MAE 230C Formula Sheet

$$\mathbf{u}\cdot\nabla\mathbf{u} = \frac{1}{2}\nabla(\mathbf{u}\cdot\mathbf{u}) - \mathbf{u}\times(\nabla\times\mathbf{u}), \quad \nabla\times(\nabla\times\boldsymbol{\omega}) = \nabla(\nabla\cdot\boldsymbol{\omega}) - \nabla^2\boldsymbol{\omega}, \quad \nabla\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A}) + (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B}$$

$$\nabla^2(\,) = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial(\,)}{\partial r}\right] + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\frac{\partial(\,)}{\partial\theta}\right] + \frac{1}{r^2\sin^2\theta}\frac{\partial^2(\,)}{\partial\phi^2}, \quad \sin(a\pm b) = \sin a\cos b \pm \cos a \sin b$$

$$\cos(a\pm b) = \cos a\cos b \mp \sin a\sin b, \quad \tan(a\pm b) = \frac{\tan a \pm \tan b}{1\mp\tan a\tan b}, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\frac{D}{Dt}\int_{\text{CM}}\rho\,dV = \frac{\partial}{\partial t}\int_{\text{CV}}\rho\,dV + \int_{\text{CS}}\rho(\mathbf{u}\cdot\mathbf{n})\,dS = 0, \quad \frac{D}{Dt}\int_{\text{CM}}\rho\mathbf{u}\,dV = \frac{\partial}{\partial t}\int_{\text{CV}}\rho\mathbf{u}\,dV + \int_{\text{CS}}\rho\mathbf{u}(\mathbf{u}\cdot\mathbf{n})\,dS = -\int_{\text{CS}}p\mathbf{n}\,dS + \int_{\text{CS}}\underline{\underline{\boldsymbol{\tau}}}\cdot\mathbf{n}\,dS$$

All eqs for Fixed CV: $$\frac{D}{Dt}\int_{\text{CM}}\rho e_0\,dV = \frac{\partial}{\partial t}\int_{\text{CV}}\rho e_0\,dV + \int_{\text{CS}}\rho e_0(\mathbf{u}\cdot\mathbf{n})\,dS = -\int_{\text{CS}}p(\mathbf{u}\cdot\mathbf{n})\,dS + \int_{\text{CS}}\underline{\underline{\boldsymbol{\tau}}}\cdot\mathbf{n}\cdot\mathbf{u}\,dS + \int_{\text{CV}}\nabla\cdot\mathbf{q}\,dV$$

Mass: $$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0, \quad \partial_0\rho + \partial_j(\rho u_j) = 0, \quad \boxed{\frac{D\rho}{Dt} = -\rho\,\nabla\cdot\mathbf{u}}, \quad \frac{D\rho}{Dt} = -\rho\partial_j u_j$$

Mom: $$\frac{\partial(\rho\mathbf{u})}{\partial t} + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u}) = -\nabla p + \nabla\cdot\underline{\underline{\boldsymbol{\tau}}}, \quad \partial_0(\rho u_i) + \partial_j(\rho u_i u_j) = -\partial_i p + \partial_j\tau_{ij}, \quad \boxed{\rho\frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla\cdot\underline{\underline{\boldsymbol{\tau}}}}, \quad \rho\frac{Du_i}{Dt} = -\partial_i p + \partial_j\tau_{ij}$$

Vort(inv.): $$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega}\cdot\nabla)\mathbf{u} - \boldsymbol{\omega}(\nabla\cdot\mathbf{u}) - \frac{1}{\rho^2}(\nabla p\times\nabla\rho), \quad \frac{D}{Dt}\left[\frac{\boldsymbol{\omega}}{\rho}\right] = \left(\frac{\boldsymbol{\omega}}{\rho}\cdot\nabla\right)\mathbf{u} - \frac{1}{\rho^3}\left[\nabla p\times\nabla\rho\right],$$ (st.,inv.): $\nabla h_0 = T\nabla s + \mathbf{u}\times\boldsymbol{\omega}$

Energy Equations

$$\frac{\partial}{\partial t}(\rho e_0) + \nabla\cdot(\rho e_0\mathbf{u}) = \nabla\cdot(\underline{\underline{\boldsymbol{\sigma}}}\cdot\mathbf{u}) - \nabla\cdot\mathbf{q}, \quad \partial_0(\rho e_0) + \partial_i(\rho e_0 u_i) = \partial_j(\sigma_{ij}u_i) - \partial_i q_i$$

$$\rho\frac{De_0}{Dt} = \nabla\cdot(\underline{\underline{\boldsymbol{\tau}}}\cdot\mathbf{u}) - \nabla\cdot(p\mathbf{u}) - \nabla\cdot\mathbf{q}, \quad \rho\frac{De_0}{Dt} = \partial_j(\tau_{ij}u_i) - \partial_j(pu_j) - \partial_j q_j$$

$$\boxed{\rho\frac{Dh_0}{Dt} = \nabla\cdot(\underline{\underline{\boldsymbol{\tau}}}\cdot\boldsymbol{u}) - \nabla\cdot\boldsymbol{q} + \frac{\partial p}{\partial t}}, \quad \rho\frac{Dh_0}{Dt} = \partial_j(\tau_{ij}u_i) - \partial_j q_j + \partial_0 p$$

$$\rho\frac{Dh}{Dt} = \frac{Dp}{Dt} + \underline{\underline{\boldsymbol{\tau}}}:\nabla\boldsymbol{u} - \nabla\cdot\boldsymbol{q}, \quad \rho\frac{Dh}{Dt} = \frac{Dp}{Dt} + \tau_{ij}\partial_j u_i - \partial_j q_j, \quad \boxed{\rho T\frac{Ds}{Dt} = \underline{\underline{\boldsymbol{\tau}}}:\nabla\boldsymbol{u} - \nabla\cdot\mathbf{q}}$$

(1Law): $de = \delta w + \delta q$, Gibb: $de = Tds - pdv$, $dh = Tds + vdp$, $h = e + pv$, $c_v = \left[\frac{\partial e}{\partial T}\right]_v$, $c_p = \left[\frac{\partial h}{\partial T}\right]_p$, (2Law) $ds = \frac{\delta q_{\text{rev}}}{T}$

Perfect Gas (IGL): $R = 287$ J/(kg $\cdot K$) $= 1716$ ft $\cdot$ lb/(slug $\cdot\,^\circ R$), $c_p = 1004.5$ J/(kg K)

$$pv = RT \to \frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}, \quad de = c_v dT, \quad dh = c_p dT, \quad R = c_p - c_v, \quad \gamma = \frac{c_p}{c_v}, \quad c_p = \frac{\gamma R}{\gamma - 1}, \quad c_v = \frac{R}{\gamma - 1}$$

$$ds = c_p\frac{dT}{T} - R\frac{dp}{p} = c_v\frac{dT}{T} + R\frac{dv}{v}, \quad a^2 = \left(\frac{\partial p}{\partial\rho}\right)_s,$$ (IGL): $a^2 = \gamma RT = \gamma\frac{p}{\rho}$, $u^2 = \gamma M^2\frac{p}{\rho}$

Calorically Perfect Gas

$$\frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}, \quad s_2 - s_1 = c_p\ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right) = c_v\ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right), \quad e = c_v T, \quad h = c_p T$$

$$d\left(p^{\gamma-1}T^{-\gamma}\right) = 0, \quad d\left(T\rho^{1-\gamma}\right) = 0, \quad d\left(p\rho^{-\gamma}\right) = 0$$

Total Properties Definition (CPG)

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2, \quad \frac{p_0}{p} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}}, \quad \frac{\rho_0}{\rho} = \left[1 + \frac{\gamma-1}{2}M^2\right]^{\frac{1}{\gamma-1}}$$

Q1D Flow (steady, adiabatic, no viscous effects, isentropic): $dT_0 = 0, \quad dp_0 = 0, \quad d\rho_0 = 0$

$\mathbf{M < 1}$: $\boldsymbol{dA+}$: $(dp,\ d\rho,\ dT)+,\ (du,\ dM)-, \quad$ $\boldsymbol{dA-}$: $(dp,\ d\rho,\ dT)-,\ (du,\ dM)+$

$\mathbf{M > 1}$: $\boldsymbol{dA+}$: $(dp,\ d\rho,\ dT)-,\ (du,\ dM)+, \quad$ $\boldsymbol{dA-}$: $(dp,\ d\rho,\ dT)+,\ (du,\ dM)-$

$$\boxed{\begin{array}{l} \dfrac{d\rho}{\rho} + \dfrac{du}{u} + \dfrac{dA}{A} = 0, \quad dp + \rho u du = 0, \quad dh + u du = 0, \quad \dfrac{dA}{A} = (M^2-1)\dfrac{du}{u} \\ \rho_1 u_1 A_1 = \rho_2 u_2 A_2, \quad p_1 A_1 + \rho_1 u_1^2 A_1 + \displaystyle\int_{A_1}^{A_2} p dA = p_2 A_2 + \rho_2 u_2^2 A_2, \quad h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2 \end{array}}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{2}{\gamma+1}\left[1 + \frac{\gamma-1}{2}M^2\right]\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Adia Energy: $a_*^2 = \dfrac{2(\gamma-1)}{\gamma+1}\left[\dfrac{a^2}{\gamma-1} + \dfrac{u^2}{2}\right]$, Identities: $\dfrac{du^2}{u^2} = \dfrac{dM^2}{M^2} + \dfrac{dT}{T}, \quad \dfrac{du}{u} = \dfrac{1}{2}\dfrac{du^2}{u^2}$, Cramer: $x_i = \dfrac{\det(A_i(\mathbf{b}))}{\det(A)}$

M def: $\dfrac{dV}{V} = \dfrac{dM}{M} + \dfrac{1}{2}\dfrac{dT}{T}$

Fanno Flow: 1D-Adiabatic Flow w/ Irreversibilities (Non-Isentropic): $dT_0 = 0$

$\mathbf{M < 1}$: $(ds,\ du,\ dM)+, \quad (dT,\ dp,\ d\rho,\ dp_0)-, \quad$ $\mathbf{M > 1}$: $(ds,\ dT,\ dp,\ d\rho)+, \quad (dp_0,\ du,\ dM)-$

$$\boxed{\frac{d\rho}{\rho} + \frac{du}{u} = 0, \quad dp + \rho u du = -\rho T ds, \quad dh + u du = 0, \quad \rho_1 u_1 = \rho_2 u_2, \quad p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 + ?, \quad h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}}$$

$$\frac{dT}{T} + (\gamma-1)M^2\frac{du}{u} = 0, \quad \frac{dT}{ds} = -\frac{T}{c_v}\left(\frac{M^2}{1-M^2}\right), \quad \frac{ds}{R} = \left(1-M^2\right)\frac{du}{u}, \quad \frac{s-s_*}{c_p} = \ln\left[M^{\frac{\gamma-1}{\gamma}}\left(\left[\frac{2}{\gamma+1}\right]\left[1 + \frac{\gamma-1}{2}M^2\right]\right)^{-\frac{\gamma+1}{2\gamma}}\right]$$

$$\frac{T_2}{T_1} = \frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}, \quad \frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\left[\frac{2+(\gamma-1)M_2^2}{2+(\gamma-1)M_1^2}\right]^{\frac{1}{2}}, \quad \frac{p_2}{p_1} = \frac{M_1}{M_2}\left[\frac{2+(\gamma-1)M_1^2}{2+(\gamma-1)M_2^2}\right]^{\frac{1}{2}}, \quad \frac{p_{02}}{p_{01}} = \frac{M_1}{M_2}\left[\frac{2+(\gamma-1)M_2^2}{2+(\gamma-1)M_1^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

$$\frac{T}{T_*} = \frac{\gamma+1}{2+(\gamma-1)M^2}, \quad \frac{p}{p_*} = \frac{1}{M}\left[\frac{\gamma+1}{2+(\gamma-1)M^2}\right]^{1/2}, \quad \frac{\rho}{\rho_*} = \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{1/2}, \quad \frac{p_0}{p_{0_*}} = \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

1-D Rayleigh Flow: (Reversible Heat Addition, Non-adiabatic + Non-Isentropic)

$\mathbf{M < 1}$: $\boldsymbol{\delta q+}$: $(ds,\ du,\ dM,\ dT_0)+,\ (dp,\ d\rho,\ dp_0)-$, $dT \pm (M = \gamma^{-1/2})$ $\boldsymbol{\delta q-}$: $(ds,\ du,\ dM,\ dT_0)-,\ (dp,\ d\rho,\ dp_0)+,\ dT\mp$

$\mathbf{M > 1}$: $\boldsymbol{\delta q+}$: $(ds\,,dp\,,d\rho\,,dT_0,\ dT) + (dp_0\,,du,\ dM)-,$ $\boldsymbol{\delta q-}$: $(ds,\ dp,\ d\rho,\ dT_0,\ dT) - (dp_0,\ du,\ dM)+$

$$\boxed{\frac{d\rho}{\rho} + \frac{du}{u} = 0, \quad dp + \rho u du = 0, \quad dh + u du = \delta q, \quad \rho_1 u_1 = \rho_2 u_2, \quad p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2, \quad h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}}$$

$$\frac{dT}{ds} = \frac{T}{c_p}\left[\frac{1-\gamma M^2}{1-M^2}\right], \quad \frac{\delta q}{h} = \left(1-M^2\right)\frac{du}{u}, \quad \frac{dT}{T} = \frac{\delta q}{h} - (\gamma-1)M^2\frac{du}{u}$$

$$\frac{p}{p_*} = \frac{1+\gamma}{1+\gamma M^2}, \quad \frac{T}{T_*} = M^2\left(\frac{\gamma+1}{1+\gamma M^2}\right)^2, \quad \frac{\rho}{\rho_*} = \frac{1}{M^2}\left(\frac{1+\gamma M^2}{\gamma+1}\right), \quad \frac{p_0}{p_{0*}} = \left[\frac{1}{\gamma+1}\left(2+(\gamma-1)M^2\right)\right]^{\frac{\gamma}{\gamma-1}}\left(\frac{\gamma+1}{1+\gamma M^2}\right)$$

$$\frac{T_0}{T_{0*}} = \frac{M^2(\gamma+1)}{(1+\gamma M^2)^2}\left[2+(\gamma-1)M^2\right], \quad \frac{s-s_*}{c_p} = \ln\left[M^2\left(\frac{\gamma+1}{1+\gamma M^2}\right)^{\frac{\gamma+1}{\gamma}}\right]$$

Stationary Normal Shocks (Adiabatic): $\Delta\rho+, \Delta p+, \Delta T+, \Delta T_0 = 0, \Delta u-, \Delta M-, \Delta p_0-$

$$\boxed{\rho_1 u_1 = \rho_2 u_2, \quad p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2, \quad h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}}$$

any gas: $h_2 - h_1 = \frac{1}{2}(p_2 - p_1)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$, $\quad e_2 - e_1 = \frac{p_2 + p_1}{2}(v_1 - v_2)$, $\quad CPG\ \frac{\rho_2}{\rho_1} = \frac{1 + \frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1}}{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}$, $\quad \frac{T_2}{T_1} = \frac{p_2}{p_1}\left[\frac{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}{1 + \frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1}}\right]$

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}, \quad \frac{T_2}{T_1} = \left[\frac{2 + (\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right]\left[1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)\right], \quad \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2 + (\gamma-1)M_1^2}, \quad \frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)$$

$$\frac{p_{02}}{p_{01}} = \left[\frac{(\gamma+1)M_1^2}{2 + (\gamma-1)M_1^2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

Unsteady Shock Waves

$$\frac{\Delta u}{a_1} = \frac{2}{\gamma+1}\left[M_s - \frac{1}{M_s}\right] = \frac{1}{\gamma}\left[\frac{p_2}{p_1} - 1\right]\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}\right]^{\frac{1}{2}}, \quad M_s^2 = 1 + \frac{\gamma+1}{2\gamma}\left[\frac{p_2}{p_1} - 1\right]$$

Finite Waves (Isentropic Equations of Motion)

$$\boxed{\frac{\partial\rho}{\partial t} + \mathbf{u}\cdot\nabla\rho + \rho\nabla\cdot\mathbf{u} = 0, \quad \frac{\partial\mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} + \frac{a^2}{\rho}\nabla\rho = 0}$$

One-Dimensional Specialization

$$\text{mass: } \frac{1}{a^2}\left(\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x}\right) + \rho\frac{\partial u}{\partial x} = 0, \quad \frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0, \quad \text{mom: } \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0, \quad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{a^2}{\rho}\frac{\partial\rho}{\partial x} = 0$$

$$\text{mass: } \rho_1\frac{\partial\tilde{s}}{\partial t} + \rho_1\left(\frac{\partial u}{\partial x} + \tilde{s}\frac{\partial u}{\partial x}\right) + \rho_1 u\frac{\partial\tilde{s}}{\partial x} = 0, \quad \text{mom: } \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{a^2}{\tilde{s}+1}\frac{\partial\tilde{s}}{\partial x} = 0$$

$$\frac{dx}{dt} = u \pm a, \quad du = \pm\frac{dp}{\rho a}, \quad J_\pm = u \pm \frac{2a}{\gamma-1}, \quad a = \frac{\gamma-1}{4}(J_+ - J_-), \quad u = \frac{1}{2}(J_+ + J_-), \quad \tilde{s} = \frac{\rho'}{\rho_1} = \frac{\rho - \rho_1}{\rho_1}$$

$$\frac{p}{p_1} = \left(\frac{\rho}{\rho_1}\right)^\gamma = (1 + \tilde{s})^\gamma, \quad \frac{T}{T_1} = (1 + \tilde{s})^{\gamma-1}$$

Left-Facing, Simple-Centered Expansion Wave

$$\frac{a}{a_1} = 1 - \frac{\gamma-1}{2}\left(\frac{u}{a_1}\right), \quad \frac{T}{T_1} = \left[1 - \frac{\gamma-1}{2}\left(\frac{u}{a_1}\right)\right]^2, \quad \frac{p}{p_1} = \left[1 - \frac{\gamma-1}{2}\left(\frac{u}{a_1}\right)\right]^{\frac{2\gamma}{\gamma-1}}, \quad \frac{\rho}{\rho_1} = \left[1 - \frac{\gamma-1}{2}\left(\frac{u}{a_1}\right)\right]^{\frac{2}{\gamma-1}}$$

$$u = \frac{2}{\gamma+1}\left(a_1 + \frac{x}{t}\right)$$

Right-Facing, Isentropic Compression Wave

$$\frac{a}{a_1} = 1 + \frac{\gamma-1}{2}\left(\frac{u}{a_1}\right), \quad \frac{T}{T_1} = \left[1 + \frac{\gamma-1}{2}\left(\frac{u}{a_1}\right)\right]^2, \quad \frac{p}{p_1} = \left[1 + \frac{\gamma-1}{2}\left(\frac{u}{a_1}\right)\right]^{\frac{2\gamma}{\gamma-1}}, \quad \frac{\rho}{\rho_1} = \left[1 + \frac{\gamma-1}{2}\left(\frac{u}{a_1}\right)\right]^{\frac{2}{\gamma-1}}$$

$$\text{Shock Tube: } \frac{p_4}{p_1} = \frac{1 + \frac{2\gamma_1}{\gamma_1+1}\left(M_s^2 - 1\right)}{\left[1 - \frac{a_1}{a_4}\frac{\gamma_4-1}{\gamma_1+1}\left(M_s - \frac{1}{M_s}\right)\right]^{\frac{2\gamma_4}{\gamma_4-1}}} = \frac{p_2}{p_1}\left[1 - \frac{(\gamma_4 - 1)\left(\frac{a_1}{a_4}\right)\left(\frac{p_2}{p_1} - 1\right)}{\sqrt{2\gamma_1\left[2\gamma_1 + (\gamma_1 + 1)\left(\frac{p_2}{p_1} - 1\right)\right]}}\right]^{\frac{-2\gamma_4}{\gamma_4-1}}$$

Acoustic Theory (Isentropic)

$$\boxed{\begin{array}{c}\text{mass: } \dfrac{\partial \rho'}{\partial t} + \rho_1 \nabla \cdot \mathbf{u}' = 0, \quad \dfrac{\partial \tilde{s}}{\partial t} + \nabla \cdot \mathbf{u}' = 0, \quad \text{mom: } \dfrac{\partial \mathbf{u}'}{\partial t} + \dfrac{a_1^2}{\rho_1}\nabla \rho' = 0, \quad \dfrac{\partial \mathbf{u}'}{\partial t} + a_1^2 \nabla \tilde{s} = 0 \\ \dfrac{\partial^2 \mathbf{u}'}{\partial t^2} - a_1^2 \nabla(\nabla \cdot \mathbf{u}') = 0, \qquad \dfrac{\partial^2 \tilde{s}}{\partial t^2} - a_1^2 \nabla^2 \tilde{s} = 0\end{array}}$$

One-Dimensional-Rectilinear Specialization

$$\text{mass: } \frac{\partial \rho'}{\partial t} + \rho_1 \frac{\partial u'}{\partial x} = 0, \quad \frac{\partial \tilde{s}}{\partial t} + \frac{\partial u'}{\partial x} = 0, \quad \text{mom: } \rho_1 \frac{\partial u'}{\partial t} + a_1^2 \frac{\partial \rho'}{\partial x} = 0, \quad \frac{\partial u'}{\partial t} + a_1^2 \frac{\partial \tilde{s}}{\partial x} = 0$$

$$\frac{\partial^2 \rho'}{\partial t^2} - a_1^2 \frac{\partial^2 \rho'}{\partial x^2} = 0, \quad \frac{\partial^2 u'}{\partial t^2} - a_1^2 \frac{\partial^2 u'}{\partial x^2} = 0, \quad \frac{\partial^2 \tilde{s}}{\partial t^2} - a_1^2 \frac{\partial^2 \tilde{s}}{\partial x^2} = 0$$

$$\boxed{\tilde{s} = F(\xi) + G(\eta)}, \ u' = f(\xi) + g(\eta), \ \xi = x - a_1 t, \ \eta = x + a_1 t, \ f = a_1 F, \ g = -a_1 G, \ u' = \pm a_1 \tilde{s} = \pm \frac{p'}{\rho_1 a_1}, \ p' = \pm \gamma p_1 \tilde{s}$$

$$\text{Linearized Isentropic: } \frac{p}{p_1} = 1 + \gamma \tilde{s}, \quad \frac{T}{T_1} = 1 + (\gamma - 1)\tilde{s}, \quad \boxed{u' = a_1[F(\xi) - G(\eta)], \quad p' = \gamma p_1 [F(\xi) + G(\eta)]}$$

Spherical Acoustics

$$\frac{\partial^2 (r\tilde{s})}{\partial t^2} - a_1^2 \frac{\partial^2 (r\tilde{s})}{\partial r^2} = 0, \quad \tilde{s} = \frac{1}{r}\mathcal{F}(r - a_1 t) + \frac{1}{r}\mathcal{G}(r + a_1 t) = \frac{1}{r} F\left(t - \frac{r}{a_1}\right) + \frac{1}{r} G\left(t + \frac{r}{a_1}\right)$$

Oblique Shock Theory

$$M_{n1} = M_1 \sin(\beta), \quad M_2 = \frac{M_{n2}}{\sin(\beta - \theta)}, \quad \tan(\theta) = 2\cot(\beta)\left[\frac{M_1^2 \sin^2(\beta) - 1}{M_1^2(\gamma + \cos(2\beta)) + 2}\right], \quad M_1^2 \sin^2 \beta - 1 = \frac{\gamma + 1}{2} M_1^2 \frac{\sin\beta \sin\theta}{\cos(\beta - \theta)}$$

$$\text{Mach: } \mu = \sin^{-1}\left(\frac{1}{M}\right)$$

Steady, Homentropic, Homenthalpic, 2-D Flow

$$\boxed{\mathbf{u} \cdot [\mathbf{u} \cdot \nabla \mathbf{u}] - a^2 \nabla \cdot \mathbf{u} = 0}$$

2D Specialization

$$[u^2 - a^2]\frac{\partial u}{\partial x} + [v^2 - a^2]\frac{\partial v}{\partial y} + uv\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right] = 0, \ [u^2 - a^2]\phi_{xx} + 2uv\phi_{xy} + [v^2 - a^2]\phi_{yy} = 0, \ \frac{dy}{dx} = \frac{\frac{uv}{a^2} \pm \sqrt{M^2 - 1}}{\frac{u^2}{a^2} - 1} = \tan[\theta \pm \mu]$$

$$\frac{dn}{ds} = \pm \frac{1}{\sqrt{M^2 - 1}} = \tan(\pm\mu), \quad d\theta \mp \sqrt{M^2 - 1}\frac{dV}{V} = 0, \quad d\nu = \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma - 1}{2}M^2}\frac{dM}{M}, \quad \theta \mp \nu = C,$$

$$\nu(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1}\left(\sqrt{\frac{\gamma - 1}{\gamma + 1}(M^2 - 1)}\right) - \tan^{-1}(\sqrt{M^2 - 1})$$

Perturbation Theory (Subsonic + Supersonic)

$$\left(1 - M_\infty^2\right)\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0, \quad \boxed{(1 - M_\infty^2)\frac{\partial^2 \phi'}{\partial x^2} + \frac{\partial^2 \phi'}{\partial y^2} + \frac{\partial^2 \phi'}{\partial z^2} = 0}, \quad c_p = \frac{2}{\gamma M_\infty^2}\left[\frac{p}{p_\infty} - 1\right]$$

$$c_p = \frac{2}{\gamma M_\infty^2}\left\{\left[1 + \frac{\gamma - 1}{2} M_\infty^2 \left(1 - \frac{\mathbf{u} \cdot \mathbf{u}}{U_\infty^2}\right)\right]^{\frac{\gamma}{\gamma - 1}} - 1\right\}$$

Linearized Pressure Coefficient

$$\text{2D: } c_p = -2\frac{u'}{U_\infty} = -\frac{2}{U_\infty}\frac{\partial \phi'}{\partial x}, \quad \text{3D: } c_p = -2\frac{u'}{U_\infty} - \frac{v'^2 + w'^2}{U_\infty^2}, \quad \text{BC: } \left.\frac{\partial \phi'}{\partial y}\right|_{y=0} = U_\infty \frac{dy_w}{dx}$$

2D-Supersonic Perturbation Theory

$$\lambda^2 = M_\infty^2 - 1, \quad \boxed{\frac{\partial^2 \phi'}{\partial x^2} - \frac{1}{\lambda^2}\frac{\partial^2 \phi'}{\partial y^2} = 0}, \quad \phi = f(x - \lambda y) + g(x + \lambda y)$$

$$f'(x - \lambda y_1) - g'(x + \lambda y_1) = -\frac{U_\infty}{\lambda}\frac{dy_{w1}}{dx}, \quad f'(x - \lambda y_2) - g'(x + \lambda y_2) = -\frac{U_\infty}{\lambda}\frac{dy_{w2}}{dx}$$

General Airfoil Drag and Lift Coefficient Formula

$$C_d = \frac{1}{c}\int_{LE}^{TE}\left(C_{pu}\frac{dy_u}{dx} - C_{pl}\frac{dy_l}{dx}\right)dx, \quad C_l = \frac{1}{c}\int_{LE}^{TE}\left(C_{pl} - C_{pu}\right)dx$$

Supersonic Thin Airfoil Theory

$$C_{pu} = \frac{2}{\lambda}\frac{dy_u}{dx}, \quad C_{pl} = -\frac{2}{\lambda}\frac{dy_l}{dx}, \quad C_l = -\frac{2}{\lambda c}\int_0^c \left[\frac{dy_l}{dx} + \frac{dy_u}{dx}\right]dx = \frac{4}{\lambda}\alpha,$$

$$C_d = \frac{2}{\lambda c}\int_0^c \left[\left(\frac{dy_u}{dx}\right)^2 + \left(\frac{dy_l}{dx}\right)^2\right]dx = \frac{1}{\lambda c}\int_0^c \left[4\left(\frac{dy_c}{dx}\right)^2 + \left(\frac{dh}{dx}\right)^2\right]dx$$

Similarity Rules

$$\phi(x,y) = \frac{1}{\sqrt{1 - M_1^2}}\frac{U_1}{U_i}\frac{\tau_1}{\tau_i}\Phi\left(x, y\sqrt{1 - M_1^2}\right), \quad C_p(x,y) = \frac{1}{\sqrt{1 - M_1^2}}\frac{\tau_1}{\tau_i}C_{pi}\left(x, y\sqrt{1 - M_1^2}\right)$$