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ELEC 6061 - Real-Time Computer Control Systems

Discrete-Time Controller Design For a Two–DOF Helicopter

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1 Abstract

This project is based on Quansar's two-degree-of-freedom helicopter, which has two DC motor propellers for controlling pitch angle θ and yaw angle ψ. Discrete-time controllers are designed for each of these two propellers using the root locus method. The two controllers designed are simulated separately using Matlab to analyze and compare the design specifications with the actual simulation results. At the same time, detailed information related to the design is listed. After ensuring that both controllers meet the design specifications, the pitch and yaw channels are coupled and the resulting effects are compared and analyzed.

2 Introduction

The two-degree-of-freedom helicopter is based on a fixed base with two propellers driven by DC motors, which control the elevation motion of the pitch channel and the lateral motion of the yaw channel of the helicopter, respectively [1]. This structure can help to understand the dynamics control laws that can be applied to the development of related tools such as aerospace. The helicopter has nonlinear and open-loop instabilities in its dynamics, and these factors allow for diverse approaches to controller design. Due to the potential of the structure for military and civil applications, several stable control methods have been designed such as fuzzy control techniques [2] and root locus methods [3]. Among them, the root locus method has a simple method of placing the poles and zeros, and the designed system has better stability and control ability, which can be more flexible to meet various design specifications. In this project, the root locus method will be used to design the controller.

3 Problem Statement

In this project, the problem to be solved is to design a single-input, single-output discrete-time controller for pitch and yaw channels, respectively. The design method is the root locus method, which keeps the root locus of the closed-loop system within the desired domain by setting the complex zeros and poles reasonably. The design specifications to be met during the design process are shown in Table 1. According to the design specification, the corresponding design parameters can be confirmed, such as the sampling rate and desired poles in the s and z planes. The approximate location of the poles is first calculated based on the angle and magnitude criterion of the root locus, followed by trial and error to obtain the controller parameters that conform to the design specification. The step response diagrams of the input and disturbance will be used to judge whether the designed controller meets the design specification. The two controllers are then coupled, and the coupled controller is simulated and analyzed. Finally, the designed controllers are evaluated using the corresponding criterion.

Table 1: The design specifications

DS1	Overshoot	$\leq 20\%$
DS2	Settling time	≤ 15 *sec*
DS3	Rise time	≤ 2 *sec*
DS4	Steady–state error for step reference input	0
DS5	Steady–state output in response to step disturbance	0
DS6	Settling time - step disturbance	≤ 15 *sec*

4 Design and Analysis: Pitch Channel

The controller of the pitch channel is designed using the root locus method of the discrete-time system. The block diagram for the pitch channel is shown in Figure 1.

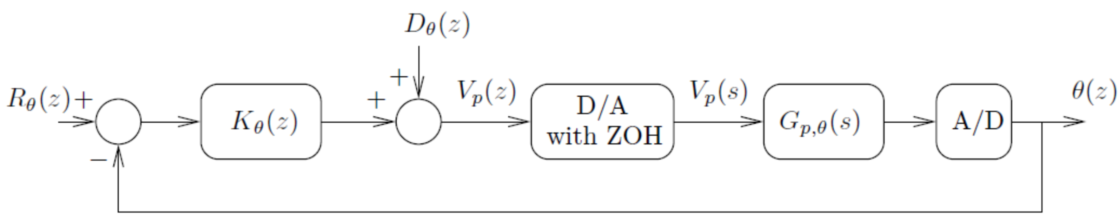

Figure 1: The block diagram for the pitch channel.

The plant for the pitch channel is shown in Equation 1. Equation 1 shows that the poles of $G_{p,\theta}(s)$ are $s_{1,2} = -0.1415 \pm 1.6508j$. The process and results of the design are described below.

$$G_{p,\theta}(s) = \frac{\theta(s)}{V_p(s)} = \frac{37.2021}{s^2 + 0.2830s + 2.7452} \tag{1}$$

4.1 A1: Sampling rate

According to the design specifications in Table 1, the designed pitch controller needs to have an overshoot of less than or equal to 20%, a settling time of less than or equal to 15 seconds, and a rise time of less than or equal to 2 seconds. The equations involved in these three design specifications are as follows.

$$Overshoot : M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \tag{2}$$

$$Settling\ time : t_s = \frac{4.6}{\zeta\omega_n} \tag{3}$$

$$Rise\ time : t_r = \frac{1.8}{\omega_n} \tag{4}$$

Among them, we can get $\zeta \geq 0.46$ according to Equation 2 and $\omega_n \geq 0.9$ according to Equation 4. In order to make the designed controller comply with the design specifications, in this project, the selected parameter is $\zeta = 0.5$ and $\omega_n = 1$. Generally, a sampling rate of 10 samples in closed-loop rise time is preferred, i.e., $T = \frac{rise\ time}{10} = 0.2$s.

4.2 A2: Zero–order–hold discrete equivalent

Using Matlab's $c2d()$ function, the expression in the s-domain can be transformed into an expression in the z-domain. In the design of the pitch channel, the zero–order hold discrete equivalent of the plant obtained is shown in Equation 5.

$$G_{p,\theta}(z) = \frac{0.7236z + 0.71}{z^2 - 1.839z + 0.945} \tag{5}$$

The corresponding poles are $z_{1,2} = 0.9196 \pm 0.3152j$ and the corresponding zeros are $z = -0.9812$.

4.3 A3: Design the controller

Based on the values of the parameters ζ and ω_n confirmed in section 4.1, the desired locations of dominant closed-loop poles in the s-plane can be found as $s_{1,2} = -0.5 \pm 0.866j$ and the corresponding desired locations of dominant closed-loop poles in the z-plane as $z_{1,2} = 0.8913 \pm 0.1559j$.

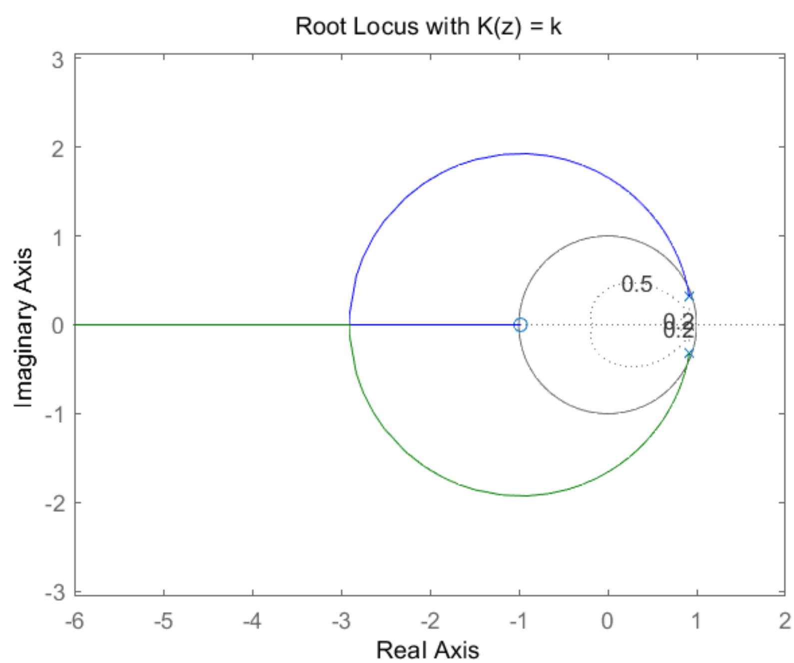

Figure 2: The root locus with $K_{p,\theta}(z) = \text{k}$.

The root locus is first plotted in Figure 2. As can be seen from the figure, most of the locus are outside the unit circle. In order to make the designed controller conform to the design specification, complex zeros and poles need to be added. Also, since $G_{p,\theta}(z)$ has no poles at 1, in order to meet DS4 and DS5, the controller $K_{p,\theta}(z)$ needs to have at least one poles at 1.

The complex zeros $z_{1,2} = 0.9199 \pm 0.2727j$ are added. First, the angle and magnitude criteria are applied to calculate the values of the pole and gain k. Subsequently, the designed controller is fine-tuned by the controlSystemDesigner for further trials and errors. The calculation process is shown in section 4.3.1 and the final controller details are shown in section 4.3.2.

4.3.1 The angle and magnitude criteria

Assume $K_{p,\theta}(z) = k\frac{z^2-1.8398z+0.9206}{(z-1)(z-a)}$, $G_{p,\theta}(z) = \frac{0.7236z+0.71}{z^2-1.8392z+0.945}$

$z = 0.8913 + 0.1559j$ satisfied C.L. sys C.E. $\Rightarrow 1 + K_{p,\theta}(z)G_{p,\theta}(z) = 0$

$\Rightarrow \frac{0.7236(z-0.9199-0.2727j)(z-0.9199+0.2727j)(z+0.9812)}{(z-1)(z-a)(z-0.9196-0.3152j)(z-0.9196+0.3152j)} = -\frac{1}{k}$

$\Rightarrow$ ∠(z-0.9199-0.2727j) + ∠(z-0.9199+0.2727j) + ∠(z+0.9812) - ∠(z-1) - ∠(z-a) - ∠(z-0.9196-0.3152j) - ∠(z-0.9196+0.3152j) $= -180°$

- $\angle(z - 0.9199 - 0.2727j)$ = -0.0286 - 0.1168j $\Rightarrow |z - 0.9199 - 0.2727j| = 0.1203$
  $tan\theta = \frac{0.0286}{0.1168} \Rightarrow \theta = -90° - 13.7589° = -103.7589°$
- $\angle(z - 0.9199 + 0.2727j)$ = -0.0286 + 0.4286j $\Rightarrow |z - 0.9199 + 0.2727j| = 0.4296$
  $tan\theta = \frac{0.0286}{0.4286} \Rightarrow \theta = 90° + 3.8176° = 93.8176°$
- $\angle(z + 0.9812)$ = 1.8725 + 0.1559j $\Rightarrow |z + 0.9812| = 1.879$
  $tan\theta = \frac{0.1559}{1.8725} \Rightarrow \theta = 4.7593°$
- $\angle(z - 1)$ = -0.1087 + 0.1559j $\Rightarrow |z - 1| = 0.1901$
  $tan\theta = \frac{0.1087}{0.1559} \Rightarrow \theta = 124.8858°$
- $\angle(z - 0.9196 - 0.3152j)$ = -0.0283 - 0.1593j $\Rightarrow |z - 0.9196 - 0.3152j| = 0.1618$
  $tan\theta = \frac{0.0283}{0.1593} \Rightarrow \theta = -100.0736°$
- $\angle(z - 0.9196 + 0.3152j)$ = -0.0283 + 0.4711j $\Rightarrow |z - 0.9196 + 0.3152j| = 0.4719$
  $tan\theta = \frac{0.0283}{0.4711} \Rightarrow \theta = 93.4378°$

$\Rightarrow -103.7589° + 93.8176° + 4.7593° - 124.8858° - \theta_a + 100.0736° - 93.4378° = -180°$

$\Rightarrow \theta_a = 56.568°$

- $\angle(z - a)$ = 0.8913 - a + 0.1559j and $\theta_a = 56.568°$
  $tan\theta_a = \frac{0.1559}{0.8913-a} \Rightarrow a = \frac{0.8913tan\theta_a - 0.1559}{tan\theta_a} = 0.7884$
  $\Rightarrow \angle(z - a)$ = 0.1029+0.1559j $\Rightarrow |z - a| = 0.1868$

Since $1 + K_{p,\theta}(z)G_{p,\theta}(z) = 0$

$\Rightarrow k = \frac{|z-1||z-a||z-0.9196-0.3152j||z-0.9196+0.3152j|}{0.7236|z-0.9199-0.2727j||z-0.9199+0.2727j||z+0.9812|} = 0.0386$

$\Rightarrow K_{p,\theta}(z) = \frac{0.0386(z^2-1.8398z+0.9206)}{(z-1)(z-0.7884)}$

4.3.2 The designed controller

After trial and error, the pole that conforms to the design specification can be obtained as $z = 0.1929$ and gain k as 0.272. In this case, the root locus is shown in Figure 3.

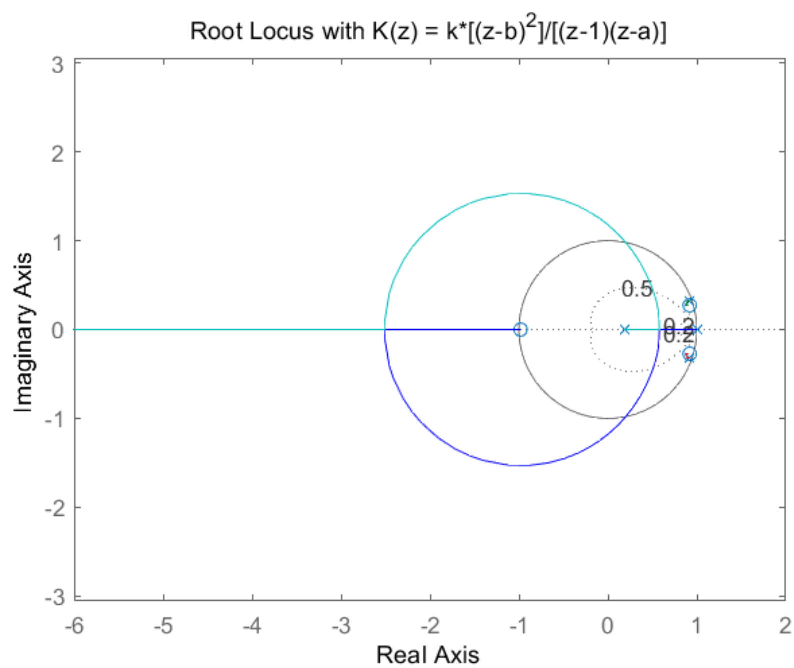

Figure 3: The root locus with $K_{p,\theta}(z) = \frac{k(z-b)^2}{(z-1)(z-a)}$.

4.4 A4: Transfer functions of the controller and the closed-loop system

The transfer function of the controller designed for the pitch channel is shown in Equation 6. From this equation, the zeros of $K_{p,\theta}(z)$ can be obtained as $z_{1,2} = 0.9199 \pm 0.2727j$ and the poles as $z_1 = 1$, $z_2 = 0.1929$.

$$K_{p,\theta}(z) = \frac{0.272(z^2 - 1.8398z + 0.9206)}{(z-1)(z-0.1929)} = \frac{0.272z^2 - 0.5004z + 0.2504}{z^2 - 1.1929z + 0.1929} \tag{6}$$

The transfer function of the closed-loop system is shown in Equation 7. From this equation, the zeros of $Gcl_{p,\theta}(z)$ can be obtained as $z_{1,2} = 0.9199 \pm 0.2727j$, $z_3 = -0.9812$ and the poles as $z_{1,2} = 0.8866 \pm 0.2751j$, $z_{3,4} = 0.531 \pm 0.3686j$.

$$Gcl_{p,\theta}(z) = \frac{0.1968z^3 - 0.169z^2 - 0.1741z + 0.1778}{z^4 - 2.835z^3 + 3.163z^2 - 1.656z + 0.3601} \tag{7}$$

4.5 A5: The response to unit step reference input

Given the unit step reference input $r_\theta[n] = 1[n]$ and $d_\theta[n] = 0$. The response of the closed-loop system can be obtained in Figure 4.

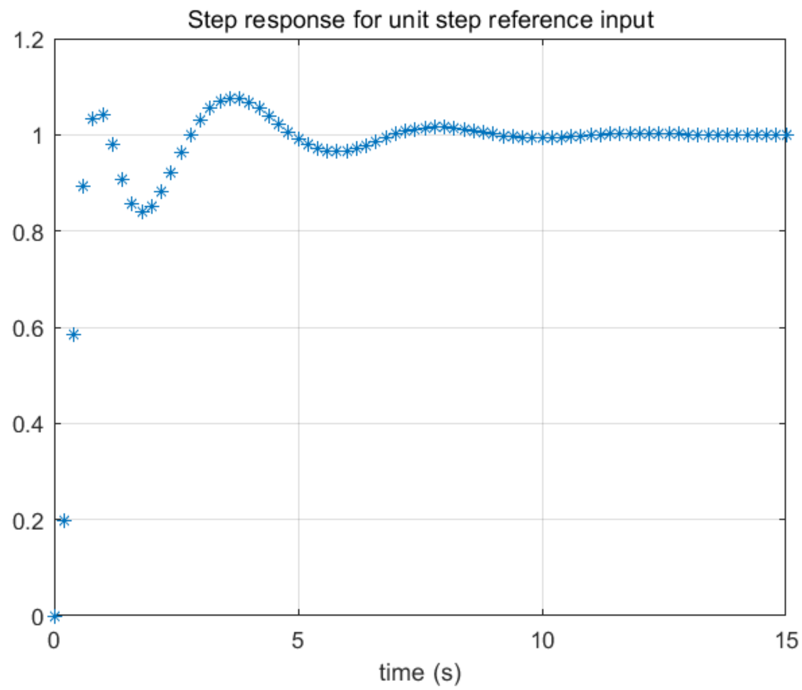

Figure 4: The step response of $Gcl_{p,\theta}(z)$ for unit step reference input.

4.5.1 A7: Comment on the design specifications

According to the response diagram in Figure 4, the relevant parameters can be obtained as shown in Table 2. The data in the table shows that the designed controller complies with DS1, DS2, and DS3, and the structure of the controller also complies with design specifications DS4 and DS5.

Table 2: The step reference information of $Gcl_{p,\theta}(z)$

	System $Gcl_{p,\theta}(z)$	**Design Specifications**
DS1: Overshoot	7.58%	$\leq 20\%$
DS2: Settling time	6.6 sec	$\leq 15\ sec$
DS3: Rise time	0.6 sec	$\leq 2\ sec$

4.6 A6: The response to unit step disturbance

Given the unit step disturbance $r_\theta[n] = 0$ and $d_\theta[n] = 1[n]$. The response of the closed-loop system can be obtained in Figure 5.

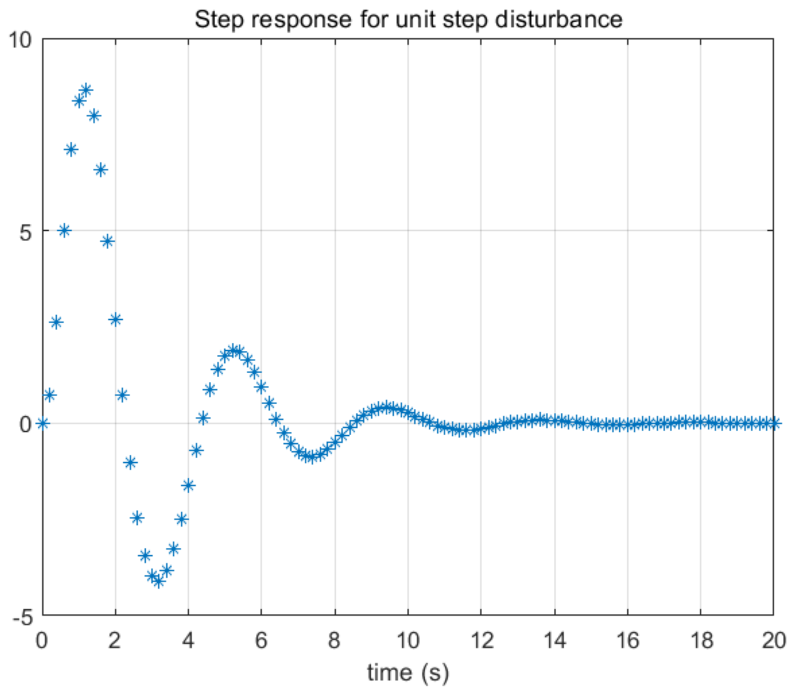

Figure 5: The step response of $Gcl_{p,\theta}(z)$ for unit step disturbance.

4.6.1 A7: Comment on the design specifications

The max settling time can be obtained as 1.8834. Thus, $0.02\theta_{max} = 0.0377$. According to the response diagram in Figure 5, it is clear that the response to step disturbance is settled within 15 s. So that the controller can meet all the design specifications.

4.7 A8: The motor voltage to step reference input

Given the unit step reference input $r_\theta[n] = 1[n]$ and $d_\theta[n] = 0$. The motor voltage to step reference input can be obtained in Figure 6.

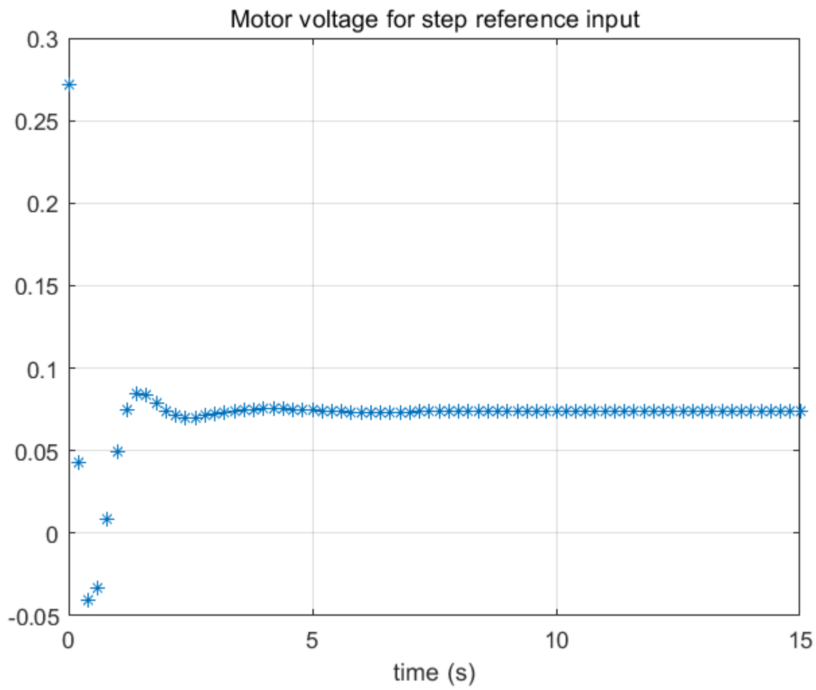

Figure 6: The motor voltage of $v_p[n]]$.

5 Design and Analysis: Yaw Channel

The controller of the yaw channel is designed using the root locus method of the discrete-time system. The block diagram for the yaw channel is shown in Figure 7.

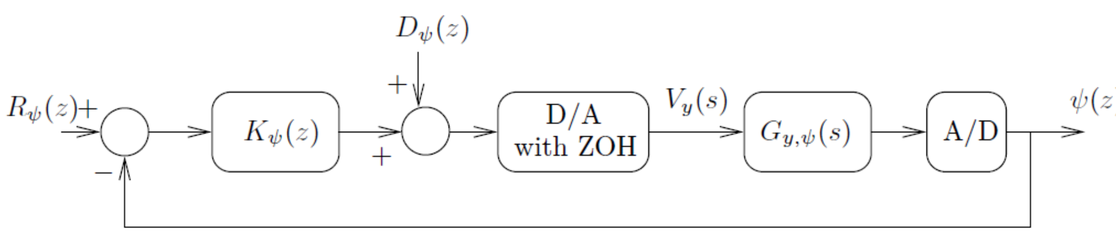

Figure 7: The block diagram for the yaw channel.

The plant for the yaw channel is shown in Equation 8. Equation 8 shows that the poles of $G_{y,\psi}(s)$ are $s_1 = 0$, $s_2 = -0.2701$. The process and results of the design are described below.

$$G_{y,\psi}(s) = \frac{\psi(s)}{V_y(s)} = \frac{7.461}{s(s+0.2701)} \tag{8}$$

5.1 A1: Sampling rate

The sampling period selected for the yaw channel is the same as that of the pitch channel. That is, T=0.2 s.

5.2 A2: Zero–order–hold discrete equivalent

In the design of the yaw channel, the zero–order hold discrete equivalent of the plant obtained is shown in Equation 9.

$$G_{y,\psi}(z) = \frac{0.1466z + 0.144}{z^2 - 1.9474z + 0.9474} \tag{9}$$

The corresponding poles are $z_1 = 1$, $z_2 = 0.9474$ and the corresponding zeros are $z = -0.9822$.

5.3 A3: Design the controller

The desired locations of dominant closed-loop poles in the s-plane and the corresponding desired locations of dominant closed-loop poles in the z-plane are the same as those in section 4.3.

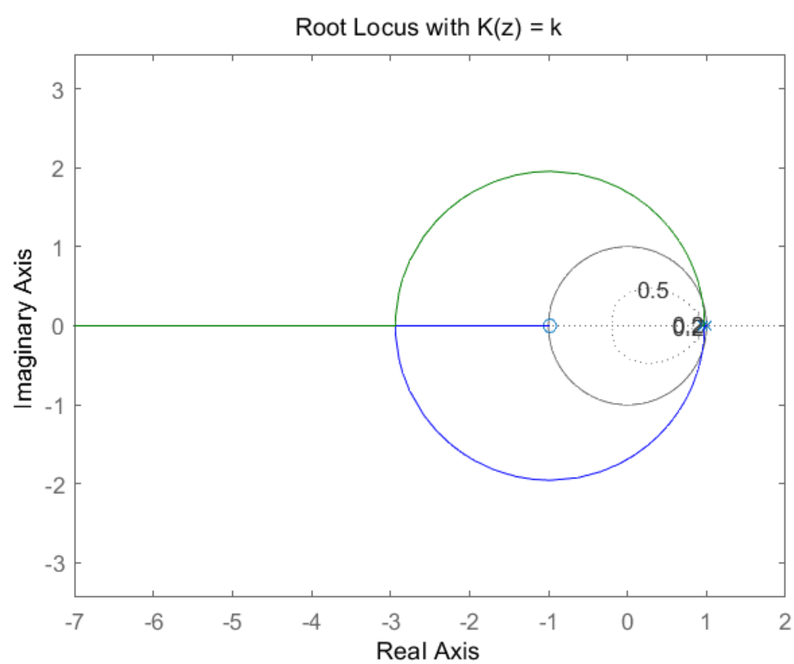

Figure 8: The root locus with $K_{y,\psi}(z) =$ k.

The root locus is first plotted in Figure 8. Use the same approach to design a controller that can meet the design specifications. Since $G_{y,\psi}(z)$ has one pole at 1, it is already met DS4 and DS5.

The complex zeros $z_{1,2} = 0.9352 \pm 0.0693j$ are added. The angle and magnitude criteria are first applied to calculate the values of the pole and gain k. Subsequently, the designed controller is fine-tuned by the controlSystemDesigner for further trials and errors. The calculation process is shown in section 5.3.1 and the final controller details are shown in section 5.3.2.

5.3.1 The angle and magnitude criteria

Assume $K_{y,\psi}(z) = k\frac{z^2-1.8704z+0.8794}{(z-1)(z-a)}$, $G_{y,\psi}(z) = \frac{0.1466z+0.144}{z^2-1.9474z+0.9474}$
$z = 0.8913 + 0.1559j$ satisfied C.L. sys C.E. $\Rightarrow 1 + K_{y,\psi}(z)G_{y,\psi}(z) = 0$
$\Rightarrow \frac{0.1466(z-0.9352-0.0693j)(z-0.9352+0.0693j)(z+0.9822)}{(z-1)(z-a)(z-1)(z-0.9474)} = -\frac{1}{k}$
$\Rightarrow$ ∠(z-0.9352-0.0693j) + ∠(z-0.9352+0.0693j) + ∠(z+0.9822) - ∠(z-1) - ∠(z-a) - ∠(z-1) - ∠(z-0.9474) $= -180°$

- $\angle(z - 0.9352 - 0.0693j)$ = -0.0439 + 0.0866j $\Rightarrow |z - 0.9352 - 0.0693j| = 0.0971$
  $tan\theta = \frac{0.0439}{0.0866} \Rightarrow \theta = 90° + 26.8817° = 116.8817°$
- $\angle(z - 0.9352 + 0.0693j)$ = -0.0439 + 0.2252j $\Rightarrow |z - 0.9352 + 0.0693j| = 0.2294$
  $tan\theta = \frac{0.0439}{0.2252} \Rightarrow \theta = 90° + 11.0308° = 101.0308°$
- $\angle(z + 0.9822)$ = 1.8735 + 0.1559j $\Rightarrow |z + 0.9822| = 1.88$
  $tan\theta = \frac{0.1559}{1.8735} \Rightarrow \theta = 4.7568°$
- $\angle(z - 1)$ = -0.1087 + 0.1559j $\Rightarrow |z - 1| = 0.1901$
  $tan\theta = \frac{0.1087}{0.1559} \Rightarrow \theta = 124.8858°$
- $\angle(z - 0.9474)$ = -0.0561 + 0.1559j $\Rightarrow |z - 0.9474| = 0.1657$
  $tan\theta = \frac{0.0561}{0.1559} \Rightarrow \theta = 90° + 19.7911° = 109.7911°$

$\Rightarrow 116.8817° + 101.0308° + 4.7568° - 124.8858° - \theta_a - 124.8858° - 109.7911° = -180°$
$\Rightarrow \theta_a = 43.1066°$

- $\angle(z - a)$ = 0.8913 - a + 0.1559j and $\theta_a = 43.1066°$
  $tan\theta_a = \frac{0.1559}{0.8913-a} \Rightarrow a = \frac{0.8913tan\theta_a - 0.1559}{tan\theta_a} = 0.7247$
  $\Rightarrow \angle(z - a) = 0.1666 + 0.1559 \Rightarrow |z - a| = 0.2282$

Since $1 + K_{y,\psi}(z)G_{y,\psi}(z) = 0$
$\Rightarrow k = \frac{|z-1||z-a||z-1||z-0.9474|}{0.7236|z-0.9352-0.0693j||z-0.9352+0.0693j||z+0.9822|} = 0.2226$
$\Rightarrow K_{p,\theta}(z) = \frac{0.2226(z^2-1.8704z+0.8794)}{(z-1)(z-0.7247)}$

5.3.2 The designed controller

After trial and error, the pole that conforms to the design specification can be obtained as $z = -0.1986$ and gain k as 2.58. In this case, the root locus is shown in Figure 9.

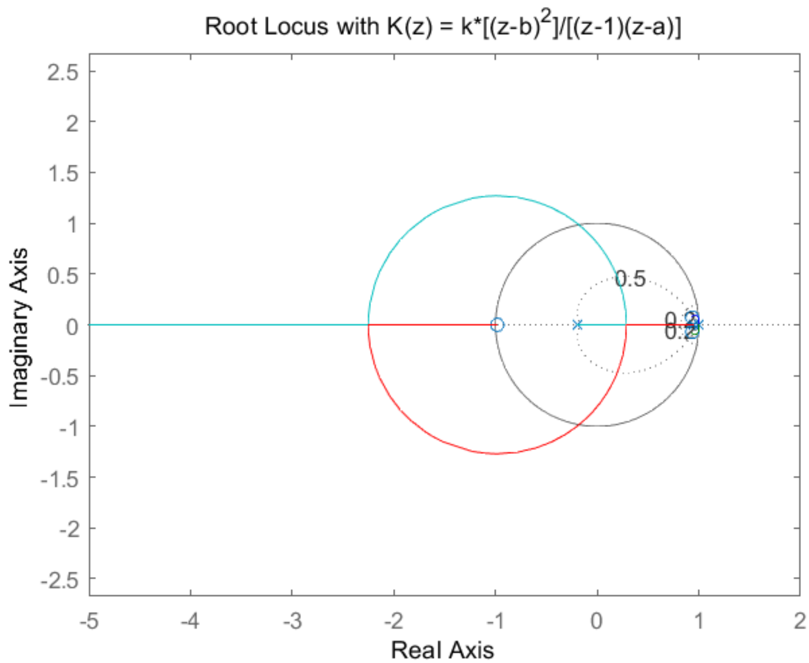

Figure 9: The root locus with $K_{y,\psi}(z) = \frac{k(z-b)^2}{(z-1)(z-a)}$.

5.4 A4: Transfer functions of the controller and the closed-loop system

The transfer function of the controller designed for the yaw channel is shown in Equation 10. From this equation, the zeros of $K_{y,\psi}(z)$ can be obtained as $z_{1,2} = 0.9352 \pm 0.0693j$ and the poles as $z_1 = 1$, $z_2 = -0.1986$.

$$K_{p,\theta}(z) = \frac{2.58(z^2 - 1.8704z + 0.8794)}{(z-1)(z+0.1986)} = \frac{2.58z^2 - 4.826z + 2.269}{z^2 - 0.8014z - 0.1986} \tag{10}$$

The transfer function of the closed-loop system is shown in Equation 11. From this equation, the zeros of $Gcl_{y,\psi}(z)$ can be obtained as $z_{1,2} = 0.9352 \pm 0.0693j$, $z_3 = -0.9822$ and the poles as $z_{1,2} = 0.9352 \pm 0.0779j$, $z_{3,4} = 0.2501 \pm 0.3076j$.

$$Gcl_{y,\psi}(z) = \frac{0.3781z^3 - 0.3359z^2 - 0.3621z + 0.3266}{z^4 - 2.371z^3 + 1.974z^2 - 0.7346z + 0.1385} \tag{11}$$

5.5 A5: The response to unit step reference input

Given the unit step reference input $r_\psi[n] = 1[n]$ and $d_\psi[n] = 0$. The response of the closed-loop system can be obtained in Figure 10.

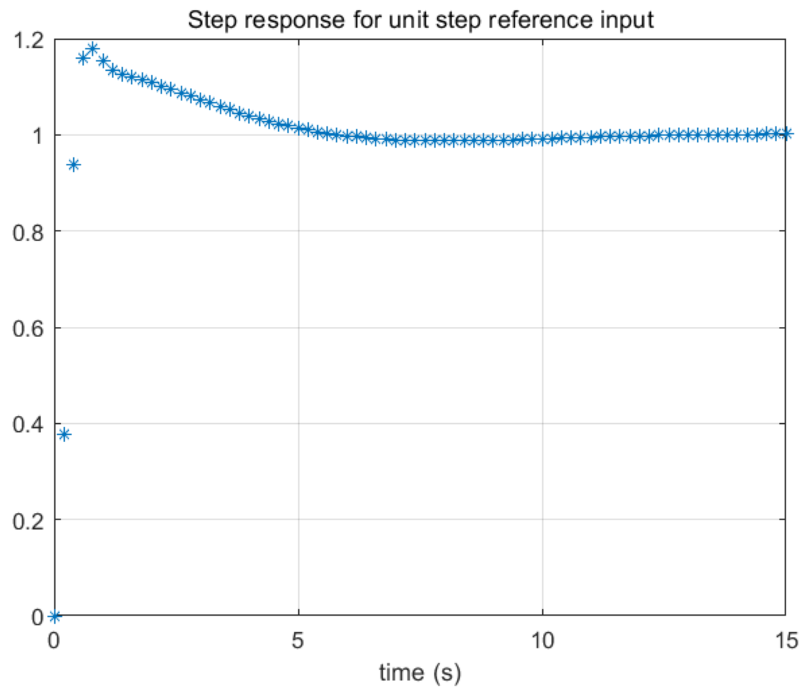

Figure 10: The step response of $Gcl_{y,\psi}(z)$ for unit step reference input.

5.5.1 A7: Comment on the design specifications

According to the response diagram in Figure 10, the relevant parameters can be obtained as shown in Table 3. The data in the table shows that the designed controller complies with DS1, DS2, and DS3, and the structure of the controller also complies with design specifications DS4 and DS5.

Table 3: The step reference information of $Gcl_{y,\psi}(z)$

	System $Gcl_{y,\psi}(z)$	**Design Specifications**
DS1: Overshoot	18.0036%	$\leq 20\%$
DS2: Settling time	4.8 sec	≤ 15 *sec*
DS3: Rise time	0.2 sec	≤ 2 *sec*

5.6 A6: The response to unit step disturbance

Given the unit step disturbance $r_\psi[n] = 0$ and $d_\psi[n] = 1[n]$. The response of the closed-loop system can be obtained in Figure 11.

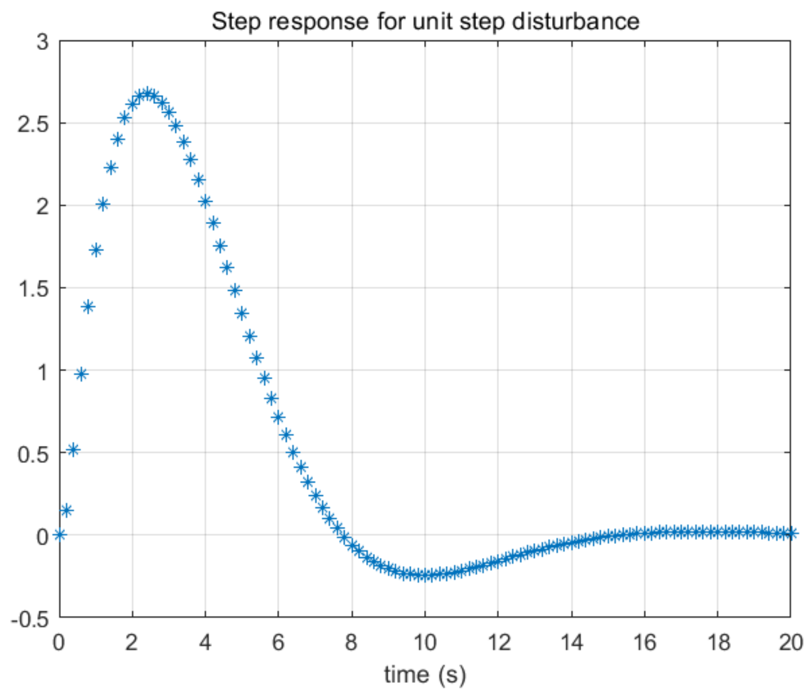

Figure 11: The step response of $Gcl_{y,\psi}(z)$ for unit step disturbance.

5.6.1 A7: Comment on the design specifications

The max settling time can be obtained as 2.6759. Thus, $0.02\theta_{max} = 0.0535$. According to the response diagram in Figure 11, it is clear that the response to step disturbance is settled within 15 s so that the controller can meet all the design specifications.

6 Cross-coupling

Both controllers designed for the pitch and yaw channels can meet the design specifications. Then the two controllers can be coupled to evaluate the effects of the entire system. The block diagram of the coupling is shown in Figure 12.

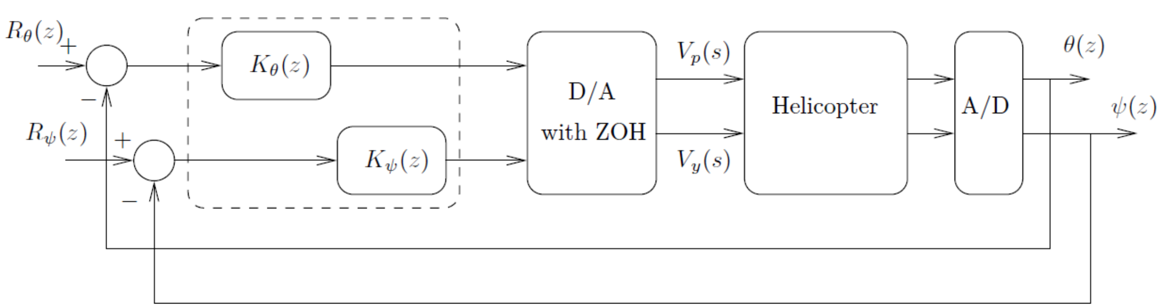

Figure 12: The block diagram for the entire system.

6.1 B1&B2: Response to a step input

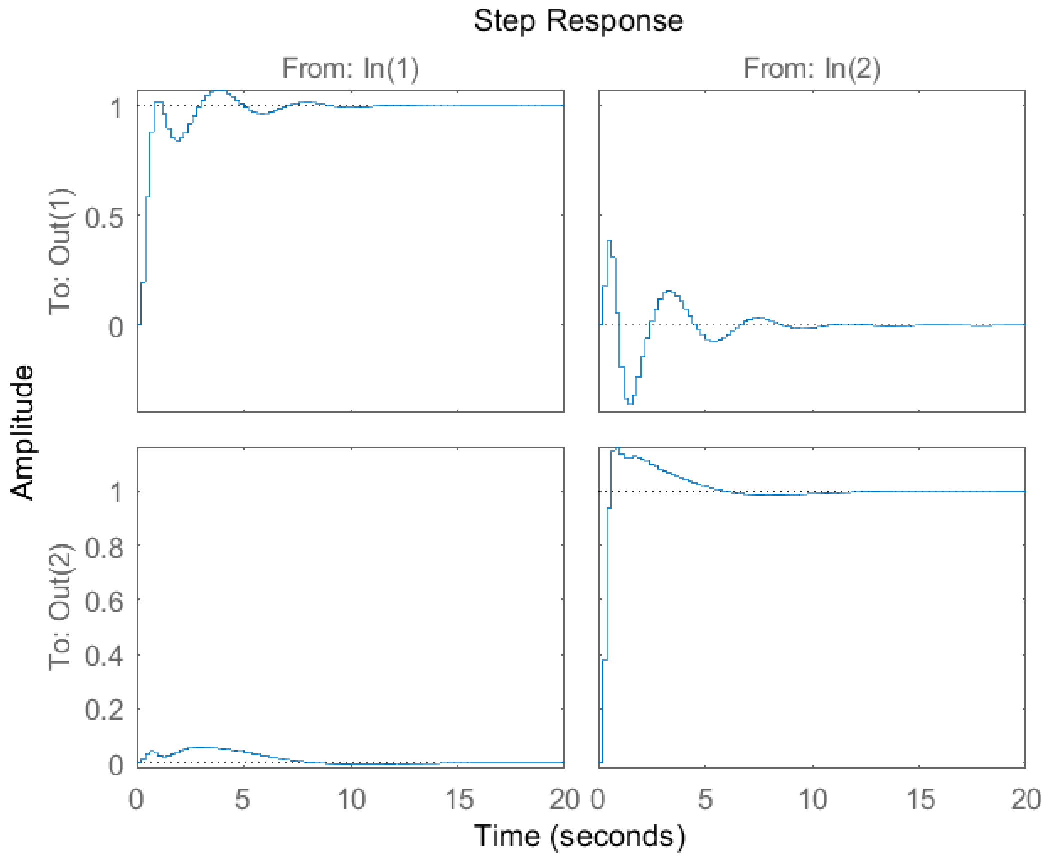

Figure 13: The response to a step input.

The pitch θ and yaw ψ in response to a step input in both pitch and yaw reference input are shown in Figure 13. The simulation conditions corresponding to these four plots are

- Input(1) to Output(1): pitch response for step input in pitch reference input $r_\theta[n] = 1[n], r_\psi[n] = 0$
- Input(1) to Output(2): yaw response for step input in pitch reference input $r_\theta[n] = 1[n], r_\psi[n] = 0$
- Input(2) to Output(1): pitch response for step input in yaw reference input $r_\theta[n] = 0, r_\psi[n] = 1[n]$
- Input(2) to Output(2): yaw response for step input in yaw reference input $r_\theta[n] = 0, r_\psi[n] = 1[n]$

6.2 C: Examine the effects of cross-coupling

Based on the step-corresponding diagrams obtained in Figure 13, the effect of cross-coupling on the entire system can be evaluated in terms of overshoot, settling time, and rise time.

6.2.1 C2: Maximum deviation of the response from the steady-state error

- Overshoot

The percentage of overshoot for the response of pitch to step pitch input and the response of yaw to step yaw input respectively are

$$\begin{cases} PO_p = 7\% \\ PO_y = 16\% \end{cases} \tag{12}$$

which both are less than the step response of the pitch(7.58%) and yaw channel(18.0036%) when analyzed individually.

6.2.2 C2: Speed

- Rise time

For both the pitch and yaw channel, the rise time of step response for unit step reference input and unit step disturbance respectively to both step pitch and yaw input is

$$\begin{cases} TR_{pr} = 0.4s \\ TR_{pd} = 0s \end{cases} \tag{13}$$

$$\begin{cases} TR_{yr} = 0.2s \\ TR_{yd} = 0s \end{cases} \tag{14}$$

which are the more or the same amount of the condition individually analyzed(0.6s and 0.2s respectively).

- Settling time

For both the pitch and yaw channel, the settling time of step response to step reference input and unit step disturbance respectively to both step pitch and yaw input is

$$\begin{cases} TS_{pr} = 0s \\ TR_{pd} = 7.2s \end{cases} \tag{15}$$

$$\begin{cases} TS_{yr} = 0s \\ TS_{yd} = 8.4s \end{cases} \tag{16}$$

which are all more than that of when the system is analyzed individually(6.6s and 4.8s respectively).

6.2.3 C2: Difference between the desired or reference input and the actual output

- Steady-state error for step reference input

$$\begin{cases} e_{ss} = \frac{1}{1+k_p} \\ K_p = \lim\limits_{z\to 1} G(z) K(z) \end{cases} \tag{17}$$

then,$e_{ss} = 0$

6.2.4 C2: Difference between the original and new steady state

- Steady-state output in response to a step disturbance

$$\theta_{ss} = \lim_{z\to 1} \frac{G(z)}{1 + G(z) K(z)} \tag{18}$$

then, $\theta_{ss} = 0$

6.2.5 C3: Verdict on cross-coupling

In summary, cross-coupling can make the system stable to some extent. The effect of cross-coupling is different for different specifications, as it can be assumed that the system after cross-coupling has approximately the same effect as the system used separately.

7 Conclusions

The main task in this project is to design the controller for both pitch and yaw channels, which means the appropriate gain k, zeros and poles of the controller should be substituted into the formula. To approach the ideal controller, sufficient trial and error are considered instead of only focusing on canceling the exact zeros and poles of the plant. It can be verified through simulation experiments that the designed controller meets the design specifications. The relevant parameters of the step response can be used to analyze the effect of the cross-coupling of the two channels on the overall system. Cross-coupling of the two channels reveals that cross-coupling has different extents of enhancement or degradation for different design specifications.

References

[1] Quanser, "2 dof helicopter - quanser." [Online]. Available: http://www.quanser.com/products/2-dof-helicopter/.

[2] B. Kadmiry and D. Driankov, "A fuzzy gain-scheduler for the attitude control of an unmanned helicopter," *IEEE Transactions on fuzzy systems*, vol. 12, no. 4, pp. 502–515, 2004.

[3] L. A. Omeiza, S. A. Tijjani, and A. O. Daniel, "Discrete–time controller design for pitch channel," *International Journal for Research in Applied Science and Engineering Technology*, pp. 8(5), 2192–2202, 2020.

A Appendix

```
clear; clc; close all;

%% Design specifications
zeta = 0.5;     %damping ratio, zeta >= 0.46 (option by
    ourselves)
ts = 15;
tr = 2;     %tr = 1.8/wn, wn >= 0.9
wn = 1;     %zeta*wn >= 0.306 (option by ourselves)
T = 0.2;     %T = tr/10##########A1

%% Pitch Channel
% G_p,zeta(s) transfer function
num_p = [37.2021];
den_p = [1 0.2830 2.7452];
Gs_p = tf(num_p, den_p)     %transfer function of Gs**********
    Q2
Gs_poles_p = pole(Gs_p)     %poles of Gs**********Q2

% Zero-order-hold discrete equivalent, Gz transfer function
Gz_p = c2d(Gs_p, T)     %transfer function of Gz##########A2
Gz_zeros_p = zero(Gz_p)     %zeros of Gz##########A2
Gz_poles_p = pole(Gz_p)     %poles of Gz##########A2

% Desired locations of dominant closed-loop poles
disp('Desired pole in s-domain');
s_d = -zeta*wn + wn*sqrt(1 - zeta.^2)*i     %desired poles of s
    -domain##########A3
disp('Desired pole in z-domain');
z_d = exp(s_d*T)     %desired poles of s-domain##########A3

% Plot with K(z) = k (doesn't work)
figure(1)
rlocus(Gz_p)
title('Root Locus with K(z) = k')
zgrid(zeta, wn*T)
axis equal

% controlSystemDesigner(Gz_p);

% z = 0.8913 + 0.1559i;
% gn = rad2deg( angle( z + 0.9812));
% gd = rad2deg( angle( z - 0.9196 - 0.3152i)) + rad2deg( angle
    ( z - 0.9196 + 0.3152i));
```

```matlab
% kn = rad2deg( angle( z - 0.9199 - 0.2727i)) + rad2deg( angle( z - 0.9199 + 0.2727i));
% kd = rad2deg( angle( z - 1));
% theta = 180 + gn - gd - kd + kn
% a = real(z) - imag(z)/tan(theta*pi/180)    %theta<90
% % a = real(z) - imag(z)*tan((theta-90)*pi/180)    %theta>90
% upm = abs(z - 0.9199 - 0.2727i) * abs(z - 0.9199 + 0.2727i) * 0.7236 * abs(z + 0.9812);
% downm = abs(z - 1) * abs(z - a) * abs(z - 0.9196 - 0.3152i) * abs(z - 0.9196 + 0.3152i);
% k = downm/upm

k_p = 0.272;
Kz_p = tf(k_p*[1 -1.8398 0.9206], [1 -1.1929 0.1929], T)    % transfer function of Kz###############A4
Kz_zeros_p = zero(Kz_p)    %zeros of Kz###############A4
Kz_poles_p = pole(Kz_p)    %poles of Kz###############A4
Gol_p = series(Kz_p, Gz_p);
figure(2)
rlocus(Gol_p)
title('Root Locus with K(z) = k*[(z-b)^2]/[(z-1)(z-a)]')
zgrid(zeta, wn*T)
axis equal

% Closed-loop system
Gcl_p = feedback(Gol_p, 1)    %transfer function of G_cl ###############A4
Gcl_zeros_p = zero(Gcl_p)    %zeros of Gcl###############A4
Gcl_poles_p = pole(Gcl_p)    %poles of Gcl###############A4

% Step response to unit step reference input
tfinal = 15;
Rz_p = tf([1 0], [1 -1], T);
[y,t] = step(Gcl_p, tfinal);
figure(3)
plot(t,y,'*')
grid
xlabel('time (s)')
title('Step response for unit step reference input')    %plot of input###############A5
stepinfo(Gcl_p)    %design specs###############A7

% Step response to unit step disturbance
tfinal_D = 20;
Dz_p = tf([1 0], [1 -1], T);
```

```
%Gcl_D = Gcl/Kz;
Gcl_D_p = feedback(Gz_p, Kz_p);
[y,t] = step(Gcl_D_p, tfinal_D);
figure(4)
plot(t,y,'*')
grid
xlabel('time (s)')
title('Step response for unit step disturbance')    %plot of disturbance##########A6
stepinfo(Gcl_D_p)    %design specs##########A7

% Motor voltage to step reference input
Gcl_M = Gcl_p/Gz_p;
[y,t] = step(Gcl_M, tfinal);
figure(5)
plot(t,y,'*')
grid
xlabel('time (s)')
title('Motor voltage for step reference input')    %plot of motor voltage##########A8

%% Yaw Channel
% G_y,zeta(s) transfer function
num_y = [7.461];
den_y = [1 0.2701 0];
Gs_y = tf(num_y, den_y)    %transfer function of Gs**********Q2
Gs_poles_y = pole(Gs_y)    %poles of Gs**********Q2

% Zero-order-hold discrete equivalent, Gz transfer function
Gz_y = c2d(Gs_y, T)    %transfer function of Gz##########A2
Gz_zeros_y = zero(Gz_y)    %zeros of Gz##########A2
Gz_poles_y = pole(Gz_y)    %poles of Gz##########A2

% Desired locations of dominant closed-loop poles
disp('Desired pole in s-domain');
s_d = -zeta*wn + wn*sqrt(1 - zeta.^2)*i    %desired poles of s-domain##########A3
disp('Desired pole in z-domain');
z_d = exp(s_d*T)    %desired poles of s-domain##########A3

% Plot with K(z) = k (doesn't work)
figure(6)
rlocus(Gz_y)
title('Root Locus with K(z) = k')
```

```
zgrid(zeta, wn*T)
axis equal

% controlSystemDesigner(Gz_y);

% z = 0.8913 + 0.1559i;
% gn = rad2deg( angle( z + 0.9822));
% gd = rad2deg( angle( z - 1)) + rad2deg( angle( z - 0.9474));
% kn = rad2deg( angle( z - 0.9352 - 0.0693i)) + rad2deg( angle( z - 0.9352 + 0.0693i));
% kd = rad2deg( angle( z - 1));
% theta = 180 + gn - gd - kd + kn
% a = real(z) - imag(z)/tan(theta*pi/180)    %theta<90
% % a = real(z) - imag(z)*tan((theta-90)*pi/180)    %theta>90
% upm = abs(z - 0.9352 - 0.0693i) * abs(z - 0.9352 + 0.0693i) * 0.1466 * abs(z + 0.9822);
% downm = abs(z - 1) * abs(z - a) * abs(z - 1) * abs(z - 0.9474);
% k = downm/upm

k_y = 2.58;
Kz_y = tf(k_y*[1 -1.8704 0.8794], [1 -0.8014 -0.1986], T)    % transfer function of Kz##########A4
Kz_zeros_y = zero(Kz_y)    %zeros of Kz##########A4
Kz_poles_y = pole(Kz_y)    %poles of Kz##########A4
Gol_y = series(Kz_y, Gz_y);
figure(7)
rlocus(Gol_y)
title('Root Locus with K(z) = k*[(z-b)^2]/[(z-1)(z-a)]')
zgrid(zeta, wn*T)
axis equal

% Closed-loop system
Gcl_y = feedback(Gol_y, 1)    %transfer function of G_cl ##########A4
Gcl_zeros_y = zero(Gcl_y)    %zeros of Gcl##########A4
Gcl_poles_y = pole(Gcl_y)    %poles of Gcl##########A4

% Step response to unit step reference input
tfinal = 15;
Rz_y = tf([1 0], [1 -1], T);
[y,t] = step(Gcl_y, tfinal);
figure(8)
plot(t,y,'*')
grid
```

```
xlabel('time (s)')
title('Step response for unit step reference input')    %plot of input##########A5
stepinfo(Gcl_y)    %design specs##########A7

% Step response to unit step disturbance
tfinal_D = 20;
Dz_y = tf([1  0], [1  -1], T);
Gcl_D_y = feedback(Gz_y, Kz_y);
[y,t] = step(Gcl_D_y, tfinal_D);
figure(9)
plot(t,y,'*')
grid
xlabel('time (s)')
title('Step response for unit step disturbance')    %plot of disturbance##########A6
stepinfo(Gcl_D_y)    %design specs##########A7

%% Coupling of pitch and yaw channels
ss(Kz_p);
ss(Kz_y);
sys_k = append(Kz_p, Kz_y);
A = [      0         1     0      0;
     -2.7451   -0.2829  0      0;
           0         0  0      1;
           0         0  0 -0.2701];
B = [     0         0   ;
     37.2021   3.5306;
          0         0   ;
       2.3892   7.461];
C = [1  0  0  0;
     0  0  1  0];
D = [0  0;
     0  0];
sys_plant = ss(A, B, C, D);
sys_plant_d = c2d(sys_plant, T);
openloopsys_d = series(sys_k, sys_plant_d);
closeloopsys_d = feedback(openloopsys_d,eye(2));    %eye(2) generate identity matrix
figure(10)
step(closeloopsys_d)    %coupling of pitch and yaw##########B1&B2
```