Question no 1: Estimated time to solve 40 minutes

I- b

II- c

III- a

IV-d

V-b

VI- e

VII- a

VIII-d

IX-b

X- c

XI- e

XII-b

XIII- e

XIV-b

XV- a

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix} = LU$$

Question no 2:

Sal Quo à def Heun's (f, t-initial, t-final, y-initial, h) t= t-initial. n=int(t-final-t-initial)/h) time values = [t] y-values = [y] for i in range (1, n+1)

K1 = f(t,y) 12= f(+h, Y+K1h) y = y + (K1/2 + K2/2) h t = t+h time-values append (t)
y-values append (y) return time-value, y-values. Example: 1/25t+y2/ y(0)=0.

Qno3- CLO 2 5+5 Marks

a) Find the truncation error of $e^{0.5}$ when the approximation of $e^{0.5}$ is the summation of the first four terms of the McLaurin's (special case of Taylor's series when $x_0 = 0$) series of $e^{0.5}$.

b) Evaluate $f(t) = t^3 - 6.1t^2 + 3.2t + 1.5$ at t = 4.71 using three-digit arithematic (rounding and chopping). Assume the true value f(4.71) = -14.263899 Calculate relative error for both chopping and rounding.

Question no Z.

Question no Z.

$$e^{3} = 1 + n + \frac{n^{2} + n^{3}}{2!} \cdot (2.5 \text{ Madd}) \cdot 4$$
 $e^{1} = 1 + n + \frac{n^{2} + n^{3}}{2!} \cdot (2.5 \text{ Madd}) \cdot 4$
 $e^{1} = 1 + n + \frac{n^{2} + n^{3}}{2!} \cdot (2.5 \text{ Madd}) \cdot 4$
 $e^{1} = (2.887 \times 10^{3}) \cdot (2.5 \text{ Madd}) \cdot 6$
 $e^{1} = (2.887 \times 10^{3}) \cdot (2.5 \text{ Madd}) \cdot 6$
 $e^{1} = (4.71)^{3} - (6.1)(4.71)^{2} + 3.2(4.71) + 1.5$
 $e^{1} = (4.71)^{3} - (6.1)(4.71)^{2} + 3.2(4.71) + 1.5$
 $e^{1} = 104 - 135 + 15.1 + 1.5$
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Qno4 – CLO 2 5+5 Marks

a) With the help of the below mentioned nodes, find the second Lagrange interpolating poynomial for $f(t) = \frac{1}{t}$; $t_0 = 2$, $t_1 = 2.75$ and $t_2 = 4$

b) A computer scientist wants to estimate the total time it would take to process a large dataset using different algorithms. The time taken by each algorithm is given by the following function: $T(n) = 2n^4 + 5n + 10$, where n is the size of the dataset.

- i) Using simple Trapezoidal Rule and simple Simpson's Rule estimate the total processing time for the dataset if the size of the dataset ranges from 0 to 2.
- ii) Also calculate bound error for simple Trapezoidal and Simpson's rule and compare with actual error

Question no 4. (a)

$$P_{2}(t) = L_{0}(t)f(t_{0}) + L_{1}(t)f(t_{1}) + L_{2}(t)f(t_{2})$$

$$L_{0}(t) = \frac{(t_{1}-2.75)(t_{1}-4)}{(2-2.75)(2-4)} = \frac{t_{1}^{2}-6.75t_{1}}{(1 \text{ Mark})}$$

$$L_{1}(t) = \frac{(t_{1}-2)(t_{1}-4)}{(2.75-2)(2.75-4)} = \frac{t_{1}^{2}-6t_{1}+8}{-0.9375}$$

$$L_{2}(t) = \frac{(t_{1}-2)(t_{1}-2.75)}{(4-2)(4-2.75)} = \frac{t_{1}^{2}-4.75t_{1}+6.5}{(1 \text{ Mark})}$$

$$P_{2}(t) = \frac{1}{2} \left(\frac{t_{1}-2}{t_{1}-2.75}\right) + \frac{1}{2.75} \left(\frac{t_{1}-6t_{1}+8}{2.75}\right) + \frac{1}{2.75} \left(\frac{t_{1}-6t_{1}+8}{2.75}\right)$$

$$= \frac{(0.333+0.16-1.4222)t_{1}^{2}+(-2.25+2.327-0.475)t_{1}+3.67-3.103+0.55}{2.9385}$$

$$L_{2}(t) = \frac{(0.333+0.16-1.4222)t_{1}^{2}+(-2.25+2.327-0.475)t_{1}+3.67-3.103+0.55}{2.9385}$$

 $\frac{1}{\sqrt{\frac{1}{2}}} \frac{1}{\sqrt{\frac{1}{2}}} = \frac{1}{\sqrt{\frac{1}{2}}} \frac{1}{\sqrt{\frac{1}{2}}} = \frac{1}{\sqrt{\frac{1}{2}}} \frac{1}{\sqrt{\frac{1}{2}}} = \frac{1}{\sqrt{\frac{1}{2}}} \frac{1}{\sqrt{\frac{1}{2}}} = \frac{1}{\sqrt{\frac{1}{2}}} =$ $\int_{0}^{2} \frac{1}{1} (n) c \ln 2 \int_{0}^{2} \left[52 + 4(17) + 10 \right] = 43.33.$ $\int_{-\infty}^{\infty} \frac{1}{1} du = \int_{-\infty}^{\infty} \frac{2}{1} 2n^{4} + 5n + 10 du = -42.8$ $T''(n) = 2.4n^2$ T''(n) = 48Error bound for Trap = (2)3 x96=64 9 / Sim z (1) x 48=0.533. Actual error Trap 2 1428-48/25.2 Adnal espor Sim = 142.8-43.33/ = 0.533

A computer scientist is studying the performance of a computer algorithm and wants to estimate the execution time (in seconds) based on the input size (in number of elements). The scientist collects data from four different experiments, where the input size and execution time are recorded. The dataset is as follows:

Input Size (x)	2	3	4
Expected time (y)	7	10	13

In this dataset, the scientist has measured the execution time for different input sizes. The input size (x) represents the number of elements in the dataset, while the execution time (y) represents the time taken by the algorithm to process the dataset.

The scientist needs assistance to apply gradient descent method on the above data set to find the optimal parameters for the function. For this you are supposed to help him by considering the following: $h_{\theta} = \sum_{j=0}^{j=1} \theta_j x_j, where \ let \ x_0 = \mathbf{1}$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{i=m} (h_{\theta_0 \theta_1}(x_{1j}) - y_j)^2 \quad let \ \boldsymbol{\theta_0} = \boldsymbol{\theta_1} = \mathbf{0}(initially)$$

$$\theta_j = \theta_j - \propto \frac{\partial y}{\partial \theta_i}$$
 let $\propto = 0.05$

Note: Just perform three iterations.

	Iteration	theta		J	Partial deri	vatives	
	1	[0. 0.]	 	53	[-1032.]		i
į		[0.5 1.6]	į	11.6983	[-4.7	-15.03333333]	İ
	3	[0.735	2.35166667]	2.58216	[-2.21	-7.06222222]	
	4	[0.8455	2.70477778]	0.570025	[-1.04016667	-3.31731481]	

<u>Qno6 – CLO 2 4+6 Marks</u>

For the following linear system,

$$-3x_1 + x_2 + 12x_3 = 5$$

$$6x_1 - x_2 - x_3 = 31$$

$$6x_1 + 9x_2 + x_3 = 4$$

- (i) Compute true/actual solution of above system(Use Calculator).
- (ii) Without any arrangement, solve the above system numerically by using Gauss-Seidal method with $X^{(0)} = (0,0,0)^t$ and perform three iterations. Compare the results with part(i) and state whether the above system is convergent or not for Gauss-Seidal Method.

Note: If the answer of above part ((ii)) is divergent then work on the below part ((iii)) otherwise leave it.

(iii) Make the above system convergent by making suitable arrangement and then solve it by Gauss-Seidal method with $X^{(0)}=(0,0,0)^t$ and stops when $\|X^{(k+1)}-X^{(k)}\|_{\infty}<0.01$

Solution			
	0#1		
i) Actua	N Solution = (L	1-972225-3	08383
		1.916	
	(2 Mark	6)	
i) Ib	erative Scheme	(1 Mar	k)
	-July 2 = (5	- X2 - 12x3)7-3
	X 2 (K-H) = (73-31	
	7(3(K+1) = (- 72 - 1223 - 73 - 1223 - 24 - 23 - 31	(k+1)
	K=0,1,2		
Ite	valinas 12 P	Manles)	
K	1 7((K+1)	X2(K+1)	7(3(KH))
0	1-1.66667	-41	383
(1516.66667	8686	-87270
2	- 346186-3333	-1989879	1998603
	134.97222	4>-3.08383	1 +> 1.91667
	Divergenc	e	

-			Duie_	
(35		& Ear 3	are interes	changed and
	then	er Ove	interchanges	1 with ev O
	(1 Ma	uk)		
	New Sys	item is,	6x1-x	2-73=31
	0		6x1+9	X2+ X3 = 4
			-321+7	12+1223=5
	above 's	ystem is	strictly dia	gonally
	duminan	t =0 It	will Conve	rse
-	The altime.	Scheme is	(1 Mo	ink)
	$\chi_1 =$	(31+2	2+23),1	6
	X (K+1)	· (4 - 6x	(Karl) (3) (9
	723 =	= (5+3)	(K3-1) / (K3	12
I	terations	(3 Marks)		
K	7,(141)	76 (K+1)	7(3	112(c) - x(c) 16
_	5.16667	-3	1.95833	-
0	4.99306	-3.10185		0-17361
2	4.97026	-3.08278		0.62280
	4.97223	-3.08328		0.60197
3	4 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	3-08330	11.91667	< 0.01
_	1			,

<u>Qno7 – CLO 2 4+6 Marks</u>

For the following linear system,

$$x_1 + x_2 + x_3 = 2$$

 $x_1 + x_2 = 3$
 $x_1 + x_3 = 4$

- a) Write Co-efficient matrix A for above system
- b) Given that absolute dominant eigen value of A is $\lambda_{Absolute\ dominant}^{(Actual\ value)} = \sqrt{2} + 1$, find the approximate absolute dominant eigen value $\lambda_{Absolute\ dominant}^{(Approximated\ value)}$ by using Power method with $X^{(0)} = (-1,0,1)^t$ and stops when $\left| \lambda_{Absolute\ dominant}^{(Actual\ value)} \lambda_{Absolute\ dominant}^{(Approximated\ value)} \right| < 0.05$

(Ruestion not)

A (K+1)
$$X_{K+1} = AX_{K}$$
 1 Mark.

(Rich) $X_{K+1} = AX_{K}$ 1 Mark.

(Rich) $X_{K+1} = AX_{K} = X_{K+1} = X_{K+1$

Sumaci	red table	
K 2 CKHI)		stopping Ceicleia
0' -1	(0,1,0)	3.41421
2	(1,1,0)	0.41421
3 25	(1,0.8,0.6)	0.08579.
4 (2.4.)	(1,0.75,0.67)	0.0142160.05
absolute orgen vo	dominant lul of A.	
Macks disteiba	lion	K20 and K21
Aterations P	serfound sent	keo and keld oened at. Marks.
2 mails	teralistics 6	Marks.
(c = d, 3, the		

Qno8 - CLO 2 4+1+1+2+2 Marks

For the following linear system,

$$x_1 + 2x_2 - x_3 = 2$$

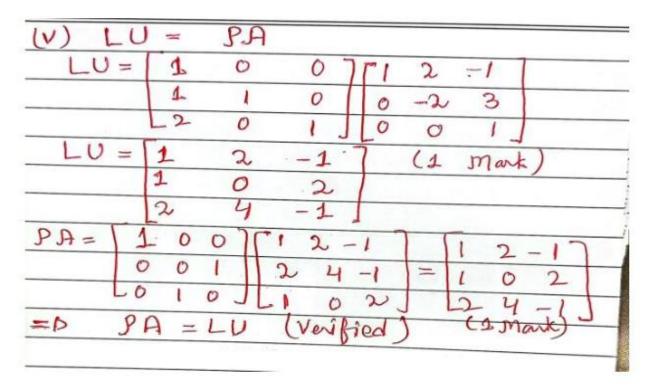
$$2x_1 + 4x_2 - x_3 = 3$$

$$x_1 + 2x_3 = 2$$

(i) Write the co-efficient matrix A for above system and find the upper triangular matrix U corresponding to A by applying row elementary operations on A.

- (ii) With the help of part (i), write the permutation matrix P
- (iii) If there is no row interchange performed in part(i) then $P = \underline{\hspace{1cm}}$ matrix.
- (iv) With the help of part(i) construct lower triangular matrix L ($l_{ii}=1, for\ all\ i$) such that PA=LU
- (v) Verify PA = LU

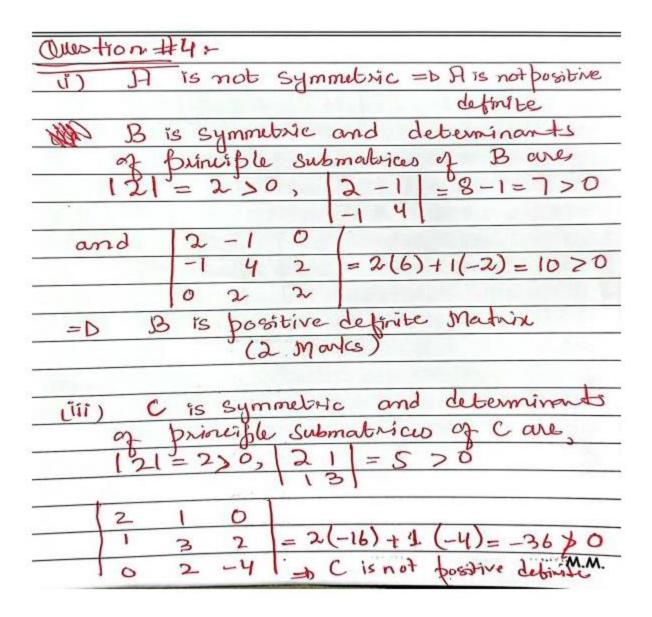
دن ه=	T1 2 -	17 (1 mark)
	2 4 -	
	L1 10 2	
R2 - 2R		
R3-R	1 0 0 2	L (2 Marks)
~	Lo -2 :	3
R 23	1 2 -1	
~	0 -2 3	= U (1 Mark)
	Lo 0 1	
(11) $S =$	1007	(1 Mark)
	0 0 1	
	Lo 1 0]	
("") P=		
	0 1 0	= Identity Matrix
	LO 0 1	(I mark)
(YY) L	= 1 0	0 7
	1 1	0 (2 Marks)
	120	1,

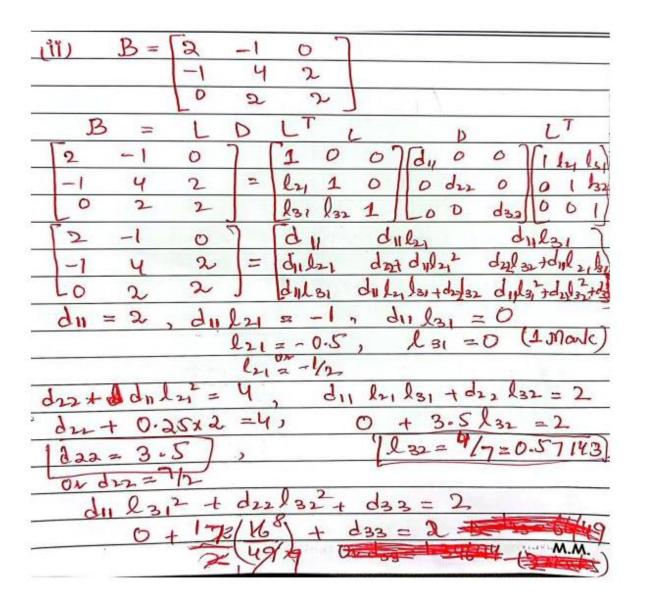


Qno9 – CLO 2 2+3+1+3+1 Marks

Consider,
$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$
, $B = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 2 \\ 0 & 2 & -4 \end{bmatrix}$

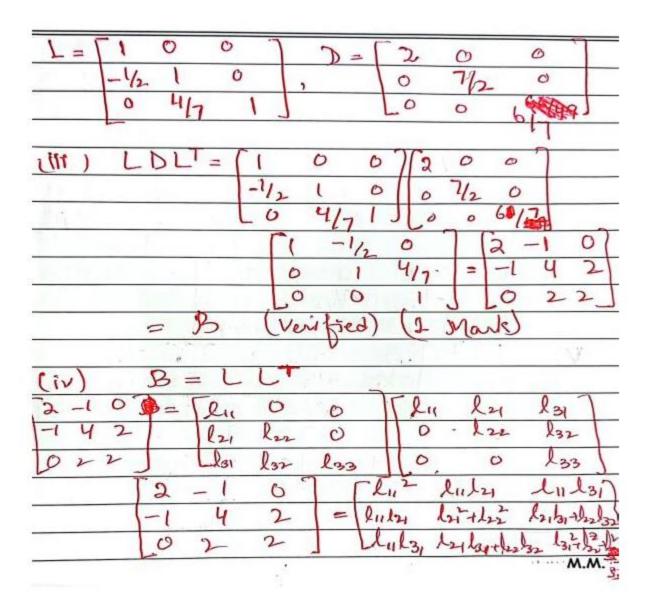
- (i) Identify from matrices A, B and C which one is positive definite?
- (ii) Decompose the matrix (answer of part(i)) into LDL^T where $l_{ii} = 1$ and $d_{ii} > 0$ for all i
- (iii) Verify part(ii) decomposition
- (iv) Decompose the matrix (answer of part(i)) into Cholesky decomposition (LL^T) wher $l_{ii} \neq 0$, for all i)
- (v) Verify part(iv) decomposition

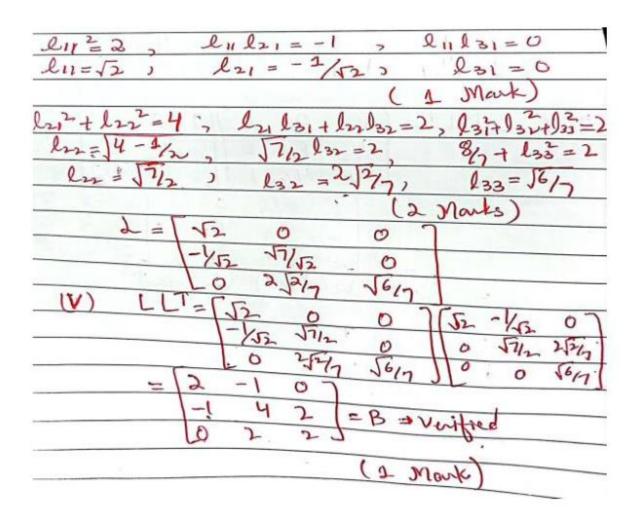




d33 = 6/

(2 Marks)





2+3+1+3+1 Marks Qno10 -

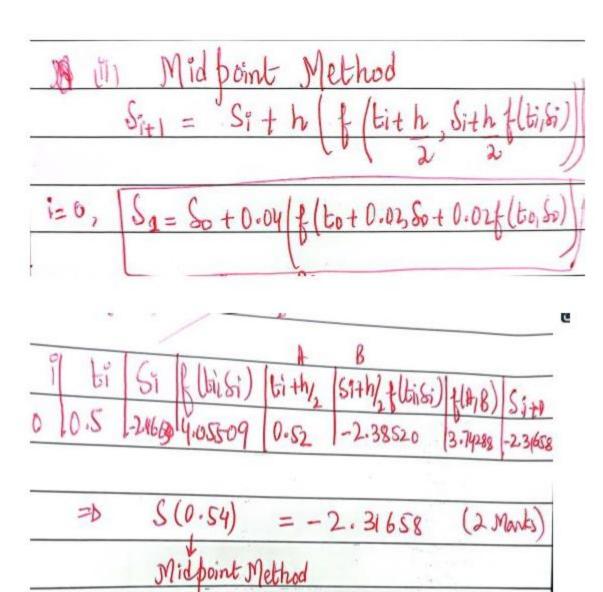
A particle is moving with velocity $v(t) = \frac{ds}{dt}(1+t) = s^2$, s(0.5) = -2.46630, where s is displacement at any time t. Find the displacement $s(\mathbf{0}.\mathbf{54})$ with h=0.04 by using

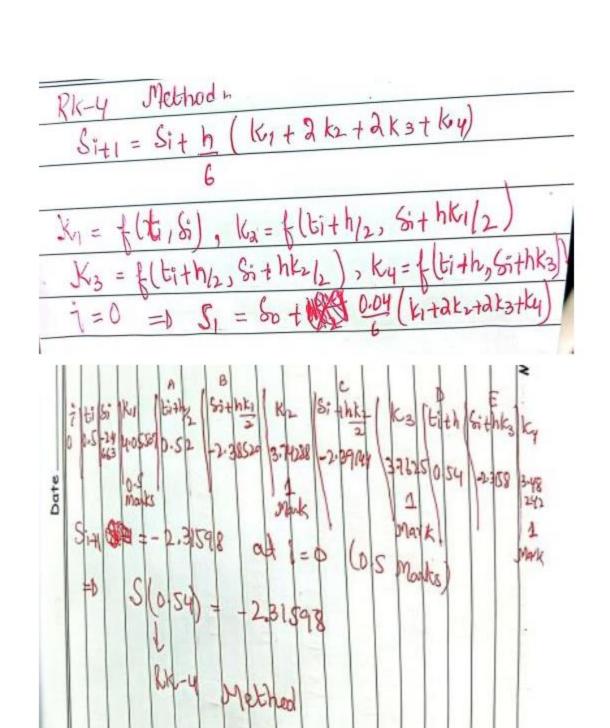
(ii) Midpoint Method and (iii) RK-4 Method (Classical approach). (i) Heun's Method

Given that $s(t) = -\frac{1}{\ln(t+1)}$ is the actual solution, compute actual error in each part (i),(ii) and (iii) also and comments on the result.

alustion #5 8 Heun's Method & ů) Itt f(tis) = s2/1+t1 f(ti, s;) | ti+1 | s;+0.04 f(ti, si) | f(A,B) ti -2.46630 4.05509 0.54 -2.30410 5:41 => S, (0.84) = -2.31625

Herm's (2 Marks)





T	Solution = S(t) = -	1 (t-11)
	= S(0.54) = -2	.31598
Acti	in Exers: (1 Mark)	
H	eun's Method = 0.0002	7
J	2k-4 (classia) Hipproach) = 000	(upto 5)
Com	rents = (1 Mark)	Plaus)
RK.	-4 Method gives better this quarton as compared of	approximation