CS766 Homework1

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Problem 1:

a.

The image will still be a circle.

proof:

assume the point in the circle is (x_0, y_0, z_0), and the image point is (x_i, y_i, z_i). Since magnification $m = \frac{f}{z_0}$, and z_0 remains unchanged in the circle, so the image will still be a circle and its diameter will be m times.

b.

Magnification:

$m_1 = \frac{f}{z_0}$

$m_2 = \frac{f}{2z_0}$

Since $\frac{Area_i}{Area_0} = m^2$

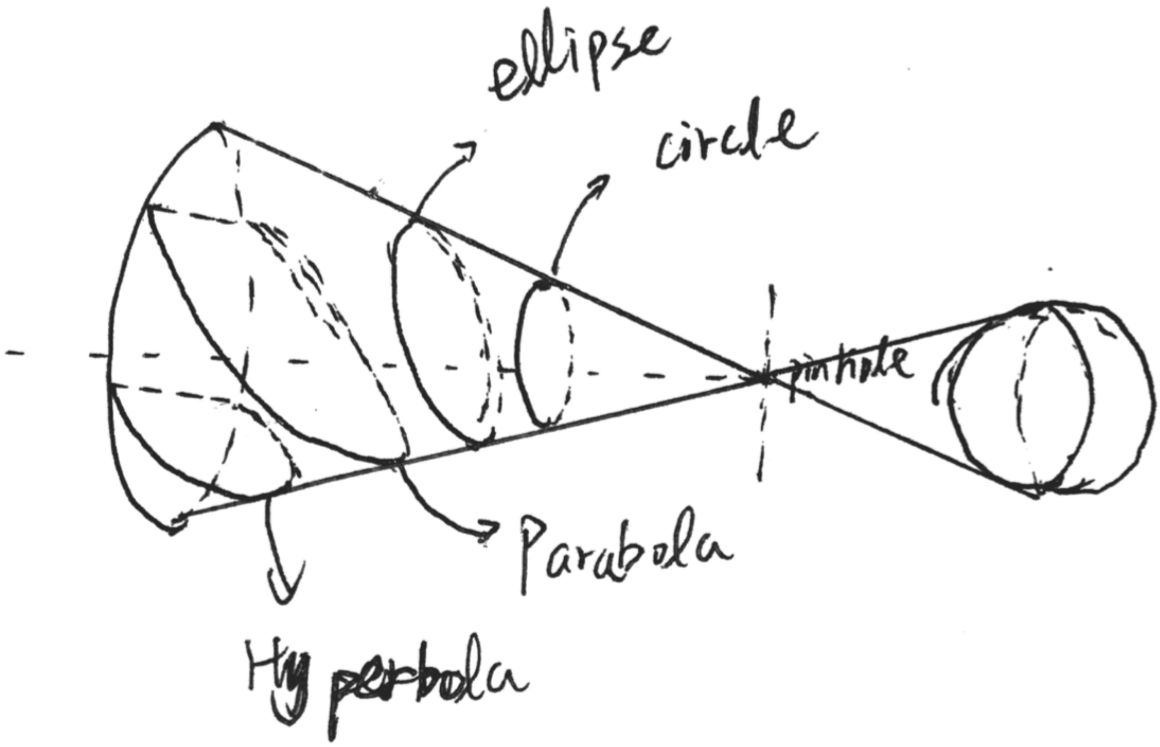
$\frac{Area_{i1}}{Area_{i2}} = \frac{m1^2}{m2^2} = 4$

So $Area_{i2} = Area_{i1}/4 = 0.25mm^2$

c.

The shape of the image will depends on the position of the sphere to the pinhole.

The image will be circle, ellipse, parabola or hyperbola as position changes., because we can treat the position change of sphere as the tilt of the image plane, i.e. the image will be decided by the conic section which formed by the plane and the cone.



Problem2:

According to Gaussian Lens Law,

$\frac{1}{i} + \frac{1}{H} = \frac{1}{f}$ $-(1)$

$\frac{1}{i'} + \frac{1}{o} = \frac{1}{f}$

when $o \to +\infty, i' = f$ $-(2)$

For the lens with aperture D and Blur circle with diameter c, From similar triangles:

$\frac{c}{D} = \frac{|i'-i|}{i'}$

$c = \frac{D}{i'}|i' - i| = \frac{f}{Ni'}|i' - i|$

$|1 - \frac{i}{i'}| = \frac{NC}{f}$

$\because i' < i$

$\therefore i = i'(1 + \frac{NC}{f}) = f + NC$

Take above equation into (1):

$\frac{1}{i} + \frac{1}{H} = \frac{1}{f}$

$H = \frac{if}{i-f} = \frac{f^2}{NC} + f$

Problem3:

a.

According to Gaussian Lens Law:

$\frac{1}{d} + \frac{1}{k} = \frac{1}{f}$

the distance $d = \frac{kf}{k-f}$

b&c.

build the coordinate system of x-y, and x-axis is the optical axis of the lens, the origin is the middle point of the lens, therefore the tilted scene line can be represented as:

$x - k = y \cdot tan\theta$ (with contraints in two ends).

According to gaussian lens law, for each point (x, y)in tilt line and the corresponding image point (x', y'), we have $\frac{1}{x} + \frac{1}{x'} = \frac{1}{f}$, take $(k, 0)$ into the equation, we can know that the correspond image point is$(\frac{kf}{k-f}, 0)$.

So the image line is

$x' - x_0{}' = y' \cdot tan\Phi$

$\frac{1}{tan\Phi} = \frac{y'}{x'-x_0{}'} = \frac{1}{tan\theta} \frac{x-k}{y} \frac{y'}{x'-x_0{}'}$ –(1)

since $\frac{1}{x} + \frac{1}{x'} = \frac{1}{f} = \frac{1}{k} + \frac{1}{x_0{}'}$

$\frac{x-k}{x'-x_0{}'} = \frac{xk}{x_0{}'x'}$

Using above equation and similar triangles (1) becomes

$\frac{1}{tan\Phi} = \frac{1}{tan\theta} \frac{k}{x_0{}'} \frac{y'(x-k)}{y(x'-x_0{}')} = \frac{1}{tan\theta} \frac{k}{\frac{kf}{k}} 1$

$tan\Phi = tan\theta \cdot \frac{f}{k-f}$