(1)

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Hafiz Rayyan Alam

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Chapter # 02

2.96 p.100, 2.101

2.96

Sol (0.4)(0.2) + 0.3(0.1) + 0.2(0.5) + 0.3(0.2)

$\Rightarrow 0.27$

2.100

$$P(C|H) = \frac{P(H|C)P(C)}{P(H|C)P(C) + P(H|B)P(B) + P(A|H)P(A)}$$

$$= \frac{(0.5)(0.23)}{(0.5)(0.23) + 0.49(0.37) + 0.39(0.42)} = \frac{0.115}{0.4433}$$

$= 0.126$

2.101

$$= \frac{(0.6)(0.75)}{(0.6)(0.75) + (0.3)(0.25)} = \frac{0.45}{0.525}$$

$= 0.857$

Chapter # 03

3.1, 3.9, 3.11, 3.20, 3.13 | 3.39, 3.44, 3.49, 3.57, 3.64, 36

3.1

X: Discrete
Y: Continuous
M: Continuous
N: Discrete
D: Discret
Q: Continuous

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3.9 $f(x) = \begin{cases} \frac{2(x+2)}{5} & 0 < x < 1 \\ 0 & \text{else where.} \end{cases}$

a) Show $P(0 < x < 1) = 1$

$\frac{2}{5}\int_0^1 (x+2)\,dx \Rightarrow \frac{2}{5}\left[\frac{x^2}{2} + 2x\right]_0^1$

$= \frac{2}{5}\left(\frac{1}{2} + 2\right) \Rightarrow \frac{2}{5}\left(\frac{5}{2}\right) \Rightarrow 1.$

b) $P(1/4 < x < 1/2)$

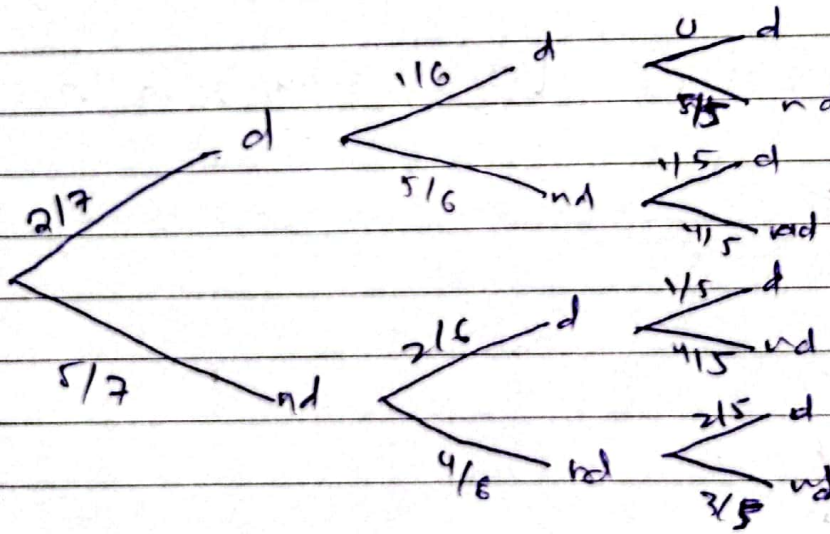
$= \frac{2}{5}\int_{1/4}^{1/2} (x+2) \Rightarrow \frac{2}{5}\left(\frac{(1/2)^2}{2} + 2(1/2)\right) - \frac{2}{5}\left(\frac{(1/4)^2}{2} + 2(1/4)\right)$

$= 0.0375$

3.11

7 tvs 2 detected 3 Random Selected

x No of defect Pd of x, Express as tree diagram



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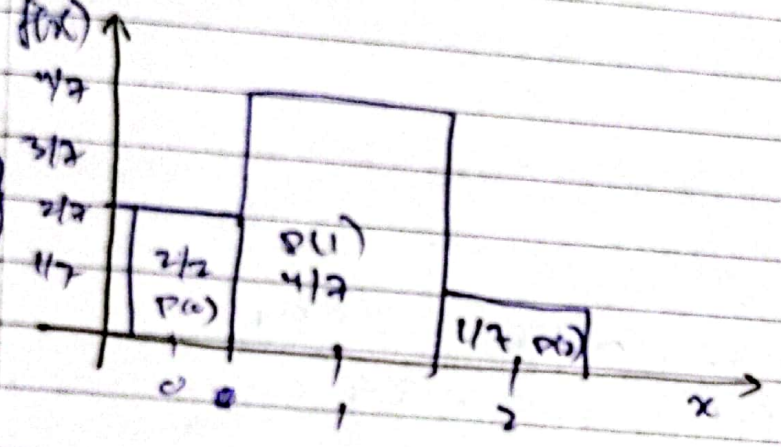
x	0	1	2
f(x)	3/7	4/7	1/7

$P(0) = {}^{(n\,n\,n)}\cdot \frac{{}^{3}C_3}{{}^{7}C_3} = \frac{1}{7}$

$P(1) = \frac{{}^{3}C_2 \times {}^{2}C_1}{{}^{7}C_3} = \frac{4}{7}$

$P(2) = \frac{{}^{3}C_1 \times {}^{2}C_2}{{}^{7}C_3} = \frac{1}{7}$



3.20 $F(x)$ & Evaluate $P(3 \le x < 4)$

$f(x) = \frac{2}{27}(1+x)$ $\quad 2 < x < 5$

$$\frac{2}{27}\int_{2}^{x}(1+x) \Rightarrow \frac{2}{27}\left[x + \frac{x^2}{2}\right]_2^x$$

$$= \frac{2}{27}\left(x + \frac{x^2}{2}\right) - \frac{2}{27}\left(2+2\right)$$

$$F(x) = \frac{2}{7}\left(x + \frac{x^2}{2}\right) - \frac{8}{27}$$

$$P(3 \le x \le 4) = \left[\frac{2}{7}\left(x + \frac{x^2}{2}\right) - \frac{8}{27}\right]_3^4$$

$$= \frac{7}{27}$$

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$$= \left[\frac{2}{27}\left(4+\frac{4^2}{2}\right) - \frac{8}{27}\right] - \left[\frac{2}{27}\left(3+\frac{3^2}{2}\right) - \frac{5}{27}\right]$$

$$= \frac{1}{3} \rightarrow P(3 \le x \le 4)$$

3.13

x	0	1	2	3	4
f(x)	0.41	0.37	0.16	0.05	0.01

$$\sum f(x) = 1$$

$F(0) = 0.41$

$F(1) = 0.41 + 0.37 = 0.78$

$F(2) = 0.41 + 0.37 + 0.16 = 0.94$

$F(3) = 0.94 + 0.05 = 0.99$

$F(4) = 0.99 + 0.01 = 1$

X	0	1	2	3	4
F(x)	0.41	0.78	0.94	0.99	1

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	x = 0	x = 1	x = 2	x = 3
y = 0	0	3/70	9/70	3/70
y = 1	2/70	12/70	12/70	2/70
y = 2	3/70	9/70	3/70	0

$$P(x+y \leq 2) = \sum_{y=0}^{2} \sum_{x=0}^{2-y} (P(x=x, y=y))$$

$$= \sum_{0}^{2} \sum_{0}^{2-y} (f(x,y))$$

$$= (f(0,0) + f(1,0) + f(2,0)) + (f(0,1) + f(1,1)) + (f(0,2))$$

$$= \left(0 + \frac{3}{70} + \frac{9}{70}\right) + \left(\frac{2}{70} + \frac{12}{70}\right) + \left(\frac{3}{70}\right)$$

$$= \frac{1}{2}$$

$$f(x,y) = \begin{cases} k(x^2+y^2) & 30 \leq x < 50 \ , \ 30 \leq y < 50 \\ 0 & \text{else} \end{cases}$$

$k = ?$

$$k \int_{30}^{50} \int_{30}^{50} x^2 + y^2 \, dx \, dy = 1$$

$$k \int_{30}^{50} \left(\frac{50^3}{3} + 50y^2\right) - \left(\frac{30^3}{3} + 30y^2\right) = 1$$

Now,

$$k \int_{30}^{50} \left(\frac{98000}{3} + 20y^2\right) dy = 1$$

$$k \left[\frac{98000}{3} y + \frac{20y^3}{3}\right]_{30}^{50} = 1$$

$$k \left(\frac{98000}{3}(50) + \frac{20(50)^3}{3}\right) - \frac{98000}{3}(30) + \frac{20(30)^3}{3}$$

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$k\left[\frac{20}{3}(50)^3 + \frac{98000}{3}(50) - \frac{20}{3}(30)^3 - \frac{98000}{3}(30)\right]$

$\frac{2940000}{3} - 278000 \quad \frac{3920000}{3} k$

$$K = \frac{3}{3920000}$$

b) $P(30 \le x \le 40 \text{ & } 40 \le y \le 50)$

$$\int_{30}^{40}\int_{40}^{50} \frac{3}{3920000}(x^2+y^2)\,dy\,dx \quad \text{| inner then outer}$$

$$= \frac{3}{3920000}\int_{30}^{40}\left[x^2y + \frac{y^3}{3}\right]_{40}^{50} dx$$

$$= \frac{3}{3920000}\int_{30}^{40}\left(50x^2 + \frac{y^3}{3}\right)_{40}^{50} dx$$

$$= \frac{3}{3920000}\int_{30}^{40}\left(10x^2 + \frac{61000}{3}\right) dx$$

$$= \frac{3}{3920000}\left[\frac{10}{3}(40)^3 + \frac{61000}{3}(40) - \frac{10}{3}(30)^3 - \frac{61000}{3}(30)\right]$$

$$= \frac{3}{3920000} \cdot \frac{980000}{3}$$

$$= \frac{1}{4}$$

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$$\int_{30}^{40}\int_{30}^{90} \frac{3}{3920000}\left(x^2+y^2\right)\,dx\,dy$$

$$\frac{3}{3920000}\left(\frac{10}{3}\cdot 40^3 + \frac{37000}{3}(40) - \frac{10}{3}\cdot 30^3 - \frac{37000}{3}(30)\right)$$

$$= \frac{3}{196}\left(\frac{37400000}{3}\right)$$

$$= \frac{37}{196}$$

(99)

f(x,y)	1	2	3	Total
1	0.05	0.05	0.1	0.2
3	0.05	0.10	0.35	0.5
5	0.00	0.2	0.1	0.3
Total	0.1	0.35	0.55	1

(y) (x)

Marginal of y

$$P(Y=3 \mid X=2) = \frac{0.10}{0.35} = \frac{2}{7}$$

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(57) $\begin{cases} kxy^2z & 0<x,y<1 \quad 0<z<2 \\ 0 & \text{else} \end{cases}$

a) $k = ?$

$$k \int_0^1 \int_0^1 \int_0^2 xy^2 z \, dz \, dy \, dx = 1$$

$$\Rightarrow k \int_0^1 \int_0^1 xy^2 \left| \frac{z^2}{2} \right|_0^2 dy \, dx \Rightarrow k \int_0^1 \int_0^1 xy^2 \cdot \left| \frac{z^2}{2} \right|_0^2$$

$$= k \int_0^1 \int_0^1 xy^2 \cdot 2 \cdot \Rightarrow 2k \int_0^1 \int_0^1 xy^2 = 1$$

$$= 2k \int_0^1 x \left| \frac{y^3}{3} \right|_0^1 \Rightarrow 2k \int_0^1 x \cdot \frac{1}{3} = 1$$

$$= \frac{2k}{3} \left| \frac{x^2}{2} \right|_0^1 = 1$$

$$= \frac{2k}{3} \left(\frac{1}{2} \right) \Rightarrow \frac{2k}{6} = 1$$

$$k = \frac{6}{2} \Rightarrow \boxed{k = 3}$$

b) $P\left(0<x<\frac{1}{4}, \frac{1}{2}<y<1, 1<z<2\right)$

$$= \int_0^{1/4} \int_{1/2}^1 \int_1^2 f(x,y,z) \, dz \, dy \, dx$$

$$= \int_0^{1/4} \int_{1/2}^1 \left(\frac{3xy^2 z^2}{2} \right) \Big|_1^2$$

$$= \int_0^{1/4} \int_{1/2}^1 \left(6xy^2 - \frac{3xy^2}{2} \right) dy \, dx$$

$$= \int_0^{1/4} \int_{1/2}^1 \left(\frac{9xy^2}{2} \right) dy \, dx$$

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$$= \int_0^{1/4} \left(\left(\frac{3xy^3}{2} \right)_{1/2}^{1} \right) dx \Rightarrow \int_0^{1/4} \left(\left(\frac{3x}{2} - \frac{3x\left(\frac{1}{2}\right)^3}{2} \right) \right) dx$$

$$= \int_0^{1/4} \frac{21x}{16} \, dx$$

$$= \left| \frac{21}{16} x \right|_0^{1/4} \Rightarrow \boxed{\frac{21}{512}}$$

Q64 -

a) $P(0 < x < 1/2 \, , \, 0 < y < 1/2)$

$$= \int_0^{1/2} \int_0^{1/2} \frac{3}{2}(x^2 + y^2) \, dy \, dx$$

$$= \frac{3}{2} \int_0^{1/2} \left(x^2 y + \frac{y^3}{3} \Big|_0^{1/2} \right) dx$$

$$= \frac{3}{2} \int_0^{1/2} \frac{1}{2} x^2 + \frac{1}{24} \, dx \Rightarrow \frac{3}{2} \left[\frac{x^3}{6} + \frac{x}{24} \right]_0^{1/2}$$

$$\boxed{= 1/16}$$

b) $P(0.75 < x < 1 \, , \, 0 < y < 1)$

$$= \frac{3}{2} \int_{0.75}^{1} \int_{0}^{1} (x^2 + y^2) \, dy \, dx \Rightarrow \frac{3}{2} \int_{0.75}^{1} x^2 y + \frac{y^3}{3} \Big|_0^1 \, dx$$

$$= \frac{3}{2} \int_{3/4}^{1} x^2 + \frac{1}{3} \Rightarrow \frac{3}{2} \left[\frac{x^3}{3} + \frac{1}{3} x \right]_{3/4}^{1}$$

$$\boxed{= \frac{53}{128}}$$

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4.7

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.4 & 1 \le x < 3 \\ 0.6 & 3 \le x < 5 \\ 0.8 & 5 \le x < 7 \\ 1.0 & 7 \ge x \end{cases}$$

a) Prob mass function of x

x	1	3	5	7
f(x)	0.4	0.2	0.2	0.2

4.12

b) $P(5 \le x < 7)$

↳ $P(5), P(6), P(7)$.

$= 0.2 + 0 + 0.2 \Rightarrow \boxed{= 0.4}$

Chapter # 04

4.1, 4.7, 4.12, 4.20 | 4.34, 4.45, 4.58

4.1

x	0	1	2	3	4
f(x)	0.41	0.37	0.16	0.05	0.01

$E(x) = \text{Exp} \Rightarrow 0(0.41) + 1(0.37) + 2(0.16) + 3(0.05) + 4(0.01)$

$= 0.88$

4.7

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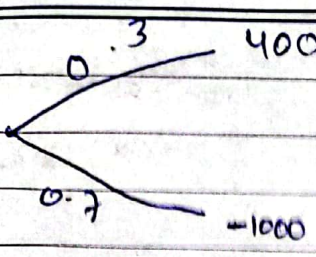
4.7



x	4000	-1000
P(x)	0.3	0.7

Mean
$= 4000(0.3) + (-1000)(0.7)$
$= 500$

4.12 Profit $5000

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1 \\ 0, & \text{else} \end{cases}$$

$$E(x) = \int x f(x) \Rightarrow 2\int_0^1 x(1-x)$$

$$= 2\int_0^1 x - x^2$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 \Rightarrow 2\left[\frac{1}{2} - \frac{1}{3}\right] = 2[$$

$$= \frac{1}{3}$$

Avg profit $= \frac{5000}{3} \rightarrow 1666.67$

(0.0)

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Q20 $f(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & \text{else} \end{cases}$ find output d. $g(x) = e^{2x/3}$

$$E(x) = \int_{-\infty}^{\infty} g(x) f(x)$$

$$= e^{-2x/3} e^{-x} \Rightarrow \int_0^{\infty} e^{\frac{2x}{3} - x}$$

$$= \int_0^{\infty} e^{-1/3 x} \Rightarrow \left[e^{-1/3 x} \div \left(\frac{-1}{3}\right) \right]$$

$$\Rightarrow -3 e^{-1/3} \Big|_0^{\infty}$$

$$= 3 \quad \text{Ans)}$$

4.34

x	-2	3	5
f(x)	0.3	0.2	0.5

$$\sigma^2 = (X - \sigma)^2 (f(x)) = E(x^2) - (\mu)^2$$

$E(x^2) \Rightarrow \Sigma n f(n)$

$= (-2)^2 (0.3) + (3)^2 (0.2) + 5^2 (0.5)$

$= 15.5$ ~~$E(X) = 2.5$~~

$\sigma^2 = 15.5 - (2.5)^2$

$= 9.25$

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(4.5) Covariance of x & y

$\sigma_{xy} = E(XY) - \mu_x \mu_y$

f(x,y)	1	2	3	Total
1	0.05	0.05	0.10	0.2
3	0.05	0.10	0.35	0.5
5	0.00	0.20	0.10	0.3
	0.1	0.35	0.55	1

$E(xy) = (1)(1)(0.05) + 2(1)(0.05)$
$+ 3(1)(0.10) + 2(3)(0.10) + (3)(3)(0.35)$
$+ (1)(5)(0) + (2)(5)(0.20) + (3)(5)(0.10)$
$= 7.85$

$\mu_x = 1(0.1) + 2(0.35) + 3(0.55) = 2.45$
$\mu_y = 1(0.2) + 3(0.5) + 5(0.3) = 3.2$

$\sigma_{xy} = 7.7 - 2.4(3.2) = 0.01$

(b) $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 \le x \le 2 \\ 0, & \text{else} \end{cases}$

$E[g(x) \pm h(x)] = E(g(x)) \pm E(h(x))$

$E(x) = \int_0^1 x(x)\,dx + \int_1^2 (2-x)x\,dx$

$= \frac{x^3}{3}\Big|_0^1 + \frac{2x^2}{2} - \frac{x^3}{3}\Big|_1^2$

$= \frac{1}{3} + \left[\left(\frac{2(4)}{2} - \frac{8}{3}\right) - \left(\frac{2}{2} - \frac{1}{3}\right)\right]$

$= 1$

~~E(x~~

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$$E(x^2) = \int x^2 f(x)$$

$$= \int_0^1 x^2 f(x)\,dx + \int_1^2 x^2 (2-x)\,dx$$

$$\frac{x^4}{4}\Big|_0^1 + \frac{2}{3}x^3 - \frac{1}{4}x^4\Big|_1^2$$

$$= \frac{7}{6}$$

$$E(Y) = E(60x^2 + 39x)$$

$$= 60(E(x^2)) + 39E(x)$$

$$= 60\left(\frac{7}{6}\right) + 39(1)$$

$$= 109$$