EXERCISE 2.

1. For each of the following average revenue functions determine the total revenue (R) and the marginal revenue (M.R). Evaluate thier values at the stated demand.

(i) $P = 12 - 3X$ at $X = 1, \dfrac{3}{2}, 2, 3$

(ii) $P = 20 - \dfrac{1}{3}X$ at $X = 3, 15, 30, 36$

(ii) $P = 20 - \dfrac{1}{5}X^2$ at $X = 2, 5, 7, 8$

2. For each of the following demand functions determine the average revenue (A.R), total revenue (R), and marginal revenue (M.R). Evaluate their values at the stated demand.

(i) $X = 18 - \dfrac{2}{3}P$ at $X = 2, 4, 6, 9, 10$

(ii) $5X = 20 - 2P$ at $X = \dfrac{1}{2}, 1, \dfrac{3}{2}, 2, \dfrac{5}{2}$

(iii) $2X = 3 - \sqrt{P}$ at $X = \dfrac{1}{4}, \dfrac{1}{2}, 1, \dfrac{3}{2}, 2$

3. For each of the following demand functions determine the average (A.R), total revenue (R), and marginal revenue (M.R).

(i) $X = 10\,e^{-3P}$ (ii) $3X = 12 - \sqrt{P}$

(iii) $X = \sqrt{\dfrac{15-P}{2}}$ (iv) $(X+5)(P+3) = 12$

4. For each of the following cost functions obtain expressions for average and marginal cost.

(i) $C = 0.05X^2 + 5X + 100$

(ii) $C = 0.3X^3 - 2X^2 + 8X + 80$

(iii) $C = 5X^2 + 3X + 50$

(iv) $\quad C = \sqrt{3X^3 + 10X^2}$

(v) $\quad C = (a+bX)^2$

5.   For each of the cost functions given in problem 4, above, determine the values of average, total and marginal costs at X = 5 units.

6.   For each of the following cost functions determine the average cost (A.C), and marginal cost (M.C).

(i) $\quad C = 3X^2 + 2X + 5$ (ii) C $= aX^2 + bX + c$

(iii) $\quad C = (aX + bX^2)^2$

7.   Given the average cost function

$$A.C = 0.3X^2 - 5X + 8 + \frac{180}{X}$$

determine the toal cost and marginal cost and evaluate the average total and marginal costs at an output of (i) 15 units (ii) 20 units.

8.   For each of the following cost functions obtain expressions for the average cost and marginal cost, evaluate them at the stated points

(i)   C $= 0.04X^2 + 3X + 150$ at   X $= 100$

(ii)   C $= 0.2X^3 - 3X^2 + 5X + 120$ at   X $= 10$

(iii) C $= aX^2 + bX$ at   X $= b/a$

(iv) C $= (aX + b)^{1/2}$ at   X $= b/a$

(v)   C $= (X^3 - 2X)(3X + 12)$ at   X $= 2$

9.   Delermine the marginal propensity to consume for the following consumption functions, where C denotes the consumption expenditure, Y denotes the gross income, Y_d the net income after tax, while, T represents the tax receipt.

(i)   C $= 1200 + 0.65\,Y$

(ii)   C $= 1000 + 0.65\,Y_d$ where $Y_d = Y - 80$

...ions for the elasticity of total cost and the elasticity of average cost.

18. For each of the cost functions given in problem 8, above, determine the elasticity fo toal cost and the elasticity of average cost at the stated points.

19. For each of the total cost functions given in problem 4, above evaluate the elasticity of total cost and the elasticity of average cost at $X = 5$.

ANSWERS TO EXERCISE 2

1. (i) $R = 12X - 3X^2$; $9, \dfrac{45}{4}, 12, 9$

     $M.R = 12 - 6X$; $6, 3, 0, -6$

    (ii) $R = 20X - \dfrac{1}{3} X^2$; $57, 225, 2300, 288$

     $M.R = 20 - \dfrac{2}{3} X$; $18, 10, 0, -4$

    (iii) $R = 30X - \dfrac{1}{5} X^3$; $\dfrac{292}{5}, 125, \dfrac{701}{5}, \dfrac{688}{5}$

     $M.R = 30 - \dfrac{3}{5} X^2$; $\dfrac{138}{5}, 15, \dfrac{3}{5}, \dfrac{-42}{5}$

2. (i) $A.R = P = 27 - \dfrac{3}{2} X$; $24, 21, 18, \dfrac{27}{2}, 12$

     $R = 27X - \dfrac{3}{2} X^2$; $48, 8, 108, \dfrac{24\,3}{2}, 120$

     $M.R = 27 - 3X$; $21, 15, 9, 0, -3$

(ii) A.R $= P = 10 - \dfrac{5}{2} X^2$; $\dfrac{35}{4}, \dfrac{15}{2}, \dfrac{2}{4}, 5\dfrac{15}{4}$

$\qquad\quad$ R $= 10X - \dfrac{5}{2} X^2$; $\dfrac{35}{8}, \dfrac{15}{2}, \dfrac{75}{8}, 10, \dfrac{75}{8}$

$\qquad\quad$ M.R $= 10 - 5X$; $\dfrac{15}{2}, 5, \dfrac{5}{2}, 0, \dfrac{-5}{2}$

(iii) A.R $= P = (3 - 2X)^2$; $\dfrac{25}{4}, 4, 1, 0, 1$

$\qquad\quad$ R $= 4X^3 - 12X^2 + 9X$; $\dfrac{25}{16}, 2, 1, 0, 2$

$\qquad\quad$ M.R $= 12X^2 - 24X + 9$; $\dfrac{15}{4}, 0, -3, 0, 9$

3. (i) $\quad$ A.R $= P = \dfrac{1}{3} \log_e \dfrac{10}{X}$

$\qquad$ R $= \dfrac{X}{3} \log_e \dfrac{10}{X}$

$\qquad$ M.R $= \dfrac{1}{3} [\log_e \dfrac{10}{X} - 1]$

(ii) $\quad$ A.R $= P = (12 - 3X)^2$

$\qquad$ R $= 9X^3 - 72X^2 + 144X$

$\qquad$ M.R $= 27X^2 - 144X + 144$

(iii) $\quad$ A.R $= P = 15 - 2X^2$

$\qquad$ R $= 15X - 2X^3$

$\qquad$ M.R $= 15 - 6X^2$

(iv) $\quad$ A.R $= P = \dfrac{12}{X + 5} - 3$

$\qquad$ R $= \dfrac{12X}{X + 5} - 3X$

$\qquad$ M.R $= \dfrac{60}{(X + 5)^2} - 3$

4. (i) $\quad$ A.C $= 0.05X + 5 + 100/X$

$\qquad$ M.C $= 0.1X + 5$

(ii) $A.C = 0.3X^2 - 2X + 8 + 80/X$

$M.C = 0.9X^2 - 4X + 8$

(iii) $A.C = 5X + 3 + 50/X$

$M.C = 10X + 3$

(iv) $A.C = \sqrt{3X + 10}$

$M.C = \dfrac{9X + 20}{2\sqrt{3X + 10}}$

(v) $A.C = \dfrac{a^2}{X} + b^2X + 2ab$

$M.C = 2b(a + bX)$

5. (i) $A.C = 25.25$ $T.C = 126.25$ $M.C = 5.5$

(ii) $A.C = 21.5$ $T.C = 107.5$ $M.C = 10.5$

(iii) $A.C = 38$ $T.C = 190$ $M.C = 53$

(iv) $A.C = 5$ $T.C = 25$ $M.C = 6.5$

(v) $A.C = 0.2a^2 + 5b^2 + 2ab$ $T.C = (a + 5b)^2$

$M.C = 2b(a + 5b)$

6. (i) $A.C = 3X + 2 + 5/X$ $M.C = 6X + 2$

(ii) $A.C = aX + b + c/X$ $M.C = 2aX + b$

(iii) $A.C = X(a + bX)^2$ $M.C = 2(a + 2bX)(aX + bX^2)$

7. $A.C = 0.3X^2 - 5X + 8 + 180/X$; $\dfrac{25}{2}$ 37

$T.C = 0.3X^3 - 5X^2 + 8X + 180$; $\dfrac{375}{2}$, 740

$M.C = 0.9X^2 - 10X + 8$; $\dfrac{121}{2}$, 168

8. (i) $A.C = 0.04X + 3 + 150/X$; 8.5

$M.C = 0.08X + 3$; 11

(ii) $A.C = 0.2X^2 - 3X + 5 + 120/X$; 7

$M.C = 0.6X^2 - 6X + 5$; 5

(iii) $A.C = aX + b$; $2b$

$M.C = 2aX + b$; $3b$

(iv) $A.C = \dfrac{(aX + b)^{1/2}}{X}$; $\dfrac{a\sqrt{2b}}{b}$

$M.C = \dfrac{a}{2\sqrt{aX+b}}$; $\dfrac{a}{2\sqrt{2b}}$

(v) $A.C = 3X^3 + 12X^2 - 6X - 24$; 36

$M.C = 12X^3 + 36X^2 - 12X - 24$; 192

9. (i) $MPC = 0.65$ (ii) $MPC = 0.65$

(iii) $MPC = 0.5525$

10. (i) $M.U_x = y + 3$; 11

$M.U_y = X + 5$; 7

(ii) $M.U_x = 2(X+3)(y+2)^3$; 4116

$M.U_y = 3(X+3)^2(Y+2)^3$; 5292

(iii) $M.U_x = 2 + \dfrac{3Y}{\sqrt{6XY}}$; $\dfrac{7}{2}$

$M.U_y = 3 + \dfrac{3X}{\sqrt{6XY}}$; 4

11. (i) $\dfrac{2X}{X-1}$; ∞ , $4, \dfrac{5}{2}$

(ii) $\dfrac{X(2X+3)}{X^2+3X+5}$; $\dfrac{5}{9}, \dfrac{14}{15}, \dfrac{13}{9}$

(iii) $\dfrac{12X+1}{(2X-1)(3X+2)}$; $\dfrac{13}{5}, \dfrac{25}{24}, \dfrac{61}{153}$

(iv) $\dfrac{4X^2}{X^4-1}$; $\infty, \dfrac{16}{15}, \dfrac{100}{624}$

12. (i) $2X+1$ (ii) $2(X+1)$

(iii) $2X\log_e b$ (iv) $a-bX$

13. (i) $\dfrac{4-X}{X}$; 2. 5