MA415 Assignment 2

Zirui Liu
U75461502
liuzirui@bu.edu

Exercise2 Matrices

1.

(a).

```
> a <- matrix (c(1, 5, -2, 1, 2, -1, 3, 6, -3), nr=3)
> a
     [,1] [,2] [,3]
[1,]    1    1    3
[2,]    5    2    6
[3,]   -2   -1   -3
> a%*%a%*%a
     [,1] [,2] [,3]
[1,]    0    0    0
[2,]    0    0    0
[3,]    0    0    0
```

(b).

```
> a[, 3] <- a[, 2] + a[, 3]
> a
     [,1] [,2] [,3]
[1,]    1    1    4
[2,]    5    2    8
[3,]   -2   -1   -4
```

2.

```
> b <- matrix(c(10, -10, 10), b=T, nc=3, nr=15)
> b
      [,1] [,2] [,3]
 [1,]   10  -10   10
 [2,]   10  -10   10
 [3,]   10  -10   10
 [4,]   10  -10   10
 [5,]   10  -10   10
 [6,]   10  -10   10
 [7,]   10  -10   10
 [8,]   10  -10   10
 [9,]   10  -10   10
[10,]   10  -10   10
[11,]   10  -10   10
[12,]   10  -10   10
[13,]   10  -10   10
[14,]   10  -10   10
[15,]   10  -10   10
```

```
> crossprod(b)
      [,1]  [,2]  [,3]
[1,]  1500 -1500  1500
[2,] -1500  1500 -1500
[3,]  1500 -1500  1500
```

3.

```
> matE <- matrix(0, nr=6, nc=6)
>
> matE
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,]    0    0    0    0    0    0
[2,]    0    0    0    0    0    0
[3,]    0    0    0    0    0    0
[4,]    0    0    0    0    0    0
[5,]    0    0    0    0    0    0
[6,]    0    0    0    0    0    0
> row(matE)
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,]    1    1    1    1    1    1
[2,]    2    2    2    2    2    2
[3,]    3    3    3    3    3    3
[4,]    4    4    4    4    4    4
[5,]    5    5    5    5    5    5
[6,]    6    6    6    6    6    6
> col(matE)
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,]    1    2    3    4    5    6
[2,]    1    2    3    4    5    6
[3,]    1    2    3    4    5    6
[4,]    1    2    3    4    5    6
[5,]    1    2    3    4    5    6
[6,]    1    2    3    4    5    6
> row(matE)-col(matE)
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,]    0   -1   -2   -3   -4   -5
[2,]    1    0   -1   -2   -3   -4
[3,]    2    1    0   -1   -2   -3
[4,]    3    2    1    0   -1   -2
[5,]    4    3    2    1    0   -1
[6,]    5    4    3    2    1    0
> matE[abs(row(matE)-col(matE))==1] <- 1
> matE
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,]    0    1    0    0    0    0
[2,]    1    0    1    0    0    0
[3,]    0    1    0    1    0    0
[4,]    0    0    1    0    1    0
[5,]    0    0    0    1    0    1
[6,]    0    0    0    0    1    0
```

4.

```
> a <- 0:4
> A <- outer(a,a,"+")
> A
     [,1] [,2] [,3] [,4] [,5]
[1,]    0    1    2    3    4
[2,]    1    2    3    4    5
[3,]    2    3    4    5    6
[4,]    3    4    5    6    7
[5,]    4    5    6    7    8
```

5.

(a).

```
> a <- outer(0:4, 0:4, "+")%%5
> a
     [,1] [,2] [,3] [,4] [,5]
[1,]    0    1    2    3    4
[2,]    1    2    3    4    0
[3,]    2    3    4    0    1
[4,]    3    4    0    1    2
[5,]    4    0    1    2    3
```

(b).

```
> b <- outer(0:9, 0:9, "+")%%10
> b
      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
 [1,]    0    1    2    3    4    5    6    7    8     9
 [2,]    1    2    3    4    5    6    7    8    9     0
 [3,]    2    3    4    5    6    7    8    9    0     1
 [4,]    3    4    5    6    7    8    9    0    1     2
 [5,]    4    5    6    7    8    9    0    1    2     3
 [6,]    5    6    7    8    9    0    1    2    3     4
 [7,]    6    7    8    9    0    1    2    3    4     5
 [8,]    7    8    9    0    1    2    3    4    5     6
 [9,]    8    9    0    1    2    3    4    5    6     7
[10,]    9    0    1    2    3    4    5    6    7     8
```

(c).

```
> c <- outer(0:8, 0:8, "-")%%9
> c
     [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
 [1,]    0    8    7    6    5    4    3    2    1
 [2,]    1    0    8    7    6    5    4    3    2
 [3,]    2    1    0    8    7    6    5    4    3
 [4,]    3    2    1    0    8    7    6    5    4
 [5,]    4    3    2    1    0    8    7    6    5
 [6,]    5    4    3    2    1    0    8    7    6
 [7,]    6    5    4    3    2    1    0    8    7
 [8,]    7    6    5    4    3    2    1    0    8
 [9,]    8    7    6    5    4    3    2    1    0
```

6.

```
> yVec <- c(7, -1, -3, 5, 17)
> Mat <- matrix(0, nr=5, nc=5)
> Mat <- abs(col(Mat) - row(Mat)) + 1
> solve(Mat, matrix(yVec, nc=1))
     [,1]
[1,]   -2
[2,]    3
[3,]    5
[4,]    2
[5,]   -4
```

7.

(a).

```
> apply(aMat, 1, function(x){sum(x>4)})
[1] 4 7 6 2 6 7
```

(b).

```
> which(apply(aMat, 1, function(x){sum(x>4)}==2))
[1] 4
```

(c).

```
> aMatCol <- colSums(aMat)
> which(outer(aMatCol, aMatCol, "+") > 75, arr.ind=T)
     row col
[1,]   2   2
[2,]   6   2
[3,]   8   2
[4,]   2   6
[5,]   8   6
[6,]   2   8
[7,]   6   8
[8,]   8   8
```

8.

(a).

```
> sum((1:20)^4 * sum(1/(4:8)))
[1] 639215.3
```

(b).

```
> sum((1:20)^4 / (3+outer(1:20, 1:5, "*")))
[1] 89912.02
```

(c).

```
> sum(outer(1:10, 1:10, function(i,j){(i>=j)*i^4/(3+i*j)}))
[1] 6944.743
```

Exercise3 Simple Functions

1.

(a).

```
> tmpFn1 <- function(xVec)
+ {
+     return(xVec^(1:length(xVec)))
+ }
> a <- c(1, 2, 3, 4, 5)
> b <- tmpFn1(a)
> b
[1]    1    4   27  256 3125
> tmpFn2 <- function(xVec2)
+ {
+     n = length(xVec2)
+     return(xVec2^(1:n)/(1:n))
+ }
> a <- c(1, 2, 3, 4, 5)
> b <- tmpFn2(a)
> b
[1]   1   2   9  64 625
```

(b).

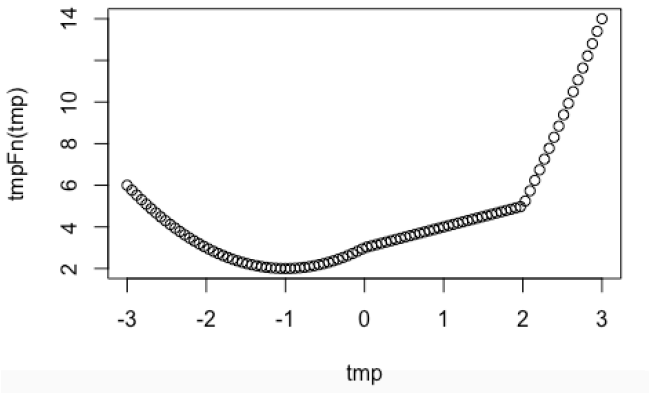
```
> tmpFn3 <- function(xVec3){
+     n = length(xVec3)
+     return(1+sum(xVec3^(1:n)/(1:n)))
+ }
> tmpFn3(1:3)
[1] 13
```

2.

```
> tmpFn <- function(xVec){
+     n <- length(xVec)
+     (xVec[-c(n-1, n)] + xVec[-c(1, n)] + xVec[-c(1, 2)]) / 3
+ }
> tmpFn(c(1:5,6:1))
[1] 2.000000 3.000000 4.000000 5.000000 5.333333 5.000000 4.000000 3.000000 2.000000
```

3.

```
> tmpFn <- function(x){
+     ifelse(x<0, x^2 + 2*x + 3, ifelse(x<2, x+3, x^2 + 4*x - 7))
+ }
> tmp <- seq(-3, 3, len=100)
> plot(tmp, tmpFn(tmp))
```



4.

```
> tmpFn <- function(mat){
+     mat[mat%%2 == 1] <- 2 * mat[mat%%2 == 1]
+     mat
+ }
```

5.

```
> tmpFn <- function(n, k){
+     tmp <- diag(k, nr = n)
+     tmp[abs(row(tmp) - col(tmp)) == 1] <- 1
+     tmp
+ }
```

6.

```
> quadrant <- function(alpha){
+     1 + floor(alpha/90)%%4
+ }
> quadrant(40)
[1] 1
> quadrant(100)
[1] 2
> quadrant(190)
[1] 3
```

7.

(a).

```
> weekday <- function(day, month, year)
+ {
+     month <- month - 2
+     if(month <= 0){
+         month <- month + 12
+         year <- year - 1
+     }
+     cc <- year %% 100
+     year <- year %% 100
+     tmp <- floor(2.6*month - 0.2) + day + year + year %% 4 + cc %% 4 - 2 * cc
+     c("Sunday", "Monday", "Tuesday", "Wednesday", "Thursday", "Friday", "Saturday")[1 + tmp%%7]
+ }
> weekday(29, 7, 1997)
[1] "Tuesday"
```

(b).

```
> weekday2 <- function(day, month, year){
+     flag <- month <= 2
+     month <- month - 2 + 12 * flag
+     year <- year - flag
+     cc <- year %% 100
+     year <- year %% 100
+     tmp <- floor(2.6 * month - 0.2) + day + year + year %% 4 + cc %% 4 - 2*cc
+     c("Sunday", "Monday", "Tuesday", "Wednesday", "Thursday", "Friday", "Saturday")[1 + tmp%%7]
+ }
> weekday(c(18, 23, 12), c(2, 3, 4), c(1998, 2003, 2012))
[1] "Wednesday" "Tuesday"   "Monday"
```

8.

(a).

```
> testLoop <- function(n){
+     xVec <- rep(NA, n-1)
+     xVec[1] <- 1
+     xVec[2] <- 2
+     for(j in 3:(n-1) )
+         xVec[j] <- xVec[j-1] + 2/xVec[j-1]
+     xVec
+ }
> testLoop(10)
[1] 1.000000 2.000000 3.000000 3.666667 4.212121 4.686941 5.113659 5.504768 5.868090
```

(b).

```
> testLoop2 <- function(yVec){
+     n <- length(yVec)
+     sum(exp(1:n))
+ }
>
> testLoop2(10)
[1] 2.718282
```

9.

(a).

```
quadmap <- function(start, rho, niter){
  xVec <- rep(NA, nither)
  xVec[1] <- start
  for(i in 1:(nither-1)){
    xVec[i+1] <- rho * xVec[i] * (1-xVec[i])
  }
  x
}
```

(b).

```
quad2 <- function(start, rho, eps = 0.02){
  x1 <- start
  x2 <- rho*x1*(1-x1)
  nither <- 1
  while(abs(x1-x2) >= eps){
    x1 <- x2
    x2 <- rho*x1*(1-x1)
    niter <- niter + 1
  }
  niter
}
```

10.

(a).

```
tmpAcf <- function(xVec){
  xc <- xVec - mean(xVec)
  denom <- sum(xc^2)
  n <- length(x)
  r1 <- sum(xc[2:n] * xc[1:(n-1)])/denom
  r2 <- sum(xc[3:n] * xc[1:(n-2)])/denom
  list(r1 = r1, r2 = r2)
}
```

(b).

```
tmpAcf <- function(x, k){
  xc <- x - mean(x)
  denom <- sum(xc^2)
  n <- length(x)
  tmpFn <- function(j){sum(xc[(j+1):n] * xc[1:(n-j)])/denom}
  c(1, sapply(1:k, tmpFn))
}
```