Lab 1: exponential growth

NRES 470/670

Spring 2019

In this lab we will have an opportunity to do some simple population modeling using three different software packages: MS Excel, R, and InsightMaker. I encourage you to work in groups!

Nomenclature for Population Ecology

First of all, we need a symbol to represent the population size. This is N!

ΔN represents the change in population size, $N_{t+1} - N_t$

The famous "BIDE" equation is a way to break down ΔN into components.

$\Delta N = B + I - D - E$ (Eq. 1)

where B represents the number of births, I reprents the number of immigrants, D represents the number of deaths, and E represents the number of emigrants.

If we ignore immigration and emigratrion, then the BIDE equation simplifies to:

$\Delta N = B - D$ (Eq. 2)

Now let's focus on B and D. The important thing to recognize is that the number of births and deaths in a population is not constant.

What does the number of births depend on?

What **is** more likely to be constant is the per-capita rate of producing offspring, or dying. Does this make sense? These per-capita rates are often expressed as lower case letters. So b represents per-capita births, and d represents per-capita deaths.

$b = \frac{B_t}{N_t}$ (Eq. 3)

–or–

$B_t = b \cdot N_t$

The letter t of course represents time, which could be years, or minutes, or decades, or whatever! So the above equation could be described as follows: "the number of births at a given time is equal to the per-capita birth rate times the total population size at that time"

Similarly,

$D_t = d \cdot N_t$ (Eq. 4)

Okay, we're almost there.

If $\Delta N = B - D$ (Eq. 5)

then

$\Delta N = b \cdot N_t - d \cdot N_t$ (Eq. 6)

which is equal to

$\Delta N = (b - d) \cdot N_t$ (Eq. 7)

which could also be written:

$\Delta N = r \cdot N_t$ (Eq. 8)

Where r represents the difference between the per-capita birth and death and death rates. If r is positive, then births are greater than deaths and the population grows. If r is negative then deaths exceed births and the population declines.

We can use *calculus notation* to consider the instantaneous change in population size:

$\frac{\partial N}{\partial t} = r \cdot N \qquad \text{(Eq. 9)}$

This is probably the most fundamental equation of population ecology.

If you integrate this equation, you get an equation that describes the population size at any time t:

$N_t = N_0 e^{rt} \qquad \text{(Eq. 10)}$

That is, population size at time t is equal to the population size at time zero (initial abundance) multiplied by the base of the natural logarithm, e to the rt power.

There you have it! Now you can compute population growth!

A couple more quick notes:

The greek symbol lambda (λ), is used to represent the finite rate of growth, or $\frac{N_{t+1}}{N_t}$. Lambda is what you multiply the current population size by to compute the population size in the next time step.

$N_{t+1} = N_t + B - D \qquad \text{(Eq. 11)}$

$N_{t+1} = N_t + b_d \cdot N_t - d_d \cdot N_t \qquad \text{(Eq. 12)}$

NOTE: I am using the d subscript to indicate that the per-capita birth and death rates now represent discrete growth (these rates only apply once per time step and are not compounded continuously)

$N_{t+1} = N_t + (b_d - d_d) \cdot N_t \qquad \text{(Eq. 13)}$

$N_{t+1} = N_t + r_d \cdot N_t \qquad \text{(Eq. 14)}$

$N_{t+1} = N_t \cdot (1 + r_d) \qquad \text{(Eq. 15)}$

$N_{t+1} = \lambda \cdot N_t \qquad \text{(Eq. 16)}$

NOTE:

Okay now that we got that over with, let's start the lab! The first software we will use is our old friend, MS **Excel**!

Exponential growth in Excel

1. Open the Excel spreadsheet ExpGrowthExcel.xlsx. To download this file, right click on the link and select "*Save link as..*". In the first column, we have a timestep of 1 year for 30 years. In the second column, we have an initial population size (N_0) of 100 individuals. We also have a per-capita rate of increase (r) that is currently set at 0.1 per year. Assume for now that r represents r_d, or the *discrete rate of increase*.
2. To generate N_t for the remaining time steps, we need to apply our knowledge of population ecology. Specifically we need to apply equation 15, above (assuming discrete population growth). Do this by clicking in the empty N(t=1) cell (position B3), typing '=', clicking on N(t=0), completing the equation and hitting enter. As you do this, you should see the equation you are creating appear in the "functions bar".
3. You can fill the remainder of the cells using the same equation for the other time steps by clicking and dragging the small square at the bottom of the N(2) cell that appears when the cell is selected.
4. What happened? We are not seeing a growing population here- actually it seems quite flat! this is surely not what we want! Click on the N(3) cell to see what equation is being used to calculate the cell value. The equation is B3*(1+D4). The B3 part is correct - we want to calculate the N(3) population

size using the N from the previous timestep - but the D4 part is incorrect. We always want to use the same r which is always in the same cell. You can see that when you drag down an equation as we have done, Excel adds 1 for each row so that the equation references the same relative positions in the spreadsheet for each new cell you want to calculate. We like that Excel did that for N, but not for r, so we can tell Excel to stay in the same row (row 3) for r by inserting a dollar sign in our equation.

5. In the N(2) cell, edit the equation in the "functions bar" so that there is a dollar sign in D3 (i.e., 'D$3' instead of 'D3') (or just use the F4 shortcut).
6. Now drag the equation down again, and you should have a population size in row 32 of 1744.94 (representing the population size at year 30!).
7. Now we will plot our population against time. Select both columns of data, and select the *scatter plot* (or "line plot") option under the 'Insert' menu. A plot of N by Time will automatically appear. You can change the r value, the data and chart will automatically adjust.

Exercise 1 (Excel-related problems)

Please provide short answers to the following questions, and **provide your Excel spreadsheet to back up your answers**.

- ***Short answer (1a.)*** Apply equation 10 (above) to compute expected population size in year 30 (now you are assuming that the per-capita rate of growth in cell D3 represents *continuous* and not *discrete* growth). Do you get the same answer? Why or why not? What about year 100?
- ***Short answer (1b.)*** What are the *units* of the per-capita rate of population growth, r?
- ***Short answer (1c.)*** What if the time step for your simulation were in months instead of years? How Would r change (i.e., assuming r represents 0.1 growth per year, what is the growth per month)? Try it in Excel! Use both methods (eq. 15 vs equation 10) to compute population size in year 30. (NOTE: when you are applying equation 10, you should now use the monthly rate instead of the annual growth rate, and you should use months and not years to represent t) Do you see any difference in final population size between the two methods (discrete vs continuous growth)? Is the difference greater or less than it was with a 1-year time step? Why or why not?
- ***Short answer (1d.)*** What is the difference between continuous population growth and discrete population growth? Can you think of at least one case where continuous growth would be a more biologically realistic model than discrete growth? Can you think of at least one case where discrete growth would be a more biologically realistic model than continuous growth? Explain.

Exponential growth in R

R is probably the most common software used by modern ecologists and conservation biologists for data analysis and simulation. There is a little bit of a learning curve with R, and I appreciate InsightMaker in many ways for making it easy to get started with programming and modeling, but R is much more powerful and widely used than InsightMaker. For that reason, I will try to integrate R into this class as much as I can. We will do more with R when we get into data analysis!

SET UP

Open the R software from the program menu or desktop.

PROCEDURE

STEP I: Set up R!

Go to website http://cran.r-project.org/. This is the source for the free, public-domain R software and where you can access R packages, find help, access the user community, etc.

STEP II. Take some time to get familiar with R

Take a quick look at the R manual, Introduction to R. To jump into the deep end of the pool, try to implement the steps in Appendix A, located here. You might not understand everything right away, but you have the link, so you can return to this!

If you already have basic R expertise, this is your opportunity to help your peers develop the level of comfort and familiarity with R that they will need to perform data analysis and programming tasks in this course.

Depending on whether you are already familiar with R, you may also find the remainder of this document useful as you work your way through the course (and there are many other good introductory R resources available online… let me know if there is one you particularly like and I will add it to the course website (Links page). As you work your way through this tutorial (on your own pace), please ask the instructor or your peers if you are uncertain about anything.

Set up the workspace

The first thing we usually do when we start an R session is we set up the workspace. That is, we load **packages**, assign key parameters and initialize variables.

In this case, setting up the workspace is easy. We just need to define our parameter of interest - r -, and set up a **vector** to represent the years of interest.

We can store data in memory by assigning it to an "object" using the assignment operator **<-**. For example, this would assign the object "x" the value of 5.

```
x <- 5      #Assign the value of 5 to the object "x"
x           #Print the value of the object "x"
```

Note that any text after a pound sign (#) is not evaluated by R. These are *comments* and are intended to help you follow the code. You should always include comments in any code that you write- we humans tend to read and understand written language better than computer code!

Let's assign our per-capita population growth rate, r (but this could be called anything), and our initial population size to an object called **N0** (that is, population size at time 0), and the number of years to simulate.

```
r <- 0.1      #Assign the value of 0.1 to the object "r", or per-capita growth rate
N0 <- 100     #Assign the value of 100 to the object "N0", or initial population size
nyears <- 50 #Assign the value of 50 to the object "nyears", or the number of time steps to simulate
```

If we want to know what the population size is at the next time step, we can simply multiply N0 by (1+r).

```
N0 * (1+r)    #Multiplies the value stored in the object "N0" by 1 plus the value stored in the object "
```

```
## [1] 110
```

How can we find the population size for the next 50 years? Let's first make an object that is a vector of years using the **seq()** or "sequence" function.

```r
years <- seq(from=0, to=nyears, by=1)   #Creates a sequence of numbers from 0 to the value stored in th
years                                   #Print the value of the object "years" that you just created.
```

```
##  [1]  0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22
## [24] 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45
## [47] 46 47 48 49 50
```

Now, let's build a storage structure to store simulated population size over this time period

```r
N <- numeric(nyears+1)    #Make an empty storage vector. The numeric() function takes the contents with
N                         #Prints the contents of the object "N".
```

```
##  [1] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## [36] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

Run the simulation!

Then we can use a **for loop** (a very powerful computer programming trick) to automatically generate the population size for each of those years (note the similarity in the equation inside the for loop to Expression 1.15 in Gotelli).

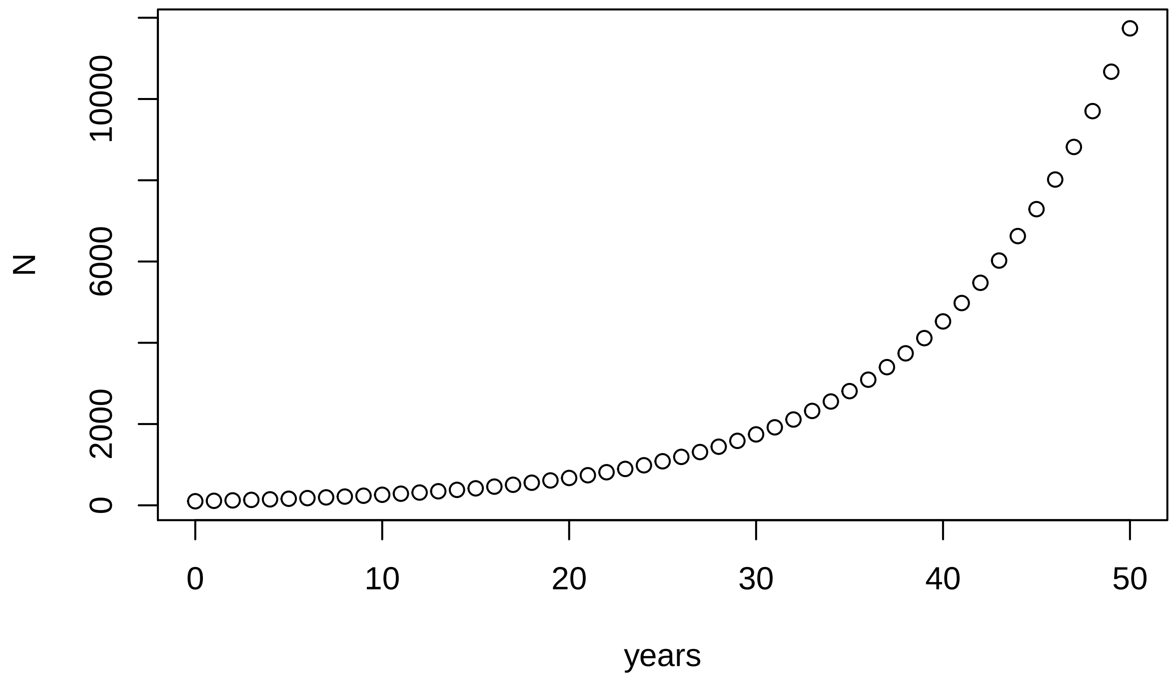
```r
N[1] <- N0                # The brackets [] are used to indicate the position of an element within a ve
lambda <- 1 + r           # (1 + r) is equal to lambda, the finite rate of growth.  This stores the res
for (i in 2:(nyears+1)){  # This for-loop will run through the line of code between the curly brackets
  N[i] <- lambda*N[i-1]   # This takes the [ith - 1] element of "N", multiplies that element by the val
}                         # This ends the for-loop.
N                         # Now print the contents of the object "N".
```

```
##  [1]   100.0000   110.0000   121.0000   133.1000   146.4100   161.0510
##  [7]   177.1561   194.8717   214.3589   235.7948   259.3742   285.3117
## [13]   313.8428   345.2271   379.7498   417.7248   459.4973   505.4470
## [19]   555.9917   611.5909   672.7500   740.0250   814.0275   895.4302
## [25]   984.9733  1083.4706  1191.8177  1310.9994  1442.0994  1586.3093
## [31]  1744.9402  1919.4342  2111.3777  2322.5154  2554.7670  2810.2437
## [37]  3091.2681  3400.3949  3740.4343  4114.4778  4525.9256  4978.5181
## [43]  5476.3699  6024.0069  6626.4076  7289.0484  8017.9532  8819.7485
## [49]  9701.7234 10671.8957 11739.0853
```

Plotting

Let's plot our population size against time.

```r
plot(N~years)   #This plot() function tells R to plot the y variable by the x variable. "N" is the y va
```



Exercise 2 (R-related problems)

Please provide short answers to the following questions, and **provide your R code to back up your answers**.

- ***Short answer (2a.)*** Tweak the above code to run for 100 years. Plot your results. What is the final population size?
- ***Short answer (2b.)*** Change r to 0.5 and run again for 100 years (and plot the results). What is the final population size now?
- ***Short answer (2c.)*** Try to tweak the value of r such that the final population size after 100 years is 1000. What is the value of r? After you solve it by trial and error, can you solve this problem analytically using Eq. 10 above? Show your calculations!
- ***Short answer (2d.)*** Change the value of r to -0.1. How long until the population goes extinct? Explain your answer.

Exponential growth in InsightMaker

You should already have created a (free) account in insightmaker, and you should already know the basics about how to set up and run a model.

1. Click "Create New Insight" to start a new model (click "Clear this Demo" to clear the canvas and have an open workspace). Save the blank model by clicking the "Save" button.

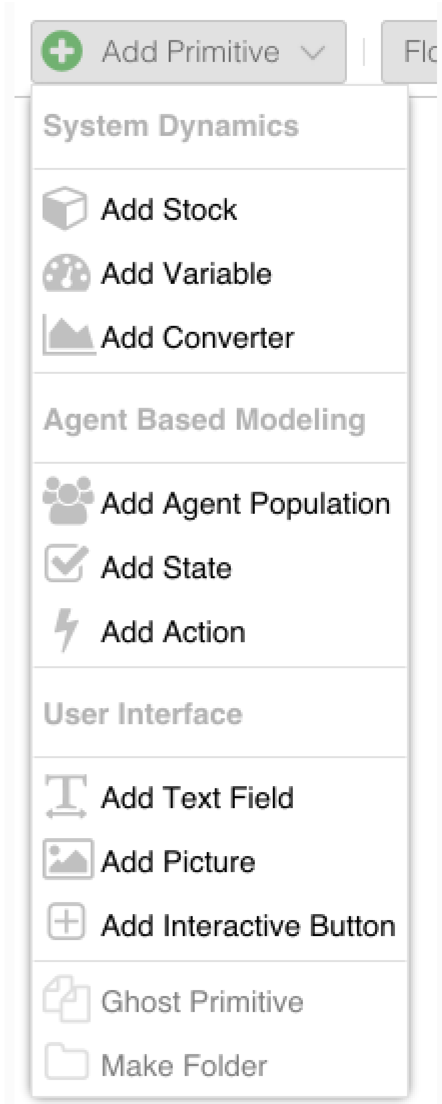


Figure 1:

2. Create a new [Stock] named *Population* using the "**Add Primitive**" button at top left ("Primitive" is just a computer-sciencey term referring to basic building blocks of a computer programming language). You can name the [Stock] and configure it in the properties tab at the right.
3. Change the Initial Value of *Population* to 100.
4. Create a new [Flow] going from empty space to the primitive *Population* (make sure the **Flow/Transitions** button is activated instead of **Links** at the top, hover over *Population* until an arrow appears, click and drag to create the [Flow], use the **Reverse Connection Direction** button to change the flow direction). Name the flow *Births*.
5. Create a new [Flow] going from *Population* to empty space. Name the flow *Deaths*.
6. The model diagram should now look something like this:
7. Change the **Flow Rate** property of *Births* to 0.16 ∗ [Population]. This represents the total number of individuals entering [Population] in each time step.

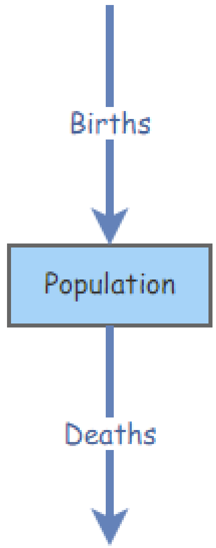


Figure 2:

8. Change the **Flow Rate** property of *Deaths* to 0.10 ∗ [Population]. This represents the total number of individuals leaving [Population] in each time step.

Can you already tell whether this is a growing or declining population? (just a quick thought question, not part of the written lab!)

9. Run the model by clicking the **Simulate** button. We can change how the simulation is run by clicking the **Settings** button (left of Save). We can also change the settings of how the plot is created by clicking the **Configure** button within the simulation results window.

Exercise 3 (InsightMaker problems)

Please provide short answers to the following questions, and (when prompted) **provide your "Insights" to back up your answers** (easiest would just be to save them as "public insights" and share them with your instructors via email- we will go over this in class).

First, tweak the above model so that per-capita (discrete) birth rate (b_d) and death rate (d_d) are separate elements of the model (using the "Variable" primitive). Your Insight should look something like this:

To enable easy manipulation of these variables, change the **Show Value** Slider option of *Birth Rate* (in the properties window) to Yes. Change the **Slider Max value** to 1, the **Slider Min value** to 0, and the **Slider Step value** to 0.01. Do the same for *Death Rate* and *Population* (initial abundance N_0). For abundance, set the step size to 1 so we don't have fractional individuals! Now click on the white space of your model; you should now see the Birth Rate, Population and Death Rate sliders on the info tab. Change the slider values of the rates a few times, re-running the simulation each time. When you are confident that your model is working right, share it with your instructor and TA (add public link to your Word document).

- ***Short answer (3a.)*** Starting with a growing population, can you come up with two different scenarios in which *Population* is neither growing nor declining, by only changing one of the sliders from the starting conditions? Explain your answer.

Clone your previous Insight before you move on to the next problem (otherwise any changes you make will carry over to your answer to the previous problem!). In general, always clone your Insights after you have copied a link to an insight into your lab write-up.

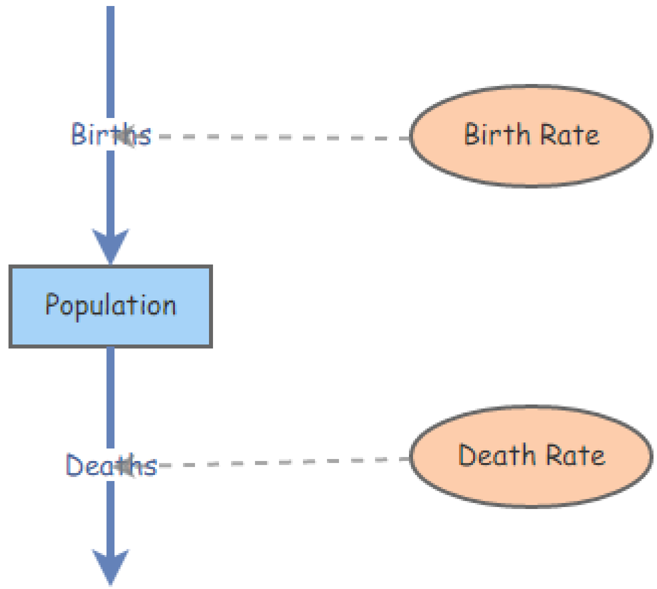


Figure 3:

The simulations that you ran above produced population growth curves that were very smooth, but we all know that populations don't grow in this manner because of **stochasticity** (randomness). Let's add some randomness to our vital rates! Change the **Show Value Slider** option of the Birth Rate primitive to No. Change the Equation to 0.1 + RandNormal(0, 0.1). The RandNormal() function generates a normally distributed random number with *mean (average) equal to the first argument* and *standard deviation equal to the second argument*. This means that in each time step we have a slightly different birth rate. Since we specified a *mean* of zero, we are not actually changing the average birth rate! Do the same for *Death Rate*. Run multiple simulations (at least 10) and look for patterns. Use the **Compare Results** tool to compare and provide output. When you are confident the model is right, add a link to your new InsightMaker model to your instructor and TA (add public link to your Word document).

- ***Short answer (3b.)*** Is it possible to have a declining population even when *Birth Rate* and *Death Rate* are the same on average? Is it possible to have a declining population when *Birth Rate* is greater than *Death Rate* on average? Explain your reasoning!

Checklist for Lab 1 completion

*Please bundle the Word, Excel, and R code files into one Zip file and submit via WebCampus. The InsightMaker models should be shared by saving your Insights as "public" and sharing the URL link with your instructors (these URLs should be embedded in your Word document).

Due Feb. 2 before lab

- Word document with short answers
  - **Exercise 1**
    - *Short answer (1a.)*
    - *Short answer (1b.)*
    - *Short answer (1c.)*
    - *Short answer (1d.)*
  - **Exercise 2**
    - *Short answer (2a.)*

    - *Short answer (2b.)*
    - *Short answer (2c.)*
    - *Short answer (2d.)*
  - **Exercise 3**
    - *Short answer (3a.)*
    - *Short answer (3b.)*
- Excel file
  - **Exercise 1**
    - Your Excel file should show that you were able to successfully use formulas to calculate N_t for each time step (year and month) and show a plot of N by Time.
- R file
  - **Exercise 2**
    - Your R code should show that you were able to (a) adapt the given code to run for 100 years, and can display a plot of the results;
    - (b) change r to 0.5 and run for 100 years and plot the results;
    - (c) identify a value of r that gives a population size of 1000 after 100 years; and
    - (d) change r to -0.1 and run until the population goes extinct- and plot the results.
- InsightMaker models
  - **Exercise 3**
    - Your first model should show that you were able to alter the given model as instructed so that the simulation runs correctly. This should be shared with Kevin and Danielle (embed InsightMaker link in your Word document) .
    - Your second model should include stochasticity in *Birth Rate* and *Death Rate*. This should be shared with Kevin and Danielle (embed InsightMaker link in your Word document).