Question no 1: Estimated time to solve 40 minutes

I- b

II- c

III- a

IV- d

V- b

VI- e

VII- a

VIII-d

IX-b

X- c

XI- e

XII-b

XIII- e

XIV- b

XV- a

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix} = LU$$

Question no 2:

Sol Q no 2.

```
def Heun's(f, t-initial, t-final, y-initial, h)
    t = t-initial.
    y = y-initial.
    n = int((t-final - t-initial)/h)
    time values = [t]
    y- values = [y]
    for i in range (1, n+1)
        K1 = f(t, y)
        K2 = f(t+h, y+K1 h)
        y = y + (K1/2 + K2/2) h.
        t = t + h
        time-values.append (t)
        y-values append (y)
        y-values.
    return time-values, y-values.
```

Example:

$$y' = 5t + y^2, \quad y(0) = 0$$

Qno3– **CLO 2** **5+5 Marks**

a) Find the truncation error of $e^{0.5}$ when the approximation of $e^{0.5}$ is the summation of the first four terms of the McLaurin's (special case of Taylor's series when x_0 =0) series of $e^{0.5}$.

b) Evaluate $f(t) = t^3 - 6.1t^2 + 3.2t + 1.5\ at\ t = 4.71$ using three-digit arithematic (rounding and chopping). Assume the true value $f(4.71) = -14.263899$ Calculate relative error for both chopping and rounding.

Q no 3.

Question no 3

a) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ (2.5 Marks)

Error = $\left| e^{0.5} - \left(1 + 0.5 + \frac{0.5^2}{2!} + \frac{(0.5)^3}{3!}\right)\right|$

Error = (2.887×10^{-3}) (2.5 Marks)

b) $f(t) = t^3 - 6.1t^2 + 3.2t + 1.5$

at $t = 4.71$

$f(t) = (4.71)^3 - (6.1)(4.71)^2 + 3.2(4.71) + 1.5$

$= 104 - 6.1 \times 22.2 + 15.1 + 1.5$

$= 104 - 135 + 15.1 + 1.5$

$= -14.4$ (rounding). (1.5 Mark)

$f(t) = (4.71)^3 - (6.1)(4.71)^2 + 3.2(4.71) + 1.5$

$= 104 - 135 + 15.0 + 1.5$

$= -14.5$ (chopping). (1.5 Mark)

Error:-

$\left(\frac{-14.263899 + 14.4}{-14.263899}\right) = 9.54 \times 10^{-3}$ (rounding) (1 Mark)

$\left(\frac{-14.263899 + 14.5}{-14.263899}\right) = 1.7 \times 10^{-2}$ (chopping) (1 Mark)

Qno4 – **CLO 2** **5+5 Marks**

a) With the help of the below mentioned nodes, find the second Lagrange interpolating poynomial for $f(t) = \frac{1}{t}$; $t_0 = 2$, $t_1 = 2.75$ *and* $t_2 = 4$

b) A computer scientist wants to estimate the total time it would take to process a large dataset using different algorithms. The time taken by each algorithm is given by the following function: $\boldsymbol{T(n) = 2n^4 + 5n + 10}$, where *n* is the size of the dataset.

i) Using simple Trapezoidal Rule and simple Simpson's Rule estimate the total processing time for the dataset if the size of the dataset ranges from 0 to 2.

ii) Also calculate bound error for simple Trapezoidal and Simpson’s rule and compare with actual error

Question no 4. (a)

$$P_2(t) = L_0(t) f(t_0) + L_1(t) f(t_1) + L_2(t) f(t_2)$$

$$L_0(t) = \frac{(t-2.75)(t-4)}{(2-2.75)(2-4)} = \frac{t^2 - 6.75t + 11}{1.5}$$

(1 Mark)

$$L_1(t) = \frac{(t-2)(t-4)}{(2.75-2)(2.75-4)} = \frac{t^2 - 6t + 8}{-0.9375}$$

(1 Mark)

$$L_2(t) = \frac{(t-2)(t-2.75)}{(4-2)(4-2.75)} = \frac{t^2 - 4.75t + 5.5}{2.5}$$

(1 Mark)

$$P_2(t) = \frac{1}{2}\left(\frac{t^2 - 6.75t + 11}{1.5}\right) + \frac{1}{2.75}\left(\frac{t^2 - 6t + 8}{-0.9375}\right) + \frac{1}{4}\left(\frac{t^2 - 4.75t + 5.5}{2.5}\right)$$

$$= (0.333 + 0.1 - 1.4222)t^2 + (-2.25 + 2.327 - 0.475)t + 3.67 - 3.103 + 0.55$$

$$= -0.9889t^2 - 0.398t + 1.117$$ ← 2 Mark

(b) i)

$T(n) = 2n^4 + 5n + 10$

$$\int_0^2 T(n)\,dn = \frac{2}{2}\left[52 + 10\right] = 62 \;;\; h = 2 - 0 = 2$$

Trap

$$\int_0^2 T(n)\,dn = \frac{1}{3}\left[52 + 4(17) + 10\right] = 43.33$$

Simpson

$$\int_0^2 T_n\,dn = \int_0^2 2n^4 + 5n + 10\,dn = 42.8$$

actual value

ii)

$T''(n) = 24n^2 \qquad T^{(iv)}(n) = 48$

Error bound for Trap $= \dfrac{(2)^3}{12} \times 96 = 64$

" " Sim $= \dfrac{(1)^5}{90} \times 48 = 0.533$

Actual error Trap $= |42.8 - 48| = 5.2$

Actual error Sim $= |42.8 - 43.33| = 0.533$

Qno5 – CLO 2 10 Marks

A computer scientist is studying the performance of a computer algorithm and wants to estimate the execution time (in seconds) based on the input size (in number of elements). The scientist collects data from four different experiments, where the input size and execution time are recorded. The dataset is as follows:

Input Size (x)	2	3	4
Expected time (y)	7	10	13

In this dataset, the scientist has measured the execution time for different input sizes. The input size (x) represents the number of elements in the dataset, while the execution time (y) represents the time taken by the algorithm to process the dataset.

The scientist needs assistance to apply gradient descent method on the above data set to find the optimal parameters for the function. For this you are supposed to help him by considering the following:

$h_\theta = \sum_{j=0}^{j=1} \theta_j x_j, where\ let\ \boldsymbol{x_0} = \mathbf{1}$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{i=m} (h_{\theta_0\theta_1}(x_{1j}) - y_j)^2 \quad let\ \boldsymbol{\theta_0} = \boldsymbol{\theta_1} = \mathbf{0}(initially)$$

$$\theta_j = \theta_j - \propto \frac{\partial y}{\partial \theta_j} \qquad let\ \propto = \mathbf{0.05}$$

Note: Just perform three iterations.

Qno.5,-

$$J = \frac{1}{6}\left[(\theta_0 + 2\theta_1 - 7)^2 + (\theta_0 + 3\theta_1 - 10)^2 + (\theta_0 + 4\theta_1 - 13)^2\right]$$

$$J = \frac{1}{6}\left[3\theta_0^2 + 29\theta_1^2 + 18\theta_0\theta_1 - 60\theta_0 - 192\theta_1 + 318\right]$$ (2 Marks)

$$\frac{\partial J}{\partial \theta_0} = \frac{1}{6}\left[6\theta_0^2 + 18\theta_1 - \cancel{192}\ 60\right]$$ (1 Mark)

$$\frac{\partial J}{\partial \theta_1} = \frac{1}{6}\left[58\theta_1 + 18\theta_0 - 192\right]$$ (1 Mark)

2 Marks for each iteration.

Iteration	theta	J	Partial derivatives
1	[0. 0.]	53	[-10. -32.]
2	[0.5 1.6]	11.6983	[-4.7 -15.03333333]
3	[0.735 2.35166667]	2.58216	[-2.21 -7.06222222]
4	[0.8455 2.70477778]	0.570025	[-1.04016667 -3.31731481]

Qno6 – CLO 2 4+6 Marks

For the following linear system,

$$-3x_1 + x_2 + 12x_3 = 5$$
$$6x_1 - x_2 - x_3 = 31$$
$$6x_1 + 9x_2 + x_3 = 4$$

(i) Compute true/actual solution of above system(Use Calculator).

(ii) Without any arrangement, solve the above system numerically by using Gauss-Seidal method with $X^{(0)} = (0,0,0)^t$ and perform three iterations. Compare the results with part(i) and state whether the above system is convergent or not for Gauss-Seidal Method.

Note: If the answer of above part ((ii)) is divergent then work on the below part ((iii)) otherwise leave it.

(iii) Make the above system convergent by making suitable arrangement and then solve it by Gauss-Seidal method with $X^{(0)} = (0,0,0)^t$ and stops when $\left\|X^{(k+1)} - X^{(k)}\right\|_\infty < 0.01$

Solution :-

Q # 1

i) Actual Solution = (4.97222, -308333, 1.91667)

(2 Marks)

ii) Iterative Scheme (1 Mark)

$$x_1^{(k+1)} = (5 - x_2^{(k)} - 12x_3^{(k)}) / -3$$

$$x_2^{(k+1)} = 6x_1^{(k+1)} - x_3^{(k)} - 31$$

$$x_3^{(k+1)} = 4 - 6x_1^{(k+1)} - 9x_2^{(k+1)}$$

$k = 0, 1, 2$

Iterations (3 Marks)

k	$x_1^{(k+1)}$	$x_2^{(k+1)}$	$x_3^{(k+1)}$
0	-1.66667	-41	383
1	1516.66667	8686	-87270
2	-346186.3333	-1989879	1998603
	$\nrightarrow$ 4.97222	$\nrightarrow$ -3.08333	$\nrightarrow$ 1.91667

Divergence

Date

(iii) Eq ② & Eq ③ are interchanged and then new eq ② is interchanged with eq ①

(1 Mark)

New system is,

$$6x_1 - x_2 - x_3 = 31$$
$$6x_1 + 9x_2 + x_3 = 4$$
$$-3x_1 + x_2 + 12x_3 = 5$$

above system is strictly diagonally dominant $\Rightarrow$ It will converge

Iterative Scheme is, (1 Mark)

$$x_1^{(K+1)} = (31 + x_2^{(K)} + x_3^{(K)}) / 6$$
$$x_2^{(K+1)} = (4 - 6x_1^{(K+1)} - x_3^{(K)}) / 9$$
$$x_3^{(K+1)} = (5 + 3x_1^{(K+1)} - x_2^{(K+1)}) / 12$$

Iterations (3 Marks)

K	$x_1^{(K+1)}$	$x_2^{(K+1)}$	$x_3^{(K+1)}$	$\|x^{(k+1)} - x^{(k)}\|_\infty$
0	5.16667	-3	1.95833	-
1	4.99306	-3.10185	1.92342	0.17361
2	4.97026	-3.08278	1.91613	0.02280
3	4.97223	-3.08328	1.91666	0.00197
	~~4.97223~~	~~-3.08334~~	~~1.91667~~	< 0.01

M.M.

Qno7 – CLO 2 4+6 Marks

For the following linear system,

$$x_1 + x_2 + x_3 = 2$$
$$x_1 + x_2 = 3$$
$$x_1 + x_3 = 4$$

a) Write Co-efficient matrix A for above system

b) Given that absolute dominant eigen value of A is $\lambda_{Absolute\ dominant}^{(Actual\ value)} = \sqrt{2} + 1$, find the approximate absolute dominant eigen value $\lambda_{Absolute\ dominant}^{(Approximated\ value)}$ by using Power method with $X^{(0)} = (-1,0,1)^t$ and stops when $\left|\boldsymbol{\lambda_{Absolute\ dominant}^{(Actual\ value)} - \lambda_{Absolute\ dominant}^{(Approximated\ value)}}\right| < \mathbf{0.05}$

Question no 7:

i) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ 2 Marks.

ii) $\lambda^{(k+1)} X_{k+1} = A X_k$ 1 Mark.

$k=0$ $\lambda^{(1)} X_1 = A X_0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = -1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\lambda^{(1)} = -1$ $X_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$k=1$ $\lambda^{(2)} X_2 = A X_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$k=2$ $\lambda^{(3)} X_3 = A X_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 1 \\ 0.5 \end{bmatrix}$

$k=3$ $\lambda^{(4)} X_4 = A X_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 2 \\ 1.5 \end{bmatrix} = 2.5 \begin{bmatrix} 1 \\ 0.8 \\ 0.6 \end{bmatrix}$

$k=4$ $\lambda^{(5)} X_5 = A X_4 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0.8 \\ 0.6 \end{bmatrix} = \begin{bmatrix} 2.4 \\ 1.8 \\ 1.6 \end{bmatrix} = 2.4 \begin{bmatrix} 1 \\ 0.75 \\ 0.67 \end{bmatrix}$

Summarized table

k	$\lambda^{(k+1)}$	X_{k+1}	Stopping Criteria
0	-1	$(0,1,0)$	3.41421
1	1	$(1,1,0)$	1.41421
2	2	$(1,1,0.5)$	0.41421
3	2.5	$(1,0.8,0.6)$	0.08579
4	$\boxed{2.57}$	$(1,0.75,0.67)$	$0.01421 < 0.05$

absolute dominant eigen value of A.

Marks distribution

Iterations performed at $k=0$ and $k=1$ 2 marks. Iterations performed at $k=2, 3$, and 4 contains 6 Marks.

Qno8 – CLO 2 4+1+1+2+2 Marks

For the following linear system,

$$x_1 + 2x_2 - x_3 = 2$$
$$2x_1 + 4x_2 - x_3 = 3$$
$$x_1 + 2x_3 = 2$$

(i) Write the co-efficient matrix A for above system and find the upper triangular matrix U corresponding to A by applying row elementary operations on A.

(ii) With the help of part (i), write the permutation matrix P

(iii) If there is no row interchange performed in part(i) then $P =$ ____ matrix.

(iv) With the help of part(i) construct lower triangular matrix L ($l_{ii} = 1, for\ all\ i$) such that $PA = LU$

(v) Verify $PA = LU$

(i) $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -1 \\ 1 & 0 & 2 \end{bmatrix}$ (1 Mark)

$\underset{\sim}{R_2 - 2R_1}$ $\underset{\sim}{R_3 - R_1}$ $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 1 \\ 0 & -2 & 3 \end{bmatrix}$ (2 Marks)

$\underset{\sim}{R_{23}}$ $\begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix} = U$ (1 Mark)

(ii) $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (1 Mark)

(iii) $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = Identity Matrix (1 Mark)

(iv) $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ (2 Marks)

(v) $LU = PA$

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$LU = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 2 \\ 2 & 4 & -1 \end{bmatrix}$$ (1 mark)

$$PA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -1 \\ 1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 2 \\ 2 & 4 & -1 \end{bmatrix}$$

$\Rightarrow$ $PA = LU$ (verified) (1 mark)

Qno9 – CLO 2 2+3+1+3+1 Marks

Consider, $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 2 \\ 1 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix}$ $and\ C = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 3 & 2 \\ 0 & 2 & -4 \end{bmatrix}$

(i) Identify from matrices A, B and C which one is positive definite?

(ii) Decompose the matrix (answer of part(i)) into LDL^T where $l_{ii} = 1\ and\ d_{ii} > 0\ for\ all\ i$

(iii) Verify part(ii) decomposition

(iv) Decompose the matrix (answer of part(i)) into $Cholesky\ decomposition\ (LL^T)\ wher\ l_{ii} \neq 0, for\ all\ i)$

(v) Verify part(iv) decomposition

Question #4:-

(i) A is not symmetric $\Rightarrow$ A is not positive definite

(ii) B is symmetric and determinants of principle submatrices of B are

$$|2| = 2 > 0, \quad \begin{vmatrix} 2 & -1 \\ -1 & 4 \end{vmatrix} = 8-1 = 7 > 0$$

and

$$\begin{vmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{vmatrix} = 2(6)+1(-2) = 10 > 0$$

$\Rightarrow$ B is positive definite Matrix

(2 Marks)

(iii) C is symmetric and determinants of principle submatrices of C are,

$$|2| = 2 > 0, \quad \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 5 > 0$$

$$\begin{vmatrix} 2 & 1 & 0 \\ 1 & 3 & 2 \\ 0 & 2 & -4 \end{vmatrix} = 2(-16)+1(-4) = -36 \not> 0$$

$\Rightarrow$ C is not positive definite

M.M.

(ii) $$B = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

$$B = L\, D\, L^T$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix} = \overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}} \overset{D}{\begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix}} \overset{L^T}{\begin{bmatrix} 1 & l_{21} & l_{31} \\ 0 & 1 & l_{32} \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{11}l_{21} & d_{11}l_{31} \\ d_{11}l_{21} & d_{22}+d_{11}l_{21}^2 & d_{22}l_{32}+d_{11}l_{21}l_{31} \\ d_{11}l_{31} & d_{11}l_{21}l_{31}+d_{22}l_{32} & d_{11}l_{31}^2+d_{22}l_{32}^2+d_{33} \end{bmatrix}$$

$d_{11} = 2$, $d_{11}l_{21} = -1$, $d_{11}l_{31} = 0$

$l_{21} = -0.5$, $l_{31} = 0$ (1 Mark)

or $l_{21} = -1/2$

$d_{22} + d_{11}l_{21}^2 = 4$, $d_{11}l_{21}l_{31} + d_{22}l_{32} = 2$

$d_{22} + 0.25 \times 2 = 4$, $0 + 3.5\, l_{32} = 2$

$\boxed{d_{22} = 3.5}$ or $d_{22} = 7/2$, $\boxed{l_{32} = 4/7 = 0.57143}$

$d_{11}l_{31}^2 + d_{22}l_{32}^2 + d_{33} = 2$

$0 + \frac{7}{2}\left(\frac{16}{49}\right) + d_{33} = 2$

M.M.

$d_{33} = 6/7$ (2 Marks)

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & 4/7 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7/2 & 0 \\ 0 & 0 & 6/7 \end{bmatrix}$$

(iii) $$LDL^T = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & 4/7 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7/2 & 0 \\ 0 & 0 & 6/7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1/2 & 0 \\ 0 & 1 & 4/7 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

$= B$ (Verified) (1 Mark)

(iv) $B = LL^T$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix} = \begin{bmatrix} l_{11}^2 & l_{11}l_{21} & l_{11}l_{31} \\ l_{11}l_{21} & l_{21}^2+l_{22}^2 & l_{21}l_{31}+l_{22}l_{32} \\ l_{11}l_{31} & l_{21}l_{31}+l_{22}l_{32} & l_{31}^2+l_{32}^2+l_{33}^2 \end{bmatrix}$$

M.M.

$l_{11}^2 = 2$, $l_{11} l_{21} = -1$, $l_{11} l_{31} = 0$

$l_{11} = \sqrt{2}$; $l_{21} = -1/\sqrt{2}$; $l_{31} = 0$

(1 Mark)

$l_{21}^2 + l_{22}^2 = 4$; $l_{21} l_{31} + l_{22} l_{32} = 2$, $l_{31}^2 + l_{32}^2 + l_{33}^2 = 2$

$l_{22} = \sqrt{4 - 1/2}$, $\sqrt{7/2}\, l_{32} = 2$, $8/7 + l_{33}^2 = 2$

$l_{22} = \sqrt{7/2}$; $l_{32} = 2\sqrt{2/7}$, $l_{33} = \sqrt{6/7}$

(2 Marks)

$$L = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & \sqrt{7}/\sqrt{2} & 0 \\ 0 & 2\sqrt{2/7} & \sqrt{6/7} \end{bmatrix}$$

(V) $$LL^T = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & \sqrt{7/2} & 0 \\ 0 & 2\sqrt{2/7} & \sqrt{6/7} \end{bmatrix} \begin{bmatrix} \sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & \sqrt{7/2} & 2\sqrt{2/7} \\ 0 & 0 & \sqrt{6/7} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix} = B \Rightarrow \text{Verified}$$

(1 Mark)

Qno10 – CLO 2 2+3+1+3+1 Marks

A particle is moving with velocity $v(t) = \frac{ds}{dt}(1+t) = s^2$, $s(0.5) = -2.46630$, where s is displacement at any time t. Find the displacement $\boldsymbol{s(0.54)}$ with $h = 0.04$ by using

(i) Heun's Method (ii) Midpoint Method and (iii) RK-4 Method (Classical approach).

Given that $s(t) = -\frac{1}{\ln(t+1)}$ is the actual solution, compute actual error in each part (i),(ii) and (iii) also and comments on the result.

Question #5 :

i) Heun's Method :-

$$\frac{ds}{dt} = \frac{S^2}{1+t}$$

$f(t,S) = S^2/1+t$, $S_0 = -2.46630$, $t_0 = 0.5$

$S_{i+1} = S_i + 0.04/2 \left(f(t_i, S_i) + f(t_{i+1}, S_i + 0.04 f(t_i, S_i)\right)$

$i=0$, $S_1 = S_0 + \frac{0.04}{2}\left(f(t_0, S_0) + f(t_1, S_0 + 0.04 f(t_0, S_0)\right)$

i	t_i	S_i	$f(t_i, S_i)$	t_{i+1} (A)	$S_i + 0.04 f(t_i, S_i)$ (B)	$f(A,B)$
0	0.5	-2.46630	4.05509	0.54	-2.30410	3.44731

S_{i+1}
$\hookrightarrow -2.31625 \Rightarrow S_1(0.54) = -2.31625$

Heun's (2 Marks)

(ii) Midpoint Method

$$S_{i+1} = S_i + h\left(f\left(t_i + \frac{h}{2}, S_i + \frac{h}{2} f(t_i, S_i)\right)\right)$$

$i = 0$, $$\boxed{S_1 = S_0 + 0.04\left(f\left(t_0 + 0.02, S_0 + 0.02 f(t_0, S_0)\right)\right)}$$

i	t_i	S_i	$f(t_i, S_i)$	A: $t_i + h/2$	B: $S_i + \frac{h}{2} f(t_i, S_i)$	$f(A, B)$	S_{i+1}
0	0.5	-2.4663	4.05509	0.52	-2.38520	3.74288	-2.31658

$\Rightarrow$ $\underbrace{S(0.54)}_{\text{Midpoint Method}} = -2.31658$ (2 Marks)

RK-4 Method :

$$S_{i+1} = S_i + \frac{h}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$k_1 = f(t_i, S_i)$, $k_2 = f(t_i + h/2, S_i + hk_1/2)$

$K_3 = f(t_i + h/2, S_i + hk_2/2)$, $k_4 = f(t_i + h, S_i + hk_3)$

$i = 0 \Rightarrow S_1 = S_0 + \frac{0.04}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

i	t_i	S_i	k_1	A $t_i + h/2$	B $S_i + \frac{hk_1}{2}$	k_2	C $S_i + \frac{hk_2}{2}$	k_3	D $t_i + h$	E $S_i + hk_3$	k_4
0	0.5	-2.4663	4.05507 0.5 Marks	0.52	-2.38529	3.74288 1 Mark	-2.39144	3.7625 1 Mark	0.54	-2.3158	3.48242 1 Mark

$S_{i+1} = -2.31598$ at $i = 0$ (0.5 Marks)

$\Rightarrow S(0.54) = -2.31598$
↓
RK-4 Method

True Solution $= s(t) = -\dfrac{1}{\ln(t+1)}$

$= s(0.54) = -2.31598$

Actual Errors :- (1 Mark)

Heun's Method $= 0.00027$
Midpoint Method $= 0.0006$
RK-4 (Classical Approach) $= 0.000003 \approx 0$ (upto 5 decimal places)

Comments :- (1 Mark)

RK-4 Method gives better approximation
M.M. in this question as compared to other two methods