5th lecture overview

Lecture overview

Properties of regular languages

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$N_1 = \{ wcw^R \}$

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Properties of regular languages

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Properties of regular languages

Language is not regular

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There is no finite automaton which accepts the language

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2^{Σ^*} ***set of all languages over the alphabet*** Σ

Regular languages

$RL \subset 2^{\Sigma^*}$

Properties of regular languages

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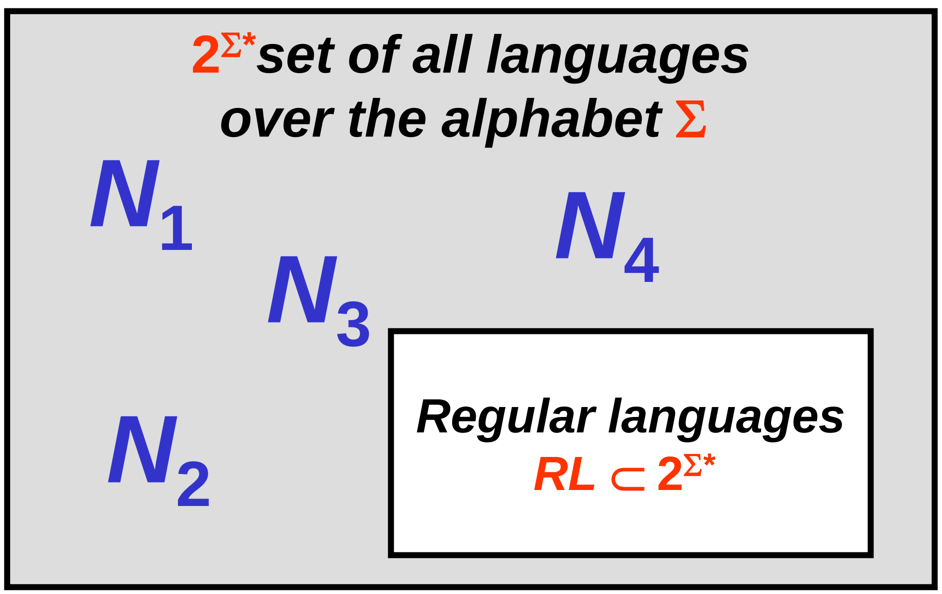

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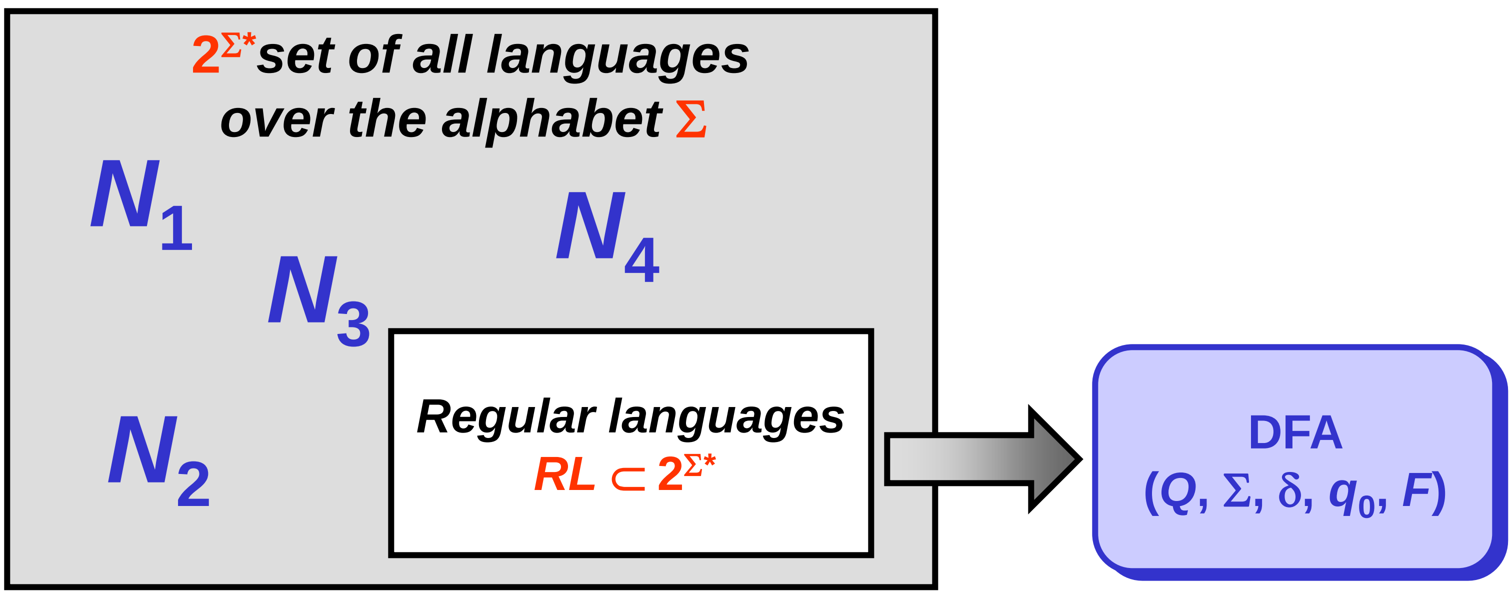

Closure properties of regular languages

A language class is *closed under a certain operation*

if applying this operation to any language(s) in the class gives a language in the same class.

Closure properties of regular languages

Closure properties of regular languages

- **Union, concatenation and Kleene's operator**

Closure properties of regular languages

- Union, concatenation and Kleene's operator
  - Based on the definition of regular expressions

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  - **There is an algorithm to construct an ε-NFA from a regular expression**

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  - **DFA $M=(Q, \Sigma, \delta, q_0, F)$,** $L(M)$

Closure properties of regular languages

- **Union, concatenation and Kleene's operator**

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- **Complement**

  - **DFA $M=(Q, \Sigma, \delta, q_0, F)$,** $\quad L(M)$
  - **DFA $M'=(Q, \Sigma, \delta, q_0, Q \backslash F)$,** $\quad L(M)^c$

Closure properties of regular languages

- Union, concatenation and Kleene's operator
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- Intersection

Closure properties of regular languages

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  - **DFA $M=(Q, \Sigma, \delta, q_0, F)$, $L(M)$**
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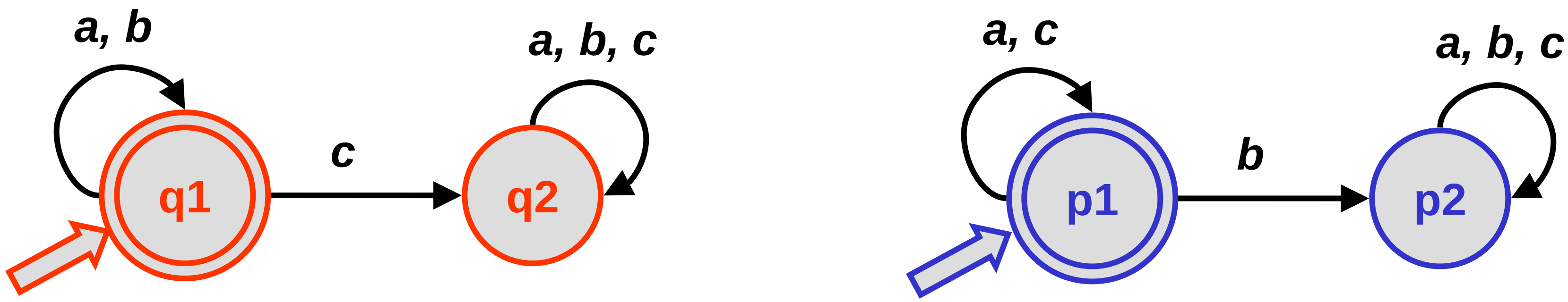
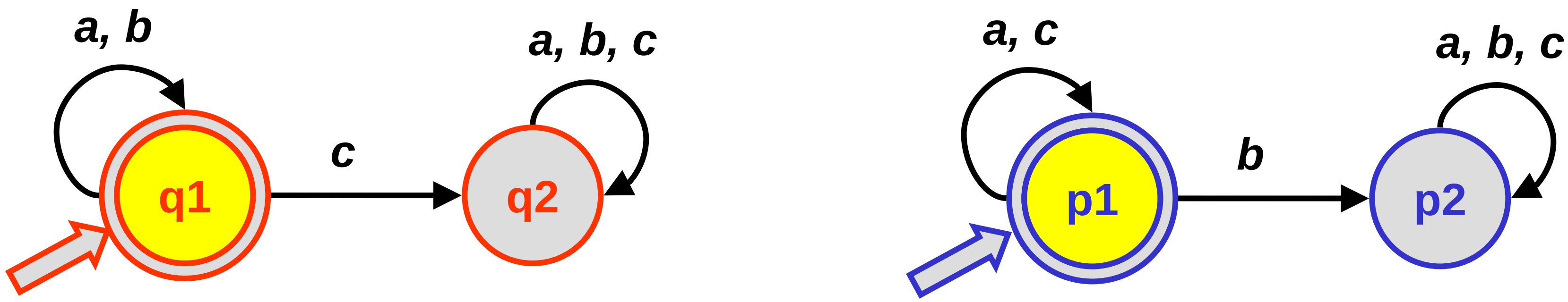
- **Intersection**
  - **De Morgan's rule: $L \cap N = ((L \cap N)^c)^c = (L^c \cup N^c)^c$**

Intersection



Intersection



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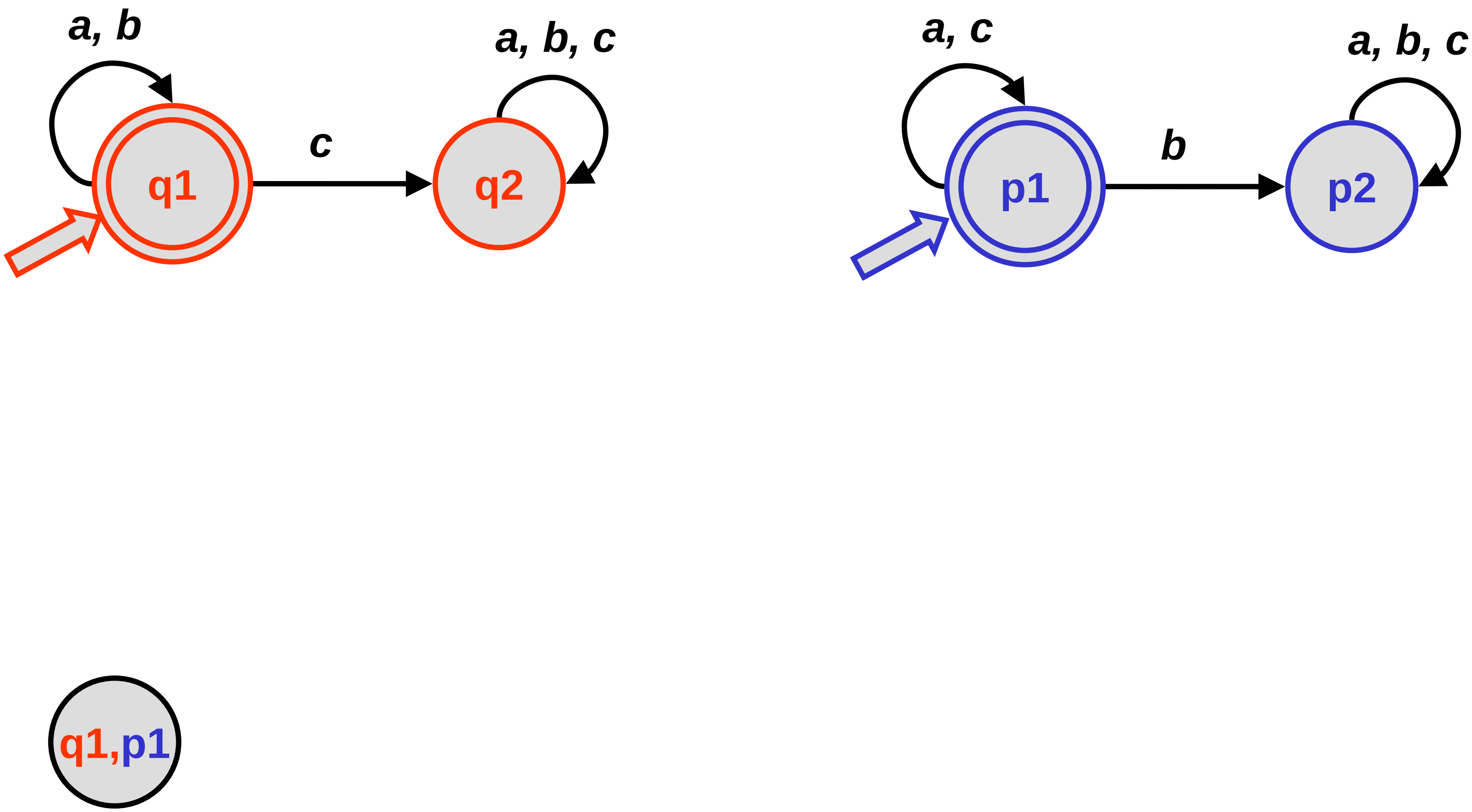

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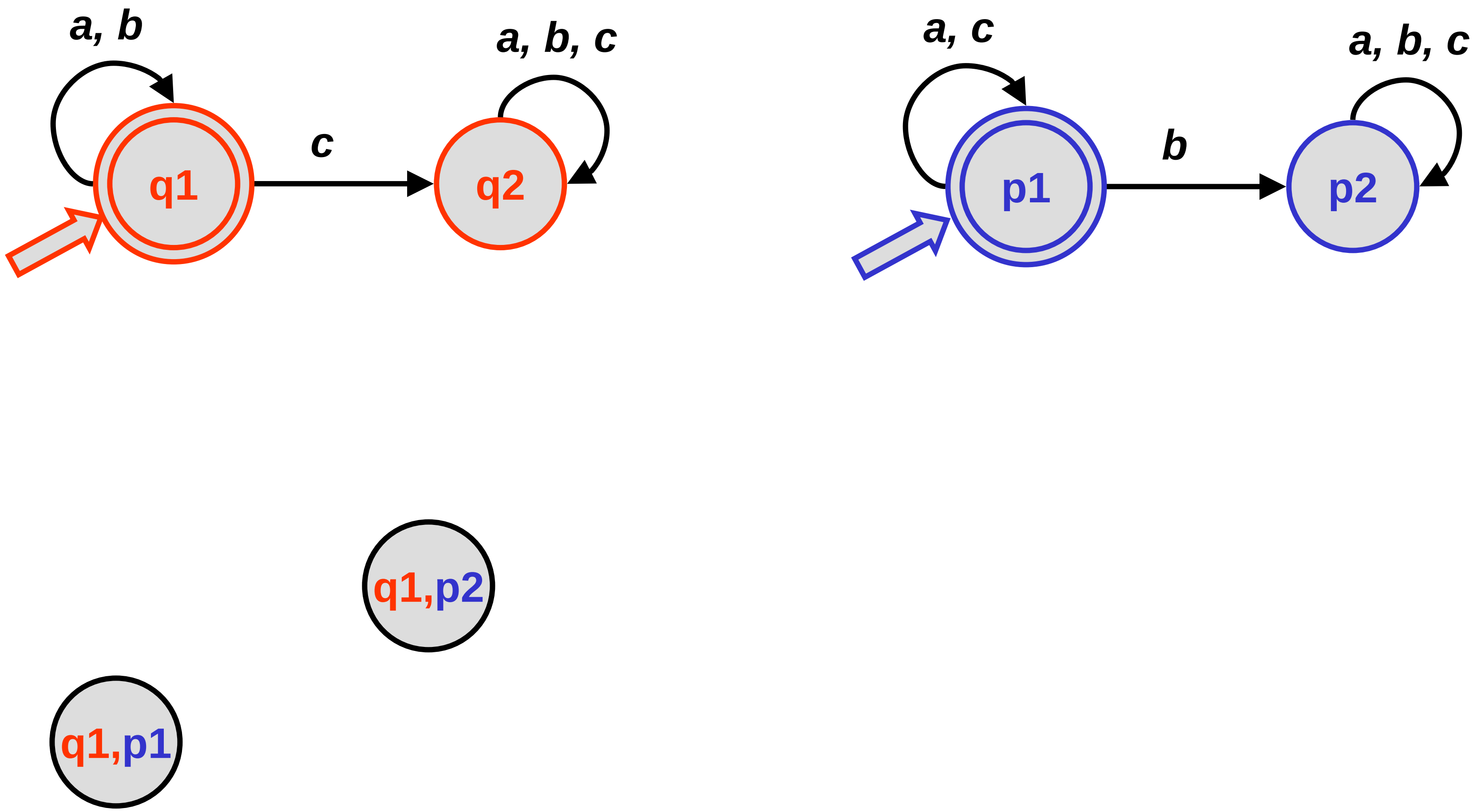

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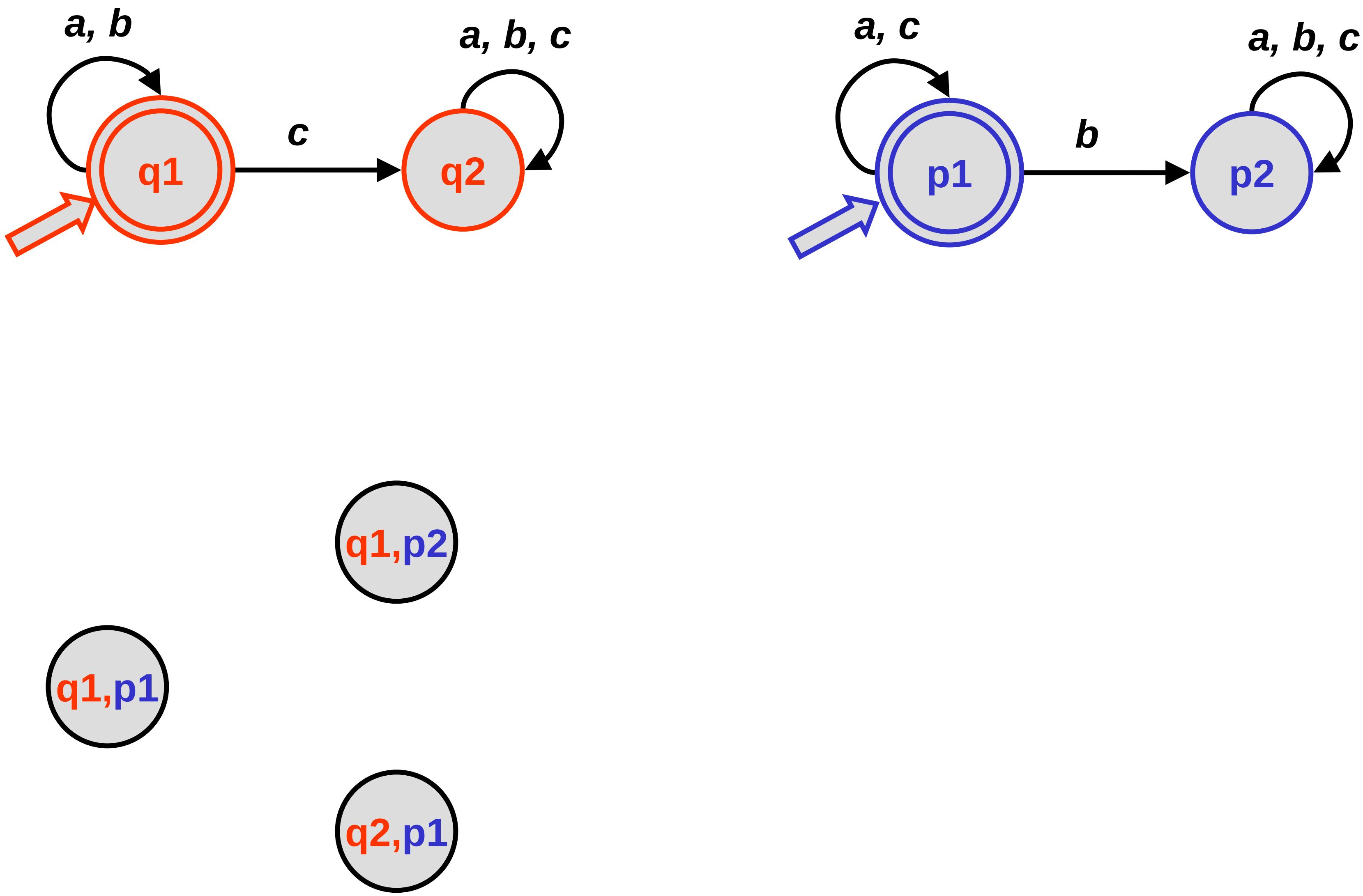

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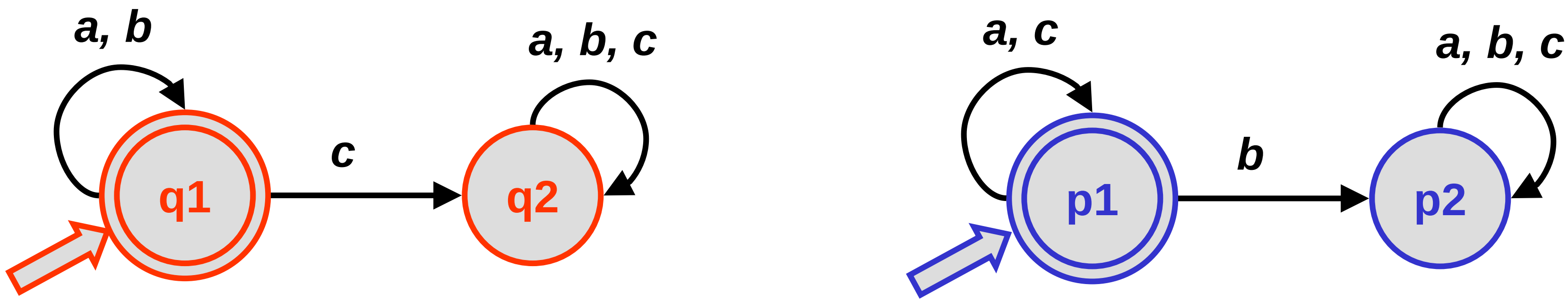

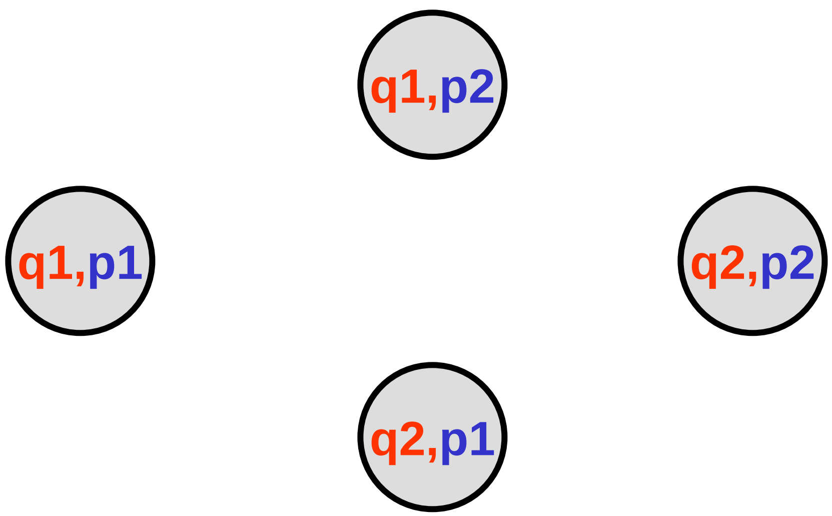

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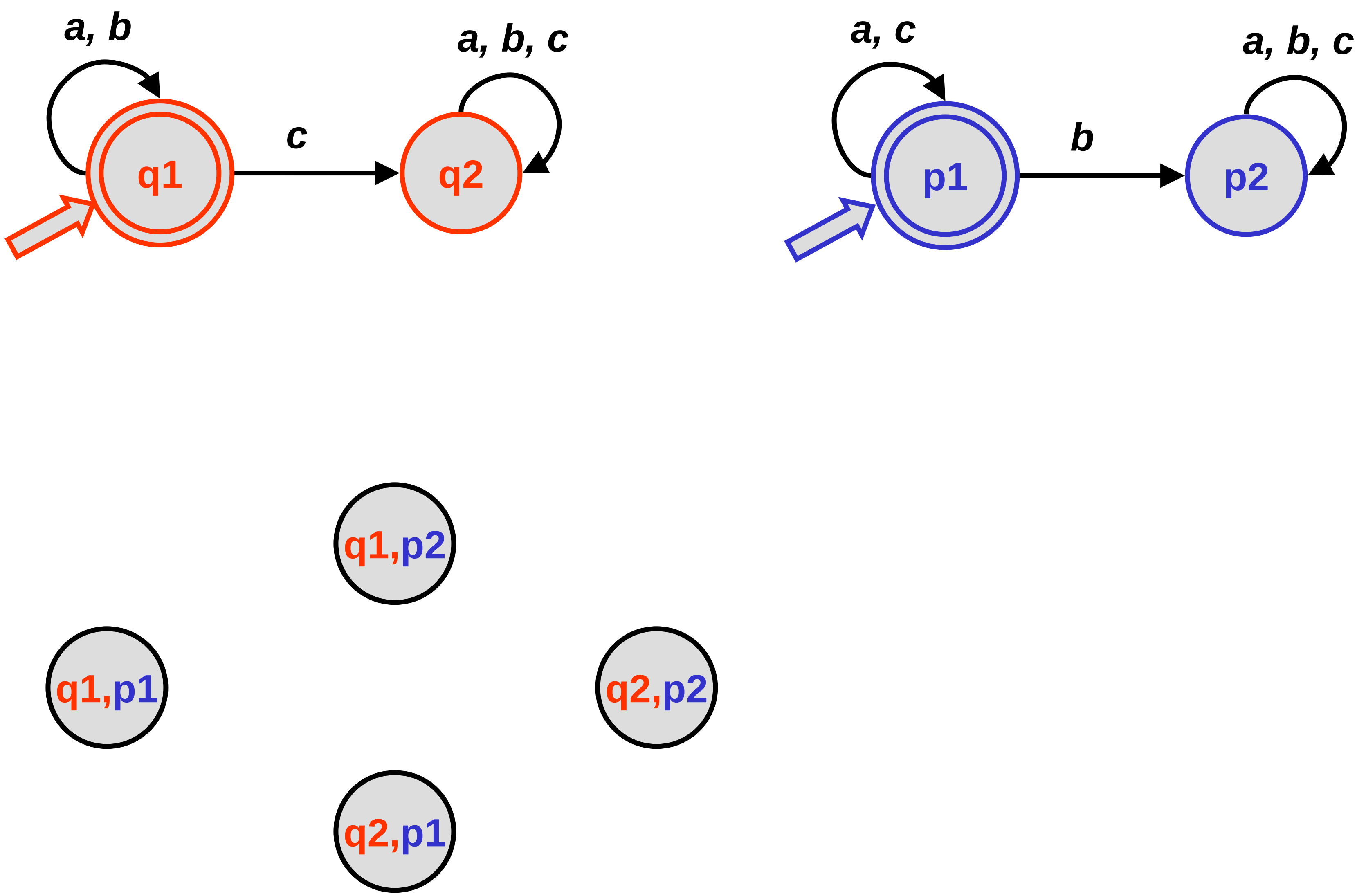

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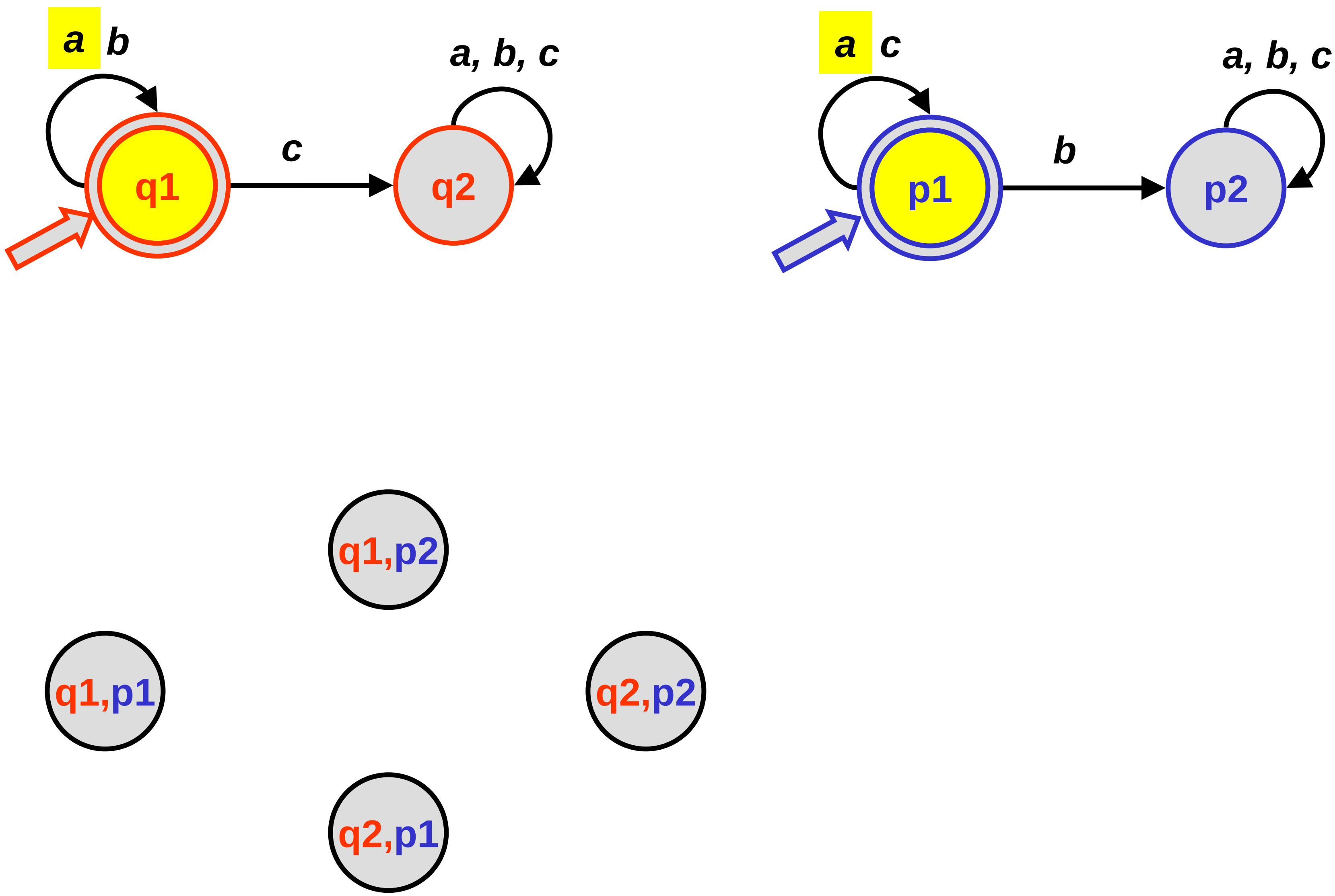

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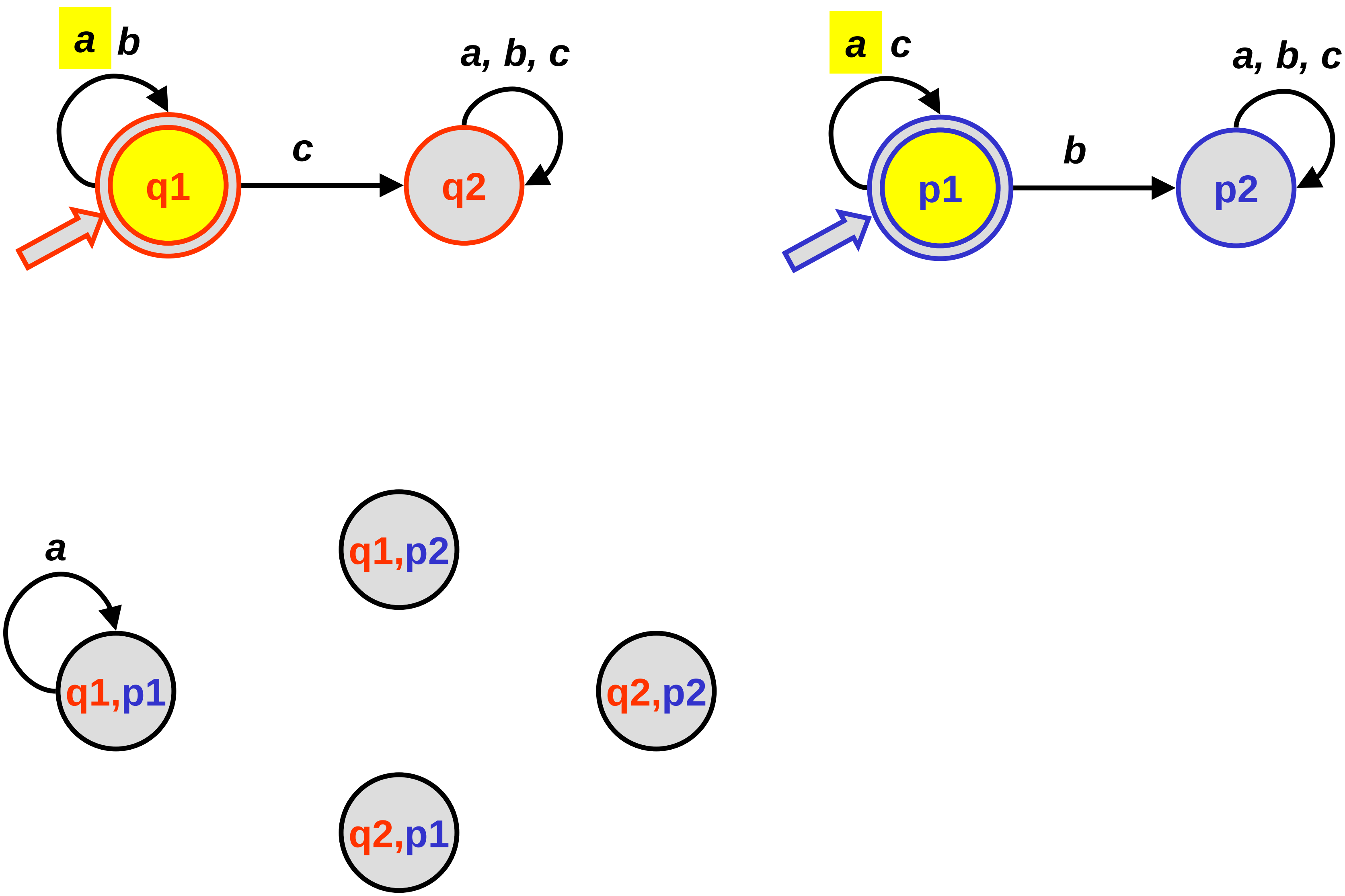

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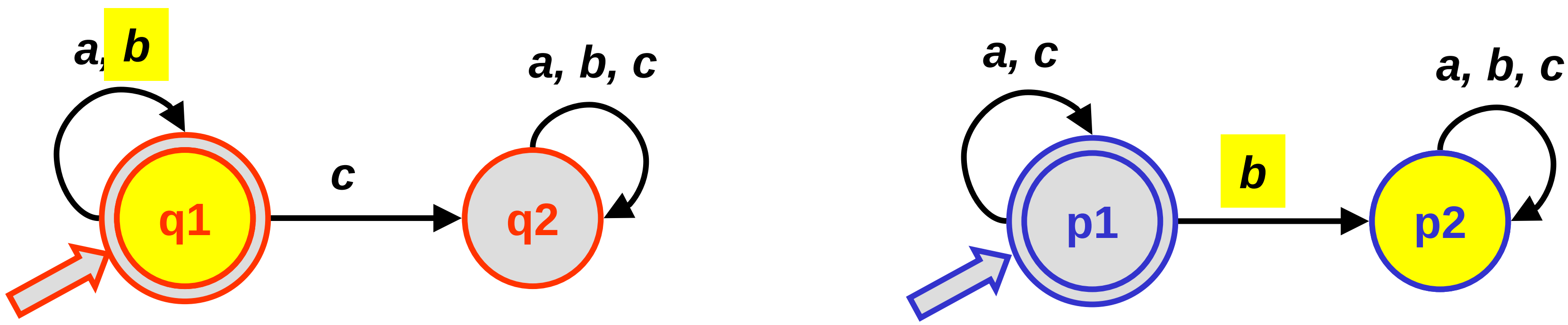

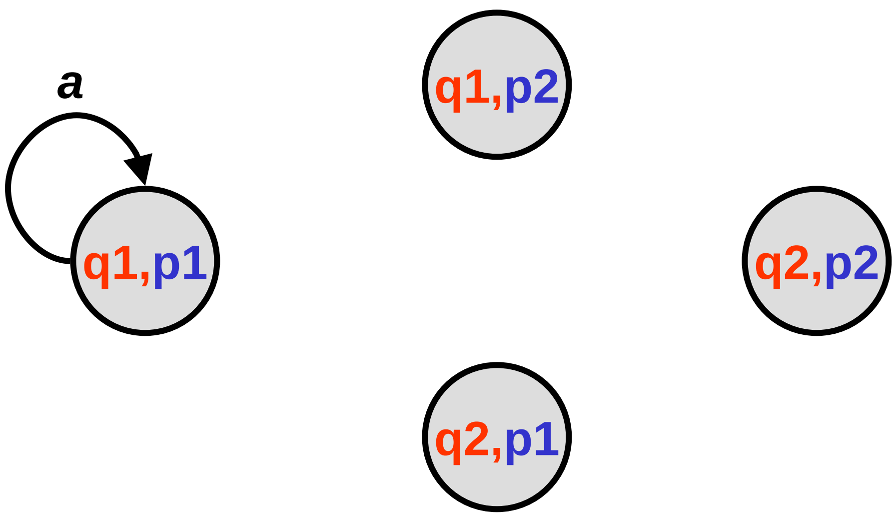

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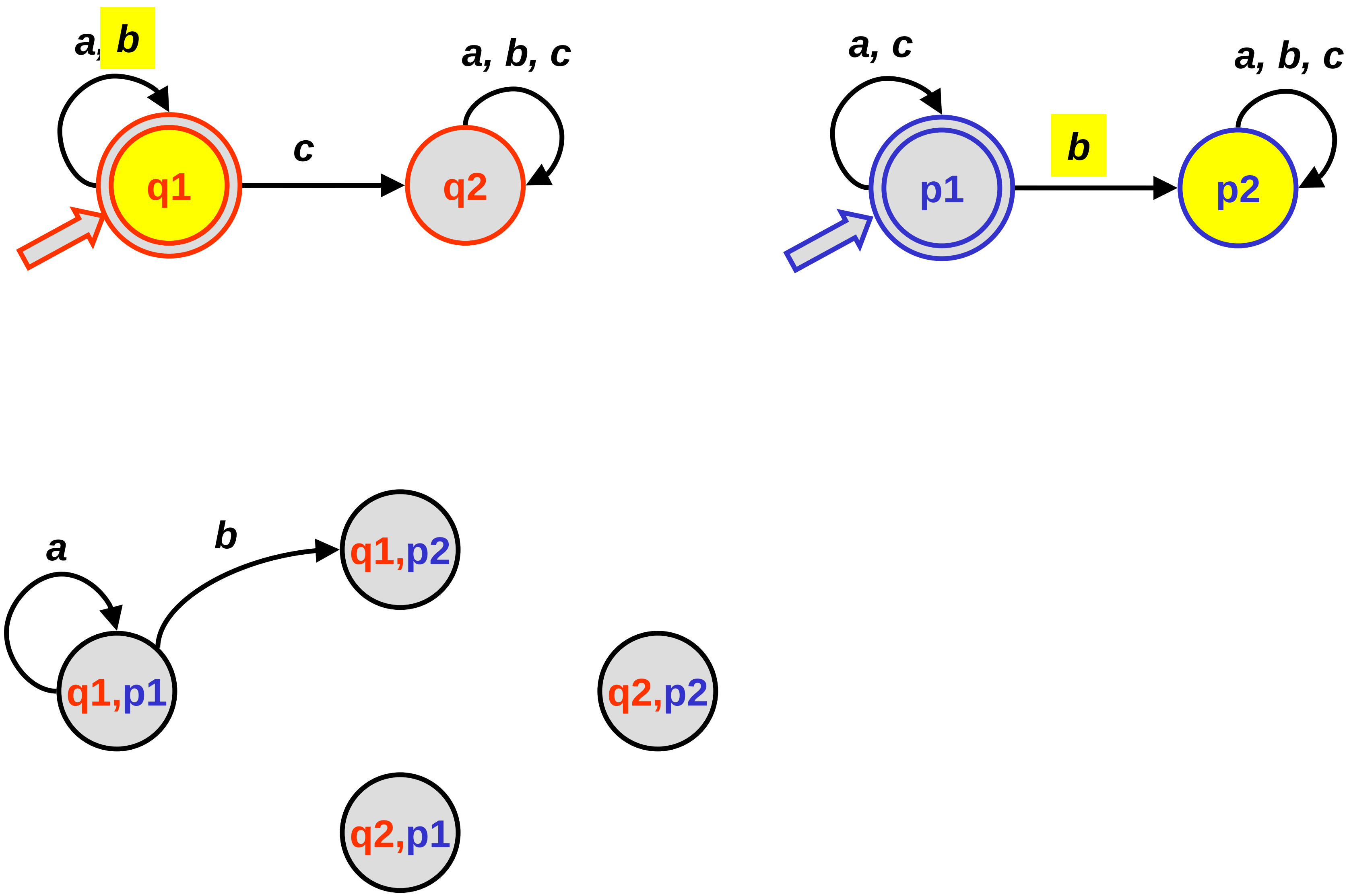

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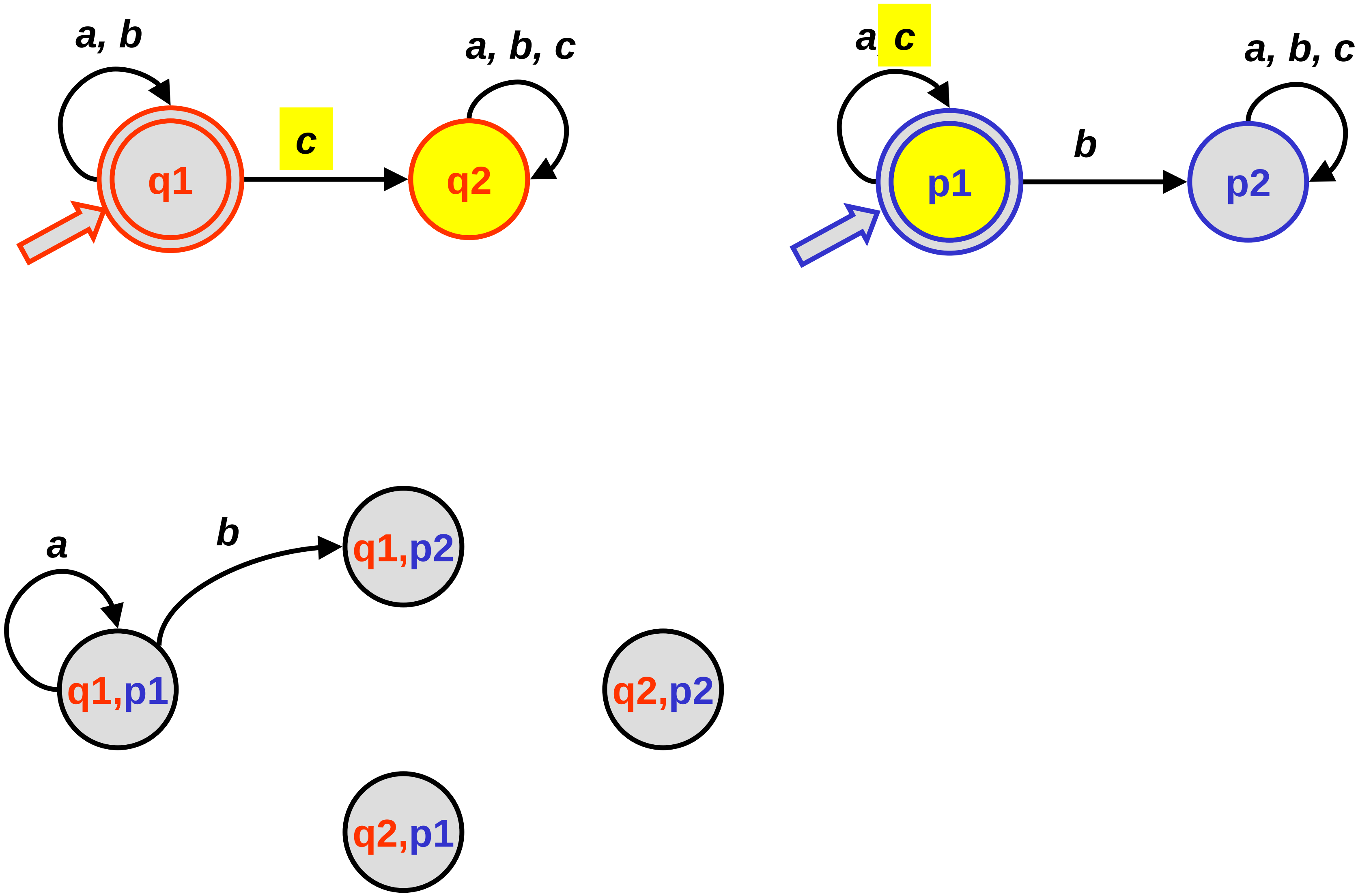

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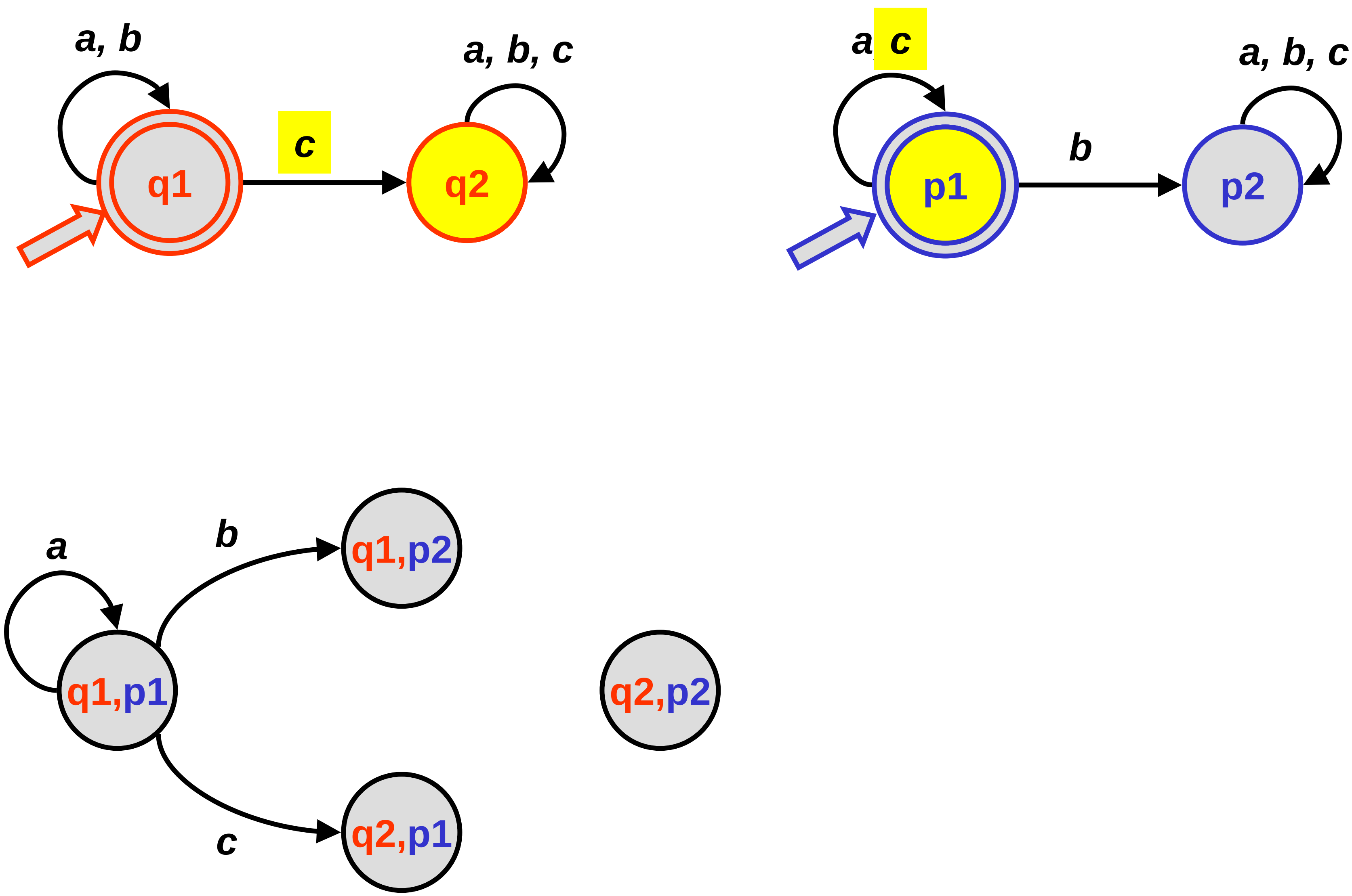

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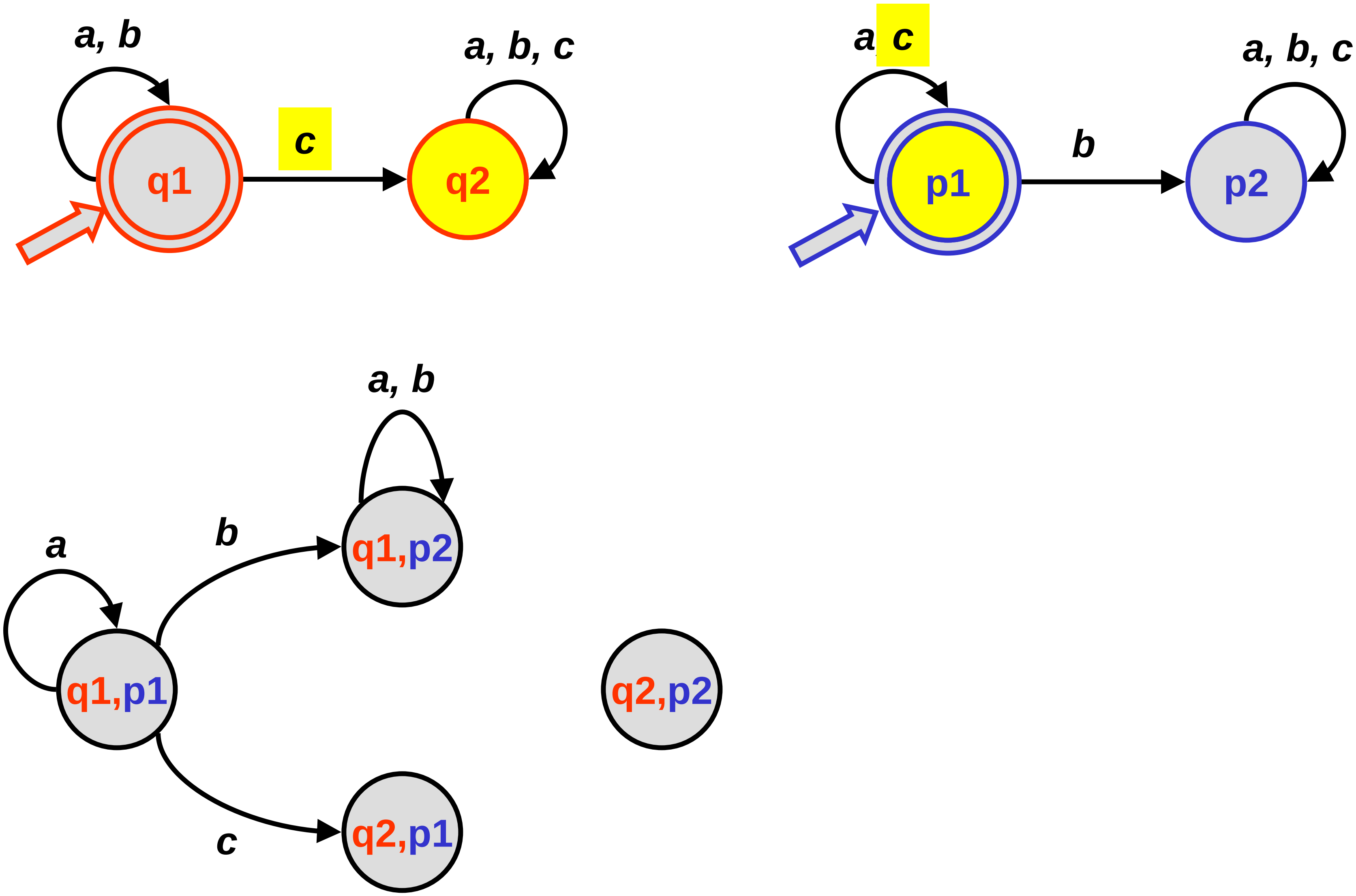

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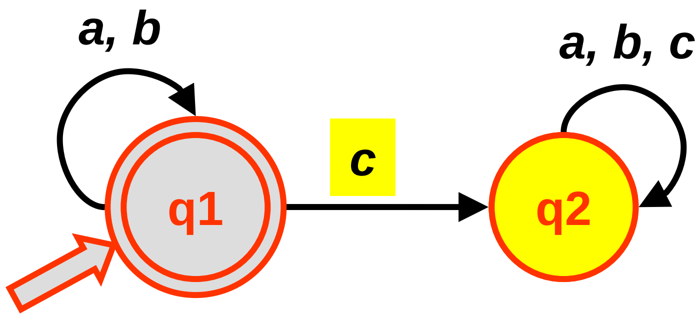

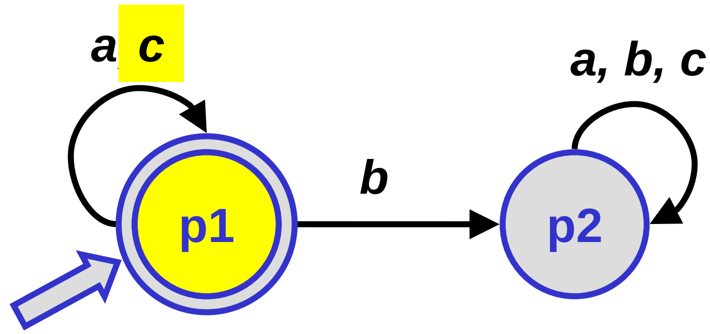

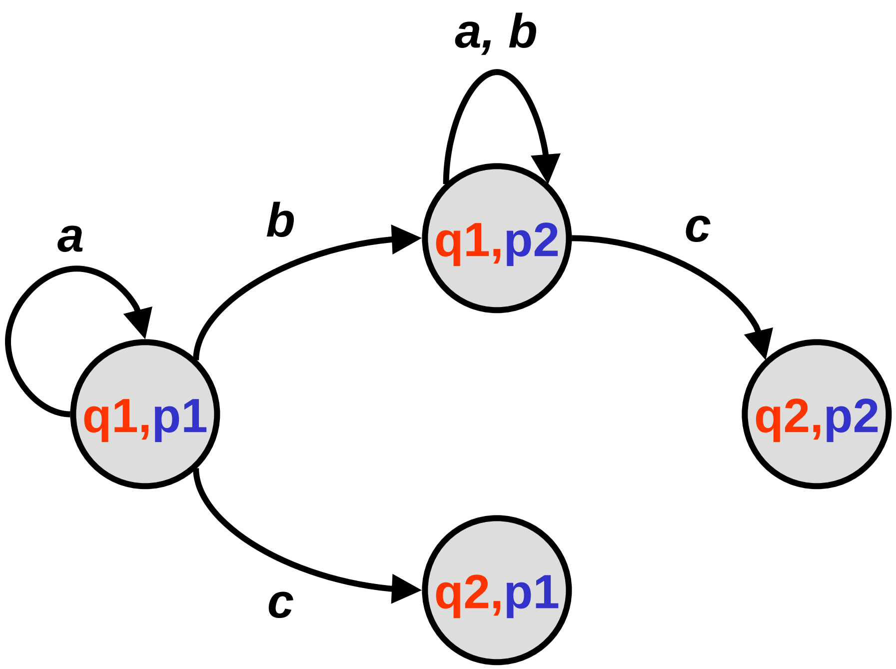

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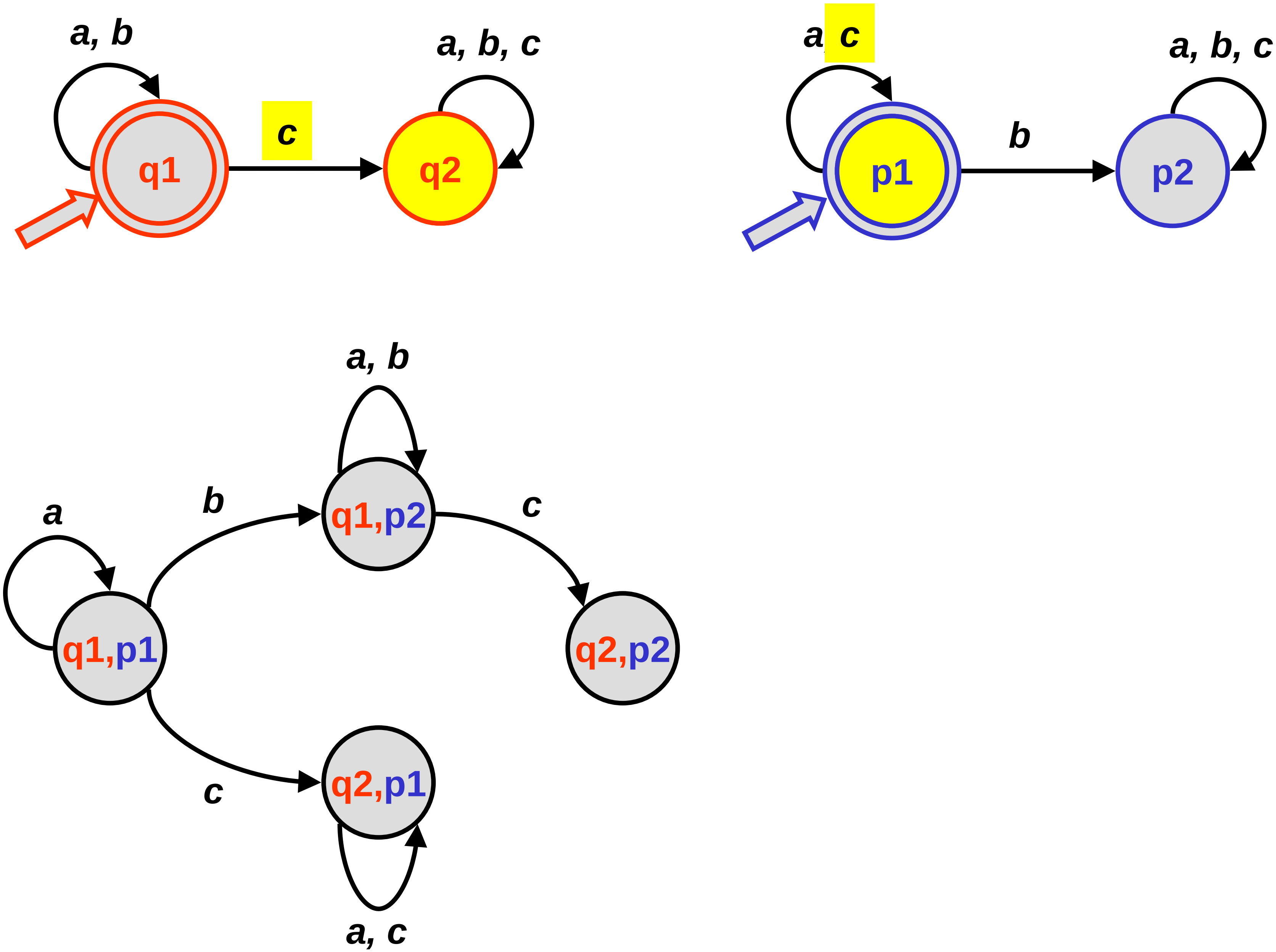

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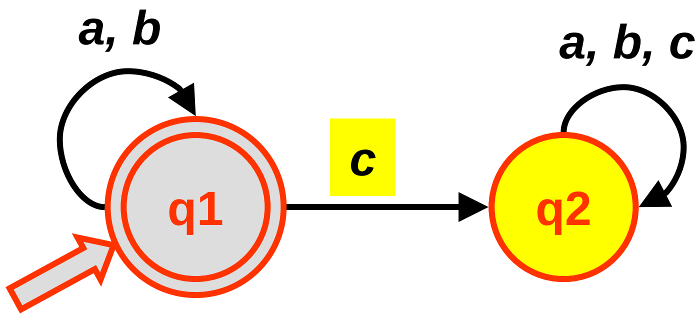

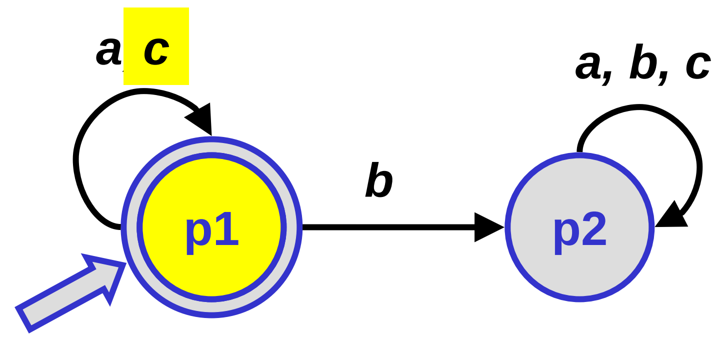

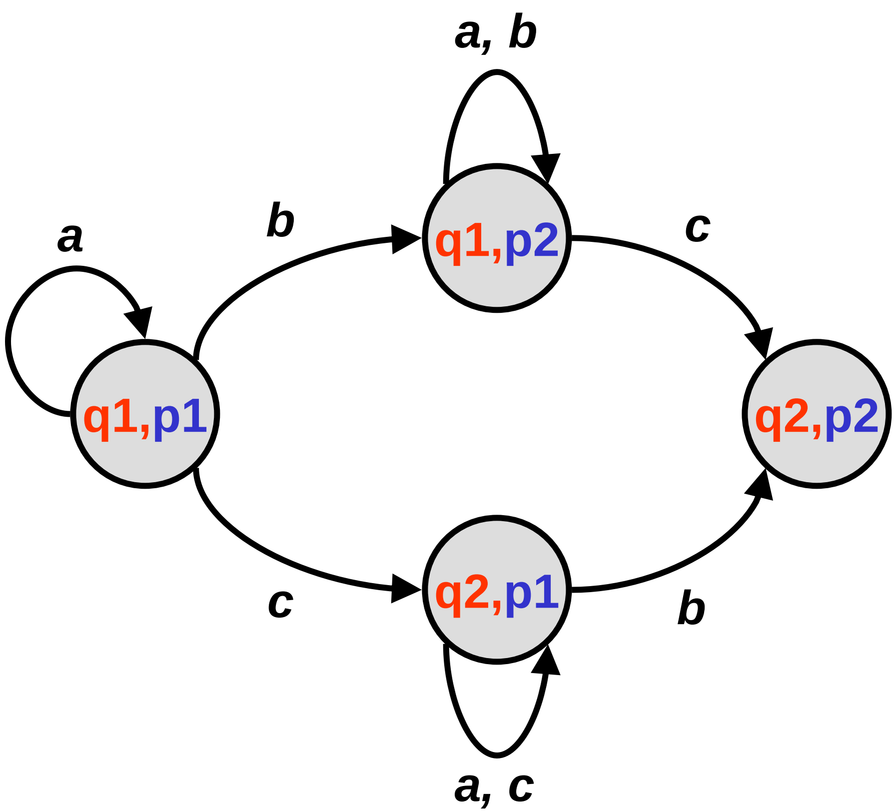

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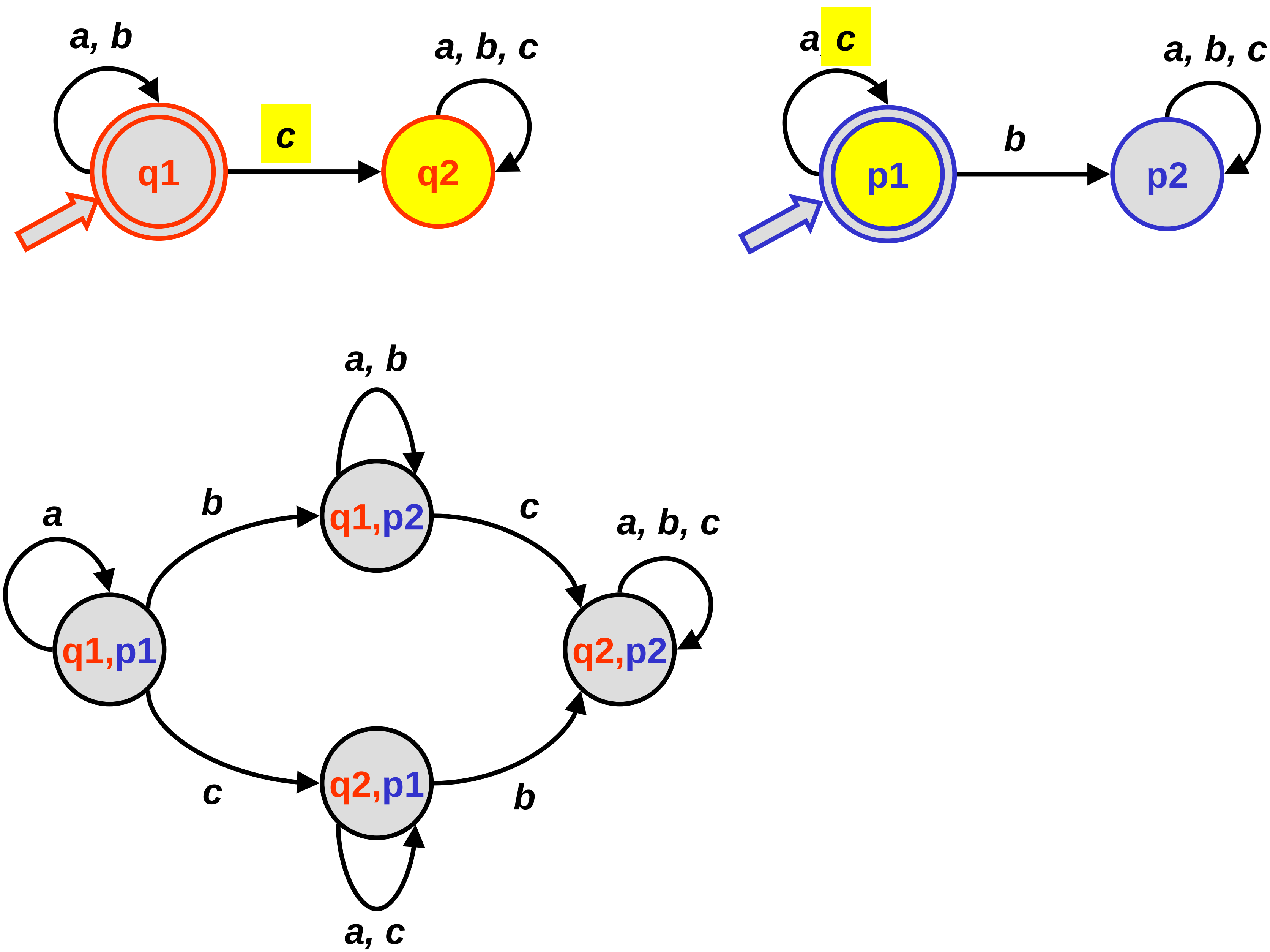

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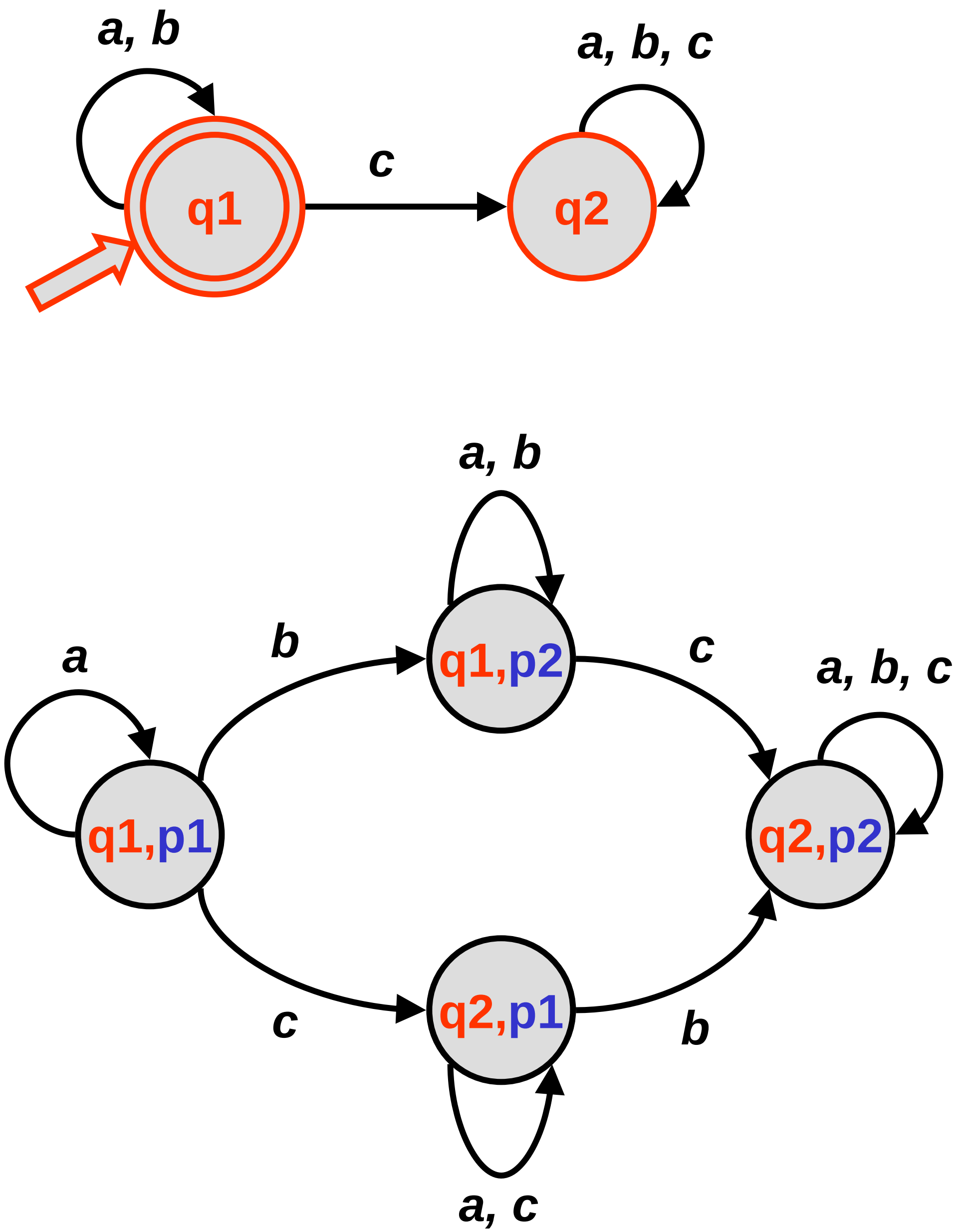

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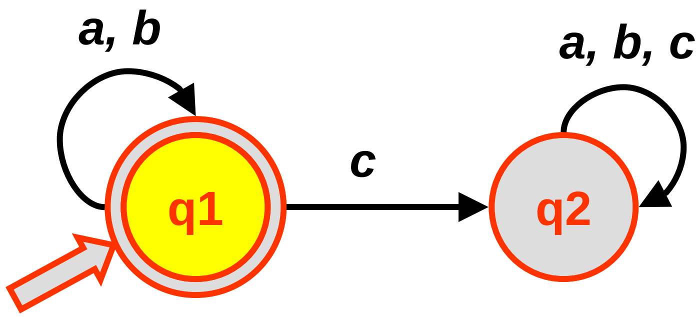

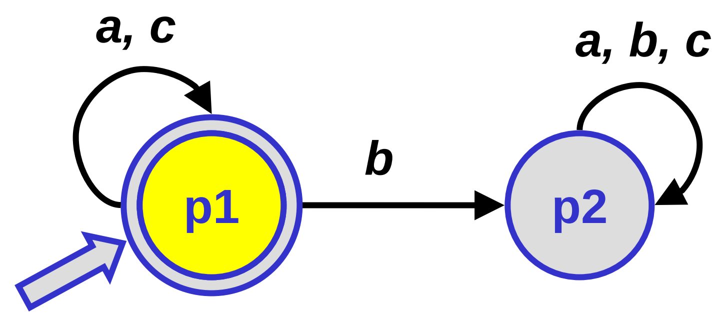

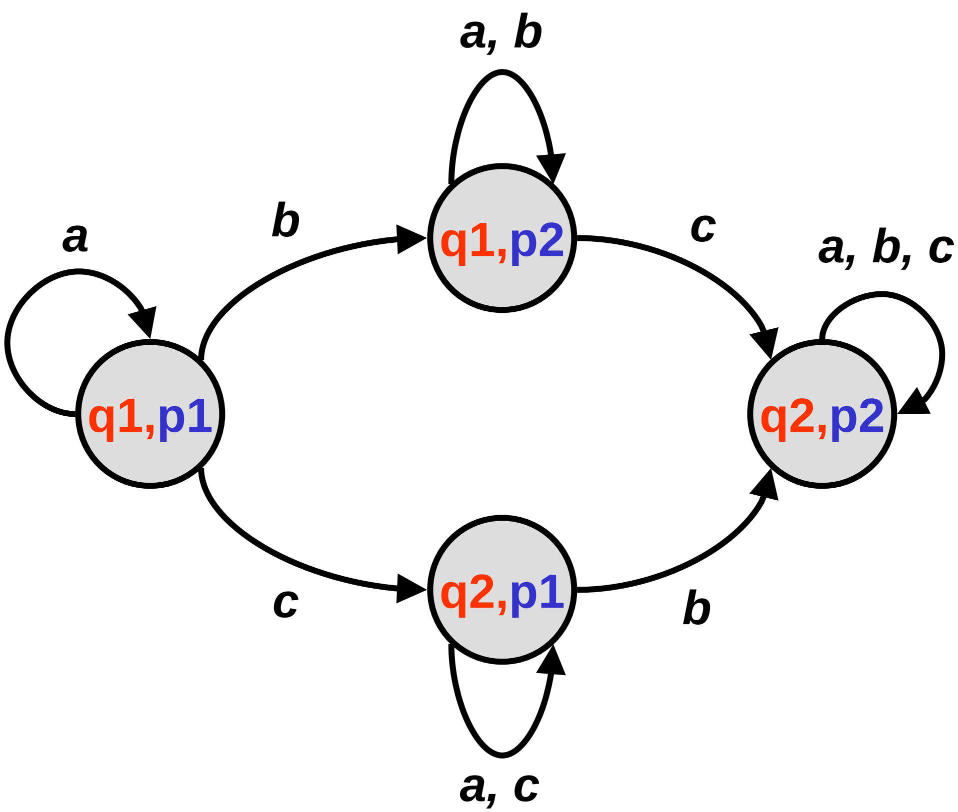

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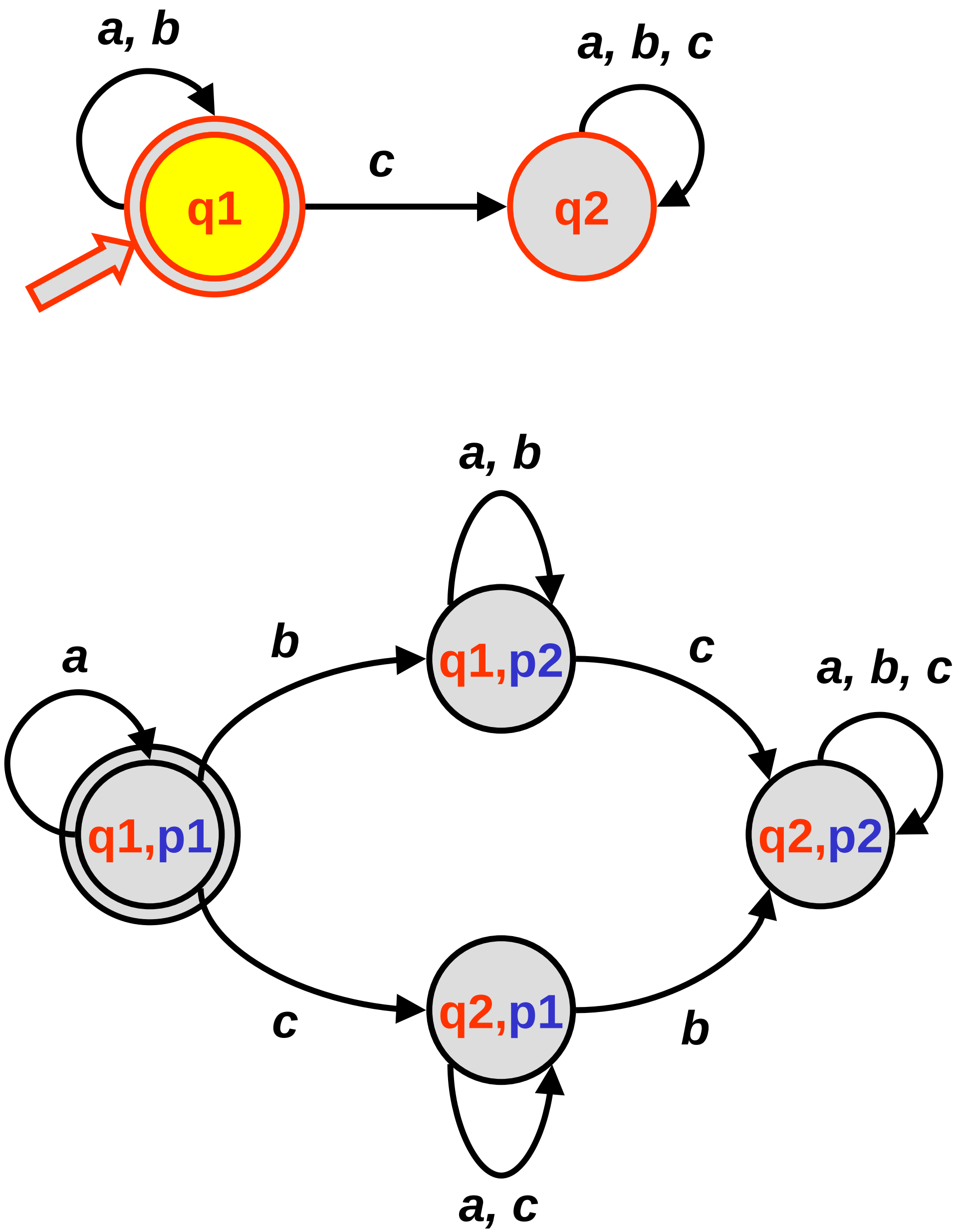

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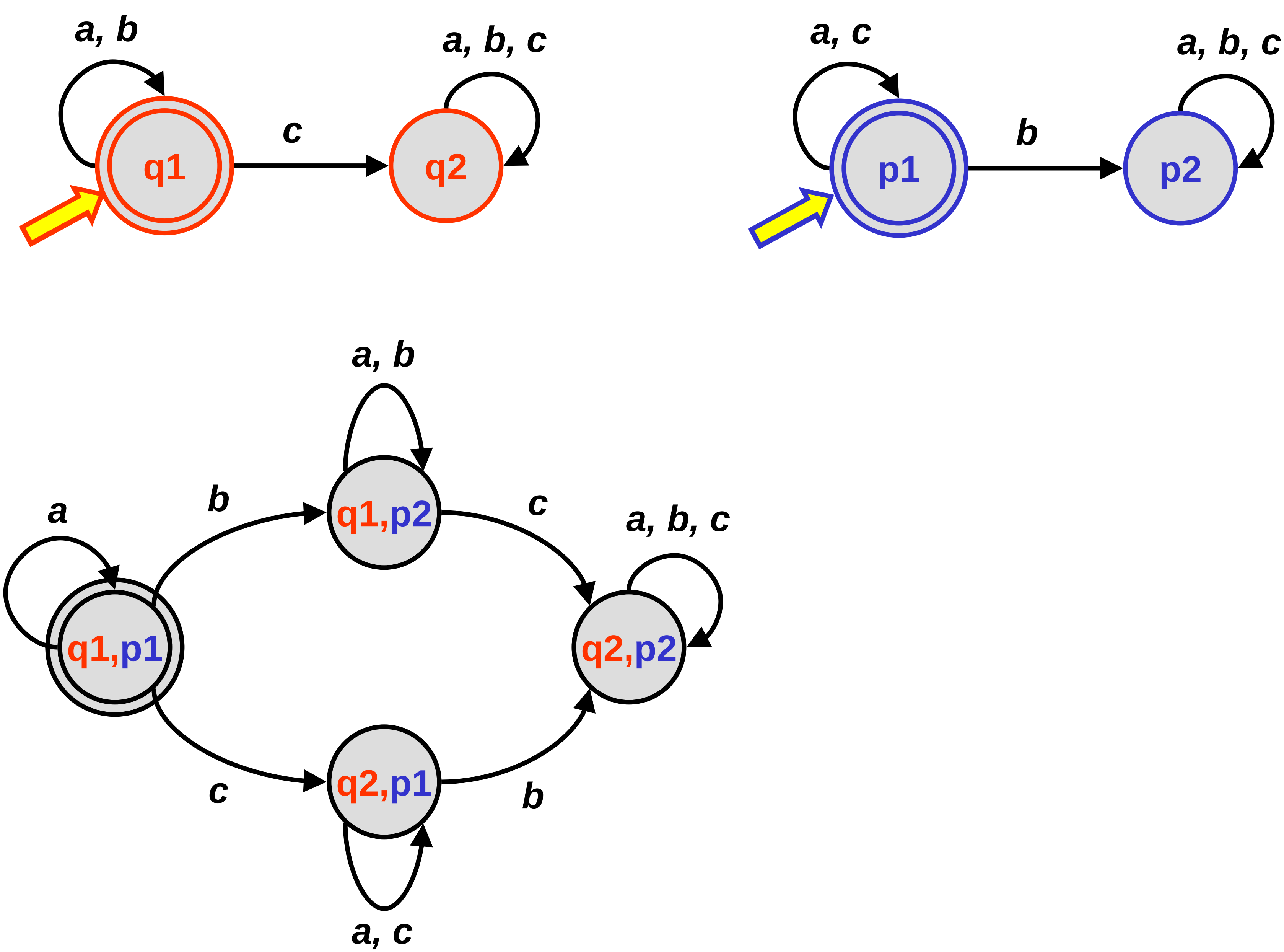

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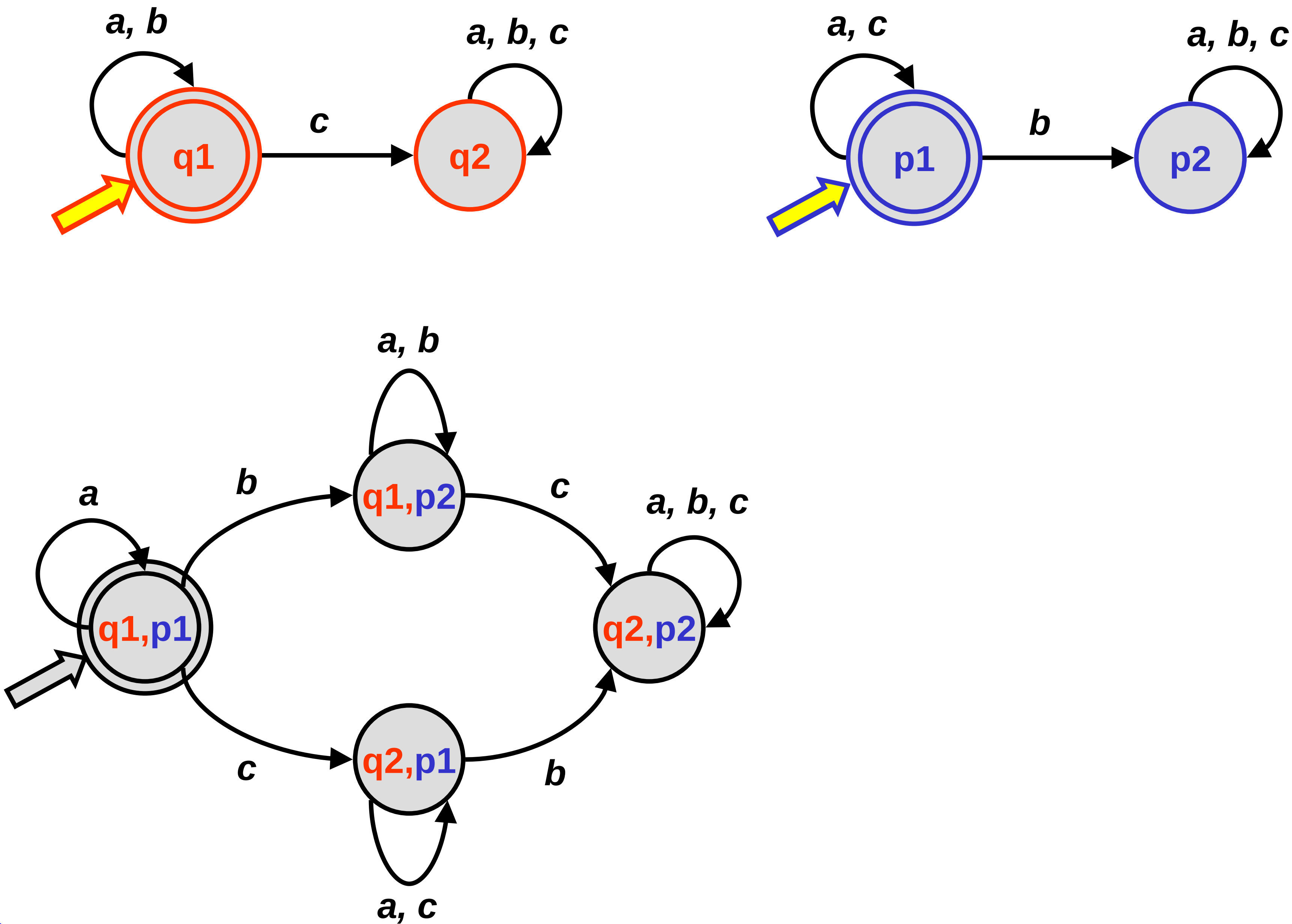

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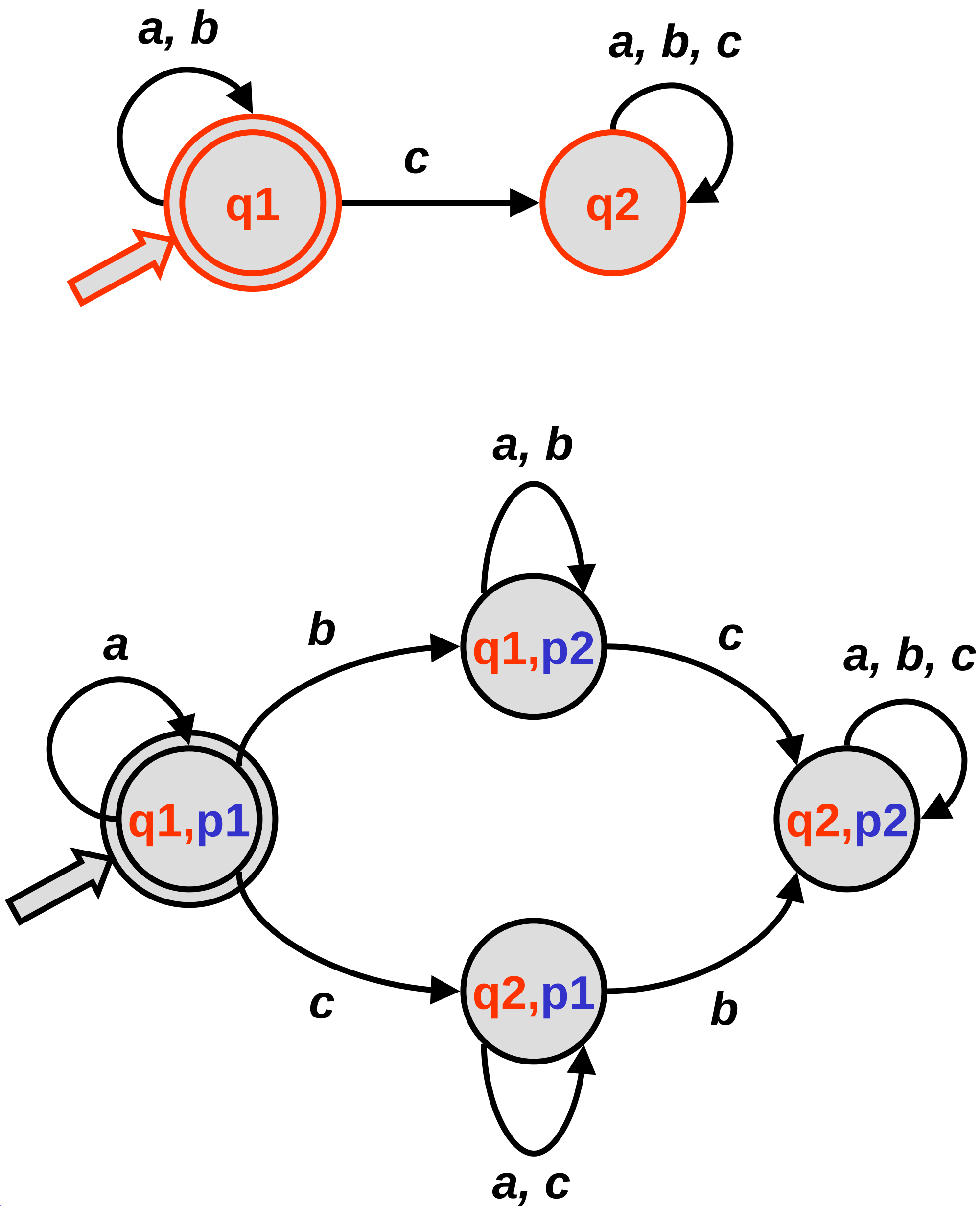

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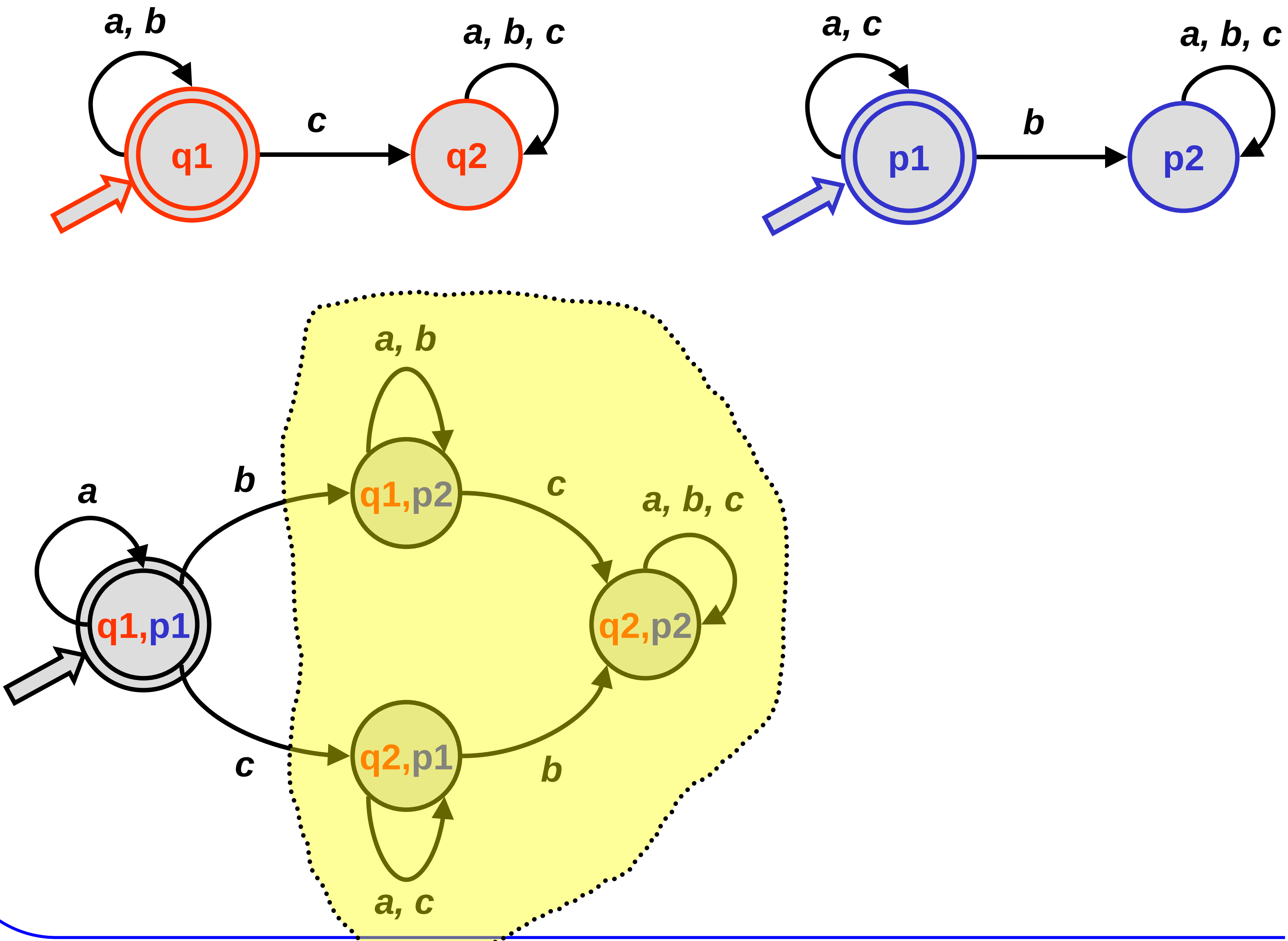

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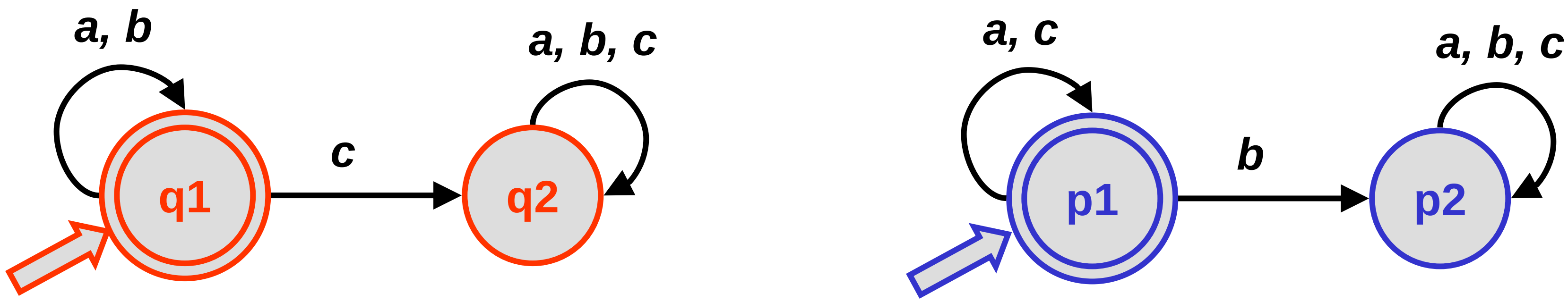
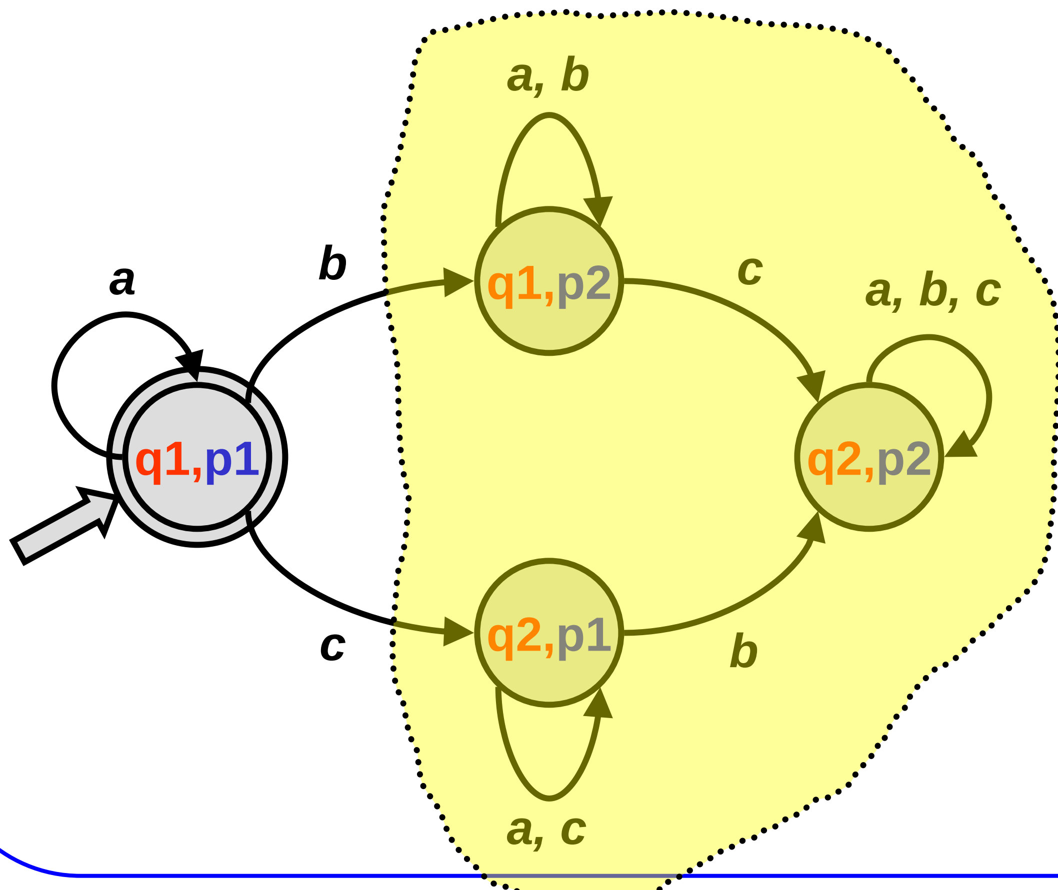
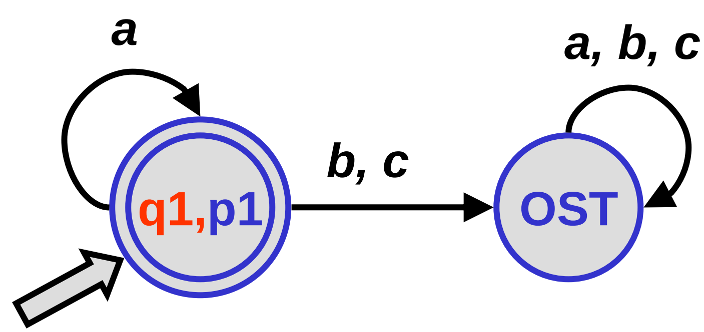




Intersection after minimization



Intersection

Intersection

DFA $M_1=(Q_1, \Sigma, \delta_1, q_1, F_1)$

Intersection

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DFA $M_2=(Q_2, \Sigma, \delta_2, p_1, F_2)$

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DFA $M_2=(Q_2, \Sigma, \delta_2, p_1, F_2)$

$$L(M) = L(M_1) \cap L(M_2)$$

Intersection

DFA $M_1=(Q_1, \Sigma, \delta_1, q_1, F_1)$ DFA $M_2=(Q_2, \Sigma, \delta_2, p_1, F_2)$

$$L(M) = L(M_1) \cap L(M_2)$$

1) $Q = Q_1 \times Q_2$

Intersection

DFA $M_1=(Q_1, \Sigma, \delta_1, q_1, F_1)$ DFA $M_2=(Q_2, \Sigma, \delta_2, p_1, F_2)$

$$L(M) = L(M_1) \cap L(M_2)$$

---

1) $Q = Q_1 \times Q_2$

2) $q_0 = [q_1, p_1]$

Intersection

DFA $M_1=(Q_1, \Sigma, \delta_1, q_1, F_1)$

DFA $M_2=(Q_2, \Sigma, \delta_2, p_1, F_2)$

$$L(M) = L(M_1) \cap L(M_2)$$

1) $Q = Q_1 \times Q_2$

2) $q_0 = [q_1, p_1]$

3) $F = F_1 \times F_2$

Intersection

DFA $M_1=(Q_1, \Sigma, \delta_1, q_1, F_1)$ DFA $M_2=(Q_2, \Sigma, \delta_2, p_1, F_2)$

$$L(M) = L(M_1) \cap L(M_2)$$

1) $Q = Q_1 \times Q_2$

2) $q_0 = [q_1, p_1]$

3) $F = F_1 \times F_2$

4) $\delta([q,p], a) = [\ \delta_1(q, a), \delta_2(p,a)\ ]$

Intersection

Intersection

	a	*b*	*c*	
q1	***q1***	***q1***	***q2***	**1**
q2	***q2***	***q2***	***q2***	**0**

Intersection

	a	b	c	
q1	q1	q1	q2	1
q2	q2	q2	q2	0

	a	b	c	
p1	p1	p2	p1	1
p2	p2	p2	p2	0

Intersection

	a	*b*	*c*	
*q*1	*q*1	*q*1	*q*2	1
*q*2	*q*2	*q*2	*q*2	0

	a	*b*	*c*	
*p*1	*p*1	*p*2	*p*1	1
*p*2	*p*2	*p*2	*p*2	0

[*q*1, *p*1]

[*q*1, *p*2]

[*q*2, *p*1]

[*q*2, *p*2]

Intersection

	a	*b*	*c*	
*q*1	*q*1	*q*1	*q*2	1
*q*2	*q*2	*q*2	*q*2	0

	a	*b*	*c*	
*p*1	*p*1	*p*2	*p*1	1
*p*2	*p*2	*p*2	*p*2	0

	a	*b*	*c*
[*q*1, *p*1]			
[*q*1, *p*2]			
[*q*2, *p*1]			
[*q*2, *p*2]			

Intersection

	a	*b*	*c*	
*q*1	*q*1	*q*1	*q*2	1
*q*2	*q*2	*q*2	*q*2	0

	a	*b*	*c*	
*p*1	*p*1	*p*2	*p*1	1
*p*2	*p*2	*p*2	*p*2	0

	a	*b*	*c*
[*q*1, *p*1]			
[*q*1, *p*2]			
[*q*2, *p*1]			
[*q*2, *p*2]			

Intersection

	a	*b*	*c*	
*q*1	*q*1	*q*1	*q*2	1
*q*2	*q*2	*q*2	*q*2	0

	a	*b*	*c*	
*p*1	*p*1	*p*2	*p*1	1
*p*2	*p*2	*p*2	*p*2	0

	a	*b*	*c*	
[*q*1, *p*1]				1
[*q*1, *p*2]				
[*q*2, *p*1]				
[*q*2, *p*2]				

Intersection

	a	b	c	
*q*1	*q*1	*q*1	*q*2	1
*q*2	*q*2	*q*2	*q*2	0

	a	b	c	
*p*1	*p*1	*p*2	*p*1	1
*p*2	*p*2	*p*2	*p*2	0

	a	b	c	
[*q*1, *p*1]				1
[*q*1, *p*2]				0
[*q*2, *p*1]				0
[*q*2, *p*2]				0

Intersection

	a	*b*	*c*	
*q*1	*q*1	*q*1	*q*2	1
*q*2	*q*2	*q*2	*q*2	0

	a	*b*	*c*	
*p*1	*p*1	*p*2	*p*1	1
*p*2	*p*2	*p*2	*p*2	0

	a	*b*	*c*	
[*q*1, *p*1]				1
[*q*1, *p*2]				0
[*q*2, *p*1]				0
[*q*2, *p*2]				0

Intersection

	a	*b*	*c*	
*q*1	*q*1	*q*1	*q*2	1
*q*2	*q*2	*q*2	*q*2	0

	a	*b*	*c*	
*p*1	*p*1	*p*2	*p*1	1
*p*2	*p*2	*p*2	*p*2	0

	a	*b*	*c*	
[*q*1, *p*1]	[*q*1, *p*1]			1
[*q*1, *p*2]				0
[*q*2, *p*1]				0
[*q*2, *p*2]				0

Intersection

	a	*b*	*c*	
*q*1	*q*1	*q*1	*q*2	1
*q*2	*q*2	*q*2	*q*2	0

	a	*b*	*c*	
*p*1	*p*1	*p*2	*p*1	1
*p*2	*p*2	*p*2	*p*2	0

	a	*b*	*c*	
[*q*1, *p*1]	[*q*1, *p*1]	[*q*1, *p*2]	[*q*2, *p*1]	1
[*q*1, *p*2]	[*q*1, *p*2]	[*q*1, *p*2]	[*q*2, *p*2]	0
[*q*2, *p*1]	[*q*2, *p*1]	[*q*2, *p*2]	[*q*2, *p*1]	0
[*q*2, *p*2]	[*q*2, *p*2]	[*q*2, *p*2]	[*q*2, *p*2]	0

Substitution

Substitution

A regular language given by the expression:

Substitution

A regular language given by the expression:

$r = 0^*(0+1)1^*$

Substitution

A regular language given by the expression:

$r = 0^*(0+1)1^*$

Substitution *R*

Substitution

A regular language given by the expression:

$r = 0^*(0+1)1^*$

Substitution *R*

Character 0

Substitution

A regular language given by the expression:

r = 0*(0+1)1*

Substitution *R*

Character 0

f*(0) = *a

Substitution

A regular language given by the expression:

$r = 0^*(0+1)1^*$

Substitution *R*

Character 0

$f(0) = a$

Character 1

Substitution

A regular language given by the expression:

$r = 0^*(0+1)1^*$

Substitution *R*

Character 0

$f(0) = a$

Character 1

$f(1) = b^*$

Substitution

A regular language given by the expression:

$r = 0^*(0+1)1^*$

Substitution R

Character 0

$f(0) = a$

Character 1

$f(1) = b^*$

We obtain a regular expression $f(R)$

Substitution

A regular language given by the expression:

$r = 0^*(0+1)1^*$

Substitution *R*

Character 0

$f(0) = a$

Character 1

$f(1) = b^*$

We obtain a regular expression *f*(*R*)

$f(\;0\;\;*\;\;(\;\;0\;\;+\;\;1\;\;)\;\;1\;\;\;*\;) =$

Substitution

A regular language given by the expression:

$r = 0^*(0+1)1^*$

Substitution R

Character 0

$f(0) = a$

Character 1

$f(1) = b^*$

We obtain a regular expression $f(R)$

$$f(\ 0\ ^*\ (\ 0\ +\ 1\ )\ 1\ ^*\ ) =$$
$$(f(0))^*\ (\ (f(0)) + (f(1))\ )\ (f(1))^* =$$

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A regular language given by the expression:

$r = 0^*(0+1)1^*$

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Character 0

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$f(1) = b^*$

We obtain a regular expression $f(R)$

$$f(\;0\;{}^*\;(\;0\;+\;1\;)\;1\;{}^*\;) =$$
$$(f(0))^*\;(\;(f(0)) + (f(1))\;)\;(f(1))^* =$$
$$a^*\;(\;a + b^*\;)\;(b^*)^* =$$

Substitution

A regular language given by the expression:

$r = 0^*(0+1)1^*$

Substitution R

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$f(0) = a$

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$$f(\ 0\ ^*\ (\ 0\ +\ 1\ )\ 1\ ^*\ ) =$$
$$(f(0))^*\ (\ (f(0)) + (f(1))\ )\ (f(1))^* =$$
$$a^*\ (\ a + b^*\ )\ (b^*)^* =$$
$$a^*(a+b^*)(b^*)^*$$

Regular definitions

Regular definitions

$d_1 \rightarrow r_1$

Regular definitions

$d_1 \rightarrow r_1$
$d_2 \rightarrow r_2$

Regular definitions

$d_1 \rightarrow r_1$
$d_2 \rightarrow r_2$
$- - -$

Regular definitions

$d_1 \rightarrow r_1$
$d_2 \rightarrow r_2$
$---$
$d_n \rightarrow r_n$

Regular definitions

$d_1 \rightarrow r_1$
$d_2 \rightarrow r_2$
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r_i are regular expresions over alphabet $\Sigma \cup \{d_1, d_2, \ldots, d_{i-1}\}$

Regular definitions

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---

letter $\rightarrow$ **A+B+ ... +Z+a+b+... +z**

Regular definitions

$d_1 \rightarrow r_1$
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$d_n \rightarrow r_n$

r_i are regular expresions over alphabet $\Sigma \cup \{d_1, d_2, \ldots, d_{i-1}\}$

<u>*letter*</u> $\rightarrow$ **A+B+ ... +Z+a+b+... +z**

<u>*digit*</u> $\rightarrow$ **0+1+ ... +9**

Regular definitions

$d_1 \rightarrow r_1$
$d_2 \rightarrow r_2$
- - -
$d_n \rightarrow r_n$

r_i ***are regular expresions over alphabet*** $\Sigma \cup \{d_1, d_2, \ldots , d_{i-1}\}$

<u>*letter*</u> → A+B+ ... +Z+a+b+... +z

<u>*digit*</u> → 0+1+ ... +9

<u>*variable*</u> → <u>*letter*</u> (<u>*letter*</u> + <u>*digit*</u>)*

Regular definitions

$d_1 \to r_1$
$d_2 \to r_2$
- - -
$d_n \to r_n$

r_i ***are regular expresions over alphabet*** $\Sigma \cup \{d_1, d_2, \dots , d_{i\text{-}1}\}$

<u>*letter*</u> $\to$ **A+B+ ... +Z+a+b+... +z**

<u>*digit*</u> $\to$ **0+1+ ... +9**

<u>*variable*</u> $\to$ <u>*letter*</u> **(** <u>*letter*</u> **+** <u>*digit*</u> **)***

<u>*variable*</u> $\to$ **(A+B+ ... +Z+a+b+... +z)((A+B+ ... +Z+a+b+... +z)+(0+1+ ... +9))***

Regular definitions

$d_1 \rightarrow r_1$
$d_2 \rightarrow r_2$
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---

digit $\rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$

Regular definitions

$d_1 \rightarrow r_1$
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$d_n \rightarrow r_n$

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$\underline{digit} \rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9$

$\underline{number} \rightarrow \underline{digit}\ \underline{digit}\ ^*$

Regular definitions

$d_1 \rightarrow r_1$
$d_2 \rightarrow r_2$
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$d_n \rightarrow r_n$

r_i ***are regular expresions over alphabet*** $\Sigma \cup \{d_1, d_2, \ldots, d_{i-1}\}$

---

digit $\rightarrow$ 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
number $\rightarrow$ *digit* *digit* *
decimals $\rightarrow$. *number* | ε

Regular definitions

$d_1 \rightarrow r_1$
$d_2 \rightarrow r_2$
$---$
$d_n \rightarrow r_n$

r_i are regular expresions over alphabet $\Sigma \cup \{d_1, d_2, \ldots, d_{i-1}\}$

---

digit	$\rightarrow$ 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
number	$\rightarrow$ ***digit digit****
decimals	$\rightarrow$. ***number*** | ε
exponent	$\rightarrow$ (E (+ | – | ε) ***number***) | ε

Regular definitions

$d_1 \rightarrow r_1$

$d_2 \rightarrow r_2$

- - -

$d_n \rightarrow r_n$

r_i ***are regular expresions over alphabet*** $\Sigma \cup \{d_1, d_2, \dots , d_{i-1}\}$

---

digit	$\rightarrow$ 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
number	$\rightarrow$ digit digit *
decimals	$\rightarrow$. number | ε
exponent	$\rightarrow$ (E (+ | – | ε) number) | ε
constant	$\rightarrow$ number decimals exponent

Pumping lemma

Pumping lemma

DFA *M* has *n* states

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Input string $a_1 a_2$ --- a_m , $m > n$

Pumping lemma

DFA M has n states

Input string $a_1 a_2$ --- a_m , $m > n$

Sequence of states $q_0 \;\; q_1 \quad \ldots \quad q_m$

Pumping lemma

DFA M has n states

Input string $a_1 a_2$ --- a_m , $m > n$

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$0 \leq j < k \leq m$ *and* $q_j = q_k$

Pumping lemma

DFA M has n states

Input string $a_1 a_2$ --- a_m , $m > n$

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$0 \leq j < k \leq m$ *and* $q_j = q_k$

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -} \; q_{j\text{-}1} \xrightarrow{a_j} q_j = q_k$$

Pumping lemma

DFA M has n states

Input string $a_1 a_2$ --- a_m , $m > n$

Sequence of states $q_0 \ \ q_1 \ \ \ldots \ \ q_m$

$0 \leq j < k \leq m$ *and* $q_j = q_k$

Substring $u = a_1 a_2$ --- a_j

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -} \ q_{j\text{-}1} \xrightarrow{a_j} q_j = q_k$$

Pumping lemma

DFA M has n states

Input string $a_1 a_2 \text{---} a_m$, $m > n$

Sequence of states $q_0 \;\; q_1 \;\; \ldots \;\; q_m$

$0 \leq j < k \leq m$ *and* $q_j = q_k$

Substring $u = a_1 a_2 \text{---} a_j$

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -} \; q_{j-1} \xrightarrow{a_j} q_j = q_k$$

$$a_{j+1} \quad q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -} \; q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \quad a_k$$

Pumping lemma

DFA M has n states

Input string $a_1\, a_2$ --- a_m , $m > n$

Sequence of states $q_0\;\; q_1\;\; \ldots\;\; q_m$

$0 \le j < k \le m$ *and* $q_j = q_k$

Substring $u = a_1\, a_2$ --- a_j

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \cdots\; q_{j-1} \xrightarrow{a_j} q_j = q_k$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \cdots\; q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \xrightarrow{a_k} q_k$$

Substring $v = a_{j+1} a_{j+2}$ --- a_k

Pumping lemma

DFA M has n states

Input string $a_1 a_2 \text{---} a_m$, $m > n$

Sequence of states $q_0 \;\; q_1 \;\; \ldots \;\; q_m$

$0 \le j < k \le m$ ***and*** $q_j = q_k$

Substring $u = a_1 a_2 \text{---} a_j$

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -} \; q_{j-1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \text{- - -} \; q_{m-1} \xrightarrow{a_m} q_m$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -} \; q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \xrightarrow{a_k} q_k$$

Substring $v = a_{j+1} a_{j+2} \text{---} a_k$

Pumping lemma

DFA *M* has *n* states

Input string $a_1 a_2 \text{---} a_m$, $m > n$

Sequence of states $q_0 \; q_1 \; \dots \; q_m$

$0 \leq j < k \leq m$ *and* $q_j = q_k$

Substring u $= a_1 a_2 \text{---} a_j$

Substring w $= a_{k+1} a_{k+2} \text{---} a_m$

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -} \; q_{j\text{-}1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \text{- - -} \; q_{m\text{-}1} \xrightarrow{a_m} q_m$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -} \; q_{k\text{-}2} \xrightarrow{a_{k\text{-}1}} q_{k\text{-}1} \xrightarrow{a_k} q_k$$

Substring v $= a_{j+1} a_{j+2} \text{---} a_k$

Pumping lemma

Substring $u = a_1\, a_2$ --- a_j

Substring $w = a_{k+1}\, a_{k+2}$ --- a_m

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \cdots\ q_{j-1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \cdots\ q_{m-1} \xrightarrow{a_m} q_m$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \cdots\ q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \xrightarrow{a_k} q_k$$

Substring $v = a_{j+1} a_{j+2}$ --- a_k

Pumping lemma

The following input strings are accepted:

Substring u **=** $a_1 a_2 \text{---} a_j$

Substring w **=** $a_{k+1} a_{k+2} \text{---} a_m$

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -}\ q_{j-1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \text{- - -}\ q_{m-1} \xrightarrow{a_m} q_m$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -}\ q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \xrightarrow{a_k} q_k$$

Substring v **=** $a_{j+1} a_{j+2} \text{---} a_k$

Pumping lemma

The following input strings are accepted:

$u\ w$

Substring $u = a_1\, a_2 \text{ --- } a_j$

Substring $w = a_{k+1}\, a_{k+2} \text{ --- } a_m$

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -}\ q_{j-1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \text{- - -}\ q_{m-1} \xrightarrow{a_m} q_m$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -}\ q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \xrightarrow{a_k} q_k$$

Substring $v = a_{j+1} a_{j+2} \text{ --- } a_k$

Pumping lemma

The following input strings are accepted:

u w

Substring u $= a_1\, a_2$ --- a_j

Substring w $= a_{k+1}\, a_{k+2}$ --- a_m

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -}\ q_{j\text{-}1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \text{- - -}\ q_{m\text{-}1} \xrightarrow{a_m} q_m$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -}\ q_{k\text{-}2} \xrightarrow{a_{k\text{-}1}} q_{k\text{-}1} \xrightarrow{a_k} q_k$$

Substring v $= a_{j+1}a_{j+2}$ --- a_k

Pumping lemma

The following input strings are accepted:

u w

u v w

Substring u **=** $a_1 a_2 \text{---} a_j$

Substring w **=** $a_{k+1} a_{k+2} \text{---} a_m$

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -}\ q_{j-1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \text{- - -}\ q_{m-1} \xrightarrow{a_m} q_m$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -}\ q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \xrightarrow{a_k} q_k$$

Substring v **=** $a_{j+1} a_{j+2} \text{---} a_k$

Pumping lemma

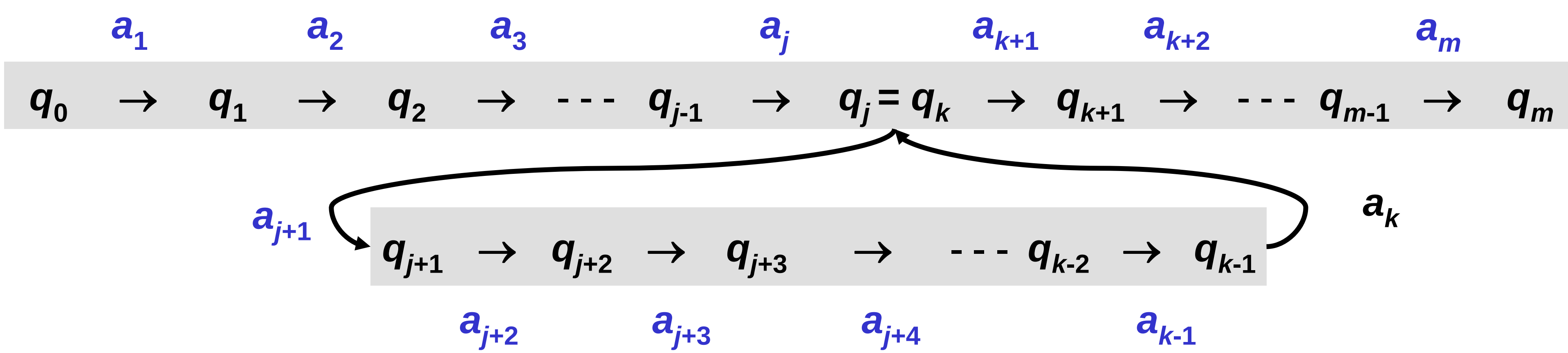
The following input strings are accepted:

$$u\ w$$

$$u\ v\ w$$

Substring $u = a_1\, a_2 \text{ --- } a_j$

Substring $w = a_{k+1}\, a_{k+2} \text{ --- } a_m$



Substring $v = a_{j+1} a_{j+2} \text{ --- } a_k$

Pumping lemma

The following input strings are accepted:

u w

u v w

Substring *u* = $a_1 a_2 \cdots a_j$

Substring *w* = $a_{k+1} a_{k+2} \cdots a_m$

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \cdots\ q_{j-1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \cdots\ q_{m-1} \xrightarrow{a_m} q_m$$

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Substring *v* = $a_{j+1} a_{j+2} \cdots a_k$

Pumping lemma

The following input strings are accepted:

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$u\ v\ w$

Substring $u = a_1\, a_2$ --- a_j

Substring $w = a_{k+1}\, a_{k+2}$ --- a_m

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -}\ q_{j-1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \text{- - -}\ q_{m-1} \xrightarrow{a_m} q_m$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -}\ q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \xrightarrow{a_k} q_k$$

Substring $v = a_{j+1} a_{j+2}$ --- a_k

Pumping lemma

The following input strings are accepted:

$$u\ w$$

$$u\ v\ w$$

$$u\ v^2\ w$$

Substring $u = a_1\, a_2 \text{ --- } a_j$

Substring $w = a_{k+1}\, a_{k+2} \text{ --- } a_m$

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -}\ q_{j-1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \text{- - -}\ q_{m-1} \xrightarrow{a_m} q_m$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -}\ q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \xrightarrow{a_k} q_k$$

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$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -}\ q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \xrightarrow{a_k} q_k$$

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Pumping lemma

The following input strings are accepted:

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Substring $u = a_1\,a_2$ --- a_j

Substring $w = a_{k+1}\,a_{k+2}$ --- a_m

$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -}\; q_{j-1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \text{- - -}\; q_{m-1} \xrightarrow{a_m} q_m$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -}\; q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \xrightarrow{a_k} q_k$$

Substring $v = a_{j+1}a_{j+2}$ --- a_k

Pumping lemma

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$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -}\ q_{j-1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \text{- - -}\ q_{m-1} \xrightarrow{a_m} q_m$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -}\ q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \xrightarrow{a_k} q_k$$

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Substring $u = a_1\, a_2 \text{ --- } a_j$

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$$q_0 \xrightarrow{a_1} q_1 \xrightarrow{a_2} q_2 \xrightarrow{a_3} \text{- - -}\ q_{j-1} \xrightarrow{a_j} q_j = q_k \xrightarrow{a_{k+1}} q_{k+1} \xrightarrow{a_{k+2}} \text{- - -}\ q_{m-1} \xrightarrow{a_m} q_m$$

$$q_j \xrightarrow{a_{j+1}} q_{j+1} \xrightarrow{a_{j+2}} q_{j+2} \xrightarrow{a_{j+3}} q_{j+3} \xrightarrow{a_{j+4}} \text{- - -}\ q_{k-2} \xrightarrow{a_{k-1}} q_{k-1} \xrightarrow{a_k} q_k$$

Substring $v = a_{j+1} a_{j+2} \text{ --- } a_k$

Pumping lemma

The following input strings are accepted:

$$u\ w$$

$$u\ v\ w$$

$$u\ v^2\ w$$

$$u\ v^i\ w$$

Substring $u = a_1 a_2 \cdots a_j$

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$$N_3 = \{\ a^k b^k\ \}$$

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Decision algorithms

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- **Non-emptiness**

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- Non-emptiness
  - **A regular language $L(M)$ is non-empty if and only if DFA M accepts a string z shorter than n, i.e. $|z|<n$.**
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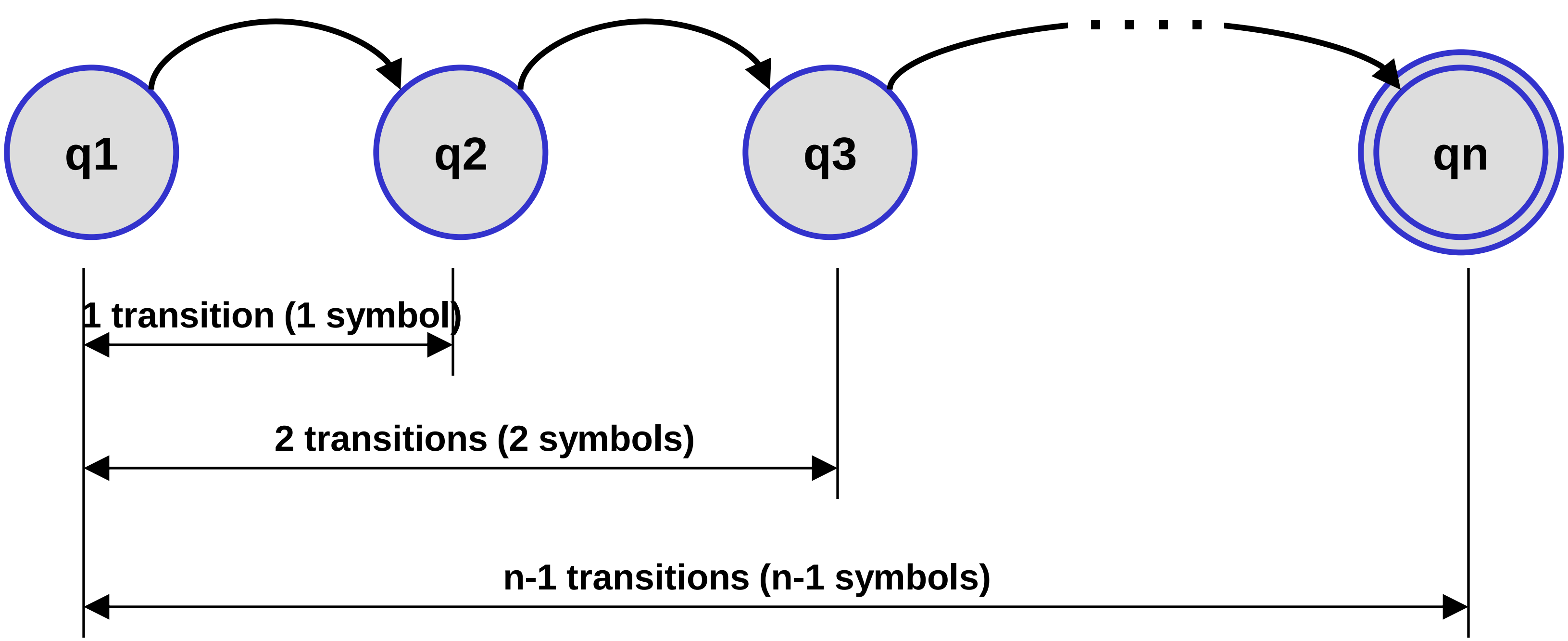

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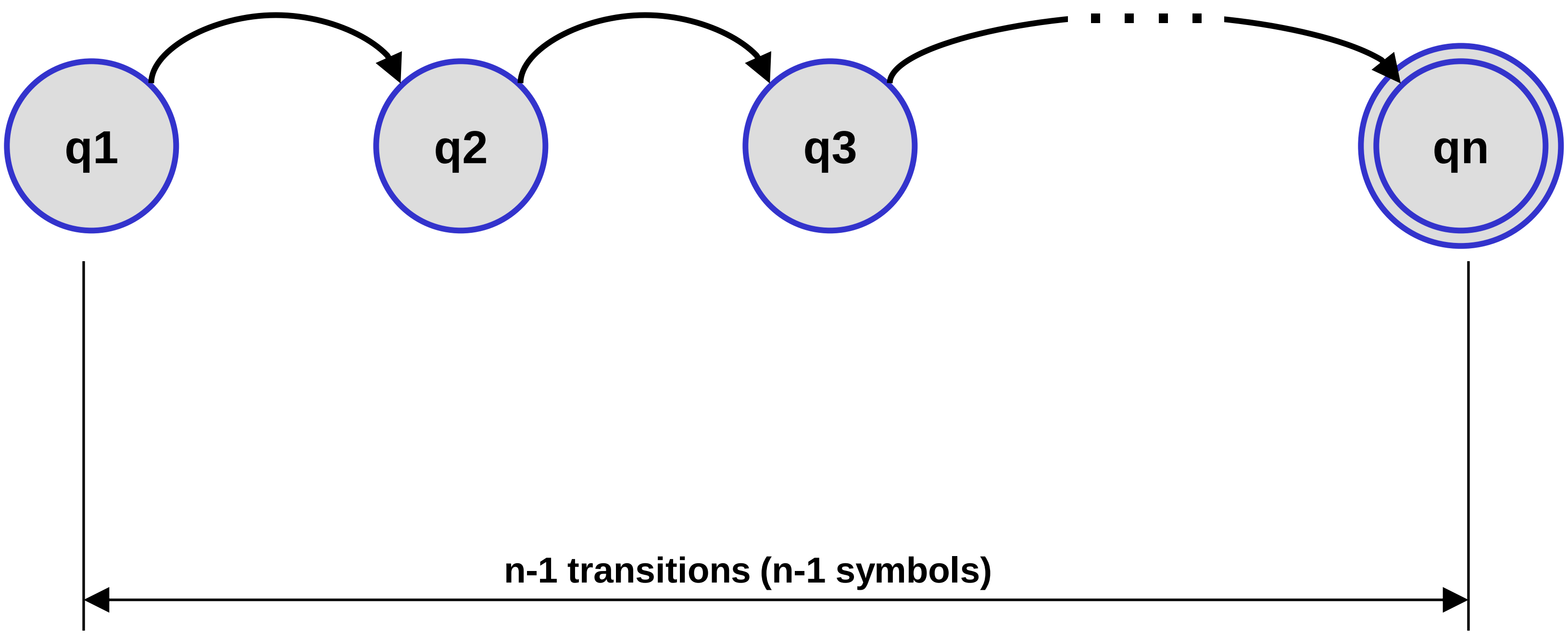

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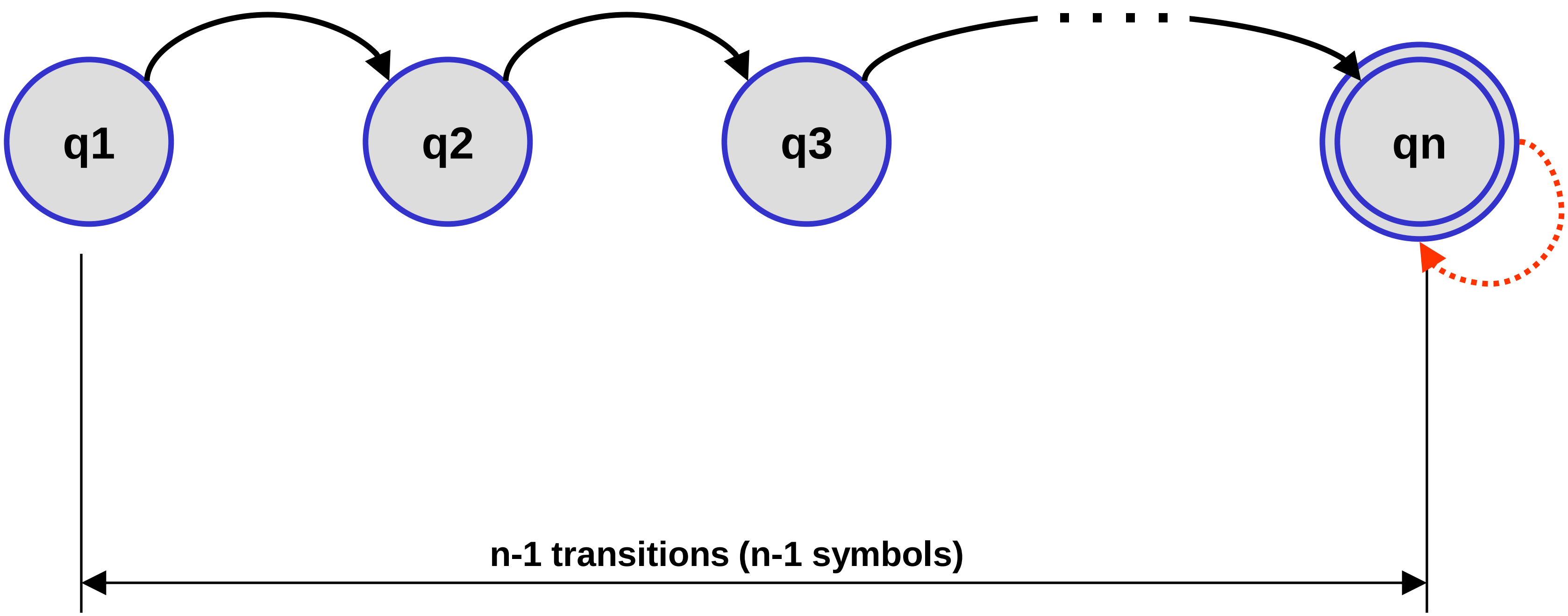

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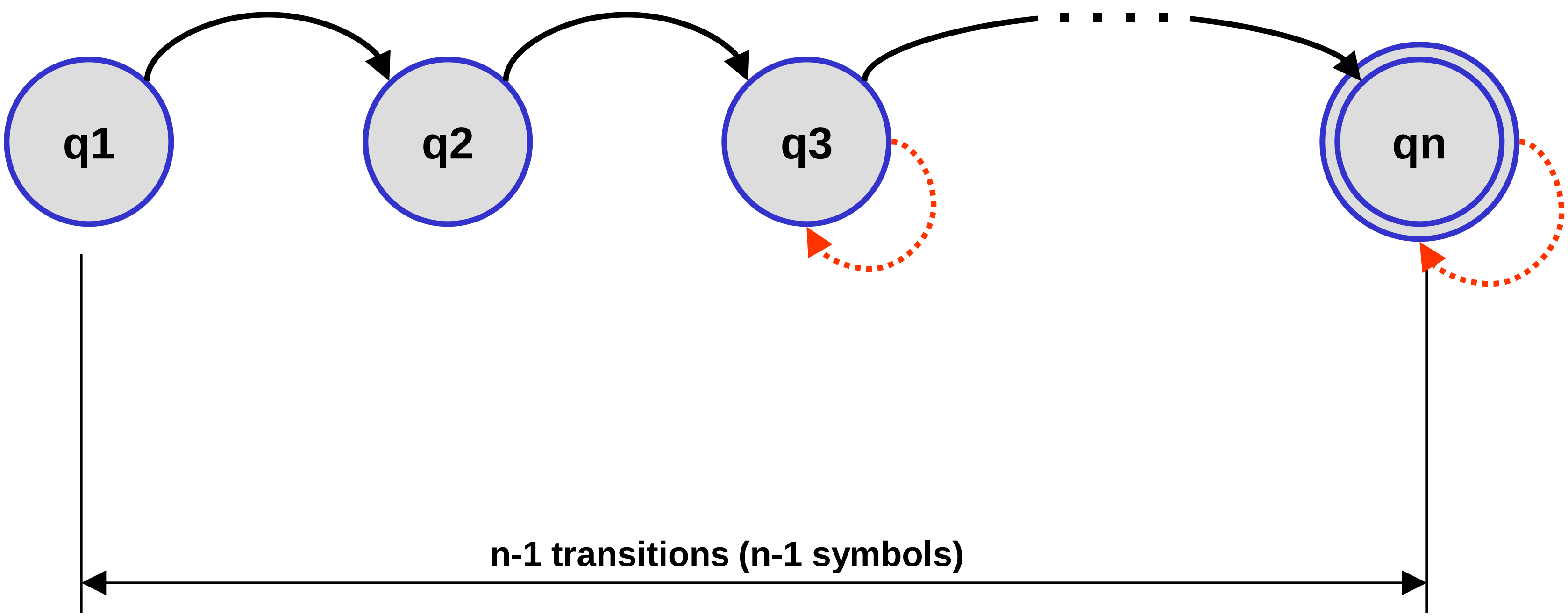

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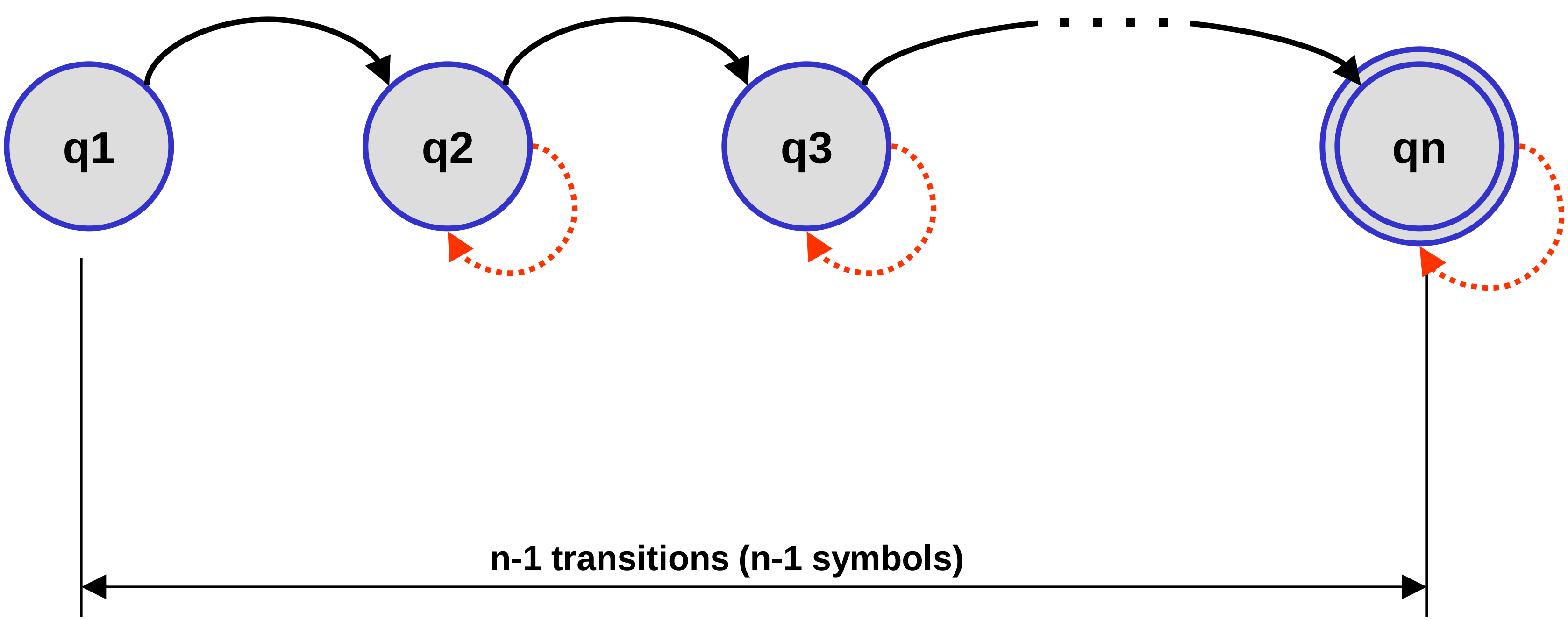

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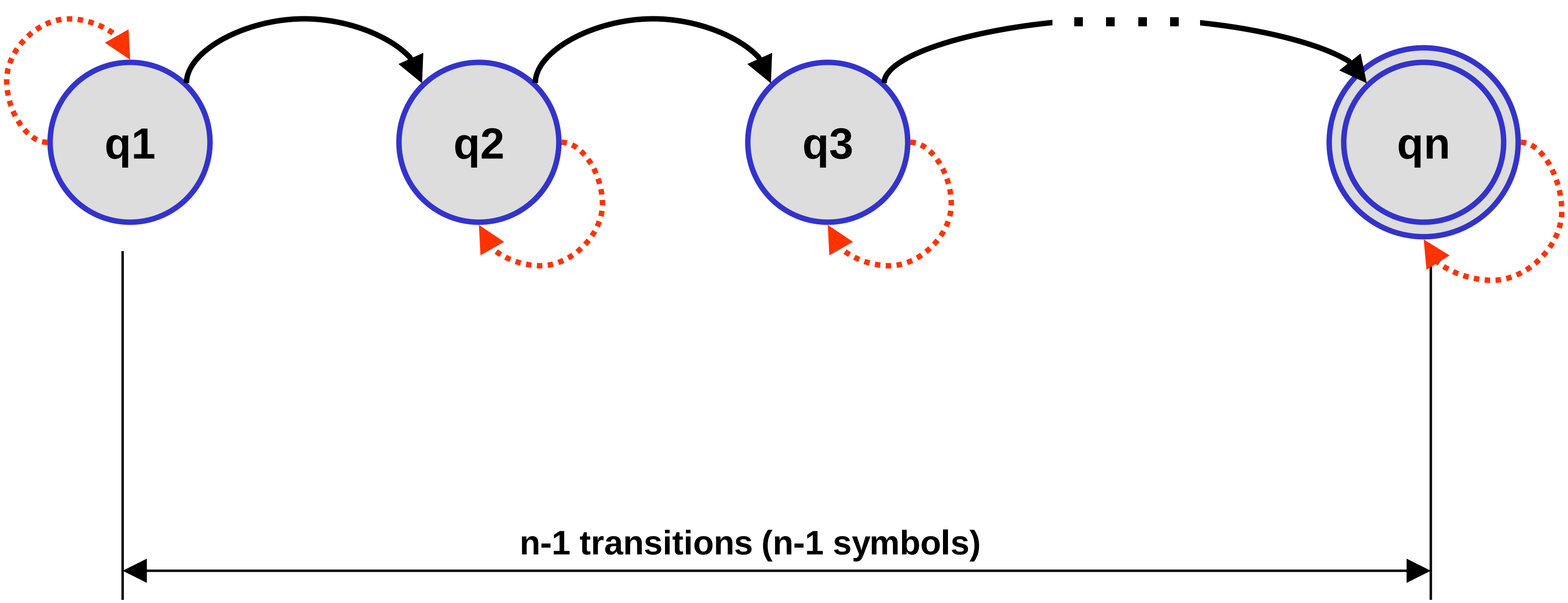
  - **A regular language *L*(*M*) is infinite if and only if DFA *M* accepts a string of length *l*, where $n \le l < 2n$.**
    - We remove all non-accepting states for which there is no sequence of transitions into an accepting state
    - If the obtained state diagram of DFA *M'* contains at least one closed loop, then the regular language *L*(*M*) is infinite.



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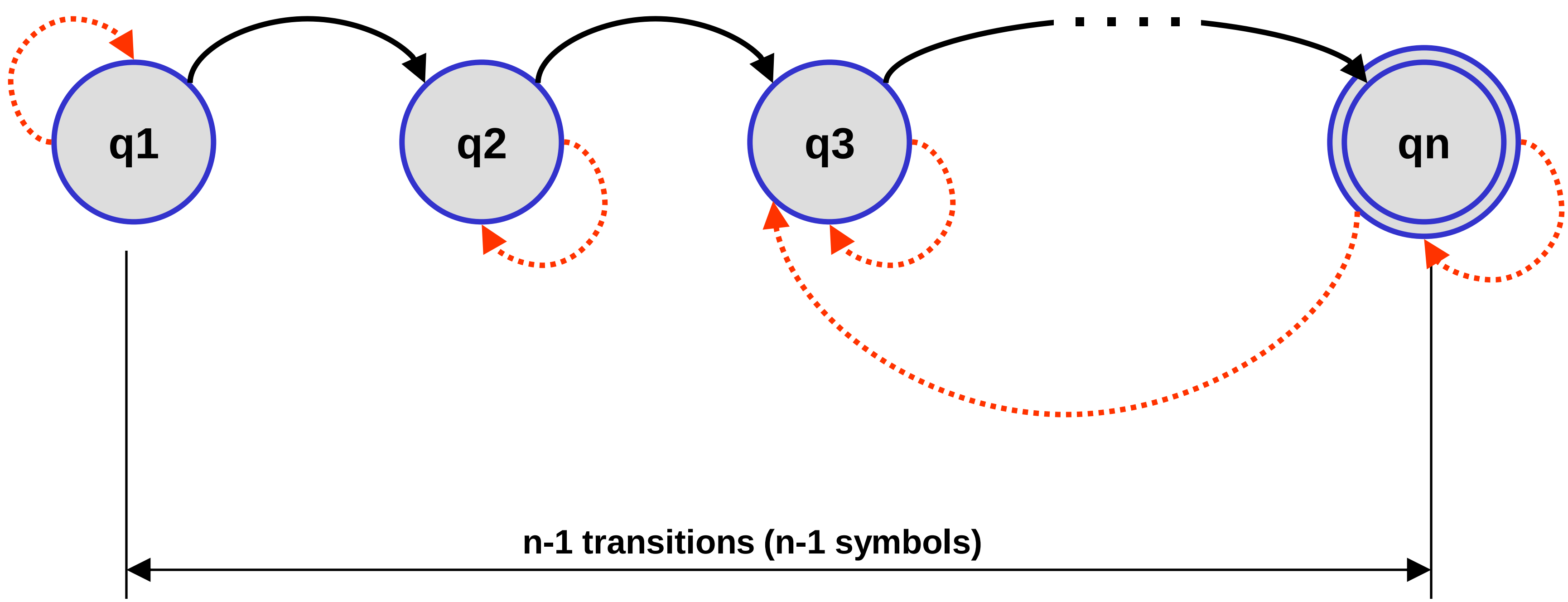

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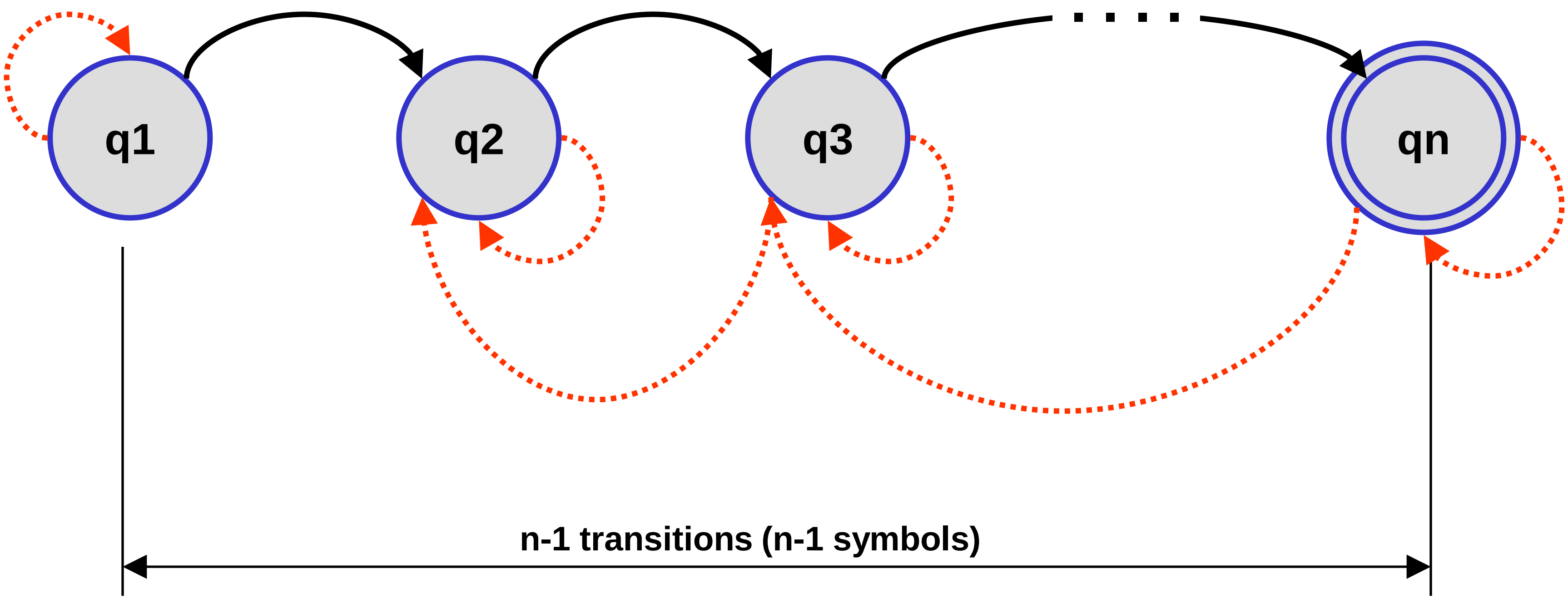

Lecture overview

Grammars

Grammars

Elements of a sentence (variables)

Grammars

Elements of a sentence (variables)

<Sentence> , <SubjectSet>, <Verb>, <ObjectSet>, <Subject>, <Object>, <Attribute>

Grammars

Elements of a sentence (variables)

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Dictionary (terminals)

Grammars

Elements of a sentence (variables)

<Sentence> , <SubjectSet>, <Verb>, <ObjectSet>, <Subject>, <Object>, <Attribute>

Dictionary (terminals)

GIRLS, CATS, WATCH, CONFUSED, SCARED, .

Grammars

Grammars

Rules for building sentences

Grammars

Rules for building sentences

1) ***<Sentence>*** → ***<SubjectSet> <Verb> <ObjectSet>*** **.**

Grammars

Rules for building sentences

1) *<Sentence>* → *<SubjectSet> <Verb> <ObjectSet>* **.**

2) *<SubjectSet>* → *<Attribute> <Subject>*

Grammars

Rules for building sentences

1) *<Sentence>* $\rightarrow$ *<SubjectSet> <Verb> <ObjectSet>* **.**

2) *<SubjectSet>* $\rightarrow$ *<Attribute> <Subject>*

3) *<ObjectSet>* $\rightarrow$ *<Attribute> <Object>*

Grammars

Rules for building sentences

1)	***<Sentence>***	→	***<SubjectSet> <Verb> <ObjectSet>*** **.**
2)	***<SubjectSet>***	→	***<Attribute> <Subject>***
3)	***<ObjectSet>***	→	***<Attribute> <Object>***
4)	***<Verb>***	→	**WATCH**

Grammars

Rules for building sentences

1)	***<Sentence>***	$\rightarrow$ ***<SubjectSet> <Verb> <ObjectSet>*** **.**
2)	***<SubjectSet>***	$\rightarrow$ ***<Attribute> <Subject>***
3)	***<ObjectSet>***	$\rightarrow$ ***<Attribute> <Object>***
4)	***<Verb>***	$\rightarrow$ **WATCH**
5)	***<Subject>***	$\rightarrow$ **GIRLS**

Grammars

Rules for building sentences

1)	***<Sentence>***	$\rightarrow$	***<SubjectSet> <Verb> <ObjectSet>*** **.**
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4)	***<Verb>***	$\rightarrow$	**WATCH**
5)	***<Subject>***	$\rightarrow$	**GIRLS**
6)	***<Subject>***	$\rightarrow$	**CATS**

Grammars

Rules for building sentences

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5)	***<Subject>***	→ **GIRLS**
6)	***<Subject>***	→ **CATS**
7)	***<Attribute>***	→ **CONFUSED**

Grammars

Rules for building sentences

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Grammars

Rules for building sentences

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Grammars

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Grammars

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Grammars

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Grammars

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<Sentence>

⇒ *<SubjectSet>* *<Verb>* *<ObjectSet>* .

Grammars

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<Sentence>

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Grammars

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⇒ *<SubjectSet>* *<Verb>* *<ObjectSet>* **.**

⇒ *<Attribute>* *<Subject>*

Grammars

1) *<Sentence>* → *<SubjectSet> <Verb> <ObjectSet>* .
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<Sentence>

⇒ *<SubjectSet> <Verb> <ObjectSet>* .

⇒ *<Attribute> <Subject> <Verb> <ObjectSet>* .

Grammars

1) <Sentence> → <SubjectSet> <Verb> <ObjectSet> .
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<Sentence>

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⇒ <Attribute> <Subject> <Verb> <ObjectSet> .

Grammars

1) *<Sentence>* → *<SubjectSet> <Verb> <ObjectSet>* .
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<Sentence>

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⇒ *<Attribute>* *<Subject>* *<Verb>* *<ObjectSet>* .

⇒ *<Attribute>* *<Object>*.

Grammars

1) *<Sentence>* → *<SubjectSet> <Verb> <ObjectSet>* .
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<Sentence>

⇒ ***<SubjectSet> <Verb> <ObjectSet>*** **.**

⇒ ***<Attribute> <Subject> <Verb> <ObjectSet>*** **.**

⇒ ***<Attribute> <Subject> <Verb> <Attribute> <Object>*****.**

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3) *<ObjectSet>* → *<Attribute> <Object>*
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<Sentence>

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⇒ *<Attribute> <Subject> <Verb> <ObjectSet>* .

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Grammars

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<Sentence>

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⇒ *<Attribute> <Subject>* WATCH *<Attribute> <Object>*.
⇒ *<Attribute>* GIRLS WATCH *<Attribute> <Object>*.

Grammars

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⇒ *<Attribute>* **GIRLS WATCH** *<Attribute>* *<Object>***.**
⇒ *<Attribute>* **GIRLS WATCH** *<Attribute>* **CATS.**
⇒ **CONFUSED GIRLS WATCH** *<Attribute>* **CATS.**
⇒ **CONFUSED GIRLS WATCH SCARED CATS.**

Grammars

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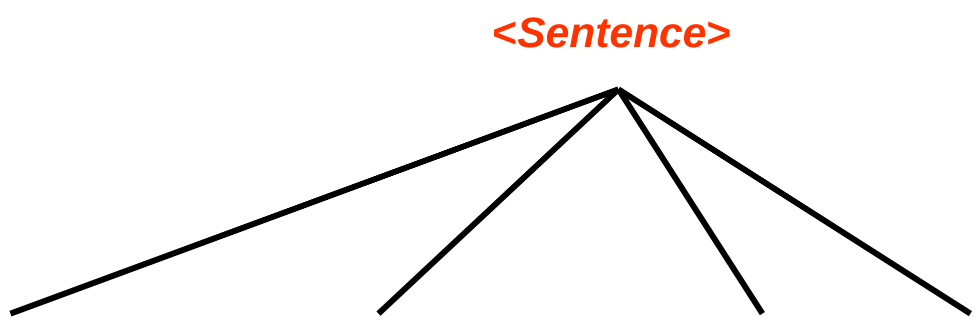

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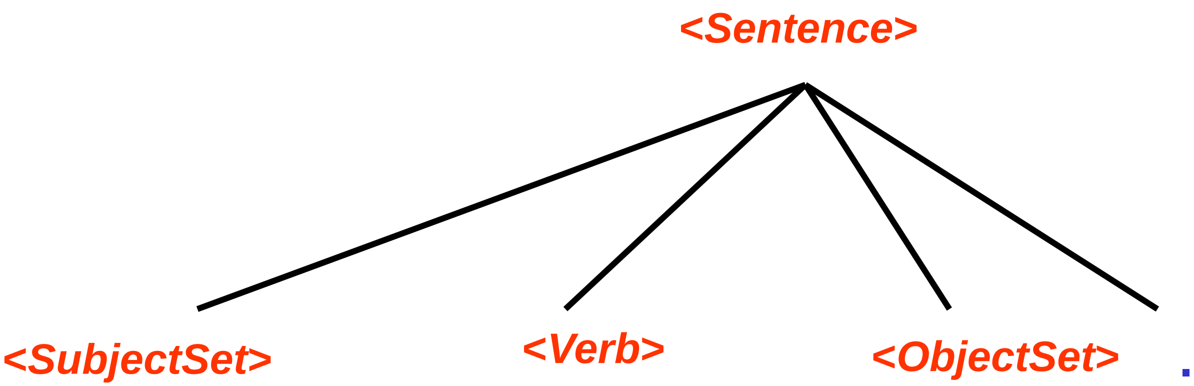

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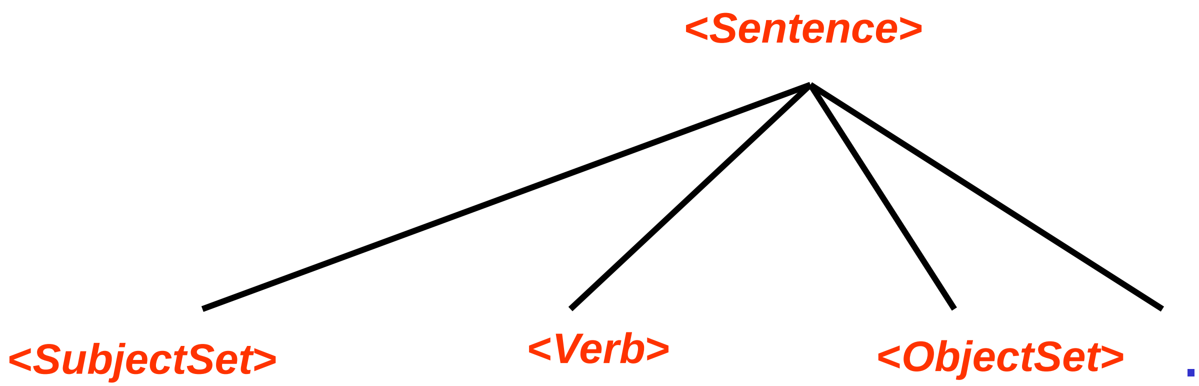

Grammars

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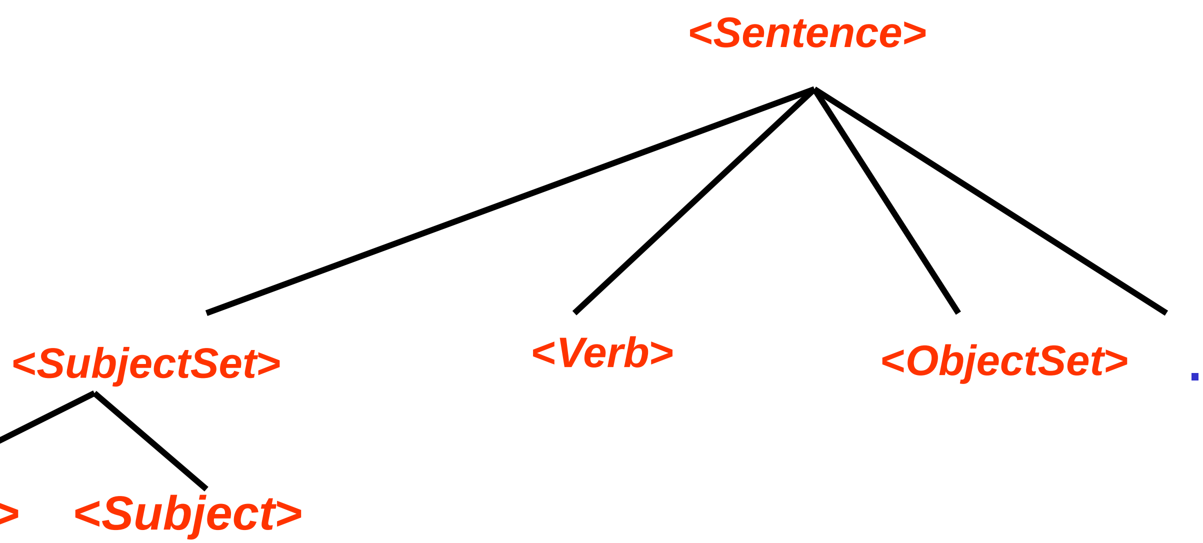

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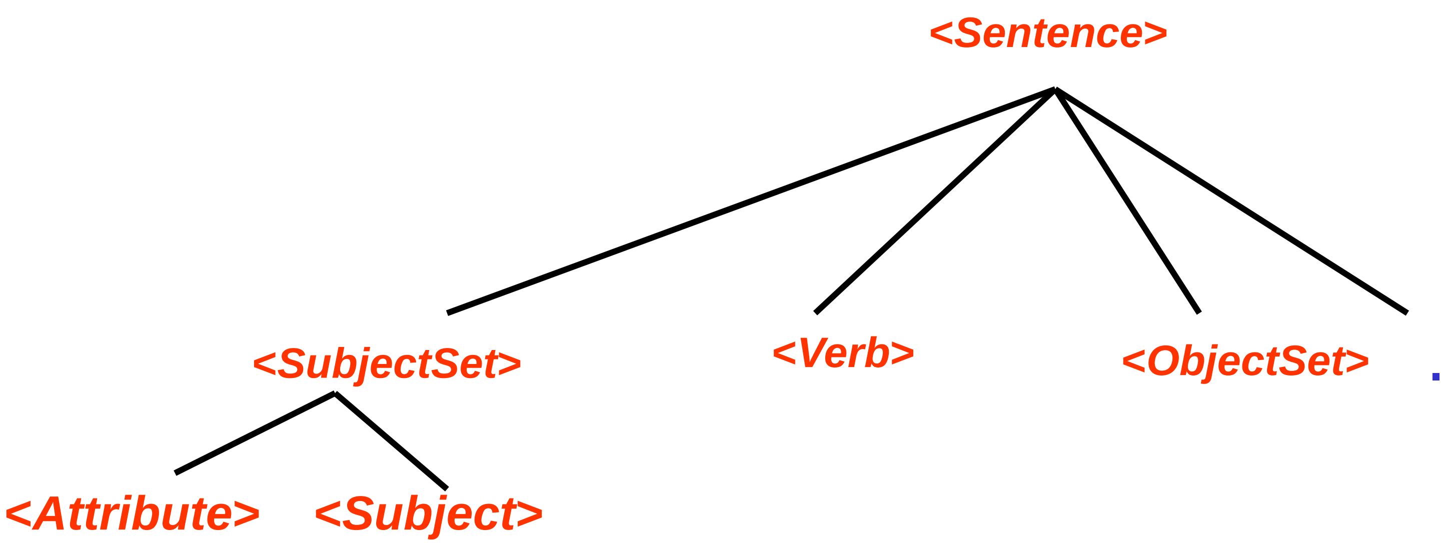

Grammars

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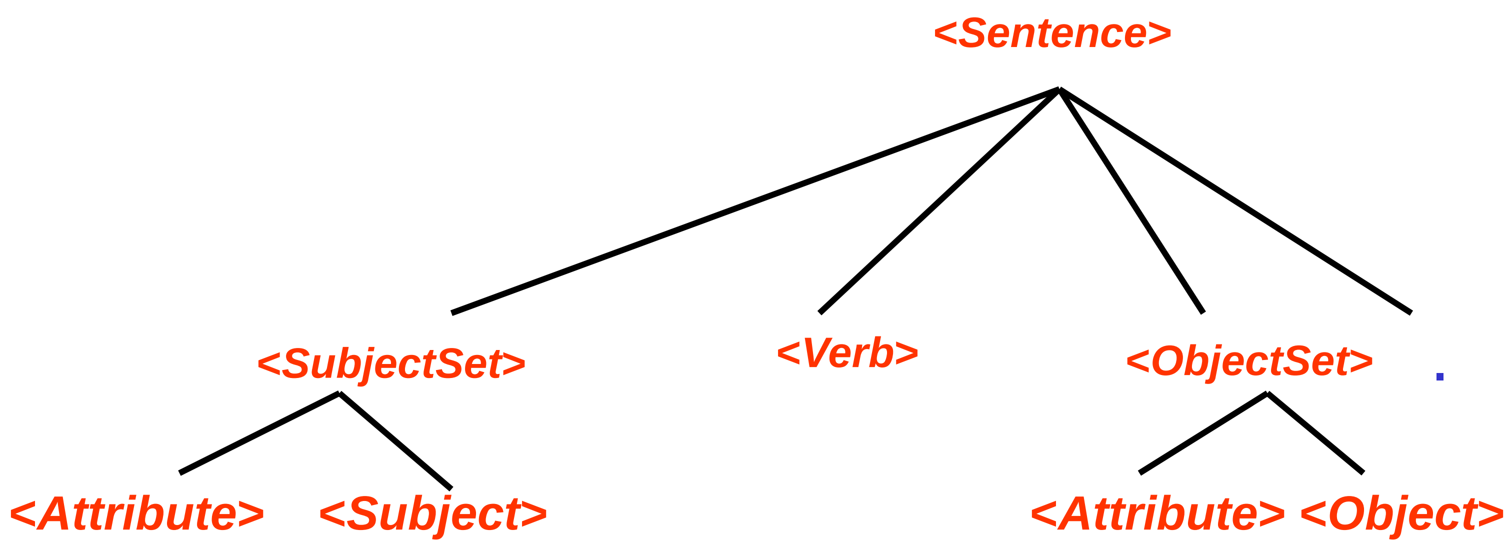

Grammars

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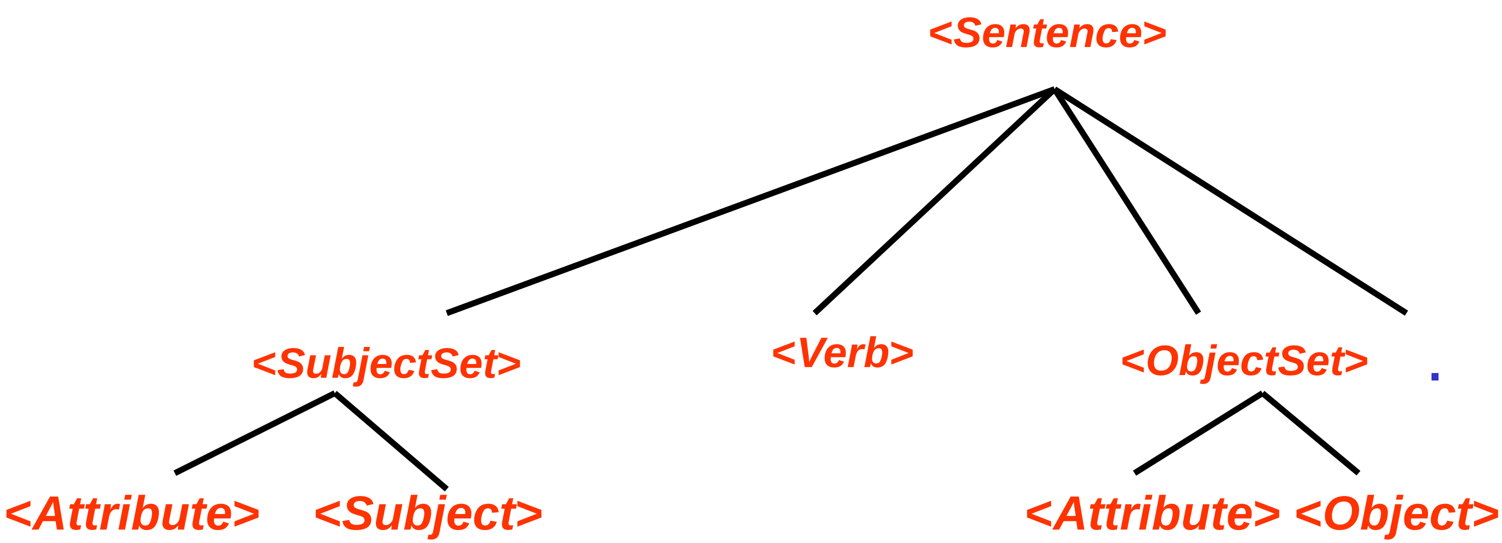

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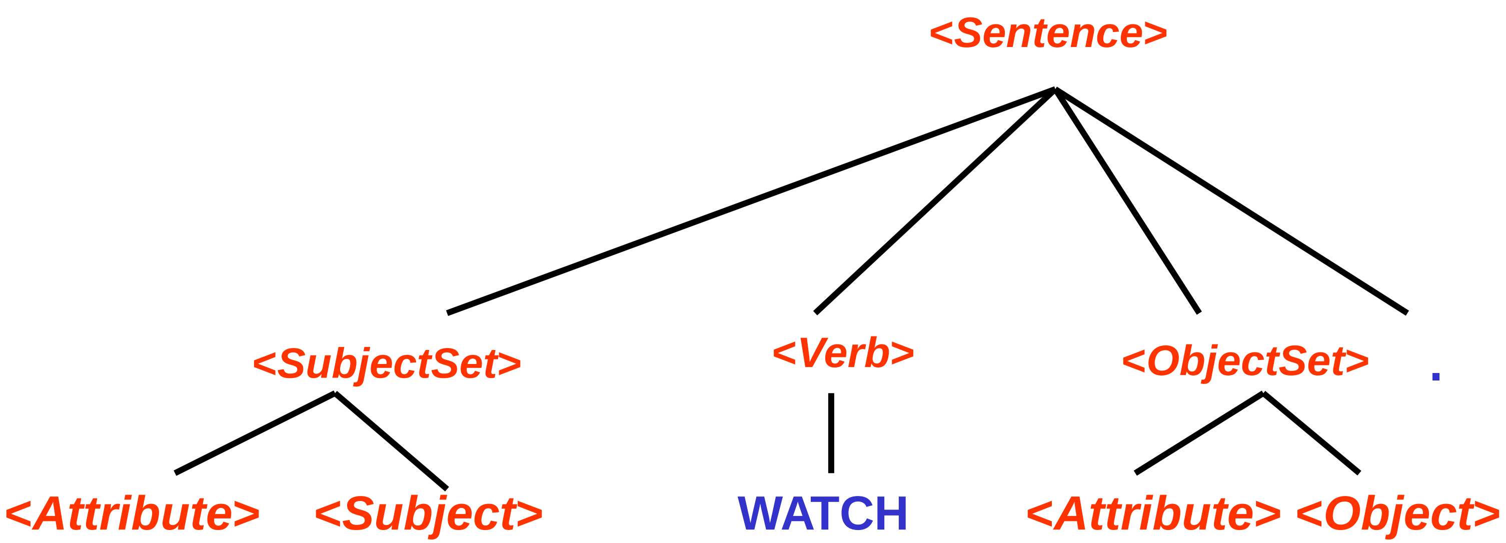
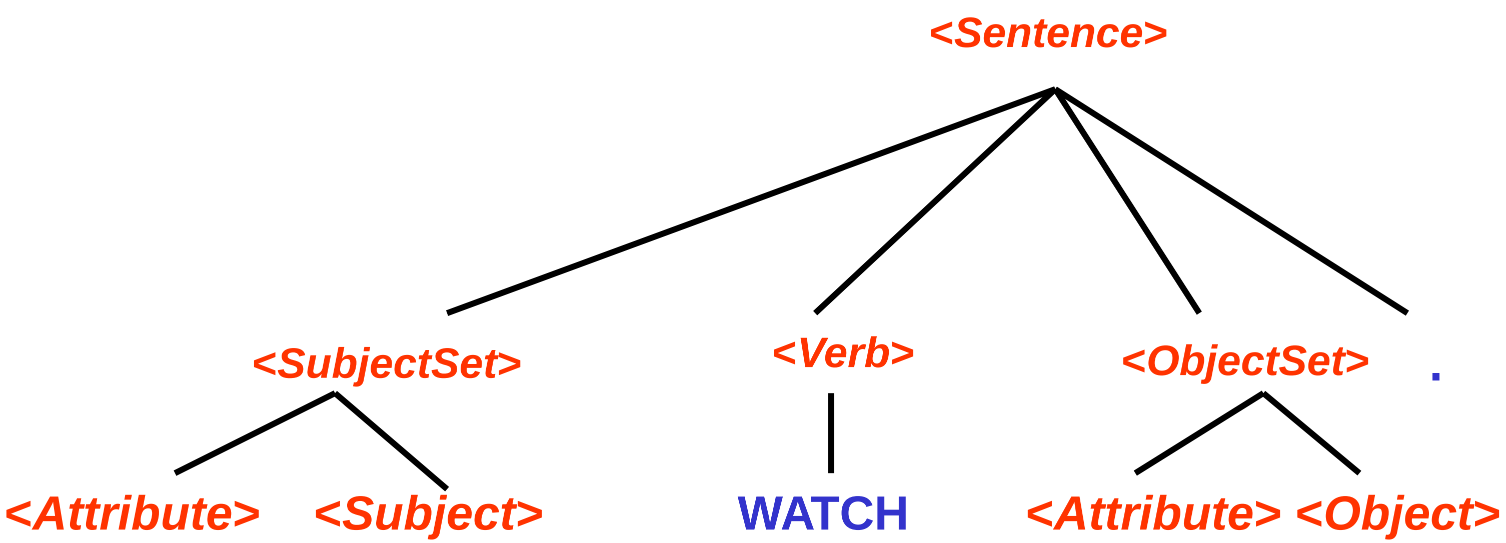
10) <Object> → CATS



Grammars

1) <Sentence> → <SubjectSet> <Verb> <ObjectSet> .
2) <SubjectSet> → <Attribute> <Subject>
3) <ObjectSet> → <Attribute> <Object>
4) <Verb> → WATCH
5) <Subject> → GIRLS
6) <Subject> → CATS
7) <Attribute> → CONFUSED
8) <Attribute> → SCARED
9) <Object> → GIRLS
10) <Object> → CATS



Grammars

1) *<Sentence>* → *<SubjectSet> <Verb> <ObjectSet>* .
2) *<SubjectSet>* → *<Attribute> <Subject>*
3) *<ObjectSet>* → *<Attribute> <Object>*
4) *<Verb>* → WATCH
5) *<Subject>* → GIRLS
6) *<Subject>* → CATS
7) *<Attribute>* → CONFUSED
8) *<Attribute>* → SCARED
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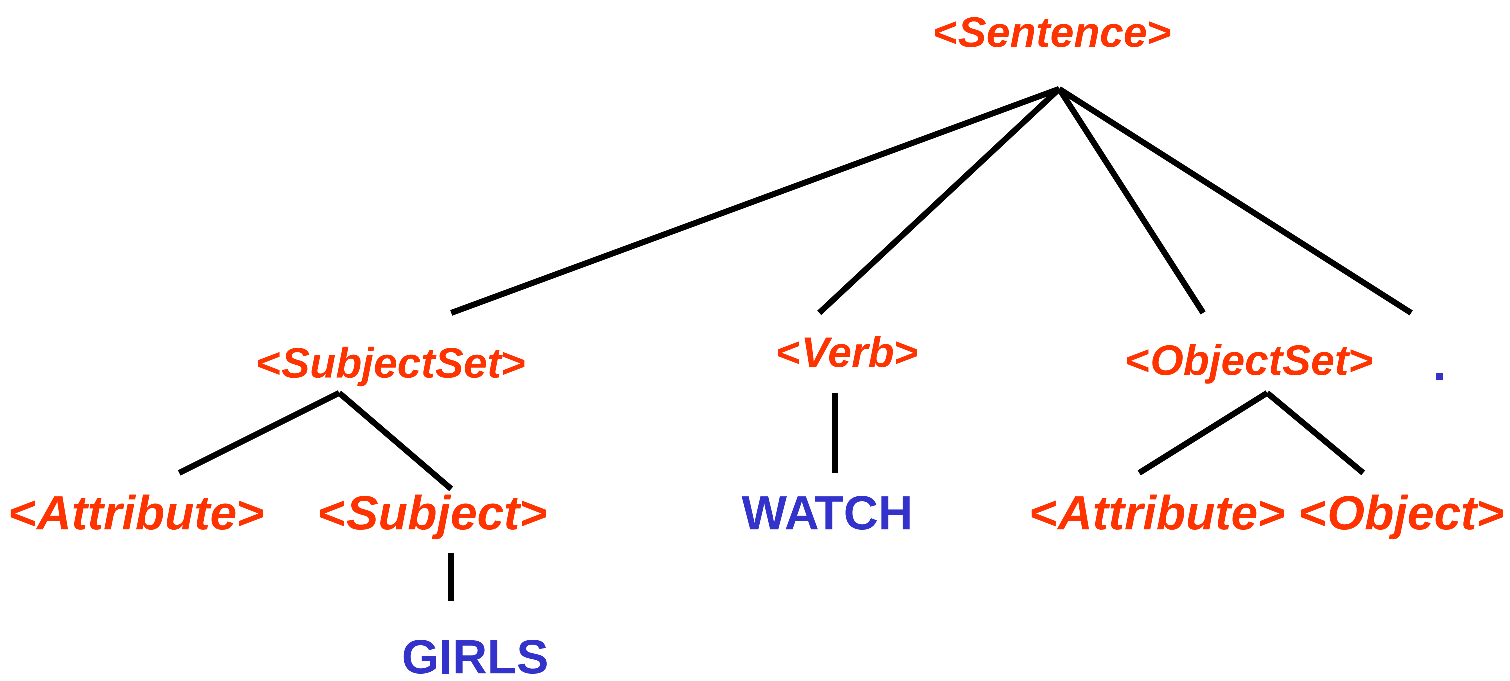
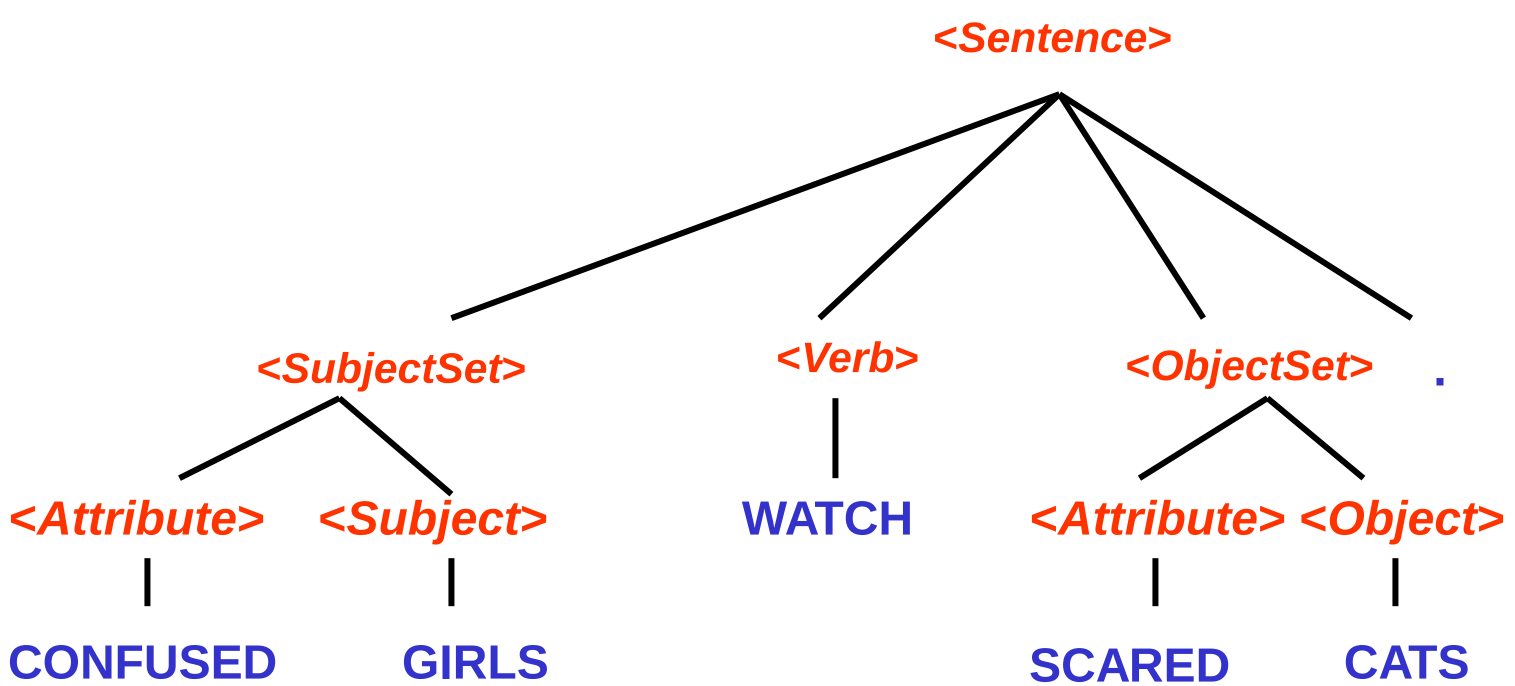
10) *<Object>* → CATS



Grammars

1) *<Sentence>* → *<SubjectSet> <Verb> <ObjectSet>* **.**
2) *<SubjectSet>* → *<Attribute> <Subject>*
3) *<ObjectSet>* → *<Attribute> <Object>*
4) *<Verb>* → **WATCH**
5) *<Subject>* → **GIRLS**
6) *<Subject>* → **CATS**
7) *<Attribute>* → **CONFUSED**
8) *<Attribute>* → **SCARED**
9) *<Object>* → **GIRLS**
10) *<Object>* → **CATS**



Grammars

Grammars

$$G = (V, T, P, S)$$

Grammars

$$G = (V, T, P, S)$$

V - finite set of variables

Grammars

$$G = (V, T, P, S)$$

V - finite set of variables

T - finite set of terminals

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$$V \cap T = \varnothing$$

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$$V \cap T = \varnothing$$

P - finite set of productions of the form

Grammars

$$G = (V, T, P, S)$$

V	- finite set of variables
T	- finite set of terminals
	$V \cap T = \emptyset$
P	- finite set of productions of the form
	$A \to \alpha$

Grammars

$$G = (V, T, P, S)$$

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A is a variable

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α is a string in $(V \cup T)^*$ or ε

Grammars

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$$A \to \alpha$$

A is a variable

α is a string in $(V \cup T)^*$ or ε

S - start symbol (a variable)

Grammars

Grammars

$G = (\{ E \},$

Grammars

G = ({ E },
{ a, $*$, $+$, (,) },

Grammars

$$G = (\ \{\ E\ \},$$
$$\{\ a,\ *,\ +,\ (,\ )\ \},$$
$$\{\ E \to E{+}E \mid E{*}E \mid (E) \mid a\ \},$$

Grammars

$$G = (\{ E \},$$
$$\{ a, *, +, (,) \},$$
$$\{ E \rightarrow E{+}E \mid E{*}E \mid (E) \mid a \},$$
$$E)$$

Grammars

$G = (\{ E \},$
$\{ a, *, +, (,) \},$
$\{ E \rightarrow E+E \mid E*E \mid (E) \mid a \},$
$E)$

$E \Rightarrow a$

Grammars

$$G = (\ \{ E \},$$
$$\{ a, *, +, (,) \},$$
$$\{ E \rightarrow E+E \mid E*E \mid (E) \mid a \},$$
$$E\ )$$

$$E \Rightarrow a$$

$$E \Rightarrow E + E \Rightarrow E + a \Rightarrow a + a$$

Grammars

$$G = (\{ E \}, \\ \{ a, *, +, (,) \}, \\ \{ E \to E+E \mid E*E \mid (E) \mid a \}, \\ E)$$

$$E \Rightarrow a$$

$$E \Rightarrow E + E \Rightarrow E + a \Rightarrow a + a$$

$$E \Rightarrow E * E \Rightarrow (E) * E \Rightarrow (E + E) * E \overset{*}{\Rightarrow} (a + a) * a$$

Grammars

Grammars

$G = (V, T, P, S)$

Grammars

$G = (V, T, P, S)$

$A \to \beta$

Grammars

$G = (V, T, P, S)$

$A \rightarrow \beta$

$\alpha\ A\ \gamma$

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Grammars

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$A \to \beta$

$\alpha A \gamma$

Grammars

$G = (V, T, P, S)$

$A \rightarrow \beta$

$$\alpha A \gamma \quad \Rightarrow$$

Grammars

$G = (V, T, P, S)$

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Grammars

$G = (V, T, P, S)$

$A \rightarrow \beta$

$$\alpha A \gamma \underset{G}{\Rightarrow}$$

Grammars

$G = (V, T, P, S)$

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Grammars

$G = (V, T, P, S)$

$A \to \beta$

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Grammars

$G = (V, T, P, S)$

$A \to \beta$

$$\alpha\, A\, \gamma \underset{G}{\Rightarrow} \alpha\, \beta\, \gamma$$

$$\alpha_1 \Rightarrow \alpha_2 \Rightarrow \alpha_3 \Rightarrow \alpha_{m-1} \Rightarrow \alpha_m$$

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Grammars

Grammars

Context-free languages CFL

Grammars

Context-free languages CFL

Regular languages

$RL \subset CFL$

Grammars

$$N = \{\, wcw^R \,\}$$

Context-free languages CFL

Regular languages

RL ⊂ CFL

Grammars

$N = \{ \; wcw^R \; \}$

Context-free languages CFL

N

Regular languages

$RL \subset CFL$

Grammars

$$N = \{\, wcw^R \,\}$$

Context-free languages CFL

N

Regular languages

$RL \subset CFL$

$$G = (V, T, P, S)$$

Grammars

$$N = \{ wcw^R \}$$

Context-free languages CFL

N

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$RL \subset CFL$

$$G = (V, T, P, S)$$

$$V = \{ S \}$$

Grammars

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Context-free languages CFL

N

Regular languages

$RL \subset CFL$

$G = (V, T, P, S)$

$V = \{\, S \,\}$

$T = \{\, a, b, c \,\}$

Grammars

$$N = \{\, wcw^R \,\}$$

$$G = (V, T, P, S)$$

$$V = \{\, S \,\}$$

$$T = \{\, a, b, c \,\}$$

$$P = \{\, S \to aSa \mid bSb \mid c \,\}$$

Grammars

$N = \{\, wcw^R \,\}$

Context-free languages CFL

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S

Grammars

$N = \{ wcw^R \}$

Context-free languages CFL

N

Regular languages

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$G = (V, T, P, S)$

$V = \{ S \}$

$T = \{ a, b, c \}$

$P = \{ S \to aSa \mid bSb \mid c \}$

$S \Rightarrow$

Grammars

Context-free languages CFL

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$$S \Rightarrow a \quad S \quad a$$

Grammars

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Context-free languages CFL

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$$S \Rightarrow aa \; S \; aa$$

Grammars

$$N = \{\ wcw^R\ \}$$

$$G = (V, T, P, S)$$

$$V = \{\ S\ \}$$

$$T = \{\ a, b, c\ \}$$

$$P = \{\ S \rightarrow aSa \mid bSb \mid c\ \}$$

$$S \Rightarrow\ aab\ S\ baa$$

Grammars

Context-free languages CFL

N

Regular languages

RL $\subset$ *CFL*

$$N = \{\ wcw^R\ \}$$

$$G = (V, T, P, S)$$

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$$S \Rightarrow aabbSbbaa$$

Grammars

$N = \{\ wcw^R\ \}$

Context-free languages CFL

N

Regular languages

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$S \Rightarrow\ aabbSbbaa$

bbaa

Grammars

Context-free languages CFL

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$$S \Rightarrow aabbSbbaa$$

$$aabb$$

Grammars

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Context-free languages CFL

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$S \Rightarrow \quad w\,S\,w^R$

Grammars

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Context-free languages CFL

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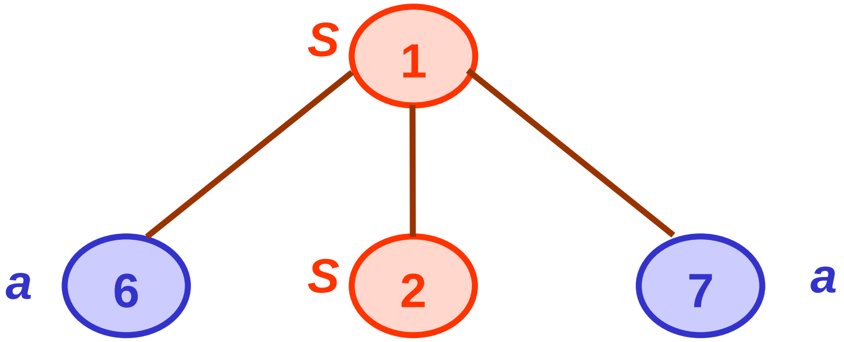

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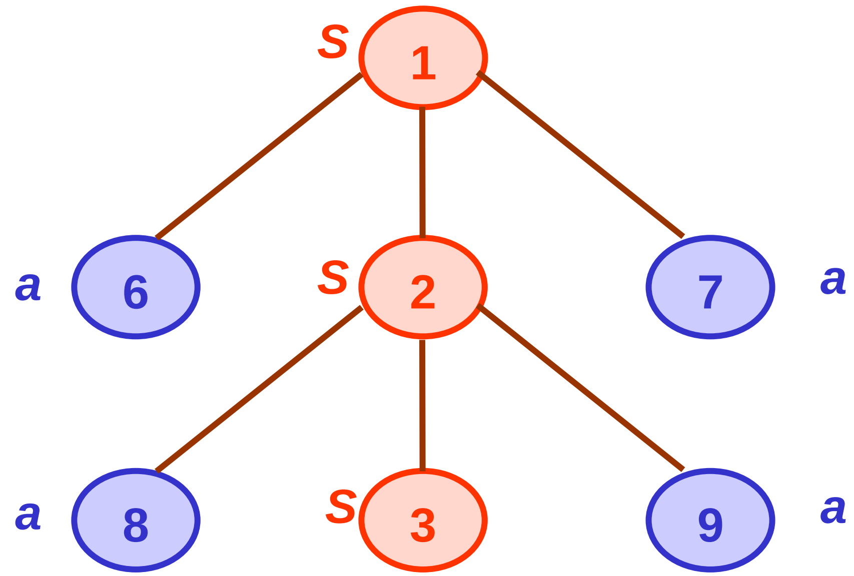

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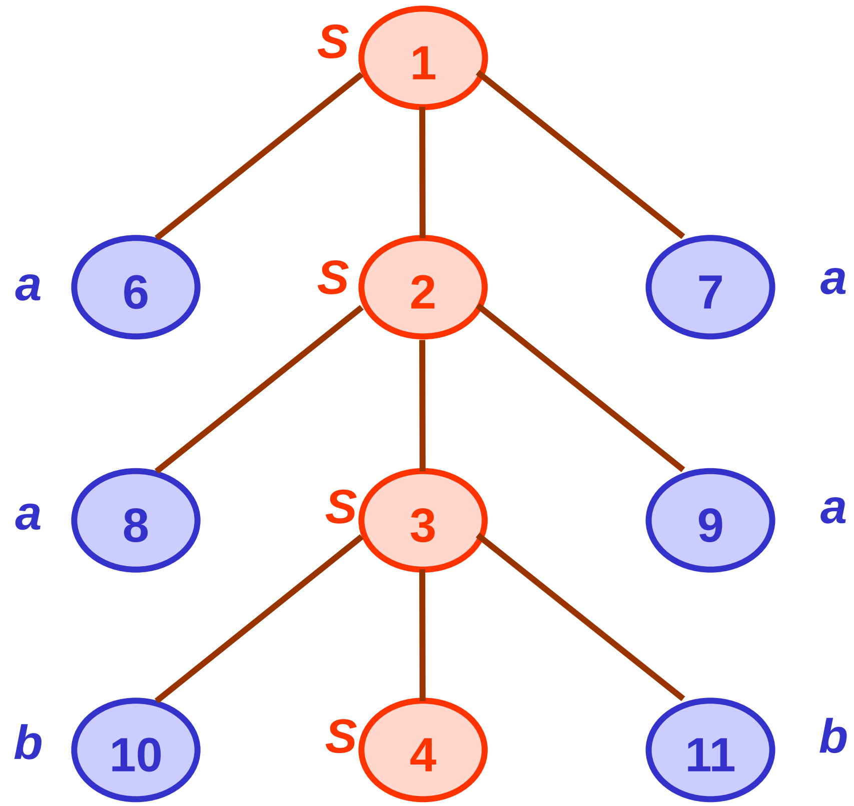

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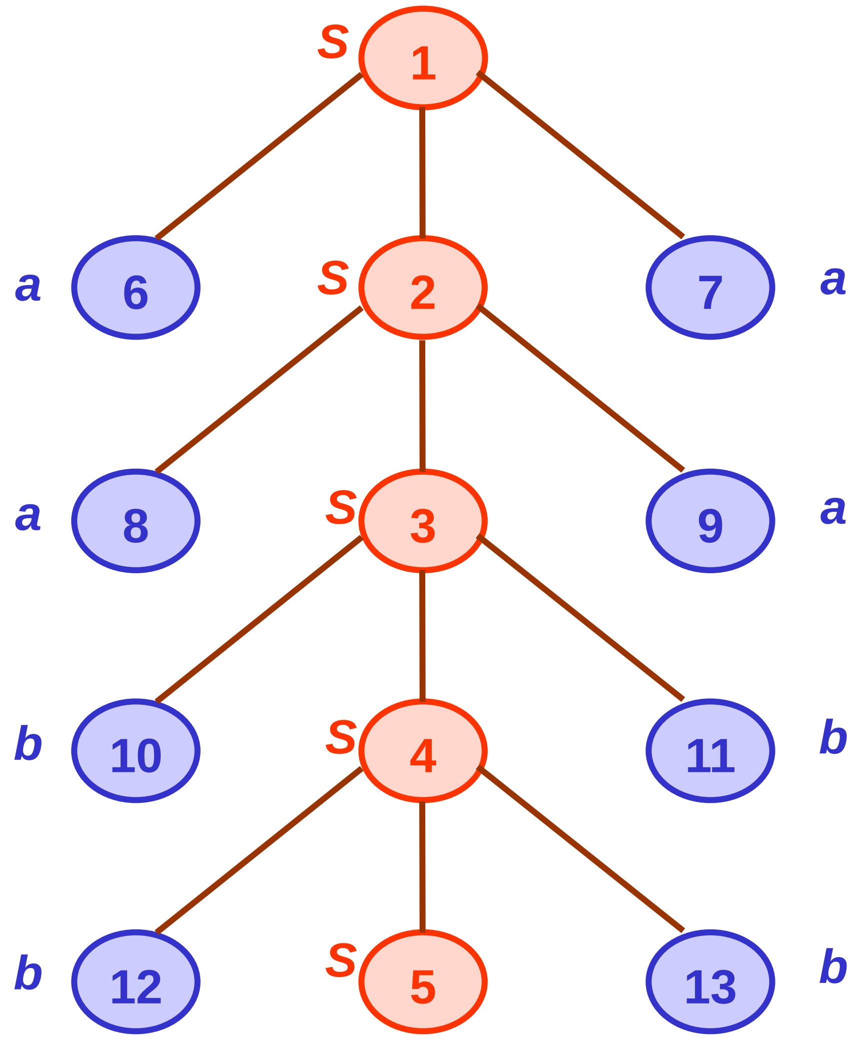

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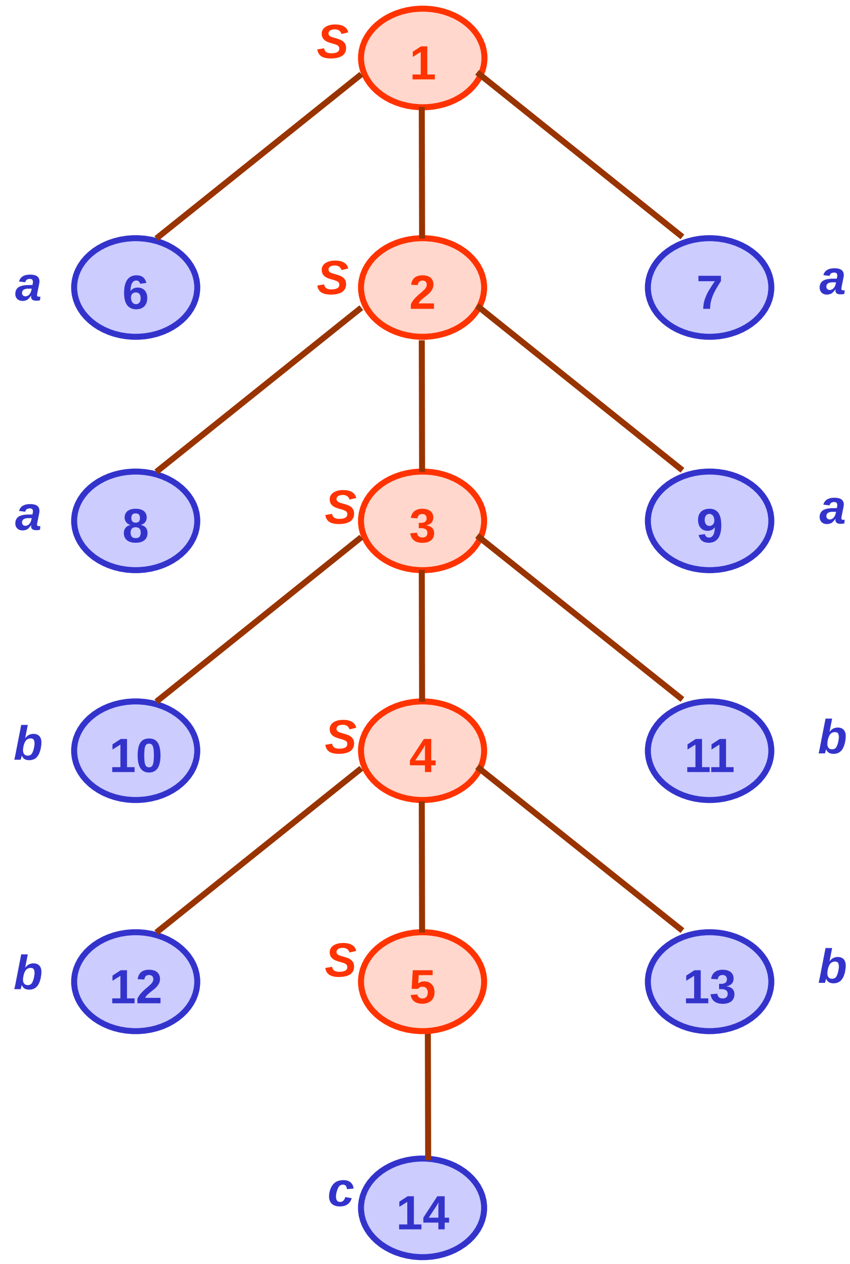

Grammars

Grammars

Parse tree **for grammar** $G = (V, T, P, S)$**:**

Grammars

Parse tree **for grammar** $G = (V, T, P, S)$**:**

1) **Nodes of the tree** - $V \cup T \cup \{\varepsilon\}$

Grammars

Parse tree **for grammar** $G = (V, T, P, S)$**:**

1) **Nodes of the tree** - $V \cup T \cup \{\varepsilon\}$

2) **Root of the tree** - S

Grammars

Parse tree **for grammar** $G = (V, T, P, S)$**:**

1) **Nodes of the tree** - $V \cup T \cup \{\varepsilon\}$

2) **Root of the tree** - S

3) **Internal nodes** - $A \in V$

Grammars

Parse tree **for grammar** $G = (V, T, P, S)$**:**

1) **Nodes of the tree** - $V \cup T \cup \{\varepsilon\}$

2) **Root of the tree** - S

3) **Internal nodes** - $A \in V$

4) **Nodes** $n_1, n_2, ..., n_k$ **- child nodes of** n

Grammars

Parse tree **for grammar $G = (V, T, P, S)$:**

1)	**Nodes of the tree**	**-** $V \cup T \cup \{\varepsilon\}$
2)	**Root of the tree**	**-** S
3)	**Internal nodes**	**-** $A \in V$
4)	**Nodes** $n_1, n_2, ..., n_k$ **Node** n	**- child nodes of** n **- symbol** A

Grammars

Parse tree **for grammar $G = (V, T, P, S)$:**

1)	**Nodes of the tree**	**-** $V \cup T \cup \{\varepsilon\}$
2)	**Root of the tree**	**-** S
3)	**Internal nodes**	**-** $A \in V$
4)	**Nodes** $n_1, n_2, \ldots, n_k$ **Node** n **Nodes** $n_1, n_2, \ldots, n_k$	**- child nodes of** n **- symbol** A **- symbols** $X_1, X_2, \ldots, X_k$

Grammars

Parse tree **for grammar** $G = (V, T, P, S)$**:**

1) Nodes of the tree - $V \cup T \cup \{\varepsilon\}$

2) Root of the tree - S

3) Internal nodes - $A \in V$

4)

Nodes $n_1, n_2, \ldots, n_k$	**- child nodes of** n
Node n	**- symbol** A
Nodes $n_1, n_2, \ldots, n_k$	**- symbols** $X_1, X_2, \ldots, X_k$
$A \to X_1 X_2 \ldots X_k$	**- production from set** P

Grammars

Parse tree **for grammar** $G = (V, T, P, S)$**:**

1)	**Nodes of the tree**	**-** $V \cup T \cup \{\varepsilon\}$
2)	**Root of the tree**	**-** S
3)	**Internal nodes**	**-** $A \in V$
4)	**Nodes** $n_1, n_2, ..., n_k$ **Node** n **Nodes** $n_1, n_2, ..., n_k$ $A \rightarrow X_1 X_2 ... X_k$	**- child nodes of** n **- symbol** A **- symbols** $X_1, X_2, ..., X_k$ **- production from set** P
5)	**Character** ε	**- a tree leaf**

Grammars

***Parse tree* for grammar $G = (V, T, P, S)$:**

1)	**Nodes of the tree**	**-** $V \cup T \cup \{\varepsilon\}$
2)	**Root of the tree**	**-** S
3)	**Internal nodes**	**-** $A \in V$
4)	**Nodes** $n_1, n_2, \ldots, n_k$ **Node** n **Nodes** $n_1, n_2, \ldots, n_k$ $A \rightarrow X_1 X_2 \ldots X_k$	**- child nodes of** n **- symbol** A **- symbols** $X_1, X_2, \ldots, X_k$ **- production from set** P
5)	**Character** ε	**- a tree leaf** **- only child of its parent**

Grammars

Parse tree **for grammar $G = (V, T, P, S)$:**

1)	**Nodes of the tree**	**-** $V \cup T \cup \{\varepsilon\}$
2)	**Root of the tree**	**-** S
3)	**Internal nodes**	**-** $A \in V$
4)	**Nodes** $n_1, n_2, \ldots, n_k$	**- child nodes of** n
	Node n	**- symbol** A
	Nodes $n_1, n_2, \ldots, n_k$	**- symbols** $X_1, X_2, \ldots, X_k$
	$A \rightarrow X_1 X_2 \ldots X_k$	**- production from set** P
5)	**Character** ε	**- a tree leaf**
		- only child of its parent
6)	**Leaves of the tree**	**- symbols from** $T \cup \{\varepsilon\}$

Grammars

Parse tree **for grammar $G = (V, T, P, S)$:**

1)	**Nodes of the tree**	**-** $V \cup T \cup \{\varepsilon\}$
2)	**Root of the tree**	**-** S
3)	**Internal nodes**	**-** $A \in V$
4)	**Nodes** $n_1, n_2, \ldots, n_k$ **Node** n **Nodes** $n_1, n_2, \ldots, n_k$ $A \rightarrow X_1 X_2 \ldots X_k$	**- child nodes of** n **- symbol** A **- symbols** $X_1, X_2, \ldots, X_k$ **- production from set** P
5)	**Character** ε	**- a tree leaf** **- only child of its parent**
6)	**Leaves of the tree**	**- symbols from** $T \cup \{\varepsilon\}$ **- make up a generated string of** $L(G)$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

Constructing a grammar for a regular language given by a DFA

	a	*b*	
S	*A*	*B*	1
A	*B*	*A*	0
B	*S*	*A*	1

Constructing a grammar for a regular language given by a DFA

	a	*b*	
S	*A*	*B*	1
A	*B*	*A*	0
B	*S*	*A*	1

$S \rightarrow aA$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \rightarrow aA$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \rightarrow aA$ $\quad$ $S \rightarrow bB$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \rightarrow aA$ $\quad$ $S \rightarrow bB$

Constructing a grammar for a regular language given by a DFA

	a	*b*	
S	*A*	*B*	1
A	*B*	*A*	0
B	*S*	*A*	1

$S \rightarrow aA \qquad S \rightarrow bB \qquad S \rightarrow \varepsilon$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \rightarrow aA$ $\quad S \rightarrow bB$ $\quad S \rightarrow \varepsilon$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \to aA \qquad S \to bB \qquad S \to \varepsilon$

$A \to aB \qquad A \to bA$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \rightarrow aA$ $\quad S \rightarrow bB$ $\quad S \rightarrow \varepsilon$

$A \rightarrow aB$ $\quad A \rightarrow bA$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \to aA \qquad S \to bB \qquad S \to \varepsilon$

$A \to aB \qquad A \to bA$

$B \to aS \qquad B \to bA \qquad B \to \varepsilon$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \rightarrow aA$ $\quad S \rightarrow bB$ $\quad S \rightarrow \varepsilon$

$A \rightarrow aB$ $\quad A \rightarrow bA$

$B \rightarrow aS$ $\quad B \rightarrow bA$ $\quad B \rightarrow \varepsilon$

String ***aba***

DFA

Grammar

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \rightarrow aA$ $\quad S \rightarrow bB$ $\quad S \rightarrow \varepsilon$

$A \rightarrow aB$ $\quad A \rightarrow bA$

$B \rightarrow aS$ $\quad B \rightarrow bA$ $\quad B \rightarrow \varepsilon$

String ***aba***

DFA $\quad S$

Grammar

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \to aA \quad S \to bB \quad S \to \varepsilon$

$A \to aB \quad A \to bA$

$B \to aS \quad B \to bA \quad B \to \varepsilon$

String ***aba***

DFA S

Grammar S

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \to aA$ $S \to bB$ $S \to \varepsilon$

$A \to aB$ $A \to bA$

$B \to aS$ $B \to bA$ $B \to \varepsilon$

String ***aba***

DFA S

Grammar S

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \to aA \qquad S \to bB \qquad S \to \varepsilon$

$A \to aB \qquad A \to bA$

$B \to aS \qquad B \to bA \qquad B \to \varepsilon$

String ***aba***

DFA $\quad S \xrightarrow{a} A$

Grammar $\quad S$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \rightarrow aA$ $\quad S \rightarrow bB$ $\quad S \rightarrow \varepsilon$

$A \rightarrow aB$ $\quad A \rightarrow bA$

$B \rightarrow aS$ $\quad B \rightarrow bA$ $\quad B \rightarrow \varepsilon$

String ***aba***

DFA $\quad S \xrightarrow{a} A$

Grammar $\quad S$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \rightarrow aA \qquad S \rightarrow bB \qquad S \rightarrow \varepsilon$

$A \rightarrow aB \qquad A \rightarrow bA$

$B \rightarrow aS \qquad B \rightarrow bA \qquad B \rightarrow \varepsilon$

String ***aba***

DFA $\quad S \xrightarrow{a} A$

Grammar $\quad S \Rightarrow aA$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \rightarrow aA$ $\quad S \rightarrow bB$ $\quad S \rightarrow \varepsilon$

$A \rightarrow aB$ $\quad A \rightarrow bA$

$B \rightarrow aS$ $\quad B \rightarrow bA$ $\quad B \rightarrow \varepsilon$

String ***aba***

DFA $\quad S \xrightarrow{a} A \xrightarrow{b} A$

Grammar $\quad S \Rightarrow aA$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \to aA \qquad S \to bB \qquad S \to \varepsilon$

$A \to aB \qquad A \to bA$

$B \to aS \qquad B \to bA \qquad B \to \varepsilon$

String aba

DFA $S \xrightarrow{a} A \xrightarrow{b} A$

Grammar $S \Rightarrow aA \Rightarrow abA$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \rightarrow aA \quad S \rightarrow bB \quad S \rightarrow \varepsilon$

$A \rightarrow aB \quad A \rightarrow bA$

$B \rightarrow aS \quad B \rightarrow bA \quad B \rightarrow \varepsilon$

String aba

DFA $\quad S \xrightarrow{a} A \xrightarrow{b} A \xrightarrow{a} B$

Grammar $\quad S \Rightarrow aA \Rightarrow abA$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \to aA \qquad S \to bB \qquad S \to \varepsilon$

$A \to aB \qquad A \to bA$

$B \to aS \qquad B \to bA \qquad B \to \varepsilon$

String *aba*

DFA $\quad S \xrightarrow{a} A \xrightarrow{b} A \xrightarrow{a} B \qquad B \in F$

Grammar $\quad S \Rightarrow aA \Rightarrow abA$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \to aA \quad S \to bB \quad S \to \varepsilon$

$A \to aB \quad A \to bA$

$B \to aS \quad B \to bA \quad B \to \varepsilon$

String ***aba***

DFA $\quad S \xrightarrow{a} A \xrightarrow{b} A \xrightarrow{a} B \quad B \in F$

Grammar $\quad S \Rightarrow aA \Rightarrow abA \Rightarrow abaB$

Constructing a grammar for a regular language given by a DFA

	a	b	
S	A	B	1
A	B	A	0
B	S	A	1

$S \to aA \qquad S \to bB \qquad S \to \varepsilon$

$A \to aB \qquad A \to bA$

$B \to aS \qquad B \to bA \qquad B \to \varepsilon$

String ***aba***

DFA $\quad S \xrightarrow{a} A \xrightarrow{b} A \xrightarrow{a} B \quad B \in F$

Grammar $\quad S \Rightarrow aA \Rightarrow abA \Rightarrow abaB \Rightarrow aba$

Constructing a grammar for a regular language given by a DFA

Constructing a grammar for a regular language given by a DFA

$G=(V, T, P, S)$

Constructing a grammar for a regular language given by a DFA

$G=(V, T, P, S)$

DFA $M=(Q, \Sigma, \delta, q_0, F)$

Constructing a grammar for a regular language given by a DFA

$G=(V, T, P, S)$

DFA $M=(Q, \Sigma, \delta, q_0, F)$

1) $T = \Sigma$

Constructing a grammar for a regular language given by a DFA

$G=(V, T, P, S)$

DFA $M=(Q, \Sigma, \delta, q_0, F)$

1) $T = \Sigma$

2) $V = Q$

Constructing a grammar for a regular language given by a DFA

$G=(V, T, P, S)$

DFA $M=(Q, \Sigma, \delta, q_0, F)$

---

1) $T = \Sigma$

2) $V = Q$

3) $S = q_0$

Constructing a grammar for a regular language given by a DFA

$G=(V, T, P, S)$

DFA $M=(Q, \Sigma, \delta, q_0, F)$

1) $T = \Sigma$

2) $V = Q$

3) $S = q_0$

4) $A \rightarrow aB$ $\delta(A, a) = B$

Constructing a grammar for a regular language given by a DFA

$G=(V, T, P, S)$

DFA $M=(Q, \Sigma, \delta, q_0, F)$

1) $T = \Sigma$

2) $V = Q$

3) $S = q_0$

4) $A \to aB$ $\quad\quad \delta(A, a) = B$

5) $A \to \varepsilon$ $\quad\quad A \in F$

Constructing a grammar for a regular language given by a DFA

Constructing a grammar for a regular language given by a DFA

DFA and grammar are equivalent

Constructing a grammar for a regular language given by a DFA

DFA and grammar are equivalent

if DFA accepts a language defined by the grammar

DFA and grammar are equivalent

if DFA accepts a language defined by the grammar

$$A \overset{*}{\Rightarrow} wB$$

Constructing a grammar for a regular language given by a DFA

DFA and grammar are equivalent

if DFA accepts a language defined by the grammar

$$A \overset{*}{\Rightarrow} wB \quad \textbf{if and only if } \delta(A, w) = B$$