Available online at www.sciencedirect.com

ScienceDirect

Procedia Engineering 181 (2017) 214 – 220

Procedia Engineering

www.elsevier.com/locate/procedia

10th International Conference Interdisciplinarity in Engineering, INTER-ENG 2016

# Analysis and Optimum Kinematic Design of a Parallel Robot

Ouafae Hamdoun[a,*], Larbi El Bakkali[a], Fatima Zahra Baghli[a]

[a]*Modeling and Simulation of Mechanical Systems Laboratory, Abdelmalek Essaadi, Faculty of Sciences, BP.2121, M'hannech, 93002, Tetouan, Morocco*

**Abstract**

Parallel robots display important advantages over their serial mechanisms in several applications where both accuracy and dynamic response are needed. Parallel manipulators have been developed extensively for applications that need high accuracy, speed and stiffness, which make them, useful in important fields. But during the designing of the parallel mechanism, it is difficult to determine the dimension of structure and the workspace for the mechanism. For this cause, kinematic optimal design is an important key in designing parallel manipulators, and was received a big attention by researchers in the past decade. Thus, because of the strong dependence of geometric performances and their parameters, the design problems for the parallel robots are more complex and the efficacy of the design method become more difficult. In view of the fact that, the robot's performance depends on numerous factors, it is difficult to say that a particular design is the only solution to a given problem, even though for a robot with only one degree of freedom and four links. If the number of links and number of DoF augment, the design becomes more difficult for the robot. Thus, it is right to know how good a mechanism may run when it is still in the design stage.

In this paper, we present first, the inverse kinematic problem and Jacobian matrix of the 3RRR parallel manipulator which is necessary for subsequent analysis, then, an optimal design study is achieved for a class of parallel robots in order to find a set of parameters that attain a good performance in terms of the important performances indexes: the workspace capabilities and dexterity. Finally, simulations results are obtained.



Peer-review under responsibility of the organizing committee of INTER-ENG 2016

*Keywords:* Inverse kinematic; Jacobian Matrix; Parallel Robot; Optimal design.

* Corresponding author. Tel.:+212-653-731613.
*E-mail address:* wafae.hamdoun@gmail.com.



Peer-review under responsibility of the organizing committee of INTER-ENG 2016
doi:10.1016/j.proeng.2017.02.374

## 1. Introduction

Parallel robots have been studied for a years ago, due to its advantages of high velocity, high precision, high stiffness, high load/weight ratio and low moving inertia and are still attracted much attention from industry and universities. In this area, the optimal design considered an important subject in designing a parallel manipulator.

For parallel robots, numerous well-defined performance indices, which are known in serial robot's field, have been developed widely and applied to the design. On the other hand, a new study [1] noted the common indices that have been applied to the optimal design of parallel robots. They are the global conditioning index [2] that is the calculation over a kind of workspace of the robot and the Jacobian matrix's condition number [3] used to improve its accuracy.

In order to avoid the unwanted effects of the maximization of the workspace [4], researchers considered the criteria of other performance inside the optimal design problem [5]. Gosselin and Angeles designed a planar 3-DoF parallel manipulator [1] and a spherical 3-DoF parallel manipulator [3] by maximizing the workspace volume while taking into account the isotropy index. Pham and Chen [6] proposed the maximization of the workspace of a parallel flexure mechanism subject to the constraints about a global index and a uniformity index of manipulability. In [7] Stamper et al. proposed to maximize the total volume of well-conditioned workspace. In the design problem, the objective function was selected as the integral of inverse condition number of the kinematic Jacobian matrix over the workspace, and the link lengths of each subchain were normalized as a constraint [8].

In the remainder of the paper, the geometric description of the mechanism and the traditional kinematic problems, e.g. inverse kinematics and Jacobian matrix are described in details. After the proposition and the definition of some indices, the optimal design problem of the 3RRR parallel robot using the recommended indices, is carried out by using Matlab software to understand its behaviour.

**Nomenclature**

| | |
|---|---|
| P(x, y) | the end-effecter position |
| $\varphi$ | the orientation of the end-effecter |
| $A_i, B_i$ *and* $C_i$ | the rotational articulations of robot's leg |
| $l_{A_i}$ | the length of Link A |
| $l_{B_i}$ | the length of Link B |
| $\theta_i$ | the input joint angles |
| k | the condition number |
| $\sigma_{max}$ | the maximum singular value of robot's Jacobian matrix |
| $\sigma_{min}$ | the minimum singular value of robot's Jacobian matrix |

## 2. Parallel robot's description

This work presented a geometric description of 3 RRR parallel robots which composed by a mobile platform (MP) and three RRR serial chains that join it to a fixed base (Fig. 1). Each RPR chain is a serial chain composed by three rotational R joints.

Allow P(x, y) be the end-effecter position in the plane and $\varphi$ its orientation. Let O be the origin of the fixed reference frame $A_i$ Points are actuated, so that the actuators are fixed to the base.

## 3. Robot's Kinematic Problem

The inverse kinematic is to find the input joint angles, $\theta_{i1}, for$ i=1,2 and 3, for a given end effector position, $p = [x, y, \varphi]$. From mechanism's geometry and from the closure equation we can set the end effector position as follows:

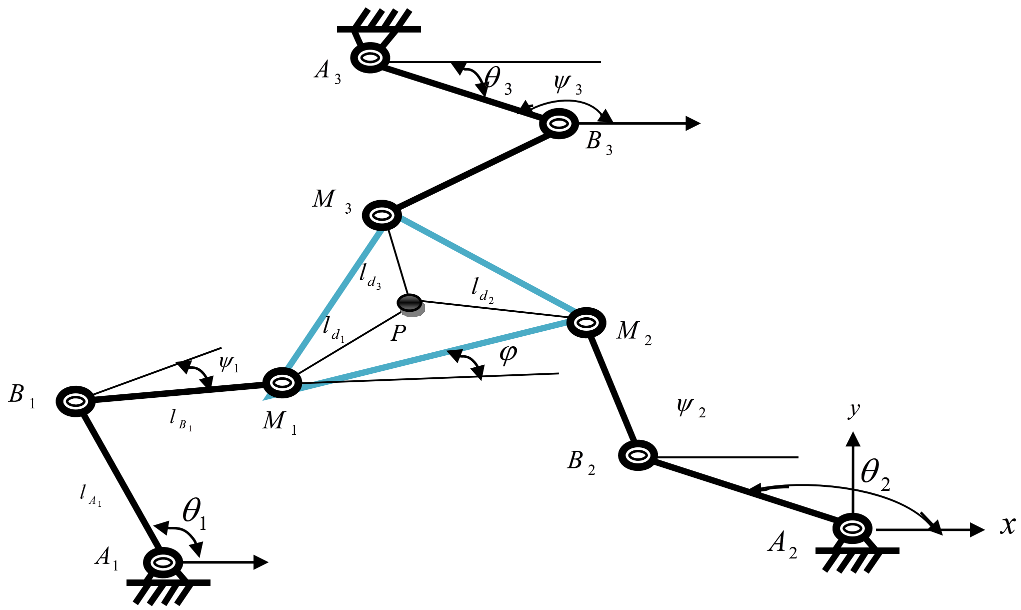


Fig. 1. Geometric structure of parallel robot.

$$P_x = l_{Ai}\cos(\theta_i) + l_{Bi}\cos(\theta_i + \psi_i) + l_{di}\cos(\sigma_i + \varphi) + x_{Ai}$$
$$P_y = l_{Ai}\sin(\theta_i) + l_{Bi}\sin(\theta_i + \psi_i) + l_{di}\sin(\sigma_i + \varphi) + y_{Ai} \tag{1}$$

where $\sigma_i$ is the angle between the lines from the point P to the joint at $M_i$ and it is considered from the line between $M_1$ and $M_2$ on the moving platform.

So $\theta_i$ are found as follows:

$$\theta_i = A\tan 2(K_i, F_i) \pm A\tan 2(\sqrt{(K_i^2 + F_i^2 - E_i^2)}, E_i) \tag{2}$$

where $i = 1, 2, 3$

$$E_i = P_x^2 + P_y^2 + l_{1i}^{\ 2} + l_{2i}^{\ 2} + l_{3i}^{\ 2} + x_{Ai}^{\ 2} + y_{Ai}^{\ 2}$$
$$-2P_x l_{3i}\cos(\sigma_i + \varphi) - 2P_x x_{Ai}$$
$$+2l_{3i}x_{Ai}\cos(\sigma_i + \varphi) - 2P_y l_{3i}\sin(\sigma_i + \varphi) -$$
$$2P_y y_{Ai} + 2l_{3i}y_{Ai}\sin(\sigma_i + \varphi) \tag{3}$$

$$K_i = 2P_y l_{Ai} - 2l_{Ai}l_{di}\sin(\sigma_i + \varphi) - 2l_{Ai}y_{Ai}$$

## 4. Jacobian Matrix

Many methods are defined to find the Jacobian matrix of the parallel mechanism including differentiation of the inverse kinematic equation. The Matrix Jacobian is defined as the matrix representing the transformation mapping the joint rates into the Cartesian velocities transformation. In fact, we present the analytical development of robot's Jacobian matrix, for each leg by differentiating (1), we can get:

$$\begin{aligned}\dot{x} &= -l_{A_i}\sin(\theta_i)\dot{\theta}_i - l_{B_i}\sin(\theta_i+\psi_i)(\dot{\theta}_i+\dot{\psi}_i) - l_{d_i}\sin(\sigma_i+\varphi)\dot{\varphi}\\ \dot{y} &= l_{A_i}\cos(\theta_i)\dot{\theta}_i + l_{B_i}\cos(\theta_i+\psi_i)(\dot{\theta}_i+\dot{\psi}_i) + l_{d_i}\cos(\sigma_i+\varphi)\dot{\varphi}\end{aligned} \tag{4}$$

We solve this equation by eliminating the passive variable $\dot{\psi}_i$ :

$$l_{A_i}\sin(\dot{\psi}_i)\dot{\theta}_i = \cos(\theta_i+\psi_i)\dot{x} + \sin(\theta_i+\psi_i)\dot{y} - l_{d_i}\sin((\dot{\theta}_i+\dot{\psi}) - (\sigma_i+\varphi))\dot{\varphi} \tag{5}$$

Thus, we can write (5) in the matrix form us follows:

$$J_\theta\dot{\Theta} = J_x\dot{\mathrm{X}} \tag{6}$$

with

$$\lambda = \theta_i + \psi_i$$

$$J_x = \begin{bmatrix}\cos(\lambda_1) & \sin(\lambda_1) & \text{-}\mathrm{l}_{\mathrm{d}_1}\sin(\lambda_1-(\sigma_1+\varphi))\\ \cos(\lambda_2) & \sin(\lambda_2) & \text{-}\mathrm{l}_{\mathrm{d}_2}\sin(\lambda_2-(\sigma_2+\varphi))\\ \cos(\lambda_3) & \sin(\lambda_3) & \text{-}\mathrm{l}_{\mathrm{d}_3}\sin(\lambda_3-(\sigma_3+\varphi))\end{bmatrix} \text{ et } \mathrm{J}_\theta = \begin{bmatrix} l_{A_1}\sin\psi_1 & 0 & 0\\ 0 & l_{A_2}\sin\psi_2 & 0\\ 0 & 0 & l_{A_2}\sin\psi_2\end{bmatrix}$$

In the expression above, we found two separate Jacobian matrices, these matrices can be combined to obtain a single matrix, which sets the inverse transformation between the input and output vilocities:

$$\dot{\Theta} = J\dot{\mathrm{X}} \tag{7}$$

with $J = J_\theta^{-1}J_x$, the Jacobian matrix of the manipulator:

$$\mathrm{J} = \begin{bmatrix}\dfrac{\cos(\lambda_1)}{l_{A_1}\sin\psi_1} & \dfrac{\sin(\lambda_1)}{l_{A_1}\sin\psi_1} & \dfrac{\text{-}\mathrm{l}_{\mathrm{d}_1}\sin(\lambda_1-(\sigma_1+\varphi)}{l_{A_1}\sin\psi_1}\\ \dfrac{\cos(\lambda_2)}{l_{A_2}\sin\psi_2} & \dfrac{\sin(\lambda_2)}{l_{A_2}\sin\psi_2} & \dfrac{\text{-}\mathrm{l}_{\mathrm{d}_2}\sin(\lambda_2-(\sigma_2+\varphi)}{l_{A_2}\sin\psi_2}\\ \dfrac{\cos(\lambda_3)}{l_{A_3}\sin\psi_3} & \dfrac{\sin(\lambda_3)}{l_{A3}\sin\psi_3} & \dfrac{\text{-}\mathrm{l}_{\mathrm{d}_3}\sin(\lambda_3-(\sigma_3+\varphi)}{l_{A_3}\sin\psi_3}\end{bmatrix}$$

## 5. Evaluation of design indexes

### *5.1. Workspace index :*

One of the mainly important indexes in the process of design of parallel robot is workspace index because of their closed loop nature which limits their workspace. Due to this, many applications relating to these parallel robots need a detailed analysis and visualization of the workspace [9]. Thus, the workspace of the 3RRR Parallel Robot is obtained by the intersection of the three circular mobility regions of the three legs, which described in the x-y plane as shown in Fig.2.

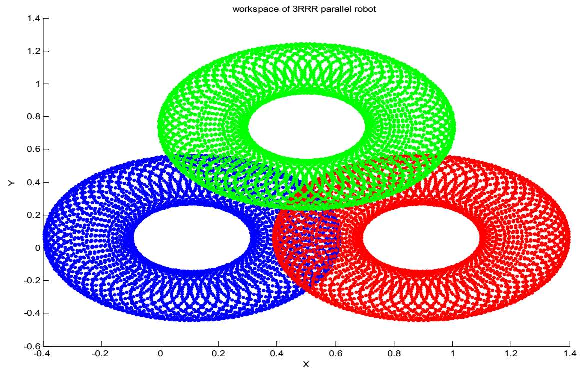


Fig. 2. Intersection between three regions of 3RRR parallel robot.

The workspace of the parallel robot was obtained by using an algorithm established using Matlab based on the solution of the inverse kinematics problem. The important advantage of this approach is that it leads for an effective quantification of workspace boundary as well as singularities, as presented in Fig.3 we can estimate the robot and its motions relative to the workspace.

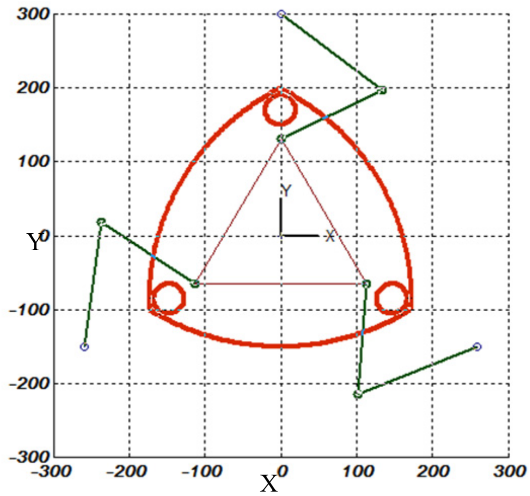


Fig. 3. 3RRR parallel robot's workspace.

*5.2. Dexterity index*

In order to make the regular workspace of the robot more effective, we introduce some on the dexterity index into the design problem to improve the quality of the regular workspace [10]. A regularly-used index for dexterity of a manipulator is the inverse condition number of the kinematic Jacobian matrix, which is defined as follows:

$$\kappa(J) = \left\|J^{-1}\right\| . \left\|J\right\| \quad (8)$$

## 6. Optimal Design

In this section, we present an optimal design for 3RRR parallel robot composed by three subchains which are identical in geometry, all the three actives joints are set in the way that, they, as vertices, compose an equilateral triangle $\left\| A_i A_j \right\| = r, i, j = 1,2,3, i \neq j$, this one consisting of the three non-actuated joints on the moving platform as its vertices is equilateral.

Let assume $l_{1i} = d$ , $l_{2i} = e$ and $l_{3i} = f$ for i=1,2,3 and $\left\| A_i A_j \right\| = r, i, j = 1,2,3, i \neq j$ .

Thus, the design parameter's vector defined as: $\beta = \left[d \;\; e \;\; f \;\; r\right]^T$

The manipulator size is normalized by: *d*=0.5021, *e*=0.4596, *f*=0.0004. So, the formulation of the optimal design problem can be written as follows in order to maximize the effective regular workspace of the robot:

find a set of optimal design parameters $\beta$ such that:

$$
\begin{aligned}
&\max_{\beta} && 1 \\
&\text{subject to} && \mathrm{K}\left(\hat{\mathrm{J}}\left(X,\theta,\beta\right)\right) \geq 0.2; \\
& && -\frac{\pi}{3} \leq \theta_1\left(\mathrm{X},\beta\right) \leq \frac{\pi}{3}; \\
& && \frac{\pi}{3} \leq \theta_2\left(\mathrm{X},\beta\right) \leq \pi; \\
& && \pi \leq \theta_3\left(\mathrm{X},\beta\right) \leq \frac{5\pi}{3}; \\
& && \mathrm{d} + \mathrm{e} + \mathrm{f} = 1; \qquad \mathrm{d, e, f} \in [0,1], \;\; \mathrm{c} \in [0,2];
\end{aligned}
\tag{9}
$$

## 7. Results and Discussions

As results, we obtain the optimal parameters *d*=0.5021, *e*=0.4596 for the maximum workspace; also, the resulting manipulator has a smaller moving platform which means that a small size of the moving platform is wanted for the maximization of effective regular workspace.

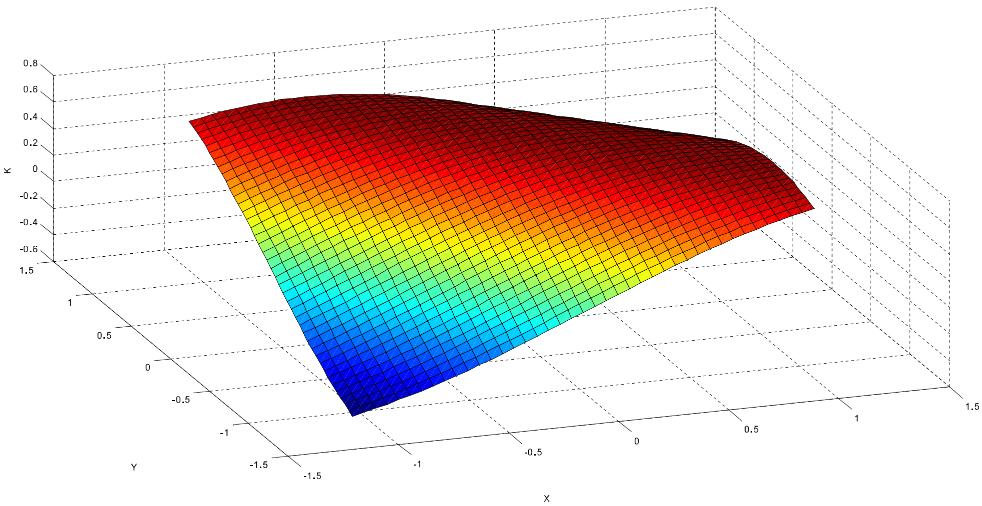


Fig. 4. The plot of the inverse condition number of the robot for $\varphi = \pi / 3$ .

Figure 4 present the plot of the inverse condition number as function of $q$ for $\varphi = \pi / 3$ .

## 8. Conclusion

In this work we present the solution of the inverse kinematics problem of the 3-RRR planar parallel robot based on the closure equation, than the Jacobian matrix is calculated, in order to calculate the condition number which is important for the optimal design of the robot. The optimal design problem is then established to find a manipulator geometry that maximizes the effective regular workspace.

## References

[1] C. Gosselin, J. Angeles, The optimum kinematic design of a spherical three-degree-of-freedom parallel manipulator. J Mech Transm Autom Des. 11(2) (1989) 202-207.
[2] J.P. Merlet, Jacobian, manipulability, condition number, and accuracy of parallel robots. ASME J. Mech. Des., 128 (2006) 199-206.
[3] C. Gosselin, J. Angeles, A global performance index for the kinematic optimization of robotic manipulators. ASME J. Mech. Des., 113 (1991) 220―226.
[4] L. Moldovan L. Geometrical Method for Description of the 6-PGK Parallel Robot's Workspace. IEEE Proc. of First International Conference on Complexity and Intelligence of the Artificial and Natural Complex Systems, Medical Applications of the Complex Systems, Biomedical Computing, 2008. CANS '08; 2008:45-51.
[5] L. Moldovan, H.S. Grif, A. Gligor, ANN Based Inverse Dynamic Model of the 6-PGK Parallel Robot Manipulator. International Journal of Computers Communication & Control, 11-1 (2016), 90-104.
[6] H.H. Pham, I-M. Chen, Optimal synthesis for workspace and manipulability of parallel flexure mechanism. In Proceeding of the 11th World Congress in Mechanism and Machine Science, 2004.
[7] R.E. Stamper, L.W. Tsai, G.C. Walsh, Optimization of a three dof translational platform for well-conditioned workspace. In Proceedings of IEEE International Conference on Robotics and Automation, 1997, 3250–3255.
[8] D. Chablat, Ph. Wenger, Architecture optimization of a 3-DoF parallel mechanism for machining applications: the Orthoglide, IEEE Trans. Robot. Autom. 19 (3) (2003) 403–410.
[9] M.Z. Huang, Design of a Planar Parallel Robot for optimal workspace and dexterity, 183 Int J Adv Robotic Sy, 2011, Vol. 8, No. 4, 176-183
[10] D. Chablat, S. Caro, R. Ur-Rehman, P. Wenger, Comparison of planar parallel manipulator architectures based on a multi-objective design optimization approch, Proceedings of the ASME 2010 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference IDETC/CIE 2010 August 15-18, 2010, Montreal, Quebec, Canada.