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Dynamic Modelling and Control of a 3-DOF Planar Parallel Robotic ($XY\theta_Z$ Motion) Platform

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Abstract

This study addresses the dynamic modelling and control of a novel three degrees of freedom (3-DOF) planar parallel robotic motion platform in the presence of parameter uncertainties and external disturbances. The proposed planar parallel motion platform is a guide way free manipulator and has two legs both with a revolute-prismatic-revolute (RPR) configuration located in the same plane connecting a moving platform. Of the two, one leg comprises of only one active prismatic joint, while the other leg consists of an active prismatic and an active rotary joint. The dynamic model has been derived using Euler-Lagrangian energy based formulation method. The proposed controller is based on computer torque control integrated with a disturbance observer. The disturbance vector comprising of disturbances due to parameter uncertainties, payload variations, frictional effects and further disturbances is estimated using a nonlinear disturbance observer incorporated with an extended Kalman filter (EKF). The observer uses only position and orientation measurements. Simulations with a typical trajectory are presented and compared with traditional controllers such as proportional integral derivative (PID) controller and computed torque controller, and the results show that superior tracking performance is achieved in the presence of parameter uncertainties and external disturbances.

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Keywords: Parallel robotic motion platform; dynamic modelling; trajectory control; disturbance observer; extended Kalman filter.

1. Introduction

Parallel kinematic machines, which are developed on the basis of parallel manipulators, have numerous advantages over their serial counterparts, one of which being able to work with ease at high speeds due to their closed kinematic

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loops (Bonev I.A., Briot S., 2007 and Bonev I.A., Gosselin C.M., Zlatanov D., 2003) which is in accord with the recent developmental trends in manufacturing and machine tool maneuvers. Thus, a high-output velocity is a crucial point for parallel manipulators to be applied into parallel kinematic machines. Although the reason for parallel robots to be more precise than serial robots is said to be because of absence of error accumulation in theory (Bonev I.A., Gosselin C.M., Zlatanov D., 2003), the real reason is that parallel robots can be built to be stiffer without being bulkier. Spatial precision positioning devices are often based on hexapods or tripods. However, when it comes to planar three-degree-of-freedom (3-DOF) positioning, virtually all commercial so-called XY-Theta positioning tables are based on the well-known sandwich serial configuration setup. When paralleled with serial manipulators, the following are the potential advantages of parallel architectures: higher kinematical precision, lighter weight and better stiffness, greater load bearing, stable capacity and suitable position of arrangement of actuators. In spite of these advantages, they do have some limitations like restricted workspace and complicated singularities (Bonev I.A., Briot S., 2007, Bonev I.A., Gosselin C.M., Zlatanov D., 2003 and Becker O., Hesselbach J., Raatz A., Wrege J., 2004).

Recently, more general approaches have been presented in this field. Merlet (Merlet J.P., 1996) solved the forward pose kinematics problem for a broad class of planar parallel manipulators. Williams et al. (Reinholtz C.F., Williams R.L., 1998) analysed the dynamics and the control of a planar three-degree-of-freedom parallel manipulator at Ohio University, while Yang et al. (Chen I.M., Chen W., Yang G., 2002) concentrated on the singularity analysis of a class of 3-RRR planar parallel robots which was developed in its laboratory. Bonev et al. (Bonev I.A., Gosselin C.M., Zlatanov D., 2003) described several types of singular configurations by studying the direct kinematics model of a 3-RPR planar parallel robot with actuated base joints. Among the few existing planar 3-DOF parallel robot prototypes that do not employ flexures, one is based on symmetric 3-PRP architecture (Company O., Fournier A., Pierrot F., Ronchi S., 2004), where the base actuators form an equilateral triangle and the linear guides of the platform form a star shaped pattern. This robot has a very limited workspace though. In contrast, the 3-RRP robot first disclosed in (Liechti R., Scheidegger A., 2003), then studied in (Bonev I.A., Yu A., Zsombor-Murray P., 2006), a first prototype of which was reported in (Hufnagel T., Muller A., 2012), offers unlimited rotation in addition to excellent stiffness in the vertical direction. However, the accuracy realizable in such a robot is questionable, since it relies on the use of a perfectly circular rail. The only commercially available parallel XY-Theta positioning table, manufactured by Hephaest Seiko is also based on the 3-PRP architecture, but its design is asymmetric (Cleghorn W.L., Mills J.K., Zhang Z., 2010). Redundant actuation of parallel manipulators is deemed a promising approach to improve their kinematic and dynamic properties. It can, in particular, eliminate the singularities and thus increase the usable range of motion (Aradyfio D.D., Qiao D., 1985). Thus, redundantly actuated parallel manipulators have attracted more and more attention.

Based on the analysis of the literature, it was found that the previous attempts on robotic planar positioning system had limited success due to the following reasons: (a) Small workspace, short stroke lengths and less dexterity (b) Bulky structure or heavier, and high cost (c) Usage of guide ways / rail (lead screw with rotary actuator) for X and Y movements (d) Coupled system dynamics and non-linear control schemes (e) Most of the systems are limited to only positioning applications (not used for the trajectory tracking applications)

The review of literature on the present status of parallel robotic planar positioning platform has clearly shown the importance of optimum design and controller development in realizing a sophisticated parallel robotic XY θ_z stage and there are plenty of opportunities and lots of potentials to develop/create a better motion stage. In this work, a new parallel robot is proposed, which can deliver accurate (planar) movements, is partially decoupled and has a relatively large workspace and low cost. It is expected that the proposed system would overcome most of the limitations listed above and would possibly perform better than the conventional (serial link) platforms and existing parallel platforms. The novelty of this parallel robot lies in its ability to achieve the decoupled state by employing legs of a different kinematic structure and act as a single degree of freedom system (similar to that of four-bar chain). The proposed system does not require any guide ways for the X and Y movements. The single moving element structure of the proposed parallel robot system is expected to overcome the problems of the conventional multi-axis stages.

The remainder of this paper is organized as follows. The materials and methods of the proposed system are discussed in Section 2. In Section 3, a detailed performance analysis along with results is discussed. Finally, Section 3 summarizes the conclusions of this study.

2. Materials and Methods

The proposed parallel robot has two legs and one work table (end effector) as shown in Fig. 1 (as a conceptual design in solid model form). Both legs have two degrees of freedom (RPR); out of this one leg has two active joints (i.e., one rotary and one prismatic joint) and one passive joint (rotary) and, other leg has one active joint (prismatic joint) and two passive joints (both rotary). These two legs are connected with the work table by two passive rotary joints. Totally, the proposed parallel robotic manipulator has six links (including the fixed link-frame) and six single degree of freedom joints (four revolute pairs and two prismatic pairs). Based on the Gruebler's mobility criterion, the mobility of the planar robotic manipulator is three. i.e., the proposed robotic manipulator has three degrees of freedom, one rotary input and two translation inputs as shown in Fig.2 (b).

2.1 Kinematic model of the 3-DOF (RPR) parallel robot

Referring Fig.2 (b), the frame AD is fixed at the base so that this link considered as a reference link/frame. The parallel robotic manipulator has three task space variables: X and Y translations and rotation about Z axis (θ_z). These task space variables are dependent on three joint space (system) variables: joint angle (α) and two joint lengths (d_1 and d_2). α is the angle (active joint angle) between the link 2 (AE) and the x axis, measured counter clockwise. d_1 is the active joint variable associated with the first leg and is defined as the distance between the joint E (on leg one) and joint B (on the end-effector) as illustrated in Fig. 2. Similarly, d_2 is the active joint variable associated with the second leg and is defined as the distance between the joint F (on leg two) and joint C (on the end-effector) as illustrated in Fig. 2. Forward kinematics refers to the use of the kinematic equations of a robot to compute the position (x and y) and orientation (θ) of the end-effector/work table (task space parameters) from specified values for the joint parameters (joint angle and joint distances). The forward kinematic equations of the 3-DOF parallel robotic manipulator are as given below

$$\begin{aligned}x &= (L_2 + d_1) \cos \alpha + L_p \cos \theta \\y &= (L_2 + d_1) \sin \alpha + L_p \sin \theta \quad (1) \\ \theta &= \tan^{-1} \left[\frac{C_y - B_y}{C_x - B_x} \right]\end{aligned}$$

Where, L_1 , L_2 , L_3 and L_4 are the link lengths of the link 1, link 2, link 3 and link 4, respectively. L_p is the distance between point B and the moving platform centroid. Inverse kinematics transforms the motion plan into joint actuator trajectories for the robot. The inverse kinematic equations of the 3-DOF parallel robotic manipulator are as follows:

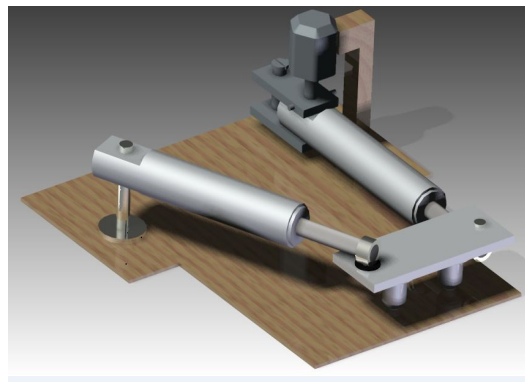


Fig. 1. Solid model representation of the proposed robotic manipulator

$$\alpha = \tan^{-1} \left[\frac{B_y}{B_x} \right]$$

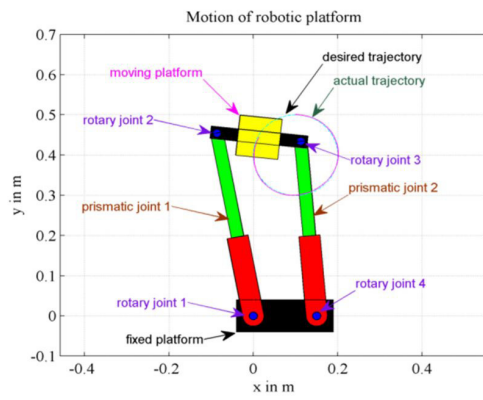
$$\beta = \tan^{-1} \left[\frac{C_y}{L_1 - C_x} \right]$$

$$d_1 = \frac{B_x}{\cos \alpha} - L_2$$

$$d_2 = \frac{L_1 - C_x}{\cos \beta} - L_4$$

(2)

Where, $B_x = x - L_p \cos \theta$, $B_y = y - L_p \sin \theta$
 $C_x = x + (L_3 - L_p) \cos \theta$, $C_y = y + (L_3 - L_p) \sin \theta$



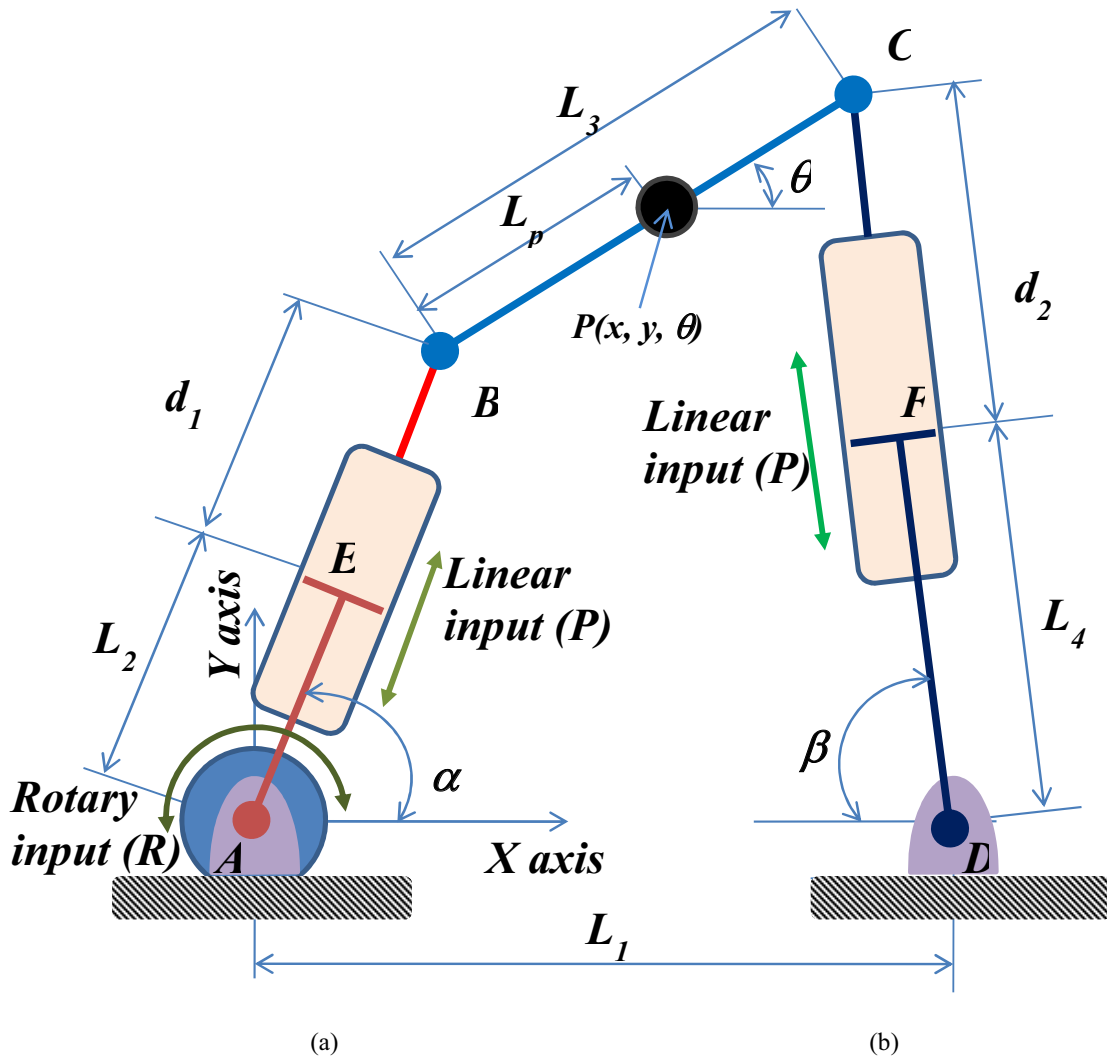


Fig. 2. (a) Joint arrangements of the proposed robotic manipulator, (b) Schematic diagram of the parallel robotic manipulator

2.2 Dynamic model of the 3-DOF (RPR) parallel robot

This section describes the relationship between joint forces, torques and joint space parameters. The dynamic model of the proposed has been developed through the Euler-Lagrangian formulation method based on energy approach. The total kinetic energy of the proposed manipulator has been calculated as the sum of the kinetic energy of the mobile platform and the kinetic energy of each of the legs by considering each of them as single 3-DOF (RPR) serial manipulator. The potential energy of the platform is zero since the proposed manipulator is planar and motion is restricted to the horizontal plane. The dynamic equations of motion of the proposed robotic platform are nonlinear and coupled, and it can be expressed as follows:

$$M(\zeta)\ddot{\zeta} + C(\zeta, \dot{\zeta})\dot{\zeta} = \tau + \tau_{dis} \quad (3)$$

Where, $\ddot{\zeta} = [\ddot{d}_1 \quad \ddot{d}_2 \quad \ddot{\alpha}]^T$ is the vector of joint accelerations. $\dot{\zeta} = [\dot{d}_1 \quad \dot{d}_2 \quad \dot{\alpha}]^T$ is the vector of joint velocities.

$\varsigma = [d_1 \ d_2 \ \alpha]^T$ is the vector of joint variables. $M(\varsigma)$ is the inertia matrix. $C(\varsigma, \dot{\varsigma})$ is the Coriolis and centripetal matrix. $\tau = [f_1 \ f_2 \ \tau_3]^T$ is the vector of joint control inputs. f_1 and f_2 are the joint input forces of the prismatic joints 1 and 2, respectively. τ_3 is the joint input torque of the rotary joint. $\tau_{dis} = \tau_{edis} + \tau_{idis}$ is the vector of internal and external disturbances. τ_{idis} is the vector comprising of internal disturbances (e.g., due to system process noises (parameter uncertainties), measurement noises, etc.) and τ_{edis} is the vector comprising of external disturbances (e.g., due to payload variations, vibrations, reactions, etc.).

2.3 Controller design of the 3-DOF (RPP) parallel robot

This subsection describes a controller integrated with a disturbance observer for the proposed motion platform. As system uncertainties, and external disturbances such as pay load variations and friction (time damping) have a significant impact on system tracking performance in case of the parallel planar manipulators, many researchers have been keen on synthesizing advanced control methods for improving the system robustness under the assumption that the known system parameters are accurate and disturbances are directly measured by force sensors.

Disturbance observer: The proposed disturbance observer estimates the disturbance vector based on the known system dynamics (in general, inaccurate system parameters), available estimated displacement and velocity vectors which are obtained from the EKF based on displacement or positional measurements (with sensor noises). The proposed disturbance observer model is given as follows:

$$\begin{aligned}\hat{\tau}_{dis} &= z + \mu(\varsigma, \dot{\varsigma}) \\ \dot{z} &= -\Gamma(\varsigma, \dot{\varsigma})z + \Gamma(\varsigma, \dot{\varsigma})(C(\varsigma, \dot{\varsigma}) - \tau - \mu(\varsigma, \dot{\varsigma})) + M^{-1}(\varsigma)(\ddot{\varsigma} + \beta\dot{\varsigma})\end{aligned}\quad (4)$$

Where, $\hat{\tau}_{dis}$ is the estimated disturbance vector. z is the vector of auxiliary variables. β is an arbitrary positive constant, i.e., $\beta > 0$. $\Gamma(\varsigma, \dot{\varsigma})$ is the observer gain matrix. $\mu(\varsigma, \dot{\varsigma})$ is the observer vector.

2.4 Extended Kalman filter

The proposed system gets the positions/orientations of the mobile platform from measurements systems (camera and vision systems) and the joint positions of the manipulator using joint position sensors (i.e., potentiometers), respectively. It is assumed that no explicit velocity information is provided to the EKF. The estimator integrates knowledge of the known plant and measurement dynamics, statistics of the process and measurement noise, and an initial condition, to estimate the state of the system. The well-known, most widely used nonlinear filter and recognized EKF is used as a state estimator in this study. Based on the system dynamics and available displacement measurements, the complete system states including the displacement and velocity vectors of the robotic manipulator are obtained using the EKF.

Let us define a state vector as follows:

$$\chi = [\varsigma^T \ \dot{\varsigma}^T]^T$$

Such that $\dot{\chi} = f(\chi, t)$ and

$$f(\chi, t) = \begin{bmatrix} \dot{\varsigma} \\ M(\varsigma)^{-1}(\tau + \hat{\tau}_{dis} - C(\varsigma, \dot{\varsigma})\dot{\varsigma}) \end{bmatrix}\quad (5)$$

Uncertainties in the system and the actuator dynamics are modeled using random variables and the combined uncertainties are grouped together into a random variable vector ϖ .

The system dynamics of the augmented state vector can be represented as follows:

$$\dot{\chi} = f(\chi, t) + \varpi\quad (6)$$

The measurement model can be expressed as:

$$\kappa = h(\chi, t) + \zeta \quad (7)$$

where, κ is the measurement vector, $h(\chi, t)$ is the measurement function or vector of measurements, and ζ is the vector of measurement noises present in the vehicle positions and the manipulator joint positions.

2.5 Controller

The proposed robotic control vector along with estimated disturbance is given as follows:

$$\tau = M(\hat{\zeta})\left(\ddot{\zeta}_d + K_D\dot{\tilde{\zeta}} + K_P\tilde{\zeta}\right) + C\left(\hat{\zeta}, \dot{\hat{\zeta}}\right)\dot{\hat{\zeta}} - \hat{\tau}_{dis} \quad (8)$$

Where K_P and K_D are the proportional and derivative gain matrices of the proposed controller and chosen as symmetric positive definite matrices. $\hat{\zeta}$ and $\dot{\hat{\zeta}}$ are the estimated position vector and velocity vector of the robotic motion platform, respectively. $\tilde{\zeta} = \zeta_d - \hat{\zeta}$ is the vector of position tracking errors. $\dot{\tilde{\zeta}} = \dot{\zeta}_d - \dot{\hat{\zeta}}$ is the vector of velocity tracking errors. ζ_d , $\dot{\zeta}_d$ and $\ddot{\zeta}_d$ are the vectors of the desired (reference) positions, velocities, and accelerations, respectively. $\hat{\tau}_{dis}$ is the estimated vector of disturbances.

The block diagram that corresponds to the proposed controller is shown in Fig. 3. This block diagram consists of three sub-sections: the user input block, the controller block and the robotic system. In the user input block, the desired manipulation task is defined (such as type of operation, desired target task space positions, duration of the task and other particulars) and based on these information, the trajectory planner derives the time trajectories of the desired task space position vector (which is considered as a reference (input) vector for the proposed controller). The entire controller block segment calculates a control vector based on the proposed control law along with the estimated disturbance vector. The disturbance observer block estimates the disturbance vector using control input vector, sensor signals and partial system dynamics (as described as earlier in this section). Robotic system block is composed of three sub blocks: the robotic platform block representing the actual system, the sensor systems block which measures the system states such as joint positions and mobile platform position variables and the disturbances block which represents the external disturbances which the system encounters.

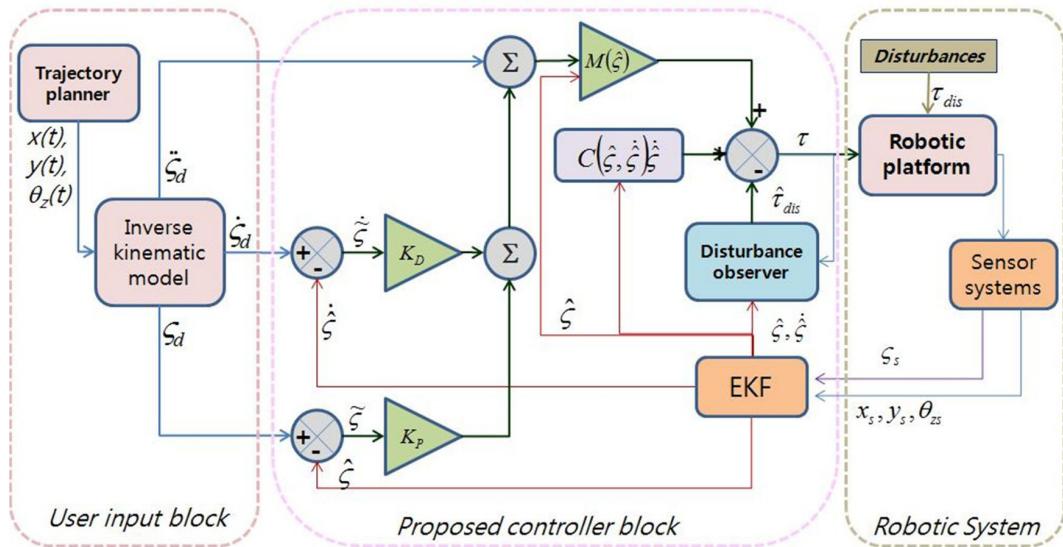


Fig. 3. Block diagram of the proposed controller

3. Results and Discussions

Computer based numerical simulations are performed to explore and demonstrate the performance of the proposed robotic platform and control scheme. The proposed system parameters for this study are as follows: The link lengths of the platform are: $L_1 = 0.15$ m, $L_2 = 0.25$ m, $L_3 = 0.2$ m and $L_4 = 0.25$ m. The end effector point is located at center of L_3 link (i.e., $L_p = L_3/2 = 0.1$ m) (mobile platform). The individual link inertias and masses are obtained using its geometry. To confirm the effectiveness of the proposed system the proposed scheme is compared with standard control schemes, specifically the proportional integral derivate (PID) control scheme and computed torque control (CTC) scheme. The controller gain matrices of all three controllers are tuned such a way that all three controller performances are quite satisfactory in the ideal situation and almost equal in terms of their tracking errors. During this task, an unknown disturbance vector (in addition to that of parameter uncertainties and noises) is added for more realistic simulating conditions. The uncertainties of the system parameters are assumed to be 10%. During the performance analysis, sensory noises were introduced into the position and orientation measurements, i.e., mean Gaussian noises of 0.01 m with a standard deviation of 0.01 m for the position measurements, and a mean of 0.5° with a standard deviation of 0.5° for the orientation measurements. In addition, the manipulator initial velocities are set to zero and the estimated system vectors are random values, while the initial desired and actual positions and orientations are the same.

Task: Circular trajectory tracking

In this task, the robotic platform starts from any arbitrary position (for the simulation it has been set as: $[0.1, 0.5]$ m) and it should follow a circular trajectory (with the circle radius set as: 0.1 m). The desired time trajectory in task space is given as follows:

$$\begin{aligned} x(t) &= 0.1 + 0.1 \sin 6t \text{ m} \\ y(t) &= 0.4 + 0.1 \cos 2t \text{ m} \\ \theta_z(t) &= 45 - 0.026t \end{aligned} \quad (9)$$

Detailed performance results of the simulations for this task are shown in Figs. 4–7. Figure 4 shows the desired and actual task space trajectories of the manipulator end effector point (mobile platform centroid) and the results show

that the performance of the proposed controller is remarkable at adapting to uncertainties and disturbances while compared to PID and computed torque control. The manipulator task space position tracking error vector and joint position tracking errors are presented in Fig. 5. For better understanding, system performance indexes such as norm of the task space tracking errors ($\|\tilde{\mathcal{P}}\|$) and norm of the joint space position tracking errors ($\|\tilde{\zeta}\|$) during this task are plotted in Figs. 6. As shown in these figures, the time trajectories of the norm of task space tracking errors and the time trajectories of the norm of the vector of joint space position errors are less in the proposed controller as compared to that of PID and CTC schemes. From the results, it is noted that the observer performance is quite satisfactory in estimating the system parameters.

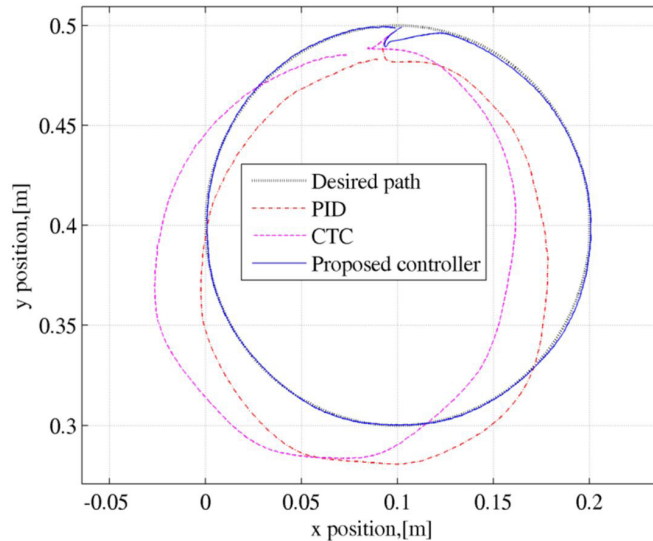


Fig. 4 Task space (xy) trajectories for the square shape trajectory

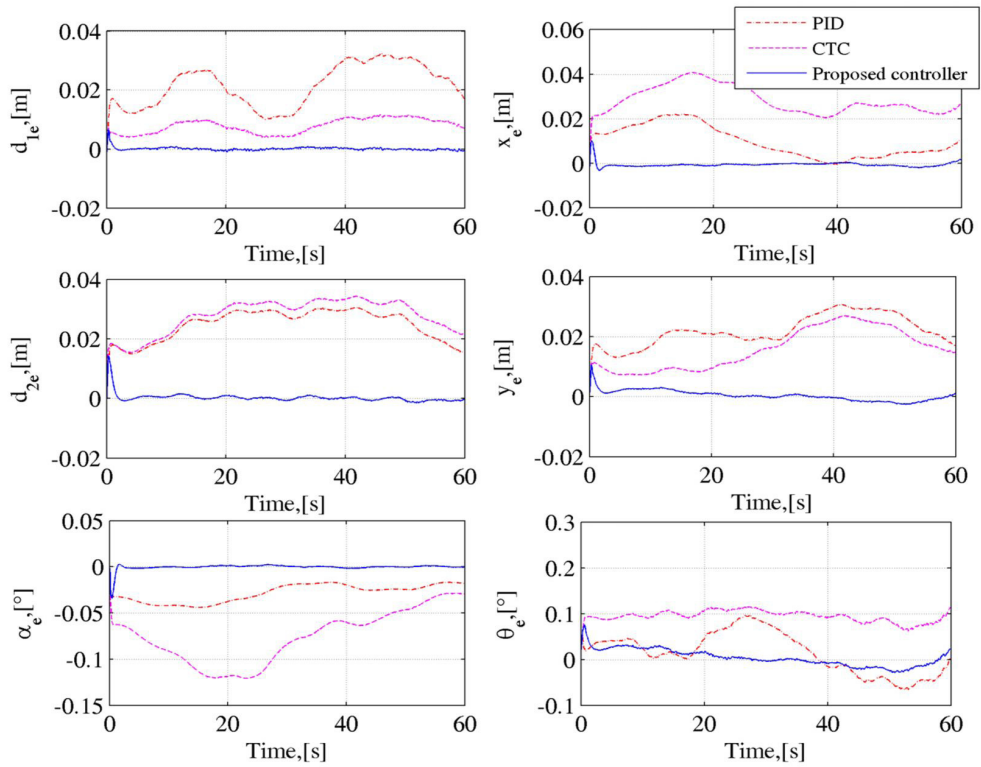


Fig. 5. Time trajectories of the joint space position errors and task space position tracking errors for the circular trajectory

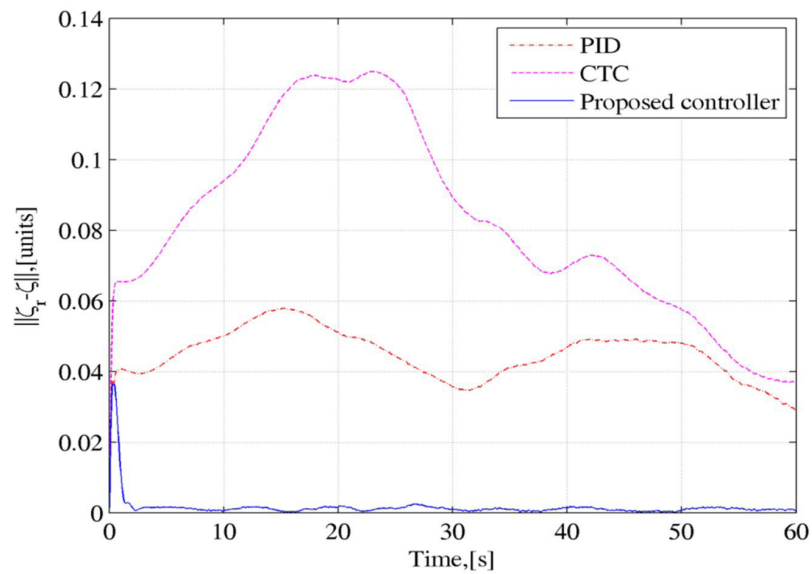


Fig. 6. Time histories of the norm of joint space position errors during circular trajectory tracking task

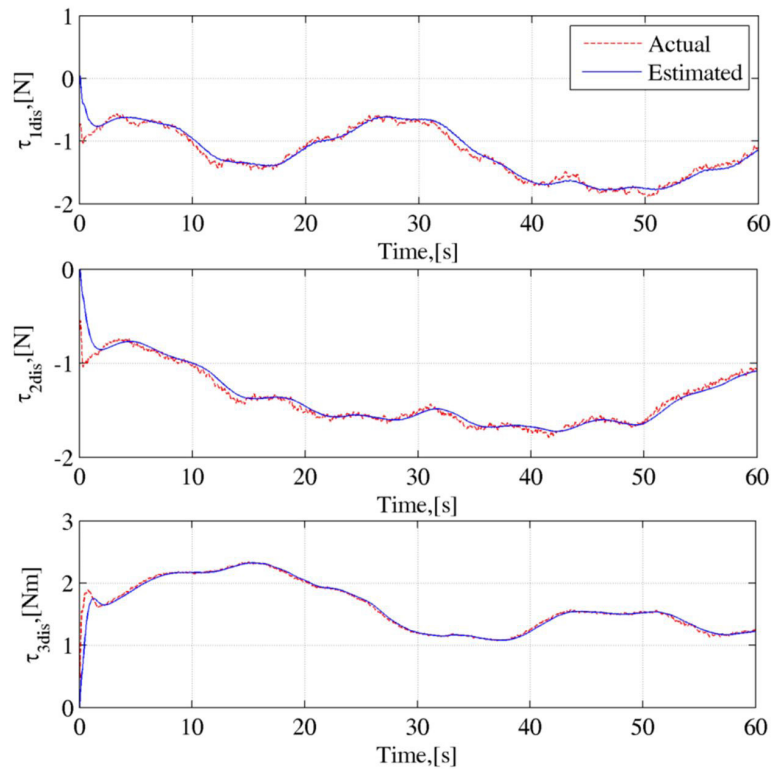


Fig. 7. Time trajectories of the disturbance estimates along with actual disturbances during the circular trajectory tracking task

Conclusions

This paper proposed a new robotic planar parallel motion platform with a disturbance observer based motion controller to perform complex manipulation tasks on incorporating joint positions trajectory information. The external disturbances due to the system dynamic changes, payload changes, external disturbances and the reaction effects during manipulation task are estimated explicitly and compensated. The effectiveness of the proposed scheme was demonstrated and established using extensive numerical simulations with appropriate manipulation tasks. The obtained results corroborate the effectiveness and robustness of the proposed scheme in terms of tracking errors in the presence of unknown external disturbances and parameter uncertainties. This work can be extended straightforwardly for spatial parallel robotic platforms where the external and internal disturbance effects can be estimated and compensated during manipulation tasks. This research particularly provides a generalized framework of the disturbance observer based nonlinear control scheme for controlling a robotic manipulator, which is crucial for achieving complex manipulation tasks for a variety of scientific and industrial applications.

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