Angle Between Vectors

$$\theta = \cos^{-1}\left(\frac{\langle \mathbf{u}, \mathbf{v}\rangle}{\|\mathbf{u}\|\|\mathbf{v}\|}\right)$$

EXAMPLE 1 | Cosine of the Angle Between Vectors in M_{22}

Let M_{22} have the standard inner product. Find the cosine of the angle between the vectors

$$\mathbf{u} = U = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = V = \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix}$$

Solution We showed in Example 6 of the previous section that

$$\langle \mathbf{u}, \mathbf{v}\rangle = 16, \quad \|\mathbf{u}\| = \sqrt{30}, \quad \|\mathbf{v}\| = \sqrt{14}$$

from which it follows that

$$\cos\theta = \frac{\langle \mathbf{u}, \mathbf{v}\rangle}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{16}{\sqrt{30}\sqrt{14}} \approx 0.78$$

Question:

Find the cosine of the angle between the vectors with respect to the Euclidean inner product.

b. $\mathbf{u} = (-1, 5, 2)$, $\mathbf{v} = (2, 4, -9)$

Solution:

$$\cos\theta = \frac{\langle \mathbf{u}, \mathbf{v}\rangle}{\|\mathbf{u}\|\,\|\mathbf{v}\|} = \frac{(-1)(2)+(5)(4)+(2)(-9)}{\sqrt{(-1)^2+5^2+2^2}\,\sqrt{2^2+4^2+(-9)^2}} = 0$$

Question:

Find the cosine of the angle between the vectors with respect to the Euclidean inner product.

3. $\mathbf{p} = -1 + 5x + 2x^2$, $\mathbf{q} = 2 + 4x - 9x^2$

Solution:

$$\cos\theta = \frac{\langle \mathbf{p},\mathbf{q} \rangle}{\|\mathbf{p}\|\,\|\mathbf{q}\|} = \frac{(-1)(2)+(5)(4)+(2)(-9)}{\sqrt{(-1)^2+5^2+2^2}\sqrt{2^2+4^2+(-9)^2}} = 0$$

Question:

Find the cosine of the angle between the vectors with respect to the Euclidean inner product.

5. $A = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$

Solution:

$$\cos\theta = \frac{\langle U,V \rangle}{\|U\|\,\|V\|} = \frac{\mathrm{tr}\left(U^T V\right)}{\sqrt{\mathrm{tr}\left(U^T U\right)}\sqrt{\mathrm{tr}\left(\mathrm{V}^T V\right)}} = \frac{(2)(3)+(6)(2)+(1)(1)+(-3)(0)}{\sqrt{2^2+6^2+1^2+(-3)^2}\sqrt{3^2+2^2+1^2+0^2}} = \frac{19}{\sqrt{50}\sqrt{14}} = \frac{19}{10\sqrt{7}}$$