Project proposal: Numerical Scheme for option pricing models

Options are financial contracts that provide the holder with the right, but not the obligation, to buy (call option) or sell (put option) an underlying asset at a predetermined price (strike price) within a specified time period. They are versatile instruments used for hedging, speculation, and portfolio management in financial markets. Given the complexity of options and their prevalence in the financial world, it is crucial to understand how to accurately price these derivatives.

The Black-Scholes-Merton model, developed by economists Fischer Black, Myron Scholes, and Robert Merton in the early 1970s, provides a mathematical framework to calculate the theoretical value of European-style options. The model is a partial differential equation known as the Black-Scholes equation

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

Where S denote the price of the underlying stock, and V denote the value of the option, while r, σ denote the risk-free rate and volatility, respectively.

A widely used numerical technique in solving the Black-Scholes equation is the finite difference method. One idea is to use the following approximations (forward difference)

$$\begin{cases} \frac{\partial V}{\partial t} \approx \frac{V(t+\delta t, S) - V(t,S)}{\delta t} \\ \frac{\partial V}{\partial S} \approx \frac{V(t, S+\delta S) - V(t,S)}{\delta S} \\ \frac{\partial^2 V}{\partial S^2} \approx \frac{V(t,S+\delta S) - 2V(t,S) + V(t,S-\delta S)}{2\delta S} \end{cases}$$

By partitioning (t, S) space into grids of point, one can start from the boundary conditions and approximate V using the difference approximations. There are different approximation schemes (ex: backward difference, Crank-Nicolson Method). Typically, these methods end up with a system of linear equations

$$Av = b$$

Where A is a matrix of band structure and v is the vector of interest.

This project aims to explore various numerical techniques for solving option pricing models, specifically focusing on methods that transform these models into systems of linear equations. We will assess different approximation schemes and apply iterative algorithms like Gauss-Seidel, Jacobi, and SOR methods, to solve these linear systems. The primary objectives are to evaluate the accuracy, speed, and stability of each method, offering insights into their practical applications in option pricing.