

# Summative assessment specification for EMATM0061

Teaching block 1 2021

## Introduction

This document contains the specification for the summative assessment on the unit EMATM0061.

**Deadline.** Your report is due on Wednesday 12th January 2022 at 13:00.

**Rules:** For the summative assessment you should not share your answers with your colleagues. The experience of solving the problems in this project will prepare you for real problem in your career as a data scientist. If someone asks you for the answer, resist! Instead, you can demonstrate how you would solve a similar problem.

**Support:** Whilst this is an independent task, there is a lot of support available if you need it. Talk to your classmates and book office hours. If you are unclear about what is required for any part of the assessment then discuss this issue with Henry Reeve in the computer lab or email [henry.reeve@bristol.ac.uk](mailto:henry.reeve@bristol.ac.uk), including the unit code EMATM00061 in the subject of your email.

**Plagiarism:** Be very careful to avoid plagiarism. For more details you should consult “Plagiarism” section within the central Blackboard page for the Data Science MSc.

**Extenuating circumstances:** For more details on the procedure for extenuating circumstances consult the “Extenuating circumstances” section within the central Blackboard page for the Data Science MSc.

**Clarity:** Clarity is highly important. Be careful to make sure you clearly explain each step in your answer. You should also include comments within your code. Your answer should clearly demarcate which part of the question you are answering.

**Programming language:** For Section A of this coursework you should use Tidyverse methods within the R programming language. For Sections B and C you can use either R, Python or Julia. Regardless of your choice of language, it is essential that your answers are clear and well-written.

**Submission structure:** You should submit a single folder containing your coursework entitled "user\_EMATM0061\_summative\_assessment", where "user" is replaced with your unique University of Bristol username. This folder should contain three separate reports corresponding to the three sections (A,B,C) of your coursework. These folders should be entitled "user\_EMATM0061\_A", "user\_EMATM0061\_B" and "user\_EMATM0061\_C", where again "user" is replaced with your unique University of Bristol username. Each of these sub-folders should contain a report and a subfolder. The reports must clearly display all your answers to the corresponding section including both explanatory pros and snippets of code where appropriate. The sub-folders should contain any data used to create the report and any supporting code (eg. .Rmd files used to create your report). Hence, the subfolder "user\_EMATM0061\_A" should contain a file entitled "user\_EMATM0061\_A\_Report" which displays all of your answers for Section A, the subfolder "user\_EMATM0061\_B" should contain a file entitled "user\_EMATM0061\_B\_Report" which displays all of your answers for Section B, and the subfolder "user\_EMATM0061\_C" should contain a file entitled "user\_EMATM0061\_C\_Report" which displays all of your answers for Section C. The compiled reports "user\_EMATM0061\_A\_Report", "user\_EMATM0061\_B\_Report" and "user\_EMATM0061\_C\_Report" can be either pdf or html documents. It is important that your approach to solving the questions is visible within these reports and you are encouraged to include pieces of clear and well-written code along with explanatory pros within the report itself.

Be careful to read each question in each section of the report.

## Section A (30 marks)

In this part of your assessment you will perform a data wrangling task with some finance data.

Note that clarity is highly important. Be careful to make sure you clearly explain each step in your answer. You should also include comments within your code. In addition, make the structure of your answer clear through the use of headings. You should also make sure your code is clean by making careful use of Tidyverse methods.\*

### A.1

Begin by downloading the csv file available within the Assessment section within Blackboard entitled “finance\_data\_EMATM0061”.

Next load the “finance\_data\_EMATM0061” csv file into R data frame called “finance\_data\_original”.

How many rows and how many columns does this data frame have?

### A.2

Generate a new data frame called “finance\_data” which is a subset of the “finance\_data\_original” data frame with the same number of rows, but only six columns:

- The first column should be called “state\_year\_code” and correspond to the “state\_year\_code” column in the csv.
- The second column should be called “education\_expenditure” and should correspond to the “Details.Education.Education.Total” column in the csv.
- The third column should be called “health\_expenditure” and should correspond to the “Details.Health.Health.Total.Expenditure” column in the csv.
- The fourth column should be called “transport\_expenditure” and should correspond to the “Details.Transportation.Highways.Highways.Total.Expenditure” column in the csv.
- The fifth column should be called “totals\_revenue” and should correspond to the “Totals.Revenue” column in the csv.
- The sixth column should be called “totals\_expenditure” and should correspond to the “Totals.Expenditure” column in the csv.

Display a subset of the “finance\_data” dataframe consisting of the first five rows and first three columns (“state\_year\_code”, “education\_expenditure”, “health\_expenditure”).

### A.3

Create a new column within the “finance\_data” data frame called “totals\_savings” which is equal to the difference between revenue and the expenditure ie. the elements of the “totals\_savings” column are equal to elements within the “totals\_revenue” minus the element within the “totals\_expenditure” column, for each row.

Your “finance\_data” data frame should now have seven columns.

Display a subset of the “finance\_data” dataframe consisting of the first three rows and the four columns “state\_year\_code”, “totals\_revenue”, “totals\_expenditure”, “totals\_savings”.

## A.4

The “state\_year\_code” column within your “finance\_data” data frame contains both a state and a year in character format connected via a double underscore.

Divide the “state\_year\_code” column into two separate columns, a “state” column and a “year” column which replace the original “state\_year\_code” column.

Your “finance\_data” data frame should now have eight columns.

Convert the states so that they appear with the first letter of each word in upper case and the remainder in lower case eg. we should see “New Hampshire” rather than “NEW HAMPSHIRE”. You may wish to use the function `str_to_title()`.

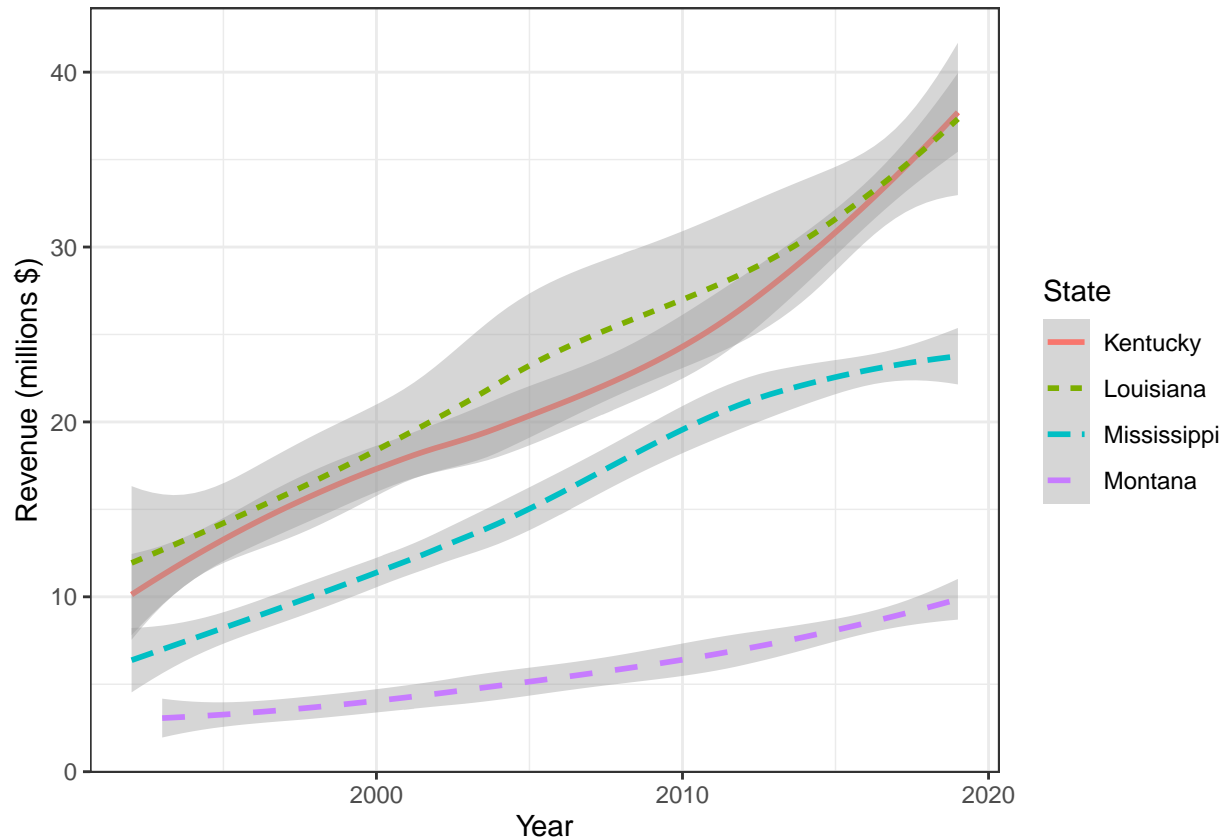
Display a subset of the “finance\_data” data frame consisting of the first three rows and the five columns “state”, “year”, “totals\_revenue”, “totals\_expenditure”, “totals\_savings”.

## A.5

Generate a plot which displays the total revenue (“total\_revenue”) as function of the year (“year”) for the following four states: Louisiana, Montana, Mississippi and Kentucky.

Display the revenue in terms of millions of dollars.

Your plot is expected to look as follows:



## A.6

Create a function called `get_decade()` which takes as input a number and rounds that number down to the nearest multiple of 10. For example, the numbers 20, 21, 22, ..., 29 would all be mapped to the output 20.

Use your `get_decade()` function to add a new column to the “finance\_data” data frame called “decade” which should give the decade corresponding to the year column. For example, the decade of the years 1990, 1991, ..., 1998, 1999 is 1990.

Your “finance\_data” data frame should now have nine columns.

Which three states had the highest mean-average savings (“totals\_savings”) over the decade starting 2000?

### Note:

- When computing the average you should disregard any years in which the information is not available i.e. the average should be taken only over those years for which the savings entry is not an “NA”.
- You should aim to use a succinct Tidyverse type solution.

## A.7

Next generate a summary data frame from the “finance\_data” data frame called “alaska\_summary” with the following properties:

Your summary data frame should correspond to rows associated with the state of Alaska. Your summary data frame should have three rows each corresponding to a decade from 1990 through to 2010 inclusive. Your data frame should also have seven columns:

- (a) “decade” – the decade (1990, 2000, 2010)
- (b) “ed\_mn” – the mean of the education expenditure in Alaska for the corresponding decade
- (c) “ed\_md” – the median of the education expenditure in Alaska for the corresponding decade
- (d) “he\_mn” – the mean of the health expenditure in Alaska for the corresponding decade
- (e) “he\_md” – the median of the health expenditure in Alaska for the corresponding decade
- (f) “tr\_mn” – the mean of the transport expenditure in Alaska for the corresponding decade
- (g) “tr\_md” – the median of the transport expenditure in Alaska for the corresponding decade

You should use Tidyverse methods to create your “alaska\_summary” data frame.

Display the “alaska\_summary” data frame.

## A.8

Create a function called `impute_by_median` which takes as input a vector numerical values, which may include some “NA”s, and replaces any missing values (“NA”s) with the median over the vector.

Next generate a subset of your “finance\_data” data frame called “idaho\_2000” which contains all those rows in which the state column takes the value “Idaho” and the “decade” column takes the value “2000” and includes the columns “year”, “education\_expenditure”, “health\_expenditure”, “transport\_expenditure”, “totals\_revenue”, “totals\_expenditure”, “totals\_savings” (i.e. all columns except “state” and “decade”).

Now apply your "impute\_by\_median" data frame to create a new data frame called "idaho\_2000\_imputed" which is based on your existing "idaho\_2000" data frame but with any missing values replaced with the corresponding median value for the that column. That is, for each of the columns "education\_expenditure", "health\_expenditure", "transport\_expenditure", "totals\_revenue", "totals\_expenditure", "totals\_savings" any missing values (given by "NA") are replaced with the median over that column.

Display a subset of your "idaho\_2000\_imputed" data frame consisting of the first five rows and the four columns "year", "health\_expenditure", "education\_expenditure" and "totals\_savings".

## Section B (30 marks)

### B.1

In this question we consider a security system at a factory. A sensor is designed to make a sound if a person walks within one metre of the gate. However, the sensor is not perfectly reliable: It sometimes makes a sound when there is no one present, and sometimes fails to make a sound when someone is present.

For simplicity we will view the passage of time as being broken down into a series of phases lasting exactly one minute. For each minute, we let  $p_0$  denote the conditional probability that the sensor makes a sound if there is *no* person within one metre of the gate, during that minute. Moreover, for each minute, we let  $p_1$  denote the conditional probability that the sensor makes a sound at least once, if there *is* at least one person present, during that minute. Suppose also that the probability that at least one person walks within one metre of the gate over any given minute is  $q$ . Again, for simplicity, we assume that  $p_0, p_1, q \in [0, 1]$  are all constant. Let  $\phi$  denote the conditional probability that at least one person has passed within one metre of the gate during the current minute, given that the alarm has made a sound during that minute.

- (a) Write a function called `c_prob_person_given_alarm` which gives  $\phi$  as a function of  $p_0, p_1$  and  $q$ .
- (b) Consider a setting in which  $p_0 = 0.05$ ,  $p_1 = 0.95$  and  $q = 0.1$ . In this case, what is  $\phi$ ?
- (c) Next consider a setting in which  $p_0 = 0.05$ ,  $p_1 = 0.95$  and generate a plot which shows  $\phi$  as we vary  $q$ . That is, you should display a curve which has  $q$  along the horizontal axis and the corresponding value of  $\phi$  along the vertical axis.

### B.2

Suppose that  $\alpha, \beta, \gamma \in [0, 1]$  with  $\alpha + \beta + \gamma \leq 1$  and let  $X$  be a discrete random variable with distribution supported on  $\{0, 1, 2, 5\}$ . Suppose that  $\mathbb{P}(X = 1) = \alpha$ ,  $\mathbb{P}(X = 2) = \beta$ ,  $\mathbb{P}(X = 5) = \gamma$  and  $\mathbb{P}(X \notin \{0, 1, 2, 5\}) = 0$ .

- (a) What is the probability mass function  $p_X : \mathbb{R} \rightarrow [0, 1]$  for  $X$ ?
- (b) Give an expression for the expectation of  $X$  in terms of  $\alpha, \beta, \gamma$ .
- (c) Give an expression for the population variance of  $X$  in terms of  $\alpha, \beta, \gamma$ .

Suppose  $X_1, \dots, X_n$  is a sample consisting of independent and identically distributed random variables with  $\mathbb{P}(X_i = 1) = \alpha$ ,  $\mathbb{P}(X_i = 2) = \beta$ ,  $\mathbb{P}(X_i = 5) = \gamma$  and  $\mathbb{P}(X_i \notin \{0, 1, 2, 5\}) = 0$  for  $i = 1, \dots, n$ . Let  $\bar{X} := \frac{1}{n} \sum_{i=1}^n X_i$  be the sample mean.

- (d) Give an expression for the expectation of the random variable  $\bar{X}$  in terms of  $\alpha, \beta, \gamma$ .
- (e) Give an expression for the population variance of the random variable  $\bar{X}$  in terms of  $\alpha, \beta, \gamma, n$ .
- (f) Create a function called `sample_X_0125()` which takes as inputs  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $n$  and outputs a sample  $X_1, \dots, X_n$  of independent copies of  $X$  where  $\mathbb{P}(X = 1) = \alpha$ ,  $\mathbb{P}(X = 2) = \beta$ ,  $\mathbb{P}(X = 5) = \gamma$  and  $\mathbb{P}(X \notin \{0, 1, 2, 5\}) = 0$ .
- (g) Suppose that  $\alpha = 0.1$ ,  $\beta = 0.2$ ,  $\gamma = 0.3$ . Use your function to generate a sample of size  $n = 100000$  consisting of independent copies of  $X$  where  $\mathbb{P}(X = 1) = \alpha$ ,  $\mathbb{P}(X = 2) = \beta$ ,  $\mathbb{P}(X = 5) = \gamma$  and  $\mathbb{P}(X \notin \{0, 1, 2, 5\}) = 0$ . What value do you observe for  $\bar{X}$ ? What value do you observe for the sample variance? Is this the type of result you expect? Explain your answer.

- (h) Once again, take  $\alpha = 0.1$ ,  $\beta = 0.2$ ,  $\gamma = 0.3$ . Conduct a simulation study to explore the behavior of the sample mean. Your study should involve 10000 trials. In each trial, you should set  $n = 100$  and create a sample  $X_1, \dots, X_n$  of independent and identically distributed random variables with  $\mathbb{P}(X_i = 1) = \alpha$ ,  $\mathbb{P}(X_i = 2) = \beta$ ,  $\mathbb{P}(X_i = 5) = \gamma$  and  $\mathbb{P}(X_i \notin \{0, 1, 2, 5\}) = 0$  for  $i = 1, \dots, n$ . For each of the 10000 trials, compute the corresponding sample mean  $\bar{X}$  based on  $X_1, \dots, X_n$ .
- (i) Generate a histogram plot which displays the behavior of the sample mean within your simulation study. Use a bin width of 0.02. The height of each bar should correspond to the number of times the sample mean took on a value within the corresponding bin.
- (j) What is the numerical value of the expectation  $\mathbb{E}(\bar{X})$  in your simulation study? What is the numerical value of the variance  $\text{Var}(\bar{X})$ ? Give your answers to 4 decimal places.

Let  $f_{\mu, \sigma} : \mathbb{R} \rightarrow [0, \infty)$  be the probability density function of a Gaussian random variable with distribution  $\mathcal{N}(\mu, \sigma^2)$ , so that the population mean is  $\mu$  and the population variance is  $\sigma^2$ .

- (k) Now append to your histogram plot an additional curve of the form  $x \mapsto 200 \cdot f_{\mu, \sigma}(x)$ , which displays a rescaled version of the probability density function of a Gaussian random variable with population mean  $\mu = \mathbb{E}(\bar{X})$  and population variance  $\sigma^2 = \text{Var}(\bar{X})$ . You may wish to consider your rescaled density  $x \mapsto 200 \cdot f_{\mu, \sigma}(x)$  displayed for a sequence of  $x$ -values between  $\mu - 4 \cdot \sigma$  and  $\mu + 4\sigma$  in increments of 0.0001. Make sure that the plot is well-presented and both the histogram and the rescaled density are clearly visible.
- (l) Discuss the relationship between the histogram and the additional curve you observe. Can you explain what you observe?

### B.3

In this question we shall use the exponential distribution to model time intervals between arrival times of birds at a bird feeder.

Let  $\lambda > 0$  be a positive real number. An exponential random variable  $X$  with parameter  $\lambda$  is a continuous random variable with density  $p_\lambda : \mathbb{R} \rightarrow (0, \infty)$  defined by

$$p_\lambda(x) := \begin{cases} 0 & \text{if } x < 0 \\ \lambda e^{-\lambda x} & \text{if } x \geq 0. \end{cases}$$

- (a) Give a formula for the the population mean and variance of an exponential random variable  $X$  with parameter  $\lambda$ .
- (b) Give a formula for the cumulative distribution function and the quantile function for exponential random variables with parameter  $\lambda$ .
- (c) Suppose that  $X_1, \dots, X_n$  is an i.i.d sample from the exponential distribution with an unknown parameter  $\lambda_0 > 0$ . What is the maximum likelihood estimate  $\hat{\lambda}_{\text{MLE}}$  for  $\lambda_0$ ?
- (d) Conduct a simulation study to explore the behavior of the maximum likelihood estimator  $\hat{\lambda}_{\text{MLE}}$  for  $\lambda_0$  on simulated data  $X_1, \dots, X_n$  generated using the exponential distribution. Consider a setting in which  $\lambda_0 = 0.01$  and generate a plot of the mean squared error as a function of the sample size. You should consider a sample sizes between 5 and 1000 in increments of 5, and consider 100 trials per sample size. For each trial of each sample size generate a random sample  $X_1, \dots, X_n$  of the exponential distribution with parameter  $\lambda_0 = 0.01$ , then compute the maximum likelihood estimate  $\hat{\lambda}_{\text{MLE}}$  for  $\lambda_0$  based upon the corresponding sample. Display a plot of the mean square error of  $\hat{\lambda}_{\text{MLE}}$  as an estimator for  $\lambda_0$  as a function of the sample size.



Now download the csv file entitled “birds\_data\_EMATM0061” from the Assessment section within Blackboard. The csv file contains synthetic data on arrival times for birds at a bird feeder, collected over a five week period. The species of bird and their arrival time are recorded.

Let's model the sequence of time differences as independent and identically distributed random variables from an exponential distribution. More, precisely, let  $Y_1, Y_2, \dots, Y_{n+1}$  denote the sequence of arrival times in seconds. Construct a new sequence of random variables  $X_1, \dots, X_n$  where  $X_i = Y_{i+1} - Y_i$  for each  $i = 1, \dots, n$ . Model the sequence of differences in arrival times  $X_1, \dots, X_n$  as independent and identically distributed exponential random variables.

- (e) Compute and display the maximum likelihood estimate of the rate parameter  $\hat{\lambda}_{MLE}$ .
- (f) Can you give a confidence interval for  $\lambda_0$  with a confidence level of 95%?

## Section C (40)

In this section you are asked to complete a Data Science report which demonstrates your understanding of a statistical method. The goal here is to choose a topic that you find interesting and explore that topic in depth. You are free to choose a topic and data set which interest you.

There will be an opportunity to discuss and get advice on your chosen direction in the computer labs.

Below are two flexible example structures you can consider for this section of your report. If you are unsure what to do, choose one of the following. Note that you should not submit more than one of the example tasks below.

### Example task 1

Investigate a particular hypothesis test eg. a Binomial test, a paired Student's t test, an unpaired Student's t test, an F test for ANOVA, a Mann-Whitney U test, a Wilcoxon signed-rank test, a Kruskal Wallis test, or some other test you find interesting.

Note that clarity of presentation is highly important. In addition, you should aim to demonstrate a depth of understanding. For this hypothesis test you are asked to do the following:

1. Give a clear description of the hypothesis test including the details of the test statistic, the underlying assumptions, the null hypothesis and the alternative hypothesis. Give an intuitive explanation for why the test statistic is useful in distinguishing between the null and the alternative.
2. Perform a simulation study to investigate the probability of type I error under the null hypothesis for your hypothesis test. Your simulation study should involve randomly generated data which conforms to the null hypothesis. Compare the proportion of rounds where a Type I error is made with the significance level of the test.
3. Apply this hypothesis test to a suitable real-world data set of your choice (some places to find data sets are described below). Ensure that your chosen data set is appropriate for your chosen hypothesis test. For example, if your chosen hypothesis test is an unpaired t-test then your chosen data set must have at least one continuous variable and contain at least two groups. It is recommended that your data set for this task not be too large. You should explain the source and the structure of your data set within your report.
4. Carefully discuss the appropriateness for your statistical test in this setting and how your hypotheses correspond to different aspects of the data set. You may want to use plots to demonstrate the validity of your underlying assumptions. Draw a statistical conclusion and report the value of your test statistic, the p-value and a suitable measure of effect size.
5. Discuss what scientific conclusions can you draw from your hypothesis test. Discuss how these would have differed if the result of your statistical test had differed. Discuss key experimental design considerations necessary for drawing any such scientific conclusion. For example, perhaps an alternative experimental design would have allowed one to draw a conclusion about cause and effect?

### Example task 2

Investigate a particular method for supervised learning. This could either be a method for regression or classification but should be a method with at least one tunable hyperparameter. You could choose one from ridge regression, k-nearest neighbour regression, a regression tree, regularized logistic regression, k-nearest neighbour classification, a decision tree, a random forest or another supervised learning technique you find interesting.

Note that clarity of presentation is highly important. In addition, you should aim to demonstrate a depth of understanding.

1. Give a clear description of the supervised learning technique you will use. Explain how the training algorithm works and how new predictions are made on test data. Discuss what type of problems this method is appropriate for.
2. Choose a suitable data set to apply this model to and perform a train, validation, test split (some places to find data sets are described below). Be careful to ensure that your data set is appropriate for your chosen algorithm. For example, if you have chosen to investigate a classification algorithm then your chosen data set must contain at least one categorical variable. Your data set for this task does not need to be large to obtain good results. The size of your data set should not exceed 100MB and you should aim to use a data set well within this limit. Your report should carefully give the source for your data. In addition describe your data set. How many features are there? How many examples? What type are each of the variables (eg. Categorical, ordinal, continuous, binary etc.?).
3. What is an appropriate metric for the performance of your model? Explore how the performance of your model varies on both the train and the validation data change as you vary the amount of training data used.
4. Explore how the performance of your model varies on both the train and the validation data change as you vary a hyperparameter.
5. Choose a hyper-parameter and report your performance based on the test data. Can you get a better understanding by using cross validation? Note that you will be graded on your understanding of the key concepts. It is far better to choose a simple hypothesis test and supervised learning algorithm, and apply sound statistical reasoning than to choose complex methods without properly demonstrating your understanding.

### Alternative tasks

You could also choose an alternative task in which you explore a statistical method or methods which interest you.

A couple of elements to bear in mind here:

- Demonstrate a solid level of understanding of the technique or techniques you consider.
- Apply your chosen method or technique to a real data set. This data must be publicly available and should not exceed 100MB in size.
- Where appropriate, use simulated data to explore and demonstrate the properties of your chosen method.
- The subject of your report should be statistical methods or techniques and their performance and behaviour. Whilst you can consider techniques motivated by a particular application, the application itself should not become the focus of your report.

If you are unsure what to do for this section you are encouraged to discuss within the computer labs.

### Note:

1. **Do not** complete and submit more than one of the above tasks. These are example tasks and you should only choose one. The goal here is to explore a topic in detail.
2. You will be graded on the level of understanding of the key concepts demonstrated within your report. With this in mind, it is far better to choose relatively simple methods, and apply sound statistical reasoning than to choose complex methods without properly demonstrating your understanding. You are welcome to include

methods not covered within the lectures, provided that they are appropriate for the task at hand and that you are able to demonstrate a clear understanding of the methods used. A small number of additional marks will be given for more advanced methods, provided that a very strong level of understanding is displayed. However, the main focus here is a clear understanding and you should not sacrifice understanding for the sake of complexity. A clear understanding of the basic concepts is paramount.

3. You do not need to use large data sets. In Sections B and C you should not use data sets larger than 100MB. The total size of the data used within sections B and C cannot exceed 200MB. This is an upper bound. You should aim to use a data set well within this limit.

## Data sets

There are a vast number of freely available data sets across the internet. Below is a few example sources. You are also welcome to use data sets from other sources. Any data you use should be freely available and accessible. The source of your data and the steps required to retrieve it should also be described within your main report.

You should also explain its structure eg. Number of rows and number of columns. You are encouraged to use tabular data throughout.

<https://www.kdnuggets.com/datasets/index.html>

<https://r-dir.com/reference/datasets.html>

<http://archive.ics.uci.edu/ml/datasets.php>

<http://lib.stat.cmu.edu/datasets/>

<http://inforumweb.umd.edu/econdata/econdata.html>

<https://lionbridge.ai/datasets/the-50-best-free-datasets-for-machine-learning/>

<https://www.kaggle.com/>

<https://www.ukdataservice.ac.uk/>

<https://data.worldbank.org/>

<https://www.imf.org/en/Data>

## **Final remarks**

Throughout your report you should emphasise:

Reproducible analysis (be careful with randomized procedures).

Clear and informative visualizations of your results.

Demonstrate a depth of understanding.

A clear writing style.