National University of Computer & Emerging Sciences, Karachi
Department of Computer Science
Final Term , Spring-2020
6^{th} July 2020, 9:00 am – 12:00 noon

Course Name / Code : 1-NUMERICAL COMPUTING / CS-325 2-NUMERICAL METHODS / MT-207	
Instructor Name: M. Jamil Usmani , Mr.Nadeem Khan , Mr.M.Shahbaz	
Student Roll No:	Section:

Instructions:

- Attempt all question. **WRITE YOUR ID ON TOP OF EVERY PAGE by your hand.** Write also **page # on every page. You should also sign on every page**
- Read each question completely before answering it. There are **8 questions and 3 pages**.
- All the answers must be solved according to the sequence given in the question paper.
- You will attempt this paper **offline**, in your **hand writing**.
- You may use **cam-scanner**, **MS lens** or any equivalent application to scan and convert your hand-written answer sheets in **a single PDF file**
- No submission will be accepted after the specified time. (**After 12:30 pm**).

Time: 180 minutes **Max Marks**: 100 points

Question 1: **[15]**

a) Let $P(x) = x^3 - 3x^2 + 3x - 1\,,\; Q(x) = ((x-3)x+3)x - 1$

Use three digit rounding to compute approximation to $P(2.19)$ and $Q(2.19)$

Calculate absolute and relative error if true values are $P(2.19) = Q(2.19) = 1.685159$

b) Consider Maclaurine series of $f(x) = cosx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

Calculate true and approximate (estimated) relative error at $x = {}^{\pi}/_{4}$

Use five number of decimal places.

c) Use difference operator to show that (any two)

i. $\Delta\nabla = \nabla\Delta = \nabla - \Delta$

ii. $\Delta = \mu\delta + \frac{\delta^2}{2}$

iii. $\mu^2 = 1 + \frac{1}{4}\delta^2$

Question 2: **[10+5]**

a) Perform two iteration and complete the following table with five decimal places. Use Secant method.

$f(x) = 3x + sinx - e^x$, $[0,1]$ $accurate\ to\ within\ \epsilon = 10^{-5}$

Iteration	$\boldsymbol{x_{n-1}}$	$\boldsymbol{x_n}$	$\boldsymbol{x_{n+1}}$	$\boldsymbol{f(x_{n+1})}$	$\boldsymbol{x_{n+1} - x_n}$
1					
2					
3					
4					
5					
6					

b) Use iterative method to approximate $7^{\frac{1}{3}}$ correct up to seven decimal , where $x_0 = 2$

Question 3: **[5+3+7]**

a) Find an interpolating polynomial for the data points $(0,1), (2,2), (3,4)$

b) Consider $\sqrt{15500} = 124.4990$, $\sqrt{15510} = 124.5392$, $\sqrt{15520} = 124.5793$ and $\sqrt{15530} = 124.6194$ Construct Simple difference table.

c) Use Newton formula to approximate $f(0.05)$ and find the absolute error.

$f(x) = e^{3x}\ for\ \ 0 \le x \le 0.4$, $h = 0.1$, Display four decimal places.

Question 4: **[5+5]**

The distance 'x' of a runner from a fixed point is measured (in meters) in given table.

Time(t)	0.2	0.4	0.6	0.8	1.0
Distance(x)	0.9798652	0.9177710	0.8080348	0.6386093	0.3843735

a) Approximate the runner's velocity at time $t = 0.2$ and $t = 1.0$ sec.

b) Approximate the runner's velocity and acceleration at time $t = 0.6$ sec.

Question 5: **[4+6]**

Find an approximation up to five decimal places to the integral $\int_0^{12} \frac{dx}{1+x^2}$, $n = 6$, use

a) Composite trapoizadal rule

b) Composite Simpson's $\frac{1}{3}rd\ and\ \frac{3}{8}th$ rules.

Question 6: **[15]**

Solve the differential equation .Show all steps and complete the following table with five number of decimal places,

$$\frac{dy}{dt} = f(t,y) = \frac{1+t}{1+y} \quad ,1 \le t \le 2 \ ,y(1) = 2\ ,step\ size\ (h) = 0.5$$

a) Modify Euler **OR** Mid-Point method
b) 4^{th} order Runge-Kutta method

Compute Absolute error for each method if true solution is $y(t) = \sqrt{t^2 + 2t + 6} - 1$

t_i	**Exact** $y_i = y(t_i)$	**Modify Euler** w_i	**4^{th} RungeKutta** w_i	**Error** $\lvert y_i - w_i \rvert$

Question 7: **[7+3]**

a) Solve $AX = b$ for the following system of linear equation

$$\begin{bmatrix} 1 & 1 & 5 \\ -3 & -6 & 2 \\ 10 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -21.5 \\ -61.5 \\ 27 \end{bmatrix} \text{, where Intial guess value } \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Perform three iteration of Gauss Seidal and approach the true solution.
Display at least five decimal places.

b) Check whether the symmetric matrix $\begin{bmatrix} 2 & 4 \\ 4 & 5 \end{bmatrix}$ is positive definite or diagonally dominant.

Question 8: **[5+5]**

a) Consider $A = \begin{pmatrix} 4 & -1 & 1 \\ -1 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}$, Determine Choleskey $\boldsymbol{LDL^t}$ factorization.

b) Solve the following linear system $\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$

ALL THE BEST