Duault,
Chloé

MS - Homework

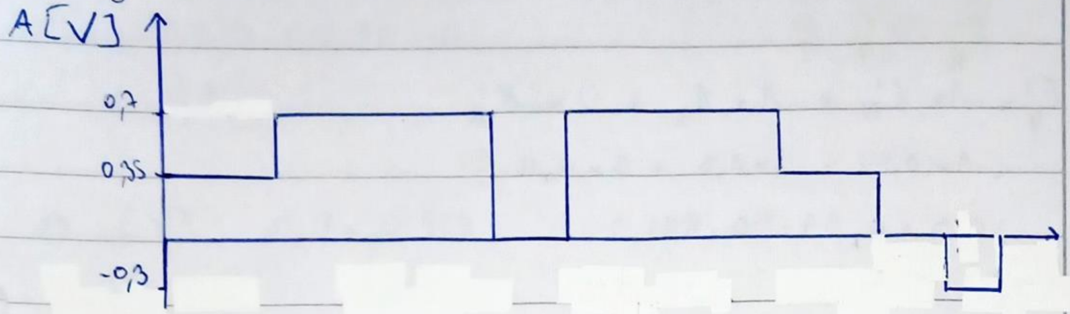
1)

a) 1-L_u = Total number of lines 819 $\qquad$ f_v = Vertical frequency = 50 Hz

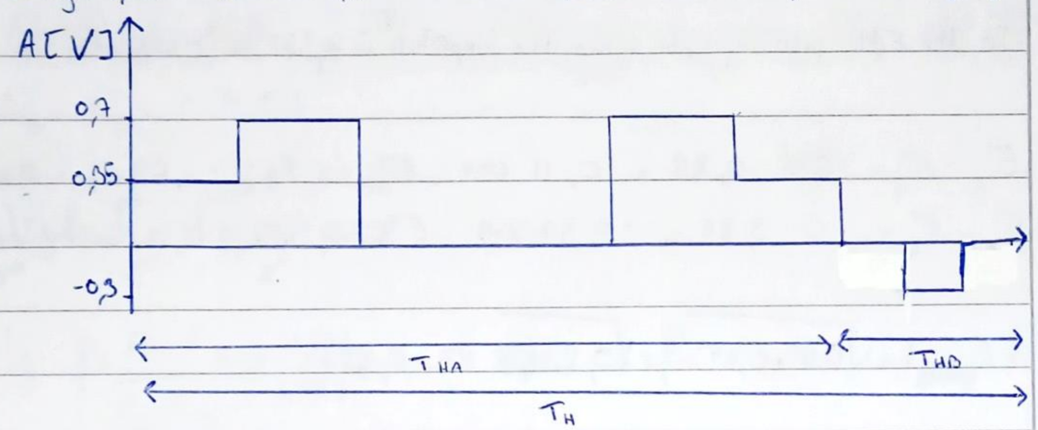
We have interlace $\qquad$ L_v = Number of active lines = 737

Duration of the horizontal blanking interval: 8,04 µs = T_{HB}

Gray = 1,5 cm $\quad$ White = 3 cm $\quad$ Black = 1cm $\quad$ White = 3cm $\quad$ Gray = 1,5cm



Gray = 1,5cm $\quad$ White = 1,75cm $\quad$ Black = 3,5cm $\quad$ White = 1,75cm $\quad$ Gray = 1,5cm



$T_H = 1/f_H \qquad f_H = L_u \cdot f_s \qquad f_s = f_v/2$

$f_s = 50/2 = 25\text{ Hz} \qquad f_H = L_u \cdot f_s = 819 \times 25 = 20\,475 \qquad T_H = 1/f_H = 1/20475 \approx 4{,}884 \cdot 10^{-5}$ sec

$= 48{,}84$ µsec

$T_{HA} = T_H - T_{HB} = 48.84 - 8{,}04 = 40{,}8$ µs

b) Duration of active part of the frame:

$T_{SA} = L_v \cdot T_{HA} = 737 \cdot 40{,}8 = 30\,069{,}6$ µs $= 30{,}0696$ ms

2)

For a yellow image: $E'_G = 1 \quad E'_R = 1 \quad E'_B = 0$

EBU / $E'_y = 0,30\, E'_R + 0,59\, E'_G + 0,11\, E'_B$

In the EBU primary color system:

$E'_y = 1 \times E'_G + 1 \times E'_R + 0 \times E'_B$

$= 1 \times 0,59 + 1 \times 0,3 + 0 \times 0,11$

$= 0,3 + 0,59 = 0,89$

$\Rightarrow$ [v] $0,89 \times 0,7 = 0,623$ volt

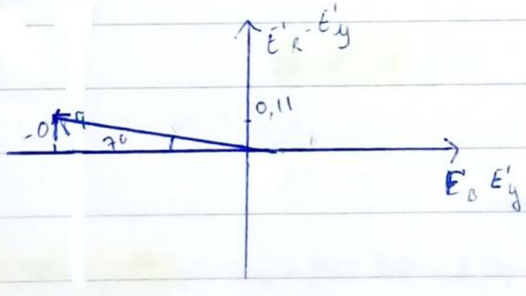
$E'_R - E'_y = 1 - 0,89 = 0,11 \iff E'_R - 0,3E'_R - 0,59E'_G - 0,11E'_B$ $\quad E'_G \approx 1, \; E'_B \approx 0$

$E'_B - E'_y = 0 - 0,89 = -0,89 \iff E'_B - 0,3E'_R - 0,59E'_G - 0,11E'_B$ $\quad E'_R \approx 1, \; E'_G \approx 1, \; E'_B \approx 0$

Amplitude = $|E_{yellow}| = \sqrt{0,11^2 + 0,89^2} = \sqrt{0,8042} = 0,897$

Color hue $\quad tg\,\varphi = \dfrac{E'_R - E'_y}{E'_B - E'_y} = \dfrac{0,11}{-0,89} \simeq -7,046°$ $\qquad \varphi = -7,046°$

Draw the color vector display:



In ITU-R BT. 709 primary color system.

$$E'_y = 0,213\,E'_R + 0,715\,E'_G + 0,072\,E'_B$$

$$E'_y = 1 \times E'_G + 1 \times E'_R + 0 \times E'_B$$

$$= 0,213 + 0,715 + 0$$

$$= 0,928$$

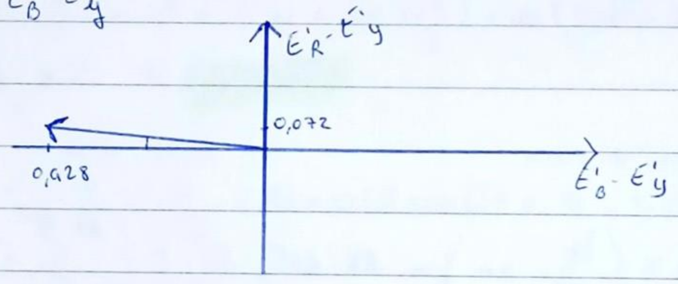
$\Rightarrow [v] = 0,7 \times 0,928 = 0,6496$ volt

$$E'_R - E'_y = 1 - 0,928 = 0,072$$

$$E'_B - E'_y = -0,928$$

Amplitude $= |E_{yellow}| = \sqrt{0,072^2 + 0,928^2} = 0,931$

Colorhue $= tg\,\varphi = \dfrac{E'_R - E'_y}{E'_B - E'_y} = \dfrac{0,072}{0,928} = -4,436 \qquad \varphi = -4,436$



3) In a 525/60 SDTV

The luminance sampling frequency signal is 18 MHz , the active picture format = 960 × 486

We use 10 bits/sample to encode the video samples

$n = 10$ bits/sample $\quad f_s = 18\ MHz \quad f_F = \frac{60}{2} = 30\ Hz$

f_s ↑ sampling frequency $\quad f_F$ ↑ frame per seconds

and $y = N_v \times L_v = 960 \times 486 = 466\,560$

In 4:4:4

$R_D = 3 \times (f_s . n) = 3 \times 18 \times 10^6 \times 10 = 540\ Mbit/sec$

Gross bit-rate = 540 Mbit/sec.

$R_N = 3 \times (N_v \times L_v) \times f_F \times n = 3 \times 960 \times 486 \times 30 \times 10$

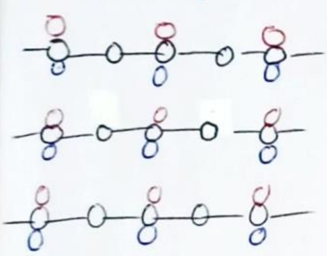
$= 419\,904\,000$ bit/s $= 419,904$ Mbit/s

Useful bit-rate = 419,904 Mbit/s

Y sample

C_R sample

C_B sample



4:2:2

$R_B = f_s . n + 2 \left(\frac{f_s}{2} \times n\right) = 18 \times 10^6 \times 10 + 2\,(9 \times 10^6 \times 10) = 360\ Mbit/sec$

$R_N = N_v . L_v . f_F . n + 2 . \left(\frac{N_v}{2} . L_v\right) . f_F . n =$

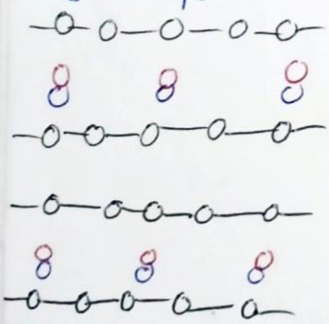
$= 960 \times 486 \times 30 \times 10 + 2 \left(\frac{960}{2} \times 486\right) \times 30 \times 10$

$= 279,936$ Mbit/s

Y sample

C_R sample

C_B sample



4:2:0

$R_D = f_s \times n + 2 \left(\frac{f_s/2}{2} \times n\right) = 18.10^6 \times 10 + 2 \left(\frac{\frac{18.10^6}{2}}{2} \times 10\right)$

$= 270$ Mbit/s

$R_N = N_v \times L_v \times f_F \times n + 2 \left(\frac{N_v}{2}\ \frac{L_v}{2}\right) \times f_F \times n$

$= 960 \times 486 \times 30 \times 10 + 2 \left(\frac{960 \times 486}{2}\right) \times 30 \times 10$

$= 209,952$ Mbit/sec

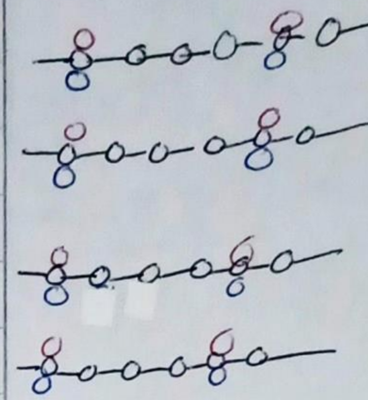
4:1:1 $R_B = f_s \times n + 2 \times (\frac{f_s}{4} \times n)$

$= 270$ Mbit/s

$R_N = N_v \times L_v \times f_F \times n + 2 \left(\frac{N_v}{4} \times L_v\right) \times f_F \times n$

$= 960 \times 486 \times 30 \times 10 + 2 \left(\frac{960}{4} \times 486\right) \times 30 \times 10$

$= 209{,}952$ Mbit/s.



4) $\underset{N_v}{1280} \times \underset{L_v}{720} / \underset{f_F}{50} / P$ HD TV

L_u = total number of lines = 750 f_s = Sampling frequency = 74,25 MHz

n = 10 bits / sample $L_v = 720$ $N_v = 1280$

T_H = Total line duration = $\frac{1}{L_u \cdot f_s} = \frac{1}{750 \times 74{,}25} = 1{,}796 \times 10^{-5} = 17{,}96\ \mu sec$

Number of samples in the active line: $N_v \times 16/9 = 1280$

Duration of active part of the image =

T_{HA} = duration of active part of a line = $\frac{N_v}{f_s} = \frac{1280}{74{,}25 \cdot 10^6} = 1{,}346 \times 10^{-5}$

$T_{SA} = T_{HA} \times L_v = 1{,}346 \times 10^{-5} \times 720 = 0{,}01241 = 12{,}41$ ms

4:2:2

Gross bit rate:

$R_B = f_s \cdot n + 2 \left(\frac{f_s}{2} \times n\right) = 74{,}25 \times 10 + 2 \left(\frac{74{,}25}{2} \times 10\right) = 1{,}485$ Gbit/s

Usefull bit rate:

$R_N = N_v \times L_v \times f_F \times n + 2 \left(\frac{N_v}{2} \times L_v\right) \times f_F \times n$

$= 1280 \times 720 \times 50 \times 10 + 2 \left(\frac{1280}{2} \times 720\right) \times 50 \times 10$

$= 921{,}6$ Mbit/s

5) a)

$$F(u,v) = \frac{2}{\sqrt{M.N}} \cdot C(u) \cdot C(v) \cdot \sum_{i=0}^{M-1} \cdot \sum_{j=0}^{N-1} \cdot f(i,j) \cdot \cos\left[\frac{(2i+1).u.\pi}{2.M}\right] \cdot \cos\left[\frac{(2j+1).v.\pi}{2.N}\right]$$

Our quantization matrix is:

2	3	5
3	4	6
5	6	7

My matrix is:

0	3	6
5	5	4
9	2	6

We know that $C(x) = \begin{cases} 1/\sqrt{2} & x = 0 \\ 1 & \text{otherwise} \end{cases}$

$$F(0,0) = \frac{2}{\sqrt{9}} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 40 = 13{,}33$$

$$F(1,0) = \frac{2}{\sqrt{9}} \times 1 \times \frac{1}{\sqrt{2}} \times \left[\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f(i,j) \cdot \cos\left[\frac{(2i+1) \times u \times \pi}{6}\right] \cdot \cos\left[\frac{(2j+1) \times v \times \pi}{6}\right]\right]$$

$$[\ ] = 0 \times \cos\left(\frac{\pi}{6}\right) \times \cos(0) + 3 \times \cos\left(\frac{\pi}{6}\right) \times \cos(0) + 6 \times \cos\left(\frac{\pi}{6}\right) \times \cos(0)$$

$$+ 5 \times \cos\left(\frac{3\pi}{6}\right) \times \cos(0) + 5 \times \cos\left(\frac{3\pi}{6}\right) \times \cos(0) + 4 \times \cos\left(\frac{3\pi}{6}\right) \times \cos(0)$$

$$+ 9 \times \cos\left(\frac{5\pi}{6}\right) \times \cos(0) + 2 \times \cos\left(\frac{5\pi}{6}\right) \times \cos(0) + 6 \times \cos\left(\frac{5\pi}{6}\right) \times \cos(0)$$

$$\approx -6{,}928$$

$$F(1,0) = \frac{2}{3} \times \frac{1}{\sqrt{2}} \times -6{,}928 \approx -3{,}266$$

$$F(2,0) = \frac{2}{3} \times 1 \times \frac{1}{\sqrt{2}} \times \left[\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f(i,j) \cdot \cos\left[\frac{(2i+1) \times u \times \pi}{6}\right] \times \cos\left[\frac{(2j+1) \times v \times \pi}{6}\right]\right]$$

$$[\ ] = 0 \times \cos\left(\frac{2\pi}{6}\right) + 3 \times \cos\left(\frac{2\pi}{6}\right) + 6 \times \cos\left(\frac{2\pi}{6}\right) + 5 \times \cos\left(\frac{6\pi}{6}\right) + 5 \times \cos\left(\frac{6\pi}{6}\right)$$

$$+ 4 \times \cos\left(\frac{6\pi}{6}\right) + 9 \times \cos\left(\frac{10\pi}{6}\right) + 2 \times \cos\left(\frac{10\pi}{6}\right) + 6 \times \cos\left(\frac{10\pi}{6}\right)$$

$$\approx -1$$

$$F(2,0) \approx \frac{2}{3} \times 1 \times \frac{1}{\sqrt{2}} \times -1 \approx -0{,}471$$

$$F(0,1) = \frac{2}{3} \times \frac{1}{\sqrt{2}} \times 1 \times \left[\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f(i,j) \cdot \cos\left[\frac{(2i+1) \cdot u \cdot \pi}{6}\right] \cdot \cos\left[\frac{(2j+1) \cdot v \cdot \pi}{6}\right] \right]$$

$$[\,] = 0 \times \cos(0) \times \cos\left(\frac{\pi}{6}\right) + 3 \times \cos(0) \times \cos\left(\frac{3\pi}{6}\right) + 6 \times \cos(0) \times \cos\left(\frac{5\pi}{6}\right) + 5 \times \cos(0) \times \cos\left(\frac{\pi}{6}\right)$$
$$+ 5 \times \cos(0) \times \cos\left(\frac{3\pi}{6}\right) + 4 \times \cos(0) \times \cos\left(\frac{5\pi}{6}\right) + 9 \times \cos(0) \times \cos\left(\frac{\pi}{6}\right) + 2 \times \cos(0) \times \cos\left(\frac{3\pi}{6}\right) + 6 \times \cos(0) \times \cos\left(\frac{5\pi}{6}\right)$$
$$\simeq -1{,}732$$

$$F(0,1) = \frac{2}{3} \times \frac{1}{\sqrt{2}} \times 1 \times (-1{,}732) \simeq -0{,}816$$

$$F(0,2) = \frac{2}{3} \times \frac{1}{\sqrt{2}} \times 1 \times \left[\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f(i,j) \times \cos\left[\frac{(2i+1) \cdot u \cdot \pi}{6}\right] \times \cos\left[\frac{(2j+1) \times v \times \pi}{6}\right] \right]$$

$$[\,] = 0 \times \cos(0) \times \cos\left(\frac{2\pi}{6}\right) + 3 \times \cos(0) \times \cos\left(\frac{6\pi}{6}\right) + 6 \times \cos(0) \times \cos\left(\frac{10\pi}{6}\right) + 5 \times \cos(0) \times$$
$$\cos\left(\frac{2\pi}{6}\right) + 5 \times \cos(0) \times \cos\left(\frac{6\pi}{6}\right) + 4 \times \cos(0) \times \cos\left(\frac{10\pi}{6}\right) + 9 \times \cos(0) \times$$
$$\cos\left(\frac{2\pi}{6}\right) + 2 \times \cos(0) \times \cos\left(\frac{6\pi}{6}\right) + 6 \times \cos(0) \times \cos\left(\frac{10\pi}{6}\right) = 5$$

$$F(0,2) = \frac{2}{3} \times \frac{1}{\sqrt{2}} \times 1 \times 5 \simeq 2{,}357$$

$$F(1,1) = \frac{2}{3} \times 1 \times 1 \times \left[\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f(i,j) \times \cos\left[\frac{(2i+1) \times u \times \pi}{6}\right] \times \cos\left[\frac{(j+1) \times v \times \pi}{6}\right] \right]$$

$$[\,] = 0 \times \cos(\pi/6) \times \cos(\pi/6) + 3 \times \cos(\pi/6) \times \cos(3\pi/6) + 6 \times \cos(\pi/6) \times \cos(5\pi/6) +$$
$$5 \times \cos(3\pi/6) \times \cos(\pi/6) + 5 \times \cos(3\pi/6) \times \cos(3\pi/6) + 4 \times \cos(3\pi/6) \times \cos(5\pi/6) +$$
$$9 \times \cos(5\pi/6) \times \cos(\pi/6) + 2 \times \cos(5\pi/6) \times \cos(3\pi/6) + 6 \times \cos(5\pi/6) \times \cos(5\pi/6) = -6{,}75$$

$$F(1,1) = 2/3 \times 1 \times 1 \times (-6{,}75) = -4{,}5$$

$$F(1,2) = 2/3 \times 1 \times 1 \times \left[\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f(i,j) \times \cos\frac{[(2i+1) \times u \times \pi]}{6} \times \cos\left[\frac{(2j+1) \times v \times \pi}{6}\right] \right]$$

$$[\,] = 0 \times \cos\left(\frac{\pi}{6}\right) \times \cos\left(\frac{2\pi}{6}\right) + 3 \times \cos\left(\frac{\pi}{6}\right) \times \cos\left(\frac{6\pi}{6}\right) + 6 \times \cos(\pi/6) \times \cos(10\pi/6) +$$
$$5 \times \cos(3\pi/6) \times \cos(2\pi/6) + 5 \times \cos(3\pi/6) \times \cos(6\pi/6) + 4 \times \cos(3\pi/6) \times \cos(10\pi/6) +$$
$$9 \times \cos(5\pi/6) \times \cos(2\pi/6) + 2 \times \cos(5\pi/6) \times \cos(6\pi/6) + 6 \times \cos(5\pi/6) \times \cos(10\pi/6)$$
$$\simeq -4{,}763$$

$$F(1,2) = \frac{2}{3} \times 1 \times 1 \times -4{,}763 \simeq -3{,}175$$

$$F(2,1)=\frac{2}{3}\times 1\times 1\times\left[\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} f(i,j)\times\cos\left[\frac{(2i+1)\times u\times\pi}{6}\right]\times\cos\left[\frac{(2j+1)\times v\times\pi}{6}\right]\right]$$

$$[\,]\ 0\times\cos\left(\frac{2\pi}{6}\right)\times\cos\left(\frac{\pi}{6}\right)+3\times\cos\left(\frac{2\pi}{6}\right)\times\cos\left(\frac{3\pi}{6}\right)+6\times\cos\left(\frac{2\pi}{6}\right)\times\cos\left(\frac{5\pi}{6}\right)+$$
$$5\times\cos\left(\frac{6\pi}{6}\right)\times\cos\left(\frac{\pi}{6}\right)+5\times\cos\left(\frac{6\pi}{6}\right)\times\cos\left(\frac{3\pi}{6}\right)+4\times\cos\left(\frac{6\pi}{6}\right)\times\cos\left(\frac{5\pi}{6}\right)+$$
$$9\times\cos\left(\frac{10\pi}{6}\right)\times\cos\left(\frac{\pi}{6}\right)+2\times\cos\left(\frac{10\pi}{6}\right)\times\cos\left(\frac{3\pi}{6}\right)+6\times\cos\left(\frac{10\pi}{6}\right)\times\cos\left(\frac{5\pi}{6}\right)$$
$$\approx -2{,}165 \qquad F(2,1)=2/3\times 1\times 1\times -2{,}165\approx -1{,}443$$

$$F(2,2)=2/3\times 1\times 1\times\left[\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} f(i,j)\times\cos\left[\frac{(2i+1)\times u\times\pi}{6}\right]\times\cos\left[\frac{(2j+1)\times v\times\pi}{6}\right]\right]$$

$$0\times\cos\left(\frac{2\pi}{6}\right)\times\cos\left(\frac{2\pi}{6}\right)+3\times\cos\left(\frac{2\pi}{6}\right)\times\cos\left(\frac{6\pi}{6}\right)+6\times\cos\left(\frac{2\pi}{6}\right)\times\cos\left(\frac{10\pi}{6}\right)+$$
$$5\times\cos\left(\frac{6\pi}{6}\right)\times\cos\left(\frac{2\pi}{6}\right)+5\times\cos\left(\frac{6\pi}{6}\right)\times\cos\left(\frac{6\pi}{6}\right)+4\times\cos\left(\frac{6\pi}{6}\right)\times\cos\left(\frac{10\pi}{6}\right)+$$
$$9\times\cos\left(\frac{10\pi}{6}\right)\times\cos\left(\frac{2\pi}{6}\right)+2\times\cos\left(\frac{10\pi}{6}\right)\times\cos\left(\frac{6\pi}{6}\right)+6\times\cos\left(\frac{10\pi}{6}\right)\times\cos\left(\frac{10\pi}{6}\right)$$
$$=3{,}25$$

$$F(2,2)\approx 2/3\times 1\times 1\times 3{,}25\approx 2{,}1667$$

The matrix of DCT coefficients is:

13,3	-0,8	2,4
-3,3	-4,5	-3,2
-0,5	-1,4	2,2

The quantized matrix is:

7	0	0
-1	-1	-1
0	0	0

b) The compressed ratio is equal to 9:4

where we have = original image : compressed image

$F'(u,v) = S(u,v) \times Q(u,v)$

$$\begin{bmatrix} 7 & 0 & 0 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 5 \\ 3 & 4 & 6 \\ 5 & 6 & 7 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 0 \\ -3 & -4 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

The formula for 2D IDCT $= f(i,j) = \frac{2}{\sqrt{M.N}} \times \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} C(u) \cdot C(v) \cdot F(u,v)$
$\times \cos\left[\frac{(2i+1) \times u \times \pi}{2 \cdot M}\right] \times \cos\left[\frac{(2j+1) \times v \times \pi}{2 \cdot N}\right]$

↳ See next page for IDCT.

-0,3	6,9	3,7
4,7	4,7	4,7
9,6	2,4	5,6

Rounded =
$$\begin{matrix} 0 & 7 & 4 \\ 5 & 5 & 5 \\ 10 & 2 & 6 \end{matrix}$$

$MSE = \frac{1}{N} \cdot \sum_{i=0}^{N-1} (x_i - x'_i)^2$

$= \frac{1}{N} \cdot [(0)^2 + (-4)^2 + (2)^2 + (0)^2 + (0)^2 + (-1)^2$
$+ (-1)^2 + (0)^2 + (0)^2] = \frac{22}{N} = 2,44$

$PSNR = 10 \log \frac{(2^n - 1)^2}{MSE} = 10 \log \frac{(2^4-1)^2}{2,44} = 19,64$

OK $F(2,2) = \frac{2}{3}\left[\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 14 \times \cos(0) \times \cos(0) + 0 + 0 + \right.$

$1 \times \frac{1}{\sqrt{2}} \times (-3) \times \cos\left(\frac{5\pi}{6}\right) \times \cos(0) + 1 \times 1 \times (-4) \times \cos\left(\frac{5\pi}{6}\right) \times \cos\left(\frac{5\pi}{6}\right) + 1 \times 1 \times (-6) \times \cos\left(\frac{5\pi}{6}\right)$

$\times \cos\left(\frac{10\pi}{6}\right) = \frac{2}{3} \times 8{,}435 \approx 5{,}62$

OK

$F(0,0) = \frac{2}{3} \times \left[\underbrace{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 14 \times \cos(0) \times \cos(0)}_{u=0 \; v=0} + \underbrace{\frac{1}{\sqrt{2}} \times 1 \times 0 \times \cos(0) \times \cos\left(\frac{\pi}{6}\right)}_{u=0 \; v=1} + \right.$

$\underbrace{\frac{1}{\sqrt{2}} \times 1 \times 0 \times \cos(0) \times \cos\left(\frac{2\pi}{6}\right)}_{u=0 \; v=2} + \underbrace{1 \times \frac{1}{\sqrt{2}} \times (-3) \times \cos\left(\frac{\pi}{6}\right) \times \cos(0)}_{u=1 \; v=0} +$

$\left. \underbrace{1 \times 1 \times (-4) \times \cos\left(\frac{\pi}{6}\right) \times \cos\left(\frac{\pi}{6}\right)}_{u=1 \; v=1} + \underbrace{1 \times 1 \times (-6) \times \cos\left(\frac{\pi}{6}\right) \times \cos\left(\frac{2\pi}{6}\right)}_{u=1 \; v=2} \right]$

$= -0{,}29$

OK

$F(1,0) = \frac{2}{3} \times \left[\underbrace{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 14 \times \cos(0) \times \cos(0)}_{u=0 \; v=0} + \underbrace{\frac{1}{\sqrt{2}} \times 1 \times 0 \times \ldots}_{u=0 \; v=1} \right.$

$\underbrace{\ldots \times 0 \ldots}_{u=0 \; v=2} + \underbrace{1 \times \frac{1}{\sqrt{2}} \times (-3) \times \cos\left(\frac{3\pi}{6}\right) \times \cos(0)}_{u=1 \; v=0}$

$\left. + \underbrace{1 \times 1 \times (-4) \times \cos\left(\frac{3\pi}{6}\right) \times \cos\left(\frac{\pi}{6}\right)}_{u=1 \; v=1} + \underbrace{1 \times 1 \times (-6) \times \cos\left(\frac{3\pi}{6}\right) \times \cos\left(\frac{2\pi}{6}\right)}_{u=1 \; v=2} \right]$

$= \frac{2}{3} \times 7 \approx 4{,}667$

$F(2,0) = \frac{2}{3} \times \left[\underbrace{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 14 \times \cos(0) \times \cos(0)}_{u=0 \; v=0} + \underbrace{0}_{u=0 \; v=1} + \underbrace{0}_{u=0 \; v=2} + \underbrace{1 \times \frac{1}{\sqrt{2}} \times -3 \times \cos\left(\frac{5\pi}{6}\right) \times \cos\left(\cancel{\frac{\pi}{6}}\right)}_{u=1 \; v=0} \right.$ (0)

OK

$\left. \underbrace{1 \times 1 \times (-4) \times \cos\left(\frac{5\pi}{6}\right) \times \cos\left(\frac{\pi}{6}\right)}_{u=1 \; v=1} + \underbrace{1 \times 1 \times (-6) \times \cos\left(\frac{5\pi}{6}\right) \times \cos\left(\frac{2\pi}{6}\right)}_{u=1 \; v=2} \right]$

$= \frac{2}{3} \times 14{,}44 \approx 9{,}62$

$F(0,1) = \frac{2}{3} \times \left[\underbrace{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 14 \times \cos(0) \times \cos(0)}_{u=0 \; v=0} + \underbrace{0}_{u=0 \; v=1} + \underbrace{0}_{u=0 \; v=2} + \right.$

$\underbrace{1 \times \frac{1}{\sqrt{2}} \times (-3) \times \cos\left(\frac{\pi}{6}\right) \times \cos(0)}_{u=1 \; v=0} + \underbrace{1 \times 1 \times -4 \times \cos\left(\frac{\pi}{6}\right) \times \cos\left(\frac{3\pi}{6}\right)}_{u=1 \; v=1}$

$\left. \underbrace{1 \times 1 \times (-6) \times \cos\left(\frac{\pi}{6}\right) \times \cos\left(\frac{6\pi}{6}\right)}_{u=1 \; v=2} \right] = \frac{2}{3} \times 10{,}36 \approx 6{,}90$

OK

$u=0 \quad v=0 \qquad\qquad u=0\ v=1 \quad u=0\ v=2$

$F(0,2) = \frac{2}{3}\Big[\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 14 \times \cos(0) \times \cos(0) + 0 + 0$

$u=1\ v=0 \qquad\qquad u=1\ v=1$

$1 \times \frac{1}{\sqrt{2}} \times (-3) \times \cos(\frac{\pi}{6}) \times \cos(0) + 1 \times 1 \times (-4) \times \cos(\frac{\pi}{6}) \times \cos(\frac{5\pi}{6})$

$u=1\ v=2$

$+ 1 \times 1 \times (-6) \times \cos(\frac{\pi}{6}) \times \cos(\frac{10\pi}{6})\Big]$

$= \frac{2}{3} \times 5{,}565 = 3{,}71$

OK

$u=0 \quad v=0 \qquad\qquad u=0\ v=1 \quad u=0\ v=2$

$F(1,1) = \frac{2}{3}\Big[\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 14 \times \cos(0) \times \cos(0) + 0 + 0$

$u=1\ v=0 \qquad\qquad u=1\ v=1$

$1 \times \frac{1}{\sqrt{2}} \times (-3) \times \cos(\frac{3\pi}{6}) \times \cos(0) + 1 \times 1 \times (-4) \times \cos(\frac{3\pi}{6})$

$u=1\ v=2$

$\times \cos(\frac{3\pi}{6}) + 1 \times 1 \times (-6) \times \cos(\frac{3\pi}{6}) \times \cos(\frac{6\pi}{6})\Big]$

$= \frac{2}{3} \times 7 = 4{,}667$

OK

$u=0\ v=0 \qquad\qquad u=0\ v=1 \qquad u=0\ v=2$

$F(1,2) = \frac{2}{3}\Big[\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 14 \times \cos(0) \times \cos(0) + 0 + 0 +$

$u=1\ v=0 \qquad\qquad u=1\ v=1$

$1 \times \frac{1}{\sqrt{2}} \times (-3) \times \cos(\frac{3\pi}{6}) \times \cos(0) + 1 \times 1 \times (-4) \times \cos(\frac{3\pi}{6}) \times \cos(\frac{5\pi}{6}) +$

$u=1\ v=2$

$1 \times 1 \times (-6) \times \cos(\frac{3\pi}{6}) \times \cos(\frac{10\pi}{6})$

$= \frac{2}{3} \times 7 \simeq 4{,}667$

OK

$u=0\ v=0 \qquad\qquad u=0\ v=1 \qquad u=0\ v=2$

$F(2,1) = \frac{2}{3}\Big[\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 14 \times \cos(0) \times \cos(0) + 0 + 0 +$

$u=1\ v=0$

$1 \times \frac{1}{\sqrt{2}} \times (-3) \times \cos(\frac{5\pi}{6}) \times \cos(0) + 1 \times 1 \times (-4) \times \cos(\frac{5\pi}{6}) \times \cos(\frac{3\pi}{6})$

$+ 1 \times 1 \times (-6) \times \cos(\frac{5\pi}{6}) \times \cos(\frac{6\pi}{6})\Big]$

$= \frac{2}{3} \times 3{,}64 = 2{,}43$