<u>Optimization of Power Generation and Distribution</u>

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Abstract

Designing an electrical power grid to be both reliable and resilient comes with its challenges; however, this problem has increasingly become of more importance in the past two decades. Our society's reliance and emphasis on technology has forced many experts to reevaluate research in the efficient management of power distribution networks, especially with events like blackouts highlighting the disastrous consequences that can evolve from being unable to provide energy to people. This study aims to optimize a local power grid with the desired goal of providing a recommended schedule on how to operate the grid based on forecasted energy demand. In our analysis, we assumed costs associated with distribution to be negligible, which allowed for a more narrowed focus on optimizing the power being generated. The problem was formulated as a mixed integer linear program (MILP) and all constraints were explicitly developed linearly using the data on hand. The extensive list of variables required us to use OpenSolver in Excel to solve the problem. Our model required the need for IF/ELSE logic; however, OpenSolver was unable to handle this, and we were forced to make some simplified assumptions. In doing so, we did reach an optimal solution, but some of the results seemed highly improbable. In a future attempt to solve this problem, we recommend a simplified approach using less data to better gauge some of these issues. We also suggest the use of a different program algorithm such as the one found in Matlab's intlinprog, which allows for the use of conditional logic statements.

Introduction

This study explores the efficient management of power distribution and generation. The overarching goal is to design a schedule that describes hour by hour which generators out of a given fleet should run, and at what capacity they should run. This goal is subject to restraints regarding maximum and minimum capacity of each generator, generator efficiency curves, and minimum load requirement as dictated by the load point (city) demands. Financially, we are trying to minimize the cost function, which is a function of natural gas price, price contracts with the city, and deviations from the predicted load, and the carbon tax from natural gas usage.

Research and optimization of electric grids such as the one studied in this report is essential to provide cheap and reliable power for the consumer. With hundreds of cities being powered by hundreds of power plants, managing the grid can become an extremely complex task due to the myriad of arrangements and configurations of power plants that can meet the load requirements. It is therefore imperative to understand how the grid can be optimized so that economic and reliable management of the grid can be assured.

Effective management and optimization of the grid can result in economic benefits. Furthermore, it will also prevent blackouts and brownouts that can result in devastating consequences. These consequences include economic shutdown, threat to public health and safety, and more. In a world placing more and more emphasis on technology, power outages prevent everyday life from proceeding, sending ripples through all aspects of modern human life. And with increasing emphasis on digital interaction due to the COVID-19 situation, the importance of reliable energy is heightened. Finally, it has been proven in many studies that access to power equates to longer life expectancy and higher living expectation.[1]

With this motivation, we embark to study how a given load schedule for a set of cities may be satisfied with given generators and power plant resources.

Problem Formulation

To conduct our analysis, a few assumptions must be made. Generating power is the limiting variable in the cost function: cost associated with distributing energy is negligible and ignored. Furthermore, we will assume the grid is independent and does not interact with other neighboring grids by importing and exporting power. Therefore, all energy produced at the turbines is consumed at the load points, and no energy leaves the system through distribution losses or foreign exports. There are only inputs and consumptions at each load point. Each generation plant runs 2 turbines in parallel (not series), which are powered only by natural gas. We will assume that natural gas supply is constant, non interruptible. For these turbines, we assume that efficiency curves and relevant data (physical properties, weather, etc.) are readily available/on hand. Further, we assume that each plant has a known natural gas storage capacity and certain production limitations. We also assume that the natural gas is only composed of methane. We assume that the load forecast provided is perfect, so we know exactly what loads need to be met at each hour. We will therefore produce an hour by hour schedule based on the provided load schedule for each city. The data given provides information on the demanded load, financial contracts with the city, and the efficiency of power plants. The following variables and indices are pertinent that we will be using to formulate the constraints and objective function in our problem. Let us describe the variables and constraints more in detail.

Firstly, the integer indices must be clarified. Since we want an hourly basis of load to each city, an hour indice is needed. There are 6 different power plants but the number of each one is clarified later. Thus, we need indices describing the different generators. There are 28 cities that we must satisfy, requiring the need for another set of indice that describe each city.

Lastly, there are different modes of efficiency that each power plant can operate at. If the plants produce electricity at many different amounts, the amount of heat (natural gas) required to produce the electricity differs at a certain level of production. Thus, an efficiency index is needed.

Secondly, let us discuss variables that require additional description in this formulation. Since all of the electricity produced by the power plants are from combustion natural gas, we need to add the constraint of using natural gas and how much volume of natural gas is used. This is taken into account by using variables C, c, V, and ΔH. The enthalpy combustion was calculated using simple stoichiometry and ideal gas relationship as seen in equation 1.

$$CH_4 + O_2 -> CO_2 + 2H_2O$$

$$\Delta H = \frac{-891\ kJ}{mole} * \frac{1 mol}{22.4\ L} * \frac{1000\ L}{1m^3} * \frac{1m^3}{(3.28)^3\ ft^3} * \frac{298\ K}{273\ K} = -\ 1230\ kJ/ft^3 \quad (Eq.\ 1)$$

Additionally, the carbon tax can be calculated by using stoichiometry as well as seen below in equation 2. This can be used to calculate the total amount of carbon tax that needs to be accounted for.

$$For\ every\ 1\ ft^3\ of\ methane,\ 1\ ft^3\ of\ CO_2\ is\ produced\ since\ it's\ a\ one\ to\ one\ mole\ ratio.$$

$$1\ ft^3\ CO_2 * \frac{0.044\ kg}{1\ mol} * \frac{2.2\ lb}{1\ kg} * \frac{1\ ton}{2000\ lb} = 6.683892083 * 10^{-5} ton \quad (Eq.\ 2)$$

Since we provide electricity at an hourly basis, we must satisfy each of the 28 cities' demand for electricity at an hourly basis. If not, we must pay a penalty for the unsatisfied demand. This is seen in equation C.1

Finally, we will discuss some of the more complex key equations and constraints. To account for the efficiency of each generator, 3 different heat rates are given to describe 3 different regimes. This results in a linear piecewise function where each heat rate defines the slope of that regime. In actuality, the generator heat curves are likely non linear, however, describing the heat curve as piecewise linear functions allows the formulation to be linear. A more robust solver can therefore be achieved. To account for these regimes, the index m is introduced to index each regime. Furthermore, since only one regime will be selected, a discrete binary variable z is used, with the constraint that the sum of the 3 z variables must equal one. This means that only one efficiency may be selected. This approach appears in the calculation of volume of natural gas consume:

$$V = \sum_{i}^{6} \sum_{j}^{23} 1000/(\Delta H\ kJ/ft^3 * 0.947817\ BTU/kJ) * x_{ij} * [z_{ij1} L_{ij1} E_{ij1} + z_{ij2} L_{ij2} E_{ij2} + z_{ij3} L_{ij3} E_{ij3}]$$

(Eq. 3)

It was also necessary to model the penalties for failing to meet demand. This is done using a binary variable y and the big M approach:

$$W = \sum_{j}^{23} \sum_{k}^{28} [(y_{kj} A(Actual_{kj} - S_{kj})] \quad \text{(Eq. 4)}$$

$$Actual_{kj} - S_{kj} \leq 99999 * y_{kj} \quad \text{(Eq. 5)}$$

A large number of 99999 is used to make the logic work. If S_{kj} (supplied power) is less than actual, then the left side is positive, but less than the big M of 99999, and then y_{kj} must be 1 for the statement to be true. Therefore, the W statement above becomes A(actual-S_{kj}), as it should be since demand was not met. The penalty is paid. If $S_{kj}$is greater than actual, then the left side is negative. Then y_{kj} could therefore be 0 or 1 for the statement to be true. However, because it is a minimization problem, y_{kj} will be selected to be 0 because that corresponds to a lower objective function. So when demand is not met, you get the penalty as y_{kj} is 1. When demand is met, you pay no penalty as y_{kj} is zero.

Note that the hourly prices for each city do not depend on the solution, and can be computed outside of the formulation. This is done in excel, and the computed independent price values are used.

Finally, it is also important to notice that generator constraints, such as max and minimum load and minimum daily operating hours are incorporated in the formulation.

The section below rigorously details every variable, equation, and constraint. The ones not mentioned in this discussion are largely straightforward.

Indices

$i \ = \ generator \ \in \{1, 2, 3, 4, 5, 6\}$
$j \ = \ hour \ \in \{0, 1, 2, \ ....., \ 21, \ 22, \ 23\}$
$k \ = \ city \in \{1, 2, \ ....., \ 26, \ 27, \ 28\}$listed in excel data file
$m \ = \ efficiency \ level \in \{1, 2, 3\}$ listed in excel data file

Variables

$C \ = \ total \ cost \ due \ to \ natural \ gas \ usage$ (\$)

$c \ = \ cost \ of \ natural \ gas \ (3.98 \ \$/1000ft^3)$

ΔH = enthalpy of combustion of methane

$V \ = \ total \ amount \ of \ natural \ gas \ used \ per \ day \ (ft^3)$

$T \ = \ total \ penalty \ from \ carbon \ tax \ (\$5/ton \ of \ CO_2)$

$Q \ = \ Revenue \ from \ electricity \ sales$ (\$)

$W \ = total \ penalty \ due \ to \ unsatisfied \ demand$ (\$)

$L_{ij} \ = \ how \ much \ power \ the \ ith \ generator \ produces \ at \ jth \ hour \ \ (MWh)$

$x_{ij} = \{1 \ if \ ith \ generator \ is \ ON \ at \ jth \ hour; \ 0 \ if \ ith \ generator \ is \ OFF \ at \ jth \ hour\}$

$P_{kj} = \ jth \ hourly \ price \ for \ city \ k \ (\$/MWh)$

$y_{kj} = \{0 \ if \ actual \ > \ typical; \ 1 \ if \ actual \ < \ typical\}$

$z_{mij} = \{1 \ \ if \ E_{ijm} \ is \ chosen; \ 0 \ if \ E_{ijm} \ if \ not \ chosen\}$

$E_{ijm} \ = \ Efficiency \ heat \ rate \ (BTU/KWh) \ of \ ith \ generator \ at \ jth \ hour \ in \ the \ mth \ efficiency \ regime$

$S_{kj} = \ energy \ supplied \ to \ city \ k \ at \ jth \ hour \ (MWh)$

$A_k, B_k \ = \ price\ parameters\ defined\ for\ each\ city$k

$Actual_{kj} = \ actual\ jth\ hour\ load\ requirement\ for\ city\ k\ (MWh)$

$Typical_{kj} = \ typical\ jth\ hour\ load\ requirement\ for\ city\ k\ (MWh)$

Of the variables listed, some of the variables were readily defined by the data given to us. A_k, B_k are parameters used to calculate the price of electricity in dollars per MWh. There's a formula readily available to us that allows us to calculate the price. This was translated into a constraint as seen in C.5

<u>Objective function</u>

$min\ C\ +\ T +\ W\ -\ Q$

$where$

$$C\ \ =\ V\ *\ \ c\ \ / 1000\ (\$) \qquad \textbf{(C.1)}$$

$$T\ =\ V\ *6.683892083\ \ *\ \ 10^{-5}\ *\ 5$$
(C.2)

$$W\ \ = \sum_{j}^{23}\ \sum_{k}^{28}[(y_{kj}\,A(Actual_{kj} - S_{kj})] \qquad (C.3)$$

$$Q\ \ =\ \ \sum_{j}^{23}\ \ \sum_{k}^{28}[P_{kj} * S_{kj}] \qquad (C.4)$$

$$P_{kj} =\ \ (1\ \ - y_{kj})\ \ *\ \ (A_k * Typical_{kj}\ \ +\ \ B_k *\ \ (Actual_{kj}\ \ -\ \ Typical_k))/Actual_{kj}\ \ +\ \ y_{kj}\ \ \ *\ \ A_k$$
(C.5)

$$V\ = \sum_{i}^{6}\ \sum_{j}^{23}\ 1000/(\Delta H\ kJ/ft^3\ *\ 0.947817\ BTU/kJ)\ *x_{ij} *\ [z_{ij1}\ L_{ij1}\ E_{ij1} + z_{ij2}\ L_{ij2}\ E_{ij2} + z_{ij3} L_{ij3}\ E_{ij3}]$$
(C.6)

<u>Constraints</u>

$$\sum_{k}^{28} S_{kj} = \sum_{i}^{6}\ L_{ij}\ for\ all\ j$$

$$Actual_{kj} - S_{kj}\ \leq\ 99999\ *\ y_{kj}$$

$$\sum_{j}^{23} x_{ij} \geq 2 \ \text{for all } i$$

$$z_{ij1} + z_{ij2} + z_{ij3} = 1$$

$$20 \leq L_{ij1} \leq 35$$

$$35 < L_{ij2} \leq 50$$

$$50 < L_{ij3} \leq 60$$

$$x_{ij} = \{0, 1\}$$

$$y_{kj} = \{0, 1\}$$

$$z_{ijm} = \{0, 1\}$$

Solution Method

To account for the vast number of variables that must be changed, Solver on Excel cannot be used. Thus, Open Solver must be used. To use Open Solver, the written formulation or program must be considered linear. Therefore, the main approach to our problem is a mixed integer linear program.

The first step to solving this MILP was to figure out the pricing of the electricity for the cities. Since the typical and actual amount were given to us, it was easy to calculate the price of electricity. Figure 1 shows an example of the typical electricity requirement and the actual electricity requirement (Column 0, 1).

		Typical	0	1
1	Allison Park	10.1	9.6	9.8
2	Altoona	21.8	20.1	19.8
3	Baldwin	9.3	9.3	9.7
4	Bethel Park	15.2	13.9	14.2
5	Bethlehem	35.2	29.9	29.8
6	Carlisle	8.8	9.7	7.6

Figure 1: Sample actual and typical electricity requirement

By using equation 3 below, the price of electricity for city k and j-th hour can be computed. This is shown in Figure 2.

$$P_{kj} = (1 - y_{kj}) * (A_k * Typical_{kj} + B_k * (Actual_{kj} - Typical_k))/Actual_{kj} + y_{kj} * A_k \qquad (Eq. 3$$

	Price	0	1
1	Allison Park	31	31
2	Altoona	28	28
3	Baldwin	31	30.922495
4	Bethel	28	28
5	Bethlehem	31	31
6	Carlisle	27.814559	28

Figure 2: Sample electricity price in ($/MWh

This process was done for all 28 cities and 24 hour. Next, we implemented a carbon tax and the amount of methane required by calculating the total amount of electricity produced by all 6 different generators. Finally, the penalty function was implemented. Initially, the penalty was modeled using an if statement that charged the penalty whenever the demand was greater than the supplied amount. However, this approach was not compatible with the simplex open solver, so the formulation was rewritten to its current state.

Open Solver was used to find the optimal solution of the mixed integer linear program (MILP). However, some of the constraints were hard to implement. It was noted that the efficiency (heat rate) exhibits a linear relationship between the power produced and heat required. However, this is a piecewise function of three different linear functions since at three different efficiency levels, the slope that transforms the amount of heat required to the electricity output differs. Without an IF/ELSE statement used in the formulation, it's hard to incorporate such constraints. Therefore, many simplifications were made to formulate a simplified, but accurate solution to the MILP. An average heat rate of 7500 BTU/kWh was used for the solution. Since all of the power plants have the same efficiency, there is no reason to distinguish the 6 different generators. Thus, we assumed to use all 6 different generators at all times, which does not require us to use the x_{ij} variable. With these simplifications, a reasonable result was produced which will be discussed in the next section.

Results and Discussion

After such simplification has been made, the following results were formulated.

Objective function = -19435.66

Q = 485710.09

T = 36119.39

V = 108078906

W = 0

This is a reasonable output as the objective function is negative which is what is desired. This tells us that the revenue gained from the pricing of the electricity exceeds the cost incurred for producing the electricity. Thus, a profit is made. The unsatisfied demand cost (W) is 0, telling us that every city at every hour was able to receive the specified amount of electricity in the load requirement data. The amount of natural gas used is close to 100 million ft^3, and the associated carbon tax (T) due to burning the natural gas is approximately $36,000. Looking at the magnitude of the numbers, the amount of natural gas burned per day seems unreasonable. Though we are not industrial chemical engineers, a high amount of natural gas seems to be used. But we have to keep in mind that this usage is for 28 cities for a single day. Furthermore, we simplified that the natural gas is composed purely of methane. This may not be the case in real life. Other organic molecules with higher enthalpy of combustion might be used to provide more energy for the power plants. We have to keep in mind that high usage of natural gas can be costly for the environment. Although the carbon tax wasn't too big, there are still moral aspects in using so much fossil fuel. Alternatives may include adding solar panels or water dams to mitigate the harmful environment effects.

However, this solution is an estimate of what the optimal solution of the actual MILP would be. The simplification was made to acquire a preliminary result which may aid us in the future. There are two key takeaways from this result: the natural gas usage is unreasonably high, possibly, all cities' electricity demands were satisfied. This is a good preliminary result to start with since we now know that with this much load produced by the power generators, all 28 cities can meet their hourly demands.

Conclusion

In summary, based on the forecasted demand schedule that was provided, we were able to optimize the grid's profits to approximately $19,500, while completely fulfilling every city's demand requirements for the day. In doing so, we earned about $485,000 in revenue from electricity sales and used about 108 million ft^3 of natural gas. Although our formulation allowed us to reach an optimal solution, there are many areas of improvement in our model that exist.

In developing the model, we made numerous simplifications and assumptions that would not hold up in the real world. For example, we assumed a non-interruptible supply of a constant priced natural gas and that the gas was solely composed of methane. However, there are supply-demand constraints that make this first assumption unreasonable. Although natural gas is composed more than 85% by volume of methane, there are other hydrocarbons and inert gasses that reside as well. This affects our calculations in the combustion of the gas, which in turn affects the tax that is paid on emissions. A rather big limitation in our model was our assumption of an averaged, constant heat rate across all the generators. Given that the heat rate data provided could not be effectively linearized and excel OpenSolver's inability to handle IF/ELSE statements, we were forced to use this averaged efficiency level. In the future, we would most likely turn to another algorithm that can handle the IF/ELSE type logic that we clearly need. A potentially viable option could be Matlab's intlinprog. Moreover, one of the challenges we encountered in regards to heat rates and efficiencies was transforming the data to a linear format. Another consideration for solving this in the future would be to use an algorithm with the ability to robustly solve non-linearities.

An issue that presents itself is the accuracy of the optimal solution that was found. Using more than 100 million ft^3 of natural gas in only one day is extremely unlikely and thus poses further considerations on the limitations of our model. Further, our model predicted zero unsatisfied demand which also seems unlikely because there exist obvious tradeoffs between the costs associated with producing energy through natural gas consumption and the costs beared from not providing energy to certain cities based on the time of the day. To further examine these issues, we would most likely need to simplify the model to encompass less cities and generators.

Works Cited

Fred Beech (2020). “Energy, Technology, and Policy” *The University of Texas at Austin.*