MULTIMEDIA SYSTEMS
First homework

1. A sequence of 4-bit symbols is given in hexadecimal notation $N_{(16)} = 9A15AD2A1AA_{(16)}$.

a. Construct a Huffman code based on the frequency of the symbols.
b. Calculate the entropy and the average code length.
c. Determine the efficiency of the code by comparing the average length of the code to the smallest possible length.
d. Determine the compression ratio of the constructed Huffman code with respect to the standard binary approach.

2. The process X is described by the probability density function:

$$f_X(x) = \begin{cases} 2x^{-2}, & x \in [2/3,1] \\ 0, & otherwise \end{cases}$$

a. Determine the associated distribution function $\Phi_X(x)$.
b. Calculate the expected value $E(x)$ and the differential entropy $h(X)$.
c. Calculate the quantization step corresponding to an output entropy of 5 bits (*assuming a high-rate case*).
d. Calculate the variance of the quantization error D for a given quantization step.

Note: Use the following expressions in the calculation:

$$\int \frac{1}{x} dx = lnx,$$

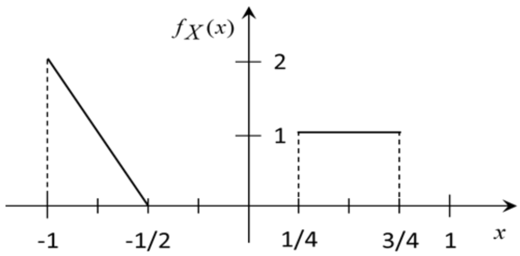
$$\int \frac{lnx}{x^2} dx = -\frac{lnx}{x} + \int \frac{1}{x^2} dx.$$

3. The time-limited discrete signal $x[n]$ is given:

$$[0] = 1, x[1] = 0, x[2] = 2, x[3] = -1, x[4] = 2.$$

a. Determine the autocorrelation of the signal $x[n]$ for shifts $j = 0, 1, 2$.
b. Determine the second order linear predictor using the autocorrelation method.
c. Determine the prediction and prediction error for samples $x[2]$ and $x[3]$.

4. The expected value of the process X whose probability density function is given by the figure is $E(x) = -1/6$. We quantize the process with a uniform scalar quantizer described by the expression $\hat{x} = round(2x)/2$.



a. Determine the output symbols and their probabilities.
b. Determine the entropy of the output symbols.
c. Determine the expected squared quantization error for each quantization class D_i and the total distortion D.
d. Calculate the SQNR.

Note - do not use the high rate quantization theory.