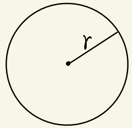
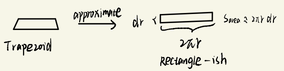
Integrals 积分

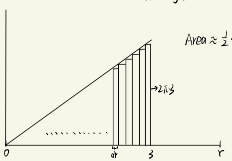
derivatives 微分

through the area of a circus



seperate the circle into numberous rings





$\text{Area} \approx \frac{1}{2}\cdot 3\cdot h = \frac{1}{2}\cdot 3\cdot 3\cdot 2\pi = \pi\cdot 3^2$

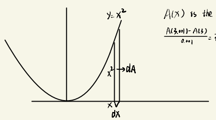
$\pi\cdot r^2$

$dA \approx x^2\, dx \qquad \frac{dA}{dx} \approx x^2$

$\frac{dA}{dx} \approx f(x)$

$dx \to 0$

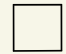
Derivative

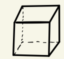


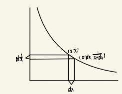
$A(x)$ is the area of graph with axis

$\frac{A(3.001)-A(3)}{0.001} = 3^2$

$f(x)=x \qquad f(x)=x^2 \qquad f(x)=x^3 \qquad f(x)=\frac{1}{x} \qquad f(x)=\sqrt{x}$

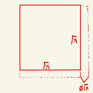






$\frac{d\frac{1}{x}}{dx} = \frac{1}{x^2 + x\cdot dx}$

$\frac{d\sqrt{x}}{dx} = \frac{1}{\sqrt{x+dx}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$



$\frac{d\sqrt{x}}{dx} = \frac{d\sqrt{x}}{2\sqrt{x}\cdot dx + \cancel{(d\sqrt{x})^2}} = \frac{1}{2\sqrt{x}}$

dx = New area

$df = d(\sqrt{x})^2 = dx = 2\sqrt{x}\, d\sqrt{x} \Rightarrow 1 = \frac{2\sqrt{x}\, d\sqrt{x}}{dx} \Rightarrow \frac{1}{2\sqrt{x}} = \frac{d\sqrt{x}}{dx}$

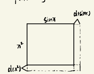
$(x+dx)^n = \underbrace{(x+dx)(x+dx)\cdots\cdots(x+dx)}_{n \text{ times}}$

nudged output

$= x^n + n\cdot x^{n-1}\cdot dx + (\text{multiple of } dx^2)$

safe to ignore anything that includes a dx raised to a power greater than one

$f(x) = g(x)\, h(x) = \text{Area}$



$\frac{d}{dx}(g\,h(x)) = \frac{dg}{dh}\,\frac{dh}{dx}$

$M(t) = a^t$

$\frac{dM}{dt} = \frac{a^{t+dt} - a^t}{dt} = \frac{a^t\cdot a^{dt} - a^t}{dt} = a^t\left(\frac{a^{dt}-1}{dt}\right)$

the defination of e : $\frac{e^{dt}-1}{dt} = 1$

$\underbrace{x^2+y^2}_{S(x)} = 5^2$

$dS = 2x\cdot dx + 2y\cdot dy$

Limits

Formal definition of a derivative $\frac{df}{dx}(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$ （有限且逼近于0的值 ✓ 无穷小的值 ✗ ）

(ϵ,δ) definition of

$y \to \epsilon$-distance y

$x \to \delta$-distance x

example: $\lim_{x\to 1} \frac{\sin(\pi x)}{x^2-1} = \frac{\cos\pi x\cdot\pi\cdot \cancel{dx}}{2x\cdot \cancel{dx}} = \frac{-\pi}{2}$

$\lim_{dx\to 0} \frac{f(x+dx)-f(x)}{dx} \Big< \begin{array}{l} \sin\pi x \Rightarrow d(\sin\pi x) = \cos\pi x\cdot\pi\cdot dx \\ x^2-1 \Rightarrow d(x^2-1) = 2x\cdot dx \end{array}$

L' Hopital 's rule

对 $f(x)$ 与 $g(x)$ 在 $x=a$ 可导，$f(a)=g(a)=0$

$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\frac{df}{dx}(a)\,dx}{\frac{dg}{dx}(a)\,dx} = \frac{f'(a)}{g'(a)}$

Antiderative

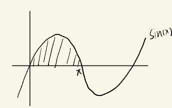
Integral 积分

$\frac{\int_a^b f(x)\,dx}{b-a} = \frac{F(b)-F(a)}{b-a}$

Average height $= \frac{\text{Area}}{\text{Width}} = \frac{\text{limit dot}}{\text{number}}$

$\frac{\int_0^\pi \sin(x)\,dx}{\pi} = \frac{\int_0^\pi \sin(x)}{\frac{\pi}{dx}}$

$\frac{dF}{dx} = f(x)$



$\frac{d\left(\frac{df}{dx}\right)}{dx} = \frac{d^2 f}{dx^2}$

Taylor series

在0附近：$P(x) = c_0 + c_1 x + c_2 x^2 + \cdots c_n x^n$

0值一致　导数一致　二阶导一致　n阶导一致

(a single point to approximation around that point)

What is a derivative

dt, ds — a tiny change

wrong : instantaneous rate of change

correct : the best constant approximation for rate of change