

Analysis of two-wheeled robot morphology for a slope environment

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Abstract—This paper considers a mathematical and simulated analysis of the Cylindabot, a robot design with an adaptive morphology with minimal actuation. The paper compares simulation against a mathematical model to both understand the dynamics of such a robot design and also provide a mathematical foundation for future designs.

I. INTRODUCTION

Two-wheeled robots are a common form of robot design due to their minimal number of actuators. A more specialised type is discussed in this paper, having large wheels that encompass the main body of the robot. The consequences of this are that the robot will be able to survive falls, handle bumps that are large compared its size and never be the wrong way up due to symmetry. Several similar robots have been previously [1], [2], [3] and [4]. In [4] the robot was able to create a mathematical model of step clearance.

In this work a set of mathematical inequalities are derived and values are then substituted into these equations to give theoretical results. Finally they are compared with results from V-rep simulations [5]. These mathematical models help confirm results from simulation, give a theoretical derivation and can be quickly adapted for different robots.

II. INTERNALLY WEIGHTED WHEELS

For continuous climbing of a slope, the centre of mass has to be further forward than the point of contact (Fig 1). Once the robot is rolling up the slope, the motors will keep the mass at this point so it will continue to climb.

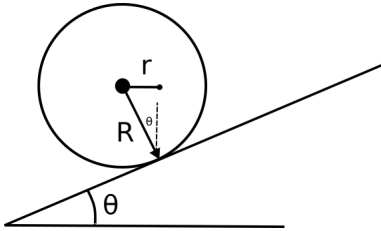


Fig. 1. Wheel on slope to show balance point

$$\frac{r}{R} > \sin\theta \quad (1)$$

In eqn. (1) r = radius of center of mass and R = radius of wheel. If this condition holds, the robot will be able to climb that slope.

III. TAILED TWO-WHEELED ROBOT

The addition of a tail allows the robot to apply more torque through its wheels but creates an extra drag behind the robot. This extra point of contact makes the robot statically balanced and the position the centre of mass less important. The mass of the robot M_t is split between M (main body) and m (tail).

For the robot moving on the flat, assuming the robot needs only to maintain a steady speed, then the friction on the wheel simply needs to be bigger than the drag of the tail.

$$\mu_1 M > \mu_2 m \quad (2)$$

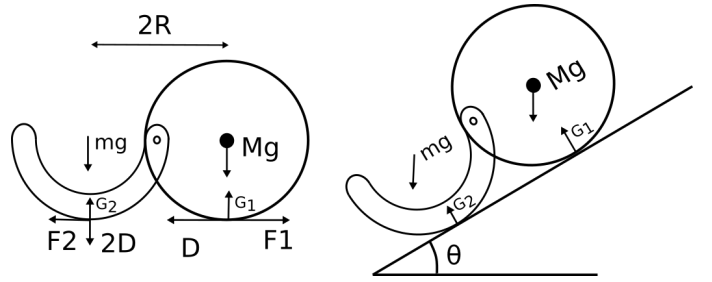


Fig. 2. Diagram of forces applied to tailed robot. D = drive force of wheels, G = ground reaction, F = friction

The moments around the two points of contact on the slope are used to calculate the dynamic requirement. Taking moments around G_1 (Fig 2) and then resolving forces the follow inequality can be derived. Here γ to represent $\tan^{-1}(0.5)$.

$$0 < \mu_1(M + m)g \cos \theta - (\mu_1 + \mu_2)G_2 - (M + m)g \sin \theta \quad (3)$$

$$\text{where: } G_2 = \frac{Mg \sin \theta + \sqrt{5}mg \cos(\theta - \gamma)}{2} \quad (4)$$

The final inequality (eqn. 3) assumes that it is desirable for the wheel not to slip. The drive force is set to zero to find the maximum angle that the wheel could grip while maintaining speed.

IV. MATHEMATICAL PREDICTIONS

The inequalities above were derived from sound theory but to obtain appropriate results, realistic parameters need to be substituted into them (listed in Table I). The most important being radius of the centre of mass (r) and frictional coefficient (μ). They are given two possible values each to inform possible designs.

TABLE I
PARAMETER TO SUBSTITUTE INTO INEQUALITIES

Term	Value
R	150mm
r	$\frac{R}{2}$ or $\frac{R}{4}$
$\mu_1 \mu_2$	0.5 or 1
m	$0.1M_t$

Firstly, consider the friction required to stop the robot sliding down the slope ignoring drive forces. Coulomb friction acts as a second requirement that must be satisfied in eqn. 5.

$$\tan \theta < \mu \quad (5)$$

$$\begin{aligned} \theta &< 26.57^\circ (\mu = 0.5) \\ \theta &< 40^\circ (\mu = 1) \end{aligned}$$

Without a tail the inequality (eqn. 1) can be used with the two centres of mass:

$$\begin{aligned} 0.5 > \sin \theta & \quad 0.25 > \sin \theta \\ \theta < 30^\circ & \quad \theta < 14.48^\circ \end{aligned} \quad (6)$$

The first of these would not be possible with $\mu = 0.5$ but is a theoretical limit for this design if the frictional coefficient was increased.

With a tail the inequality (eqn. 3) can be used as $D \rightarrow 0$:

$$\begin{aligned} \tan \theta &< \frac{4}{15} \\ \theta &< 14.9^\circ \end{aligned} \quad (7)$$

Recalculated with $\mu = 1$, this gives an angle of $\theta = 21.8$ with a tail.

V. SIMULATION RESULTS

In this section these mathematical models are compared with a similar set up created in V-rep simulations. In simulation, the slope varies from 0 to 45 degrees in increments of one with 10 runs at each angle. This means that there are 460 runs for each result given in Table II. The results are estimated by adding together the number of successful runs and using that to calculate the middle of the region where the robot fails. Two offset masses are used to set the centre of mass to different locations in Fig 3.

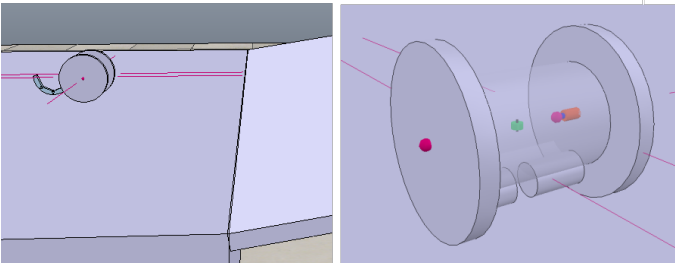


Fig. 3. Simulation setup. Left: the angle of the slope was varied and the robot with tail deployed Right: the robot with offset mass and no tail.

A. Comparison of Results

The results from Table II match well to the mathematical predictions. The mathematical predictions are an upper limit and therefore having results being just below the predictions is to be expected.

TABLE II
SLOPES - SIMULATED VERSUS MATHEMATICAL PREDICTIONS

Centre of Mass	μ	Target Speed (rad/s)			Mathematical Prediction (deg)
		1	2	3	
		Max Slope Angle (deg)			
Quarter	1	10	13	14.1	14.8
	0.5	10	13	14	14.8
Half	1	20	28	29	30
	0.5	20	26	26	26.6
Tail	1	20.9	21.9	21.6	21.8
	0.5	13.5	13.8	13.7	14.9

The speed of the robot was input into the V-rep simulation as a value of intrinsic target velocity. A proportional controller was used when the tail was not present to balance the robot. It was tuned to a target speed of 3 rad/s hence the lower speeds were not as close to the prediction.

The results for the tail were less affected by the target speed of the robot, this could be because the tail allows the movement to be more stable. With the results matching it is promising that they may translate to the real world, and be useful in simulation-based path-planning models.

VI. CONCLUSION

In this work a mathematical analysis of an internally weighted two wheeled robot, with and without a tail, was undertaken. The results from these calculations were compared to results from V-Rep simulations. Three factors were varied so that a comparison could be made. Firstly how the robot design is balanced, where the centre of mass is or whether it has a tail. The second was the frictional coefficient of wheels and finally the intrinsic target velocity. A similar approach was used for steps, however the results did not line up with predictions. The results from slope mathematical predictions are encouragingly accurate and could applied to help plan routes in unstructured terrain.

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