#### Lecture 12.

4.1.3 Extensions to the basic Turing machine

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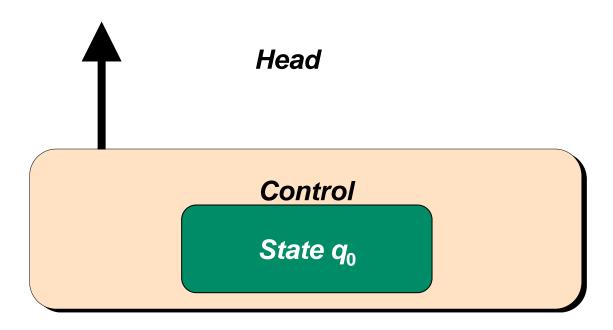


# **Extensions to the basic Turing machine**

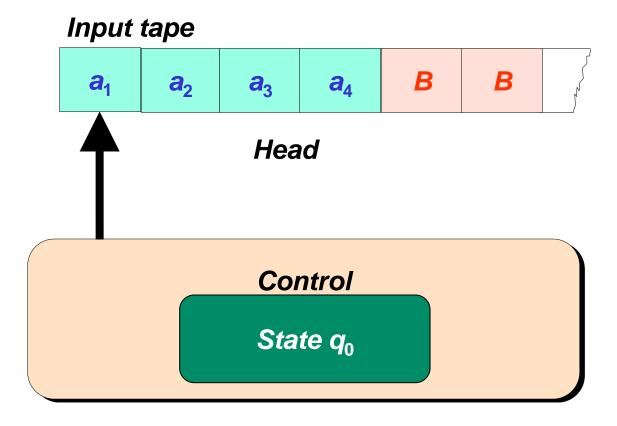
- TM with a two-sided infinite tape
- Multitape TM
- Nondeterministic TM
- TM with a multidimensional input



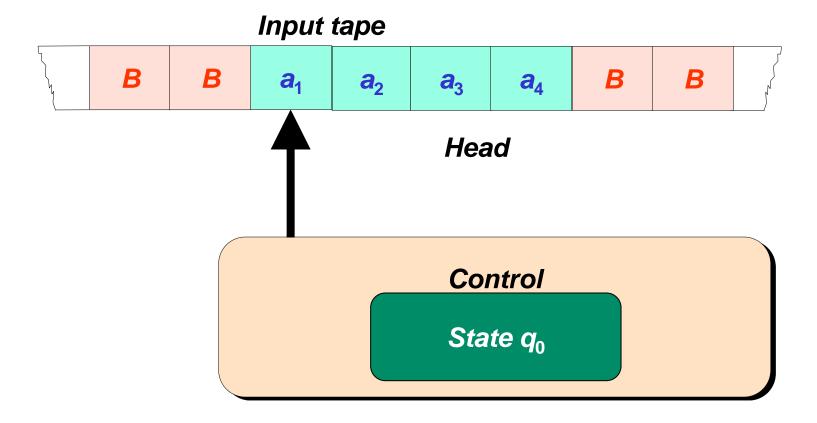
















TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$



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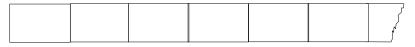
TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
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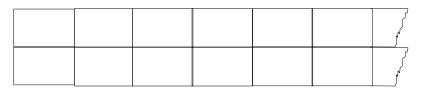
TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$





TS 
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TS 
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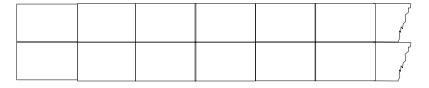




TS 
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$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Input symbols: [a, B]





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TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Input symbols: [a, B]

a	а	а	a		7
B	B	B	B		1



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

BBBBBBBBBAAABBB

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Input symbols: [a, B]

Blank cells: [B, B]

a	a	a	a		y
В	B	B	B		y S



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

B B B B B a a a B B

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Input symbols: [a, B]

Blank cells: [B, B]

а	a	a	a	B	B	
В	В	B	В	B	B	



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Input symbols: [a, B]

Blank cells: [B, B]

а	a	a	a	B	B	
B	B	B	B	B	B	

**States have two components:** 



TS 
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Input symbols: [a, B]

Blank cells: [B, B]

a	a	a	a	B	B	
В	B	B	B	B	B	

**States have two components:** 

First component: state TM M<sub>2</sub>



TS 
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TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Input symbols: [a, B]

Blank cells: [B, B]

a	a	a	a	B	B	
B	B	B	B	B	B	

**States have two components:** 

First component: state TM M<sub>2</sub>

Second component: symbol *U* or *D* 



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

BBBBBBBBAAABBA

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Input symbols: [a, B]

Blank cells: [B, B]

а	a	a	a	B	B	
B	B	B	B	B	B	

**States have two components:** 

First component: state  $TM M_2$ 

Second component: symbol *U* or *D* 

[q, U], [q, D]



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$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

B B B B B a a a B B

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Input symbols: [a, B]

Blank cells: [B, B]

a	a	a	a	B	B	
B	B	B	B	B	B	

**States have two components:** 

First component: state TM M<sub>2</sub>

Second component: symbol *U* or *D* 

[q, U], [q, D]

Tape symbols: [X, Y]



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$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

B B B B B B a a a B B

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Input symbols: [a, B]

Blank cells: [B, B]

а	a	a	a	B	B	
B	В	В	В	B	B	

**States have two components:** 

First component: state TM M<sub>2</sub>

Second component: symbol U or D

[q, U], [q, D]

Tape symbols: [X, Y]

$$F_1 = \{ [q, U], [q, D] \mid q \in F_2 \}$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

a	a	a	a	В	B	7
В	В	В	В	B	B	p



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

B B B B B a a a B B

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

a	a	a	a	В	B	p
В	В	В	В	B	B	M

 $q_1$ ,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

B B B B B a a a B B

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

а	a	a	a	B	B	h
В	В	В	В	B	B	h

 $q_1$ ,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a)=(q, X, R)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

B B B B B a a a B B

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

а	а	a	a	B	B	h
В	В	В	В	B	B	h

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If  $\delta_2(q_2, a)=(q, X, R)$ 



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TS 
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а	а	a	a	B	B	h
В	В	В	В	B	B	h

 $q_1$ ,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a) = (q, X, R)$$



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B B B B B B X a a B B

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

а	a	a	a	B	B	
В	В	В	В	B	B	

 $q_1$ ,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If  $\delta_2(q_2, a)=(q, X, R)$ 



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TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

a	a	a	a	В	B	p
В	В	В	В	B	B	M

 $q_1$ ,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a)=(q, X, R)$$

$$\delta_1(q_1, [a, B]) = ([q, U], [X, \mathfrak{C}], R)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

a	a	a	a	В	B	
B	В	В	В	B	B	7

$$q_1$$
,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a) = (q, X, R)$$

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TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	а	а	a	B	B	h
¢	В	В	В	B	B	h

 $q_1$ ,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a)=(q, X, R)$$

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TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	a	a	a	B	B	h
¢	В	В	В	B	B	h

$$q_1$$
,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a)=(q, X, R)$$

$$\delta_1(q_1, [a, B]) = ([q, U], [X, \mathfrak{C}], R)$$



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TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	a	a	a	B	B	h
¢	В	В	В	B	B	h

$$q_1$$
,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a) = (q, X, R)$$

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$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TM 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

a	a	a	a	B	B	1
В	В	В	В	B	B	4



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

B B B B B a a a B B

TM 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

а	а	a	a	B	B	h
В	В	В	В	B	B	h

 $q_1$ ,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

B B B B B a a a B B

TM 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

a	а	а	a	B	B	p
B	В	В	В	B	B	M

 $q_1$ ,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a) = (q, X, L)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

B B B B B B a a a B B

TM 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

a	a	a	a	В	B	h
В	В	В	В	B	B	, A

 $q_1$ ,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a) = (q, X, L)$$



TS 
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B B B B B X a a B B

TM 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

a	а	a	a	В	B	h
В	В	В	В	B	B	M

 $q_1$ ,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
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B B B B B X a a B B

TM 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

а	a	a	a	B	B	h
В	В	В	В	B	B	h

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а	a	a	a	B	B	My d
B	В	В	В	B	B	M

$$q_1$$
,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a) = (q, X, L)$$

$$\delta_1(q_1, [a, B]) = ([q, D], [X, \emptyset], R)$$



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$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TM 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

a	a	a	a	В	B	p
B	В	В	В	B	B	4

$$q_1$$
,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a) = (q, X, L)$$

$$\delta_1(q_1, [a, B]) = ([q, D], [X, \emptyset], R)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TM 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	а	а	a	B	B	h
¢	В	В	В	B	B	h

$$q_1$$
,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a) = (q, X, L)$$

$$\delta_1(q_1, [a, B]) = ([q, D], [X, \emptyset], R)$$



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$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TM 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	a	a	a	B	B	h
¢	B	В	В	B	B	, A

$$q_1$$
,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a) = (q, X, L)$$

$$\delta_1(q_1, [a, B]) = ([q, D], [X, \emptyset], R)$$



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TM 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	a	a	a	B	B	h
¢	В	В	В	B	B	h

$$q_1$$
,  $q_2$  – Start states TM  $M_1$ , TM  $M_2$ 

If 
$$\delta_2(q_2, a) = (q, X, L)$$

$$\delta_1(q_1, [a, B]) = ([q, D], [X, \emptyset], R)$$



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TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	, p
¢	Y	Y	Y	Y	Y	



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	and the second
¢	Y	Y	Y	Y	Y	and the same of th

Ako 
$$\delta_2(q, X)=(p, Z, A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	d S
¢	Y	Y	Y	Y	Y	a de la companya de l

Ako 
$$\delta_2(q, X) = (p, Z, A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	a de la companya de l
¢	Y	Y	Y	Y	Y	Mark Control

Ako 
$$\delta_2(q, X)=(p, Z, A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	a de la companya de l
¢	Y	Y	Y	Y	Y	Mark Control

Ako 
$$\delta_2(q, X)=(p, Z, A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

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TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	Z	X	X	X	No.
¢	Y	Y	Y	Y	Y	

Ako 
$$\delta_2(q, X)=(p, Z, A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	Z	X	X	X	y de la companya de l
¢	Y	Y	Y	Y	Y	g d

Ako 
$$\delta_2(q, X)=(p, Z, A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Ako 
$$\delta_2(q, X) = (p, Z, A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	Z	X	X	X	No.
¢	Y	Y	Y	Y	Y	N. S.

Ako 
$$\delta_2(q, X)=(p, Z, A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Ako 
$$\delta_2(q, X)=(p, Z, A)$$

$$\delta_1([q, U], [X, Y]) = ([p, U], [Z, Y], A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Ako 
$$\delta_2(q, X) = (p, Z, A)$$

$$\delta_1([q, U], [X, Y]) = ([p, U], [Z, Y], A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Ako 
$$\delta_2(q, X)=(p, Z, A)$$

$$\delta_1([q, U], [X, Y]) = ([p, U], [Z, Y], A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	, p
¢	Y	Y	Y	Y	Y	



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	
¢	Y	Y	Y	Y	Y	

Ako 
$$\delta_2(q, Y)=(p, Z, A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	
¢	Y	Y	Y	Y	Y	

Ako 
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X	X	X	X	X	X	
¢	Y	Y	Y	Y	Y	

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TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	y de la companya de l
¢	Y	Y	Y	Y	Y	d S

Ako 
$$\delta_2(q, Y)=(p, Z, A)$$



TS 
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TS 
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X	X	X	X	X	X	No.
¢	Y	Z	Y	Y	Y	

Ako 
$$\delta_2(q, Y)=(p, Z, A)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
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X	X	X	X	X	X	N. S.
¢	Y	Z	Y	Y	Y	a de la companya de l

Ako 
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TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	y S
¢	Y	Z	Y	Y	Y	y de la companya de l

Ako 
$$\delta_2(q, Y)=(p, Z, A)$$



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$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

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Ako 
$$\delta_2(q, Y)=(p, Z, A)$$

$$\delta_1([q, D], [X, Y]) = ([p, D], [X, Z], -A)$$



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$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

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X	X	X	X	X	X	y S
¢	Y	Y	Y	Y	Y	A S



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	y de la companya de l
¢	Y	Y	Y	Y	Y	d S

Ako 
$$\delta_2(q, X) = (p, Z, R)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
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X	X	X	X	X	X	y de la companya de l
¢	Y	Y	Y	Y	Y	a de la companya de l

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X	X	X	X	X	X	p
¢	Y	Y	Y	Y	Y	M

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X	X	X	X	X	X	y de la companya de l
¢	Y	Y	Y	Y	Y	d S

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Ako 
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$$\delta_1([q, U], [X, ¢]) = ([p, U], [Z, ¢], R)$$



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TS 
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X	X	X	X	X	X	No.
¢	Y	Y	Y	Y	Y	No.



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	y de la companya de l
¢	Y	Y	Y	Y	Y	a de la companya de l

Ako 
$$\delta_2(q, X) = (p, Z, L)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	y S
¢	Y	Y	Y	Y	Y	No. of the second

Ako  $\delta_2(q, X) = (p, Z, L)$ 



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	p
¢	Y	Y	Y	Y	Y	M

Ako 
$$\delta_2(q, X) = (p, Z, L)$$



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$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

X	X	X	X	X	X	y de la companya de l
¢	Y	Y	Y	Y	Y	d S

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Ako 
$$\delta_2(q, X) = (p, Z, L)$$

$$\delta_1([q, U], [X, ¢]) = ([p, D], [Z, ¢], R)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Ako 
$$\delta_2(q, X) = (p, Z, L)$$

$$\delta_1([q, U], [X, ¢]) = ([p, D], [Z, ¢], R)$$



TS 
$$M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$$

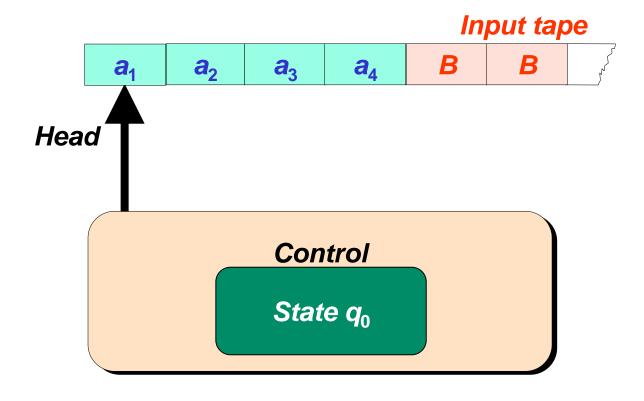
TS 
$$M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$$

Ako 
$$\delta_2(q, X) = (p, Z, L)$$

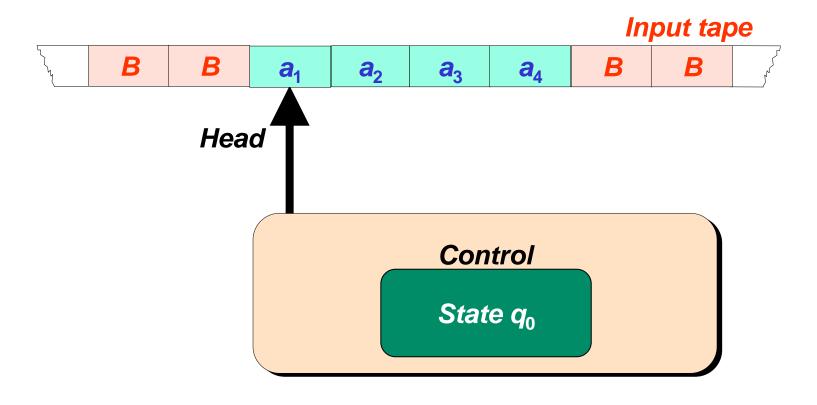
$$\delta_1([q, U], [X, ¢]) = ([p, D], [Z, ¢], R)$$



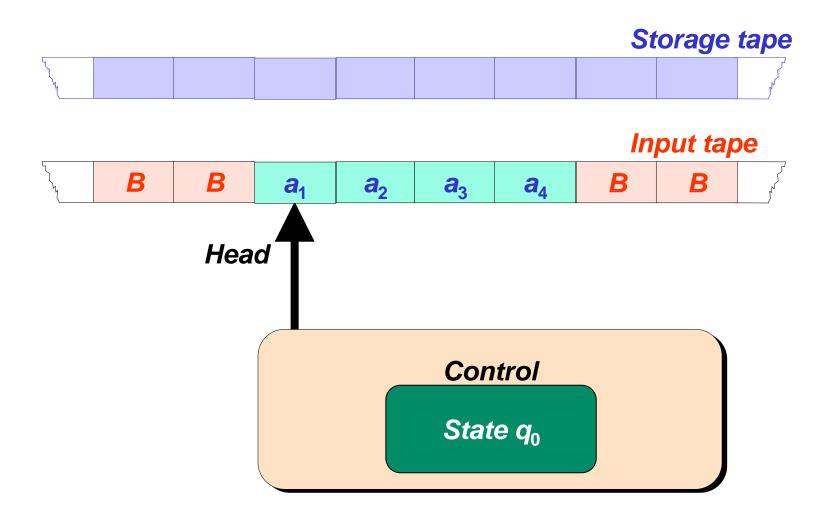




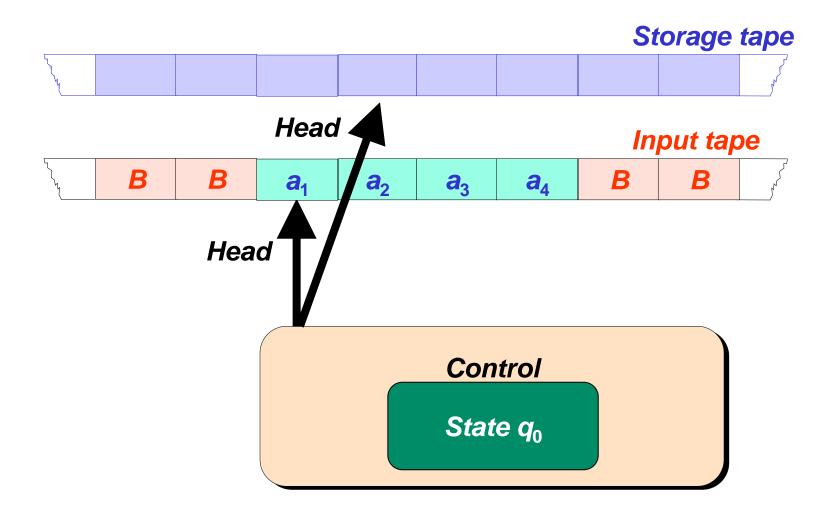




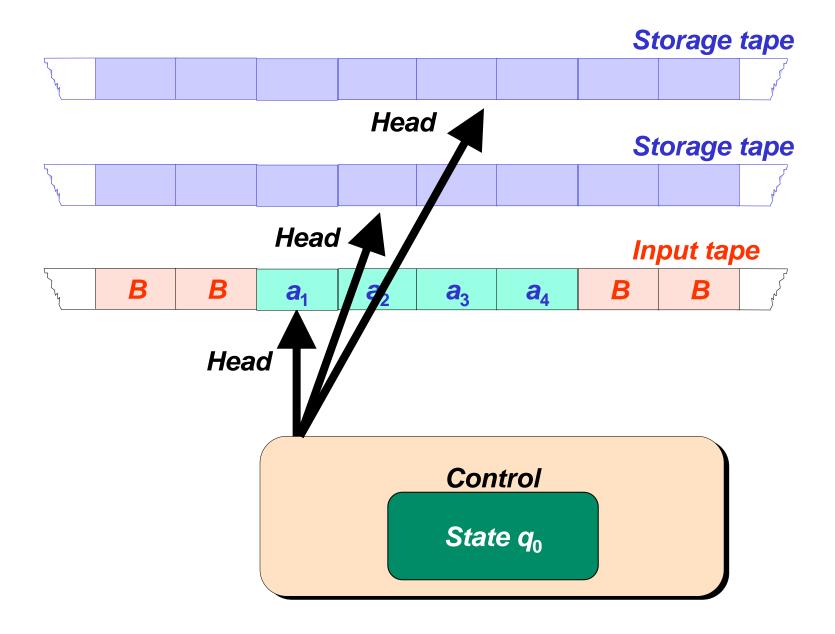
















Decision



- Decision
  - Input



- Decision
  - Input
    - 1. State



- Decision
  - Input
    - 1. State
    - 2. Reading k symbols from k tapes



- Decision
  - Input
    - 1. State
    - 2. Reading k symbols from k tapes
  - Output



- Decision
  - Input
    - 1. State
    - 2. Reading k symbols from k tapes
  - Output
    - 1. New state



- Decision
  - Input
    - 1. State
    - 2. Reading k symbols from k tapes
  - Output
    - 1. New state
    - 2. Writing *k* symbols on *k* tapes



#### Decision

- Input
  - 1. State
  - 2. Reading k symbols from k tapes
- Output
  - 1. New state
  - 2. Writing *k* symbols on *k* tapes
  - 3. Independent movement of k heads





Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Tape 1 contents

 $A_1$   $A_2$  ---  $A_i$  ---  $A_m$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Tape 1 contents

 $A_1$ 

 $A_2$ 

---

 $A_i$ 

- - -

 $A_m$ 

Tape 2 contents

 $B_1$ 

 $B_2$ 

---

 $B_i$ 

- - -

 $B_{m}$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Tape 1 contents

$$A_2$$

$$\boldsymbol{A_m}$$

Tape 2 contents

$$B_2$$

$$B_i$$

 $A_i$ 

$$B_{m}$$

Tape 3 contents



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position X ---  $A_i$  ---  $A_m$ 

Tape 2 contents  $B_1$   $B_2$  ---  $B_i$  ---  $B_m$ 

Tape 3 contents  $C_1$   $C_2$  ---  $C_i$  ---  $C_m$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$	 $\boldsymbol{A}_{m}$
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	$B_2$	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>

Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>		Ci		C <sub>m</sub>
-----------------	-----------------------	-----------------------	--	----	--	----------------



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	A <sub>2</sub>	 <b>A</b> <sub>i</sub>	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	C <sub>2</sub>	 Ci	 C <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $\boldsymbol{A_i}$	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<i>B</i> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	C <sub>1</sub>	C <sub>2</sub>	C,	

[ State TS  $M_1$ ,



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $\boldsymbol{A_i}$	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<i>B</i> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	C <sub>1</sub>	C <sub>2</sub>	C,	

[State TS M<sub>1</sub>, Counter,



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	<b>A</b> <sub>2</sub>	 A <sub>i</sub>	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<i>B</i> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 B <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci	 <b>C</b> <sub>m</sub>

[State TS M<sub>1</sub>, Counter, Tape 1 contents, ..., Tape k contents]



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$	 $\boldsymbol{A}_{m}$
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	$B_2$	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	C <sub>2</sub>	 Ci	 C <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<i>B</i> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	C <sub>1</sub>	C <sub>2</sub>	 Ci	 C <sub>m</sub>

[  $q_0$  ,



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $\boldsymbol{A}_{i}$	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	$B_2$	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 <b>C</b> <sub>i</sub>	 <b>C</b> <sub>m</sub>

[  $q_0$  , 3



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$	 $\boldsymbol{A}_{m}$
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 <b>C</b> <sub>i</sub>	 <b>C</b> <sub>m</sub>

 $[ q_0, B, B, B, B]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 <b>A</b> <sub>i</sub>	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 B <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	C <sub>2</sub>	 Ci	 <b>C</b> <sub>m</sub>

 $[ \quad q_0 \quad , \quad 3 \quad , \quad B \quad , \quad B \quad ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$	 $\boldsymbol{A}_{m}$
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci	 <b>C</b> <sub>m</sub>

 $[ q_0, B, B, B]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$	 $\boldsymbol{A}_{m}$
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci	 <b>C</b> <sub>m</sub>

 $[ q_0, B, B, B, B ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	$B_2$	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci	 <b>C</b> <sub>m</sub>

 $[ \quad m{q_0} \quad , \quad \ \ \, 2 \quad , \quad B \quad , \quad B \quad , \quad B \quad ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $\boldsymbol{A}_{i}$	 $\boldsymbol{A}_{m}$
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci	 <b>C</b> <sub>m</sub>

 $[ q_0, B, B_1, B_1, C_1 ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	A <sub>2</sub>	 Ai	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	C <sub>2</sub>	 Ci	 <b>C</b> <sub>m</sub>

 $[ q_0, B, B_1, B_1, C_1 ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $\boldsymbol{A}_{i}$	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci	 <b>C</b> <sub>m</sub>

 $[ q_0, B, B_1, B_1, C_1 ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $\boldsymbol{A}_{i}$	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	$B_2$	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 <b>C</b> <sub>i</sub>	 <b>C</b> <sub>m</sub>

 $[ q_0, B, C_1 ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	A <sub>2</sub>	 $A_i$	 $\boldsymbol{A}_{m}$
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 C <sub>i</sub>	 <b>C</b> <sub>m</sub>

 $[ q_0, B, C_1 ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $\boldsymbol{A}_{i}$	 $\boldsymbol{A}_{m}$
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci	 <b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $\boldsymbol{A}_{i}$	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<i>B</i> <sub>1</sub>	$B_2$	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 C <sub>i</sub>	 <b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 Ai	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	C <sub>2</sub>	 Ci	 <b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$	 $\boldsymbol{A}_{m}$
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	$B_2$	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 <b>C</b> <sub>i</sub>	 <b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $\boldsymbol{A}_{i}$	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	$B_2$	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci	 <b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$	 $\boldsymbol{A}_{m}$
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 C <sub>i</sub>	 <b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$	 $\boldsymbol{A}_{m}$
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	$B_2$	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	C <sub>2</sub>	 Ci	 C <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $\boldsymbol{A}_{i}$	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	$B_2$	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 <b>C</b> <sub>i</sub>	 <b>C</b> <sub>m</sub>

 $[ q , 3 , A_2 , B_i , C_1 ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 Ai	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	$B_2$	 B <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci	 <b>C</b> <sub>m</sub>

 $[ q , 3 , A_2 , B_i , C_1 ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	A <sub>2</sub>	 <b>A</b> <sub>i</sub>	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 B <sub>i</sub>	 B <sub>m</sub>
Head 3 position	X			
Tape 3 contents	C <sub>1</sub>	C <sub>2</sub>	 C <sub>i</sub>	 C <sub>m</sub>

 $[ q , 2 , A_2 , B_i , C_1 ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X		
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$	 A <sub>m</sub>
Head 2 position			 X	
Tape 2 contents	<b>B</b> <sub>1</sub>	$B_2$	 <b>Z</b> <sub>i</sub>	 <b>B</b> <sub>m</sub>
Head 3 position	X			
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 <b>C</b> <sub>i</sub>	 <b>C</b> <sub>m</sub>

 $[ q , 2 , A_2 , B_i , C_1 ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X			
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$		$\boldsymbol{A}_{m}$
Head 2 position				X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 <b>Z</b> <sub>i</sub>		<b>B</b> <sub>m</sub>
Head 3 position	X				
Tape 3 contents	<b>C</b> <sub>1</sub>	C <sub>2</sub>	 C <sub>i</sub>		<b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X			
Tape 1 contents	<b>A</b> <sub>1</sub>	A <sub>2</sub>	 Ai		A <sub>m</sub>
Head 2 position				X	
Tape 2 contents	<i>B</i> <sub>1</sub>	<b>B</b> <sub>2</sub>	 Z <sub>i</sub>		<b>B</b> <sub>m</sub>
Head 3 position	X				
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci		<b>C</b> <sub>m</sub>

 $[ q , 2 , A_2 , B_i , C_1 ]$ 



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X			
Tape 1 contents	<b>A</b> <sub>1</sub>	A <sub>2</sub>	 <b>A</b> <sub>i</sub>		A <sub>m</sub>
Head 2 position				X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 Z <sub>i</sub>		<b>B</b> <sub>m</sub>
Head 3 position	X				
Tape 3 contents	<b>C</b> <sub>1</sub>	C <sub>2</sub>	 Ci		C <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X			
Tape 1 contents	<b>A</b> <sub>1</sub>	$A_2$	 $A_i$		A <sub>m</sub>
Head 2 position				X	
Tape 2 contents	<b>B</b> <sub>1</sub>	$B_2$	 Z <sub>i</sub>		<b>B</b> <sub>m</sub>
Head 3 position	X				
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 <b>C</b> <sub>i</sub>		<b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position		X			
Tape 1 contents	<b>A</b> <sub>1</sub>	<b>Z</b> <sub>2</sub>	 Ai		A <sub>m</sub>
Head 2 position				X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 <b>Z</b> <sub>i</sub>		<b>B</b> <sub>m</sub>
Head 3 position	X				
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 C <sub>i</sub>		<b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position	X				
Tape 1 contents	<b>A</b> <sub>1</sub>	<b>Z</b> <sub>2</sub>	 $A_i$		A <sub>m</sub>
Head 2 position				X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 <b>Z</b> <sub>i</sub>		<b>B</b> <sub>m</sub>
Head 3 position	X				
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci		<b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position					
Tape 1 contents	<b>A</b> <sub>1</sub>	<b>Z</b> <sub>2</sub>	 A <sub>i</sub>		A <sub>m</sub>
Head 2 position				X	
Tape 2 contents	<b>B</b> <sub>1</sub>	<b>B</b> <sub>2</sub>	 Z <sub>i</sub>		B <sub>m</sub>
Head 3 position	X				
Tape 3 contents	C <sub>1</sub>	C <sub>2</sub>	C,		C <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position	X				
Tape 1 contents	<b>A</b> <sub>1</sub>	<b>Z</b> <sub>2</sub>	 Ai		A <sub>m</sub>
Head 2 position				X	
Tape 2 contents	<i>B</i> <sub>1</sub>	$B_2$	 Z <sub>i</sub>		<b>B</b> <sub>m</sub>
Head 3 position	X				
Tape 3 contents	<b>C</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci		<b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position	X				
Tape 1 contents	<b>A</b> <sub>1</sub>	<b>Z</b> <sub>2</sub>	 Ai		A <sub>m</sub>
Head 2 position				X	
Tape 2 contents	<i>B</i> <sub>1</sub>	<b>B</b> <sub>2</sub>	 Z <sub>i</sub>		<b>B</b> <sub>m</sub>
Head 3 position	X				
Tape 3 contents	<b>Z</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci		<b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position	X				
Tape 1 contents	<b>A</b> <sub>1</sub>	<b>Z</b> <sub>2</sub>	 A <sub>i</sub>		A <sub>m</sub>
Head 2 position				X	
Tape 2 contents	<i>B</i> <sub>1</sub>	$B_2$	 <b>Z</b> <sub>i</sub>		<b>B</b> <sub>m</sub>
Head 3 position		X			
Tape 3 contents	<b>Z</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci		<b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position	X				
Tape 1 contents	<b>A</b> <sub>1</sub>	<b>Z</b> <sub>2</sub>	 A <sub>i</sub>		A <sub>m</sub>
Head 2 position				X	
Tape 2 contents	<i>B</i> <sub>1</sub>	$B_2$	 <b>Z</b> <sub>i</sub>		<b>B</b> <sub>m</sub>
Head 3 position		X			
Tape 3 contents	<b>Z</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci		<b>C</b> <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

Head 1 position	X				
Tape 1 contents	<b>A</b> <sub>1</sub>	<b>Z</b> <sub>2</sub>	 $A_i$		A <sub>m</sub>
Head 2 position				X	
Tape 2 contents	<i>B</i> <sub>1</sub>	$B_2$	 Z <sub>i</sub>		<b>B</b> <sub>m</sub>
Head 3 position		X			
Tape 3 contents	<b>Z</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci		C <sub>m</sub>



Simulating 3-tape TM  $M_1$  using 6 tracks of the single-tape TM  $M_2$ 

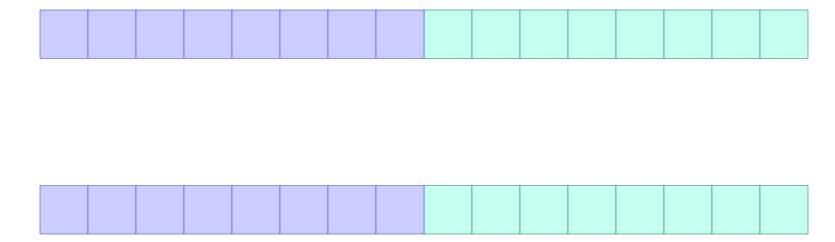
Head 1 position	X				
Tape 1 contents	<b>A</b> <sub>1</sub>	<b>Z</b> <sub>2</sub>	 A <sub>i</sub>		A <sub>m</sub>
Head 2 position				X	
Tape 2 contents	<i>B</i> <sub>1</sub>	$B_2$	 <b>Z</b> <sub>i</sub>		<b>B</b> <sub>m</sub>
Head 3 position		X			
Tape 3 contents	<b>Z</b> <sub>1</sub>	<b>C</b> <sub>2</sub>	 Ci		<b>C</b> <sub>m</sub>

[ q , 3 , B , B , B ]

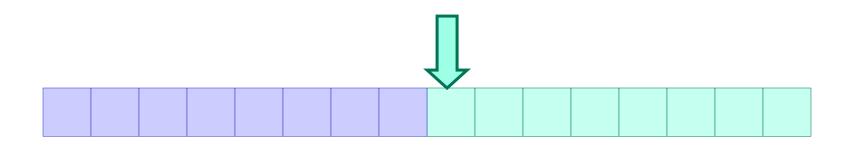


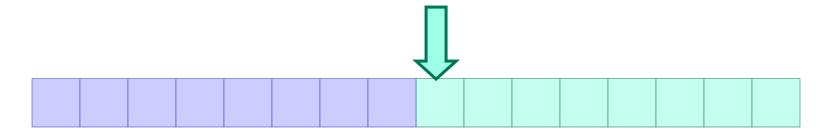






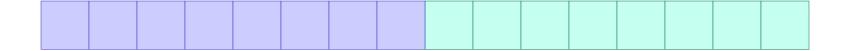


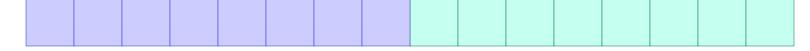




Initial step
Distance: 0







Initial step
Distance: 0

Step i





Initial step
Distance: 0

Step i





Initial step

Distance: 0

Step i





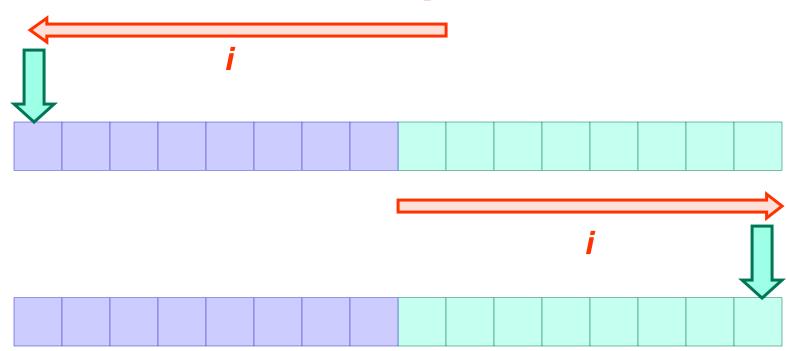
**Initial step** 

Distance: 0

Step i

Distance: 2i





**Initial step** 

Distance: 0

Step i

Distance: 2i

Simulating single transition: min. 4i head movements





**Initial step** 

Distance: 0

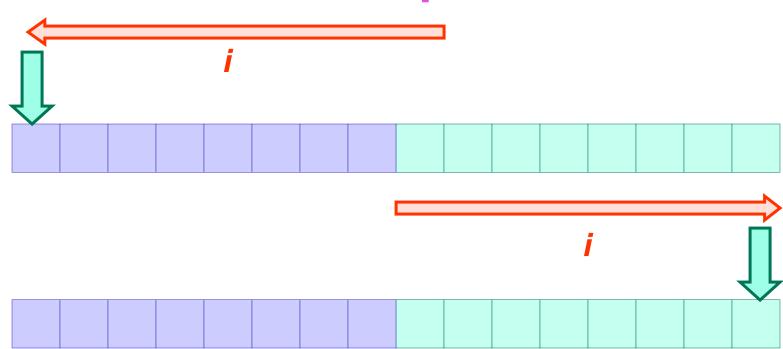
Step i

Distance: 2i

Simulating single transition: min. 4i head movements

m head movements





**Initial step** 

Distance: 0

Step i

Distance: 2i

Simulating single transition: min. 4i head movements

m head movements

$$\sum_{i=1}^{m} 4i \approx 2m^2$$





$$\{ ww^R / w \in (a+b)^* \}$$



$$\{ ww^R / w \in (a+b)^* \}$$

a	a	b	a	a	a	b	b
---	---	---	---	---	---	---	---

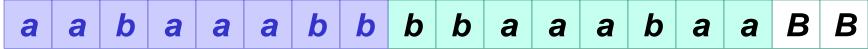


$$\{ ww^R / w \in (a+b)^* \}$$





$$\{ ww^R / w \in (a+b)^* \}$$





$$\{ ww^R / w \in (a+b)^* \}$$

a	a	b	a	a	a	b	b	b	b	a	a	a	b	a	a	B	B
В	В	В	B	В	В	В	В	В	В	В	В	В	В	В	В	B	B



$$\{ ww^R / w \in (a+b)^* \}$$

																	B
B	B	В	B	В	В	B	В	В	В	B	В	В	В	В	В	В	B



$$\{ ww^R / w \in (a+b)^* \}$$

																B	
1	В	В	B	В	B	B	В	В	В	В	В	В	В	В	В	B	B



$$\{ ww^R / w \in (a+b)^* \}$$

																	B
1	B	В	B	В	В	В	B	В	В	B	В	В	В	В	В	В	B



$$\{ ww^R / w \in (a+b)^* \}$$

																	B
<b>V</b>	B	В	B	В	В	В	B	В	В	B	В	В	В	В	В	В	B



$$\{ ww^R / w \in (a+b)^* \}$$

																	B
1	B	В	В	В	В	В	В	В	В	B	В	В	В	В	1	В	B



$$\{ ww^R / w \in (a+b)^* \}$$

																	B
<b>V</b>	B	В	B	В	В	B	В	В	В	B	В	В	В	В	1	B	B



$$\{ ww^R / w \in (a+b)^* \}$$

																	B
<b>V</b>	B	В	B	В	В	В	B	В	В	B	В	В	В	1	<b>V</b>	B	B



$$\{ ww^R / w \in (a+b)^* \}$$

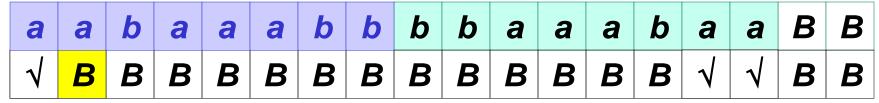




#### **Multitrack TM**

$$\{ ww^R / w \in (a+b)^* \}$$

#### Input tape



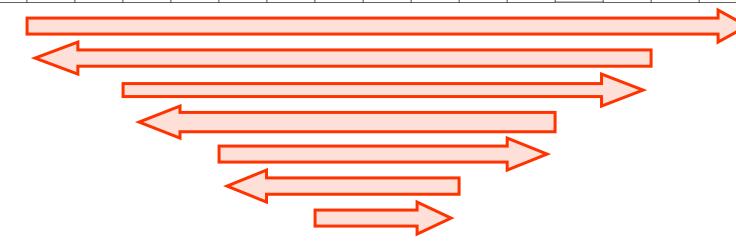


#### **Multitrack TM**

$$\{ ww^R / w \in (a+b)^* \}$$

#### Input tape

																B	
<b>\</b>	1	1	1	1	1	1	1	<b>V</b>	1	1	1	√	1	1	7	В	B





$$\{ ww^R / w \in (a+b)^* \}$$



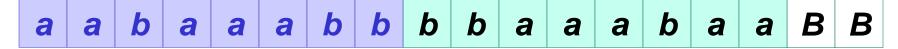
$$\{ ww^R / w \in (a+b)^* \}$$



Input tape



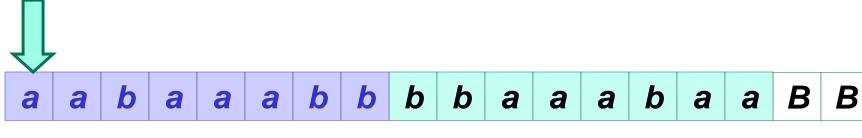
 $\{ ww^R / w \in (a+b)^* \}$ 



Input tape



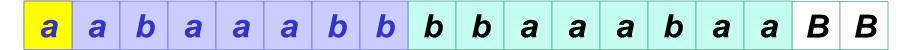
$$\{ ww^R / w \in (a+b)^* \}$$



Input tape



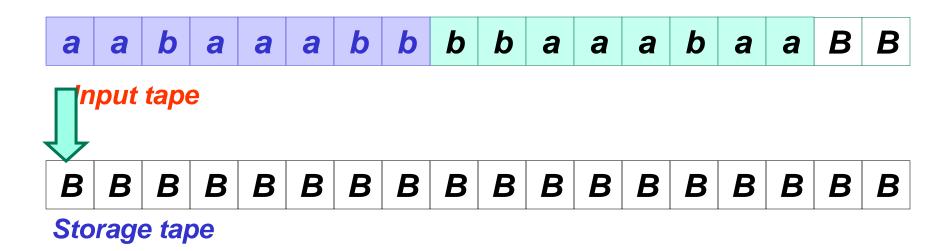
$$\{ ww^R / w \in (a+b)^* \}$$



Input tape

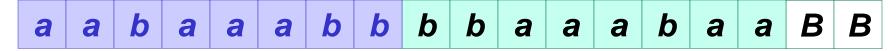


$$\{ ww^R / w \in (a+b)^* \}$$





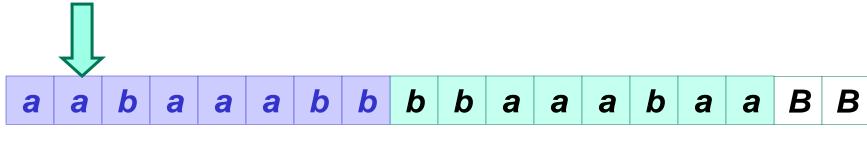
 $\{ ww^R / w \in (a+b)^* \}$ 



Input tape



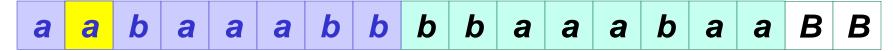
$$\{ ww^R / w \in (a+b)^* \}$$



Input tape



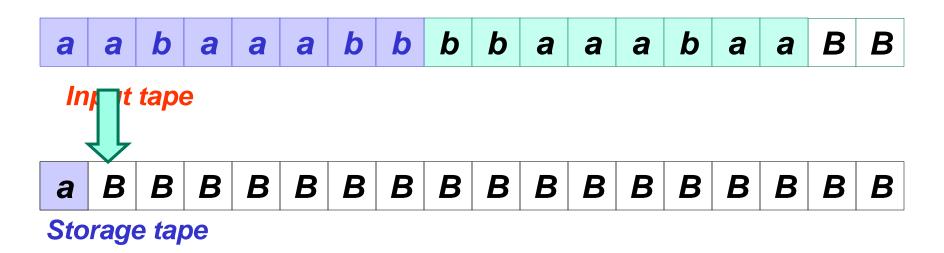
 $\{ ww^R / w \in (a+b)^* \}$ 



Input tape

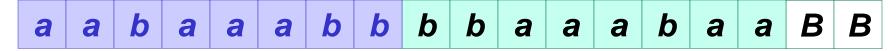


$$\{ ww^R / w \in (a+b)^* \}$$





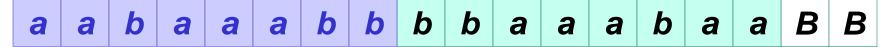
 $\{ ww^R / w \in (a+b)^* \}$ 



Input tape



 $\{ ww^R / w \in (a+b)^* \}$ 



Input tape



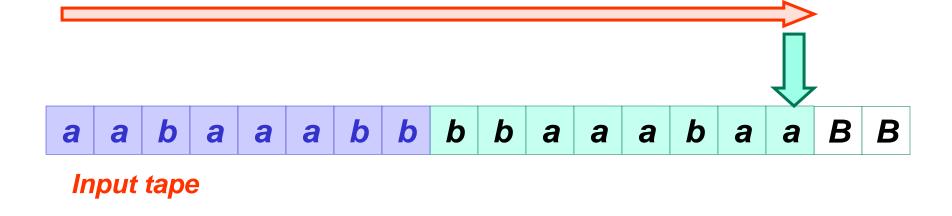
 $\{ ww^R / w \in (a+b)^* \}$ 



Input tape



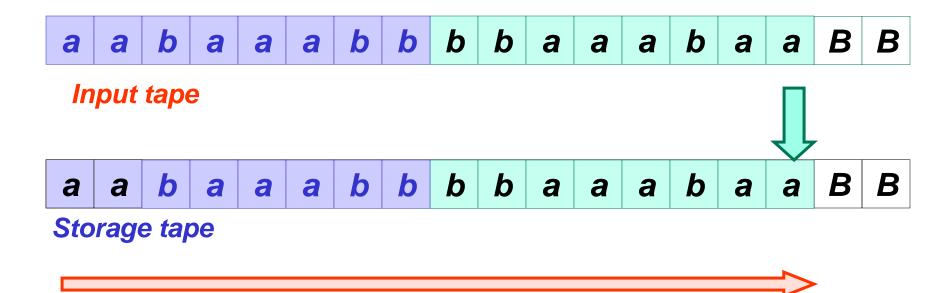
$$\{ ww^R / w \in (a+b)^* \}$$



b b b B B b b b a a a a a a a a a a Storage tape

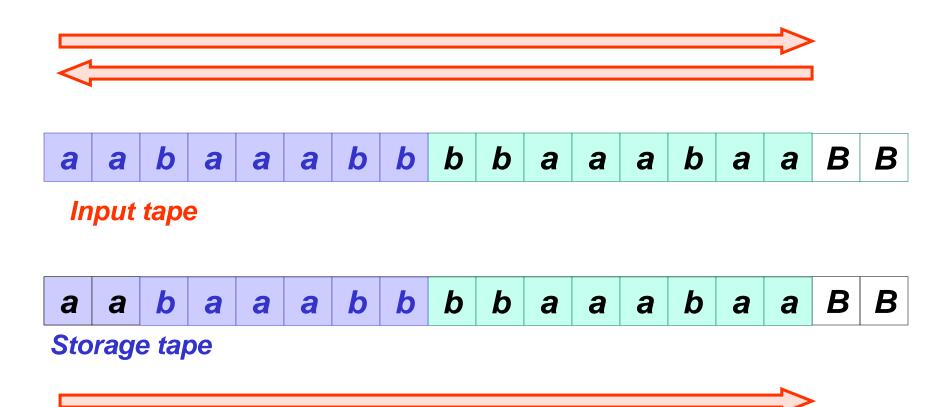


$$\{ ww^R / w \in (a+b)^* \}$$



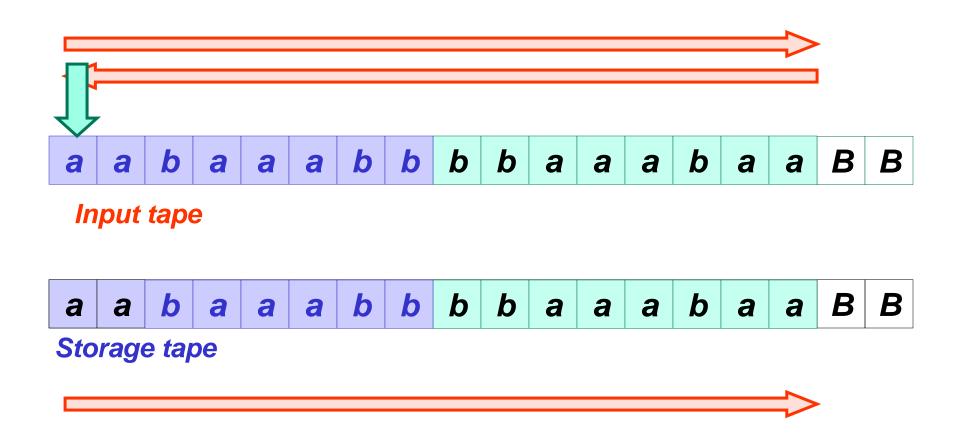


$$\{ ww^R / w \in (a+b)^* \}$$



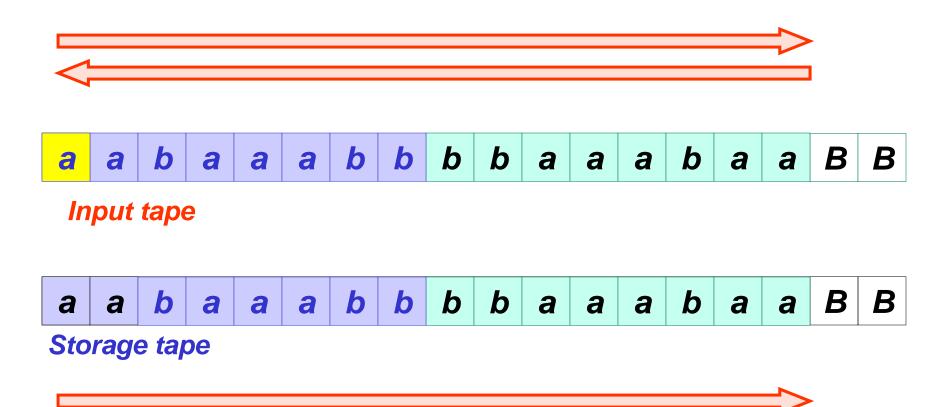


 $\{ ww^R / w \in (a+b)^* \}$ 



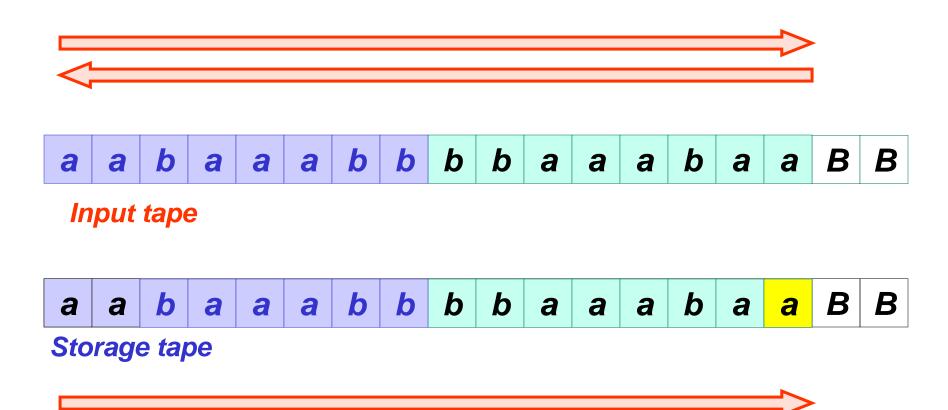


$$\{ ww^R / w \in (a+b)^* \}$$



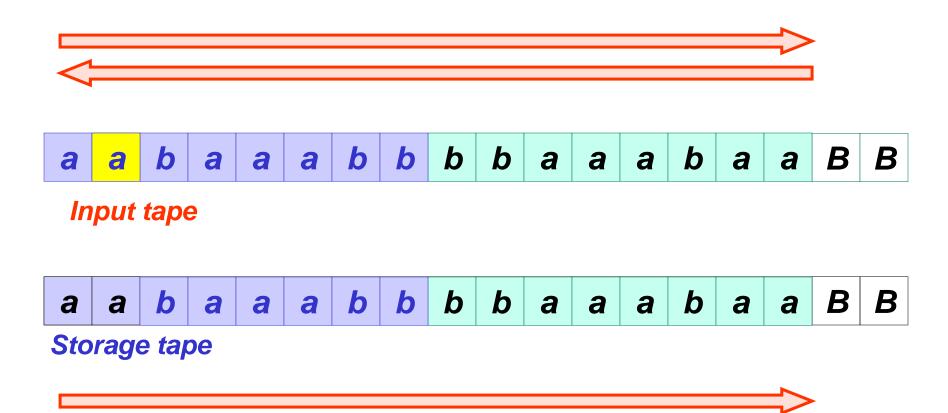


$$\{ ww^R / w \in (a+b)^* \}$$



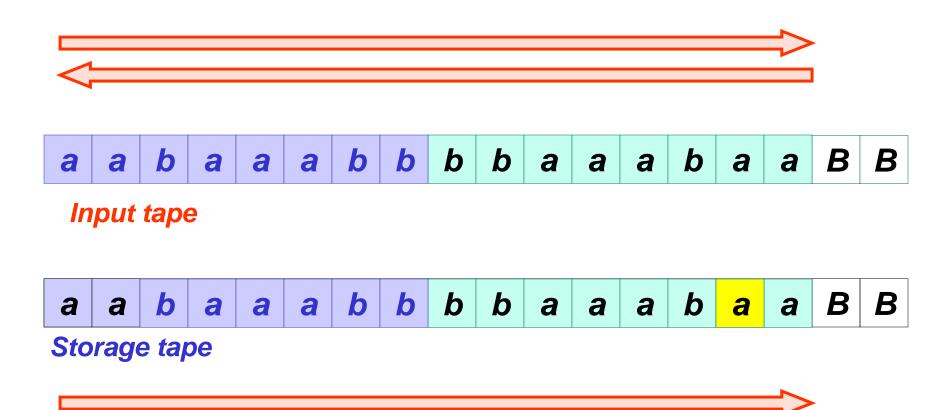


$$\{ ww^{R} / w \in (a+b)^{*} \}$$



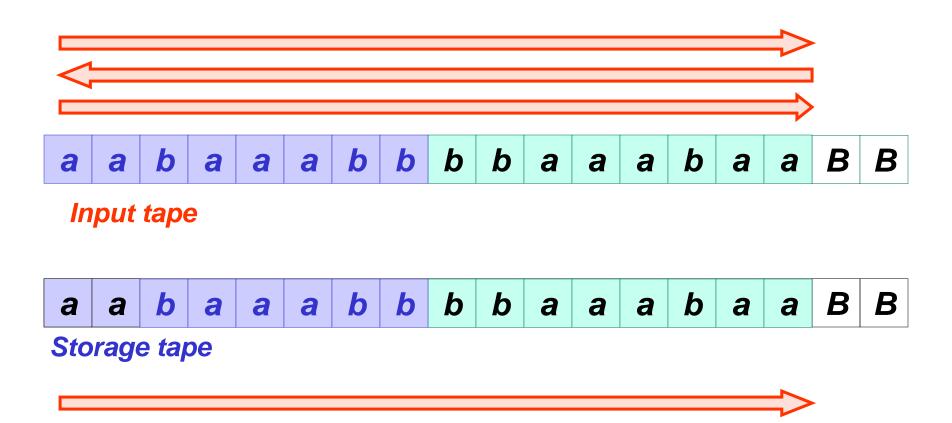


$$\{ ww^R / w \in (a+b)^* \}$$



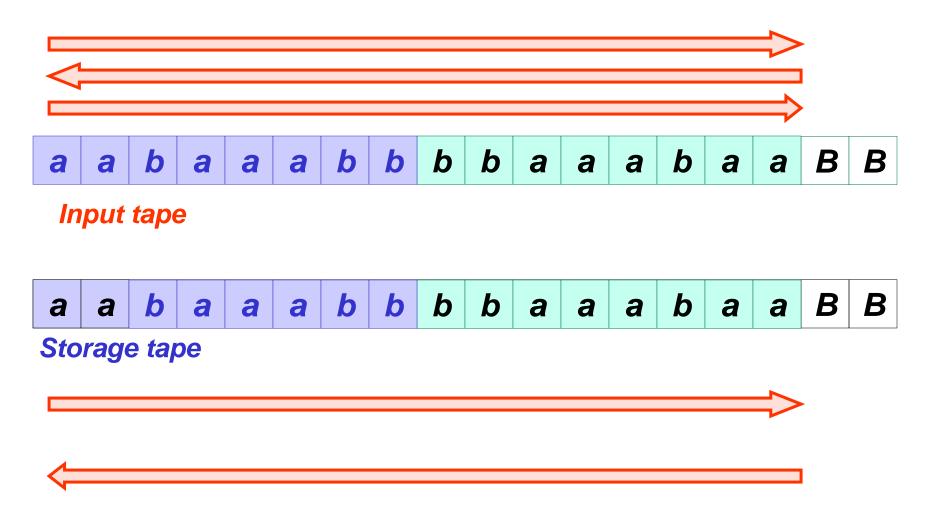


$$\{ ww^R / w \in (a+b)^* \}$$





$$\{ ww^R / w \in (a+b)^* \}$$







$$\delta(q, X) =$$



$$\delta(q, X) = \{$$



$$\delta(q, X) = \{ (p_1, Z_1, D_1), \}$$



$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., \}$$



$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., (p_k, Z_k, D_k) \}$$



$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., (p_k, Z_k, D_k) \}$$

# Equivalent to deterministic TM





$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., (p_k, Z_k, D_k) \}$$



$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., (p_k, Z_k, D_k) \}$$

$$(q, X)$$



$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., (p_k, Z_k, D_k) \}$$

$$(q, X)$$

$$(p_1, Z_1, D_1)$$

$$(p_1, X_1)$$



$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., (p_k, Z_k, D_k) \}$$

$$(q, X)$$

$$(p_1, Z_1, D_1) \qquad (p_2, Z_2, D_2)$$

$$(p_1, X_1) \qquad (p_2, X_2)$$



$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., (p_k, Z_k, D_k) \}$$

$$(q, X)$$

$$(p_1, Z_1, D_1) \qquad (p_2, Z_2, D_2) \qquad (p_k, Z_k, D_k)$$

$$(p_1, X_1) \qquad (p_2, X_2) \qquad ... \qquad (p_k, X_k)$$



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, Z_{1}, D_{1}) \qquad (p_{2}, Z_{2}, D_{2}) \qquad (p_{k}, Z_{k}, D_{k})$$

$$(p_{1}, X_{1}) \qquad (p_{2}, X_{2}) \qquad ... \qquad (p_{k}, X_{k})$$



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, Z_{1}, D_{1}) \qquad (p_{2}, Z_{2}, D_{2}) \qquad (p_{k}, Z_{k}, D_{k})$$

$$(p_{1}, X_{1}) \qquad (p_{2}, X_{2}) \qquad ... \qquad (p_{k}, X_{k})$$

$$(r_{1}, T_{1}, Y_{1})$$



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, Z_{1}, D_{1}) \qquad (p_{2}, Z_{2}, D_{2}) \qquad (p_{k}, Z_{k}, D_{k})$$

$$(p_{1}, X_{1}) \qquad (p_{2}, X_{2}) \qquad ... \qquad (p_{k}, X_{k})$$

$$(r_{1}, T_{1}, Y_{1}) \qquad (r_{2}, T_{2}, Y_{2})$$

$$(r_{1}, W_{1}) \qquad (r_{2}, W_{2})$$



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, Z_{1}, D_{1}) \qquad (p_{2}, Z_{2}, D_{2}) \qquad (p_{k}, Z_{k}, D_{k})$$

$$(p_{1}, X_{1}) \qquad (p_{2}, X_{2}) \qquad ... \qquad (p_{k}, X_{k})$$

$$(r_{1}, T_{1}, Y_{1}) \qquad (r_{2}, T_{2}, Y_{2}) \qquad (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}) \qquad (r_{2}, W_{2}) \qquad ... \qquad (r_{m}, W_{m})$$



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, Z_{1}, D_{1}) \qquad (p_{2}, Z_{2}, D_{2}) \qquad (p_{k}, Z_{k}, D_{k})$$

$$(p_{1}, X_{1}) \qquad (p_{2}, X_{2}) \qquad ... \qquad (p_{k}, X_{k})$$

$$(r_{1}, T_{1}, Y_{1}) \qquad (r_{2}, T_{2}, Y_{2}) \qquad (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}) \qquad (r_{2}, W_{2}) \qquad ... \qquad (r_{m}, W_{m})$$



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, Z_{1}, D_{1}), (p_{2}, Z_{2}, D_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(p_{1}, X_{1}), (p_{2}, X_{2}), ..., (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}), (r_{2}, W_{2}), ..., (r_{m}, T_{m}, Y_{m})$$



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, X_{1}) \qquad (p_{2}, X_{2}) \qquad ... \qquad (p_{k}, X_{k})$$

$$(r_{1}, T_{1}, Y_{1}) \qquad (r_{2}, T_{2}, Y_{2}) \qquad ... \qquad (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}) \qquad (r_{2}, W_{2}) \qquad ... \qquad (r_{m}, W_{m})$$



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, X_{1}) \qquad (p_{2}, X_{2}) \qquad ... \qquad (p_{k}, X_{k})$$

$$(r_{1}, T_{1}, Y_{1}) \qquad (r_{2}, T_{2}, Y_{2}) \qquad ... \qquad (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}) \qquad (r_{2}, W_{2}) \qquad ... \qquad (r_{m}, W_{m})$$

TS that never halts



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, X_{1}) \qquad (p_{2}, X_{2}) \qquad ... \qquad (p_{k}, X_{k})$$

$$(r_{1}, T_{1}, Y_{1}) \qquad (r_{2}, T_{2}, Y_{2}) \qquad ... \qquad (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}) \qquad (r_{2}, W_{2}) \qquad ... \qquad (r_{m}, W_{m})$$

TS that never halts

⇒ Infinite depth tree



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, X_{1}), (p_{2}, X_{2}), ..., (p_{k}, Z_{k}, D_{k})$$

$$(p_{1}, X_{1}), (p_{2}, X_{2}), ..., (p_{k}, Z_{k}, D_{k})$$

$$(r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}), (r_{2}, W_{2}), ..., (r_{m}, T_{m}, Y_{m})$$



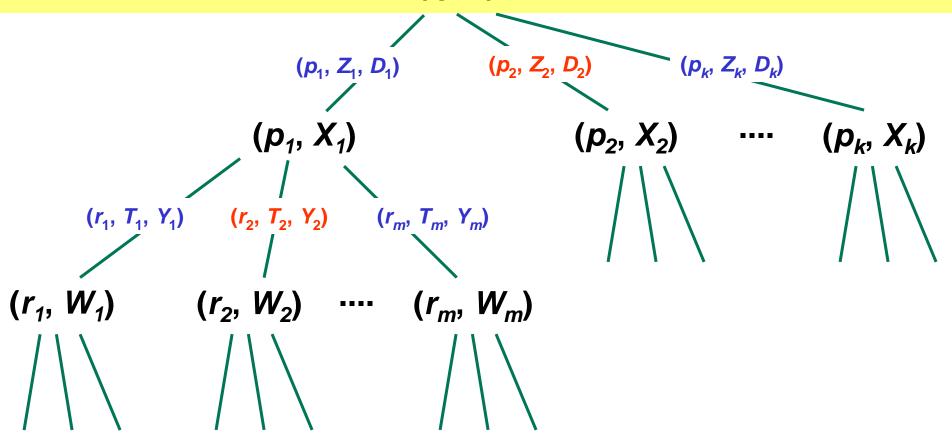
⇒ Infinite depth tree

⇒ Breadth first search



$$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_m, T_m, Y_m) \}$$

(q, X)



TS that never halts

⇒ Infinite depth tree

⇒ Breadth first search



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, X_{1}), (p_{2}, X_{2}), ..., (p_{k}, Z_{k}, D_{k})$$

$$(p_{1}, X_{1}), (p_{2}, X_{2}), ..., (p_{k}, Z_{k}, D_{k})$$

$$(r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}), (r_{2}, W_{2}), ..., (r_{m}, W_{m})$$

⇒ Infinite depth tree

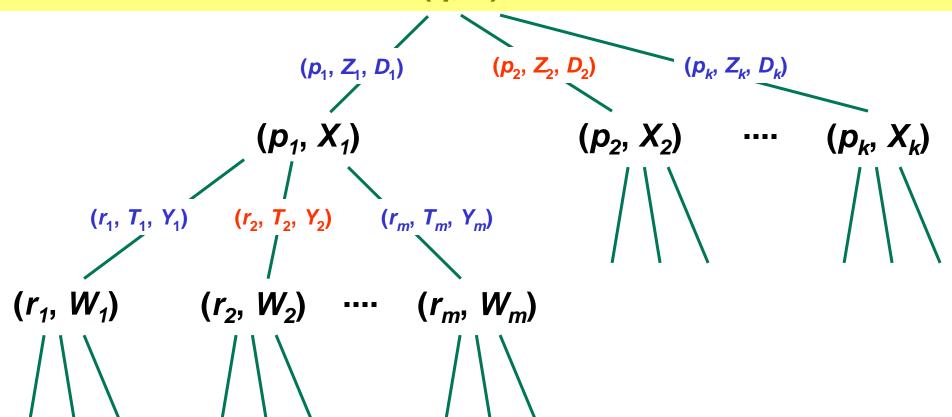


⇒ Breadth first search

TS that never halts

$$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_m, T_m, Y_m) \}$$

(q, X)



TS that never halts

⇒ Infinite depth tree

⇒ Breadth first search



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, X_{1}) \qquad (p_{2}, X_{2}) \qquad ... \qquad (p_{k}, X_{k})$$

$$(r_{1}, T_{1}, Y_{1}) \qquad (r_{2}, T_{2}, Y_{2}) \qquad ... \qquad (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}) \qquad (r_{2}, W_{2}) \qquad ... \qquad (r_{m}, W_{m})$$

⇒ Infinite depth tree



⇒ Breadth first search

TS that never halts

$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, X_{1}), (p_{2}, X_{2}), ..., (p_{k}, Z_{k}, D_{k})$$

$$(p_{1}, X_{1}), (p_{2}, X_{2}), ..., (p_{k}, X_{k})$$

$$(r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}), (r_{2}, W_{2}), ..., (r_{m}, W_{m})$$

⇒ Infinite depth tree

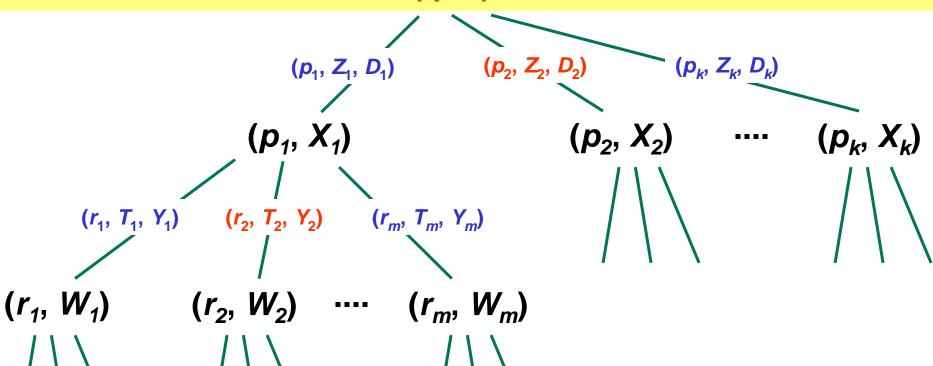


⇒ Breadth first search

TS that never halts

$$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_m, T_m, Y_m) \}$$

(q, X)



TS that never halts

⇒ Infinite depth tree

⇒ Breadth first search



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, X_{1}) \qquad (p_{2}, X_{2}) \qquad ... \qquad (p_{k}, X_{k})$$

$$(r_{1}, T_{1}, Y_{1}) \qquad (r_{2}, T_{2}, Y_{2}) \qquad ... \qquad (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}) \qquad (r_{2}, W_{2}) \qquad ... \qquad (r_{m}, W_{m})$$

⇒ Infinite depth tree



⇒ Breadth first search

TS that never halts

$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, X_{1}), (p_{2}, X_{2}), ..., (p_{k}, X_{k})$$

$$(p_{1}, X_{1}), (p_{2}, X_{2}), ..., (p_{k}, X_{k})$$

$$(r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}), (r_{2}, W_{2}), ..., (r_{m}, W_{m})$$





⇒ Breadth first search



$$\delta(p_{1}, X_{1}) = \{ (r_{1}, T_{1}, Y_{1}), (r_{2}, T_{2}, Y_{2}), ..., (r_{m}, T_{m}, Y_{m}) \}$$

$$(q, X)$$

$$(p_{1}, X_{1}) \qquad (p_{2}, X_{2}) \qquad ... \qquad (p_{k}, X_{k})$$

$$(r_{1}, T_{1}, Y_{1}) \qquad (r_{2}, T_{2}, Y_{2}) \qquad ... \qquad (r_{m}, T_{m}, Y_{m})$$

$$(r_{1}, W_{1}) \qquad (r_{2}, W_{2}) \qquad ... \qquad (r_{m}, W_{m})$$

TS that never halts

⇒ Infinite depth tree

⇒ Breadth first search



$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., (p_k, Z_k, D_k) \}$$



$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., (p_k, Z_k, D_k) \}$$

$$r = max( card( \delta(q_i, X_i)))$$



$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., (p_k, Z_k, D_k) \}$$

$$r = max( card ( \delta(q_i, X_j) ) )$$

$$n_1, n_2, ..., n_m$$



$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., (p_k, Z_k, D_k) \}$$

$$r = max( card ( \delta(q_i, X_j) ) )$$

$$n_1, n_2, ..., n_m$$
  
 $1 \le n_i \le k$ 



$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), ..., (p_k, Z_k, D_k) \}$$

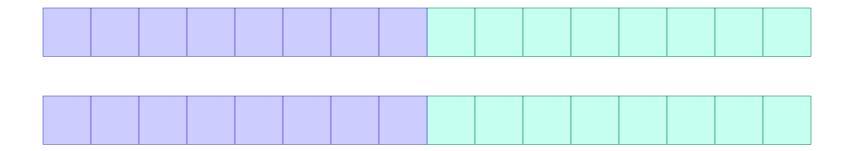
$$r = max( card ( \delta(q_i, X_j) ) )$$

$$n_1, n_2, ..., n_m$$
 $1 \le n_i \le k$ 
 $1 \le k \le r$ 

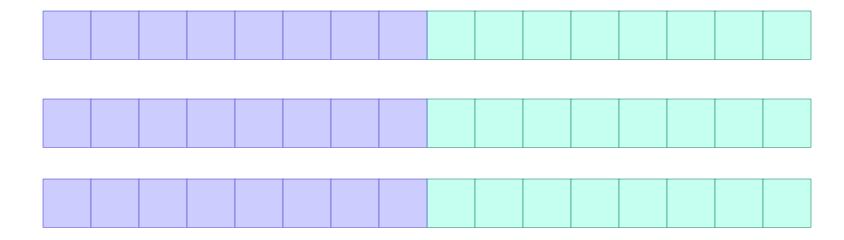




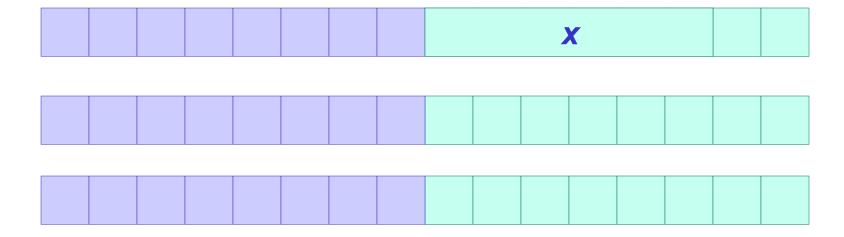




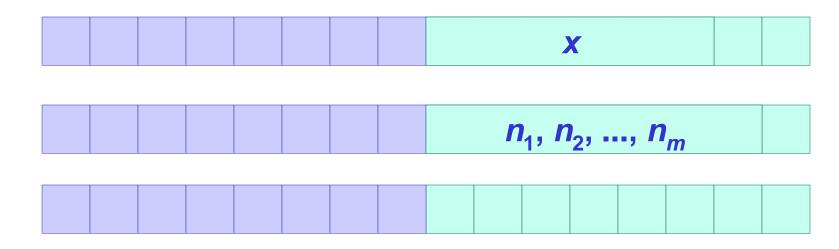




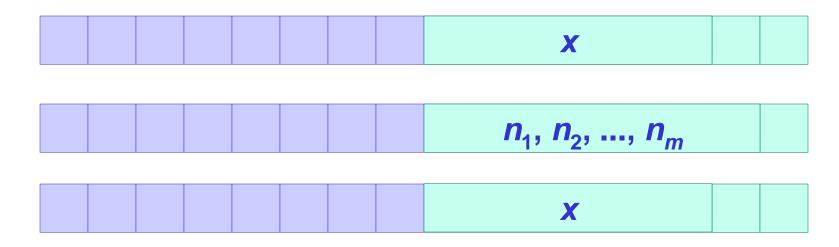




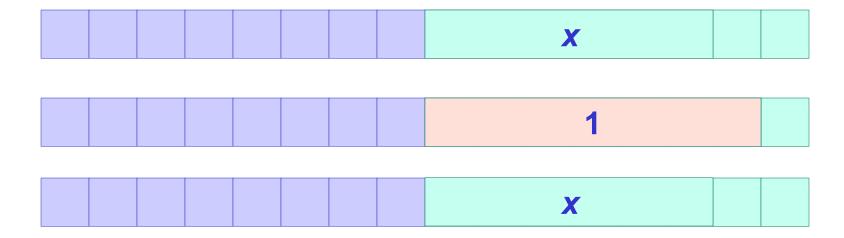




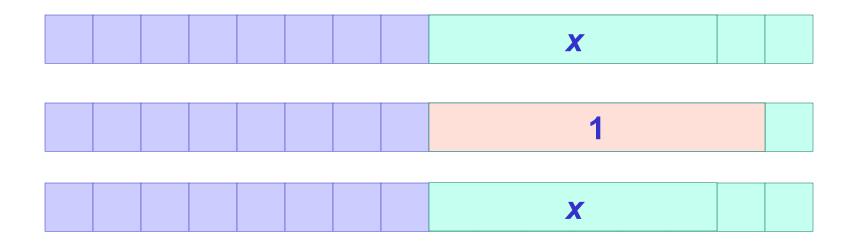








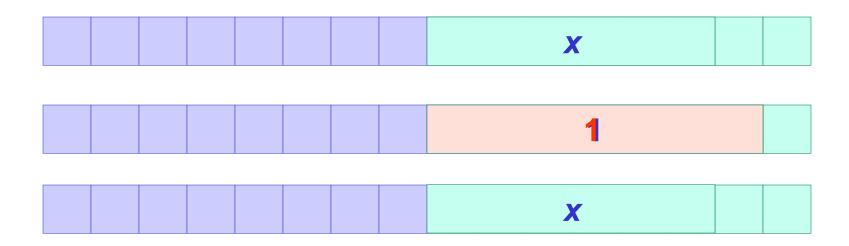




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

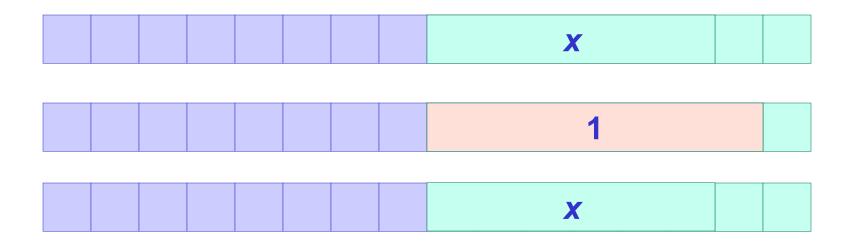




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

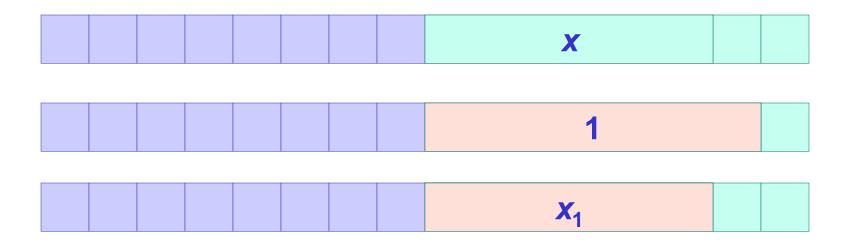




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

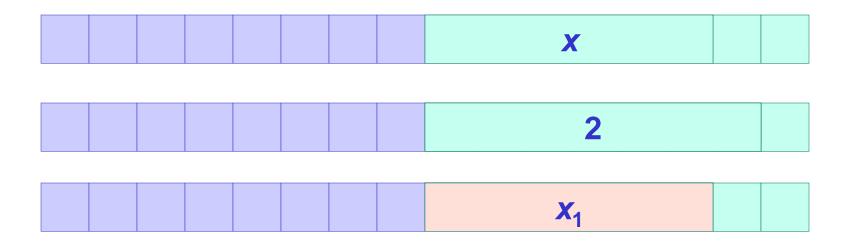




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

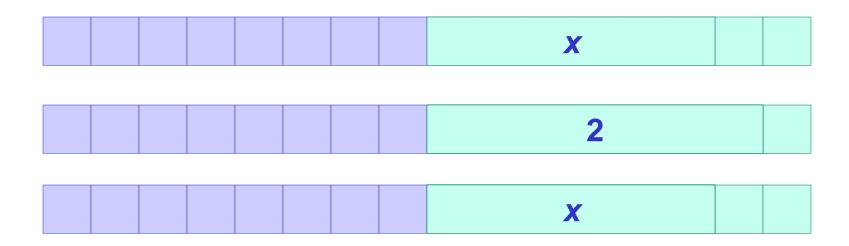




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

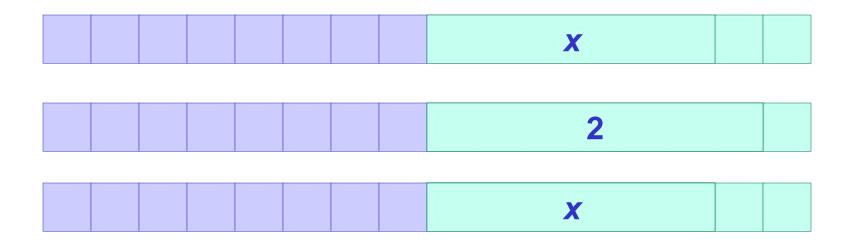




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

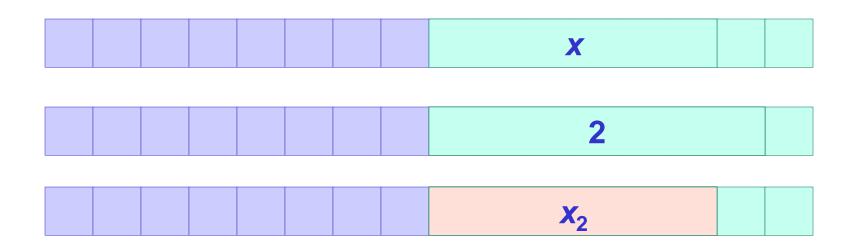




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

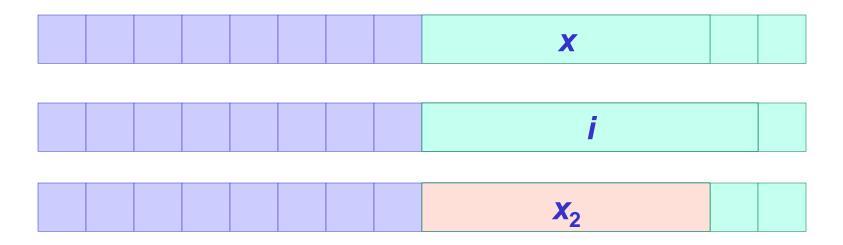




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

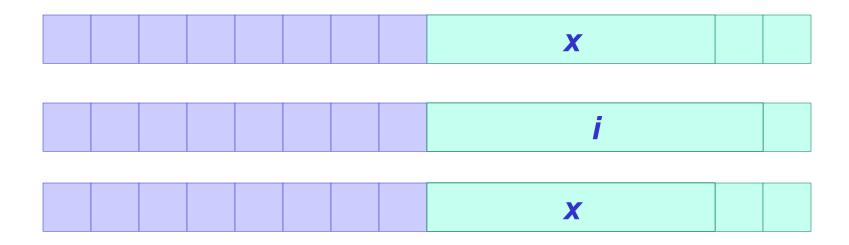




$$\delta(q_1, X_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

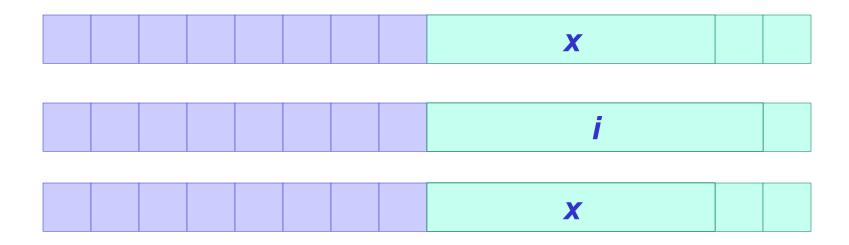




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

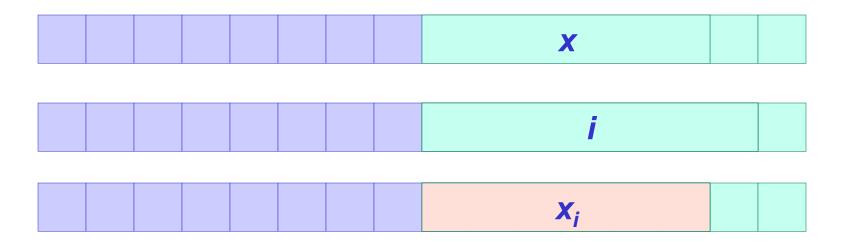
$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$





$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

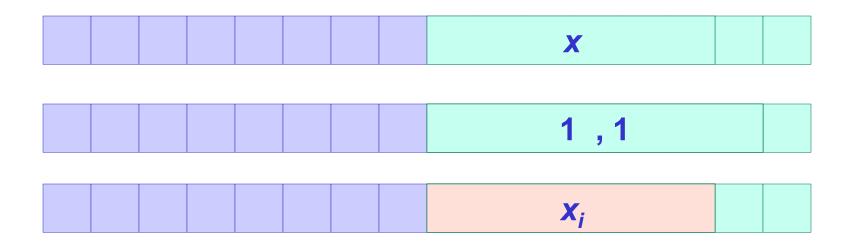




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

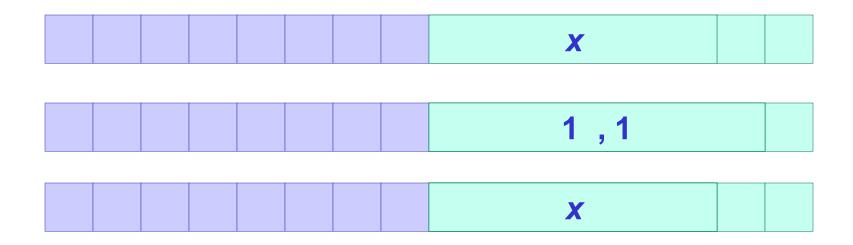




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

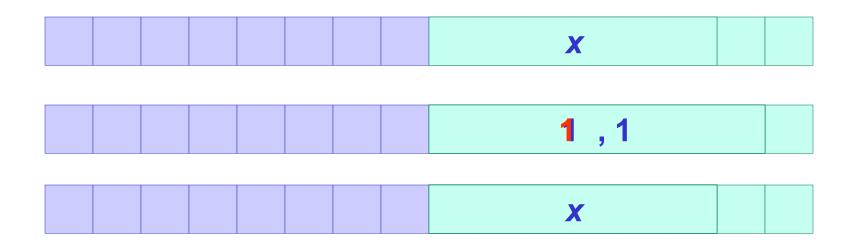




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

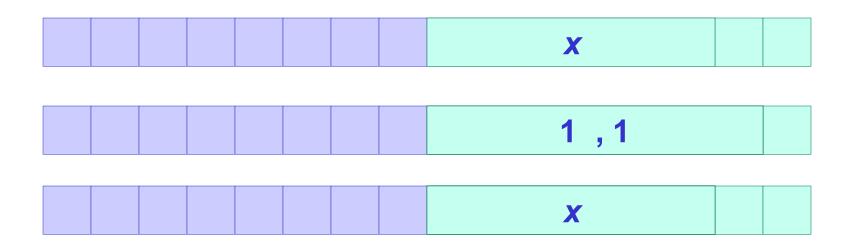




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

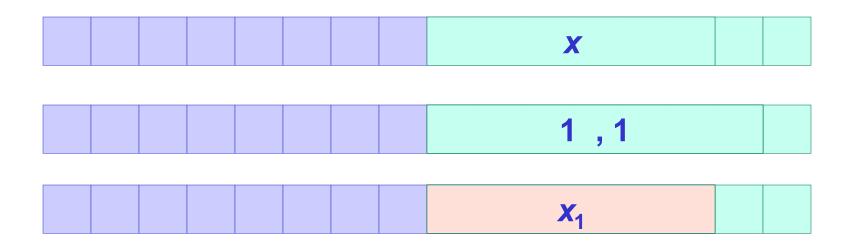




$$\delta(q_1, X_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

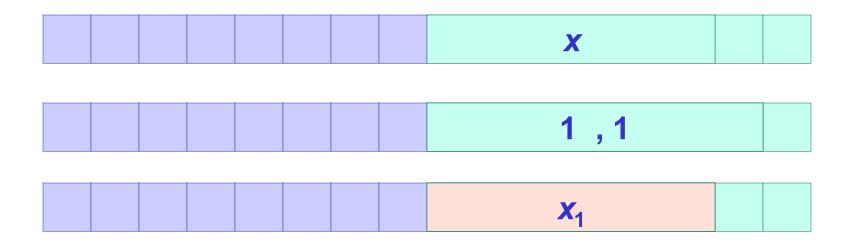




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

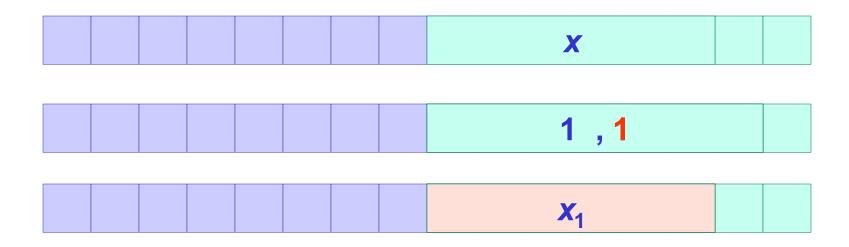




$$\delta(p_1, X_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

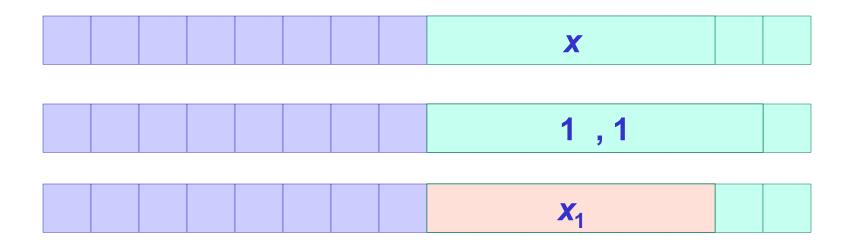




$$\delta(\boldsymbol{p}_1,\,\boldsymbol{X}_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

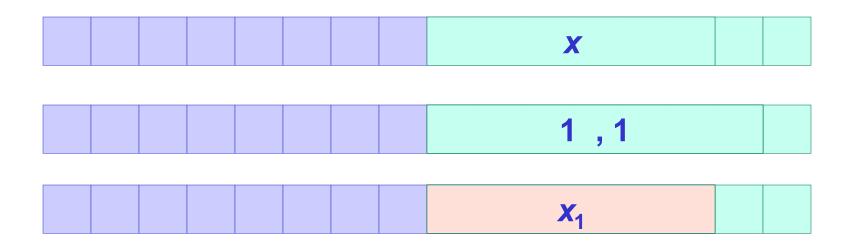




$$\delta(p_1, X_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

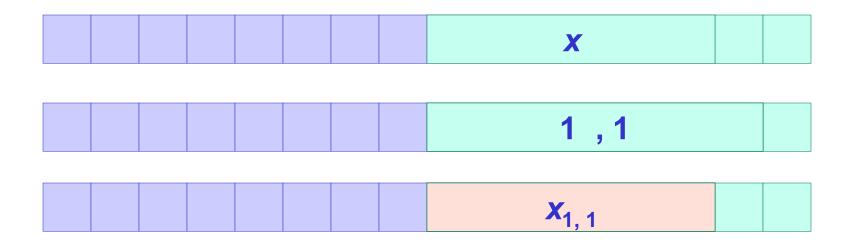




$$\delta(\boldsymbol{p}_1,\,\boldsymbol{X}_2) =$$

$$\{ (r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k) \}$$

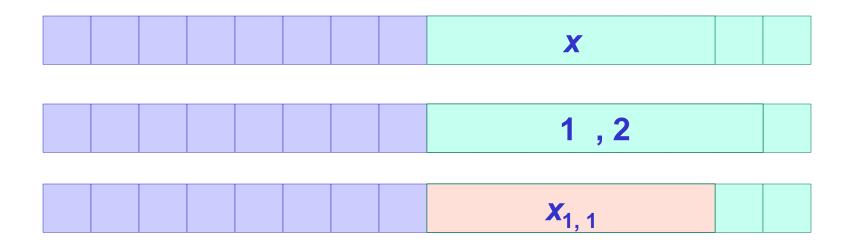




$$\delta(p_1, X_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

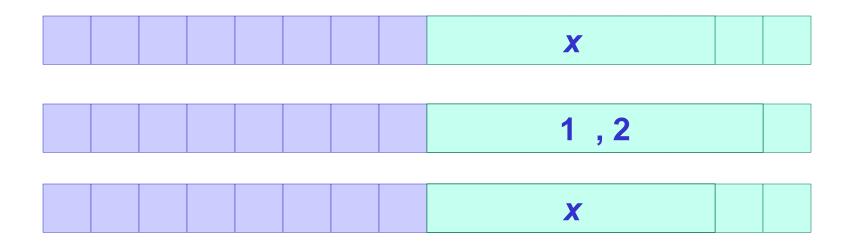




$$\delta(p_1, X_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

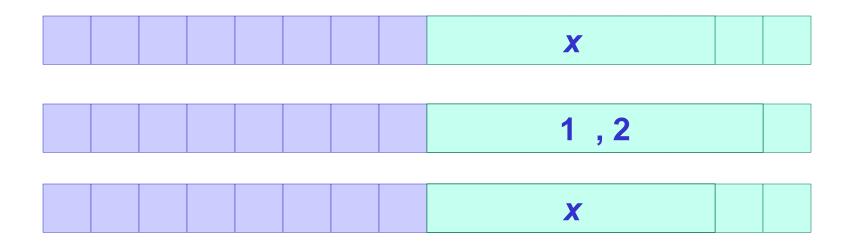




$$\delta(p_1, X_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

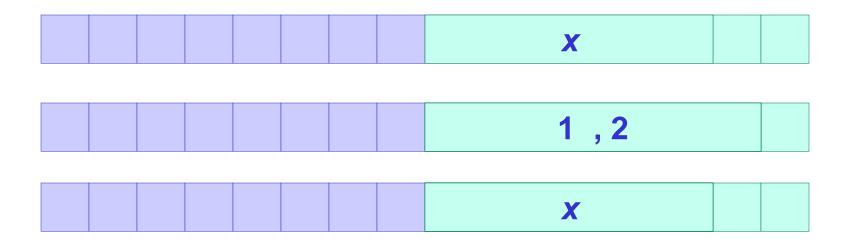




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

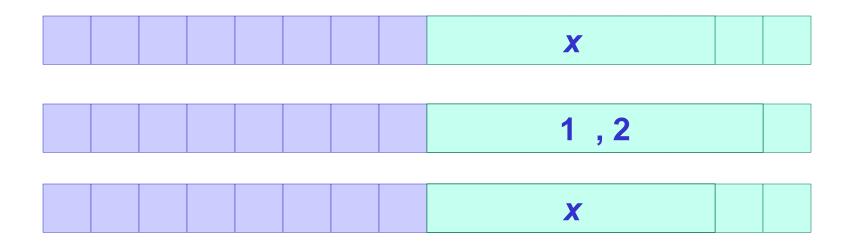




$$\delta(\boldsymbol{q}_1,\,\boldsymbol{X}_1) =$$

$$\{(p_1, Z_1, W_1), (p_2, Z_2, W_2), ..., (p_{ni}, Z_{ni}, W_{ni}), ..., (p_k, Z_k, W_k)\}$$

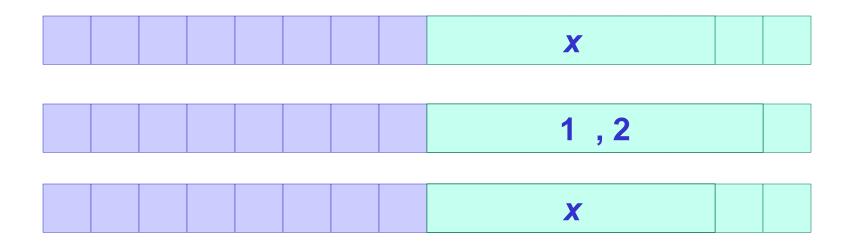




$$\delta(p_1, X_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

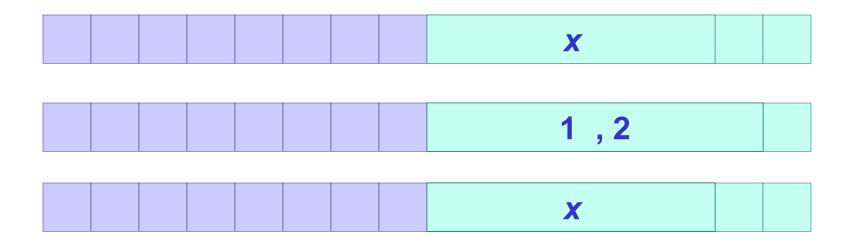




$$\delta(\boldsymbol{p}_1,\,\boldsymbol{X}_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

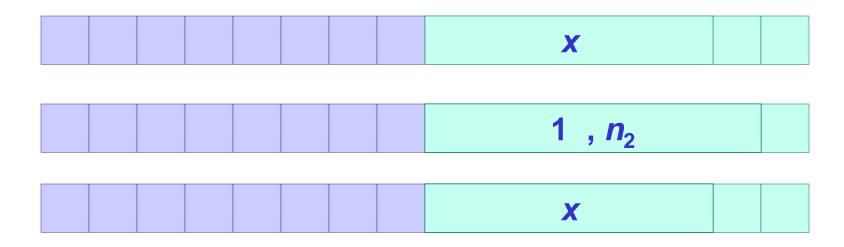




$$\delta(\boldsymbol{p}_1,\,\boldsymbol{X}_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

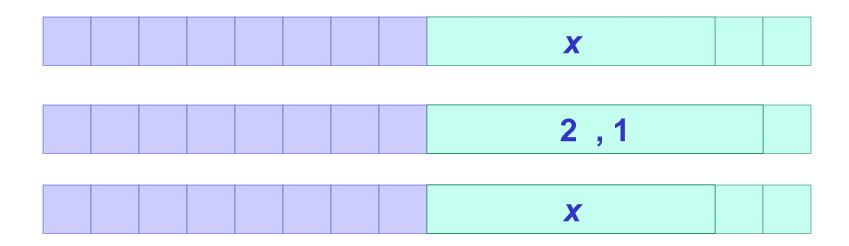




$$\delta(\boldsymbol{p}_1,\,\boldsymbol{X}_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

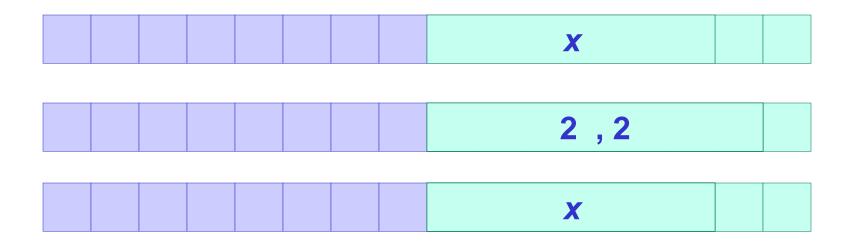




$$\delta(\boldsymbol{p}_1,\,\boldsymbol{X}_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

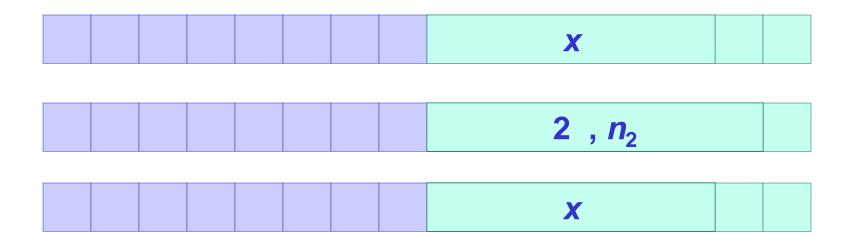




$$\delta(\boldsymbol{p}_1,\,\boldsymbol{X}_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

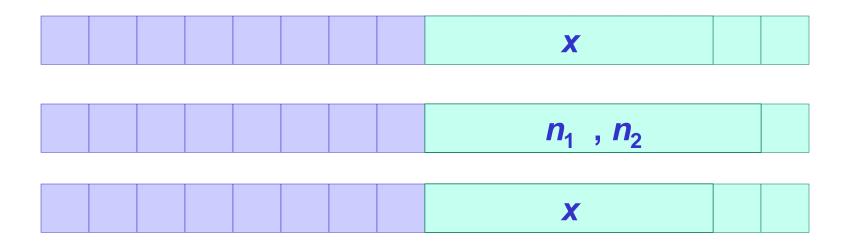




$$\delta(\boldsymbol{p}_1,\,\boldsymbol{X}_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

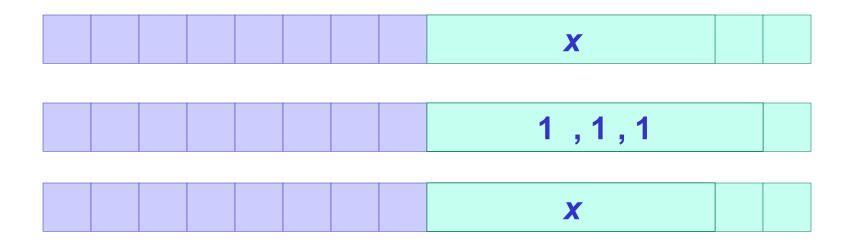




$$\delta(\boldsymbol{p}_1,\,\boldsymbol{X}_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

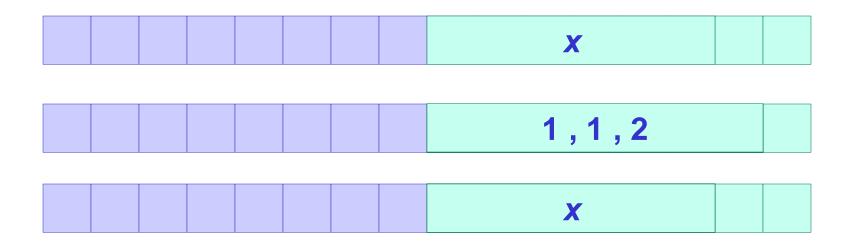




$$\delta(\boldsymbol{p}_1,\,\boldsymbol{X}_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

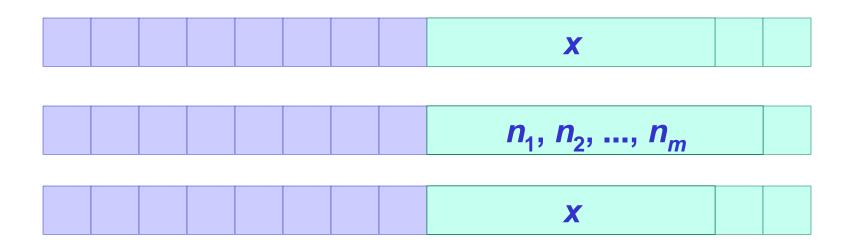




$$\delta(\boldsymbol{p}_1,\,\boldsymbol{X}_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$





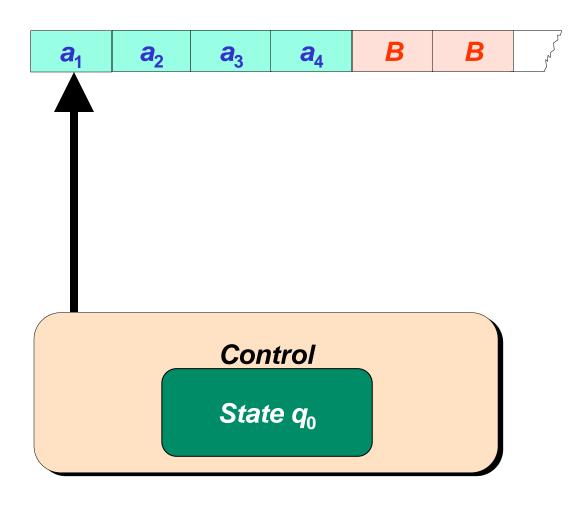
$$\delta(\boldsymbol{p}_1,\,\boldsymbol{X}_2) =$$

$$\{(r_1, T_1, Y_1), (r_2, T_2, Y_2), ..., (r_{ni}, T_{ni}, Y_{ni}), ..., (r_k, T_k, Y_k)\}$$

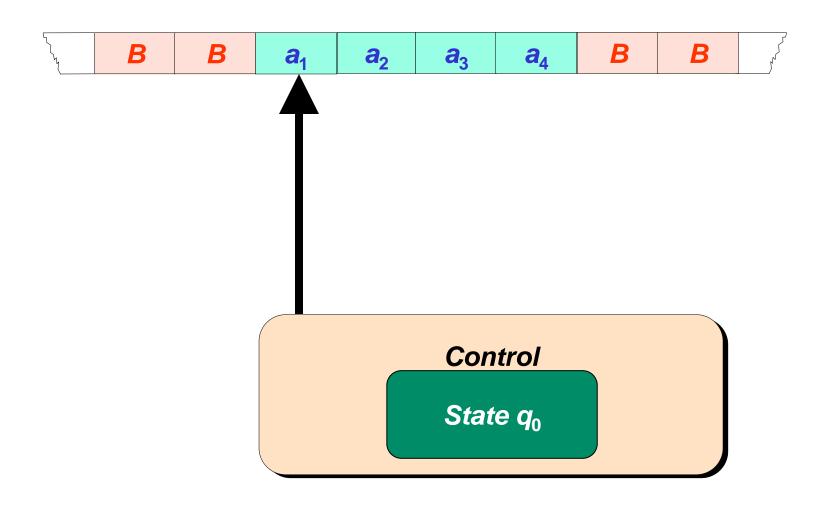


# TM with a multidimensional input

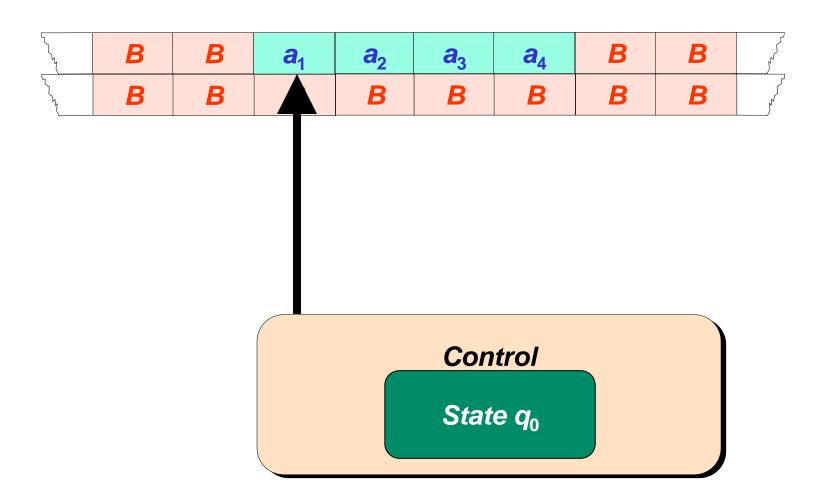












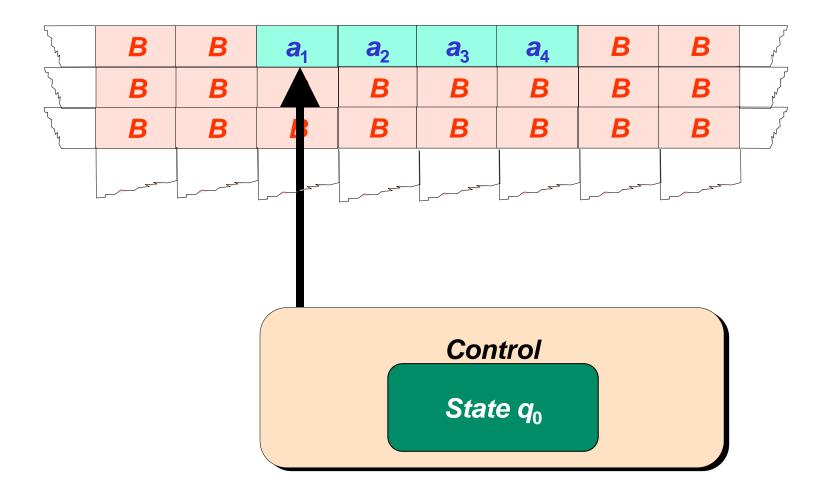




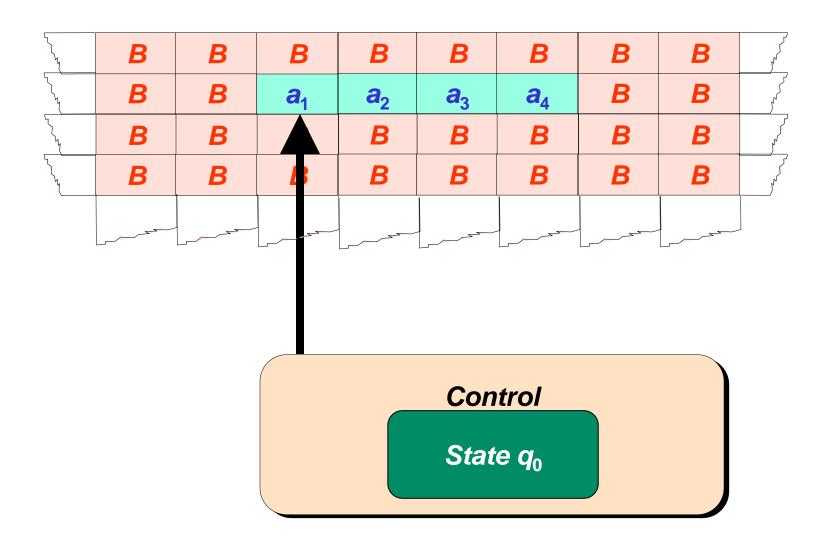
Control

State q<sub>0</sub>

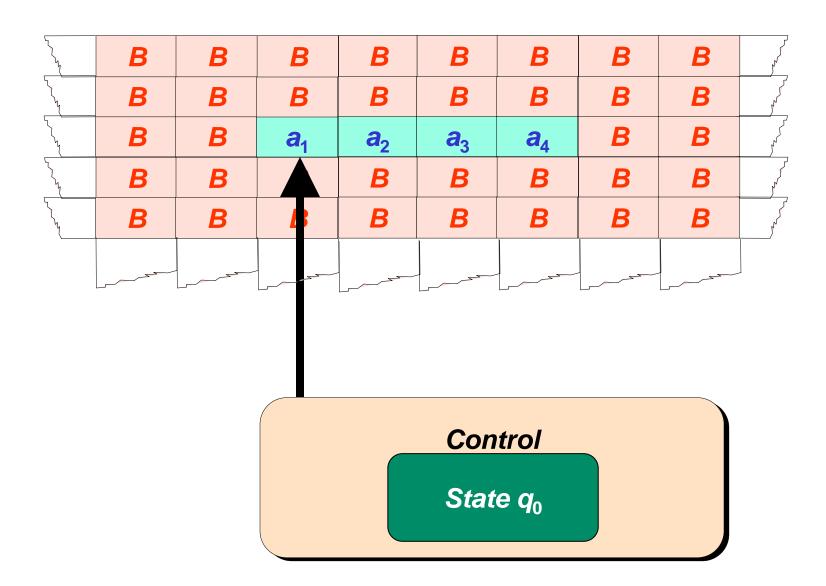




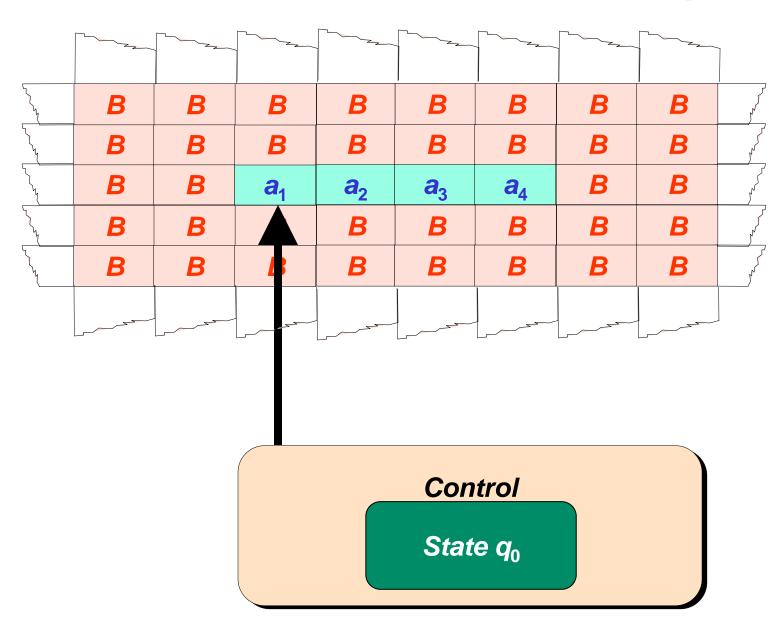
















В	В	В	a <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>



			1			
В	В	В	<b>a</b> <sub>1</sub>	В	В	B
B	В	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	<b>a</b> <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>



В	В	В	a <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	<b>a</b> <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	a <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	<b>a</b> <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \*



В	В	В	a <sub>1</sub>	В	В	В
В	B	a <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \*



В	В	В	a <sub>1</sub>	В	В	В
В	B	a <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub> \*\*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	B	<b>a</b> <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\*  $BBBa_1BBB$  \*  $BBa_2a_3a_4a_5B$  \*  $a_6a_7a_8a_9Ba_{10}B$  \*  $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$  \*\*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub> \*\*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	<b>a</b> <sub>3</sub>	a <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub> \*\*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	B	<b>a</b> <sub>2</sub>	<b>a</b> <sub>3</sub>	a <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub> \*\*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	B	<b>a</b> <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub> \*\*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	B	<b>a</b> <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	B	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	B	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	B	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>\*\*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>\*\*

Horizontal movement within the rectangle Vertical movement within the rectangle



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>\*\*

Horizontal movement within the rectangle Vertical movement within the rectangle



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	B	a <sub>2</sub>	<b>a</b> <sub>3</sub>	$a_4$	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>\*\*

Horizontal movement within the rectangle Vertical movement within the rectangle



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	$a_3$	$a_4$	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>\*\*

Horizontal movement within the rectangle Vertical movement within the rectangle



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	B	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

Horizontal movement within the rectangle Vertical movement within the rectangle



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	B	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

Horizontal movement within the rectangle Vertical movement within the rectangle



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	B	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

Horizontal movement within the rectangle Vertical movement within the rectangle



В	В	В	a <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

Horizontal movement within the rectangle Vertical movement within the rectangle



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	$a_3$	a <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>\*\*

Horizontal movement within the rectangle Vertical movement within the rectangle



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	B	a <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>\*\*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>\*\*



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	B	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>
			В			

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub> \*\*



В	В	В	a <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	B	<b>a</b> <sub>10</sub>	В
В	<b>a</b> <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>
В	В	В	В	В	В	В

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>\*\*



В	В	В	a <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	B
В	<b>a</b> <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>
В	В	В	В	В	В	В

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	B	<b>a</b> <sub>10</sub>	В
В	<b>a</b> <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>
В	В	В	В	В	В	В



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	$a_9$	B	<b>a</b> <sub>10</sub>	В
В	<b>a</b> <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>
В	В	В	В	В	В	В



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	$a_9$	B	<b>a</b> <sub>10</sub>	В
В	<b>a</b> <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	B	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>
В	В	В	В	В	В	В



В	В	В	a <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	B
В	<b>a</b> <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>
В	В	В	В	В	В	В



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	$a_3$	a <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>



В	В	В	<b>a</b> <sub>1</sub>	В	В	В
В	В	<b>a</b> <sub>2</sub>	$a_3$	a <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>



В	В	В	a <sub>1</sub>	В	В	В
В	В	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	B	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>



В	В	В	<b>a</b> <sub>1</sub>	В	В	В	
В	В	<b>a</b> <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В	
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В	
В	<b>a</b> <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	B	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>	В

\*\*  $BBBa_1BBB$  \*  $BBa_2a_3a_4a_5B$  \*  $a_6a_7a_8a_9Ba_{10}B$  \*  $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ 



В	В		<b>a</b> <sub>1</sub>	В	В	В	В
В	В	<b>a</b> <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В	В
<b>a</b> <sub>6</sub>			<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>				В

\*\* BBBa<sub>1</sub>BBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>



В	В	В	<b>a</b> <sub>1</sub>	В	В	В	В
В	В	<b>a</b> <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	B	В
<b>a</b> <sub>6</sub>			<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>	В

\*\* BBBa<sub>1</sub>BBBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>B \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>



В	В	В	<b>a</b> <sub>1</sub>	В	В	В	В
В	B	<b>a</b> <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>			В		В	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>	В

\*\* BBBa<sub>1</sub>BBBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>BB \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>B \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>



В	В	В	<b>a</b> <sub>1</sub>	В	В	В	В
В	В	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В	В
<b>a</b> <sub>6</sub>			<b>a</b> <sub>9</sub>		<b>a</b> <sub>10</sub>	В	В
В	a <sub>11</sub>		<b>a</b> <sub>13</sub>				В

\*\* BBBa<sub>1</sub>BBBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>BB \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>BB \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>



В	В	В	<b>a</b> <sub>1</sub>	В	В	В	В
В	B	<b>a</b> <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>			В		В	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>	В

\*\* BBBa<sub>1</sub>BBBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>BB \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>BB \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>B



В	В	В	<b>a</b> <sub>1</sub>	В	В	В	В
В	В	<b>a</b> <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	B	В
<b>a</b> <sub>6</sub>			<b>a</b> <sub>9</sub>	В	<b>a</b> <sub>10</sub>	В	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>	В

\*\* BBBa<sub>1</sub>BBBB \* BBa<sub>2</sub>a<sub>3</sub>a<sub>4</sub>a<sub>5</sub>BB \* a<sub>6</sub>a<sub>7</sub>a<sub>8</sub>a<sub>9</sub>Ba<sub>10</sub>BB \* Ba<sub>11</sub>a<sub>12</sub>a<sub>13</sub>Ba<sub>14</sub>a<sub>15</sub>B



В	В	В	<b>a</b> <sub>1</sub>	В	В	В	В
В	B	<b>a</b> <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В	В
<b>a</b> <sub>6</sub>	<b>a</b> <sub>7</sub>			В		В	В
В	a <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	В	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>	В



В	В	В	a <sub>1</sub>	В	В	В	В
В	В	<b>a</b> <sub>2</sub>	$a_3$	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В	В
<b>a</b> <sub>6</sub>	a <sub>7</sub>	<b>a</b> <sub>8</sub>	<b>a</b> <sub>9</sub>		<b>a</b> <sub>10</sub>	В	В
В	<b>a</b> <sub>11</sub>	<b>a</b> <sub>12</sub>	<b>a</b> <sub>13</sub>	B	<b>a</b> <sub>14</sub>	<b>a</b> <sub>15</sub>	В



В	В	В	a <sub>1</sub>	В	В	В	В
В	В	a <sub>2</sub>	<b>a</b> <sub>3</sub>	<b>a</b> <sub>4</sub>	<b>a</b> <sub>5</sub>	В	В
<b>a</b> <sub>6</sub>			<b>a</b> <sub>9</sub>		<b>a</b> <sub>10</sub>	В	В
В	a <sub>11</sub>		<b>a</b> <sub>13</sub>				В

