Optimization algorithms inspired by nature

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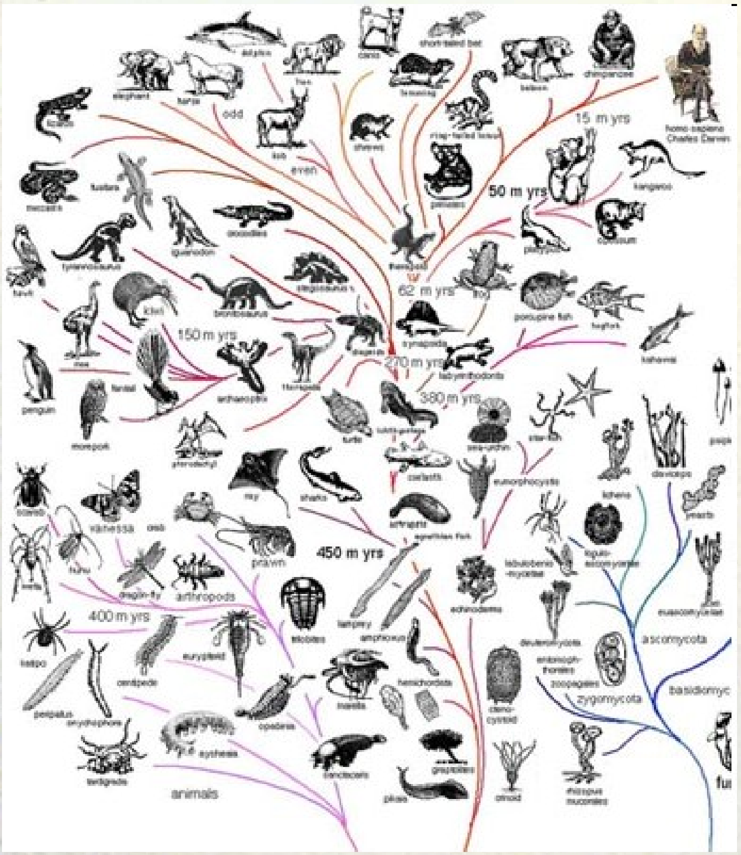
Introduction

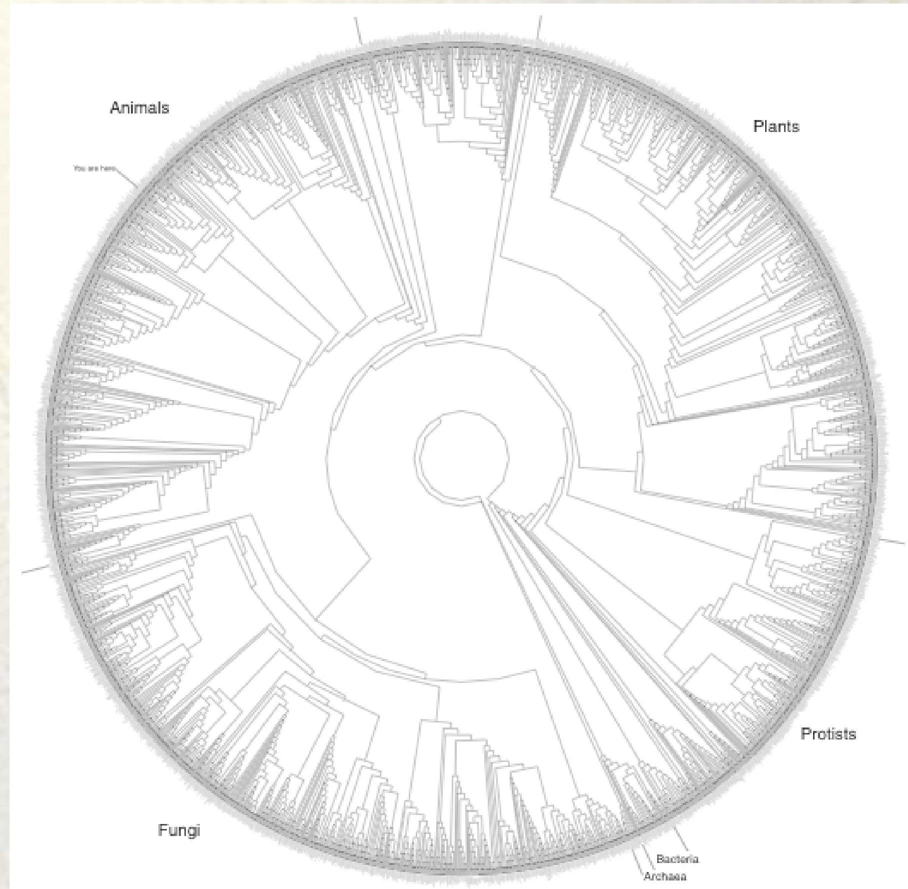
- Emerging intelligence
- Optimization problems
- Genetic algorithm
- Ant colony optimization

Emerging intelligence

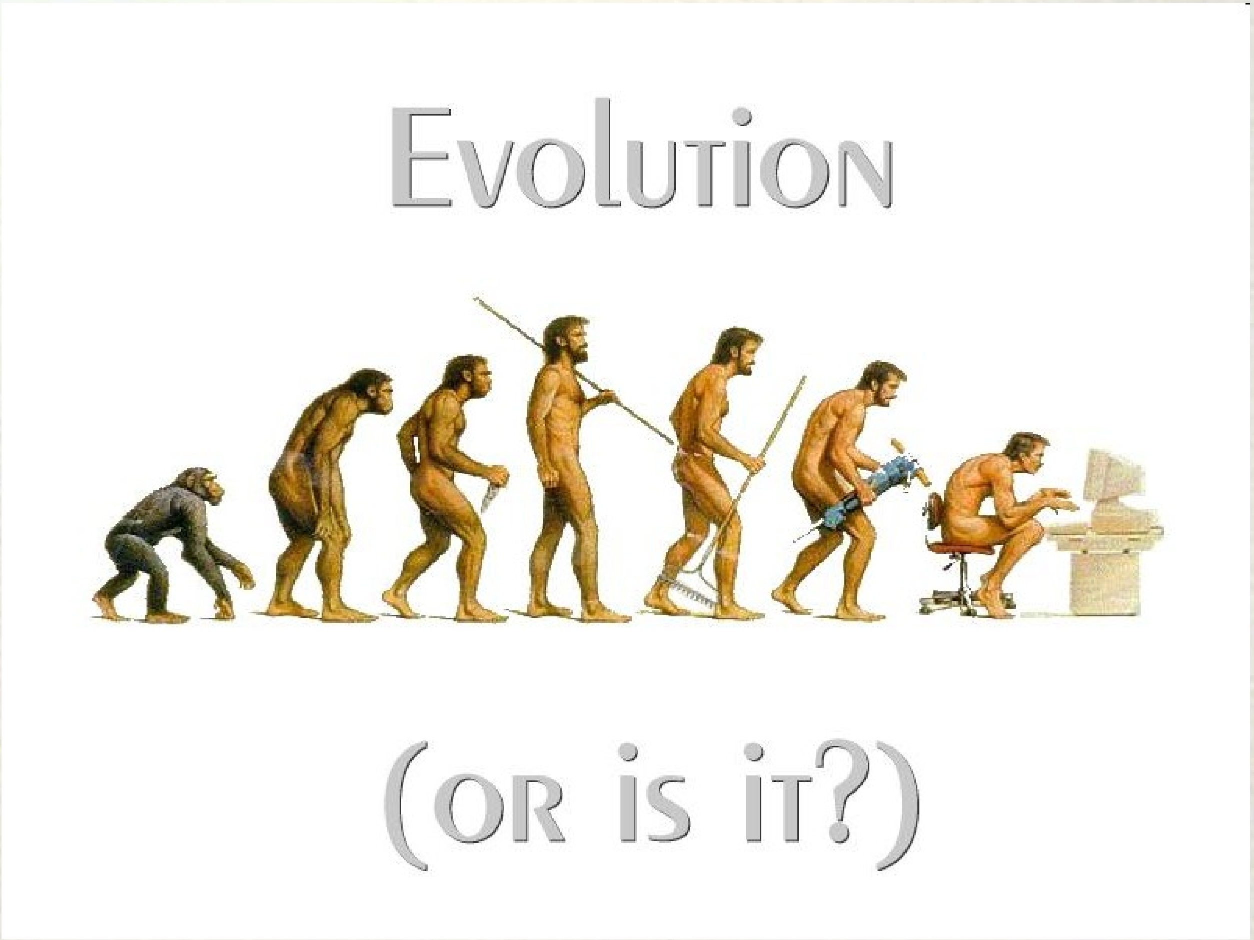
- **Emerging intelligence**
- Optimization problems
- Genetic algorithm
- Ant colony optimization

Introduction: Evolution





Introduction: Evolution



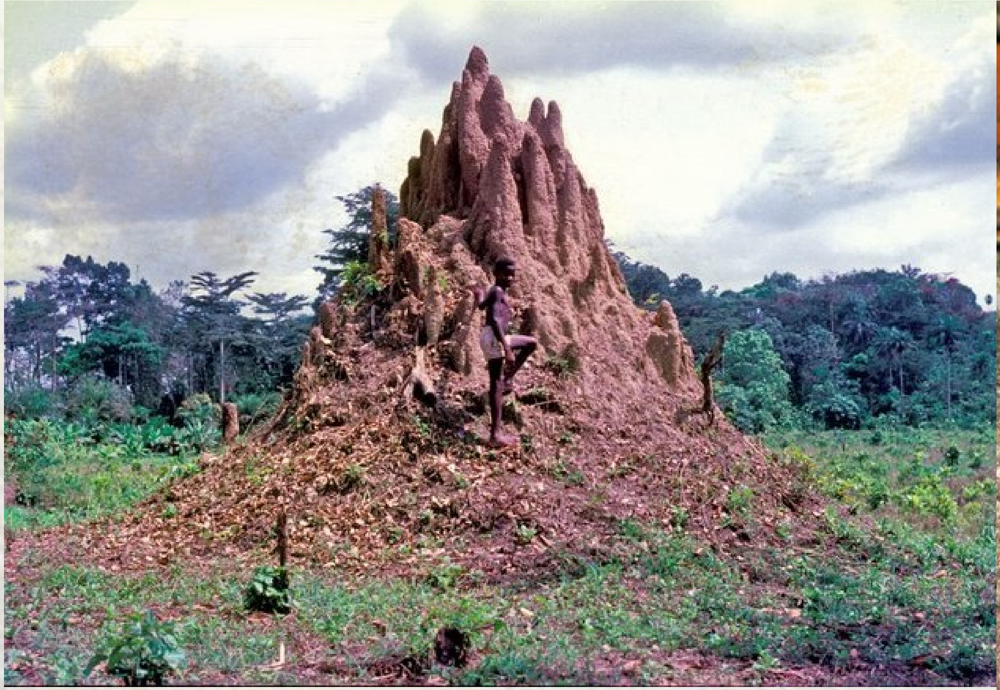
Introduction: Flock of birds

Introduction: Bea swarm

Introduction: Ant colonies

- Area of 50 m2, depth of 8m

Introduction: Ant colonies

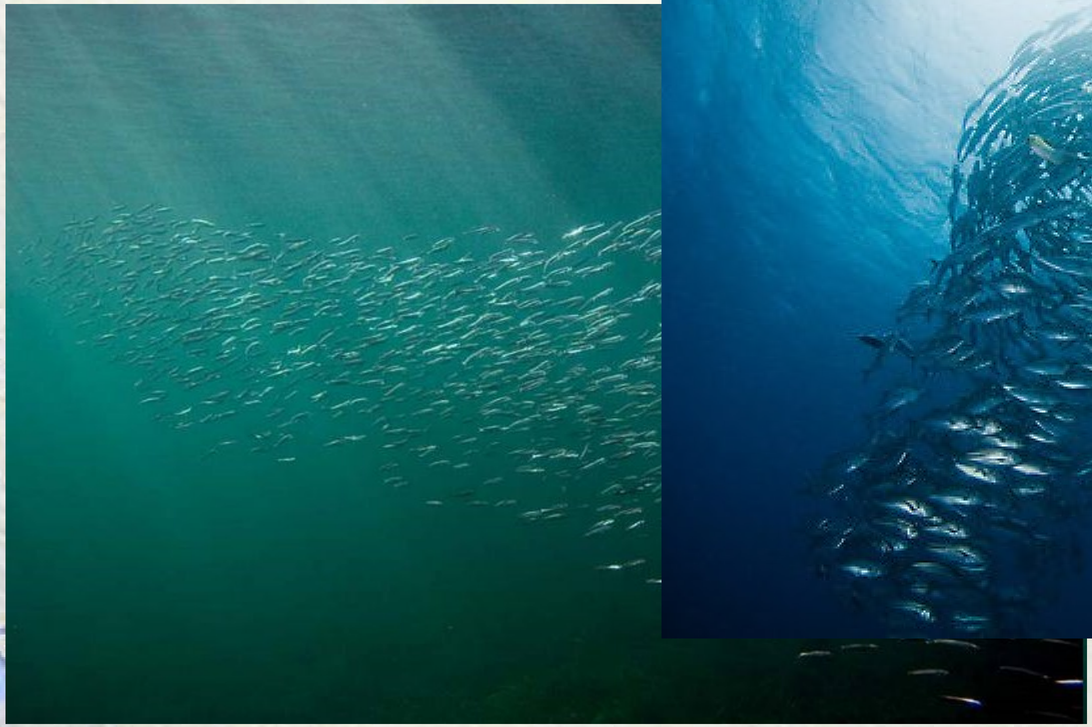


- Ant mound, path…

Introduction: Ant colonies

Introduction: Ant colonies

Introduction: School of fish



Introduction: conclusions

Dumb parts, properly connected
into a swarm, yield smart results.

Kevin Kelly
New Rules for the New Economy
Sep 1997

Introduction: conclusions

The **whole** is **greater** than

the sum of the **parts.**

Optimization problems

- Emerging intelligence
- **Optimization problems**
- Genetic algorithm
- Ant colony optimization

Optimization problems

- Optimization: the procedure of finding the best solution of a problem, the solution with the lowest price
- Typically:
  - Continuous variables
  - Combinatorial problems

Optimization problems

- State space search:
  - Find a path from S_0 to S_F
  - The solution is the **path** (e.g. 3x3 jigsaw puzzle!)
- CSP: *Constraint Satisfaction Problem*
  - A kind of state space search where the path from S_0 to S_F is not important. The solution is the final state itself.

Optimization problems

- CSP: *Constraint Satisfaction Problem*
  - Constraints that must be satisfied are defined
  - A criterion function to optimize is given

Optimization problems

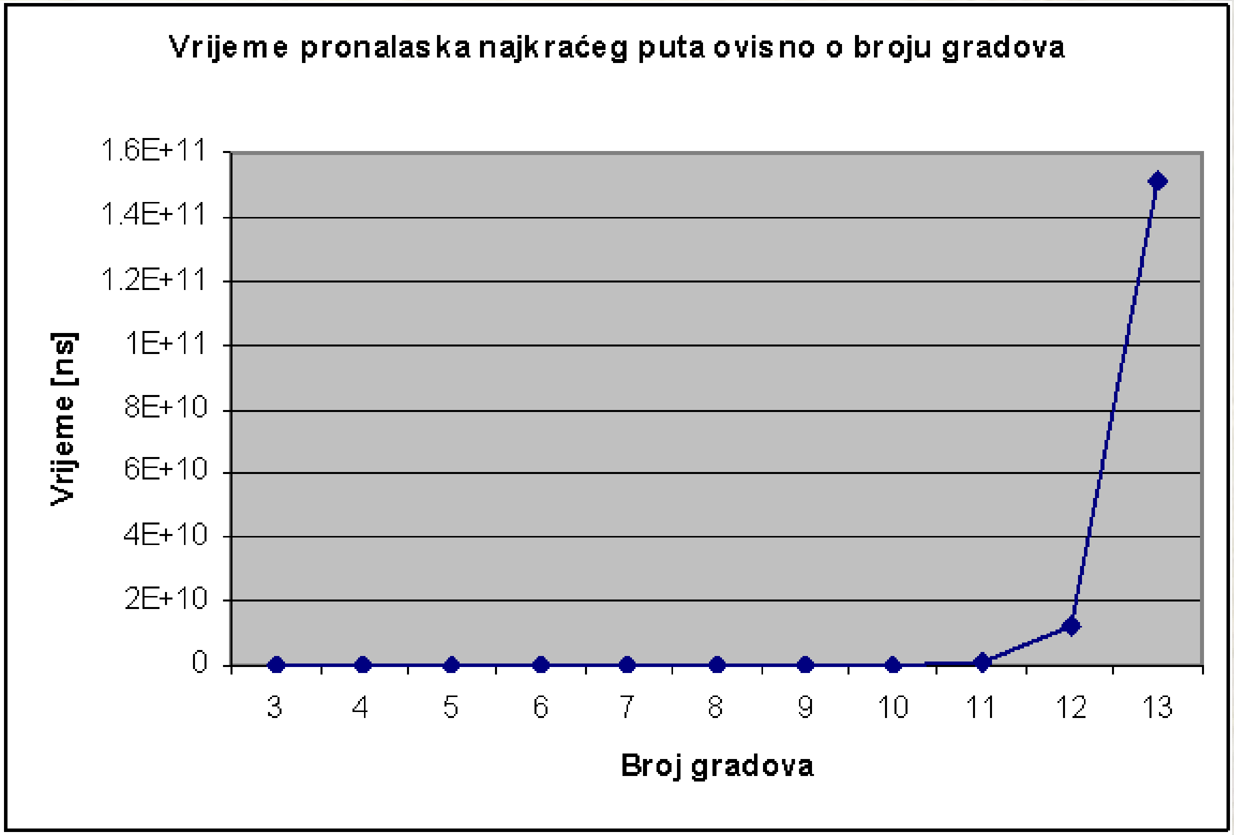
- We've seen ways to tackle combinatorial problems
  - State space search algorithms
    - Breadth first search
    - Depth first search
    - A*
    - …
- Unfortunately, often not applicable to real problems

Optimization problems

- The traveling salesman problem
  - Coordinates of n cities on the map are given
  - Find the shortest tour through all cities
  - Mathematically: find the shortest Hamiltonian cycle in the graph
  - **NP hard problem**
    (factorial complexity)

Optimization problems

- Traveling salesman problem



Optimization problems

- Traveling salesman problem
  - 12 cities, 12 sec
  - 13 cities, 2.5 min
  - 14 cities, 0.5h
  - 15 cities, 7.6h
  - 16 cities, 4.7 days
  - …
  - 500 cities, ????

Optimization problems

- Other problems
  - Scheduling unscheduled students into groups for classes
  - Midterm timetable creation
  - Lab assignments timetable creation
- Enumerating all possibilities?
  - It would take much much more time than the age of the universe

Optimization problems

- Heuristics
  - Algorithms that find *sufficiently* good solutions, usually do not guarantee optimality, and have low computational complexity (polynomial)
  - They can be
    - Construction based
    - Local search based

Optimization problems

- Heuristics
  - Contruction based
    - Build the solution incrementally
  - Local search algorithms
    - Start with a completed solution and try to incrementally make it better

Optimization problems

- Metaheuristics
  - A set of algorithmic concepts used to define heuristic methods applicable to a wide set of problems
  - A heuristic guiding problem specific heuristics

Optimization problems

- Metaheuristics
  - Simulated annealing
  - Taboo search
  - Evolutionary algorithms
  - Ant colony optimization
  - Swarm optimization
  - Artificial imunological systems
  - …

A problem ☺

- "No free lunch" Theorem, Wolpert & Macready, 1995, 1997:
  - *All algorithms seeking an optimum of a goal function behave identically with respect to any performance measure, when considered averaged over all possible goal functions*
    ...

A problem ☺

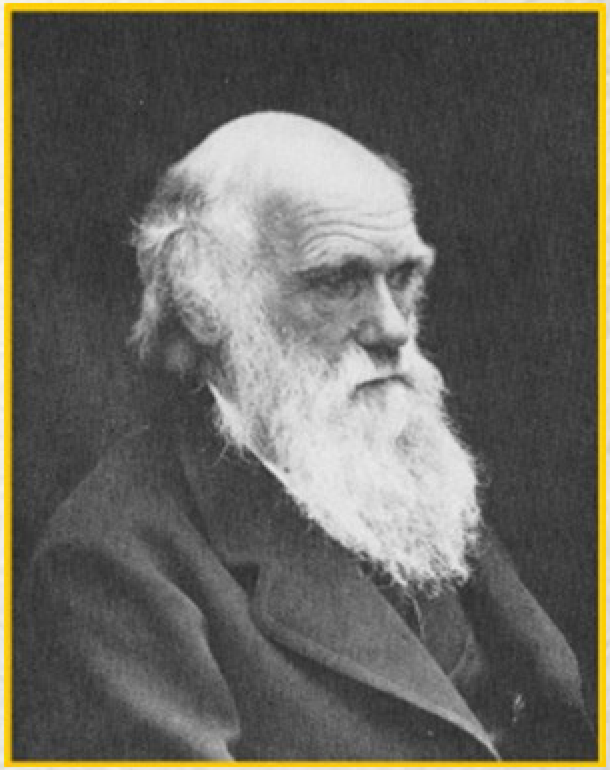
- "No free lunch“ Theorem, Wolpert & Macready, 1995, 1997:
  - *Specifically, if algoritm A is better than algorithm B on some goal functions, then, roughly speaking, there must be exactly that many different goal functions on which B is better than A.*

Genetic algorithm

- Emerging intelligence
- Optimization problems
- **Genetic algorithm**
- Ant colony optimization

Genetic algorithm

- Evolution as inspiration
- Population based algorithms
- Darwins theory about the origin of species

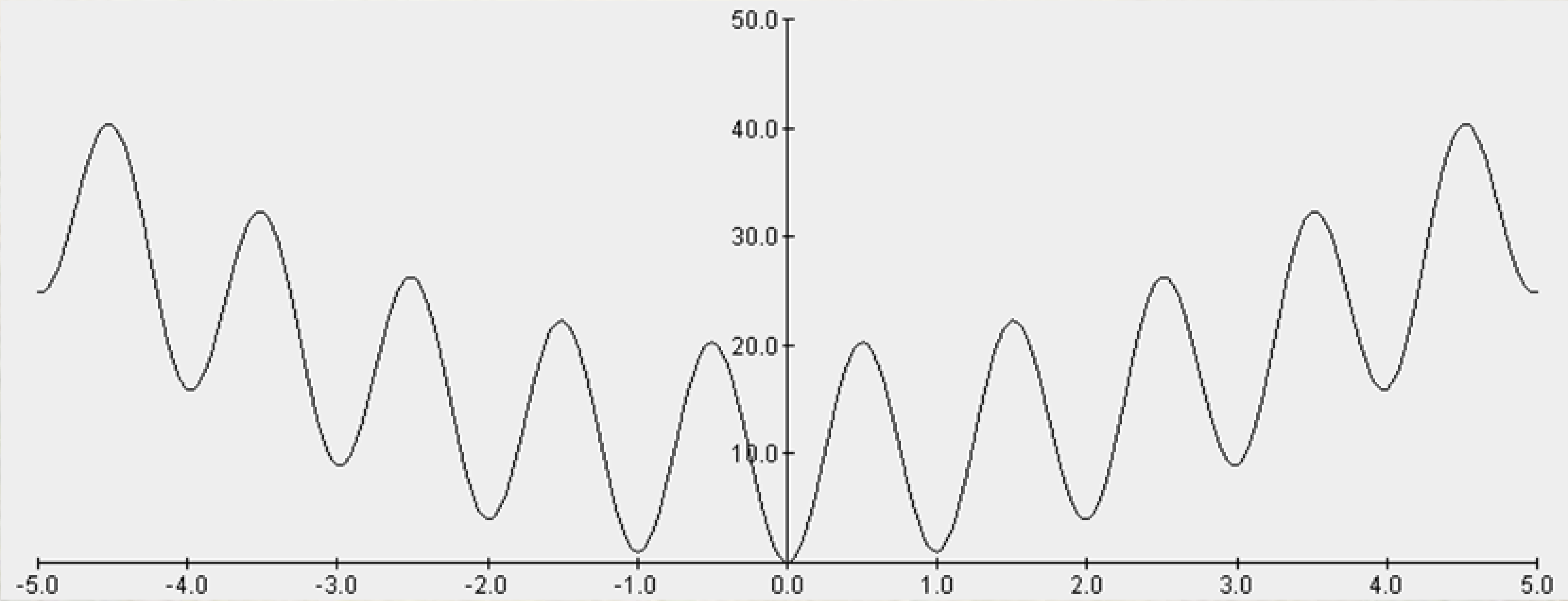


Genetic algorithm

- Main settings: Darwin
  - Fertility of species – there are always more descendants than required
  - Size of the population is roughly constant
  - Food supply is limited
  - For species that reproduce sexually there are no identical individuals, there are variations
  - Most of an individuals specific variations is passed on to its descendants

Genetic algorithm

- Example problem $f(x) = 10 + x^2 - 10 \cdot \cos(2 \cdot \pi \cdot x)$
  - Find x for which $f(x)$ is minimal



Genetic algorithm

- How does GA work?
  - There is a population of chromosomes
  - Each chromosome represents one solution to the problem
  - Each solution has a fitness
  - In our example fitness and f(x) are opposite→ higher f(x) means lower fitness

Genetic algorithm

- Implemention
  - From the current generation we iteratively create the next one
  - We select individuals that have a higher probability of creating better solutions
  - They are combined using the crossover operator
  - Resulting individuals are mutated using the mutation operator

Genetic algorithm

- Flowchart

Genetic algorithm

- Roles
  - **Selection** → selectional pressure→ speed of convergence
  - **Crossover** → searching the neighbourhood of parents
  - **Mutation** → getting out of local optima, big jumps in the solution space

Genetic algorithm

- Binary chromosome

0	1	0	0	0	1	1	1

  - A sequence of binary digits interpreted as a solution (value of a variable)
  - Three bit chromosome: 000, 001, ..., 111
  - Assuming we are observing a real variable from the interval [-2, 2], then: 000≡-2, 001≡-1.43, ..., 111≡2
  - What is the number of bits for a given precision?

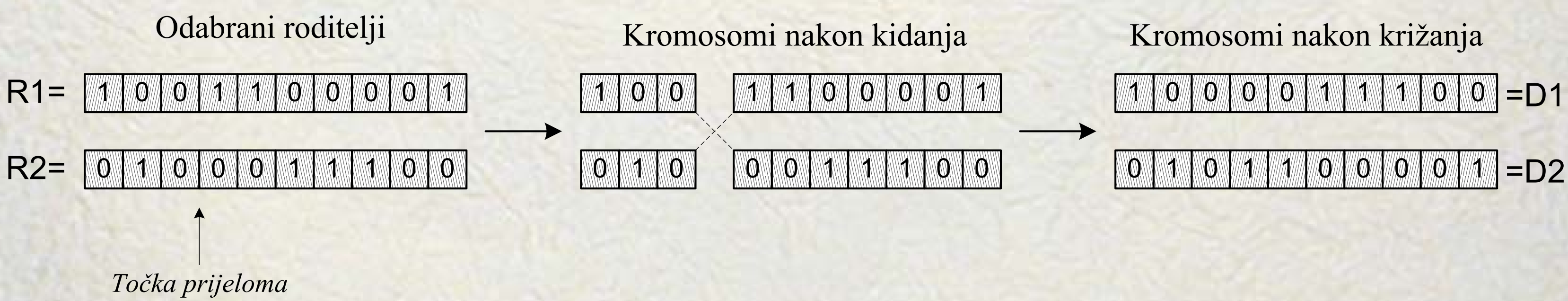
Genetic algorithm

- Binary chromosome
  - A more complex example
    - Solution for a function of three variables x, y, z

$$\underbrace{0\;1\;0\;0\;0\;1\;1\;1}_{x}\;\underbrace{0\;0\;1\;1\;0\;1\;0\;1}_{y}\;\underbrace{1\;1\;0\;0\;1\;1\;0\;0}_{z}$$

Genetic algorithm

- Crossover with one breaking point
  - Two parents are chosen
  - A breaking point is randomly chosen
  - Crossover is performed

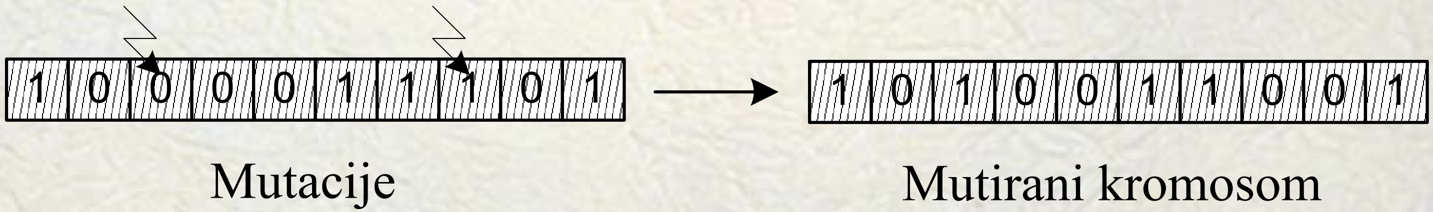


Genetic algorithm

- Other types of crossover
  - Crossover with one breaking point
  - Crossover with *n* breaking points
  - Uniform crossover
  - …

Genetic algorithm

- Mutation operator
  - Mutation probability is given
  - Each bit is flipped with that probability



  - Can introduce a huge change!

Genetic algorithm

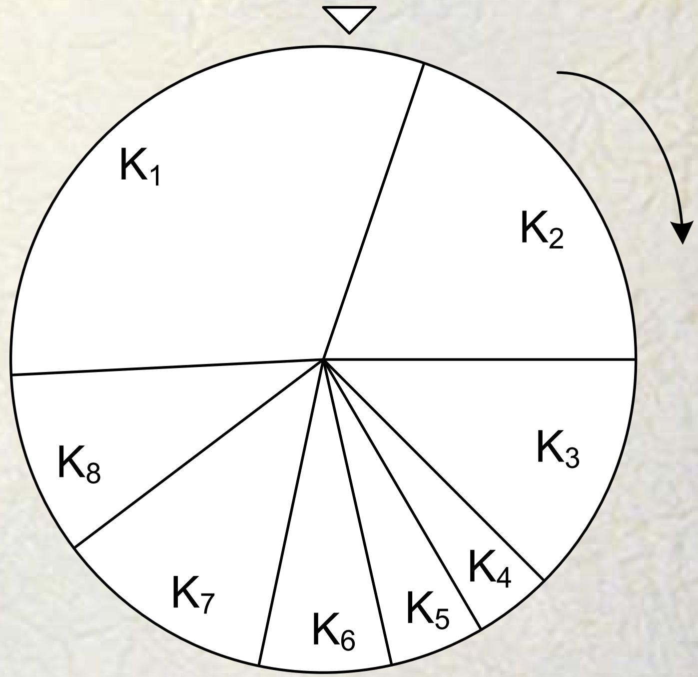
- Selection of parents
  - Proportional selection - Roulette-wheel selection
  - More fitness of an individual means higher chances in the selection process

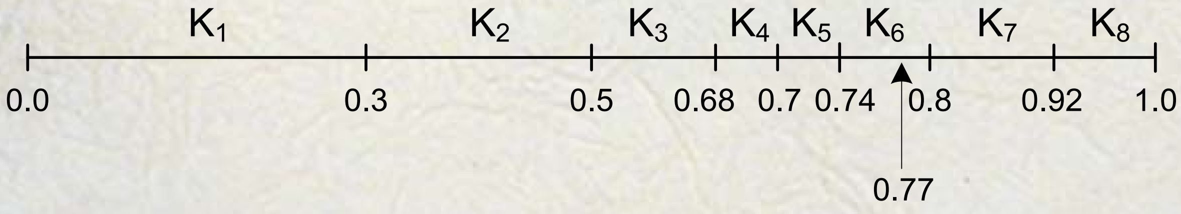
$$probSel(i) = \frac{fit(i)}{\sum_{j=1}^{n} fit(j)}$$

Genetic algorithm

- Selection of parents – proportional selection

$$len(i) = \frac{fit(i)}{\sum_{j=1}^{n} fit(j)}$$

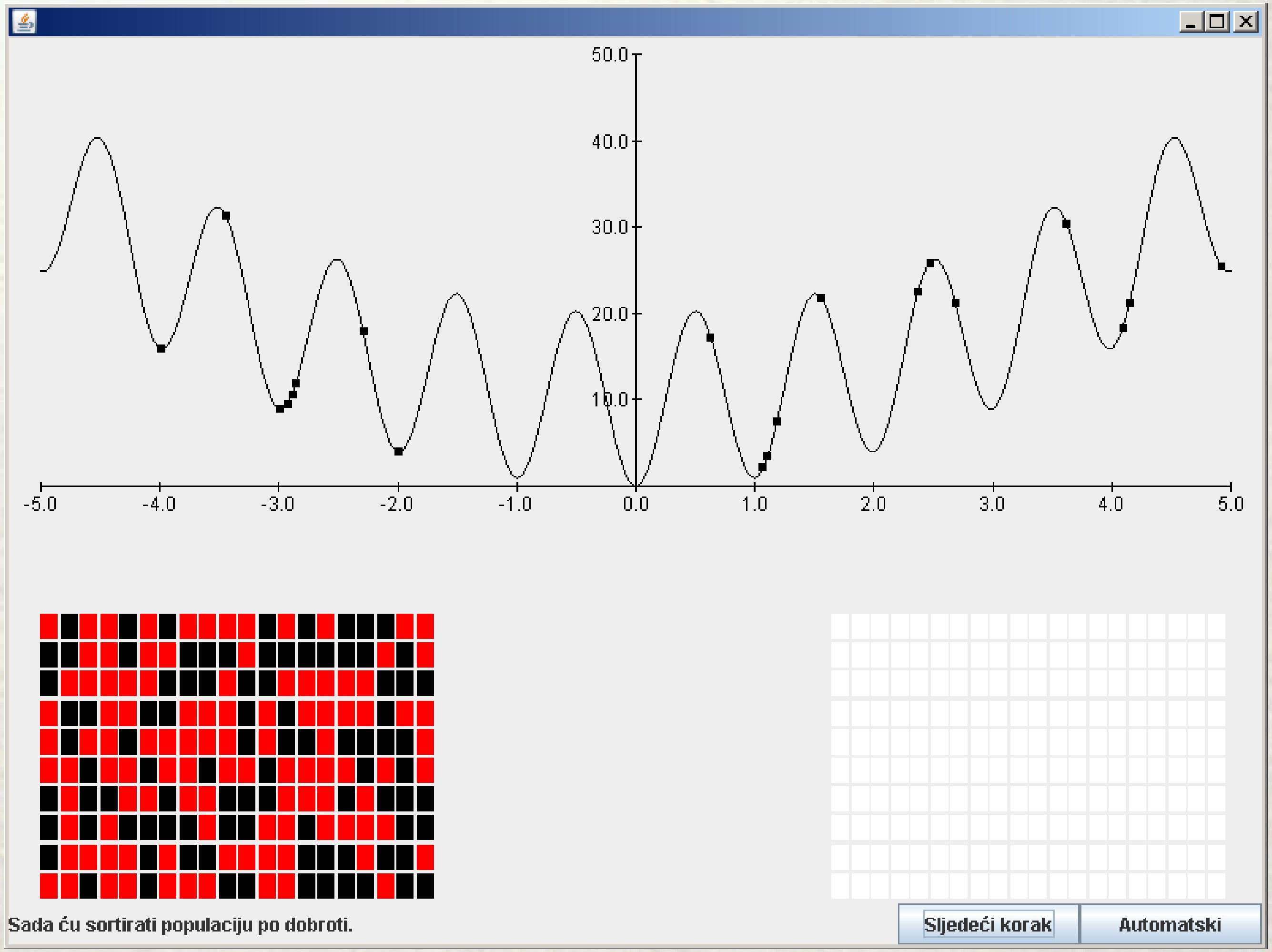




Genetic algorithm

```
P = create_initial_population(POP_SIZE)
evaluate(P)
repeat_until_done:
  new_population P' = ∅
  repeat_while size(P')<POP_SIZE
    select R1 and R2 from P
    {D1, D2} = crossover(R1, R2)
    mutate D1, mutate D2
    add D1 and D2 into P'
  end_repeat
  P = P'
  evaluate(P)
end_repeat
```

Genetic algorithm



Ant colony optimization

- Emerging intelligence
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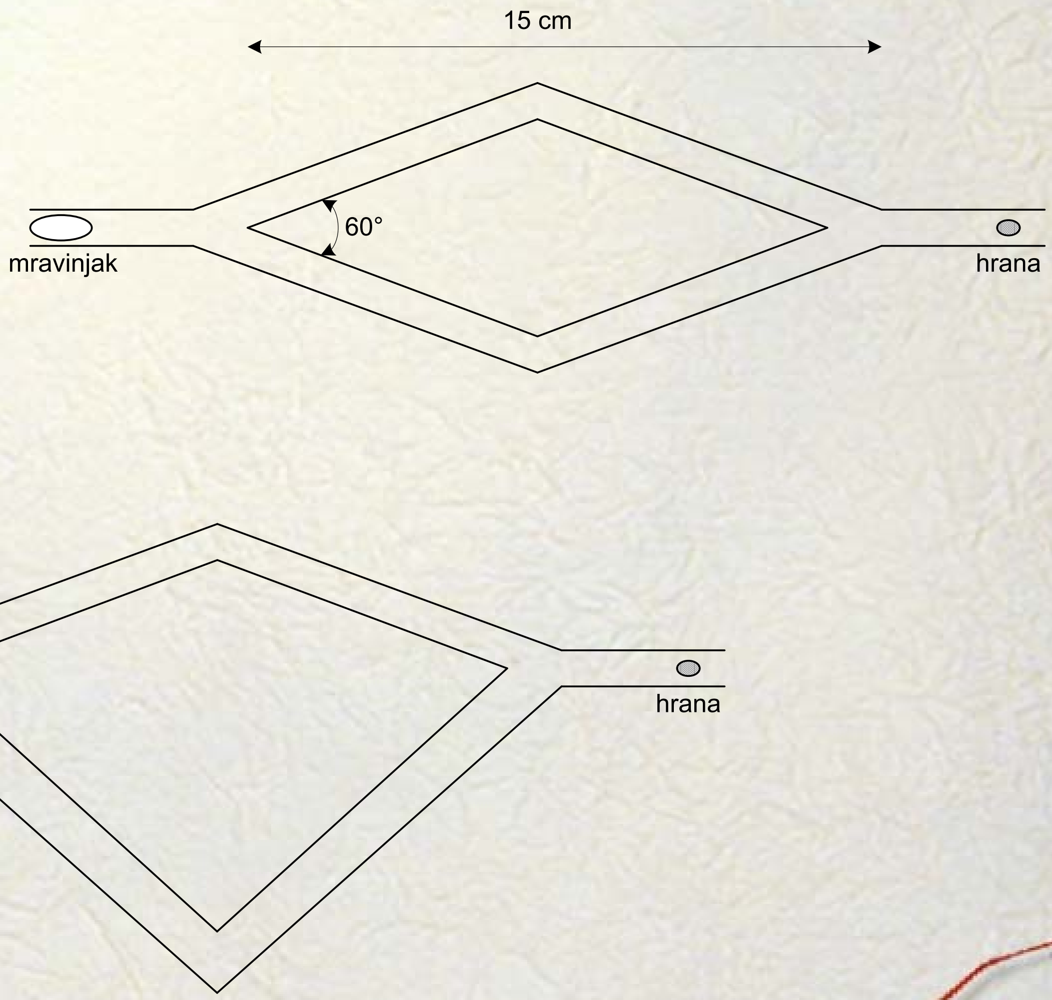
- Genetic algorithm
- **Ant colony optimization**

Ant colony optimization

- Ants exhibit interesting behavior
  - They successfully find the shortest path to food sources

Ant colony optimization

- Experiments



Ant colony optimization

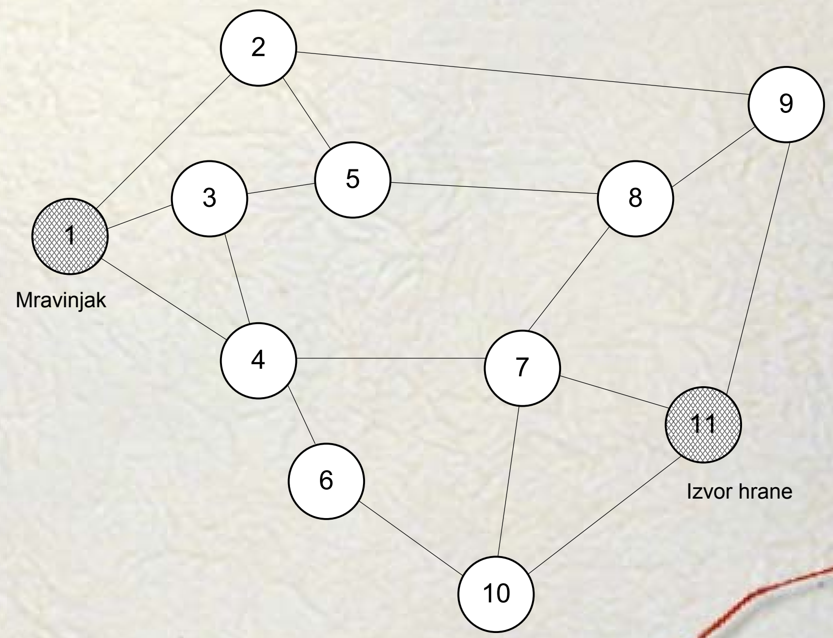
- Explanation
  - While moving ants leave a feromon trail behind
  - An ant moves randomly, but it is more likely to go in the direction where the feromon trail is stronger

Ant colony optimization

- Directly applicable to problems described by graphs
- E.g. From 1, possible next are 2,3,4

$$\tau_0 = konst$$

$$p_{ij}^k = \begin{cases} \dfrac{\tau_{ij}^{\alpha}}{\sum\limits_{l \in N_i^k} \tau_{il}^{\alpha}}, & \text{if } j \in N_i^k \\ 0, & \text{if } j \notin N_i^k \end{cases}$$

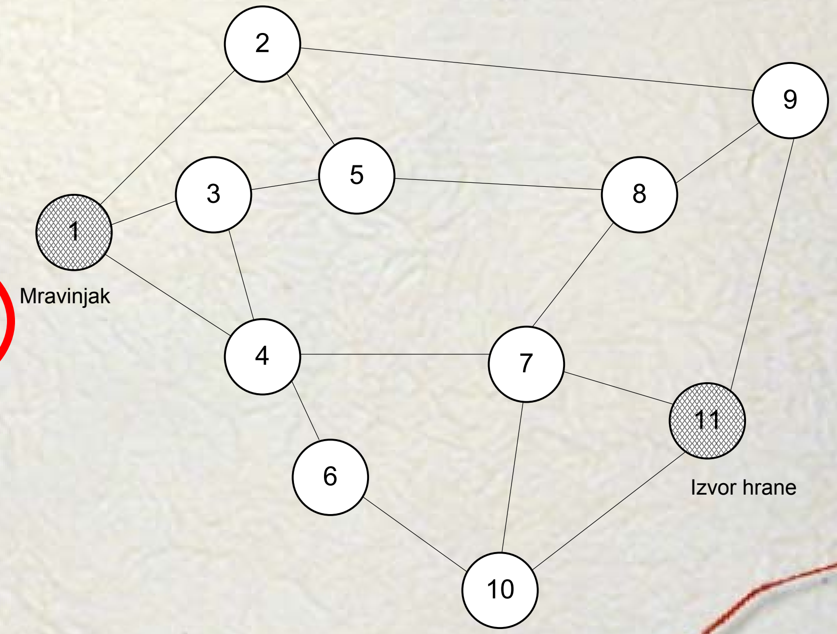


Ant colony optimization

- Ant System algorithm
  - Using heuristic information has additional benefits on performance

$$\tau_0 = \frac{m}{C^{nn}}$$

$$p_{ij}^k = \begin{cases} \dfrac{\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta}}{\sum_{l \in N_i^k} \left(\tau_{il}^{\alpha} \cdot \eta_{il}^{\beta}\right)}, & \text{ako } j \in N_i^k \\ 0, & \text{ako } j \notin N_i^k \end{cases}$$



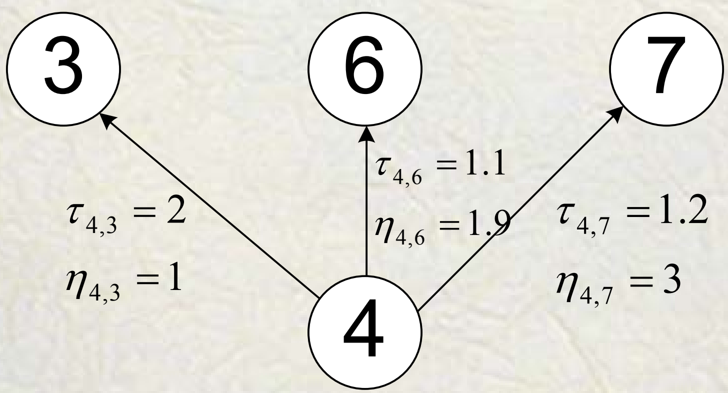
Ant colony optimization

- Ant System algorithm

```
repeat until not done
  repeat for each ant
    create solution
    evaluate solution
  end repeat
  evaporate_pheromons
  repeat for all_or_some ants
    update_pheromons
  end repeat
end repeat
```

Ant colony optimization

- Procedure: `Create_solution`
  - An ant starts from a node
  - With respect to the probabilities, a next node is chosen, then another, and so on until the ant reaches the last node



Uz α=1, β=2:

p(4→3)=11,9%

p(4→6)=23,7%

p(4→7)=64,4%

Ant colony optimization

- Procedure: `Evaluate_solution`
  - Calculates the total path length
  - Moving from one node to another is usually associated with a cost (cities → distance)

Ant colony optimization

- Procedure: `Evaporate_pheromons`
  - Lowers pheromon trails on all edges by an amount

$$\tau_{ij} \leftarrow \tau_{ij} \cdot (1 - \rho)$$

  - Geometric progression!
  - Very costly(graph has many edges)

Ant colony optimization

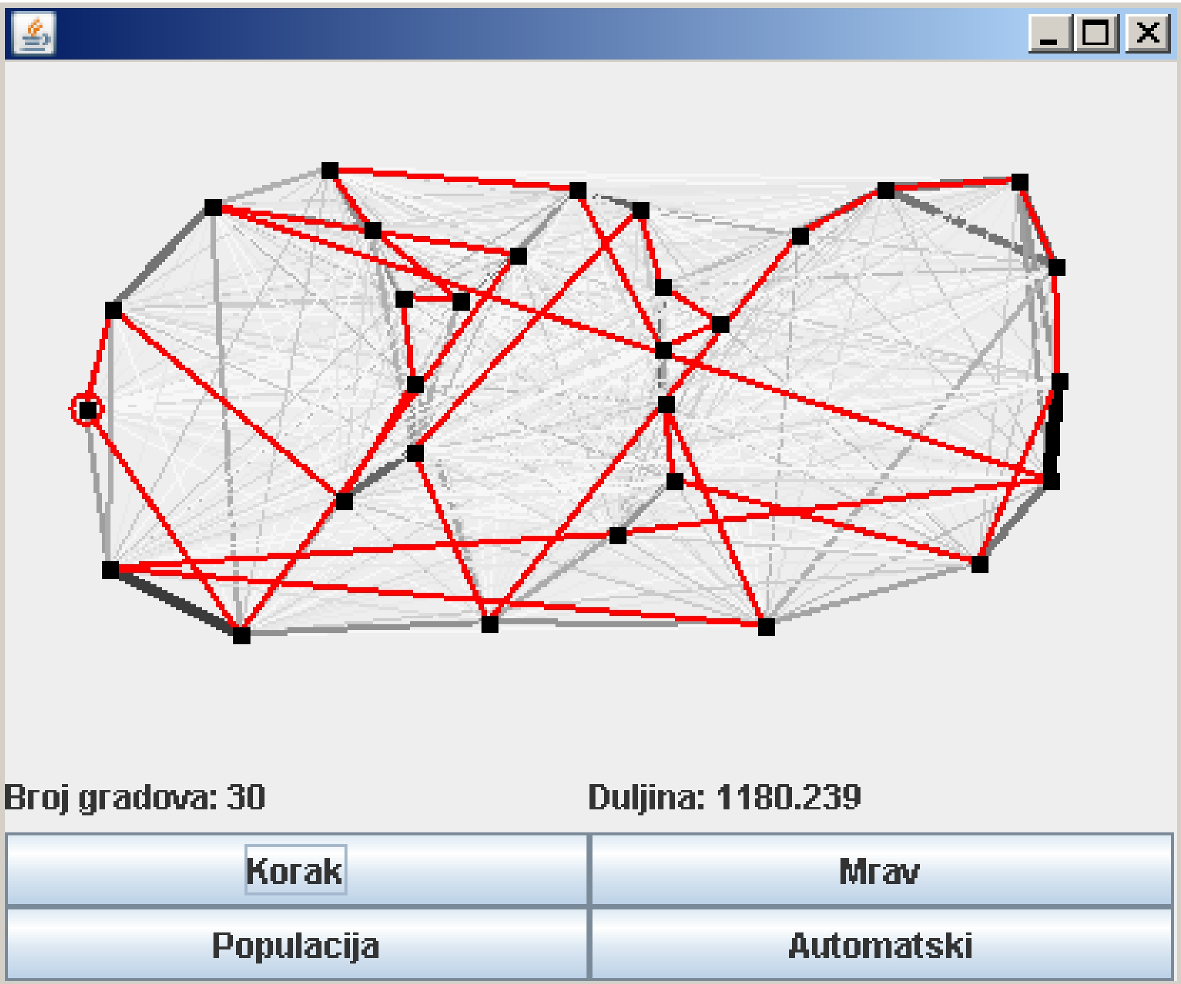
- Procedura: `Update_pheromons`
  - Funkcija za odabranog mrava dodaje nove feromonske tragove iznosa:

$$\Delta\tau_{ij}^{k} = \begin{cases} 1/C^{k}, & \text{if edge } i\text{-}j \text{ is on the path of ant } k \\ 0, & \text{otherwise} \end{cases}$$

  - Novo stanje je tada:

$$\tau_{ij} \leftarrow \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}$$

Ant colony optimization



Conclusion

- Algorithms inspired by nature – a very vivid area of research!
- Methods that can efficiently tackle problems that were previously unsolvable
- New algorithms emerging (e.g. Bee Colony Optimization, Intelligent Water Drops, …)

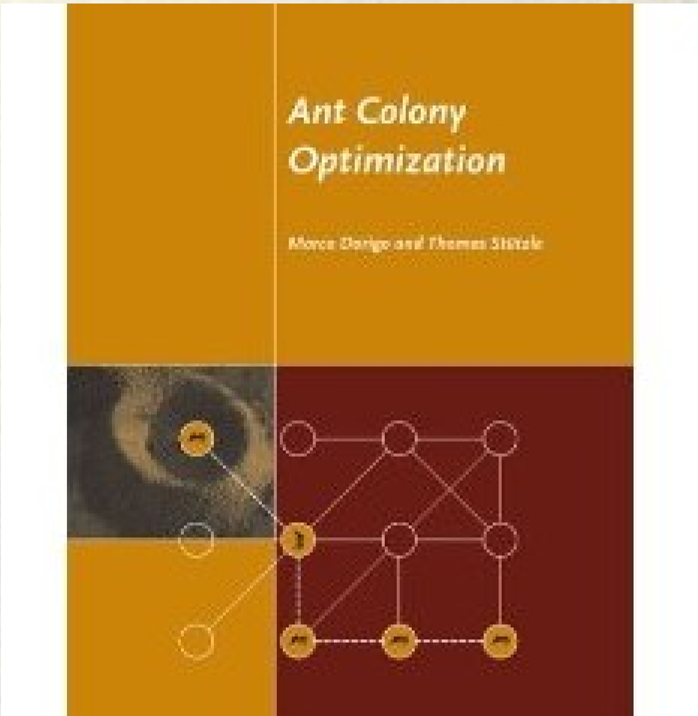
Links

- Video about an ant colony http://www.inquisitr.com/14238/holy-crap-billions-of-ants-in-one-colony/

Links

- Materials
  http://java.zemris.fer.hr/nastava/ui/

- Implementations
  http://java.zemris.fer.hr/nastava/ui/evoAlg.zip


Ant Colony
Optimization