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MASTER THESIS

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Kinetic effects in RF discharges

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“Without encroaching upon grounds appertaining to the theologian and the philosopher, the domain of natural sciences is surely broad enough to satisfy the wildest ambition of its devotees. [. . .] The work may be hard, and the discipline severe; but the interest never fails, and great is the privilege of achievement. ”

— John William Strutt, 3rd Baron Rayleigh, 1884  
*in: Address to the British Association in Montreal*

# Declaration of Authorship

I hereby certify that this thesis has been composed by me and is based on my own work, unless stated otherwise. No other person's work has been used without due acknowledgement in this thesis. All references and verbatim extracts have been quoted, and iall sources of information, including graphs and data sets, have been specifically acknowledged.

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Greifswald; August 19, 2017



# Contents



# List of Abbreviations

abbreviation	full expression
e.g.	exempli gratia; <i>for example</i>
etc.	et cetera; <i>and so on</i>
ac	alternating current
dc	direct current
rf, RF	radio frequency
ccrf	capacitively coupled radio frequency
EDF	energy distribution function
EDV	<i>german: Energieverteilungsfunktion</i> , energy distribution function
EEDF	electron energy distribution function
IEDF	ion energy distribution function
p., pp.	page, plural pages
ff.	folio; <i>on the (next) page</i> , ablative of folium ( <i>page</i> )
HWA	hard wall approximation
MS	mass spectrometer
PROES	phase resolved emission spectroscopy
MWI	microwave interferometer
AN	antenna
FC	flow controller
PIC	particle-in-cell
MCC	Monte-Carlo-Colissions

**Table 1:** List of abbreviations and their corresponding phrases. If specified, the translation or an equivalent expression is written.





# Physical Quantities

Quantity	Unit	Symbol	Dimension	Value
Speed of Light	m/s	$c_0$	$L^1T^{-1}$	$2,997 \cdot 10^8$
thermal velocity	m/s	$v_{th,j}$	$L^1T^{-1}$	
drift velocity	m/s	$v_{D,j}, u_j$	$L^1T^{-1}$	
Boltzmann constant	eV/K	$k_B$	$M^1L^2T^{-2}K^{-1}$	$8,617 \cdot 10^{-23}$
mobility	$cm^2/Vs$	$\mu_j$	$I^1T^2M^{-1}$	
planck constant	eVs	$\hbar$	$G^{-1/2}c^{6/2}\varepsilon_0^{1/2}$	$4,1345 \cdot 10^{-15} \text{ eVs}$ $6,646 \cdot 10^{-34} \text{ Js}$
kinetic temperature	eV	$T_j$	$M^1L^2T^{-2}$	$1 \text{ eV} = 1,902 \cdot 10^{-19} \text{ K}$
elementary charge	C	$e$	$I^1T^1$	$1,902 \cdot 10^{-19}$
electric charge	C	$Q, q$	$I^1T^1$	
particle mass	kg	$m_j$	$M^1$	electron: $9,109 \cdot 10^{-31}$ ion: $5,310 \cdot 10^{-26}$ anion: $5,143 \cdot 10^{-26}$
reduced mass	kg	$\mu_{j,k}$	$M^1$	
distance,location	cm	$r, \vec{r}$	$L^1$	
Debye length	cm	$\lambda_{D,j}$	$L^1$	
particle distance	cm	$\bar{b}$	$L^1$	
mean free path	cm	$s_{mfp,j}$	$L^1$	
particle density	$cm^{-3}$	$n_j$	$L^{-3}$	
Vacuum permittivity	F/m	$\varepsilon_0$	$M^{-1}L^{-3}T^{-4}A^2$	$8,854 \cdot 10^{-12}$
electrostatic potential	V	$\Phi, U$	$M^1L^2I^{-1}T^{-3}$	
electric current	As	$I, J$	$I^1$	
electric current density	$As/cm^2$	$j_j$	$I^1L^{-2}$	
electric charge density	$C/cm^3$	$\rho$	$I^1T^1L^{-3}$	

Quantity	Unit	Symbol	Dimension	Value
electric resistance	$\Omega$	$R$	$M^1 L^2 T^{-3} I^{-2}$	
electric capacity	F	$C$	$M^{-1} L^{-2} T^4 I^2$	
time	s	$t$	$T^1$	
plasma frequency	Hz	$\omega_{p,j}$	$T^{-1}$	
collisional frequency	Hz	$\nu_j$	$T^{-1}$	

**Table 2:** Physical properties in their commonly — or for this purpose most convenient — units and corresponding SI units. If not specified, the values of each quantity refer to the afore-mentioned units.

# Abstract

The Thesis Abstract is written here and usually kept to just this page. The page is kept centered vertically so it can expand into the blank space above the title too.



# Physical Properties of Low Temperature RF Plasma

In this first chapter I will provide the necessary physical background for this work about the numerical simulation of low temperature capacitively coupled radio frequency plasma. Here both the mathematical basics and method for the simulation, as well as the most important aspects about the plasma properties will be explained.

## 1.1 Plasma Physics

### 1.1.1 Capacitively Coupled Radio Frequency Plasma

The experiment where after the conducted simulations is modelled after revolves around a capacitively coupled radio frequency, low temperature plasma at low pressures of oxygen. Here, I will refer to a plasma as an globally quasi-neutral gas, consisting of freely moving charges — e.g. electrons, positively and negatively ions — and neutral gas particles. The ratio between charged and neutral species defines the *degree of ionization*, which in this case is very low. The term of global neutrality emphasizes the purpose for different length scales inside the gas itself. Hence, the associated condition of neutrality by equal densities  $n_e = n_i$  only is valid for areas larger than the so called *Debye sphere*. Inside this ball with a radius of  $\lambda_D$  the *Debye length*, the afore-mentioned neutrality is not satisfied.

The creation of a plasma is accomplished by 2 parallel metal plates, the electrodes, where on at least one an ac signal at radio frequency is applied — this kind of experimental setup is among the most common, thus being used for basic but also in-depth studies of the afore-mentioned discharges. Here, a rf signal at exactly 13,56 MHz with an amplitude between 100–1000 V will be used. This equals to a wavelength of 22,11 m for the electric field wave, which is orders of magnitude higher than the eventually simulated experiment. The use of external magnetic fields is not within the scope of this work — correspondingly, the experiment I will refer to, also did not include any kinds of magnetic confinement or manipulation.

That said, a multitude of electric setups are possible, such as coated or grounded electrodes. Therefore, different regimes of operation ensue. For example, differently driven or shaped metal plates heavily influence the charge creation process inside the plasma. In summary, the electrodes, neutral gas and electric layout resemble a dielectric hindered plate capacitor.

This simplification can be used to access important physical properties, such as an additional voltage offset on one of the electrodes or charge currents at such. A basic scheme of an asymmetric rf discharge can be seen in figure 1.1. In the case of different electrode sizes, as seen in the scheme, the potential inside the spatially restricted area between wall and discharge can change drastically. This plasma sheath forms also between grounded parts of discharge containment or probes and plasma volume. This additional direct current offset is called *self-bias* (see section 1.1.4). A dielectric displacement current between plasma sheath and volume accommodates as a result of the different time scales of particle movement (see section 1.1.5). Especially, self-bias and displacement current play a key role in the following investigations, as a capacitive coupling between electrodes and power supply is difficult to model in a numerical kinetic simulation.

A strong mathematical analysis of general plasma properties would not be suitable for this kind of work, although certain aspects will be discussed later, such as in section 1.1.4 and section 1.1.5. In comparison to other low temperature, low pressure discharges — an example could be a dielectric hindered dc discharge at high voltages, with an electrode space gap of just a couple millimeters —, radio frequency plasma are characterized by their unique transport process inside the sheath and heating mechanisms of charged species. A more in-depth discussion can be found in section 1.1.6.

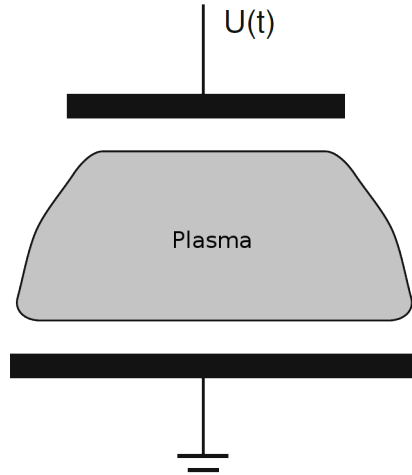
### 1.1.2 Plasma-Wall Interaction

In the discharges bulk, neutral gas particles are excited by electron collisions and radiating visible light. However, areas around, e.g. floating metal surfaces, probes and grounded walls are darker than the bulk. This is due to the low electron density and kinetic energy in this *plasma sheath*. Though areas with vanishing electron numbers can glow because of high collision efficiencies and/or frequencies.

Electrons, in general, are of a much higher mobility  $\mu_e$  and thermal velocity  $v_{th,e}$ . Hence they impinge onto walls and surfaces more often than other species, leading to a — in this case we consider an electronegative oxygen discharge, where the following can be assumed true — negative charge and potential.

#### 1.1.2.1 Child-Langmuir Law

For an asymmetric ccrf discharge, dc *self bias* and displacement current are important parts of the electric system. Hence, the *Child-Langmuir Law* as a function of those properties can be written. The rf component of the excitation is neglected.



**Figure 1.1:** Schematic of an asymmetric discharge with one grounded and one driven electrode. The rf signal is denoted with  $U(t)$ . [11]

A greatly negative charged wall at  $x = 0$  shall be a barrier for electrons of thermal velocity, e.g.  $|\Phi(0) - \Phi(d)| \ll k_B T_e / e$ . The thickness of the sheath shall be considered  $d$ . In an one-dimensional approach [11], the electron density  $n_e(x)$  can be written with a *Boltzmann* distribution function  $f_B(\Phi)$ :

$$n_e(x) = n_e(d) \cdot f_B(\Phi) = n_e(d) \cdot \exp \left\{ \left( \frac{e(\Phi(x) - \Phi(d))}{k_B T_e} \right) \right\} . \quad (1.1)$$

This means that the electron density decreases exponentially towards the negatively charged wall. It can be assumed that the sheath thickness  $d \ll s_{\text{mfp},i}$  the mean free path of the ions inside the plasma bulk. Hence, ions enter the sheath collisionless.

At the boundary between bulk and pre-sheath, the walls potential vanishes because of the plasmas shielding capabilities. Here, the ions are at  $v_{i,0}$  speeds, therefore their density becomes:

$$n_i(x) = n_i(d) \left( 1 - \frac{2e\Phi(x)}{m_i v_{i,0}^2} \right)^{-1/2} \quad (1.2)$$

Futhermore, one can assume that the kinetic energy at this point is smaller than the potential energy for the acceleration inside the pre-sheath, e.g.  $m_i v_{i,0}^2 \ll |e\Phi(x)|$ . Using *Poisson's* equation, and taking into account the ion-sheath interaction, equation 1.3 gives an equation for  $\Phi(x)$ :

$$\Delta\Phi \cong -\frac{en_i(-d)}{\varepsilon_0} \left( -\frac{2e\Phi(x)}{m_i v_{i,0}^2} \right)^{-\frac{1}{2}} \quad (1.3)$$

Solving this, and using the unpertubated ion current  $j_i = n_i(d)ev_{i,0}$ , one yields the result by *Langmuir*.

$$\Phi(x) = \left( \left( \frac{3}{4}(x+d) \right)^4 \left( \frac{j_i}{\varepsilon_0} \right)^2 \frac{m_i}{2e} \right)^{\frac{1}{3}} \quad (1.4)$$

Again, solving equation 1.4 for the current  $j_i$  yields the *Child-Langmuir Law* (see equation 1.5). This equation defines the ion current as a function of the unpertubated plasma bulk. In other words, the sheath changes its thickness in dependency of those certain discharge parameters, always satisfying the ion current defined by the *Child-Langmuir Law*.

$$j_i = \frac{4}{9}\varepsilon_0 \left( \frac{2e(\Phi(-d) - \Phi(0))^3}{m_i d^2} \right)^{\frac{1}{2}} \quad (1.5)$$

### 1.1.2.2 Surface Effects and Secondary Ion Emission

Although plasma sheath physics are influenced by bulk properties, such as temperatures and densities, the space charge area itself is mainly characterized by processes of the wall. Hence an important aspect is the absorption and re-emission of ions and electrons. Those species than have unique features, like e.g. high velocities. Let's assume the associated metal wall behind the sheath ideally absorbs all impinging, charged particles, which recombine immediately near the surface.

**Secondary Electron Emission** The discharges electrons are much faster and mobile than the other species, leading to the afore-mentioned negative charging and potential drop towards the wall. This accelerates the ion species up to *Bohm velocity* — see section 1.1.3 for a more in-depth discussion of the *Bohm criteria* and the ions sheath physics.

Continuity and charge conservation must be satisfied, hence the fluxes  $J_j$  of the species  $j$  must be equal at the sheath edge — into and out of the sheath:

$$J_e = J_i \quad (1.6)$$

The mentioned potential drop  $\Delta\Phi$  from section 1.1.2.1, beside accelerating positive ions, reflects negative charges slower than  $v \leq \sqrt{2e\Delta\Phi/m}$ . Similar to equation 1.1, the electron current towards the wall can be written. Here the first moment of the electron velocity  $\langle v \rangle$  is used, calculated in equation 1.8 with the electron energy distribution function (EEDF)  $f_e(v)$ :

$$j_e = -\frac{e}{4}n_e\langle v \rangle \exp\left(-\frac{e\Delta\Phi}{k_B T_e}\right) \quad (1.7)$$

$$\langle v \rangle = \int_{\mathbb{R}} v \cdot f_e(v) dv \quad (1.8)$$

Impinging ions are neutralized before impact by particles from the electron gas in the wall. Like before, any produced neutrals are reflected and exit the sheath collisionless — the mean free path is larger than the sheath thickness. Hence ionization is a process almost exclusively happening in the bulk or the pre-sheath.

Assuming a fast electron impacts on the wall, there is a chance of colliding with and liberating a second electron from the target. Here, the *secondary electron emission* coefficient is noted as  $\gamma$ : an impinging electron emits  $\gamma$ -many electrons from the metal. This *SEE* reduces the  $\Delta\Phi$  because of an addition charge current from the wall towards the sheath edge, therefore altering the continuity condition  $j_i = j_e$ . A new *effective potential drop*  $\Delta\Phi_{\text{eff}}$  can be written in equation 1.9. According to [1] there is a critical value  $\gamma_c$  from which on the wall potential is unstable, leading to shifting sheath edges — the sheath edge oscillates with the rf signal anyway — and strong currents from the wall.

$$\Delta\Phi_{\text{eff}} = -\frac{k_B T_e}{e} \cdot \ln\left((1 - \gamma) \sqrt{\frac{m_i}{2\pi m_e}}\right) \quad (1.9)$$

**Secondary Ion Emission** Research prior to this thesis [8] indicates that ions are produced near the surface of a metal electrode and heavily accelerated in the plasma sheath. In theory, secondary emission by surface ionization — in analogy to the surface neutralization — occurs with incident atoms of thermal energy. Hence one assumes a positively biased wall at high temperatures as the target. It's valence level is therefore broadened, giving an atom  $A$  the chance to deposit an electron at the metal. After equilibrating thermally, a positive ion is emitted by chance. This statistical process can be described by a thermodynamical equation (see equation 1.11) yielding the ionization coefficient of  $A$ . In equation 1.11 a modified approach for the *Saha-Langmuir equation* on the degree of ionization in gases can be found. Here, the surface's temperature  $T$  and average work function  $\bar{\Phi}_+$  are needed. Additionally, the ionization energy  $I(A)$  — or impact energy —, the particle number currents of both species  $j$  and  $j^+$ , corresponding statistical weights  $w$ ,  $w^+$  and reflection coefficients at the intrinsic potential



barrier  $r/r^+$  are used.

$$A \rightleftharpoons A^+ + e^- \quad (1.10)$$

$$\alpha^+(A^+) = \frac{j^+}{j} = \frac{(1-r^+)w^+}{(1-r)w} \cdot \exp\left(\frac{\bar{\Phi}_+ + e\sqrt{eV_{\text{ext}}} - I(A)}{k_B T}\right) \quad (1.11)$$

At high temperatures of, e.g. 1000 K and externally applied potentials  $V_{\text{ext}} < 1$  kV, the reflection associated *Schottky term*  $e\sqrt{eV_{\text{ext}}}$  and the corresponding coefficients  $r/r^+$  can be neglected — it appears to be just half of the thermal energy at room temperature. Though a theoretical approach is possible, there have't been accurate studies of such coefficients for temperatures around 300 K.

In addition to SIE of positive ions, a case for negative ions can be easily derived with small changes to equation 1.11: a negatively biased electrode is assumed and the average work function yields a different sign. The electron affinity of the incident particle  $B$  is noted as  $A(B)$ .

$$B + e^- \rightleftharpoons B^- \quad (1.12)$$

$$\alpha^-(B^-) = \frac{(1-r^-)w^-}{(1-r)w} \cdot \exp\left(\frac{-\bar{\Phi}_- + e\sqrt{eV_{\text{ext}}} + A(B)}{k_B T}\right) \quad (1.13)$$

Applying the former assumptions to both equations of positive and negative ions, inserting a homogeneous work function  $\Phi = \bar{\Phi}_- = \bar{\Phi}_+$  for the used substrate yields the originally derived *Saha-Langmuir equations*.

$$\alpha^+(A^+) = \frac{w^+}{w} \exp\left(\frac{\Phi - I(A)}{k_B T}\right), \quad \alpha^-(B^-) = \frac{w^-}{w} \exp\left(\frac{-\Phi + A(B)}{k_B T}\right) \quad (1.14)$$

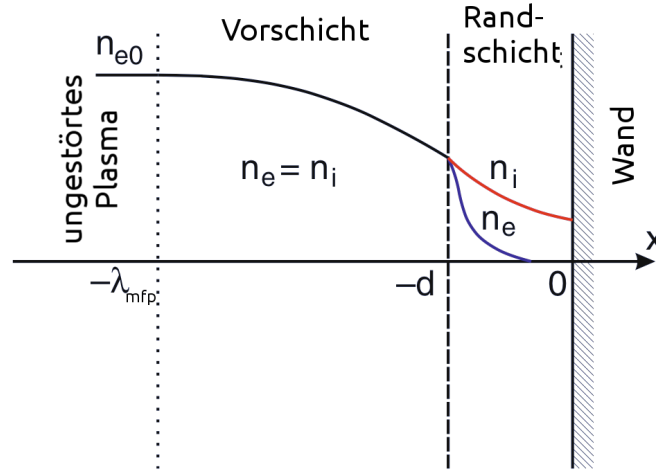
Though only considering atomic particle beams onto the wall until this point, forms similar to equation 1.14 can be derived for molecular surface interactions [7]. In case of the earlier discussed ccrf discharges, arguments like high temperatures can not be applied, hence the need for measured reflection coefficients.

Works of, e.g. [15] and [10] investigated ion beam scattering, electron loss and transport in plasma sheath environments for metal walls, especially MgO(100) surfaces. There Ustaze et al. studied incident oxygen gas particles — ions and neutrals — on magnesium oxyde surfaces. Impinging atoms became negatively charged ions, picking up electrons from the MgO of the wall. This interaction, though requiring a minimum ionization and liberation energy for the electron, is most effective at low energies  $< 1$  eV. This is due to a maximum of residence time at the target for an incoming atom. Hence it can be considered a non-resonant charge transfer process at the anion site. For an more in-depth discussion of both electron loss and capture for anion transport processes at walls one should consider [7].

Here, due to the lack of theoretical and experimental data on ion surface production, an ion beam onto the wall will result in a anion current in opposing direction of

$$j_- = \eta j_+, \quad (1.15)$$

with a corresponding efficiency of an incident positive particle  $\eta$ . One should keep in mind the reaction of the plasma sheath and potential inside to an additional anion current from the wall. Also, the same stability criteria apply for  $\eta$  as they do for the electron emission coefficient  $\gamma$ . In case of SIE beyond a critical value  $\eta_c$ , a second plasma sheath may develop, enclosing the bulk and an inner sheath and reducing transport in-between.



**Figure 1.2:** One dimensional density profiles as a function of the distance to a floating wall. Note the exponential decrease of the electron density  $n_e$  from the sheath border towards the presumably negatively charged wall. Densities already reach approximately  $0,66n_{e,0}$  inside the pre-sheath. [11]

### 1.1.3 Bohm Criteria

In section 1.1.2 the behaviour of charge particle densities inside the plasma sheath has been discussed. In contrast to the discharge volume, those densities do not satisfy the quasi-neutrality condition in a distance of  $d$  from the wall anymore. Though we know that the sheath is a spacially restricted area around electrostatic floating surfaces, a physical law concerning this circumstance has not been derived here. So the question ensues, why the area of electron depletion does not extend further into the discharge volume.

To answer this question, one has to take a look at a substitutional system. This will be a, likewise mechanical, one-body extremal problem of a point mass. In this case only kinematic potentials with inverted parabolic maxima are of interest. Therefore, in this unstable equilibrium, a small perturbation culminates into a large force on the test body.

To see the quality of this example, one has to take a look at the second order differential equation of the afore-mentioned mechanical problem and the electrostatic *Poisson's equation* (see equation 1.16).

$$m \frac{d^2 \vec{r}}{dt^2} = -\frac{dV}{d\vec{r}} \quad \Leftrightarrow \quad \Delta_{\vec{r}} \Phi = -\frac{d\Psi}{d\Phi} = f(\Phi) \stackrel{\text{Poisson's}}{=} \frac{\rho}{\varepsilon_0} \quad (1.16)$$

For an instability, the force on the test body must increase with the distance from the equilibrium, hence the equation 1.18 is used to calculate the exact velocity at which an ion is entering the sheath. This results in the first *Bohm criteria*.

$$0 > \left. \frac{d^2 \Psi}{d\Phi^2} \right|_{\Phi=0} \stackrel{\text{equation 1.16}}{=} \left. \frac{d}{d\Phi} \left( \frac{n_e(x) - n_i(x)}{\varepsilon_0} \right) \right|_{\Phi=0} = \frac{en_e(-d)}{\varepsilon_0} \left( \frac{e}{k_b T_e} - \frac{e}{m_i v_{i,0}^2} \right) \quad (1.17)$$

$$\Rightarrow v_{i,0} \geq v_{i,B} = \sqrt{\frac{k_B T_e}{m_i}} \quad (1.18)$$

Analogously you can define the so called *Mach number*  $M = v_{i,0}/v_{i,B}$ , where  $v_{i,B}$  denotes the *Bohm velocity*.

Now, to understand why the sheath does not extend further than a fixed distance  $d$  from the discharge boundary, the particle movement has to be investigated on a smaller scale. As seen above, there is an electric field in the *pre-sheath* that accelerates the ions to  $v_{i,B}$ . In addition, quasi-neutrality is still satisfied here:

$$n_i(x) = n_{i,0} \exp\left(\frac{e\Phi(x)}{k_B T_e}\right) = n_e(x) . \quad (1.19)$$

Still,  $\Phi(x)$  is the potential inside the pre-sheath from section 1.1.2 and  $n_{i,0}$  the unpertubated density from the plasma *bulk*. A greater part of the ion transport process in this area is governed by collisions with neutral gas particles, hence the velocity distribution function with the collision frequency  $\nu_{n,i}$  has to be rewritten:

$$\frac{dv_i}{dx} = \frac{\nu_{n,i} v_i^2}{v_B^2 - v_i^2} . \quad (1.20)$$

From the singularity in equation 1.20 at  $v_i = v_B$  and the knowledge of  $\Phi(x)$  at the wall, one can calculate the sheath thickness  $d$ . Furthermore, ions with velocities smaller than the Bohm velocity are being accelerated inside the pre sheath. According to equation 1.18 velocities greater than  $v_B$  are not allowed here. This is, together with equation 1.20 the reason why the ion velocity is exactly  $v_B$  at the boundary of the plasma sheath and thus a positive space-charge ensues.

$$M \geq 1 \Leftrightarrow v_i(-d) \geq v_B \quad (1.21)$$

Conclusively, at the sheath boundary equation 1.21 is satisfied.

At  $x = -d$ , both negative and positive charge density decreased to  $n_i = n_e \approx 0,66n_{e,0}$  (see figure 1.2), where the potential is approximately  $-k_B T_e/2e$  because of the currents onto the wall. In summerization, the plasma does not ‘see’ its sheath, because the ion dynamic discussed before is spatially restricted. The sheath only develops where there is electron depletion or an externally applied, negative potential.

#### 1.1.4 Self Bias Voltage

An important step towards the electric characterization of such ccrf discharges is the development of a replacement circuit, see figure 1.3a. Thus, one can define a specific impedance for a rf discharge of excitation frequency  $\omega$ . The value of  $\varepsilon_p$  resembles the permeability of the working gas between the driven and/or grounded electrode [11]. In addition, this volume has the capacity  $C_p$  — the capacity of a cubicle with a cross section  $A$ , thickness  $b$  and electron-neutral collision frequency  $\nu_{e,n}$  calculates like equation 1.22.

$$\varepsilon_p = 1 - \frac{\omega_{p,e}^2}{\omega(\omega - i\nu_{e,n})}, \quad C_p = \varepsilon_p C_0 = \varepsilon_p \varepsilon_0 \frac{A}{b} \quad (1.22)$$

$$Z_p = \left( i\omega C_p + \frac{1}{\frac{1}{\omega_{p,e}^2 C_0} (\nu_{e,n} + i\omega)} \right)^{-1} \quad (1.23)$$

The equation 1.23 represents the full electrical impedance, consisting of the inverse sum of real and imaginary resistance, as well as the capacity of the neutral gas volume. Here,  $i\omega/(\omega_{p,e}^2 C_0)$  characterizes the electrons inertia in regard to an external excitation  $\omega$ . The real

part  $\nu_{e,n}/(\omega_{p,e}^2 C_0)$  denotes the resistance by neutral particle collisions.

For high excitation frequencies, e.g. 13,56 MHz the bulk impedance can be neglected (see equation 1.23, [6]). Both sheath capacities of anode and cathode take the dominant part. Therefore, the discharge potential and voltage can be written as:

$$U(t) = U_{sb} + U_{rf} \sin(\omega t), \quad \Phi_p(t) = \overline{\Phi_p} + \Phi_{rf} \sin(\omega t) \quad (1.24)$$

Both electrodes sheath collapses completely during a full cycle of  $U_{rf}(t)$ , which is why charges can impinge onto the surface and force the plasma potential  $\Phi_p$  to equal out locally with the walls. A short circuit between plasma and sheath occurs when  $\Phi_p$  becomes negative with regard to the excitation. The equation 1.25 and figure 1.3b express this circumstance.

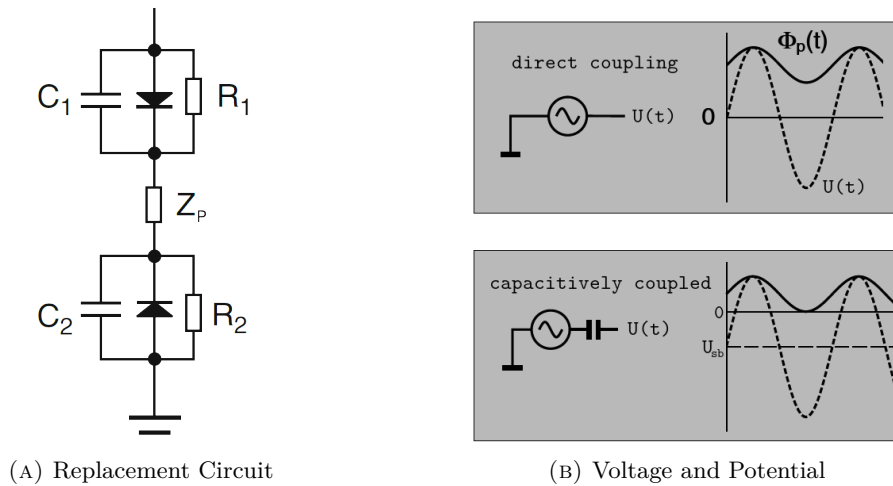
$$\Phi_p \text{ max} = \overline{\Phi_p} + \Phi_{rf} \geq U_{sb} + U_{rf}, \quad \Phi_p \text{ min} = \overline{\Phi_p} - \Phi_{rf} \geq 0. \quad (1.25)$$

If there is no special coupling between electrode and electrical driver, the equality in equation 1.25 is true. However, if a capacitive coupling is used, there can't be any net current between excitation and electrode. The capacitance can not be inverted over the course of one rf cycle. The electron currents are then equal on both electrodes, therefore shifting the minimum plasma potential to ground and the maximum to the excitation.

Finally, the dc *self bias* part  $U_{sb}$  and the mean plasma potential  $\overline{\Phi_p}$  are

$$\overline{\Phi_p} = \frac{1}{2} (U_{sb} + U_{rf}), \quad U_{sb} = \frac{C_1 - C_2}{C_1 + C_2} U_{rf}. \quad (1.26)$$

If the excitation frequency  $\omega$  is small compared to other time scales, e.g. electron and ion plasma frequencies, the electron current from the sheath  $j_L$  becomes bigger than the displacement current  $j_{dc}$ . Hence the electron current onto the driven electrode decreases by a maxwellian factor — this is a function of the thereon applied voltage — compared to the corresponding ion current. Conclusively, the electrodes sheath impedance is bigger than those of the floating walls. Together with equation 1.24 and equation 1.18 the plasma potential  $\Phi_p$  approximately vanishes,



**Figure 1.3:** Figure for an asymmetrically driven ccrf discharge. [11]

(A):  $Z_p$  denotes the impedance of the plasma bulk. A diode represents the directed electron current from the sheath into the discharge volume.  $R_j$  and  $C_j$  are the electrical properties of the positive space-charge area.

(B): Schematic of potential and voltage of a direct capacitively coupled rf discharge.

requiring only the currents onto the driven electrode to equal out. For small values of  $\omega$  equation 1.27 yields the *self bias voltage* [11]. Here,  $\mathbf{J}_0$  denotes the zeroth order *Bessel function*.

$$U_{sb} = \frac{k_B T_e}{e} \ln \left[ \mathbf{J}_0 \left( \frac{e U_{rf}}{k_B T_e} \right) \right] \quad (1.27)$$

In (insert fig:imagandreal here) the voltage's real and imaginary part are shown for an exemplary ccrf discharge. One can examine there that the self bias never disappears for excitations  $U_{rf} \neq 0$ , hence becoming an important part for capacitively coupled plasma.

### 1.1.5 Dielectric Displacement Current

Due to their higher mobility and plasma frequency  $\omega_{p,e}$ , the electron distribution can follow an external excitation with a similarly high frequency much better than the heavier ions species. Because of that, one will assume those as nearly stationary, e.g.  $\omega_{p,i} \ll \omega_{p,e}, \omega_{rf}$ . Investigating the circumstances and consequences of this relation yields the displacement current  $j_d$ .

Lets suppose there is an area of thickness  $d$  in front of a negatively charged wall, where the electron density is negligible and the corresponding ion property constant at  $n_{0,i}$ . Thus an electric field of

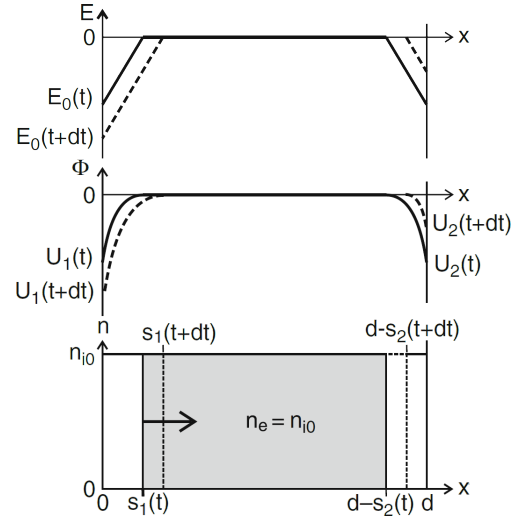
$$E_0 = -en_{0,i}d/\varepsilon_0 \quad (1.28)$$

establishes. If the wall potential now decreases due to electron bombardement or external manipulation, the sheaths border moves further inside into the discharges volume with the velocity  $u_s = ds_1/dt$ . Thus, the sheath expansion and hence charge movement creates an additional *displacement current*  $j_d$ , which is compensated with  $j_{d,e}$  the electron current from this border displacement. Hence charge conservation and continuity is satisfied [2].

$$j_d = -en_{0,i}u_s = -j_{d,e} \quad (1.29)$$

Electrons that are pushed out of this positive space-charge area then contribute to the plasma bulk density, and conclusively, to the quasi neutrality  $n_e = n_{0,i}$ . But in case of a harmonically driven discharge, the sheath in front of the opposing electrode is shrinking with  $ds_1 = -ds_2$ . Hence, the bulks spatial expansion and position are oscillating sinusoidal, or: the sheaths thickness oscillates harmonically around a mean value, e.g  $s_0$ . The associated voltage drop across the discharge [11] between the sheath potentials  $U_{1/2}$  would be

$$\Delta U = U_1 - U_2 = -\frac{2en_{i,0}s_0}{\varepsilon_0} \exp\{i\omega t\} \quad (1.30)$$



**Figure 1.4:** One dimensional density, potential and electric field for an asymmetric, harmonically driven discharge. Note the moving sheaths border. [11]

### 1.1.6 Heating Mechanisms

**Ohmic Heating** In a spatially uniform electric field that oscillates perpendicular to the electrodes harmonically, like it is the case in the bulk of a previously concerned ccrf discharge, electrons periodically gain and lose energy in the absence of collisions without any net energy gain [13]. This is due to the symmetrical de-/acceleration in the sheaths and main plasma volume over one rf cycle. Let's assume the electric field has no or a negligible component parallel to the electrodes. Hence the mean absorbed power by the electrons in an oscillating electric field is

$$\bar{P}_{\text{ohm}} = \omega_{\text{rf}} \int_0^{T_{\text{rf}}} j_{\text{tot}}(t) \cdot E(t) dt \quad (1.31)$$

$$m_e \frac{dv_e}{dt} = -eE(t) - m_e \nu_{n,e} v_e \quad (1.32)$$

The total charge current density  $j_{\text{tot}}$  is the sum of displacement current from section 1.1.5 and conduction current  $en_{e,0}v_e$ . Solving equation 1.32 for the velocity, one yields an imaginary and real part due to the bulk impedance, like it was discussed earlier in section 1.1.4. Substitution of this result [13] gives

$$\bar{P}_{\text{ohm}} = \frac{|E_0|^2 \text{Re}(\sigma_p)}{2} = \frac{|j_0|^2}{2 \text{Re}(\sigma_p)}, \quad \sigma_p = \frac{n_e e^2}{m_e(\nu_{n,e} + i\omega_p)} \quad (1.33)$$

This is, again, the total mean power dissipated into the electron species through acceleration in a harmonically oscillating electric field and neutral gas friction. The property  $\sigma_p$  is the plasma conductivity, hence yielding  $j_0 = \sigma_p E$ . This shows that power from an spatially uniform, harmonically oscillating electric field can only be transferred via collisions. Elastic electron-neutrals collision transferred into a direction perpendicular to the field and is, hence, not lost during the reversal of  $E(t)$ . Therefore the electron species gains energy during the field oscillation. This mechanism is called *ohmic heating* and takes place mainly in the plasma bulk.

**Stochastic Heating** Low-pressure, capacitively coupled rf plasma can primarily be stabilized by collisionless heating in the sinusoidally modulated discharge sheaths, like it was proposed earlier in section 1.1.2.1 and the following. Most theoretical models assume a ‘hard wall’ approximation (HWA), where the electrons are considered to collide elastically with the oscillating sheath edge. Heating power is then averaged by reverse and forward energy fluxes into and out of the sheath respectively. This gives an easy access to heating mechanisms of the proposed discharges.

The heating mechanism in such low pressure plasma is of particular importance, because collisions are rare and sheath processes are key to the sustainability of the discharge (see e.g. section 1.1.3). The afore-mentioned HWA uses the ‘*fermi acceleration*’ argument, which implies that the particles are heated or cooled due to the sheath sinusoidal oscillation and a corresponding de-/acceleration. This process, though relying on enough randomization in phase-space inside the bulk, sufficiently creates a net heating of the plasma [4, 3]. This is referred to as *stochastic heating*.

Here one will consider the sheath electric field as constant,  $E = U_{\text{sb}}/s_0$ , the bulk expansion to be  $l$  and the sheath thickness  $d$  being modulated cosinusoidal. The equation of motion for an

electron in the sheath is taken from above, canceling out the part of ohmic neutral gas heating.

$$d(t) \approx s_0 \left( 1 + \frac{U_{\text{rf}}}{U_{\text{sb}}} \cos(\omega t) \right) \quad (1.34)$$

The equation 1.32 is introduced as dimensionless with the corresponding parameters:  $\alpha = m_e \omega^2 s_0^2 / (e U_{\text{sb}})$ ,  $\beta = U_{\text{rf}} / U_{\text{sb}}$  and  $\epsilon = s_0 / l$ . Integrating yields the velocity  $\mu(\tau)$  of an electron as it moves through the sheath. The transit time  $\tau$  is considered for one pass through the sheath of the particle. It can be used to calculate the change in velocity experienced by the electron on each bounce between this oscillating and a fixed wall — this would be the lowest order *fermi acceleration*. Assuming there are two distinct points in motion, where the particle enters (index  $n$ ) and re-enters (index  $n+1$ ) the sheath, this gives, using  $\varphi_n$  the phase of the sheath, for the transit time and velocity [3]

$$\tau_n \approx \frac{2\alpha(\mu_n - \beta \cos \varphi_n)}{1 + \alpha\beta \cos \varphi_n}, \quad \mu_{n+1} = -\mu + \frac{2(\mu_n - \beta \sin \varphi_n)}{1 + \alpha\beta \cos \varphi_n} \quad (1.35)$$

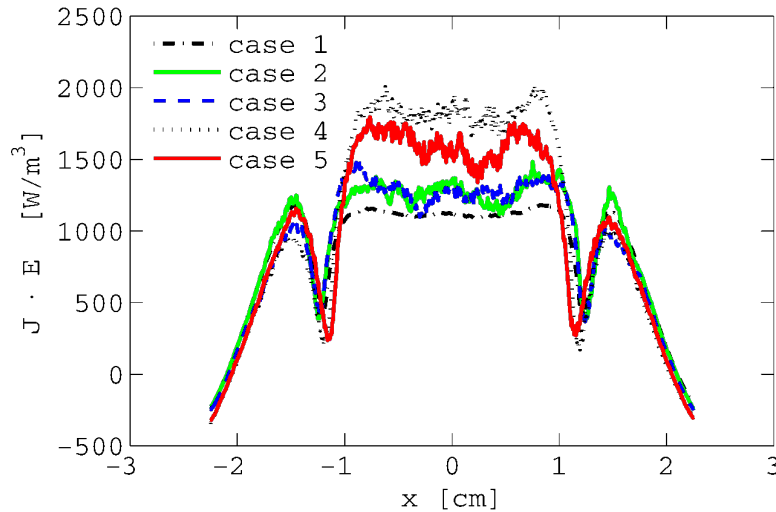
In *hamiltonian mappings*, those two variables are not canonically conjugate, hence insufficient for checking conservational properties. One has to keep that in mind when evaluating the HWA approximation. The change in velocity in one pass through the sheath becomes the impulse approximation of the *Fermi acceleration*. Here, written again with common variables.

$$\Delta v = v_{n+1} - v_n = -2\omega s_0 \frac{U_{\text{rf}}}{U_{\text{sb}}} \sin \varphi_n \quad (1.36)$$

A question to answer is what parameter defines the degree of randomization required for an ample stochastic heating in the plasma sheath. Therefore,  $K$  is defined as a function of phase-space chaos by electron energy. Additionally, a simple condition for stochastic motion is derived at the same time.

$$K = \alpha \beta \frac{U_{\text{sb}}}{\epsilon E}, \quad E < m_e \omega^2 s_0 l \frac{U_{\text{rf}}}{U_{\text{sb}}} \quad (1.37)$$

As is well known [3], chaotic motion occurs in this mapping for  $K > 1$ . It decreases with increasing energy, so the system is less stochastic at higher energies. This is due to the shrinking



**Figure 1.5:** Electron heating rate for a ccrf discharge of parallel plates at 6,7 Pa with an electrode gap of 4,5 cm at 222 V. Other corresponding plasma parameters are noted in table 1.1. [5]

Case	$\Phi_0$	$n_{e,0}/\text{m}^{-3}$	$n_{i-,0}/\text{m}^{-3}$	$n_{i+,0}/\text{m}^{-3}$	$T_{e,0}/\text{eV}$
1	101,30	$2,43 \cdot 10^{14}$	$1,17 \cdot 10^{16}$	$1,20 \cdot 10^{16}$	2,83
2	101,25	$2,29 \cdot 10^{14}$	$1,21 \cdot 10^{16}$	$1,23 \cdot 10^{16}$	2,98
3	101,75	$2,18 \cdot 10^{14}$	$1,19 \cdot 10^{16}$	$1,20 \cdot 10^{16}$	2,98
4	103,11	$1,55 \cdot 10^{14}$	$1,75 \cdot 10^{16}$	$1,78 \cdot 10^{16}$	3,59
5	102,55	$1,65 \cdot 10^{14}$	$1,70 \cdot 10^{16}$	$1,71 \cdot 10^{16}$	3,43

**Table 1.1:** Selected plasma parameters for the used cases in figure 1.5 at the center of the discharge. [5]

phase shift across the discharge volume with higher energies. Hence phase correlations between successive collisions in and with the plasma sheath reduce stochasticity.

Last, but not least, one calculates the instantaneous power dissipated into the plasma due to this heating mechanism. Here, using the sheath speed from above  $u_s$ , the electron drift velocity  $u_e$  and the maxwellian electron velocity distribution funtion  $f_v(v_e, t)$ , Lieberman [9] finds

$$P_{\text{stoc}}(t) = -2m_e \int_{u_s}^{\text{inf}} u_s(v_e - u_s)^2 f_v(v_e, t) dv = 2n_{e,0}u_e k_B T_e \sin(\omega t) \quad (1.38)$$

Clearly, this results yields not net heating if averaged over one rf cycle. That is, if the current used would be conserved in a manner the HWA is predicting it, or the electron drift velocity does not satisfy the maxwellian distribution function. Hence, there have to be deviations from the proposed theory of stochastical heating. This would be e.g. ab initio information about the EEDF, unconsidered transit time effects of the sheath electric field and neglected particle losses and current conservation.

Therefore more theoretical approaches on the heating in low-pressure, low-temperature rf plasma. For example, Surendra et al [14] put forth the idea that the compression and de-compression of the electron density volume between opposing plasma sheaths generates heat inside the bulk is responsible for the observed heating.

The electron heating profile is shown in figure 1.5. The electron heating peaks near the sheath edges are due to the stochastic heating while the main plateau in the bulk is a result of the dominantly strong ohmic heating with slow electrons and neutral gas friction.