\omega_j + \eta_a^j}{\theta} - \frac{\Re[Q(0)]}{2}\right). \tag{6}$$

- The trivial solution is unstable when at least one $\xi_{mat} + \xi_a^j < 0$

- If A is a periodic solution with one pure TW, $A_r = \sqrt{N} R_r e^{i m_r \tau + i\alpha}$ and $A_j = 0$ for $j = 1, \ldots, N$ and $j \neq r$, system (**??**) can be solved to get

$$\Im[Q(R_r)] = \frac{2(\xi_a^r + \xi_{mat})}{\theta}, \tag{7}$$

$$m_r = \frac{\omega_r + \eta_a^r}{\theta} - \frac{1}{2}\Re[Q(R_r)]. \tag{8}$$

- The aerodynamic damping is given by the expression

$$\xi_a(k) = a - b \sin\left(\frac{2\pi k}{N} + c\right). \tag{9}$$

- The bifurcation parameter is b and it modifies the intensity of the aerodynamic force

Stability Analysis

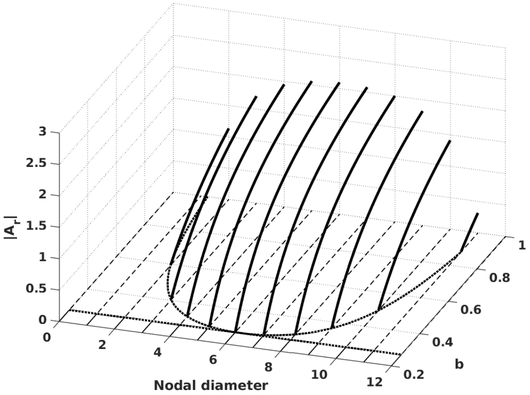


Figure 10: Bifurcation diagram of the periodic solutions for one pure TW (1/2)

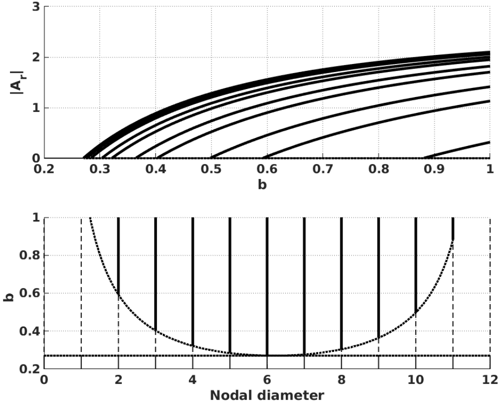


Figure 11: Bifurcation diagram of the periodic solutions for one pure TW (2/2)

- Linear stability of a pure TW with wavenumber r: TWr

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\begin{pmatrix}\vdots\\ a_{r-1}\\ a_r\\ a_{r+1}\\ \vdots\end{pmatrix} = -\frac{i}{4}Q'(R_r)R_r\left(\begin{pmatrix}\vdots\\ a_{r-1}\\ a_r\\ a_{r+1}\\ \vdots\end{pmatrix}+\begin{pmatrix}\vdots\\ \bar{a}_{r+1}\\ \bar{a}_r\\ \bar{a}_{r-1}\\ \vdots\end{pmatrix}\right)$$
$$+\begin{bmatrix} -\frac{\xi_a^1-\xi_a^r}{\theta}+i\frac{\omega_1-\omega_r+\eta_a^1-\eta_a^r}{\theta} & \cdots & 0\\ & \ddots & \\ \vdots & 0 & \vdots\\ & \ddots & \\ 0 & \cdots & -\frac{\xi_a^N-\xi_a^r}{\theta}+i\frac{\omega_N-\omega_r+\eta_a^N-\eta_a^r}{\theta}\end{bmatrix}\begin{pmatrix}\vdots\\ a_{r-1}\\ a_r\\ a_{r+1}\\ \vdots\end{pmatrix}. \tag{10}$$

Stability Analysis

- The damping differences $(\xi_a^j - \xi_a^r)$ are small
- At the beginning, the growth is given by each $\xi_a^j + \xi_{mat}$
- Decay is much slower because the exponents are one order of magnitude lower

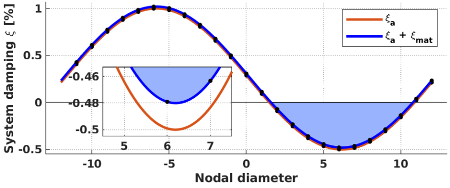


Figure 12: Aero + structural damping

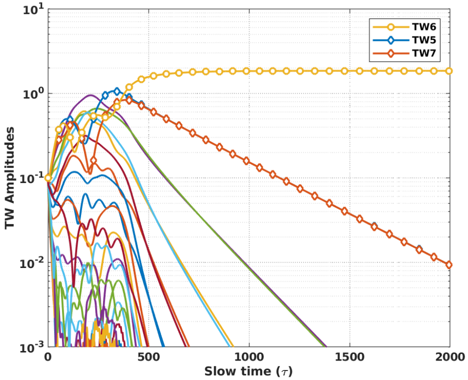


Figure 13: TW amplitude evolution (log scale)

Stability Analysis

- The periodic solution for the most flutter unstable TW is always stable
- Other TWs exhibit an unstable behaviour before becoming stable
- New final states could arise from the change in stability of the periodic branches

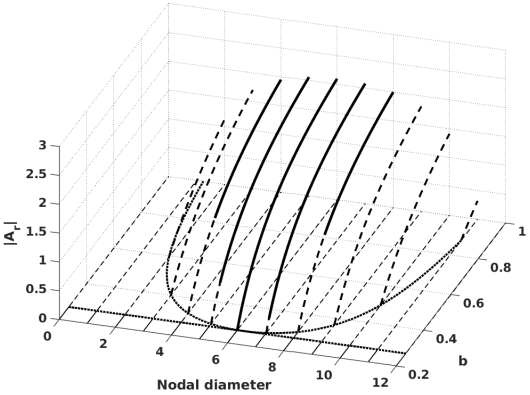


Figure 14: Bifurcation diagram with stability (periodic and trivial solutions)

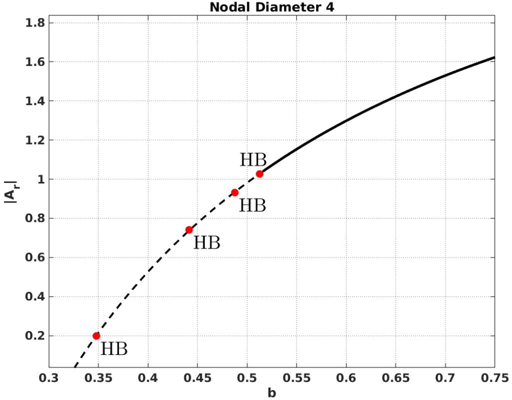


Figure 15: Bifurcation diagram for TW4

Continuation of Quasi-periodic Solutions

- Numerical continuation is performed to check for new possible final states
- The external software AUTO is used
- Continuation of the periodic branch with ND $= 4$ has a new stable quasi-periodic solution

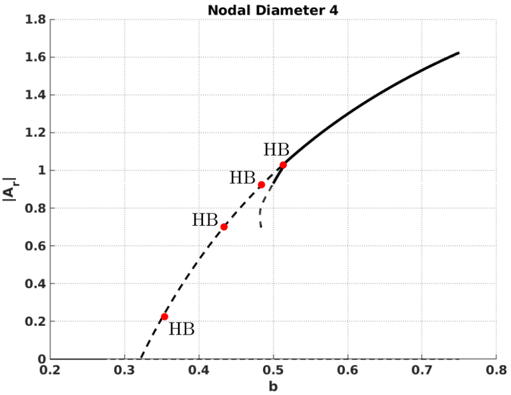


Figure 16: Continuation of the bifurcation diagram for ND $= 4$ from pure TW4

Continuation of Quasi-periodic Solutions

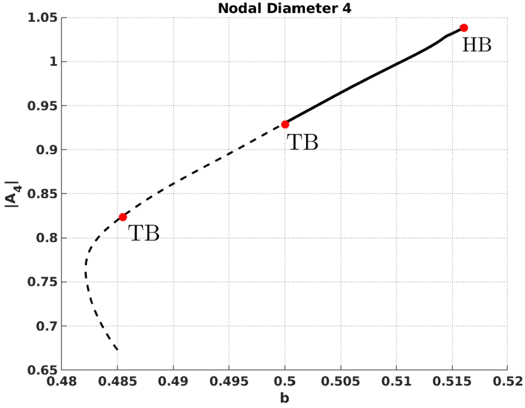


Figure 17: Continuation of the bifurcation diagram for ND = 4 from pure TW4

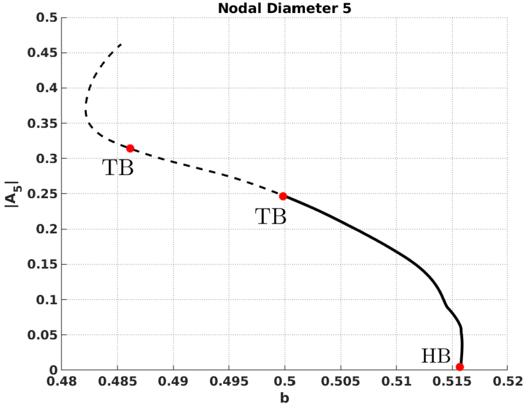
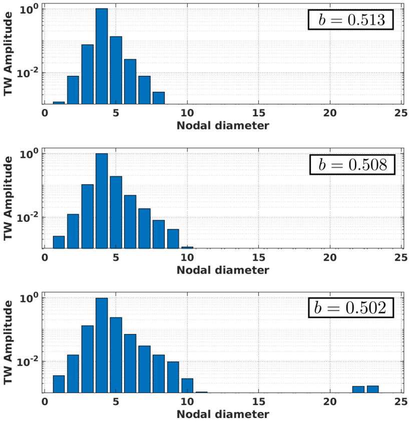


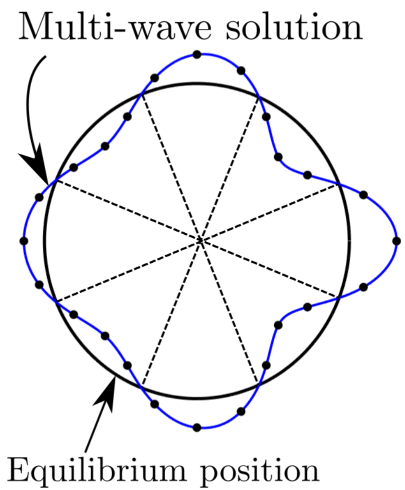
Figure 18: Continuation of the bifurcation diagram for ND = 5 from pure TW4

Continuation of Quasi-periodic Solutions

- This new solution is composed by several TWs
- A small variation of the aerodynamic damping greatly increases the amplitude of other TWs (TW5 Amplitude grows to $\sim 25\%$ of the amplitude of TW4)
- New periodic solutions arise from this new branch (Torus Bifurcation)
- Continuation from other TWs equilibrium do not present a new stable solution
- The quasi-periodic solutions have the effect of a pulse propagating along the bladed-disk



Continuation of Quasi-periodic Solutions



Conclusions

- The stability from the trivial solution was determined
- Bifurcating from the trivial solution, the periodic solutions for one pure TW were computed and their stability was obtained
- Using numerical continuation, a new stable solution were found from the previous pure TW branches. This solution is more complex because is composed by several TW
- Previous works showed the final states to be one pure TW (Martel, Corral, Rahul (2015)) or a multi-wave solution (Gross, Krack (2020)). However, they only relied on numerical integration. Generating the bifurcation diagram of the system exploring for stable solutions removes the uncertainty associated with numerical integration.
- The multi-wave solution has less maximum displacements amplitude than the pure TW solution.

References

Thank you for your attention

¿Any questions?