HW1_Statistical Machine Learning

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Question 1 (3.7-5)

We have $\hat{y}_i = x_i\hat{\beta} = x_i\Big(\frac{\sum_{i'=1}^{n} x_{i'}y_{i'}}{\sum_{j=1}^{n} x_j^2}\Big)$ so that $\hat{y}_i = \sum_{i'=1}^{n}\Big(\frac{x_i x_{i'}}{\sum_{j=1}^{n} x_j^2}\Big)y_i'$, therefore

$$a_i' = \frac{x_i x_i'}{\sum_{j=1}^{n} x_j^2}.$$

Question 2 (3.7-6)

We have the least-square linear regression as $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$, we plug in $x = \bar{x}$ and $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x}$ then we have

$$\hat{y} = \bar{y} - \hat{\beta}_1\bar{x} + \hat{\beta}_1\bar{x} = \bar{y}.$$

Therefore, the point $(\bar{x}, \bar{y})$ must be on the least-square line.