①+②: $P(Y=0) = 0.4$ ∴ $P(Y=1) = 0.6$

①+③: $P(X=0) = a+0.1$

$P(X=0) = \frac{1}{4} = 0.25$

②+④: $P(X=1) = b+0.3$

Worksheet Week 21

$a+0.1=0.25$

$a+b=0.6$

$a+b+0.4=1$

$a+b=0.6$

$a=0.15$

$b=0.45$

Problems

Q1. X and Y are two (binary) random variables. If X and Y are independent, then $P(X,Y) = P(X)P(Y)$

(a) Give an example of two random variables that are independent.

(b) Complete the probability table below in such way that the variables X and Y are independent.

$P(X|Y)$

	$X=0$	$X=1$
$Y=0$	0.7	0.3
$Y=1$	0.6	0.4

(c) Determine the missing entries (a, b) of the joint distribution in such a way that the variables X and Y are again independent.

$$P(Y=0, X=0) = 0.1$$
$$P(Y=0, X=1) = 0.3$$
$$P(Y=1, X=0) = a$$
$$P(Y=1, X=1) = b$$

$a=0.15$

$b=0.45$

Q2. Consider the following Bayesian network:

Q1 (a) X: A human is detected to be positive for Covid

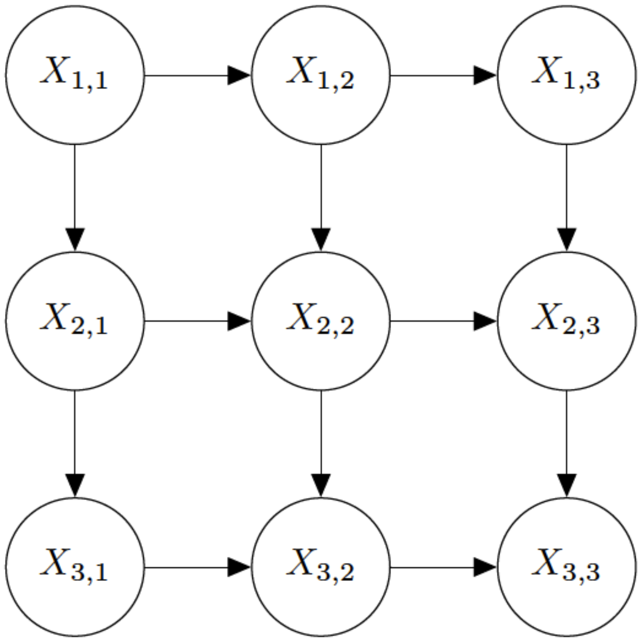
Y: The medicine is effective

(c) $P(Y=0) \cdot P(X=0) = 0.1$ ①

$P(Y=0) \cdot P(X=1) = 0.3$ ②

$P(Y=1) \cdot P(X=0) = a$ ③

$P(Y=1) \cdot P(X=1) = b$ ④

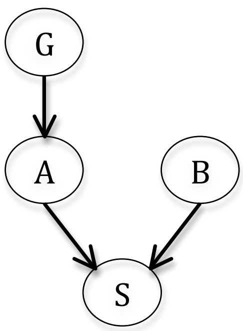


(a) Which random variables are independent of $X_{3,1}$? no nodes independent

(b) Which random variables are independent of $X_{3,1}$ given $X_{1,1}$? $X_{1,2}$ $X_{1,3}$

Q3. Solve the questions on slides 42 and 44 of the lecture slides.

Q4. A patient can have a symptom, S, that is caused by two different diseases, A and B. It is known that the presence of a gene G is important in the manifestation of disease A. The Bayes net and conditional probability tables are shown in Figure 2.



$P(G)$	
g	0.1
$\neg g$	0.9

$P(B)$	
b	0.4
$\neg b$	0.6

$P(A\|G)$		
g	a	1.0
g	$\neg a$	0.0
$\neg g$	a	0.1
$\neg g$	$\neg a$	0.9

$P(S\|A,B)$			
a	b	s	1.0
a	b	$\neg s$	0.0
a	$\neg b$	s	0.9
a	$\neg b$	$\neg s$	0.1
$\neg a$	b	s	0.8
$\neg a$	b	$\neg s$	0.2
$\neg a$	$\neg b$	s	0.1
$\neg a$	$\neg b$	$\neg s$	0.9

Figure 1: Bayes net and probability tables for Q5

(a) What is the probability that a patient has disease A

(b) What is the probability that a patient has disease A if we know that the patient has disease B

(c) What is the probability that a patient has disease A if we know that the patient has disease B AND symptom S