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Jerk-Optimal Trajectory Planning for Stewart Platform in Joint Space

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***Abstract* - A technique for jerk-optimal trajectory planning in joint space is presented in this paper. Jerk is the third derivative of joint position of the trajectory, it affects stabilization and efficiency of whole robot system significantly. As Stewart platform has characteristics of high speed and heavy haul, then jerk of every joint should be minimized to get the platform running smoothly and improve the accuracy of movement. In order to get the optimal jerk trajectory, this paper has formulated this as an optimization problem with constraint conditions of kinematic parameters and execution time. Cubic spline has been used to define the trajectory. Simulations based on SQP-F algorithm were conducted and the results showed effectiveness of the approach presented in this paper.**

Index Terms* - *Jerk-Optimal, Trajectory Planning, SQP-F Algorithm

I. INTRODUCTION

In many applications, it is necessary for actuator to move on a desired trajectory smoothly and stably. High jerk of the robot joint may wear out the robot structure, excite the resonance frequencies of the body structure, create vibrations which can damage the actuator’s structure, and heavily excite its resonance frequencies. Moreover, low-jerk trajectories can be executed more rapidly and accurately. For reasons upon, jerk-optimal trajectory planning becomes quite important

In recent years many researchers have studied on the problem about that. Lin[1] converted the jerk-optimal problem to nonlinear optimization problem and developed an algorithm to schedule the time interval. Piazzi[2] proposed a algorithm based on interval analysis which can get the global optimal solution. Secco[3] exploited a minimum jerk approach to define the trajectory in the Cartesian space which guarantees a natural movement of the finger. GasparettoA[4]presented a technique for time-jerk optimal planning of robot trajectories, this method does not take the execution time as given a priori and takes into account kinematic constraints on the robot motion. Huang[5]developed minimum-jerk trajectory planning method based on GA and obtained optimal trajectory which is fitful for high velocity and high precision dynamic control. Lin[6] presented a fast and unified method to find a minimum-jerk robot joint trajectory using PSO algorithm. Liu[7] presented an interesting method of high smooth trajectory planning method which is a combination of multi-degree splines in Cartesian space and multi-degree B-splines in joint space. This approach is an effective solution to trajectory Planning.

As parallel mechanism has larger load, faster movement and higher accuracy than serial mechanism, the jerk-optimal trajectory is quite necessary. MinHee[8] studied trajectory planning of the modified 3-degrees-of-freedom Stewart Platform mechanism in static environments. Cong[9] studied time-jerk synthetic optimization method of trajectory planning based on Cartesian space of Stewart parallel mechanism and proposed a real-time method of feasible directions to solve the non-linear programming problem of optimizing the time interval of the interpolation. Afroun[10] studied the problem of planning optimal trajectories for a Gough parallel robot. The planning process consists of searching for a motion ensuring the accomplishment of the assigned task, minimizing a cost function and satisfying various constraints inherent to the robot kinematics and dynamics. Duan[11] presented a real-time motion planning based on vibration control strategy. To determine the optimal position and orientation of the cable driven parallel manipulator, the real-time optimization is conducted according to the principle of uniform tension in the six driving cables.

There are many useful algorithm to solve the non-linear optimization problem. Liu[7] applied the sequential quadratic programming (SQP) method to solve time-optimal trajectory planning problem with linearizing the non-linear constraints. Jiang[12] used sequential quadratic programming method to solve trajectory optimization with the given range. Huang[5] used use the genetic algorithms to solve the optimal trajectory planning. Lin[6] proposed a fast and unified approach based on particle swarm optimization (PSO) with K-means clustering to solve the near optimal solution of a minimum-jerk joint trajectory.

Jerk-optimal trajectory planning considering kinematic constraints and execution time constraint in joint space is studied in this paper. As constraints functions consist of inequality and equality constraints, a improved sequential quadratic programming is used to solve the near optimal solution of a jerk-optimal joint trajectory. In this paper, formulation of the optimization problem is formulated in part II which include cubic spline interpolation technique, object function and constraints function of the optimization problem and execution of the algorithm. Simulation result is showed in part . Conclusion is summarized in part IV.

II. FORMULATION OF THE OPTIMIZATION PROBLEM

Cubic spline is chosen to define the trajectory and solve the optimization problem because the generated trajectories have continuous values of the accelerations, which ensures motors

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of the platform run smoothly. Moreover, unlike higher order polynomials, cubic spline doesn't present problems such as excessive oscillations and overshoot between any pair of reference points.

Some basic content of the cubic spline interpolation will now be briefly reviewed in this section. In our joint trajectory planning, we assume the robot manipulator has N joints and the trajectory for each joint has n knot points including the first and last point. The second and second-last knot points of the sequence are virtual points so that the initial and final conditions for velocity and acceleration can be respected. We illustrate the ith knot point of the jth joint as q_{ji} where $i=1,2,3,...,n$ and $j=1,2,...,N$. Parameter h_i is denoted as the length of the time interval between knot point i and $i+1$.

Trajectory in h_i is expressed as $Q_{ji}(t)$, then acceleration $\ddot{Q}_{ji}(t)$ can be expressed as:

$$\ddot{Q}_{ji}(\mathrm{t})=\frac{t_{i+1}-t}{h_i}\ddot{Q}_{ji}(t_i)+\frac{t-t_i}{h_i}\ddot{Q}_{ji}(t_{i+1}) . \quad (1)$$

By setting $Q_{ji}(t_i)=q_{ji}$ and $Q_{ji}(t_{i+1})=q_{j,i+1}$, the following interpolations can be obtained:

$$\begin{aligned}\dot{Q}_{ji}(t)=&\frac{-\ddot{Q}_{ji}(t_i)}{2h_i}(t_{i+1}-t)^2+\frac{\ddot{Q}_{ji}(t_{i+1})}{2h_i}(t-t_i)^2\\&+\frac{q_{j,i+1}-q_{ji}}{h_i}+\frac{h_i\ddot{Q}_{ji}(t_i)-h_i\ddot{Q}_{ji}(t_{i+1})}{6}\end{aligned} . \quad (2)$$

$$\begin{aligned}Q_{ji}(t)=&\frac{\ddot{Q}_{ji}(t_i)}{6h_i}(t_{i+1}-t)^3+\left(\frac{q_{ji}}{h_i}-\frac{h_i\ddot{Q}_{ji}(t_i)}{6}\right)(t_{i+1}-t)\\&+\frac{\ddot{Q}_{ji}(t_{i+1})}{6h_i}(t-t_i)^3+\left(\frac{q_{j,i+1}}{h_i}-\frac{h_i\ddot{Q}_{ji}(t_{i+1})}{6}\right)(t-t_i)\end{aligned} . \quad (3)$$

In Eq.2 and Eq.3, $j=1,2,...,N,\quad i=1,2,...,n-1$.

After some algebra, function followed can be obtained:

$$\begin{aligned}&\frac{h_{i-1}}{h_{i-1}+h_i}\ddot{Q}_{j,i-1}+2\ddot{Q}_{j,i}+\frac{h_i}{h_{i-1}+h_i}\ddot{Q}_{j,i+1}\\&=\frac{6}{h_{i-1}+h_i}\left(\frac{q_{j,i+1}-q_{j,i}}{h_i}-\frac{q_{j,i}-q_{j,i-1}}{h_{i-1}}\right)\end{aligned} . \quad (4)$$

where $2\le i\le n\text{-}1$.

Add four boundary conditions as followed:

$$\begin{cases}\dot{Q}_{j1}(t_1)=v_{j1},\dot{Q}_{jn}(t_n)=v_{jn}\\ \ddot{Q}_{j1}(t_1)=a_{j1},\ddot{Q}_{jn}(t_n)=a_{jn}\end{cases} . \quad (5)$$

Then Eq.4 can be expressed as:

$$\mathbf{KA}_j=\mathbf{B}_j . \quad (6)$$

Where $\mathbf{A}_j=\left[\ddot{Q}_{j1},\ddot{Q}_{j2},...,\ddot{Q}_{jn}\right]$. $\mathbf{B}_j$ is a known vector which is expressed as:

$$\mathbf{B}_j=\begin{cases}\dfrac{6}{h_1}\left(\dfrac{q_{j2}-q_{j1}}{h_1}-v_{j1}\right),i=1\\ \dfrac{6}{h_{j,i-1}+h_{j,i}}\left(\dfrac{q_{j,i+1}-q_{j,i}}{h_i}-\dfrac{q_{j,i}-q_{j,i-1}}{h_{i-1}}\right),2\le i\le n-1\\ \dfrac{6}{h_{n-1}}\left(v_{jn}-\dfrac{q_{j,n}-q_{j,n-1}}{h_{n-1}}\right),i=n\end{cases} . \quad (7)$$

$\mathbf{K}$ is non-singular and band-diagonal $n\times n$ matrix.

$$\mathbf{K}=\begin{bmatrix}2&1&&&\\ \gamma_2&2&\alpha_2&&\\ &\ddots&\ddots&\ddots&\\ &&\gamma_{n-1}&2&\alpha_{n-1}\\ &&&1&2\end{bmatrix}_{n\times n} . \quad (8)$$

$$\begin{cases}\gamma_i=\dfrac{h_{i-1}}{h_{i-1}+h_i}\\ \alpha_i=1-\gamma_i,i=2,3,...,n-1\end{cases} . \quad (9)$$

After Eq.6 is solved, we obtain the jerk of the trajectory of the jth joint by the function:

$$\dddot{Q}_{ji}=\frac{\ddot{Q}_{j,i+1}-\ddot{Q}_{ji}}{h_i} . \quad (10)$$

As most optimal trajectory planning methods are used in off-line calculation applications such as CNC system, which need massive calculation to get the trajectory. In order to simplify the calculation, trajectory can be dived into M parts. As velocity and acceleration of every joint at the starting point and stopping point is zero, and considering the characteristics of cubic spline, boundary conditions of the interpolation is defined as followed:

$$\begin{cases}k=1: & \boldsymbol{v}_1^1=0,\ \ \boldsymbol{v}_n^1=\left(\boldsymbol{q}_n^1-\boldsymbol{q}_1^1\right)/T\\ 1<k<M: & \boldsymbol{v}_1^k=\boldsymbol{v}_n^{k-1},\ \ \boldsymbol{v}_n^1=\left(\boldsymbol{q}_n^{k-1}-\boldsymbol{q}_1^{k-1}\right)/T\\ k=M: & \boldsymbol{v}_1^M=\boldsymbol{v}_n^{M-1},\ \ \boldsymbol{v}_n^M=0\end{cases} . \quad (11)$$

$$\begin{cases}k=1: & a_1^1=0,\ a_n^1=v_n^1/T\\ 1<k<M: & a_1^k=a_n^{k-1},\ a_n^k=(v_n^k-v_1^k)/T\\ k=M: & a_1^k=a_n^{k-1},a_n^k=0\end{cases} . \quad (12)$$

Limiting jerk of the joints yields several positive effects such as reduction of the stresses to the actuators and to the robot structure. Furthermore, it also contributes to a better accuracy in trajectory tracking and simplifies design of robot controllers.

In order to get the optimal jerk value, the object function followed is applied.

$$\min:\sum_{j=1}^{N}\int_0^{tf}\dddot{q}_j^{\,2}(t)dt . \quad (13)$$

Kinematics constraints of the platform should also be considered.

$$\begin{cases} \left|\dot{q}_j(t)\right| \le VC_j, \left|\ddot{q}_j(t)\right| \le WC_j, \left|\dddot{q}_j(t)\right| \le JC_j \\ \sum_{i=1}^{n-1} h_i = T, h_i \ge w_i \end{cases} . \quad (14)$$

VC_j is velocity constraint value, WC_j is acceleration constraint value and JC_j is jerk constraint value. T is execution time constraint value.

In order to apply the present technique, the object function and kinematic constraints should be expressed explicitly considering the fact that the trajectory is made of cubic spline.

Object function is expressed as followed.

$$\min : \sum_{j=1}^{N} \sum_{i=1}^{n} \frac{\left(\ddot{q}_{ji} - \ddot{q}_{j,i+1}\right)^2}{h_i} . \quad (15)$$

As equation of velocity in $[t_i, t_{i+1}]$ is quadratic polynomial. Extremum of $\dot{q}(t)$ is

$$\begin{cases} t_{j,i}^* = t_i + \dfrac{h_i \ddot{Q}_{j,i}}{\ddot{Q}_{j,i} - \ddot{Q}_{j,i+1}} \\ \dot{Q}_{j,i}^* = Q_{j,i}(t_{j,i}^*) \end{cases} . \quad (16)$$

Then the velocity constraint function can be expressed as

$$\max(\dot{Q}_{j,i}(t_i), \dot{Q}_{j,i}(t_{i+1}), \dot{Q}_{j,i}^*) \le VC_j . \quad (17)$$

where $j = 1, 2, ..., N$, $i = 1, 2, ..., n-1$.

The acceleration constraint function can be expressed as

$$\max(\ddot{Q}_{ji}) \le WC_j \quad \forall i = 1, 2, ..., n . \quad (18)$$

The jerk constraint function can be expressed as

$$\left|\frac{\left(\ddot{Q}_{ji} - \ddot{Q}_{j,i+1}\right)}{h_i}\right| \le JC_j \quad \forall i = 1, 2, ..., n-1 . \quad (19)$$

The execution time constraint function can be expressed as

$$h_i \ge w_i = \max\left(\frac{\left|q_{j,i+1} - q_{j,i}\right|}{VC_j}\right) . \quad (20)$$

Jerk optimization is to solve the minimum jerk problem under kinematics constraints during the trajectory planning in joint space, while jerk depends on the length of the time interval. Under circumstances of the certain range, the trajectory optimization problem of optimal jerk is a complex nonlinear problem seen from the optimization model. And a variety of complex constraints including inequality and equality functions were designed, then it's quite hard to get the analytical solution of the problem. So the numerical optimization algorithms should be used. Sequential quadratic programming is one of the common and effective numerical optimization algorithms. Considering the inequality and equality constraints, an improved sequential quadratic programming method (SQP-F), which is characterized by super-linear convergence, is adopted to solve the trajectory planning.

The steps for running the algorithm are summarized below:

Step1, set the kinematic constraints and execution time constraint parameter.

Step2, get a given path in the operating space, apply the kinematic inversion to get a sequence of knot points in the joint space.

Step3, chose a suitable initial solution and start the SQP-F algorithm.

III. SIMULATION AND EXPERIMENT RESULTS

The algorithm described in this paper is tested in simulation for a 6-joint robot. The input data has been taken to be the same as in [3] and the result is compared with that in [3]. Calculation program is developed in C++ language based on Visual Studio 2010 and runs on a Core I5-4590(3.3GHz) with 4G RAM.

Table I contains the values of the kinematic limits of the joints. Table II contains the values of the knot points. Figure 1 shows the jerk-optimal trajectory of the six joints and their velocity, acceleration and jerk. Execution time is 9.1S. We obtained h= [0.794, 2.412, 2.839, 2.21, 0.845], which can be validated by the results obtained in [3].

Table III contains the mean values of velocity, acceleration and jerk for all joints, compared with the results found in [3]. Approach presented in this paper yield lower mean value of velocity, acceleration and jerk.

Table I
Maximum Kinematic Constrains

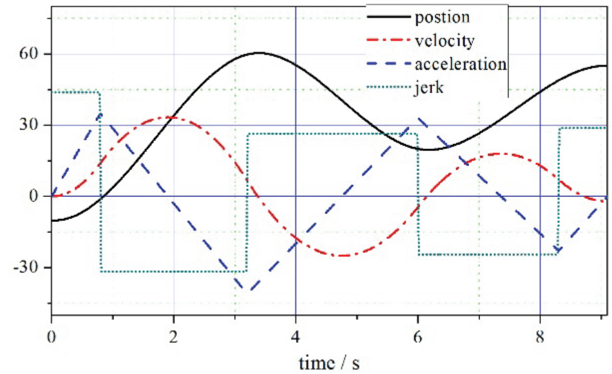
Joint Number	Velocity (°/s)	Acceleration (°/s^2)	Jerk (°/s^3)
1	100	60	60
2	95	60	66
3	100	75	85
4	150	70	70
5	130	90	75
6	110	80	70

Table II
Input Data of Trajectory Planning

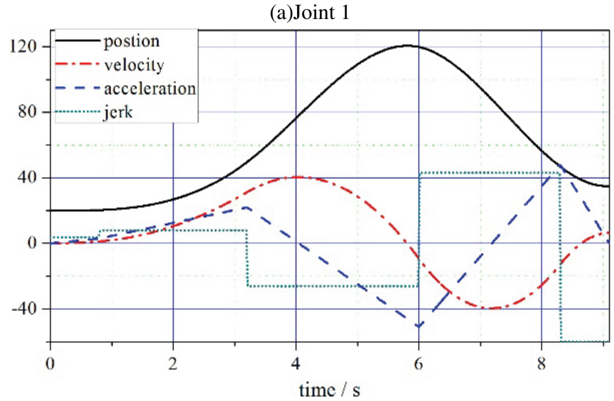
Joint Number	1	2	3	4	5	6
1	-10	Virtual Point	60	20	Virtual Point	55
2	20		50	120		35
3	15		100	-10		30
4	150		100	40		10
5	30		110	90		70
6	120		60	100		25

Table III
Input of Trajectory Planning

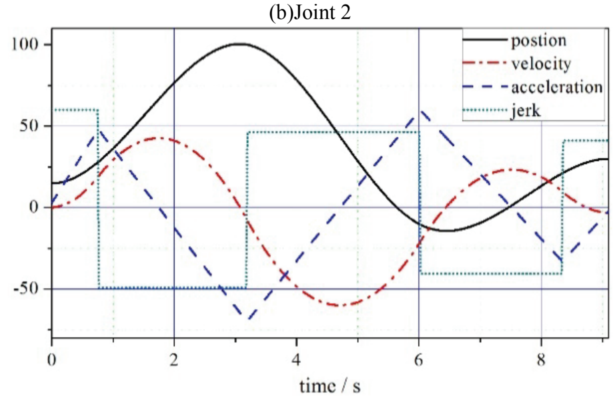
Joint Number		1	2	3	4	5	6
Our Method	V	15.97	20.35	26.11	15.31	14.17	19.15
	A	17.46	18.34	28.13	5.33	11.86	20.68
	J	29.77	27.13	46.79	5.49	16.33	35.78
Gaspa-Retto	V	16.10	20.69	26.42	15.38	14.46	19.50
	A	17.15	18.80	28.52	5.55	12.18	21.39
	J	29.24	27.68	47.35	5.75	16.38	36.97



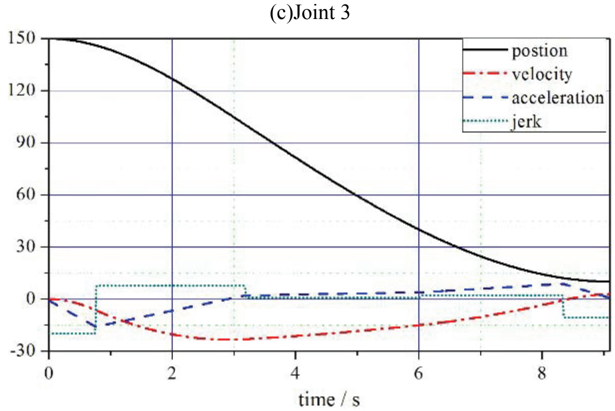
(a)Joint 1



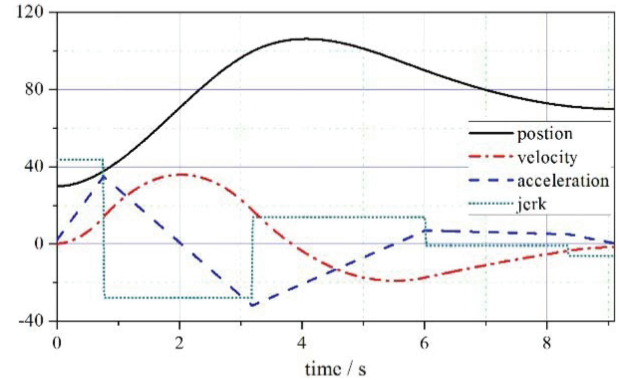
(b)Joint 2



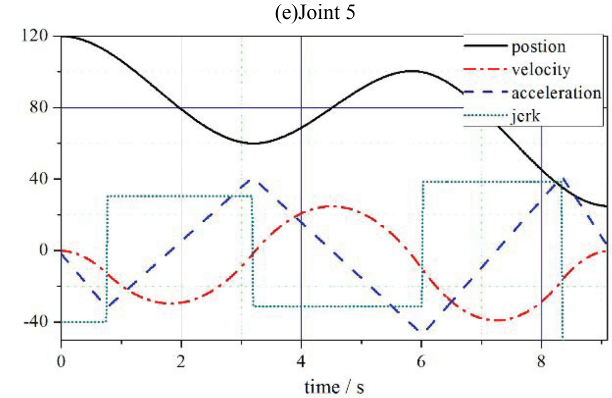
(c)Joint 3



(d) Joint 4



(e)Joint 5



(f)Joint 6

Fig.1 The Jerk-Optimal Trajectory of Joint1-6

Comparison with simulation result in reference [3] verifies the effectiveness of the algorithm presented in this paper.

Then algorithm is applied to solve the jerk optimal trajectory of Stewart platform in joint space. Figure 2 is the mechanical structure of the platform.

Figure 2
Mechanical Structure of the Stewart Platform

Table IV contains the values of the kinematic limits of the actuators. As six actuators of the platform are all the same, parameters of kinematic constraints are also the same. Table V contains the values of the knot points. Execution time between every adjacent knot is 50ms and the total execution time of the trajectory is 150ms.

Table IV
Kinematic Constraints of Stewart Platform

Velocity (mm/s)	Acceleration (mm/s^2)	Jerk (mm/s^3)
385	3000	4000

Table V
Kinematic Constrains of Stewart Platform

Joint Num	1 (mm)	2 (mm)	3 (mm)	4 (mm)
1	-1.5	-0.65	0.60	1.40
2	-0.5	0.09	-1.55	-0.92
3	1	2.30	1.59	2.90
4	2	1.03	1.26	0.30
5	-2.5	-3.41	-2.75	-3.67
6	3	2.72	1.85	1.60

Figure 3 shows the jerk-optimal trajectory of actuators of the Stewart platform. Figure 4 shows the velocity of every actuator. Figure 5 shows the acceleration of every actuator and figure 6 shows jerk of actuators.

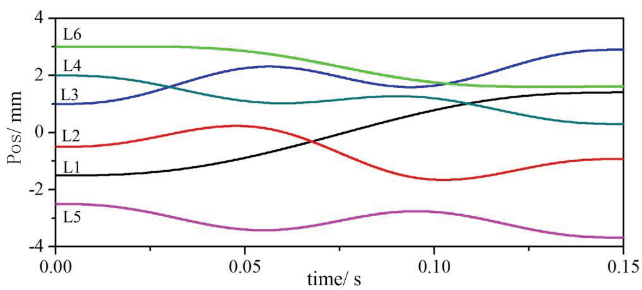


Fig. 3 The jerk-optimal trajectory of actuators

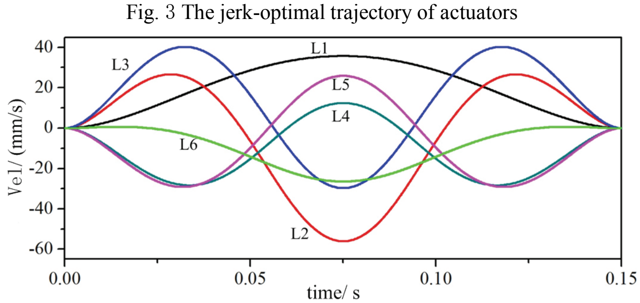


Fig. 4 Velocity of actuators of jerk-optimal trajectory

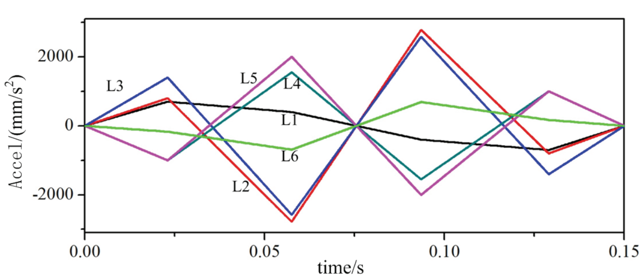


Fig. 5 Velocity of actuators of jerk-optimal trajectory

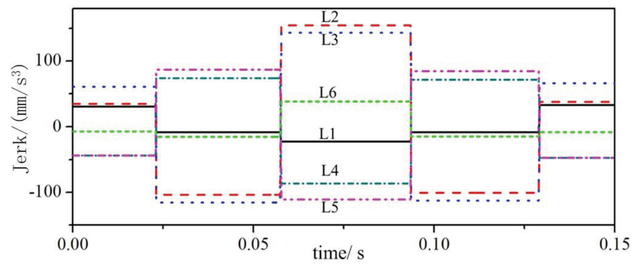


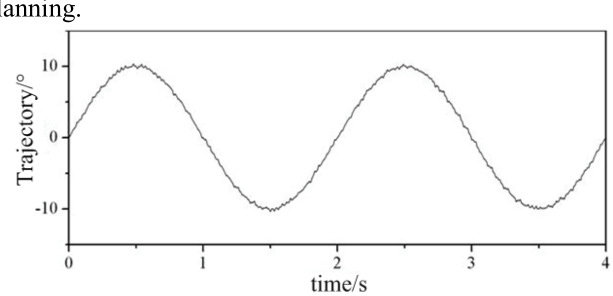
Fig. 6 Jerk of actuators of jerk-optimal trajectory

Two virtual knot points are added between the first and second knot point, the third and forth knot point. Then there are six knot points for calculation. The initial time interval is h = [25ms, 25ms, 50ms, 25ms, 25ms]. After the calculation, the time interval is h=[23.174ms, 34.267ms, 36.485ms, 35.096ms, 21.021ms], the execution time is 149.9ms. The maximum jerk of the trajectory is $154.7\,mm/s^3$, the maximum acceleration is $2884.7mm/s^2$, the maximum velocity is $-51.3mm/s$. All the results are less than the kinematic constraint values.

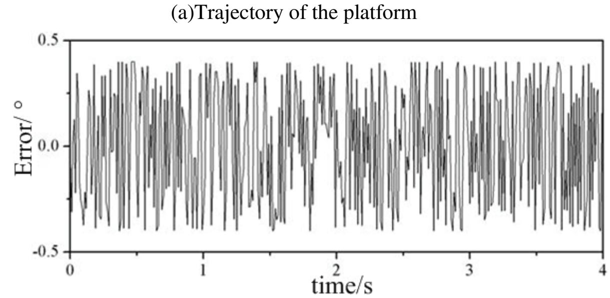
In motion control experiment, the jerk-optimal trajectory planning is compared with the cubic spline trajectory planning. Servo cycle of the servo system is 1ms and the instruction cycle is 20ms. The test signal is sinusoidal signal, frequency of the signal is 0.5Hz and amplitude is 10 deg, which can be expressed as:

$$\alpha = 10\sin(\pi t)\,. \quad (21)$$

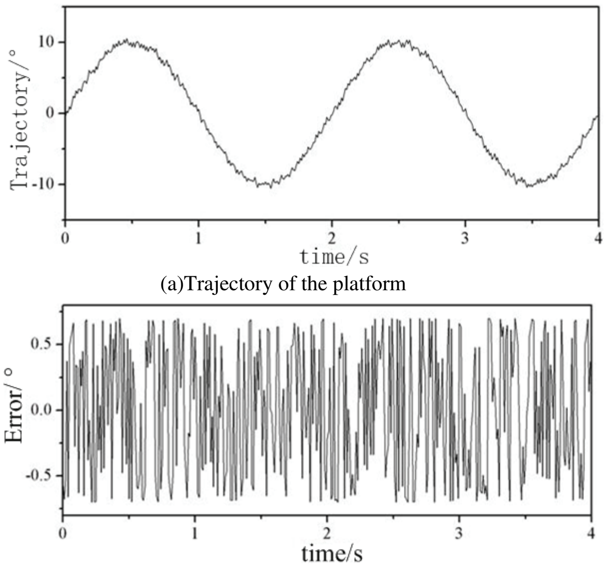
Figure 7 shows the result of jerk-optimal trajectory planning. Figure 8 shows the result of cubic spline trajectory planning.



(a)Trajectory of the platform



(b)Trajectory error of the platform
Fig. 7 Experiment result of jerk-optimal trajectory



(a)Trajectory of the platform

(b)Trajectory error of the platform

Fig. 8 Experiment result of cubic spline trajectory

For experiment result of jerk-optimal trajectory planning method, the maximum error of the roll angle is $0.47°$, while the mean error is $0.11°$. For experiment result of cubic splines trajectory planning method, the maximum error of the roll angle is $0.64°$, while the mean error is $0.18°$.

IV CONCLUSION

This paper proposed a jerk-optimal robot trajectory planning method in joint space considering kinematics constraints and execution time constraint to get robot running smoothly, ensure movement accuracy, improve the stability and robustness of the movement controller algorithm. Cubic splines are applied to define the trajectory and a special interpolation technique is used to ensure the continuity of velocity, acceleration. In order to solve the non-linear optimization problem, an improved sequential quadratic programming method (SQP-F) is adopted. The algorithm is tested in simulation experiment, taking data found in [3] as input. Comparison of the results with those obtained from [3]show that the algorithm is effective in performing an optimal trajectory planning. Then the algorithm is applied to get the jerk-optimal trajectory of Stewart platform in joint space. Finally, the jerk-optimal trajectory planning method is compared with traditional cubic spline trajectory in motion control experiment, the result shows that the track error of jerk-optimal trajectory is less than track error of cubic spline trajectory.

V ACKNOWLEDGEMENTS

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