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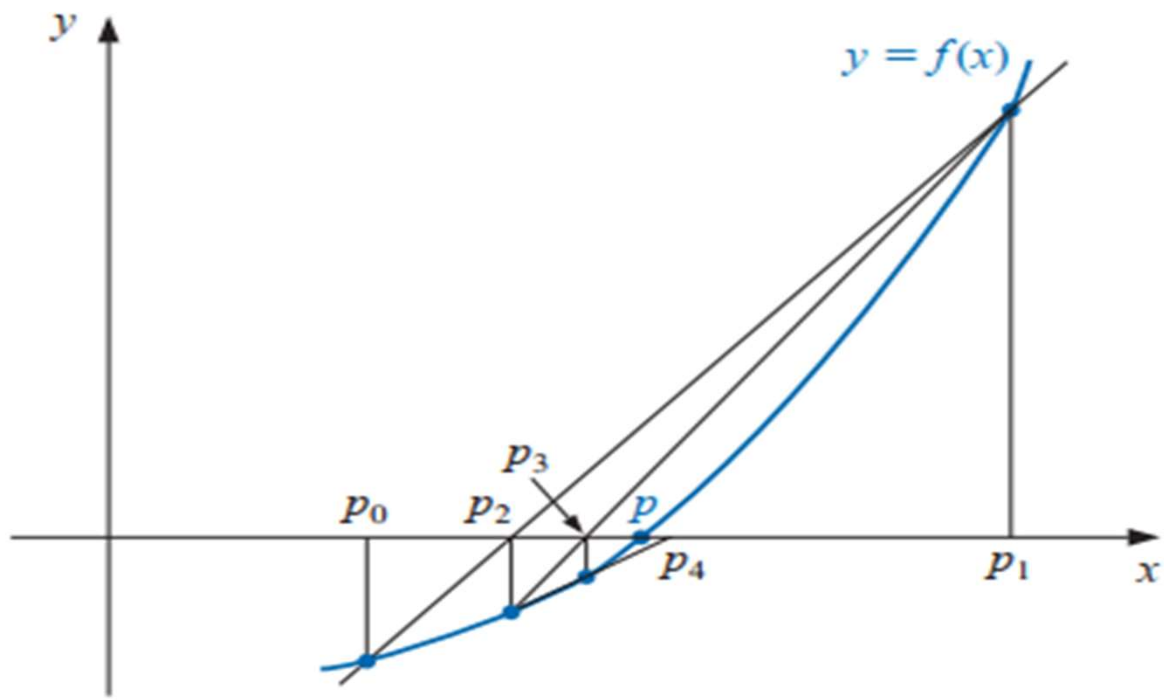
Lecture # 07 & 08

Secant & False Position Methods

(Regula Falsi or Linear Interpolation Method)

Secant Method:



Secant Method:

$$p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})}.$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

To find a solution to $f(x) = 0$ given initial approximations p_0 and p_1:

INPUT initial approximations p_0, p_1; tolerance TOL; maximum number of iterati

OUTPUT approximate solution p or message of failure.

Step 1 Set $i = 2$;
$q_0 = f(p_0)$;
$q_1 = f(p_1)$.

Step 2 While $i \le N_0$ do Steps 3–6.

Step 3 Set $p = p_1 - q_1(p_1 - p_0)/(q_1 - q_0)$. (*Compute* p_i.)

Step 4 If $|p - p_1| < TOL$ then
OUTPUT (p); (*The procedure was successful.*)
STOP.

Step 5 Set $i = i + 1$.

Step 6 Set $p_0 = p_1$; (*Update* p_0, q_0, p_1, q_1.)
$q_0 = q_1$;
$p_1 = p$;
$q_1 = f(p)$.

Step 7 OUTPUT ('The method failed after N_0 iterations, $N_0 =$', N_0);
(*The procedure was unsuccessful.*)
STOP.

Problem Statement. Use the secant method to estimate the root of $f(x) = e^{-x} - x$. Start with initial estimates of $x_{-1} = 0$ and $x_0 = 1.0$.

Solution. Recall that the true root is 0.56714329. . . .

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

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First iteration:

$$x_{-1} = 0 \qquad f(x_{-1}) = 1.00000$$

$$x_0 = 1 \qquad f(x_0) = -0.63212$$

$$x_1 = 1 - \frac{-0.63212(0 - 1)}{1 - (-0.63212)} = 0.61270 \qquad \varepsilon_t = 8.0\%$$

Second iteration:

$$x_0 = 1 \qquad f(x_0) = -0.63212$$
$$x_1 = 0.61270 \qquad f(x_1) = -0.07081$$

(Note that both estimates are now on the same side of the root.)

$$x_2 = 0.61270 - \frac{-0.07081(1 - 0.61270)}{-0.63212 - (-0.07081)} = 0.56384 \qquad \varepsilon_t = 0.58\%$$

Third iteration:

$$x_1 = 0.61270 \qquad f(x_1) = -0.07081$$

$$x_2 = 0.56384 \qquad f(x_2) = 0.00518$$

$$x_3 = 0.56384 - \frac{0.00518(0.61270 - 0.56384)}{-0.07081 - (-0.00518)} = 0.56717 \qquad \varepsilon_t = 0.0048\%$$

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Use the Secant method to find a solution to $x = \cos x$, and compare the approximations with those given in Example 1 which applied Newton's method.

n	p_n
0	0.7853981635
1	0.7071067810
2	0.7602445972
3	0.7246674808
4	0.7487198858
5	0.7325608446
6	0.7434642113
7	0.7361282565

	Newton
n	p_n
0	0.7853981635
1	0.7395361337
2	0.7390851781
3	0.7390851332
4	0.7390851332

Use the Secant method to find a solution to $x = \cos x$, and compare the approximations with those given in Example 1 which applied Newton's method.

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

Use the Secant method to find a solution to $x = \cos x$, and compare the approximations with those given in Example 1 which applied Newton's method.

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1	0.7395361337
2	0.7390851781
3	0.7390851332
4	0.7390851332

	Secant
n	p_n
0	0.5
1	0.7853981635
2	0.7363841388
3	0.7390581392
4	0.7390851493
5	0.7390851332

Root Finding Methods:

- *Bracketing methods*. As the name implies, these are based on two initial guesses that "bracket" the root—that is, are on either side of the root.
- *Open methods*. These methods can involve one or more initial guesses, but there is no need for them to bracket the root.

False Position Method:

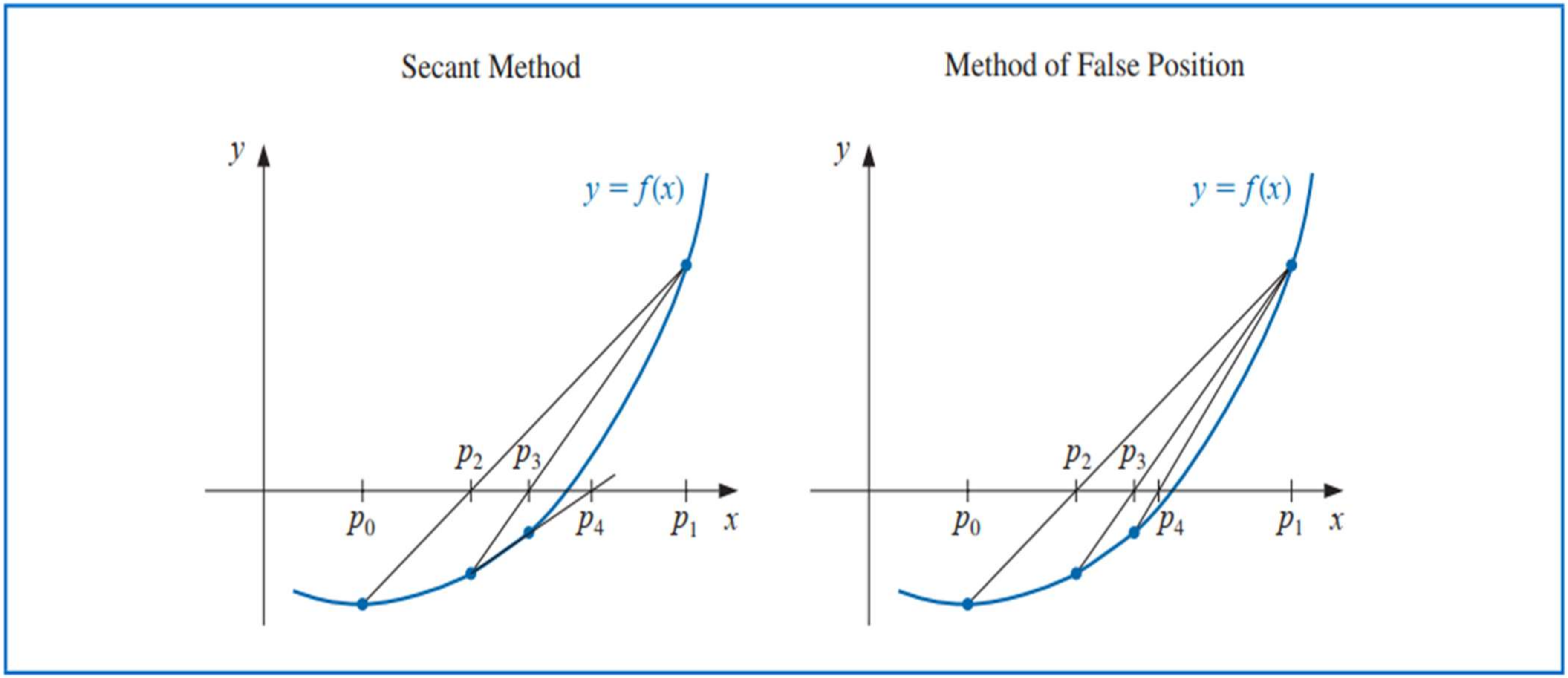
The **method of False Position** (also called *Regula Falsi*) generates approximations in the same manner as the Secant method, but it includes a test to ensure that the root is always bracketed between successive iterations

The term *Regula Falsi*, literally a false rule or false position, refers to a technique that uses results that are known to be false, but in some specific manner, to obtain convergence to a true result. False position problems can be found on the Rhind papyrus, which dates from about 1650 B.C.E.

Working/Iteration Rule:

First choose initial approximations p_0 and p_1 with $f(p_0) \cdot f(p_1) < 0$. The approximation p_2 is chosen in the same manner as in the Secant method, as the x-intercept of the line joining $(p_0, f(p_0))$ and $(p_1, f(p_1))$. To decide which secant line to use to compute p_3, consider $f(p_2) \cdot f(p_1)$, or more correctly $\text{sgn}\, f(p_2) \cdot \text{sgn}\, f(p_1)$.

- If $\text{sgn}\, f(p_2) \cdot \text{sgn}\, f(p_1) < 0$, then p_1 and p_2 bracket a root. Choose p_3 as the x-intercept of the line joining $(p_1, f(p_1))$ and $(p_2, f(p_2))$.
- If not, choose p_3 as the x-intercept of the line joining $(p_0, f(p_0))$ and $(p_2, f(p_2))$, and then interchange the indices on p_0 and p_1.


Secant Method
Method of False Position
y
$y = f(x)$
p_0
p_1
p_2
p_3
p_4
x

Algorithm:

To find a solution to $f(x) = 0$ given the continuous function f on the interval $[p_0, p_1]$ where $f(p_0)$ and $f(p_1)$ have opposite signs:

INPUT initial approximations p_0, p_1; tolerance TOL; maximum number of iterations N_0.

OUTPUT approximate solution p or message of failure.

Step 1 Set $i = 2$;
$q_0 = f(p_0)$;
$q_1 = f(p_1)$.

Step 2 While $i \le N_0$ do Steps 3–7.

Step 3 Set $p = p_1 - q_1(p_1 - p_0)/(q_1 - q_0)$. (*Compute p_i.*)

Step 4 If $|p - p_1| < TOL$ then
OUTPUT (p); (*The procedure was successful.*)
STOP.

Step 5 Set $i = i + 1$;
$q = f(p)$.

Step 6 If $q \cdot q_1 < 0$ then set $p_0 = p_1$;
$q_0 = q_1$.

Step 7 Set $p_1 = p$;
$q_1 = q$.

Step 8 OUTPUT ('Method failed after N_0 iterations, $N_0 =$', N_0);
(*The procedure unsuccessful.*)
STOP.

Example: Find the solution $x = cosx$

Solution To make a reasonable comparison we will use the same initial approximations as in the Secant method, that is, $p_0 = 0.5$ and $p_1 = \pi/4$. Table 2.6 shows the results of the method of False Position applied to $f(x) = \cos x - x$ together with those we obtained using the Secant and Newton's methods. Notice that the False Position and Secant approximations agree through p_3 and that the method of False Position requires an additional iteration to obtain the same accuracy as the Secant method. ■

	False Position	Secant	Newton
n	p_n	p_n	p_n
0	0.5	0.5	0.7853981635
1	0.7853981635	0.7853981635	0.7395361337
2	0.7363841388	0.7363841388	0.7390851781
3	0.7390581392	0.7390581392	0.7390851332
4	0.7390848638	0.7390851493	0.7390851332
5	0.7390851305	0.7390851332	
6	0.7390851332		

Example:

Consider finding the root of f(x) = x^2 - 3. Let ε = 0.01, and start with the interval [1, 2].

Example:

Consider finding the root of $f(x) = x^2 - 3$. Let $\varepsilon = 0.01$, and start with the interval [1, 2].

a	b	f(a)	f(b)	c	f(c)	Update	Step Size
1.0	2.0	-2.00	1.00	1.6667	-0.2221	a = c	0.6667
1.6667	2.0	-0.2221	1.0	1.7273	-0.0164	a = c	0.0606
1.7273	2.0	-0.0164	1.0	1.7317	0.0012	a = c	0.0044

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Solve:

$x^3 - 2x - 5 = 0$, by using regula falsi method ($\varepsilon = 0.0001$)

Root = 2.094547

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Sol:

Step	x0	x1	x2	f(x2)
1	2.000000	3.000000	2.058824	-0.390800
2	2.058824	3.000000	2.081264	-0.147204
3	2.081264	3.000000	2.089639	-0.054677
4	2.089639	3.000000	2.092740	-0.020203
5	2.092740	3.000000	2.093884	-0.007451
6	2.093884	3.000000	2.094305	-0.002746
7	2.094305	3.000000	2.094461	-0.001012
8	2.094461	3.000000	2.094518	-0.000373
9	2.094518	3.000000	2.094539	-0.000137
10	2.094539	3.000000	2.094547	-0.000051

Do Q 1-10 from Ex # 2.3