Exercise:PCA in 2D

From Ufldl

Contents

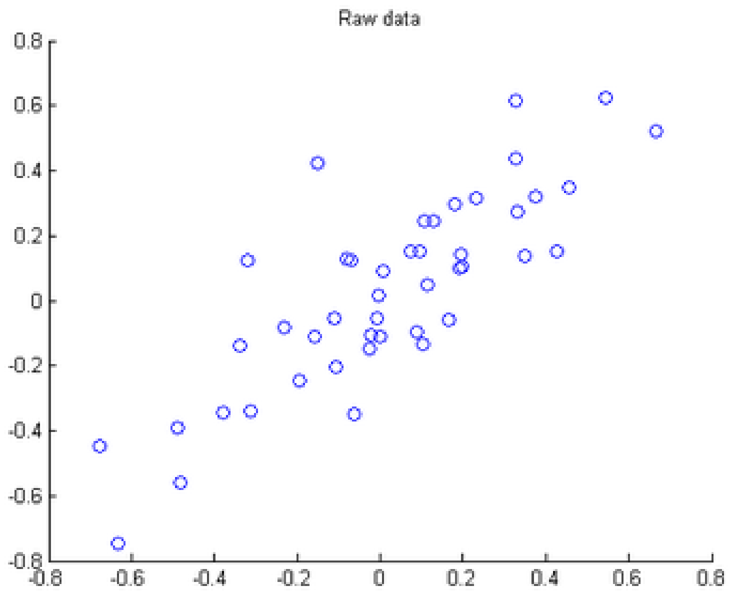
- 1 PCA, PCA whitening and ZCA whitening in 2D
  - 1.1 Step 0: Load data
  - 1.2 Step 1: Implement PCA
    - 1.2.1 Step 1a: Finding the PCA basis
    - 1.2.2 Step 1b: Check xRot
  - 1.3 Step 2: Dimension reduce and replot
  - 1.4 Step 3: PCA Whitening
  - 1.5 Step 4: ZCA Whitening

PCA, PCA whitening and ZCA whitening in 2D

In this exercise you will implement PCA, PCA whitening and ZCA whitening, as described in the earlier sections of this tutorial, and generate the images shown in the earlier sections yourself. You will build on the starter code that has been provided at pca_2d.zip (http://ufldl.stanford.edu/wiki/resources/pca_2d.zip). You need only write code at the places indicated by "YOUR CODE HERE" in the files. The only file you need to modify is `pca_2d.m`. Implementing this exercise will make the next exercise significantly easier to understand and complete.

Step 0: Load data

The starter code contains code to load 45 2D data points. When plotted using the `scatter` function, the results should look like the following:

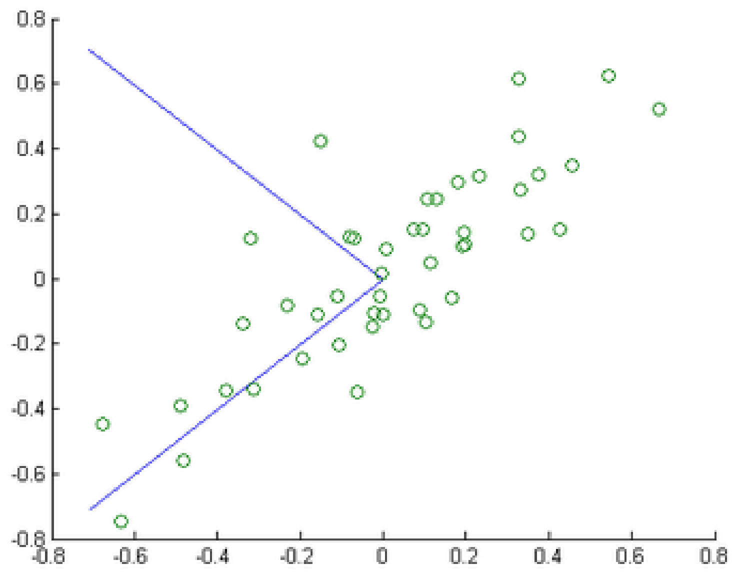


Step 1: Implement PCA

In this step, you will implement PCA to obtain x_{rot}, the matrix in which the data is "rotated" to the basis comprising $u_1, \ldots, u_n$ made up of the principal components. As mentioned in the implementation notes, you should make use of MATLAB's `svd` function here.

Step 1a: Finding the PCA basis

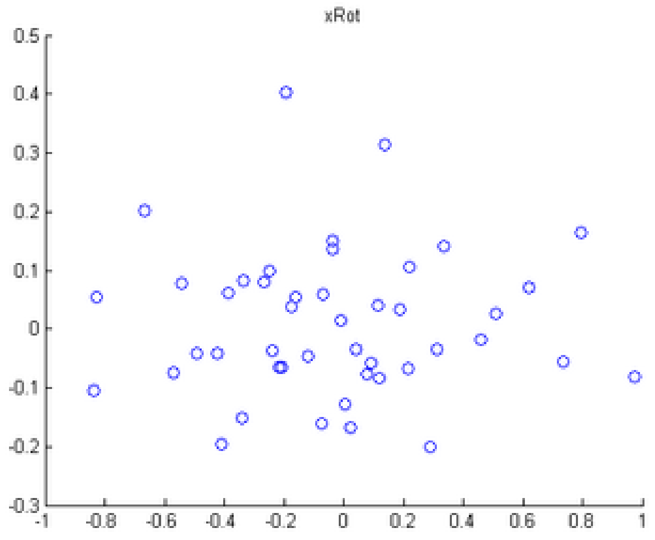
Find u_1 and u_2, and draw two lines in your figure to show the resulting basis on top of the given data points. You may find it useful to use MATLAB's `hold on` and `hold off` functions. (After calling `hold on`, plotting functions such as `plot` will draw the new data on top of the previously existing figure rather than erasing and replacing it; and `hold off` turns this off.) You can use `plot([x1,x2], [y1,y2], '-')` to draw a line between `(x1,y1)` and `(x2,y2)`. Your figure should look like this:



If you are doing this in Matlab, you will probably get a plot that's identical to ours. However, eigenvectors are defined only up to a sign. I.e., instead of returning u_1 as the first eigenvector, Matlab/Octave could just as easily have returned $-u_1$, and similarly instead of u_2 Matlab/Octave could have returned $-u_2$. So if you wound up with one or both of the eigenvectors pointing in a direction opposite (180 degrees difference) from what's shown above, that's okay too.

Step 1b: Check xRot

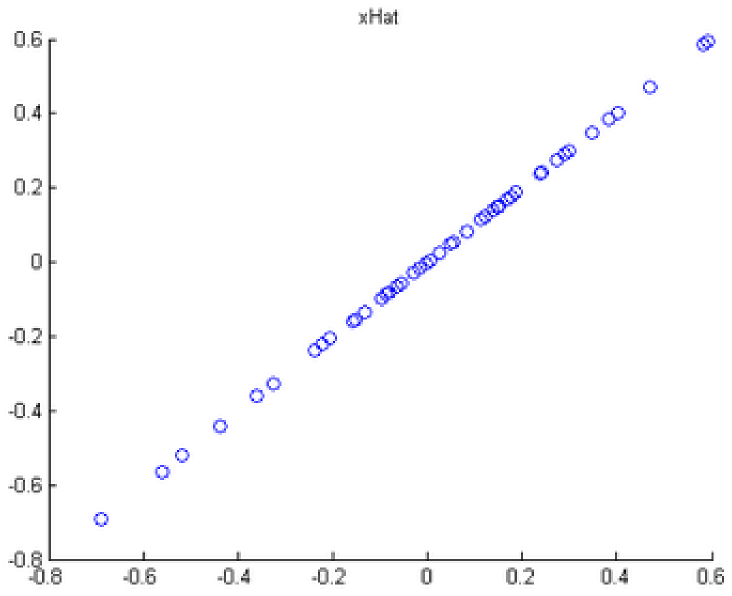
Compute `xRot`, and use the `scatter` function to check that `xRot` looks as it should, which should be something like the following:



Because Matlab/Octave could have returned $-u_1$ and/or $-u_2$ instead of u_1 and u_2, it's also possible that you might have gotten a figure which is "flipped" or "reflected" along the x- and/or y-axis; a flipped/reflected version of this figure is also a completely correct result.

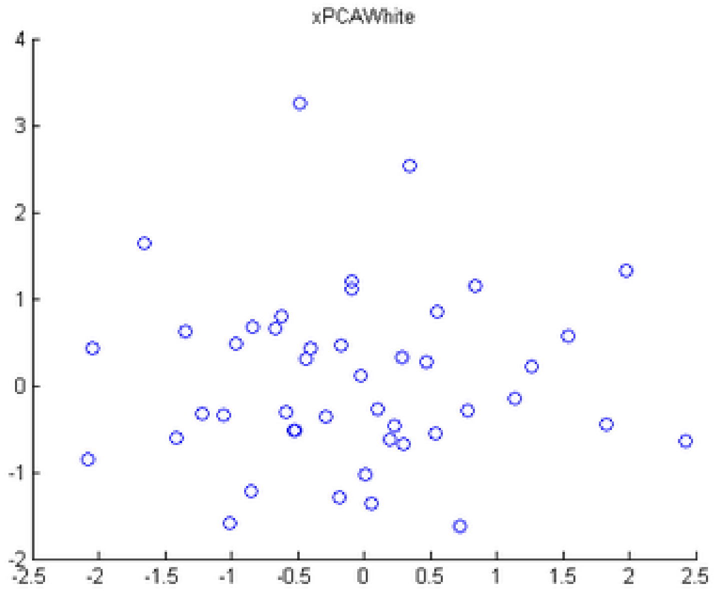
Step 2: Dimension reduce and replot

In the next step, set k, the number of components to retain, to be 1 (we have already done this for you). Compute the resulting `xHat` and plot the results. You should get the following (this figure should **not** be flipped along the x- or y-axis):



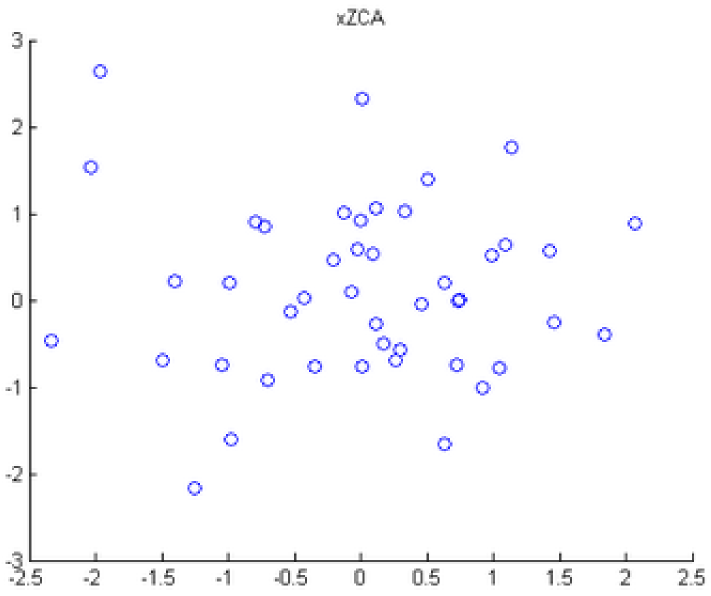
Step 3: PCA Whitening

Implement PCA whitening using the formula from the notes. Plot `xPCAWhite`, and verify that it looks like the following (a figure that is flipped/reflected on either/both axes is also correct):



Step 4: ZCA Whitening

Implement ZCA whitening and plot the results. The results should look like the following (this should not be flipped/reflected along the x- or y-axis):



PCA | Whitening | Implementing PCA/Whitening | **Exercise:PCA in 2D** | Exercise:PCA and Whitening