

Position error estimation and compensation of 3-DOF delta robot under the effect of link tolerances

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ABSTRACT

Spatial parallel mechanism is one of the prominent areas of research in robotics. Delta robots are widely used in the area of biomedical and surgical and other mechanisms aimed to enhance the movement of manipulators for various tasks in the industries. This article presents the investigation of position error in a 3-DOF delta robot under the effect of link tolerance. First, the generalized mathematical formulation is demonstrated and then the comparative estimation is presented. The forward and inverse kinematics of the manipulator is derived using the loop-closure equation. The position error is estimated by using the proposed approach. Different combinations of tolerances are considered and an effort is made to compensate for the maximum position error that occurred by varying the input angles. The proposed approach is helpful in analyzing spatial parallel configurations further it is providing a computational method for simulating complex manipulations.

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1. Introduction

Over the last two decades, theoretical and practical research in the field of parallel manipulators has accelerated dramatically. The fundamental reasons are that these processes are stronger, quicker, and more precise. Parallel industrial robots have a number of advantages over serial robots, including a compact structure, high rigidity, high repeated positioning accuracy, strong load-bearing capacity, and so on. As a result, it is used in high-speed and high-precision tasks [1]. However, parallel robots can have their own issues, earlier research concentrated on parallel mechanisms with six degrees of freedom (DOF), which have the advantages of high stiffness, low inertia, and great payload capacity. To overcome these issues, there has lately been a rising trend toward parallel manipulators with three translational degrees of freedom, which are better suited for high speed and high stiffness manipulation. Furthermore, the availability of closed-form systems allows for precise design and efficient control. One well-known example is the delta robot, which was developed by Clavel [2]. It is ideal for pick and place tasks with light items. Delta robots of various sizes have been adopted into a wide range of industrial applications,

including the food, pharmaceutical, and electronics industries [3–7].

Many researchers elaborated the area of kinematics, dynamics, and trajectory and performance analysis of the manipulator. Position accuracy is one of the important parameters for assessing parallel robot performance [8–17]. Parallel robots are becoming faster and lighter and other mechanical parameters influencing end effector position are becoming more complicated. There has been extensive study in the area of parallel robot position accuracy and compensating methods due to influence of design and manufacturing limitations [18–22]. Chen [23] developed an error model for a four-axis Delta robot and investigated the sensitivity of the mechanism error source that affects end effector position precision. Many authors have studied the influence of joint clearance on the pose error (position and orientation error) of the planar and spatial parallel mechanisms [24–30]. Genliang et al. [31] methodically explained the uncertainty that joint clearance had on the position inaccuracy of a parallel robot. Jawale and Jaiswal [32] estimated the position error of the single DOF closed chain mechanism due the tolerances using Taylor series approximation method and validated the obtained results by graphical and geometrical method. Bai et al. is concerned with the kinematic calibration of the Delta robot using distance measurements. The study

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Nomenclature

$r_{e1}, r_{e2}, r_{e3}, r_{f1}, r_{f2}$ and r_{f3} Binary links of the manipulator
 $r'_{e1}, r'_{e2}, r'_{e3}, r'_{f1}, r'_{f2}$ and r'_{f3} Link length under tolerance
 E_x, E_y , and E_z Tool attachment position
 F_{xi}, F_{yi} , and F_{zi} (F_i for $i = 1, 2, 3$) Coordinates of fixed platform
 l_b and l_e Distance between the center point and the corner edge of fixed and moving platform

θ_1, θ_2 and θ_3 Input angles (Rotary Actuator)
 E'_x, E'_y , and E'_z Deviated tool position under tolerance
 E_{xi}, E_{yi} , and E_{zi} (E_i for $i = 1, 2, 3$) Coordinates of moving platform

proposes a linearized compensator for real-time error compensation. The testing findings suggest that the approach can increase the robot's positioning accuracy [33]. Jawale and Jaiswal investigated the positional error in a P3R mechanism with joint clearance and further compensated the error using inverse kinematic based approach.[34] Peng suggested a way of managing the good that provides an effective method for enhancing robot placement accuracy. Vischer and Clavel devised an error calibration technique and used the vector method to create the Delta robot's parametric model. The robot's kinematics are calibrated using implicit calibration and semi-parametric calibration [35]. Jawale and Jaiswal studied the influence of tolerances on error estimation in P3R and 4R mechanisms. The comparison of errors provided the basis for selecting the mechanism that gave better performance [36]. The proposed work is derived to estimate the maximum positional error due to the manufacturing tolerances on the links of a spatial 3-DOF delta robot. Compensation of maximum positional error is performed using the inverse kinematics approach. The work presented in this paper is organized as follows - Section 2 deals with methodology Section 3 deals with the positional error consideration which explains the geometry of the manipulator, mathematical modeling under tolerance. Section 4 deals with the formulation of error compensation using inverse kinematics. Section 5 deals with the demonstration of sample case of design variables. Section 6 results and discussion followed by the effect of tolerances on error estimation for spatial delta manipulator. Conclusions and the scope of the future work are discussed in Section 7.

2. Methodology

The proposed methodology applied for analyzing the error is described in flow chart (Fig. 1.). A mathematical formulation is developed for a 3-DOF spatial delta robot under influence of tolerance through analytical approach. The deviations in tool attachment position under the effect of link tolerance is estimated

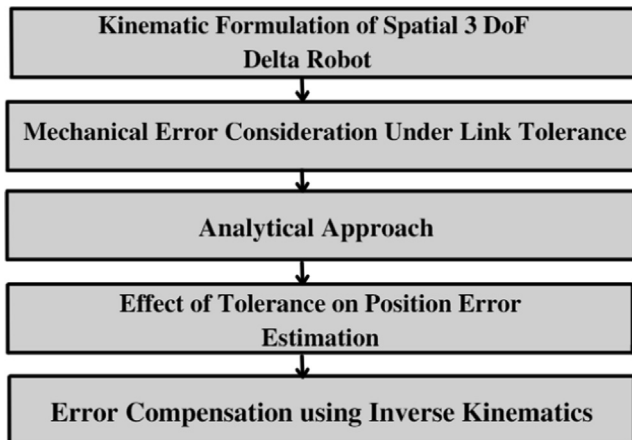


Fig. 1. Flowchart.

though moving platform of the manipulator. Later on, effect of moving platform position on mechanical error is estimated. Compensation of mechanical error of the moving platform using inverse kinematic approach is carried out.

3. Mechanical error consideration

3.1. Geometry of the manipulator

Fig 2 (a-b) shows the kinematic diagram of the delta robot with and without link tolerances respectively. The cartesian co-ordinate systems $O(x,y,z)$ and $E(x,y,z)$ are attached to the fixed base and moving base respectively as shown in Fig. 2. (a). The points $F_1F_2F_3$ lie on X-Y plane and $E_1E_2E_3$ lie on x-y plane. As shown in the Fig. 2 (a) the origin O of the fixed co-ordinate system is located at the centroid of $\Delta F_1F_2F_3$ which depicts the fixed base coordinates. Similarly, the origin E of the moving co-ordinate system is located at the centroid of $\Delta E_1E_2E_3$ which depicts the coordinates of the moving platform. Both the triangles are equilateral with the fixed base being larger in size compared to the moving platform. Furthermore, the axis of each revolute joint J_i lies on x-y plane and is perpendicular to the vector OF_i . The imaginary points J_1, J_2 and J_3 are mere projections of points J_1, J_2 and J_3 respectively and were constructed for the purpose of kinematic formulation. The notations $r'_{f1}, r'_{f2}, r'_{f3}$ and $r'_{e1}, r'_{e2}, r'_{e3}$ denote the set of upper and lower links respectively with tolerance, thus producing a deviation in the position of end effector. The $\Delta E'_1E'_2E'_3$ shows the deviation of moving platform of the delta robot due the effect of tolerances as shown in Fig. 2 (b).

3.2. Mathematical model under link tolerance

The forward kinematics of a 3-DOF parallel delta robot under the influence of link-tolerance is presented. Fig. 2 (a) shows the kinematic sketch of a delta manipulator made up of a fixed and moving platform with six binary links. The mathematical formulation of a delta robot is to obtain the three-dimensional coordinate $E(x, y, z)$ of the terminal mobile platform by knowing the angles $\theta_1, \theta_2, \theta_3$ of three active arms. The equations are derived under the influence of link tolerance as follows: -

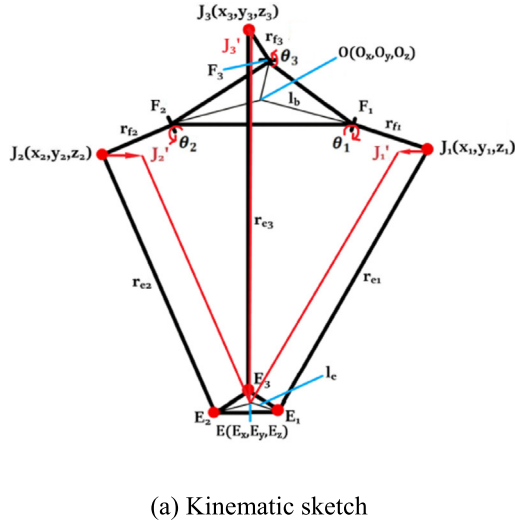
$$J'_1 = (0, -(l_b - l_e) - r_{f1}\cos\theta_1, -r_{f1}\sin\theta_1) \quad (1)$$

$$J'_2 = ((l_b - l_e + r_{f2}\cos\theta_2)(\cos 30^\circ), (l_b - l_e + r_{f2}\cos\theta_2)(\cos 60^\circ), -r_{f2}\sin\theta_2) \quad (2)$$

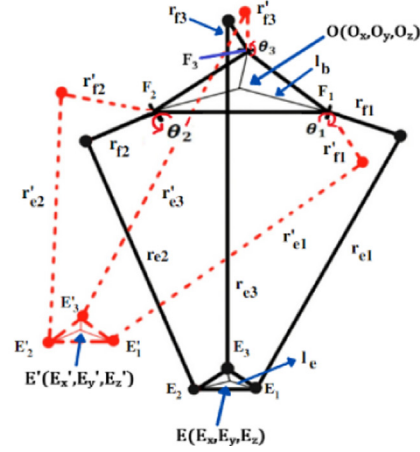
$$J'_3 = ((-l_b + l_e - r_{f3}\cos\theta_3)(\cos 30^\circ), (l_b - l_e + r_{f3}\cos\theta_3) \times (\cos 60^\circ), -r_{f3}\sin\theta_3) \quad (3)$$

$$x^2 + y^2 + z^2 - 2xx_1 - 2yy_1 - 2zz_1 = r_{e1}^2 - w_1 \quad (4)$$

$$x^2 + y^2 + z^2 - 2xx_2 - 2yy_2 - 2zz_2 = r_{e2}^2 - w_2 \quad (5)$$



(a) Kinematic sketch



(b) Deviated position and orientation under tolerance

Fig. 2. (a-b). Delta Robot.

$$x^2 + y^2 + z^2 - 2xx_3 - 2yy_3 - 2zz_3 = r_{e3}^2 - w_3 \quad (6)$$

$$x = a_1 \left(\frac{-b - \sqrt{D}}{2a} \right) + b_1 \quad (7)$$

$$y = a_2 z + b_2 \quad (8)$$

$$z = \left(\frac{-b - \sqrt{D}}{2a} \right) \quad (9)$$

4. Formulation of error compensation using inverse kinematics

The mathematical formulation derived for error compensation of the delta robot is given below.

Equation of circle with radius F_1J_1 and corresponding point J_1 on circumference and center as F_1

$$y_{j1}^2 + l_b^2 + 2y_{j1}l_b + z_{j1}^2 = r_f^2 \quad (10)$$

Equation of circle with radius $J_1E'_1$ and corresponding point J_1 on circumference and center as E'_1

$$y_{j1}^2 - 2y_{j1}(y - l_e) + z_{j1}^2 - 2z_{j1}z = r_e^2 - x^2 - z^2 - (y - l_e)^2 \quad (11)$$

$$z_{j1}^2 = r_f^2 - y_{j1}^2 - l_b^2 - 2y_{j1}l_b \quad (12)$$

$$z_{j1} = ay_{j1} + b \quad (13)$$

$$\theta_1 = \tan^{-1} \left(\frac{z_{j1}}{y_{f1} - y_{j1}} \right) \quad (14)$$

$$\text{For finding } \theta_2, \text{ we use } x = x \cos 120^\circ + y \sin 120^\circ \quad (15)$$

$$y = y \cos 120^\circ - x \sin 120^\circ \quad (16)$$

$$\text{For finding } \theta_3, \text{ we use}$$

$$x = x \cos 120^\circ - y \sin 120^\circ \quad (17)$$

$$y = y \cos 120^\circ + x \sin 120^\circ \quad (18)$$

5. Case study

The link length dimensions delta robot with $R_f = 205$ mm, $R_e = 50$ mm, $l_b = 400$ mm, $l_e = 1000$ mm, are taken for the analysis. The range of joint angles of the actuator are taken were from -40 to 80 degrees [37]. For the workspace analysis, the range is divided into 30 equal pieces. Each range has $30 + 1 = 31$ possible values. The three variables can provide a total of $31 \times 31 \times 31 = 29,791$ distinct postures. These positions and postures are then translated into a set of discrete points in three-dimensional space using the mathematical formulation of the moving platform using MATLAB Scatter plot. Further analysis is done to study its behaviour under the influence of link tolerance. The maximum positional error is studied from a range of 10 – 100 μm and the error is further compensated using the inverse kinematic approach.

6. Results & discussion

6.1. Workspace under link tolerances

Fig. 3 (a) depicts the desired reachable workspace of a spatial delta robot of the 30 intervals that were taken within the given range of actuator angles or input angles of -40 to 80 degrees. Fig. 3 (b) & Fig. 3 (c) shows the representation of the XZ and YZ plane of the workspace, respectively. Because there is a risk of collision among the robot's three follower rods during motion, but the collision of the Delta robot happens in the singular configuration, the singular configuration is restrained to avoid the rod collision.

Fig. 4 depicts the plot showing the comparison of the workspace under the influence of maximum level tolerance (i.e 100 μm) versus that of the desired position, indicating that there is a deviation from that of the desired workspace.

6.2. Position error estimation

The Table 1 shows the desired and deviated position of the moving platform of the delta robot. The maximum position error in each level of the tolerances (i.e 10 to 100 μm) is shown in the last column of the Table 1.

The 3D plot of tolerance vs position vs maximum positional error is shown in Fig. 5. It is observed that the maximum position error increases with the increase in the level of tolerances. The maximum error is obtained at 100 μm and the minimum error at the 10 μm level of tolerances. Fig. 5 indicates that the position error

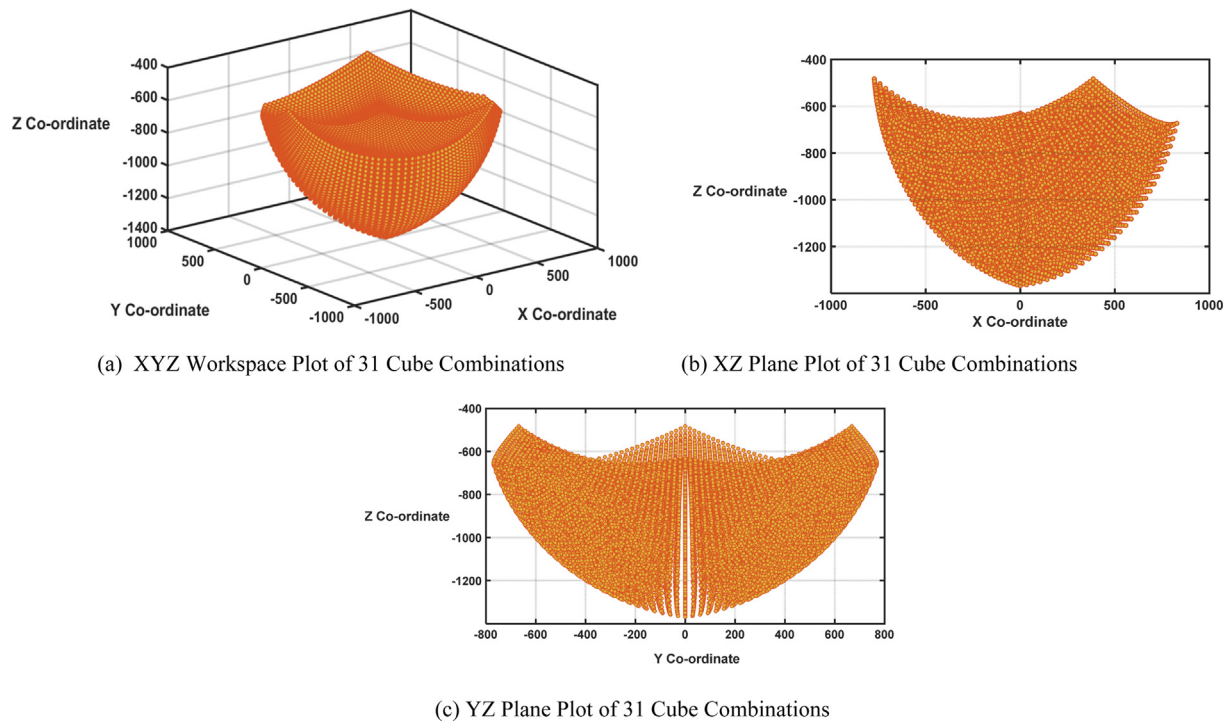
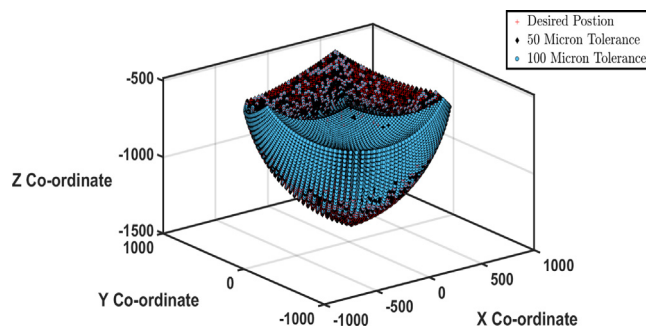


Fig. 3. (a-c) Workspace of the delta robot.



is directly proportional to the level of the link tolerances. Fig. 6 represents the 2D plot of maximum position error vs tolerance indicating for the range 10 to 100 μm .

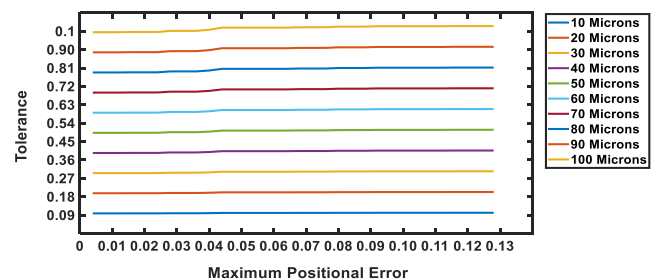
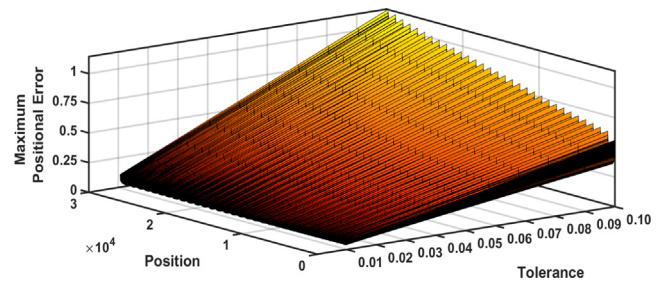


Table 1
Position error estimation under link tolerance.

Tolerance (in Microns)	Input Angles			Desired Position of moving planform			Deviated Position of moving planform			Position Error
	θ_1	θ_2	θ_3	x	y	z	x'	y'	z'	
10	80	80	80	0	0	-1368.40	-0.05	-0.09	-1368.41	0.10
20	80	80	80	0	0	-1368.40	0.22	0	-1368.42	0.22
30	80	80	80	0	0	-1368.40	-0.17	0.29	-1368.42	0.33
40	80	80	80	0	0	-1368.40	-0.22	-0.39	-1368.43	0.44
50	80	80	80	0	0	-1368.40	0.57	0	-1368.43	0.57
60	80	80	80	0	0	-1368.40	0.68	0	-1368.44	0.68
70	80	80	80	0	0	-1368.40	-0.39	0.69	-1368.45	0.79
80	80	80	80	0	0	-1368.40	-0.45	-0.79	-1368.45	0.91
90	80	80	80	0	0	-1368.40	1.02	0	-1368.46	1.02
100	80	80	80	0	0	-1368.40	-0.57	0.98	-1368.47	1.13

Table 2

Error compensation using inverse kinematics.

Tolerance (in Microns)	Input Angles			Deviated Angles (Under Tolerance)			Error Compensation		
	θ_1	θ_2	θ_3	θ'_1	θ'_2	θ'_3	$\Delta\theta_1$	$\Delta\theta_2$	$\Delta\theta_3$
10	80	80	80	80.00	80.00	79.92	0.00	0.00	0.08
20	80	80	80	79.98	80.01	80.01	-0.01	0.01	0.01
30	80	80	80	80.02	79.97	80.02	0.02	-0.02	0.02
40	80	80	80	80.02	80.02	79.97	0.02	0.02	-0.02
50	80	80	80	79.96	80.03	80.03	-0.03	0.03	0.03
60	80	80	80	79.95	80.04	80.04	-0.04	0.04	0.04
70	80	80	80	80.04	79.95	80.04	0.04	0.04	0.04
80	80	80	80	80.05	80.05	79.94	0.05	0.05	-0.05
90	80	80	80	79.93	80.06	80.06	-0.06	0.06	0.06
100	80	80	80	80.07	79.92	80.07	0.07	-0.07	0.07

6.3. Error compensation

The inverse kinematic approach is used to compensate the maximum position error for each combination of tolerances, i.e. from 10 to 100 μm . Table 2 shows the maximum error compensation required in order to obtain the desired position and nullify the effect of link tolerance on the delta robot. The error compensation of all the three input angles of the robot for each level of the tolerances (i.e 10 to 100 μm) is shown in the last three columns of Table 2. It is observed that the position error is less than or almost equal to the level of tolerances.

7. Conclusions

The proposed study assesses the positional accuracy of the delta robot while taking into consideration the randomness of link lengths. The observation of the present work is increasing maximum positional error with an increase in the level of tolerance. The positional error obtained can be linked to the term “Kinematic Reliability”. The kinematic reliability is defined as the likelihood that the actual position of the moving platform falls within a sphere, with a radius equal to a specified tolerance threshold, centered at the desired position. The limit-state function is nonlinear in order to engage the tool attachment position's dependent XYZ coordinates with regard to the random link lengths. Because of the problem's complexity, typical dependability approaches are inapplicable.

CRediT authorship contribution statement

Darren Alton Dsouza: Software, Visualization, Formal analysis, Validation, Writing – original draft. **Rayyan Muhammad Rafikh:** Visualization, Software, Formal analysis, Validation. **Ankur Jaiswal:** Conceptualization, Methodology, Supervision, Project administration, Validation, Writing – original draft.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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