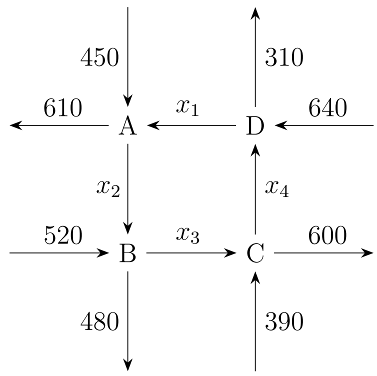
National University of Computer & Emerging Sciences, Karachi
FAST School of Computing
Final Exam FALL 2023
Monday 3^{rd}January 2024, 9:00 am - 12:00 pm

Course Code: MT-1004	Course Name: Linear Algebra
Instructor Name: Alishba Tariq, Fareeha Sultan, Dr. Khusro Mian, Mairaj Ahmed, Moheez ur Rahim, Shahid Ashraf, Usama Antuley	Total Marks:100
	Student Roll No:

(Read each question completely before answering it. There are 04 Questions and 02 pages.)

Question 1 (CLO3) 10+10+10 marks

a.) The accompanying figure shows a network of one-way streets with traffic flowing in the directions indicated. The flow rates along the streets are measured as the average number of vehicles per hour. Set up a linear system whose solution provides the unknown flow rates, and solve the system for the unknown flow rates.



b.) Let $T_A : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T_A = (x_2, x_1+x_2-3x_3)$, and let $u_1 = (0,1,1), u_2 = (2,-1,1)$ and $u_3 = (1,1,-2)$. Determine whether the set $\{T_A(u_1), T_A(u_2), T_A(u_3)\}$ spans $\mathbb{R}^2$.

c.) The first four Hermite polynomials are $1, 2x, -2+4x^2, -12x+8x^3$, Show that they form the basis for P_3

Question 2 (CLO1) 15+10 marks

a.) The vector space $\mathbb{R}^3$ has the Euclidean inner product.

i) Apply the Gram–Schmidt process to transform the basis, into an orthogonal basis $S = \{v_1, v_2, v_3\}$, $\{u_1 = (1,1,1), u_2 = (-1,1,0), u_3 = (1,2,1)\}$

ii) Normalize the orthogonal basis vectors to obtain an orthonormal basis $Q = \{q_1, q_2, q_3\}$.

iii) Express the vector $w = (2,1,0)$ as a linear combination of the vectors in S, and find the coordinate vector $(w)_S$

b.) Let M_{22} have the standard inner product. Find the cosine of the angle between the vectors $u = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $v = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$ and verify that $\langle u, v \rangle = \frac{1}{4} \parallel u+v \parallel^2 - \frac{1}{4} \parallel u-v \parallel^2$

Question 3 (CLO1) 10+15 marks

a.) Find a matrix P that diagonalizes $B = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$, and determine B^9.

b.) Express the quadratic form Q in the matrix notation $x^T A x$, where A is a symmetric matrix. Also find an orthogonal change of variables that eliminates the cross product terms in the quadratic form Q, and express Q in terms of the new variables.

$$Q = 2x_1^2 + 5x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_1x_3 - 8x_3x_2$$

Question 4 (CLO1) 20 marks

MCQs: Please write the correct choice(A,B,C,D) along with the question number on answer script, no need to show any working or writing the answer

1.) The size of the matrix for the following charecterstics equation is . $\lambda^3(\lambda^2 - 5\lambda - 6) = 0$

(A) (5,4) (B) (5,6) (C) (5,2) (D) (5,5)

2.) Which of the following matrix would not be suggested as Orthogonal Matrix.

(A) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix}$

3.) If C is a $n \times 4$ matrix and $D = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, then the second column of the matrix CD is

(A) The same as the second column of C
(B) The sum of the 1^{st} and 2^{nd} columns of C
(C) The sum of the 2^{nd} and 4^{th} columns of C
(D) The same as the third row of D

4.) How can we determine a basis comprising invertible matrices for M_{22} when a basis for M_{22} is already provided?

(A) Spanning Set (B) Trace (C) Transpose (D) Not possible

5.) If 0 is the eigen value of a matrix, then A is not

(A) Diagonalizable (B) Symmetric (C) Orthogonal (D) Invertible

6.) What can we say about two vectors whose dot product is negative

(A) The vectors are orthogonal
(B) The angle between the two vectors > 90
(C) The angle between the two vectors < 90
(D) None of the above statements is correct.

7.) What is the inner product of the vectors $(0, 1, -1)$ and $(4, 2, -3)$

(A) 0 (B) 4 (C) (0,2,3) (D) 5

8.) Choose the correct completion of the following statement: If A and B are square matrices such that $AB = I$, then zero is an eigenvalue of

(A) A but not of B (B) B but not of A (C) Both A and B (D) Neither A nor B

9.) If the augmented matrix [A|b] of a system Ax = b is row equivalent to $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$; x is

(A) The system is inconsistent.
(B) x=$(5, -2 - s, 1)$
(C) x=$(5, -2, 1)$
(D) x=$(5, -3, 1)$

10.) Let V be a vector space, and let S be a subset of V . S spans V means

(A) The elements of S are all distinct from each other
(B) S is a basis for V
(C) Every vector in V has exactly one representation as a linear combination of vectors in S
(D) Every vector in V can be expressed as a linear combination of vectors in S