National University of Computer and Emerging Sciences
Karachi Campus

Discrete Structures (CS1005)

Sessional-II Exam

Date: November 06th 2024

Course Instructors:

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Total Time (Hrs):	1
Total Marks:	30
Total Questions:	2

Roll No   Section   Student Signature

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Attempt all the questions in the given sequence.

CLO # 2: Construct formal logic proofs and/or informal, but rigorous, logical reasoning to real problems, such as predicting the behavior of software or solving problems such as puzzles.

Propositional Logic, Predicate Logic and Rules of Inference. [8 x 2 = 16 Marks] Time: 30 Minutes

Q1: a. Let S={1,2,3,4,5,6} Define a relation R={(a,b) | a divides b} on the set S.

(i) List all the ordered pairs in the relation R. Also, create a graph from the relation R.

(ii) Determine whether the relation R on S is Symmetric, Transitive, Asymmetric or Antisymmetric. Provide a justification for each of the properties you have checked.

(iii) Is R an Equivalence relation on integers?

(iv) Check properties for Partially Ordered Set (POSET) for the given relation R.

b. (i) The third term of a geometric sequence is 324 and the sixth term is 96. Find the common ratio and first term of the sequence.

(ii) Let $\{a_n\}$ be a sequence that satisfies the recurrence relation $a_n=2a_{n-1}+5$ for n=1,2,3,4,..., and suppose that $a_0=3$. Determine the values of a_1, a_2, a_3, and a_4.

(iii) Let m, n ∈ Z. Prove or disprove that if m.n is odd, then m+n is even.

(iv) If x^2-6x+5 is even then x is an odd integer. [Hint: Proof by Contraposition]

CLO # 1: Explain the key concepts of Discrete Structures such as Mathematical Logic, Sets, Permutations, Relations, Graphs and Trees etc.

Sets Theory and Functions: [7 x 2 = 14 Marks] Time: 30 Minutes

Q2: a. (i) Determine if the following two graphs, Graph G1 and Graph G2 shown in Figure 1, are isomorphic. If they are, provide a function F:V(G1)→V(G2) that defines the isomorphism. If they are not, explain why.

Graph G1: V = {a, b, c, d, e}, E = {{a, b}, {a, c}, {a, e}, {b, d}, {b, e}, {c, d}}.

Graph G2:

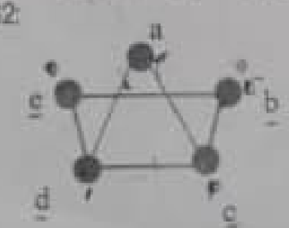

Figure 1: Graph G2

(ii) Is Graph G2 is planar? If yes redraw otherwise justify. In addition, using Euler formula, determine the number of regions in Graph G2.

(iii) Is Graph G2 is bipartite? If yes provide the disjoint sets otherwise justify.

(iv) State and explain a theorem that provides the necessary and sufficient conditions for the existence of Euler circuits and Euler paths in a graph. Additionally, determine whether each of the given graphs possesses an Euler circuit or Euler path.

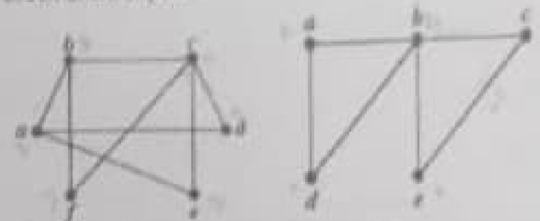

Figure 2: (a) (b)

b. (i) Use Prim's algorithm to find a minimum spanning tree starting from A for the following graph in Figure 3. Also, indicate the order in which edges are added to form each tree.

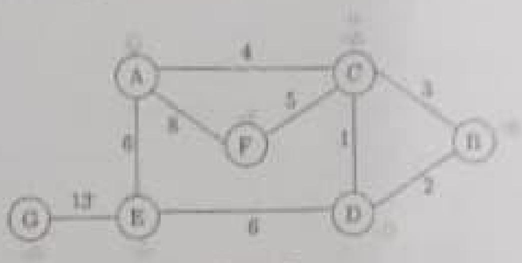

Figure 3

(ii) Consider the graph given in Figure 3. With the indicated link costs, use Dijkstra's shortest-path algorithm to compute the shortest path from **A** to all other nodes. Use table 01 given below for computation.

Table 01: Dijkstra's shortest-path algorithm

N	D(B)	D(C)	D(D)	D(E)	D(F)	D(G)
A	-	-	-	-	-	

(iii) Determine the prefix expression for the tree shown in Figure 4, and then solve the expression based on the resultant prefix expression. consider the value of a is 1, b is 2, c is 3, d is 4, e is 5 and f is 6.

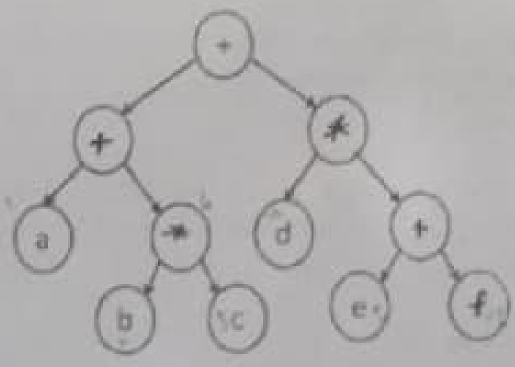

Figure 4