

# Elements of Ecology

Seventh Edition



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## Chapter 9

### Population Growth

Lecture prepared by  
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# Chapter 10 Population Growth

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- **Population growth** refers to how the number of individuals in a population increases or decreases with time
  - Individuals added via birth and **immigration**
  - Individuals removed via death and **emigration**
- Immigration and emigration occur in **open populations** but not in **closed populations**

## 10.1 Population Growth Reflects the Difference between Rates of Birth and Death

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- A (closed) population of a freshwater hydra
  - Will increase as a result of new “births” (budding, a form of asexual reproduction)
  - Will decrease as a result of some hydra death
- Birth and death are continuous
  - $b$  = the proportion of hydra producing a new individual per unit time
  - $d$  = the proportion of hydra dying



## 10.1 Population Growth Reflects the Difference between Rates of Birth and Death

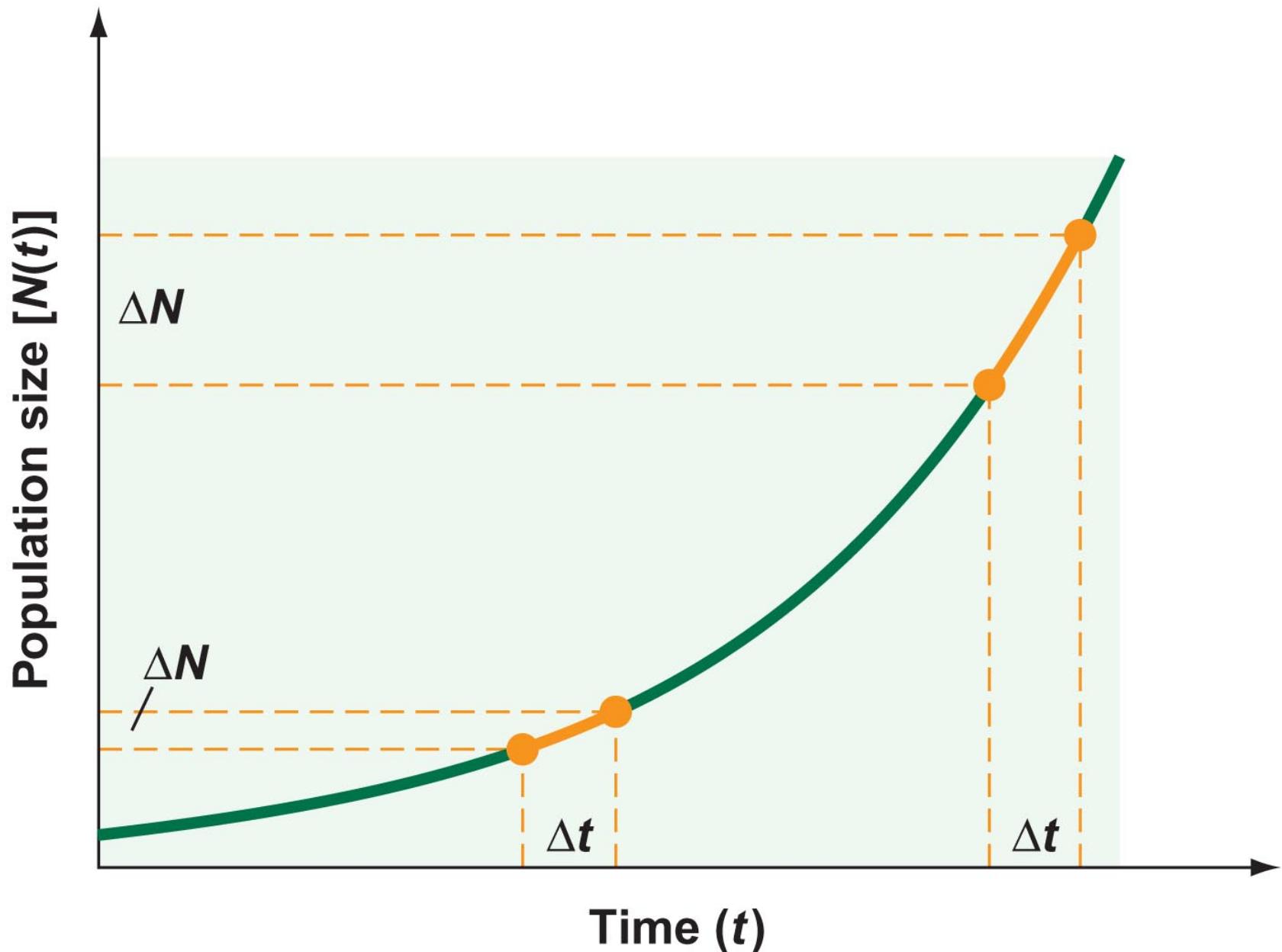
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- Population size at a particular time =  $N(t)$
- The number of hydra reproducing [ $B(t)$ ] or dying [ $D(t)$ ] over a particular time period ( $\Delta t$ ) can be calculated
  - $B(t) = bN(t)\Delta t$
  - $D(t) = dN(t)\Delta t$
- The population size ( $N$ ) at the next time period ( $t + \Delta t$ ) would be
  - $N(t + \Delta t) = N(t) + B(t) + D(t)$

## 10.1 Population Growth Reflects the Difference between Rates of Birth and Death

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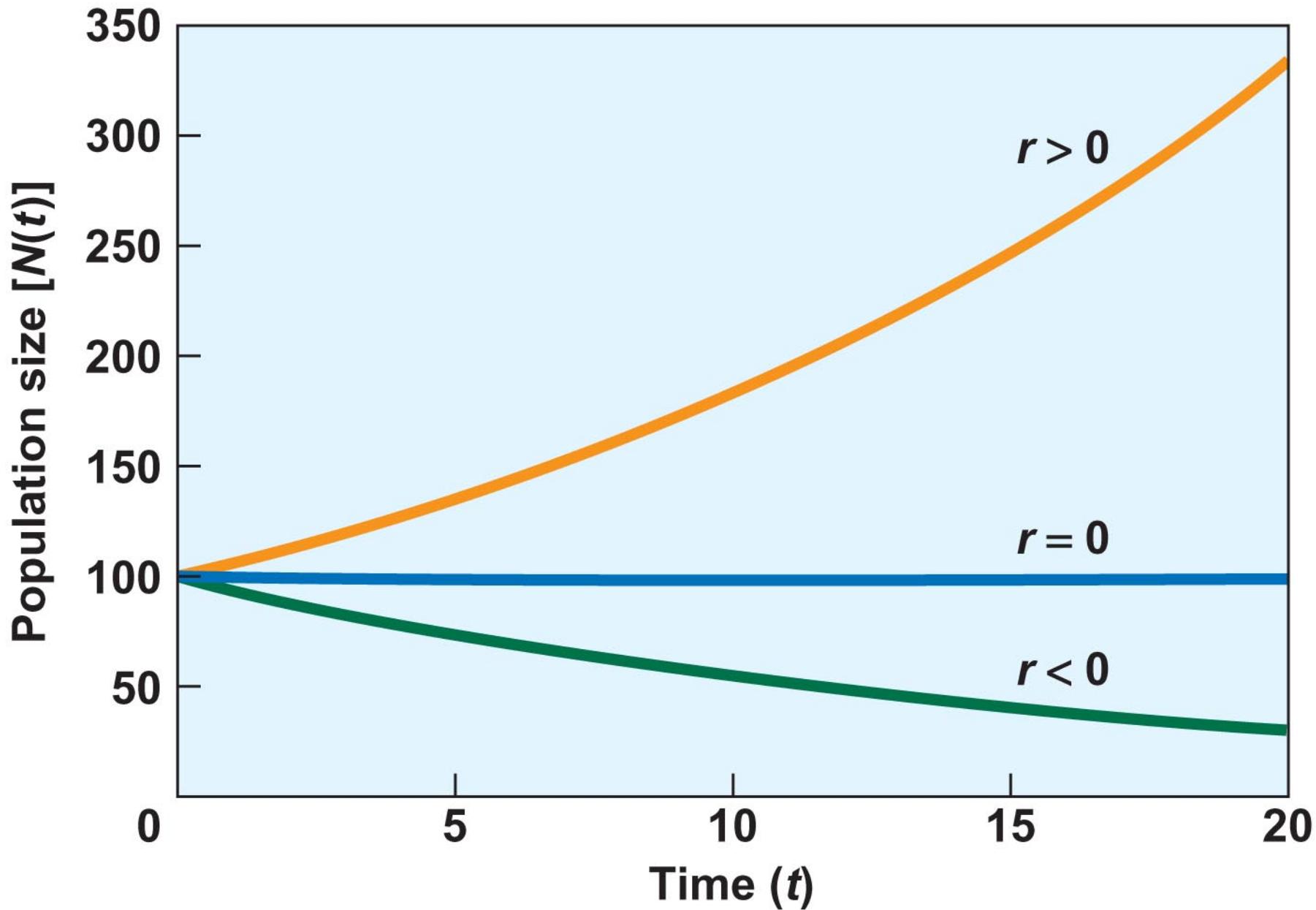
- The pattern of population size is a function of time
- Rearranging the equation:
  - $N(t + \Delta t) - N(t)/\Delta t = \Delta N/\Delta t = (b - d)N(t)$
- The relationship (slope) between  $N(t)$  and  $t$  is nonlinear (curve)



## 10.1 Population Growth Reflects the Difference between Rates of Birth and Death

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- Rate of change is best described by the derivative of the equation =  $dN/dt = (b - d)N$ 
  - This derivative expresses that as the  $\Delta t$  approaches zero and the rate of change is instantaneous
- $r = (b - d)$  = **instantaneous** (per capita) rates of birth and death (growth)
- **Exponential population growth** =  $dN/dt = rN$ 
  - Predicts the *rate* of population change through time



## 10.1 Population Growth Reflects the Difference between Rates of Birth and Death

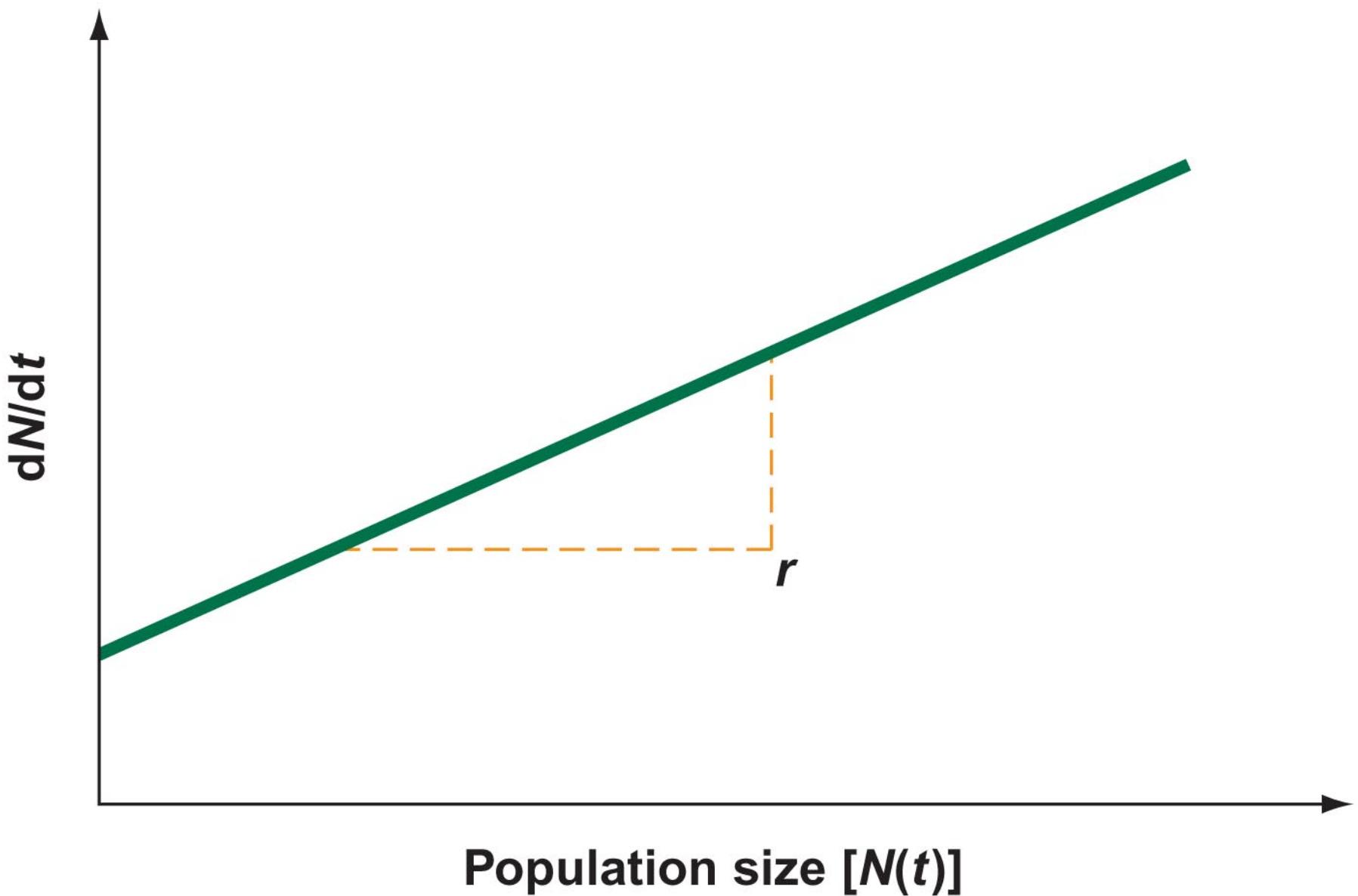
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- An alternate differential equation allows us to predict population size  $M(t)$  under conditions of exponential growth
  - $M(t) = M(0)e^{rt}$

## 10.1 Population Growth Reflects the Difference between Rates of Birth and Death

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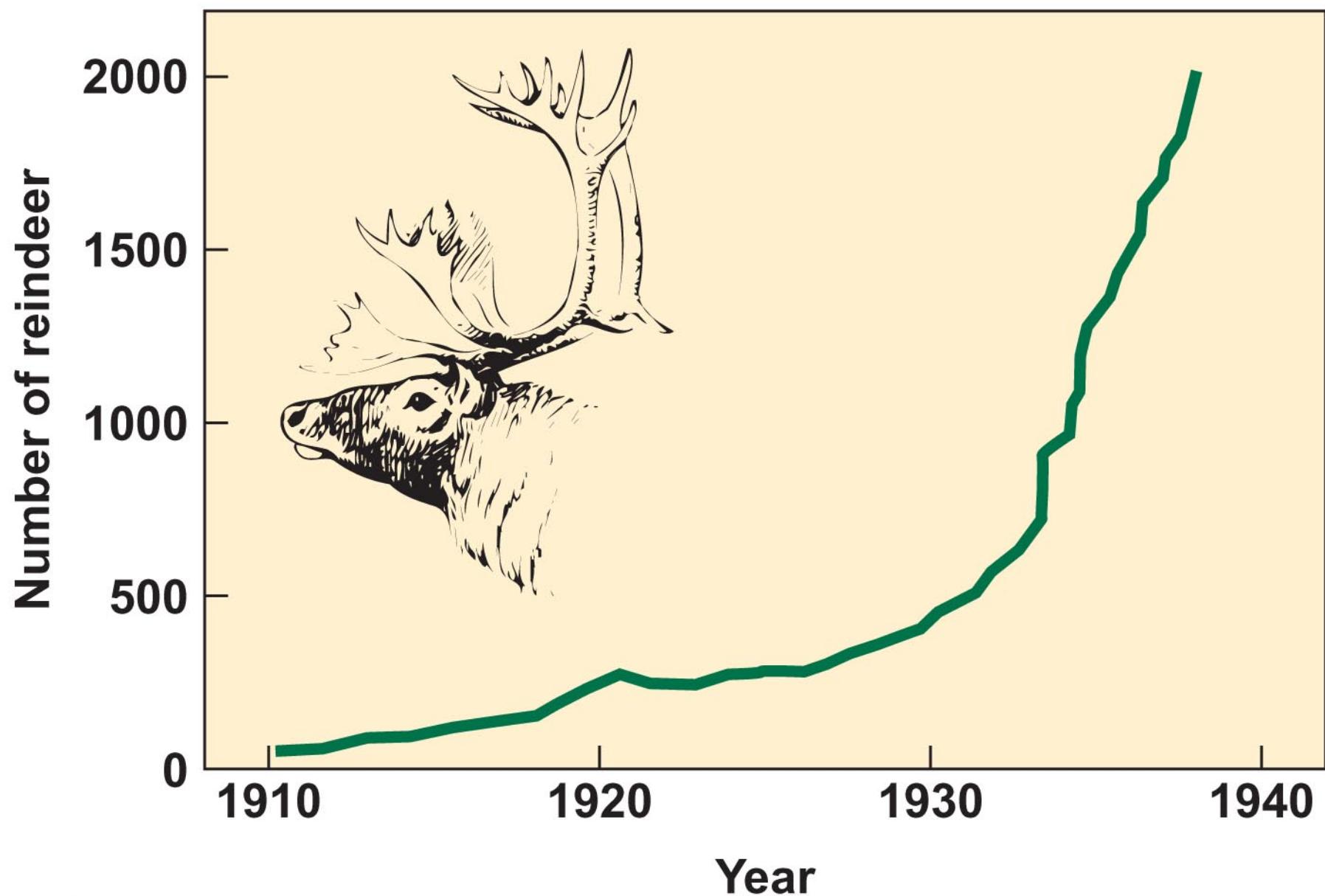
- Exponential growth rate
  - When  $r = 0$ , there is no change in population size
  - When  $r > 0$ , the population increases exponentially
  - When  $r < 0$ , the population decreases exponentially
- Exponential growth results in a continuously accelerating (or decelerating) rate of population increase (or decrease)



## 10.1 Population Growth Reflects the Difference between Rates of Birth and Death

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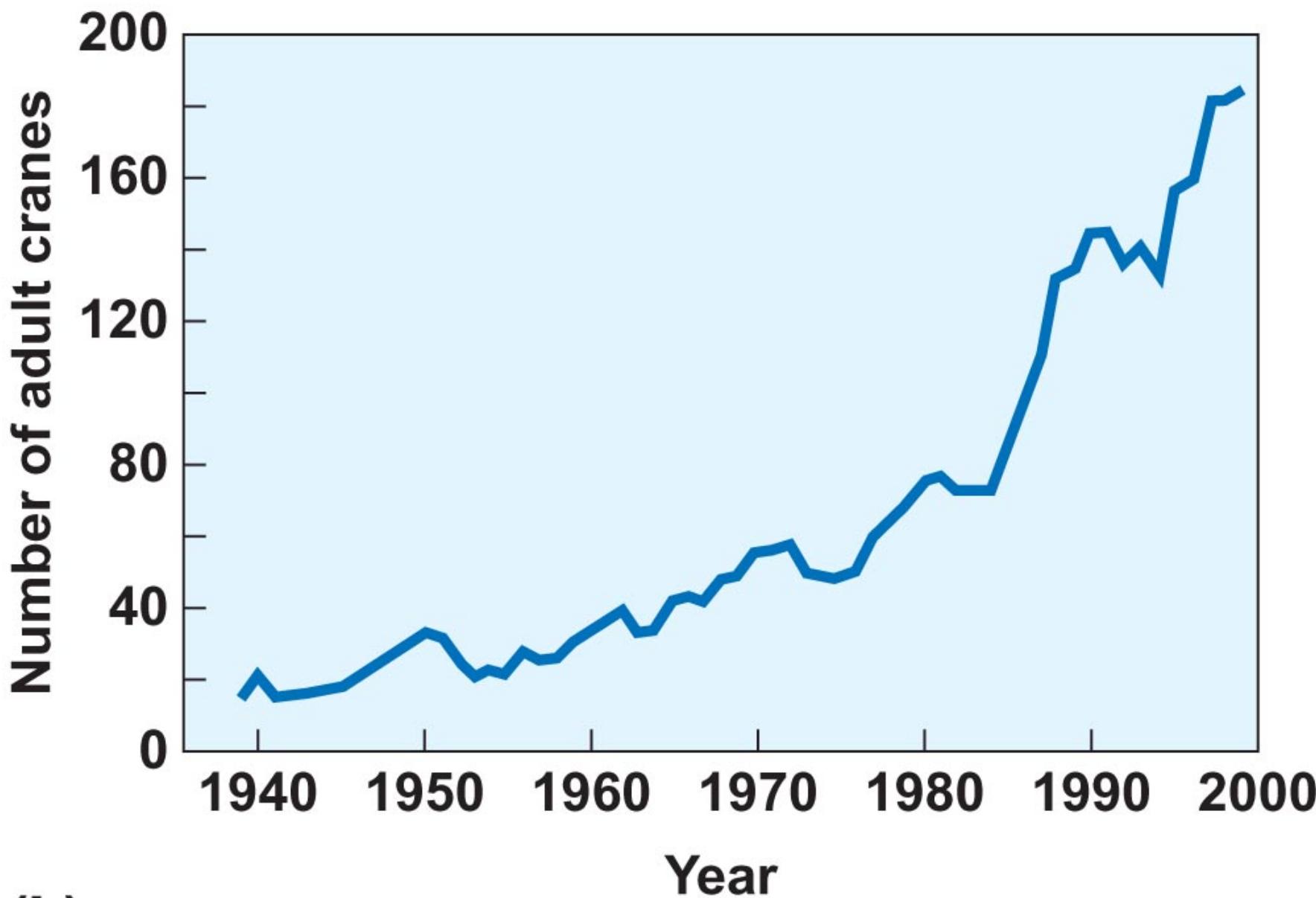
- Exponential growth rate is characteristic of populations that inhabit favorable conditions at low population densities (e.g., conditions of colonization)





(a)

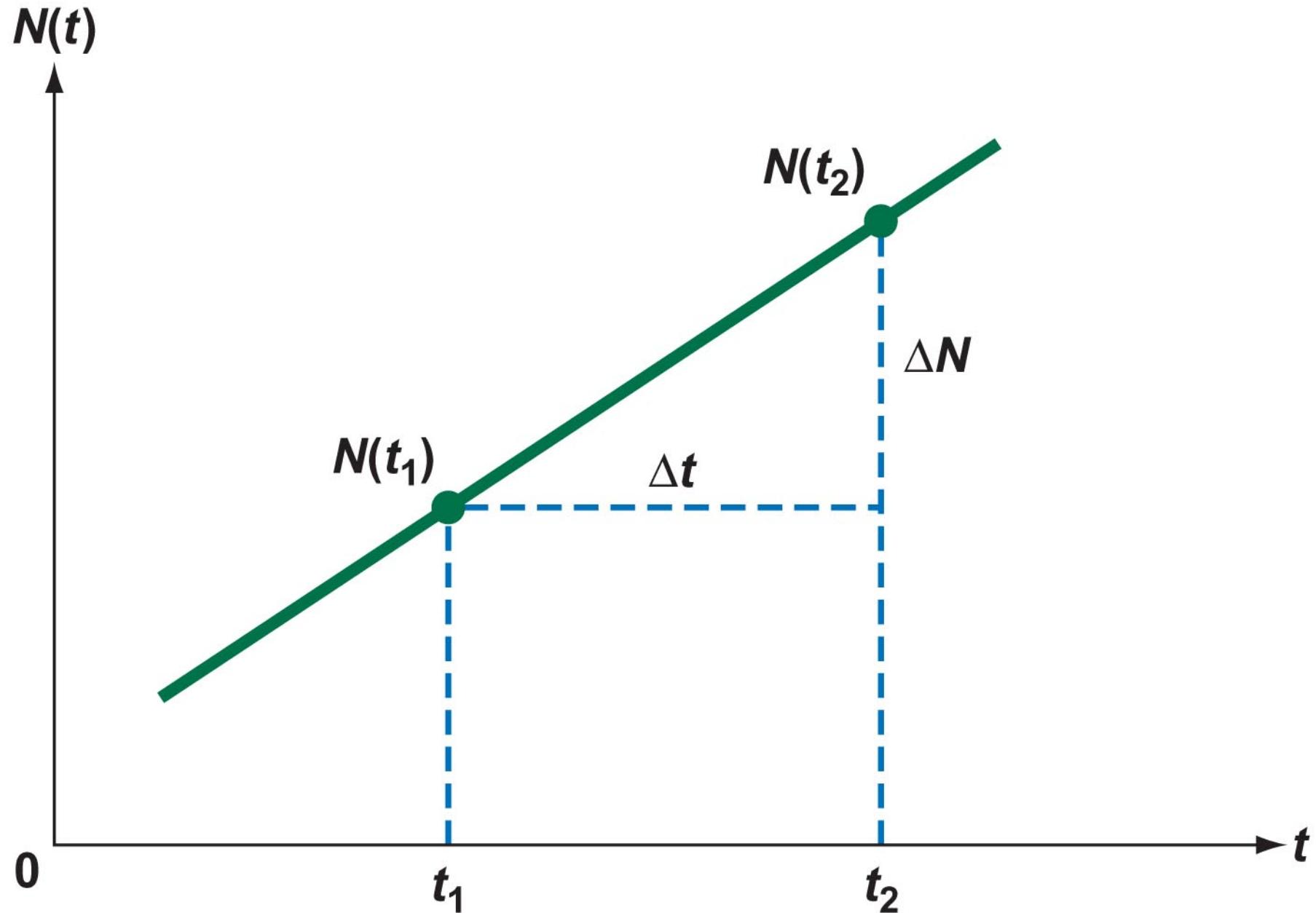
Cranes



# Quantifying Ecology 10.1 Derivatives and Differential Equations

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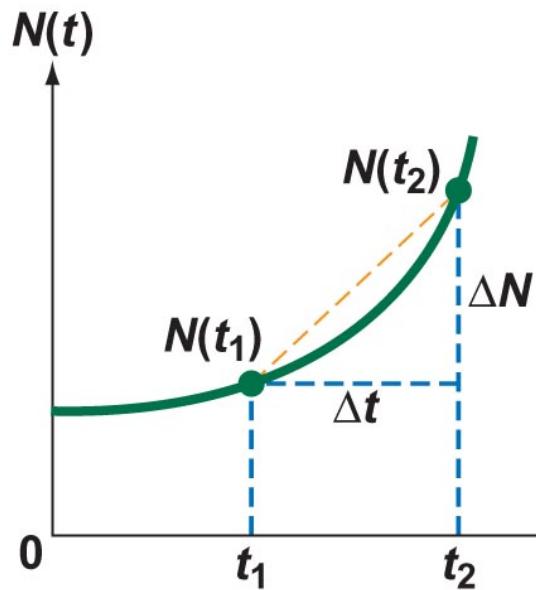
- If  $N(t)$  is a linear function of  $t$  then the resulting graph will be a straight line
- The rate of population change is given by the slope =  $s = \Delta N/\Delta t = N(t_2) - N(t_1)/t_2 - t_1$ 
  - The slope of a linear function does not depend on the value of  $t$



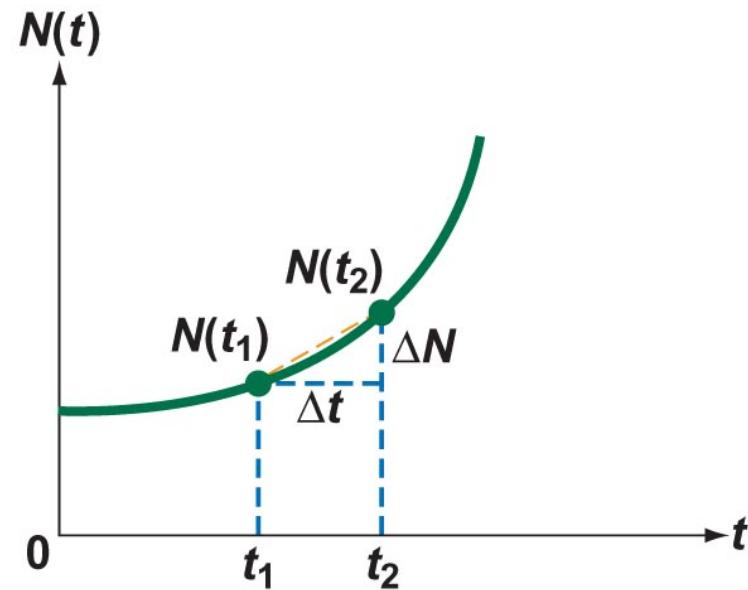
# Quantifying Ecology 10.1 Derivatives and Differential Equations

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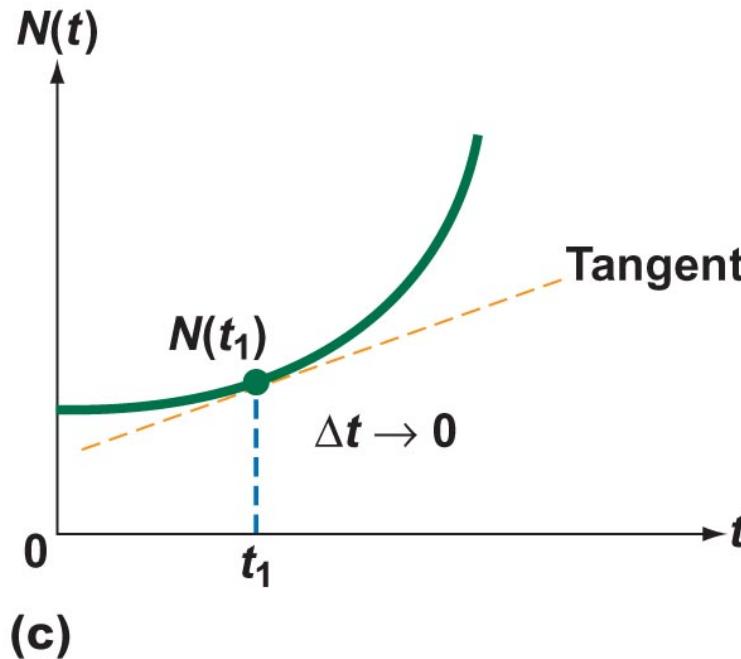
- The slope of a nonlinear function (curve) *does* depend on the value of  $t$ 
  - As  $t_2$  moves closer to  $t_1$ , the slopes vary by smaller and smaller amounts and will eventually approach a constant “limiting value”
- The slope of the function  $M(t)$  at  $t_1$  is known as the derivative of  $M(t)$  written as  $dM(t)/dt$



(a)



(b)



(c)

## 10.2 Life Tables Provide a Schedule of Age-Specific Mortality and Survival

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- Change in population abundance through time is a function of the birth and death rates ( $r$  = per capita growth rate)
- A **life table** is an age-specific account of mortality
- A **cohort** is a group of individuals born in the same period of time

## 10.2 Life Tables Provide a Schedule of Age-Specific Mortality and Survival

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- Life table of gray squirrels (*Sciurus carolinensis*)
  - $x$  = age classes
  - $n_x$  = the number of individuals from the original cohorts that are alive at the specified age ( $x$ )

$x$	$n_x$
0	530
1	159
2	80
3	48
4	21
5	5

## 10.2 Life Tables Provide a Schedule of Age-Specific Mortality and Survival

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- Life table of grey squirrels (*Sciurus carolinensis*)
  - $I_x$  = the probability at birth of surviving to any given age ( $x$ )



$x$	$n_x$	$I_x$	
0	530	1.00	$n_0 / n_0 = 530 / 530$
1	159	0.30	$n_1 / n_0 = 159 / 530$
2	80	0.15	$n_2 / n_0 = 80 / 530$
3	48	0.09	
4	21	0.04	
5	5	0.01	

## 10.2 Life Tables Provide a Schedule of Age-Specific Mortality and Survival

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- Life table of gray squirrels (*Sciurus carolinensis*)
  - $d_x$  = age-specific mortality = the difference between the number of individuals alive for any age class ( $n_x$ ) and the next older age class ( $n_{x+1}$ )

$x$	$n_x$	$d_x$	
0	530	371	$n_0 - n_1 = 530 - 159$
1	159	79	$n_1 - n_2 = 159 - 80$
2	80	32	
3	48	27	
4	21	16	
5	5	5	

## 10.2 Life Tables Provide a Schedule of Age-Specific Mortality and Survival

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- Life table of gray squirrels (*Sciurus carolinensis*)
  - $q_x$  = age-specific mortality rate = the number of individuals that died in a given time interval ( $d_x$ ) divided by the number alive at the beginning of that interval ( $n_x$ )

$x$	$n_x$	$d_x$	$q_x$	
0	530	371	0.70	$d_0 / n_0 = 371 / 530$
1	159	79	0.50	$d_1 / n_1 = 79 / 159$
2	80	32	0.40	
3	48	27	0.55	
4	21	16	0.75	
5	5	5	1.00	

**Table 10.1 | Gray Squirrel Life Table**

$x$	$n_x$	$l_x$	$d_x$	$q_x$
0	530	1.0	371	0.7
1	159	0.3	79	0.5
2	80	0.15	32	0.4
3	48	0.09	27	0.55
4	21	0.04	16	0.75
5	5	0.01	5	1.0

# Quantifying Ecology 10.3 Life Expectancy

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- Life expectancy ( $e$ ) refers to the average number of years an individual is expected to live from the time of its birth
- Using the life table for the gray squirrel
- $e_x$  = age-specific life expectancy = the average number of years that an individual of a given age ( $n_x$ ) is expected to live into the future
  - Several initial calculations are necessary to determine  $e_x$

# Quantifying Ecology 10.3 Life Expectancy

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- $Lx$  = the average number of individuals alive during the age interval  $x$  to  $x + 1$ .
  - Assumes mortality is evenly spread over the year

$x$	$n_x$	$L_x$	
0	530	344.5	$= (n_0 + n_1) / 2 = (530 + 159) / 2 = 344.5$
1	159	119.5	$= (n_2 + n_3) / 2 = (80 + 48) / 2 = 64$
2	80	64.0	
3	48	34.5	
4	21	13.0	$= (n_5 + n_6) / 2 = (5 + 0) / 2 = 2.5$
5	5	2.5	

# Quantifying Ecology 10.3 Life Expectancy

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- $T_x$  = the total years lived into the future by individuals of age class  $x$  in the population

$$\begin{aligned}
 &= L_0 + L_1 + L_2 + L_3 + L_4 + L_5 \\
 &= 344.5 + 119.5 + 64 + 34.5 + 13 + 2.5 = 578
 \end{aligned}$$

$x$	$L_x$	$T_x$
0	344.5	578.0
1	119.5	233.5
2	64.0	114.0
3	34.5	50.0
4	13.0	15.5
5	2.5	2.5

$$= L_4 + L_5 = 13 + 2.5 = 15.5$$

$$= L_5 = 2.5$$

# Quantifying Ecology 10.3 Life Expectancy

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- $e_x$  = age-specific life expectancy = the average number of years that an individual of a given age ( $n_x$ ) is expected to live into the future

$x$	$n_x$	$T_x$	$e_x$	
0	530	578.0	1.09	$= T_0 / n_0 = 578 / 530 = 1.09$
1	159	233.5	1.47	$= T_1 / n_1 = 233.5 / 159 = 1.47$
2	80	114.0	1.43	$= T_2 / n_2 = 114 / 80 = 1.43$
3	48	50.0	1.06	
4	21	15.5	0.75	
5	5	2.5	0.50	

## 10.3 Different Types of Life Tables Reflect Different Approaches to Defining Cohorts and Age Structure

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- A **cohort** or **dynamic life table** is used to track the fate of a group of individuals born at a given time
  - These individuals are followed from birth to death
- A **dynamic composite life table** constructs a cohort from individuals born over several time periods

## 10.3 Different Types of Life Tables Reflect Different Approaches to Defining Cohorts and Age Structure

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- A **time-specific life table** is a distribution of age classes during a single time period
- Several assumptions are made in this approach
  - Each age class was sampled in proportion to its numbers in the population
  - Age-specific mortality rates (and birthrates) are constant over time

## 10.3 Different Types of Life Tables Reflect Different Approaches to Defining Cohorts and Age Structure

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- Many animals (e.g., insects) live only one breeding season. Because generations do not overlap, all individuals belong to the same age class
  - $n_x$  is measured by estimating the population size several times over its annual season

# Sparse Gypsy moth







[http://en.wikipedia.org/wiki/Lyma  
ntria\\_dispar\\_dispar](http://en.wikipedia.org/wiki/Lyma<br/>ntria_dispar_dispar)



**Table 10.2 |** Life Table of a Sparse Gypsy Moth Population in Northeastern Connecticut

$x$	$n_x$	$l_x$	$d_x$	$q_x$
Eggs	450	1.000	135	0.300
Instars I–III	315	0.700	258	0.819
Instars IV–VII	57	0.127	33	0.582
Prepupae	24	0.053	1	0.038
Pupae	23	0.051	7	0.029
Adults	16	0.036	0	1.000

Source: Data from R. W. Campbell 1969.

## 10.3 Different Types of Life Tables Reflect Different Approaches to Defining Cohorts and Age Structure

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- The life table is useful for studying several areas of plant demography
  - Seedling mortality and survival
  - Population dynamics of perennial plants marked as seedlings
  - Life cycles of annual plants



*Sedum smallii* H. E. Ahles





**Table 10.3 | Life Table for a Natural Population of *Sedum smallii***

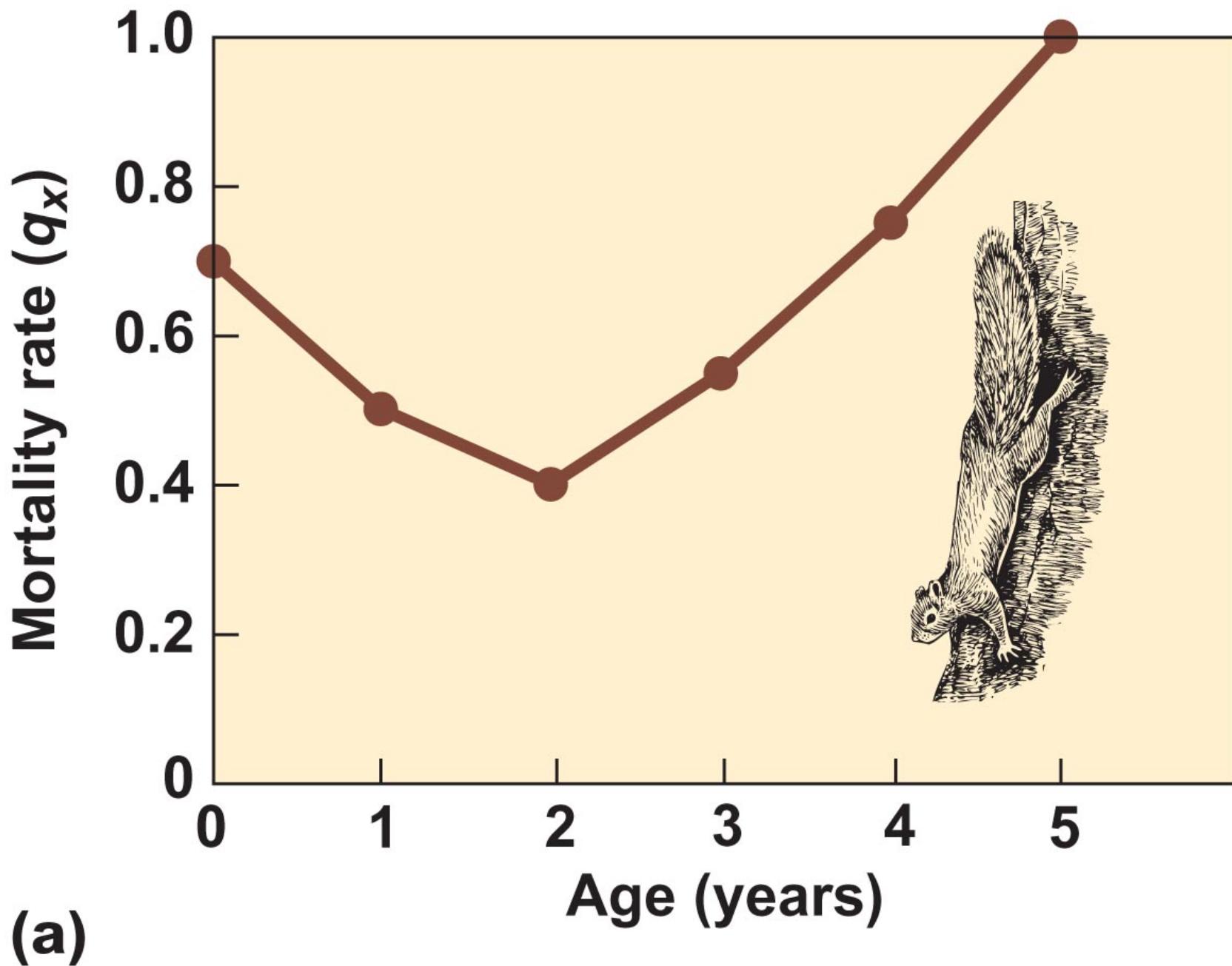
$x$	$l_x$	$d_x$	$q_x$
Seed produced	1.000	0.16	0.160
Available	0.840	0.630	0.750
Germinated	0.210	0.177	0.843
Established	0.033	0.009	0.273
Rosettes	0.024	0.010	0.417
Mature plants	0.014	0.014	1.000

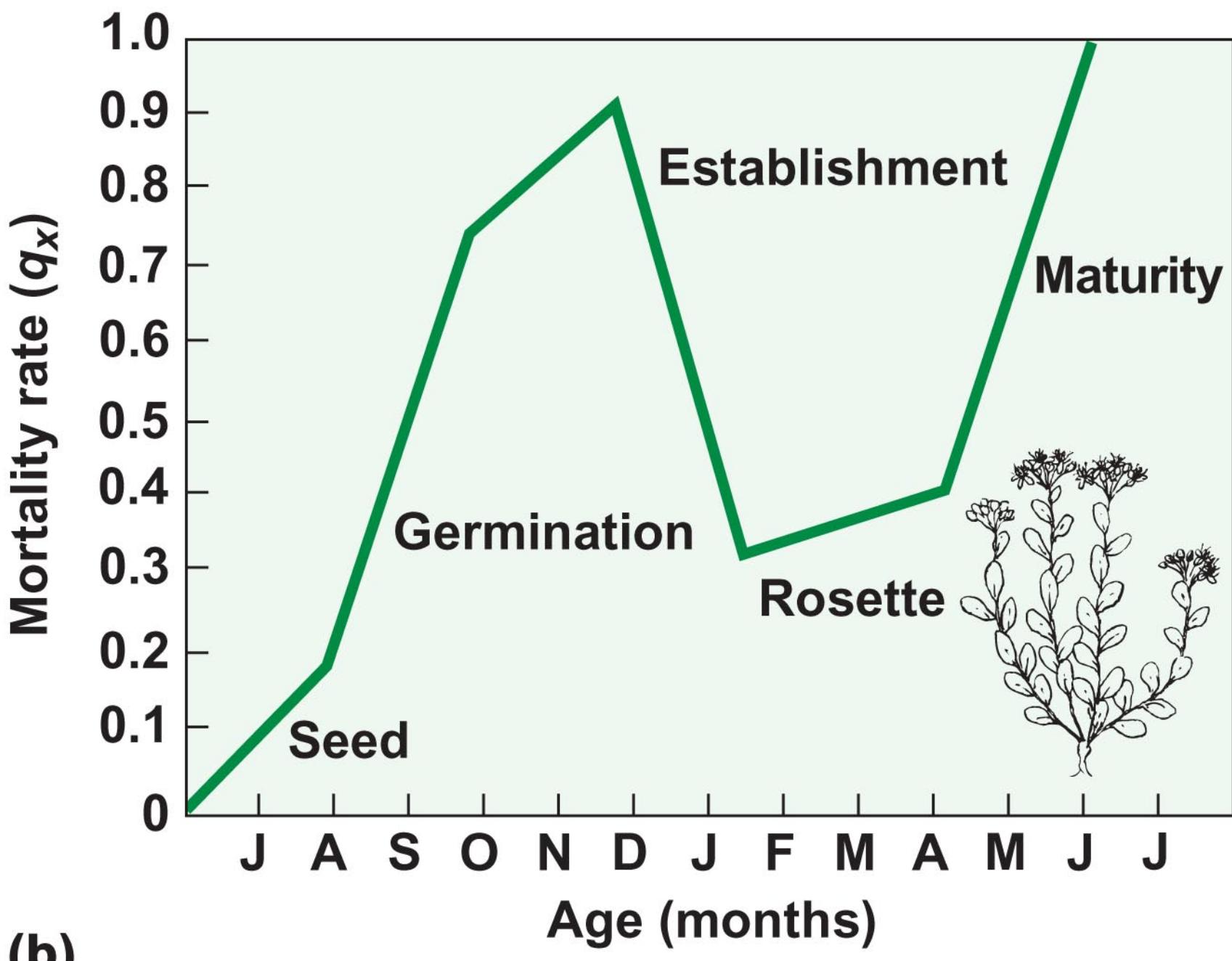
Source: Data from Sharitz and McCormick 1973.

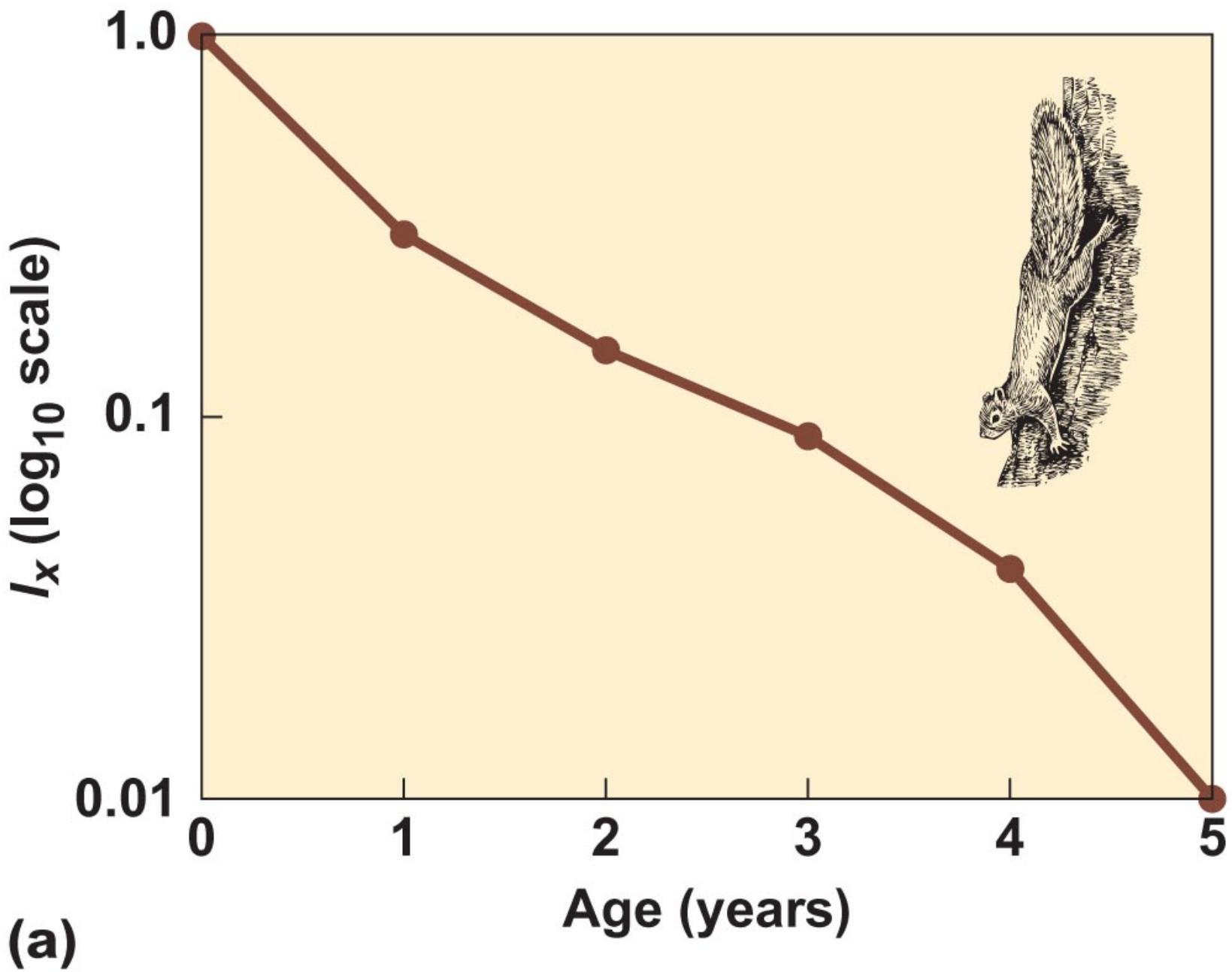
## 10.4 Life Tables Provide Data for Mortality and Survivorship Curves

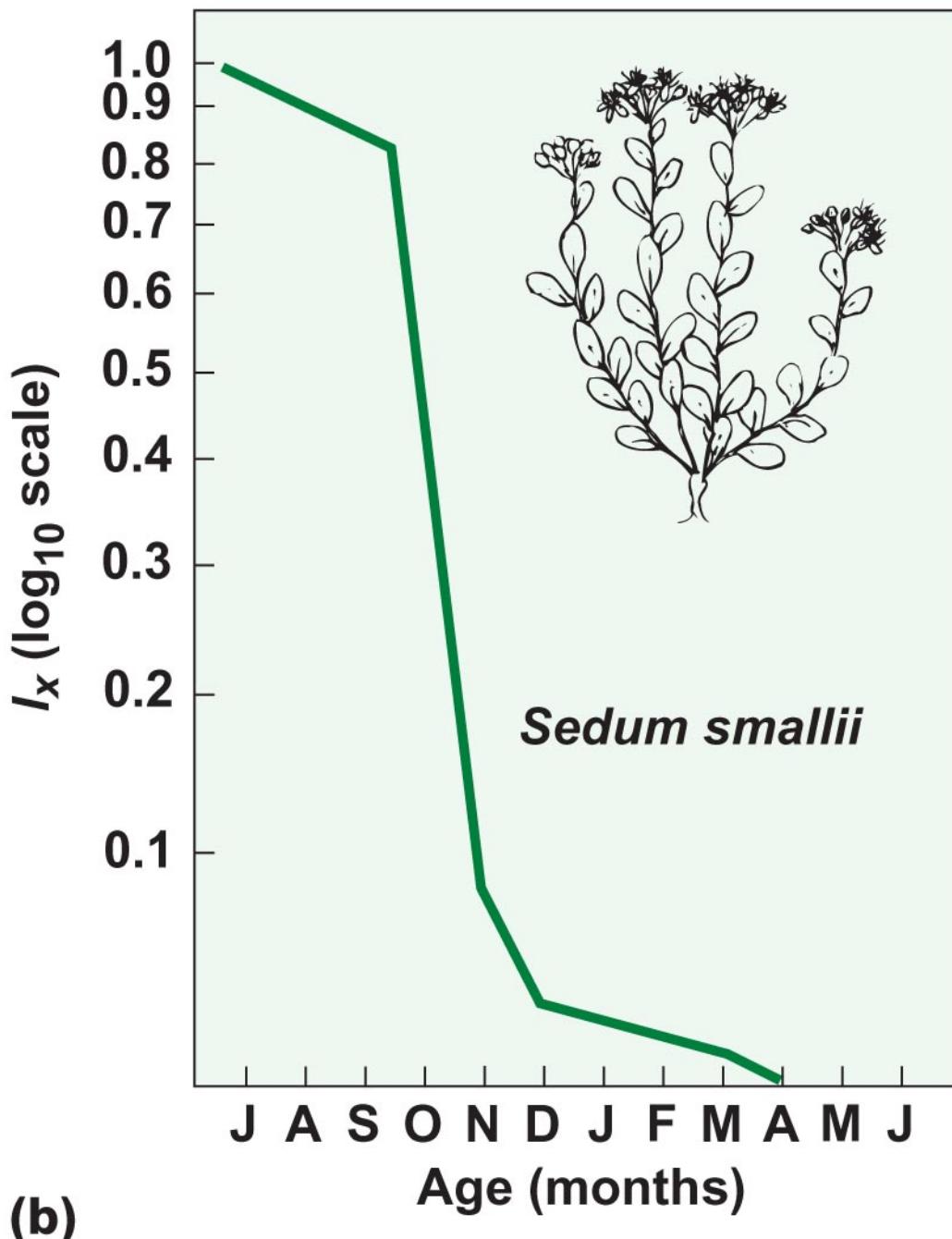
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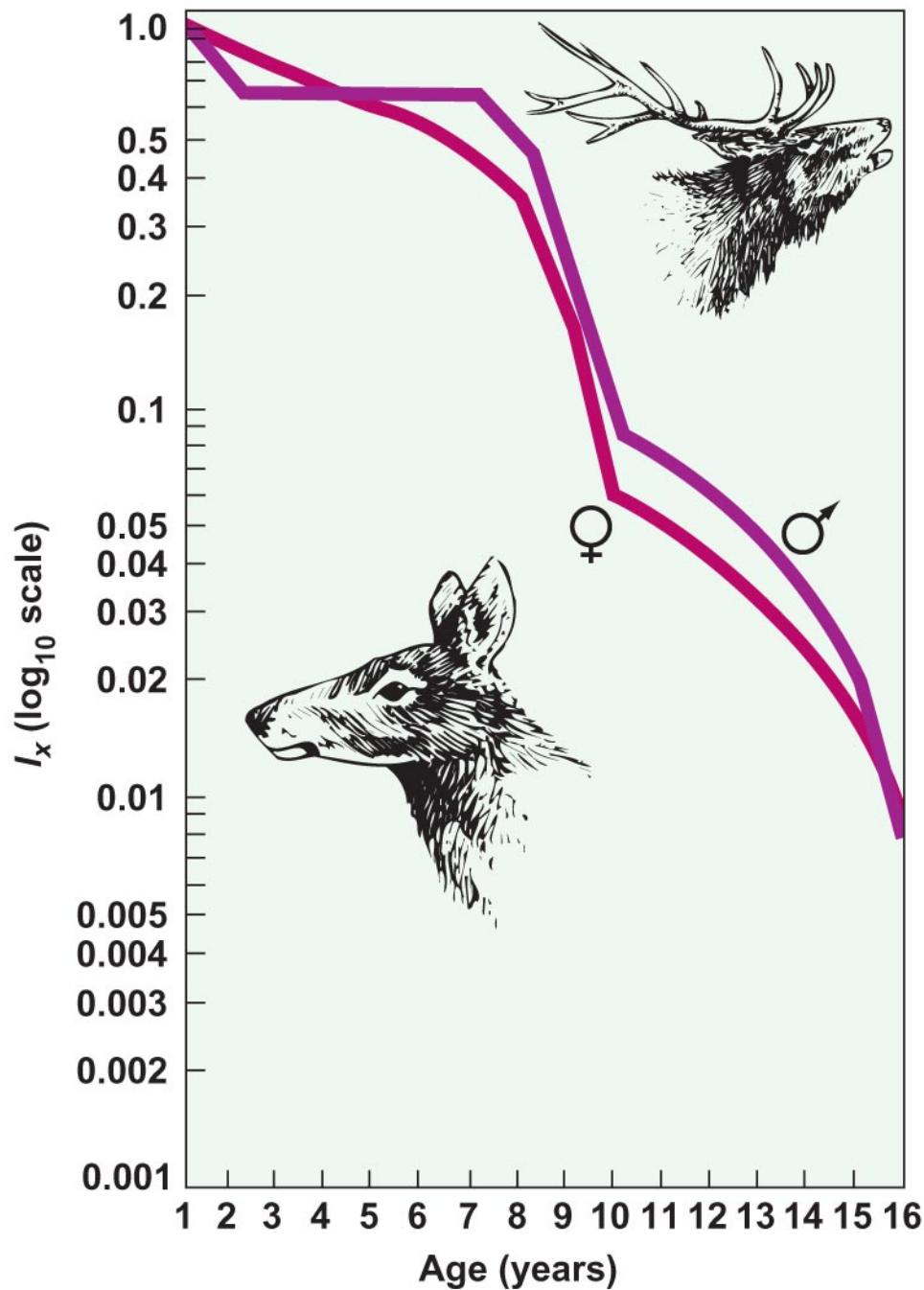
- Life table data are generally presented as:
  - A **mortality curve** that plots the  $q_x$  column against age ( $x$ )
  - A **survivorship curve** that plots the  $l_x$  column against age ( $x$ )
- Life tables and curves are based on data from one population at a specific time and under certain environmental conditions







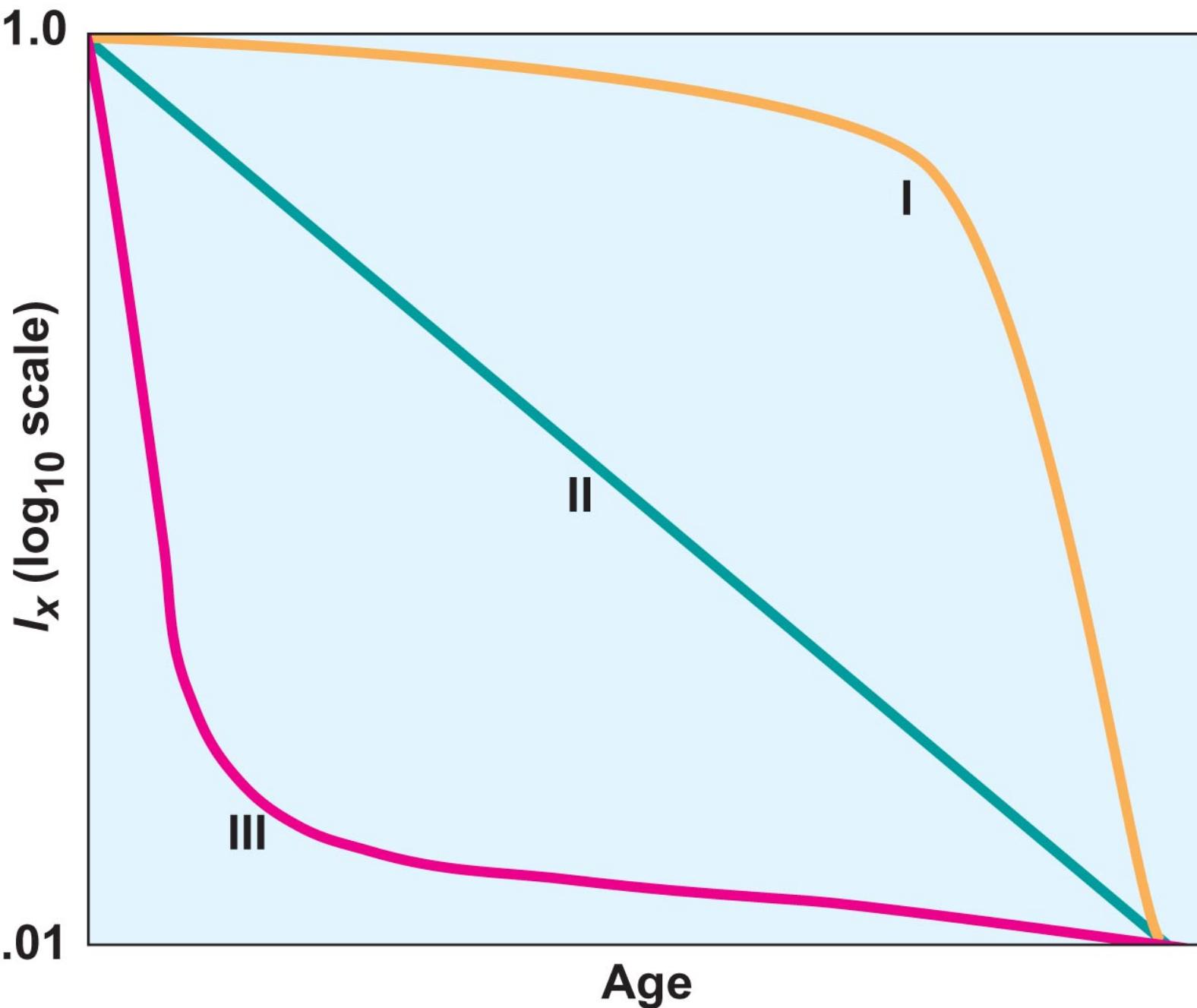


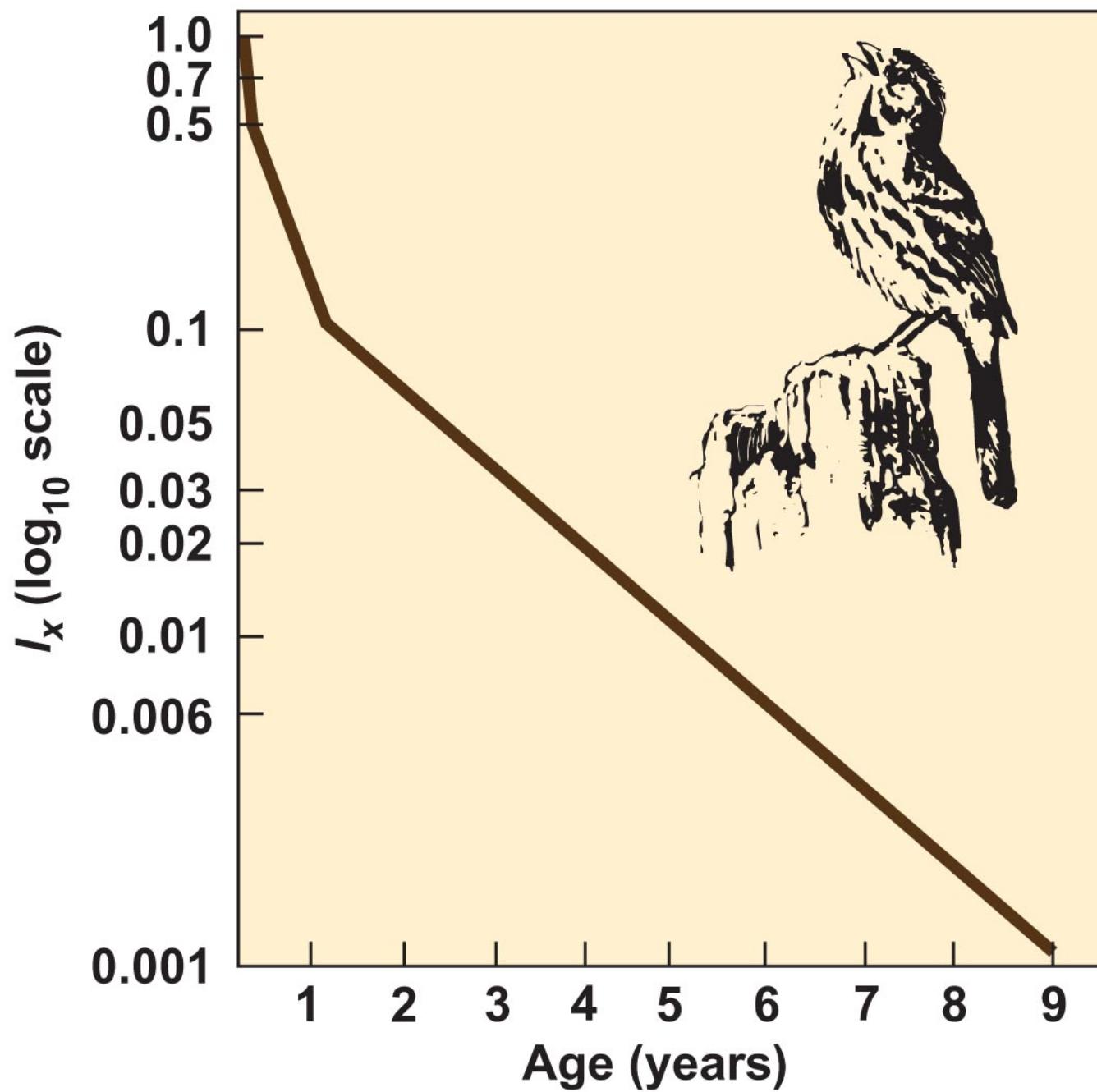


## 10.4 Life Tables Provide Data for Mortality and Survivorship Curves

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- There are three *idealized* types of survivorship curves
  - **Type I:** typical of populations in which individuals have long life spans, survival rate is high throughout the life span with heavy mortality at the end
    - Humans, other mammals, some plants
  - **Type II:** survival rates do not vary with age
    - Adult birds, rodents, reptiles, perennial plants
  - **Type III:** mortality rates are extremely high in early life
    - Fish, many invertebrates, and plants





## 10.5 Birthrate is Age-Specific

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- The **crude birthrate** is expressed as births per 1000 population per unit time
  - Only females give birth
  - Birthrate of females generally varies with age
- Birthrate is better expressed as the number of births per female of age  $x$

## 10.5 Birthrate is Age-Specific

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- $b_x$  = mean number of females born to a female in each age group
  - Continuing with the gray squirrel example
- $\Sigma$  = **gross reproductive rate** = the average number of female offspring born to a female over her lifetime

$x$	$b_x$
0	0
1	2
2	3
3	3
4	2
5	0
$\Sigma$	10

## 10.6 Birthrate and Survivorship Determine Net Reproductive Rate

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- A **fecundity table** combines the survivorship ( $l_x$ ) with the age-specific birthrates ( $b_x$ )
- $l_x b_x$  = mean number of females born in each age group, adjusted for survivorship

## 10.6 Birthrate and Survivorship Determine Net Reproductive Rate

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- $R_0$  = **net reproductive rate** = the average number of females that will be produced during a lifetime by a newborn female

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**Table 10.4 | Gray Squirrel Fecundity Table**

$x$	$l_x$	$b_x$	$l_x b_x$
0	1.0	0.0	0.00
1	0.3	2.0	0.60
2	0.15	3.0	0.45
3	0.09	3.0	0.27
4	0.04	2.0	0.08
5	0.01	0.0	0.00
$\Sigma$		10.0	1.40

## 10.6 Birthrate and Survivorship Determine Net Reproductive Rate

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- $R_0 = 1$ ; on average, females will replace themselves in the population
- $R_0 < 1$ ; females are not replacing themselves in the population
- $R_0 > 1$ ; females are more than replacing themselves in the population

## 10.7 Age-Specific Mortality and Birthrates Can Be Used to Project Population Growth

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- For simplicity, age-specific mortality ( $q_x$ ) is converted to **age-specific survival ( $s_x$ )**
  - $s_x = 1 - q_x$
- A **population projection table** can be constructed using  $s_x$  and  $b_x$



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**Table 10.5 |** Age-Specific Survival and Birthrates for Squirrel Population

$x$	$l_x$	$q_x$	$s_x$	$b_x$
0	1.0	0.7	0.3	0.0
1	0.3	0.5	0.5	2.0
2	0.15	0.4	0.6	3.0
3	0.09	0.55	0.45	3.0
4	0.04	0.75	0.25	2.0
5	0.01	1.0	0.0	0.0

## 10.7 Age-Specific Mortality and Birthrates Can Be Used to Project Population Growth

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- We will use the life and fecundity table values of the gray squirrel to illustrate a hypothetical population of squirrels introduced into an unoccupied forest
- Females are only used to construct the **population projection table**

## 10.7 Age-Specific Mortality and Birthrates Can Be Used to Project Population Growth

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- Initial population size
  - $x_0 = 20$  individuals of age 0
  - $x_1 = 10$  adults of age 1

Number of individuals  
in each age class ( $x$ )  
at year 0

$x$	$b_x$	$s_x$
0	0.0	0.30
1	2.0	0.50
2	3.0	0.60
3	3.0	0.45
4	2.0	0.25
5	0.0	0.00

	Year	
	0	1
20	27	
10	6	
		5

Total    30    38

$$= (b_1 \times 6) + (b_2 \times 5)$$

$$= (2.0 \times 6) + (3.0 \times 5)$$

$$= 12 + 15 = 27$$

$$= 20 \times s_0 = 20 \times 0.3 = 6$$

$$= 10 \times s_1 = 10 \times 0.5 = 5$$

## 10.7 Age-Specific Mortality and Birthrates Can Be Used to Project Population Growth

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- Survivorship and fecundity are determined in a similar manner for each succeeding year

**Table 10.6 | Population Projection Table, Squirrel Population**

Age	Year ( $t$ )										
	0	1	2	3	4	5	6	7	8	9	10
0	20	27	34.1	40.71	48.21	58.37	70.31	84.8	101.86	122.88	148.06
1	10	6	8.1	10.23	12.05	14.46	17.51	21.0	25.44	30.56	36.86
2	0	5	3.0	4.05	5.1	6.03	7.23	8.7	10.50	12.72	15.28
3	0	0	3.0	1.8	2.43	3.06	3.62	4.4	5.22	6.30	7.63
4	0	0	0	1.35	0.81	1.09	1.38	1.6	1.94	2.35	2.83
5	0	0	0	0	0.33	0.20	0.27	0.35	0.40	0.49	0.59
Total $N(t)$	30	38	48.2	58.14	68.93	83.21	100.32	120.85	145.36	175.30	211.25
Lambda	$\lambda$	1.27	1.27	1.21	1.19	1.21	1.20	1.20	1.20	1.20	1.20

## 10.7 Age-Specific Mortality and Birthrates Can Be Used to Project Population Growth

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- An **age distribution** for each successive year can be calculated from a population projection table
  - Age distribution is the proportion of individuals in the various age classes for any one year
- A **stable age distribution** is attained when the proportions of each age group in the population remain the same year after year (even though  $M(t)$  increases)

**Table 10.7 |** Approximation of Stable Age Distribution, Squirrel Population

Age	Proportion in Each Age Class for Year										
	0	1	2	3	4	5	6	7	8	9	10
0	.67	.71	.71	.71	.69	.70	.70	.70	.70	.70	.70
1	.33	.16	.17	.17	.20	.17	.17	.18	.18	.18	.18
2		.13	.06	.07	.06	.07	.07	.07	.07	.07	.07
3			.06	.03	.03	.04	.04	.03	.03	.03	.03
4				.02	.01	.01	.01	.01	.01	.01	.01
5					.01	.01	.01	.01	.01	.01	.01

## 10.7 Age-Specific Mortality and Birthrates Can Be Used to Project Population Growth

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- An estimate of population growth can be derived from a population projection table
- $\lambda = \text{finite multiplication rate} = N(t + 1)/N(t)$ 
  - Once the population reaches a stable age distribution, the value of  $\lambda$  remains constant
- $\lambda > 1.0$  indicates a population that is growing
- $\lambda < 1.0$  indicates a population in decline
- $\lambda = 1.0$  indicates a stable population size through time

## 10.7 Age-Specific Mortality and Birthrates Can Be Used to Project Population Growth

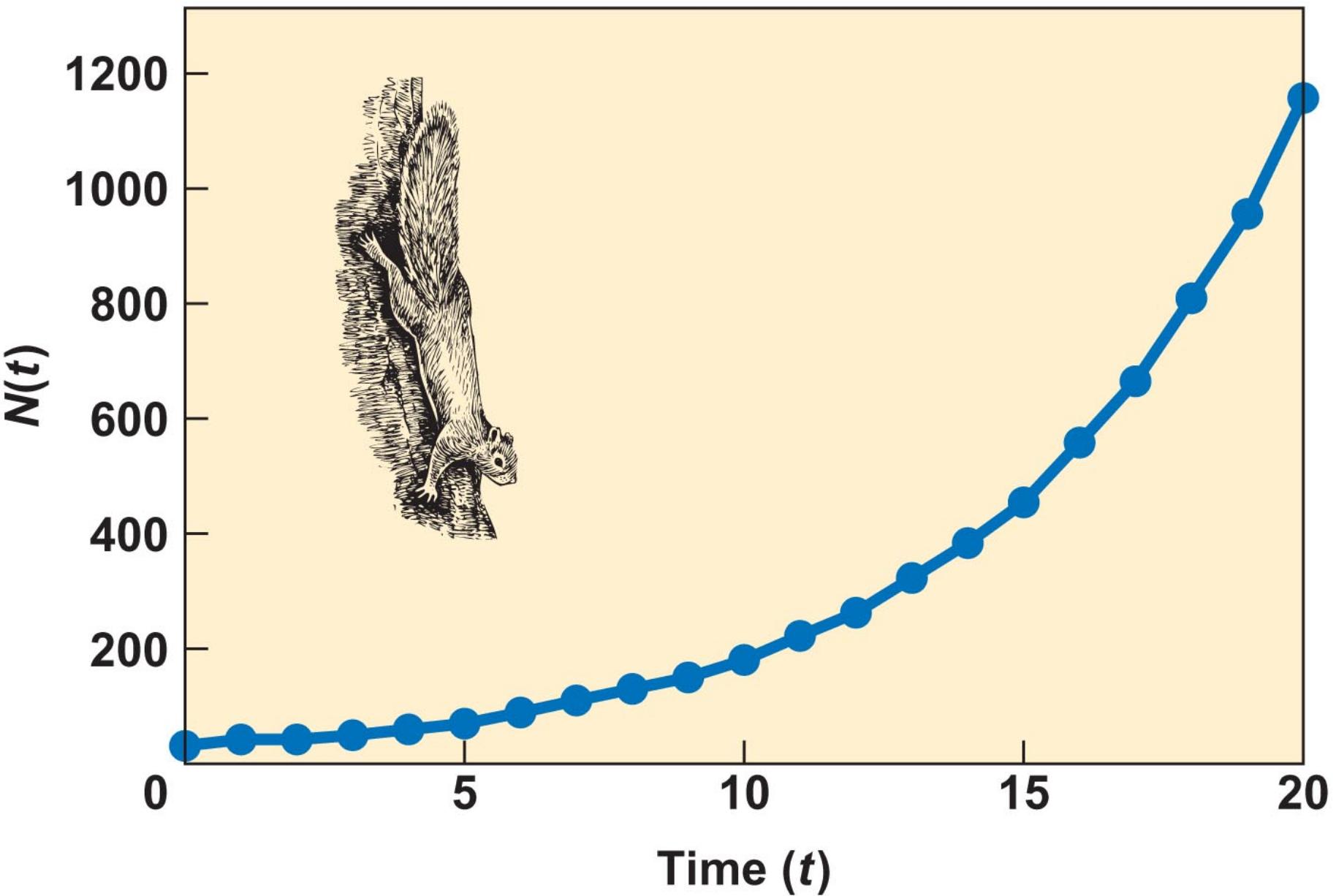
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- The population projection table demonstrates two concepts of population growth
  - $\lambda$  (estimated population growth rate) is a function of  $s_x$  and  $b_x$
  - The constant rate of population increase from year to year and the stable age distribution are results of  $s_x$  and  $b_x$  that are constant through time

## 10.7 Age-Specific Mortality and Birthrates Can Be Used to Project Population Growth

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- If  $\lambda$  does *not* vary (under conditions of stable age distribution), population size in the future can be projected
  - $N(t) = N(0) \lambda t$ 
    - Describes a pattern of population growth similar to the exponential growth model



## 10.7 Age-Specific Mortality and Birthrates Can Be Used to Project Population Growth

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- **Geometric population growth** =  $N(t) = N(0)e^{\lambda t}$ 
  - Finite
- **Exponential population growth** =  $N(t) = N(0)e^{rt}$ 
  - Continuous
- $\lambda = e^r$  or  $r = \ln\lambda$

## 10.8 Stochastic Processes Can Influence Population Dynamics

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- Population dynamics represent the combined outcome of many individual probabilities
  - Age-specific survival rates ( $s_x$ ) represent the **probability** that a female of that age will survive to the next age class
- This reality has led ecologists to develop probabilistic or stochastic models of population growth to account for these variations

## 10.8 Stochastic Processes Can Influence Population Dynamics

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- **Demographic stochasticity** is the random (stochastic) variations in birth and death rates from year to year
  - The variations in  $d$  and  $b$  cause populations to deviate from the predictions based on deterministic models
- **Environmental stochasticity** is the random variations in the environment or the occurrence of natural disasters
  - These events directly influence  $d$  and  $b$

## 10.9 A Variety of Factors Can Lead to Population Extinction

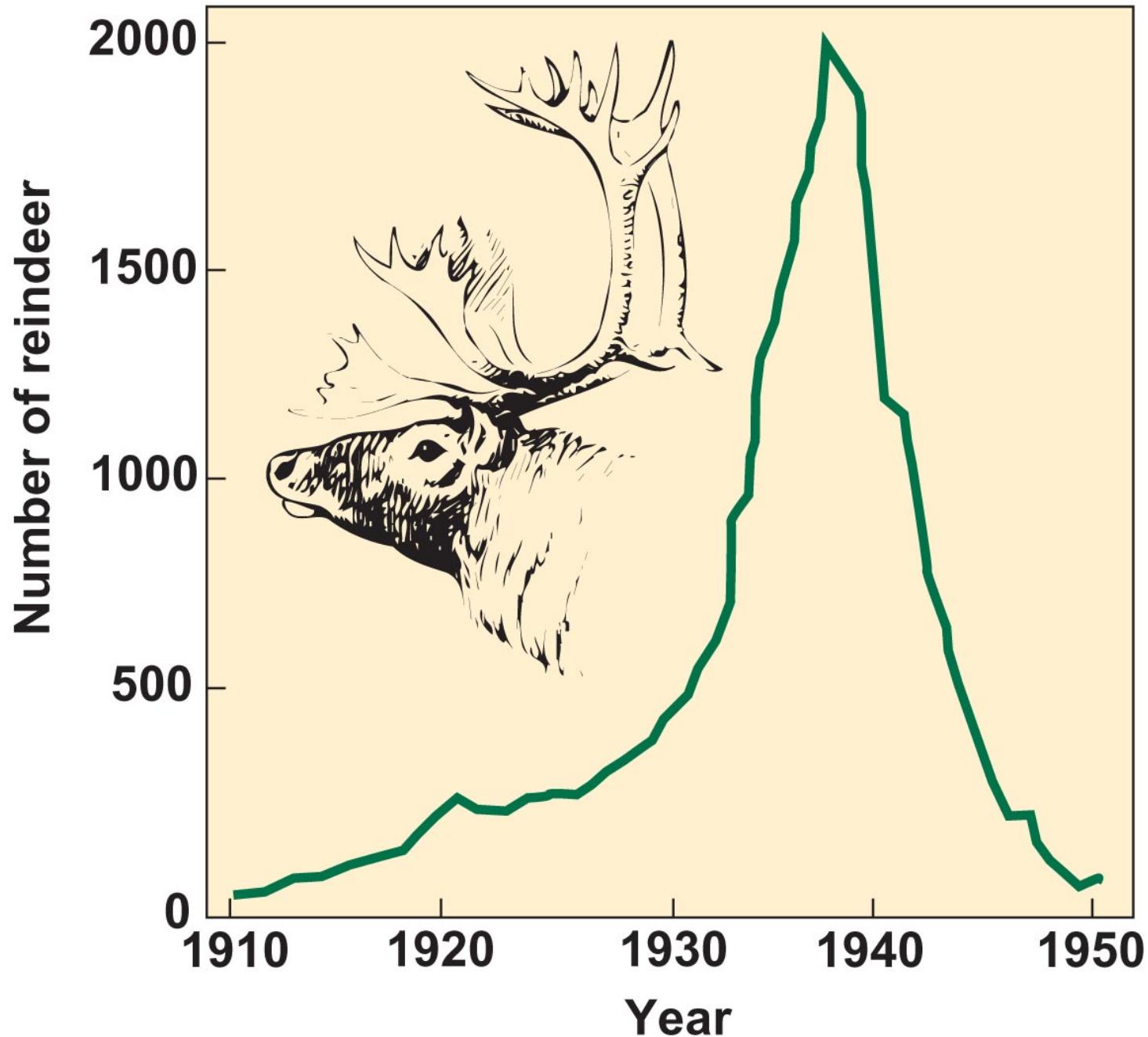
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- Under the following conditions, a population can become so small that it declines toward extinction:
  - When deaths exceed births, populations decline
  - $R_0$  becomes less than 1.0
  - $r$  becomes negative

## 10.9 A Variety of Factors Can Lead to Population Extinction

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- Under the following conditions, a population can become so small that it declines toward extinction:
  - Extreme environmental events
  - Severe shortage of resources



## 10.9 A Variety of Factors Can Lead to Population Extinction

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- Under the following conditions, a population can become so small that it declines toward extinction:
  - Introduction of a novel predator, competitor, or parasite (disease)
  - Habitat loss (due to human activities)
  - Small population size

## 10.10 Small Populations Are Susceptible to Extinction

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- Small populations are more susceptible to both demographic and environmental stochasticity
- When only a few individuals make up a population, the fate of each individual can be crucial to population survival
  - Over large territories, it can be impossible to find a mate (large cats)
  - Chemical signals will not be intercepted (insects)
  - Pollination is unlikely (plants)

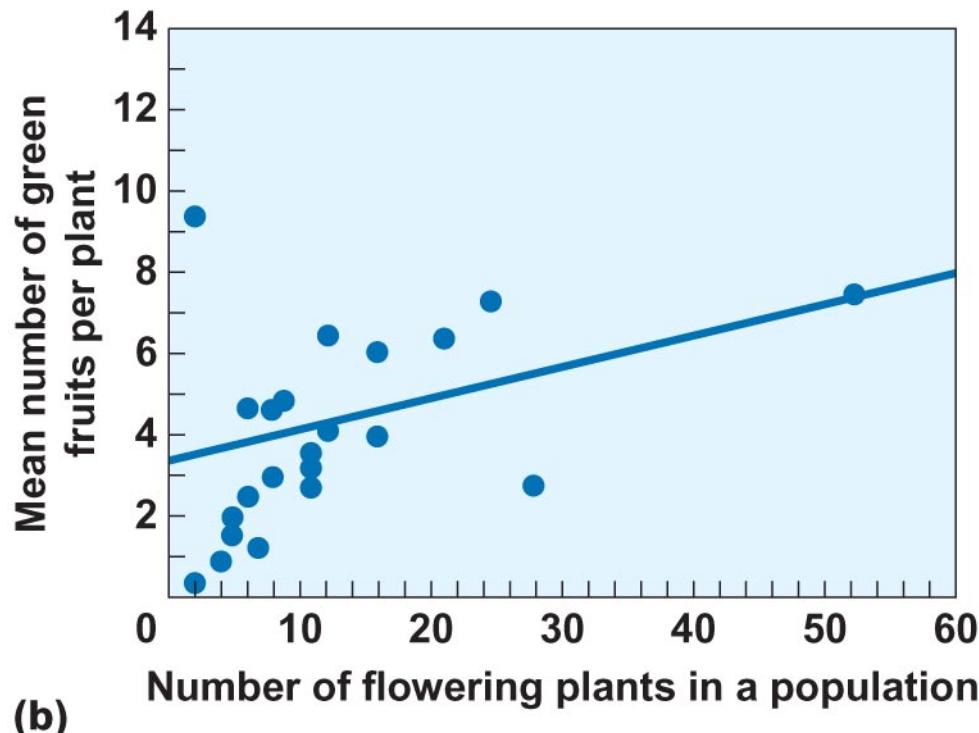
## 10.10 Small Populations Are Susceptible to Extinction

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- Hackney and McGraw (West Virginia University) examined the reproductive limitations by small population size on American ginseng (*Panax quinquefolius*)
  - Fruit production per plant declined with decreasing population size due to reduced visitation by pollination



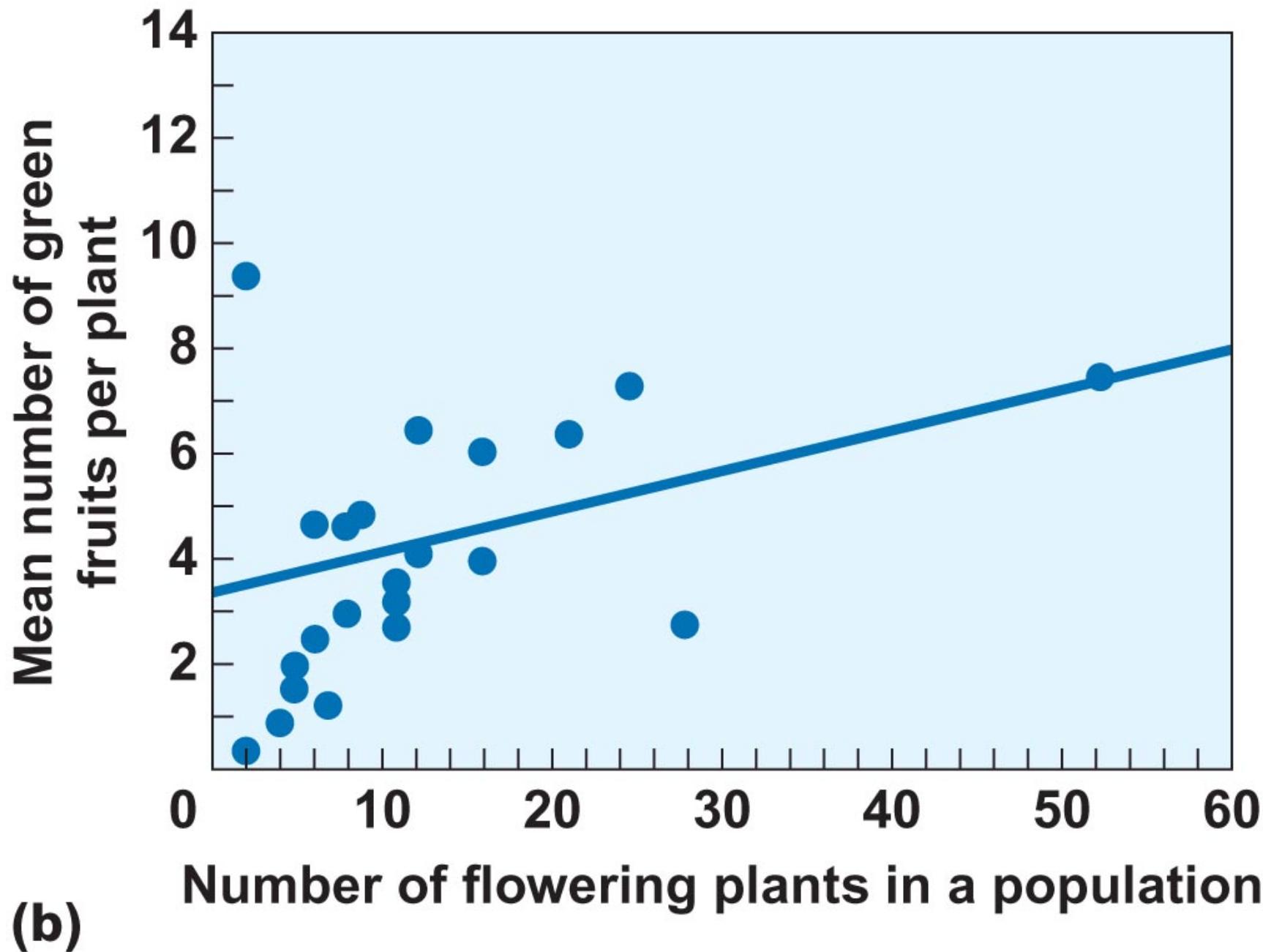
(a)



(b)



(a)



## 10.10 Small Populations Are Susceptible to Extinction

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- Small population size may result in the breakdown of social structures that are integral to successful cooperative behaviors (mating, foraging, defense)
- The **Allee effect** is the decline in reproduction or survival under conditions of low population density
- There is less genetic variation in a small population and this may affect the population's ability to adapt to environmental change