Lecture 9

3.2 PUSHDOWN AUTOMATA (PA)

- 3.2.1 PA Model
- 3.2.2 PA Definition
- 3.2.3 PA and Context-Free Grammar

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3.2 PUSHDOWN AUTOMATA (PA)

- **3.2.1 PA Model**
- **3.2.2 PA Definition**
- **3.2.3 PA and Context-Free Grammar**

Pushdown Automata (PA)

Pushdown Automata (PA)

Finite Automata

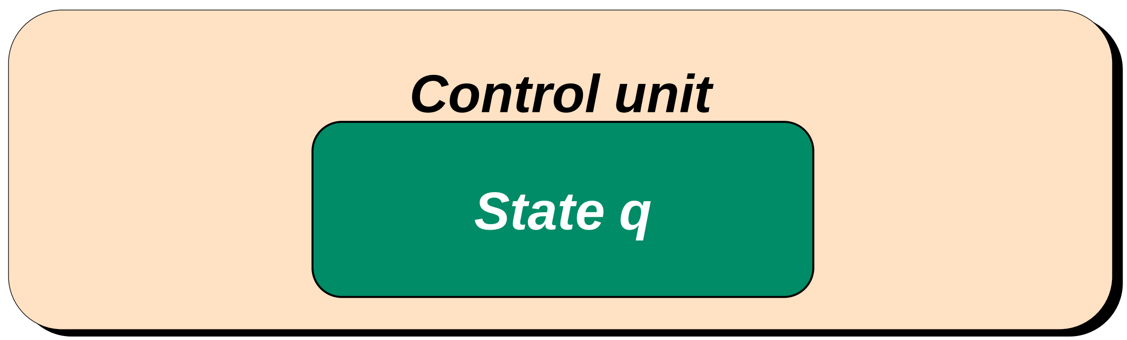
Pushdown Automata (PA)

Finite Automata

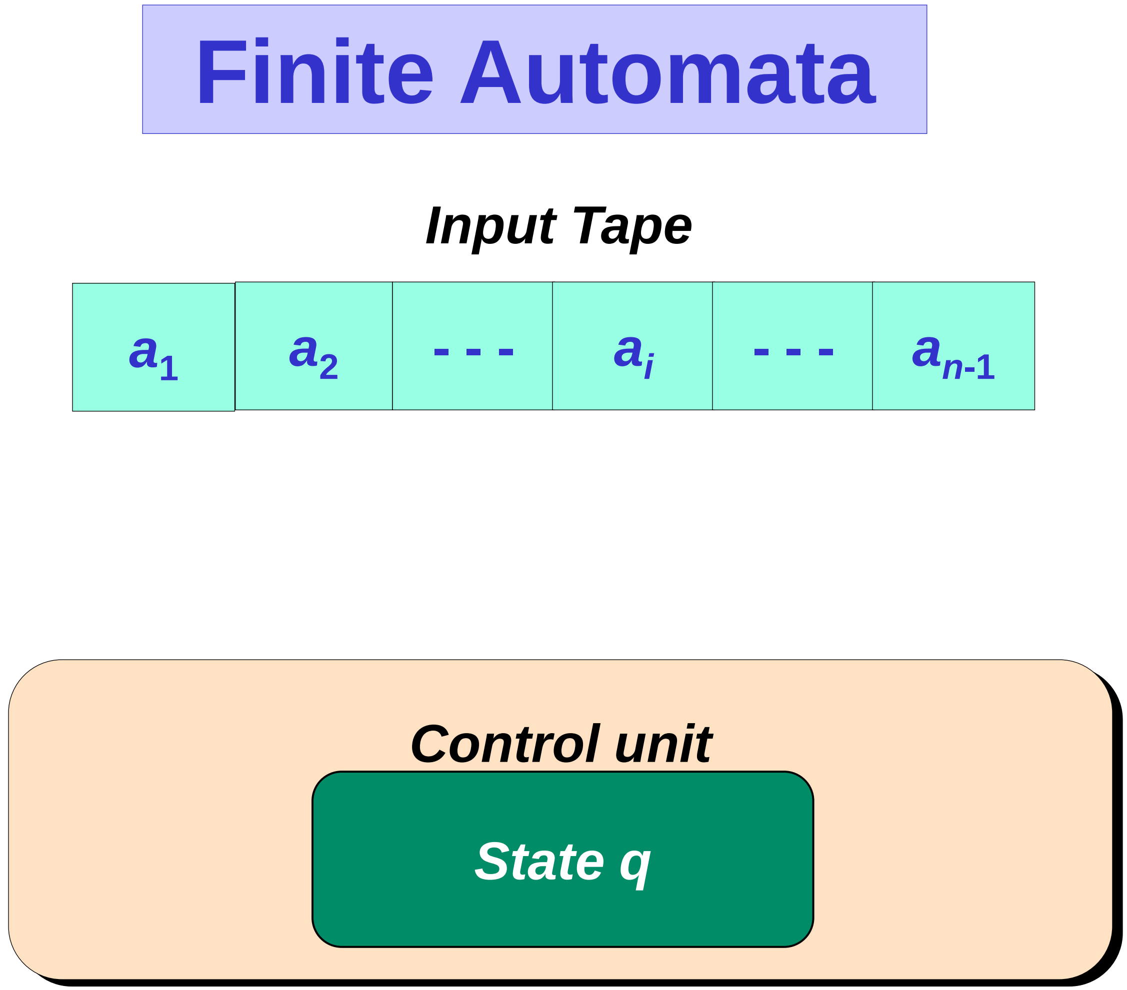
Control unit

Pushdown Automata (PA)

Finite Automata

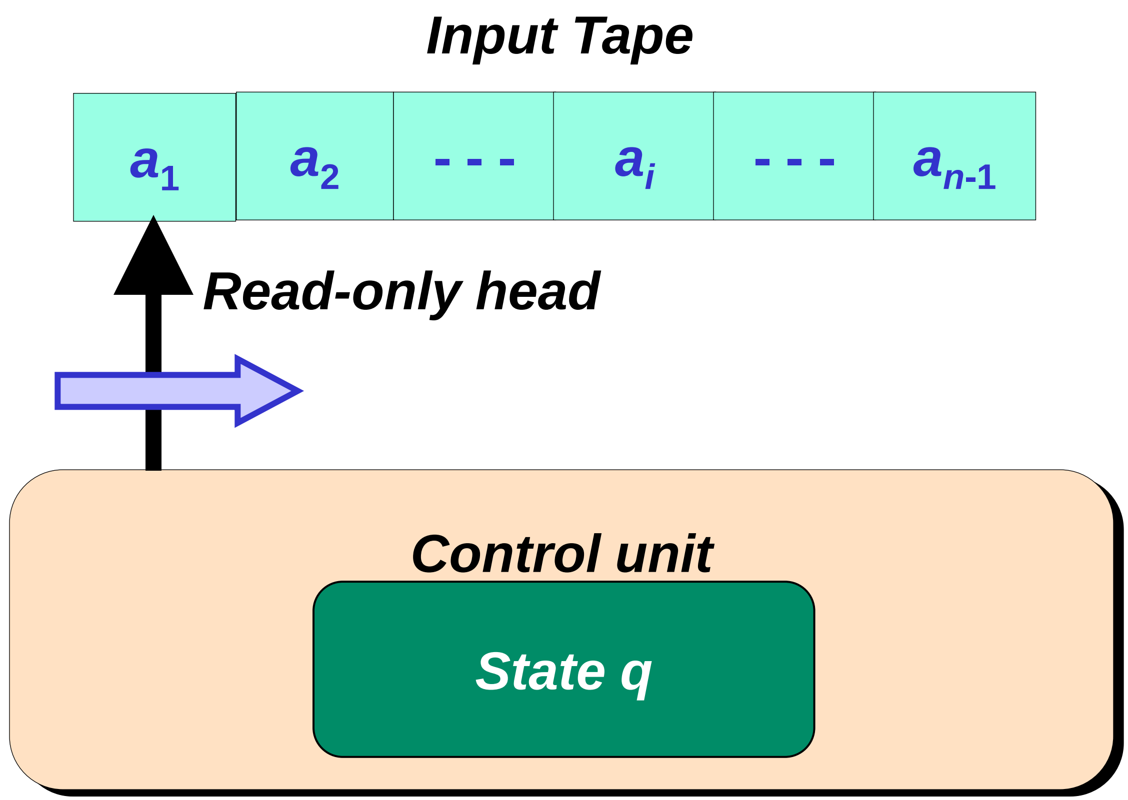


Pushdown Automata (PA)



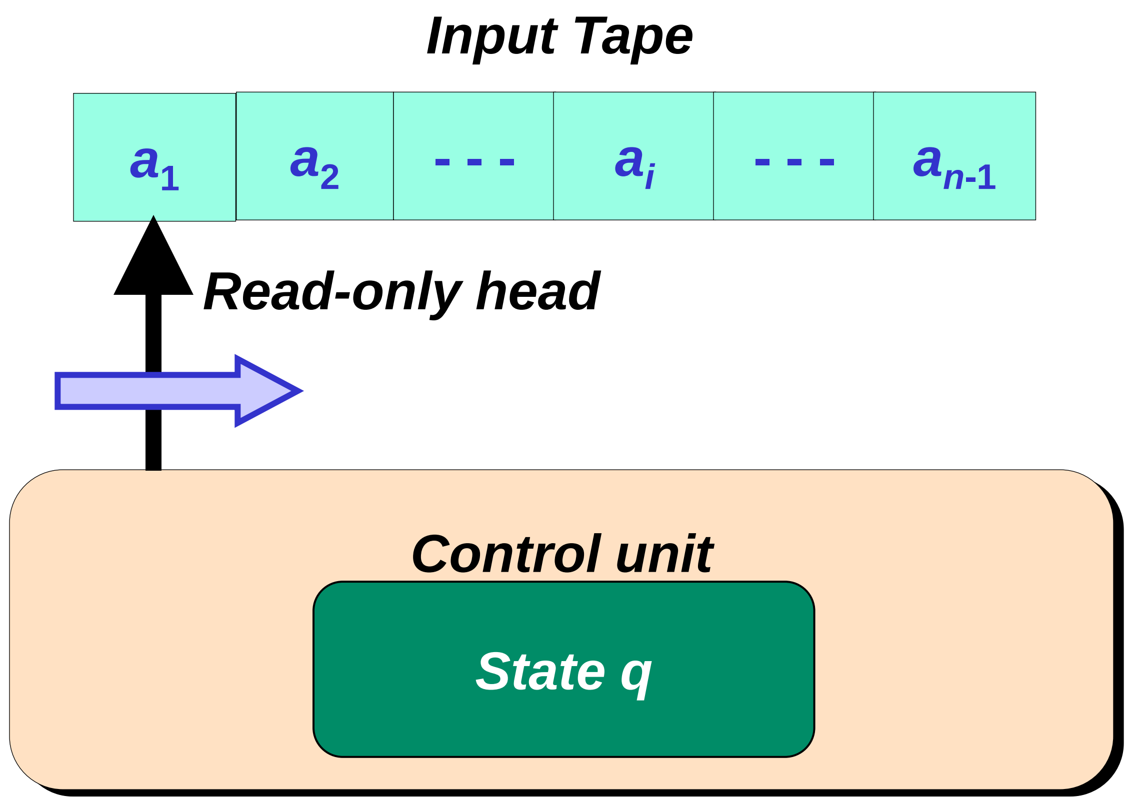
Pushdown Automata (PA)

Finite Automata



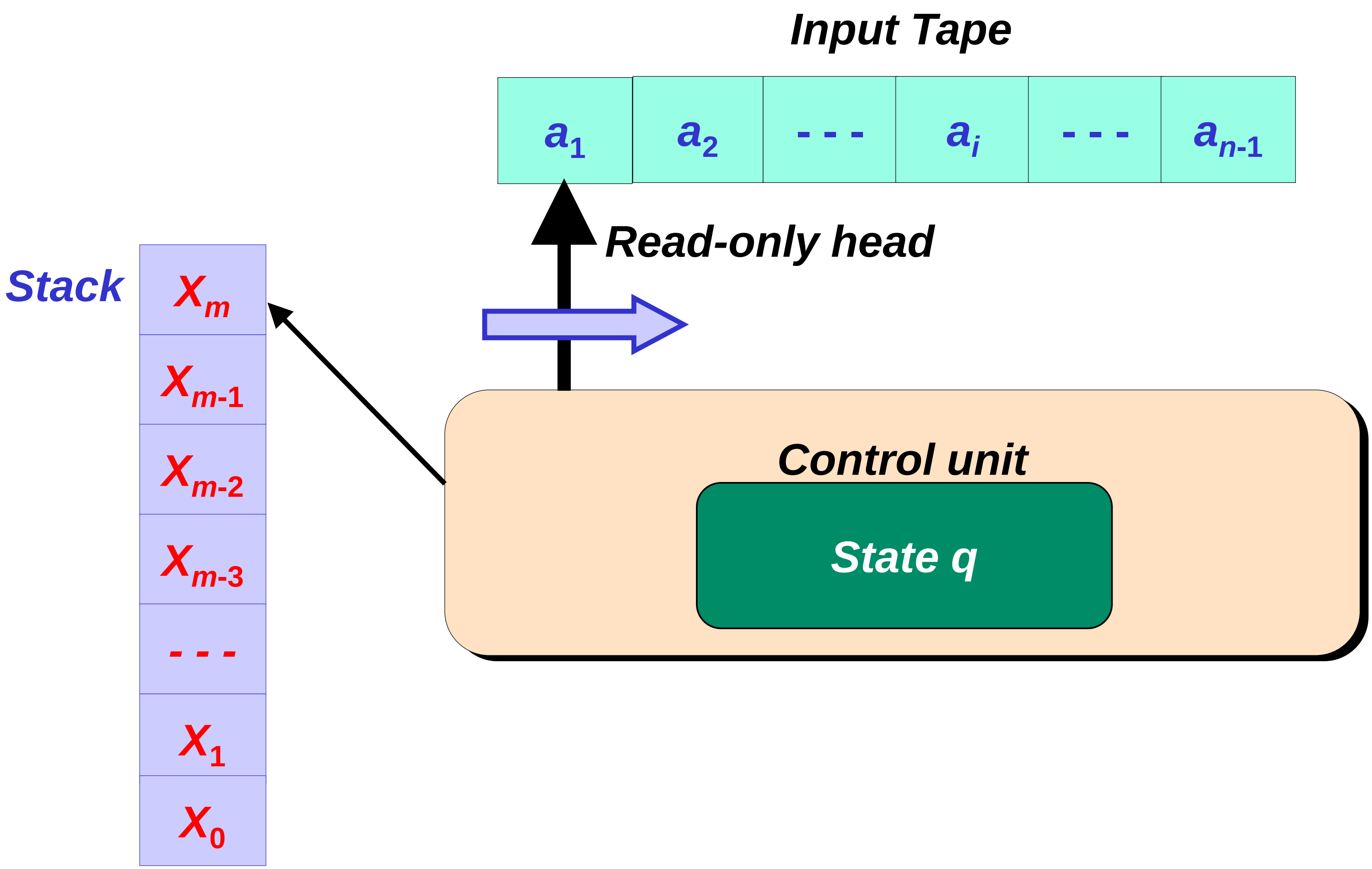
Pushdown Automata (PA)

Pushdown automata



Pushdown Automata (PA)

Pushdown automata



Pushdown Automata

Pushdown Automata

- **Decision**

Pushdown Automata

- **Decision**
  - Input

Pushdown Automata

- **Decision**
  - Input
    1. **State of control unit**

Pushdown Automata

- **Decision**
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    1. **State of control unit**
    2. **Symbol read on the input track – *not necessarily***

Pushdown Automata

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    3. **Symbol read on the top of the stack**

Pushdown Automata

- **Decision**
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  - Output

Pushdown Automata

- **Decision**
  - Input
    1. **State of control unit**
    2. **Symbol read on the input track –** ***not necessarily***
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  - Output
    1. **New state**

Pushdown Automata

- **Decision**
  - Input
    1. **State of control unit**
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  - Output
    1. **New state**
    2. **If symbol is read from the input track – *head moves to the right***

Pushdown Automata

- Decision
  - Input
    1. **State of control unit**
    2. **Symbol read on the input track –** ***not necessarily***
    3. **Symbol read on the top of the stack**

  - Output
    1. **New state**
    2. **If symbol is read from the input track –** ***head moves to the right***
    3. **Writing a new symbol at the top of the stack**

Pushdown Automata

- **Decision**
  - Input
    1. **State of control unit**
    2. **Symbol read on the input track – *not necessarily***
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  - Output
    1. **New state**
    2. **If symbol is read from the input track – *head moves to the right***
    3. **Writing a new symbol at the top of the stack**
       - **Empty string ε**

Pushdown Automata

- **Decision**
  - Input
    1. **State of control unit**
    2. **Symbol read on the input track – *not necessarily***
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    1. **New state**
    2. **If symbol is read from the input track – *head moves to the right***
    3. **Writing a new symbol at the top of the stack**
       - **Empty string ε**
       - **String of length 1**

Pushdown Automata

- **Decision**
  - Input
    1. **State of control unit**
    2. **Symbol read on the input track – *not necessarily***
    3. **Symbol read on the top of the stack**

  - Output
    1. **New state**
    2. **If symbol is read from the input track – *head moves to the right***
    3. **Writing a new symbol at the top of the stack**
       - **Empty string ε**
       - **String of length 1**
       - **String containing multiple symbols**

Pushdown Automata

Pushdown Automata

- **Two transition types**

Pushdown Automata

- Two transition types
  - State of the control unit - *q*

Pushdown Automata

- **Two transition types**
  - **State of the control unit** **- *q***
  - **Input symbol** **- *a***

Pushdown Automata

- Two transition types
  - State of the control unit - q
  - Input symbol - a
  - Symbol at the top of the stack - Z

Pushdown Automata

- Two transition types
  - State of the control unit - q
  - Input symbol - a
  - Symbol at the top of the stack - Z

  - $(q, a, Z) = (p, \gamma)$

Pushdown Automata

- Two transition types
  - State of the control unit - q
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- $(q, a, Z) = (p, \gamma)$
  - control unit changes state to new state p

Pushdown Automata

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  - $(q, a, Z) = (p, \gamma)$
    - — control unit changes state to new state p
    - — read-only head moves one position to the right

Pushdown Automata

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    - — read-only head moves one position to the right
    - — top stack symbol is replaced by the string γ

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Pushdown Automata

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- $(q, \varepsilon, Z) = (p, \gamma)$
  - control unit changes state to new state p
  - read-only remains at the same position
  - top stack symbol is replaced by the string γ

Pushdown Automata

Pushdown Automata

- **Input string acceptance decision**

Pushdown Automata

- **Input string acceptance decision**
  - **made in two possible ways**

Pushdown Automata

- **Input string acceptance decision**
  - **made in two possible ways**

  - **PA *M* that accepts by final (accepting) state:**

Pushdown Automata

- **Input string acceptance decision**
  - **made in two possible ways**

  - **PA *M* that accepts by final (accepting) state:**
    - **String is accepted if the control unit is in accepting state after it reads all symbols from the input type.**

Pushdown Automata

- **Input string acceptance decision**
  - **made in two possible ways**

  - **PA *M* that accepts by final (accepting) state:**
    - **String is accepted if the control unit is in accepting state after it reads all symbols from the input type.**
    - ***L*(*M*)**

Pushdown Automata

- Input string acceptance decision
  - made in two possible ways

  - **PA *M* that accepts by final (accepting) state:**
    - String is accepted if the control unit is in accepting state after it reads all symbols from the input type.
    - $L(M)$

  - **PA *M* that accepts by empty stack:**

Pushdown Automata

- **Input string acceptance decision**
  - **made in two possible ways**

  - **PA *M* that accepts by final (accepting) state:**
    - **String is accepted if the control unit is in accepting state after it reads all symbols from the input type.**
    - ***L*(*M*)**

  - **PA *M* that accepts by empty stack:**
    - **String is accepted if the stack remains empty after all symbols from the input type are read.**

Pushdown Automata

- **Input string acceptance decision**
  - **made in two possible ways**

- **PA *M* that accepts by final (accepting) state:**
  - **String is accepted if the control unit is in accepting state after it reads all symbols from the input type.**
  - ***L*(*M*)**

- **PA *M* that accepts by empty stack:**
  - **String is accepted if the stack remains empty after all symbols from the input type are read.**
  - ***N*(*M*)**

$N(M) = \{\ (^n\ a\ )^n\ \mid n \geq 1\}$

$N(M) = \{\ ({}^{n}\, a\ )^{n}\ \mid n \geq 1\}$

Transition	State	Input	Stack	New state	New stack top	Read-only head

$N(M) = \{\ (^n\ a\ )^n \mid n \geq 1\}$

Transition	State	Input	Stack	New state	New stack top	Read-only head
1)	q_0	(	K	q_0	AK	Move to right

$N(M) = \{ \ (^n a\ )^n \ | \ n \geq 1 \}$

Transition	State	Input	Stack	New state	New stack top	Read-only head
1)	q_0	(	*K*	q_0	*AK*	*Move to right*
2)	q_0	(	*A*	q_0	*AA*	*Move to right*

$N(M) = \{ \ (^n a\ )^n \ \mid n \geq 1\}$

Transition	State	Input	Stack	New state	New stack top	Read-only head
1)	q_0	(	K	q_0	AK	Move to right
2)	q_0	(	A	q_0	AA	Move to right
3)	q_0	a	A	q_1	A	Move to right

$N(M) = \{ \ (^n a\ )^n \ \mid n \geq 1\}$

Transition	State	Input	Stack	New state	New stack top	Read-only head
1)	q_0	(	K	q_0	AK	Move to right
2)	q_0	(	A	q_0	AA	Move to right
3)	q_0	a	A	q_1	A	Move to right
4)	q_1	)	A	q_1	ε	Move to right

$N(M) = \{ \ (^n a\ )^n \ \mid n \geq 1 \}$

Transition	State	Input	Stack	New state	New stack top	Read-only head
1)	q_0	(	K	q_0	AK	Move to right
2)	q_0	(	A	q_0	AA	Move to right
3)	q_0	a	A	q_1	A	Move to right
4)	q_1	)	A	q_1	ε	Move to right
5)	q_1	ε	K	q_1	ε	Remain at position

$N(M) = \{ \ (^n a\ )^n \ \mid n \geq 1 \}$

Transition	State	Input	Stack	New state	New stack top	Read-only head
1)	q_0	(	K	q_0	AK	Move to right
2)	q_0	(	A	q_0	AA	Move to right
3)	q_0	a	A	q_1	A	Move to right
4)	q_1	)	A	q_1	ε	Move to right
5)	q_1	ε	K	q_1	ε	Remain at position

Input tape

(	(	(	a	)	)	)

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Transition	State	Input	Stack	New state	New stack top	Read-only head
1)	q_0	(	K	q_0	AK	Move to right
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3)	q_0	a	A	q_1	A	Move to right
4)	q_1	)	A	q_1	ε	Move to right
5)	q_1	ε	K	q_1	ε	Remain at position

Input tape

(	(	(	a	)	)	)

Stack

K

$N(M) = \{ (^n a)^n \mid n \geq 1\}$

Transition	State	Input	Stack	New state	New stack top	Read-only head
1)	q_0	(	K	q_0	AK	Move to right
2)	q_0	(	A	q_0	AA	Move to right
3)	q_0	a	A	q_1	A	Move to right
4)	q_1	)	A	q_1	ε	Move to right
5)	q_1	ε	K	q_1	ε	Remain at position

Input tape

(	(	(	a	)	)	)

Control unit

State q_0

Stack

K

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5)	q_1	ε	K	q_1	ε	Remain at position

Input tape

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Control unit

State q_0

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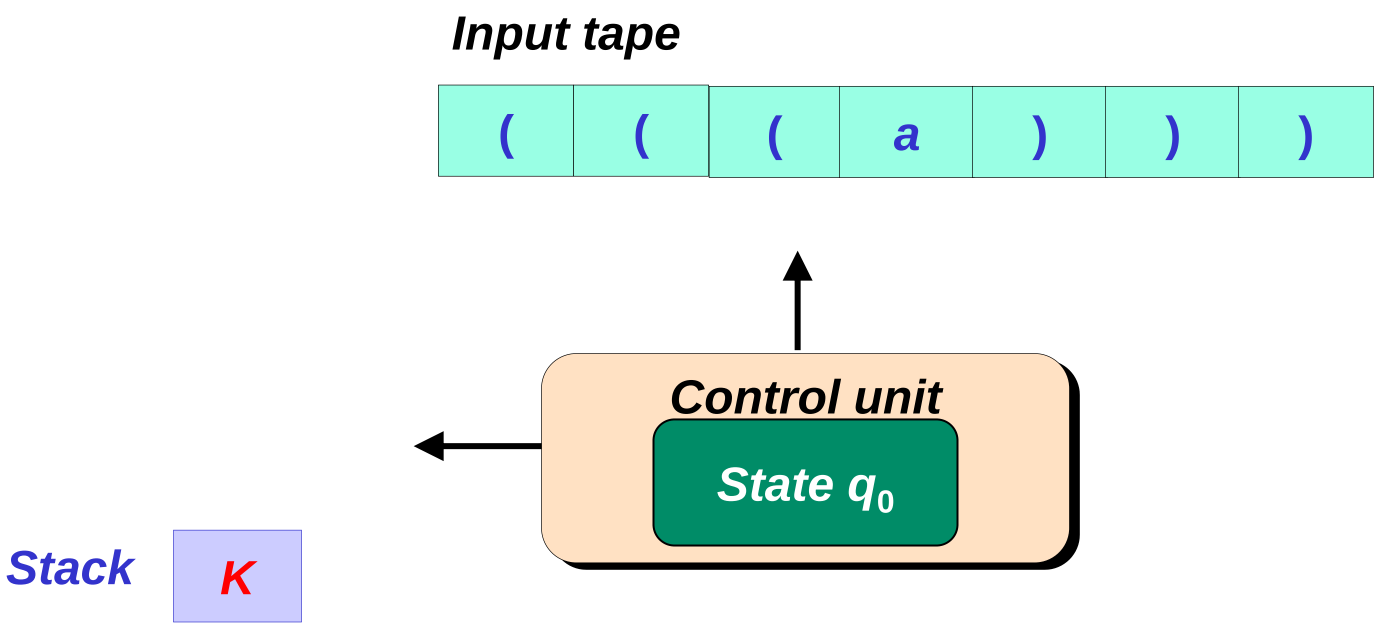
5)	q_1	ε	K	q_1	ε	Remain at position



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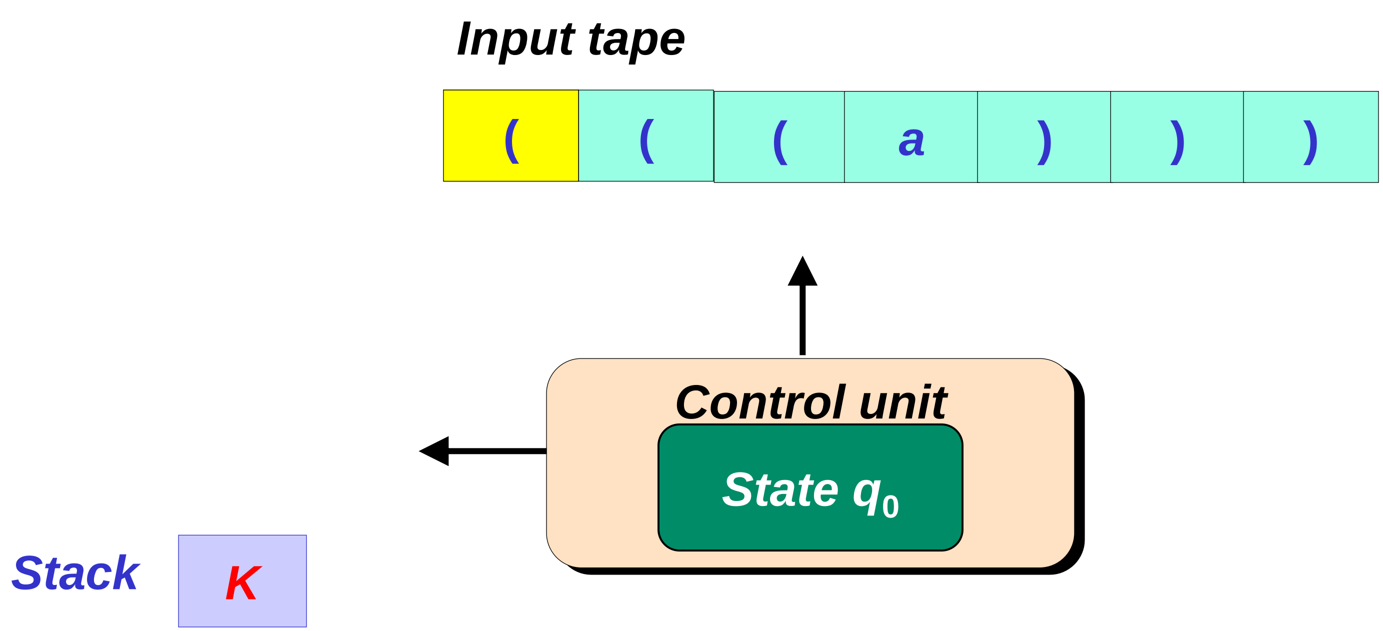

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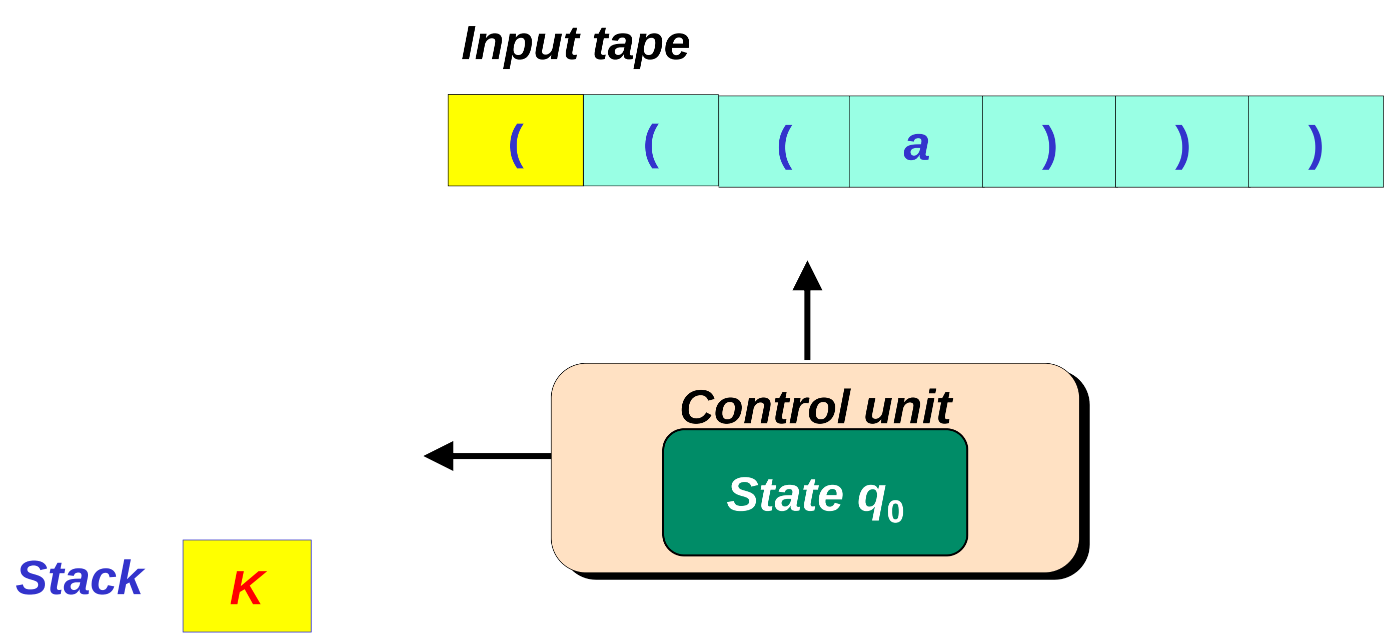

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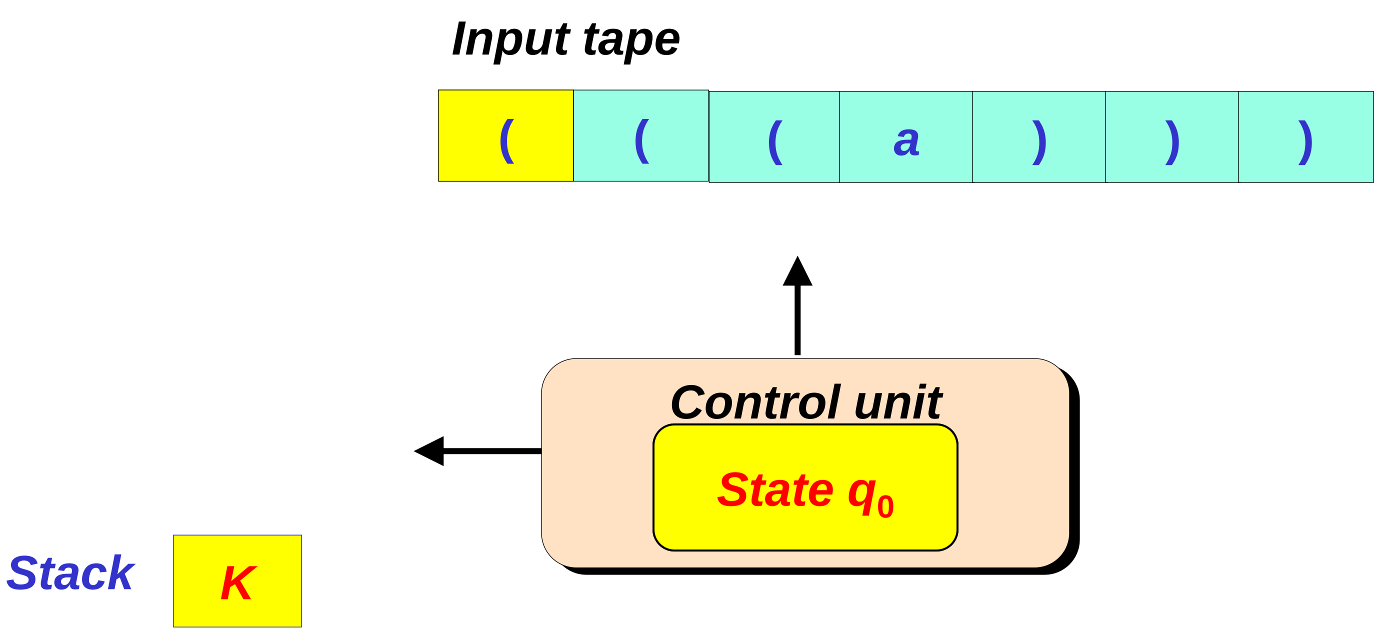

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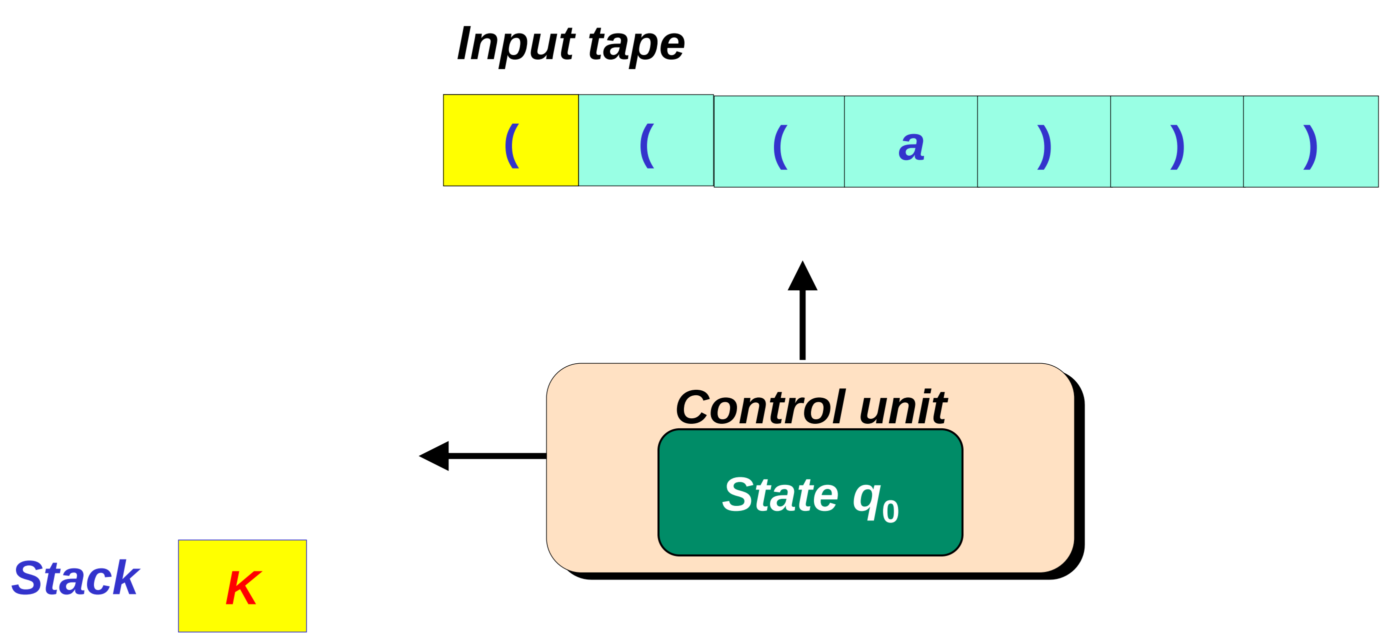

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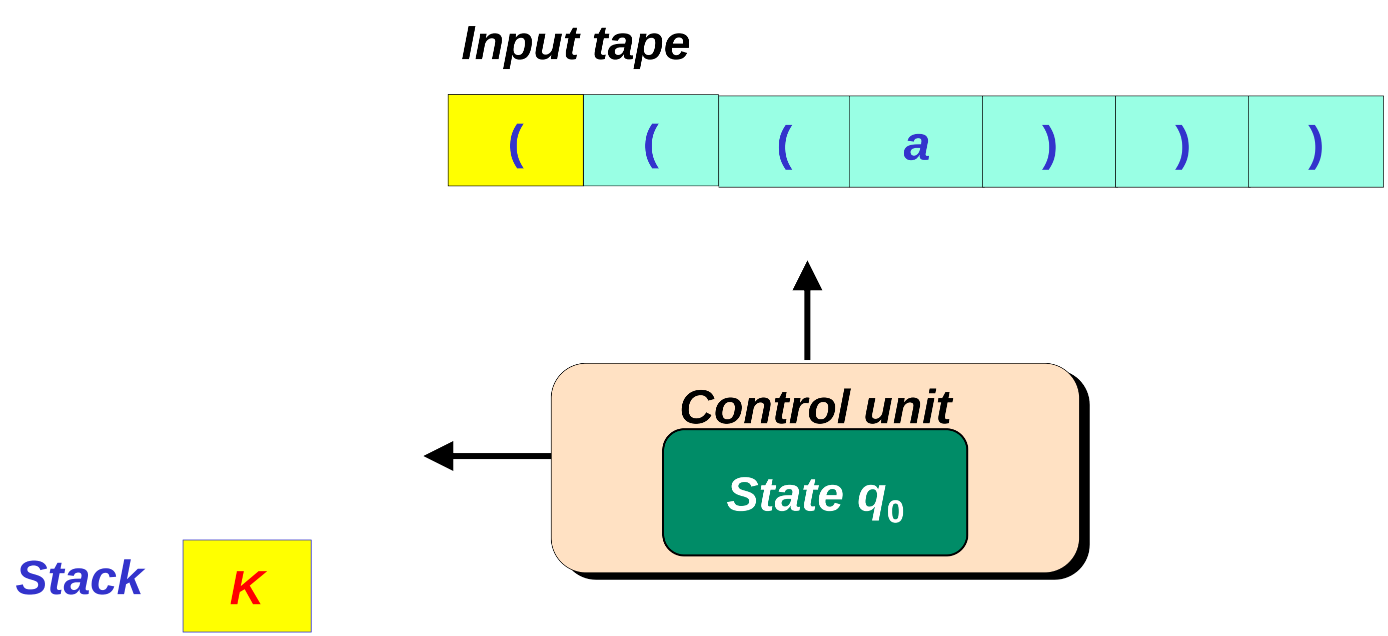

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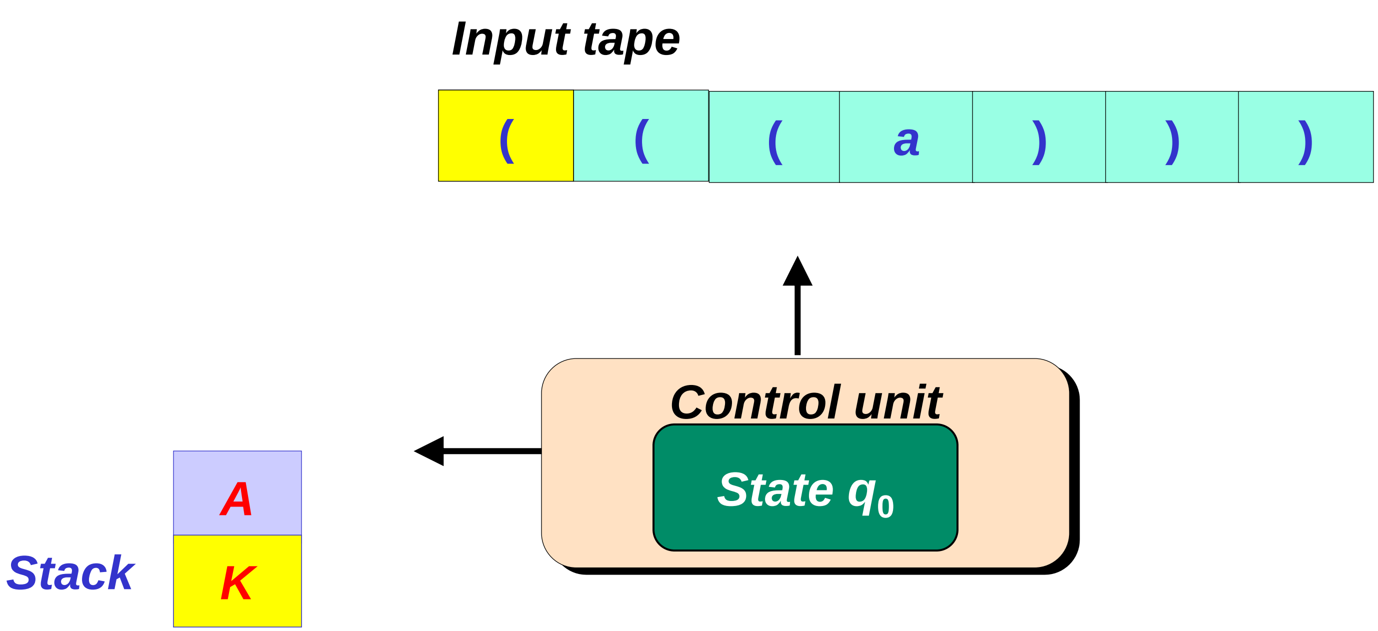

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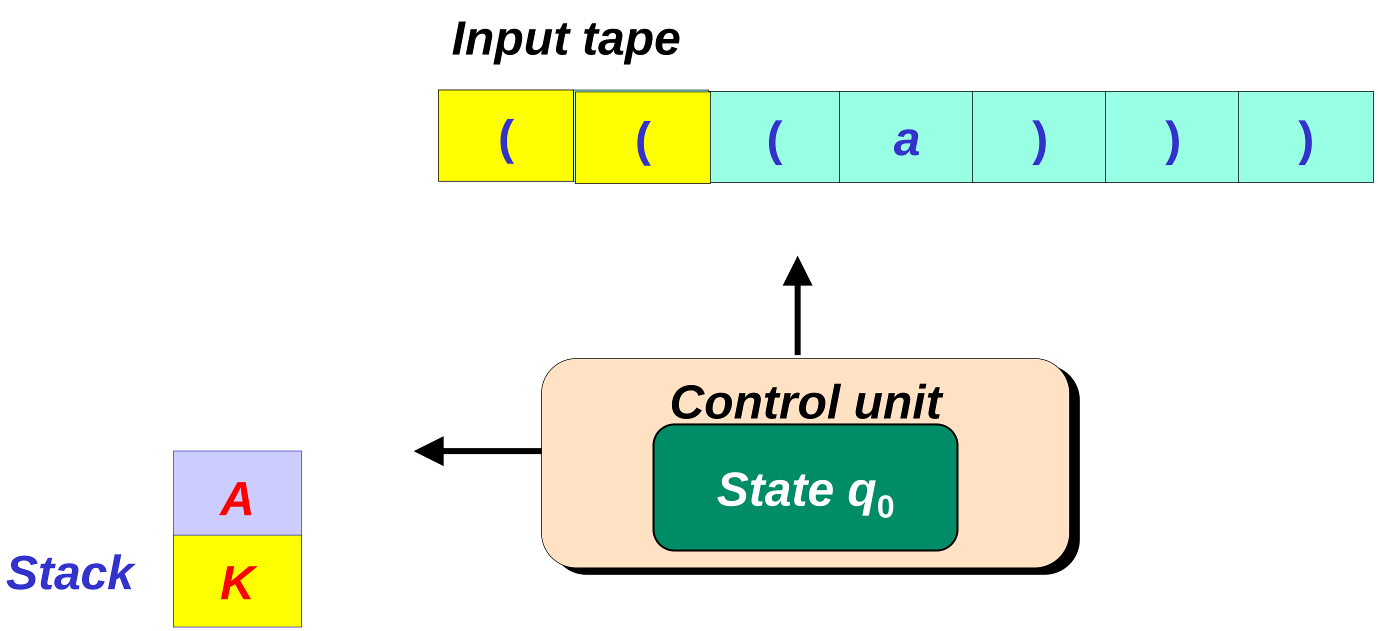

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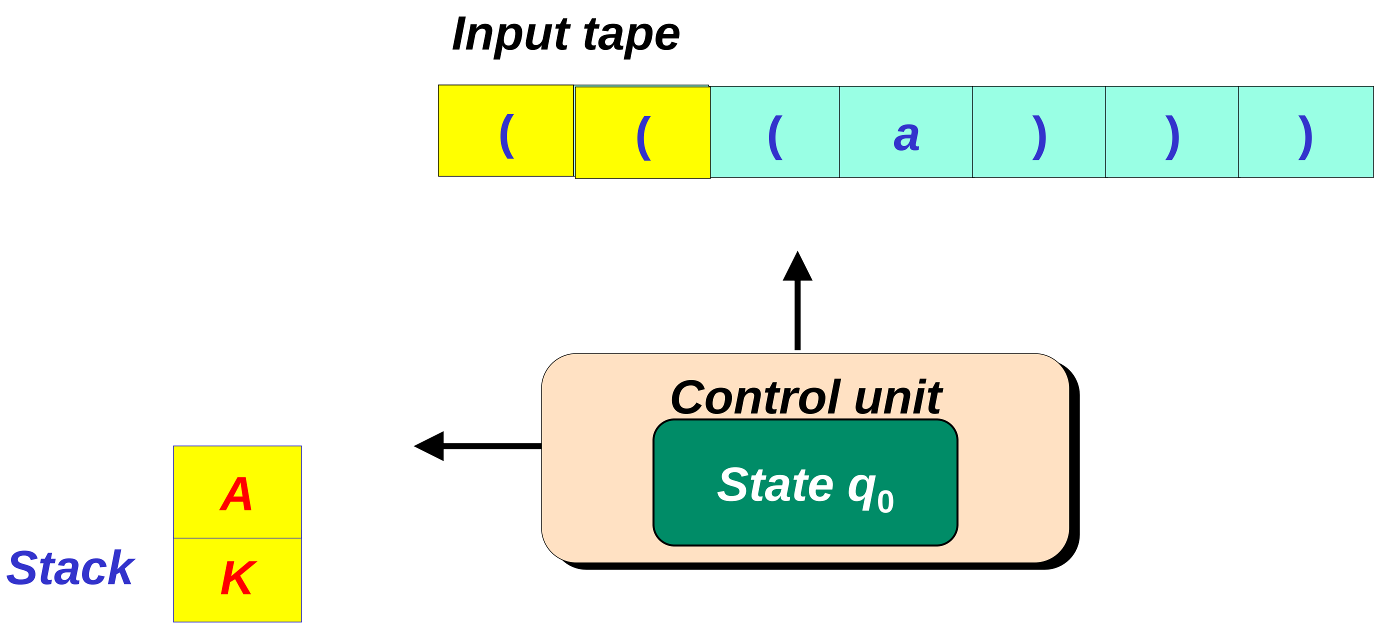

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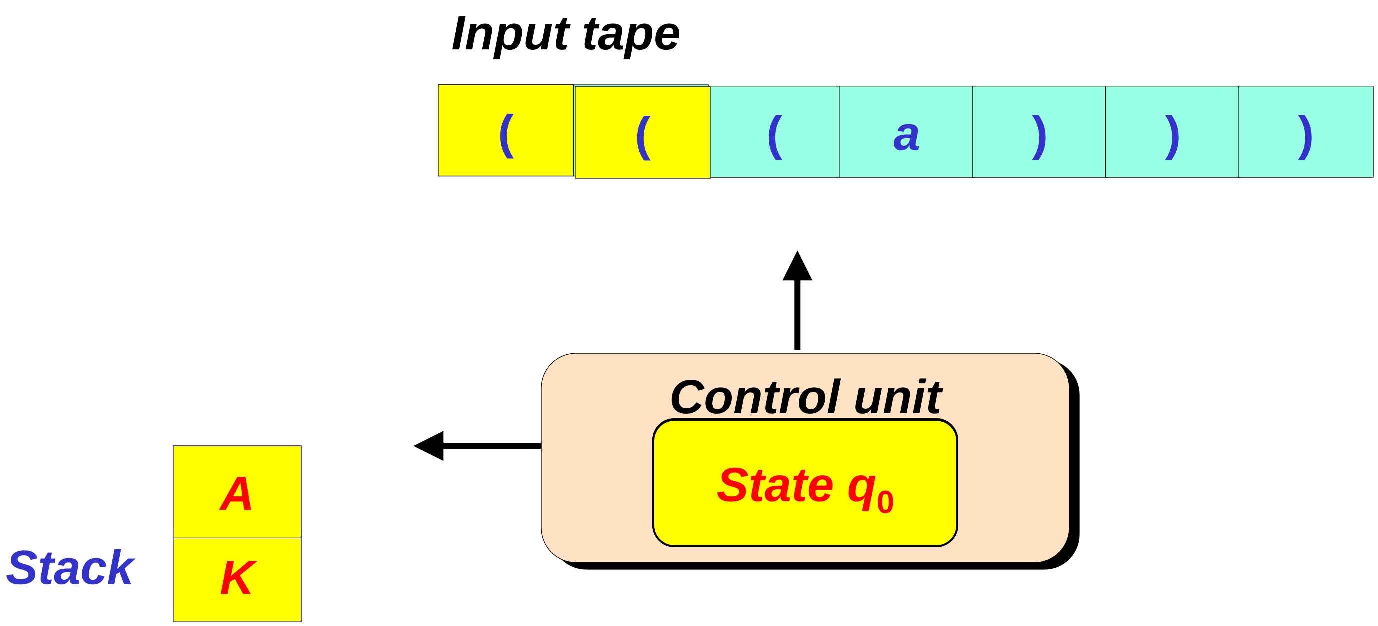

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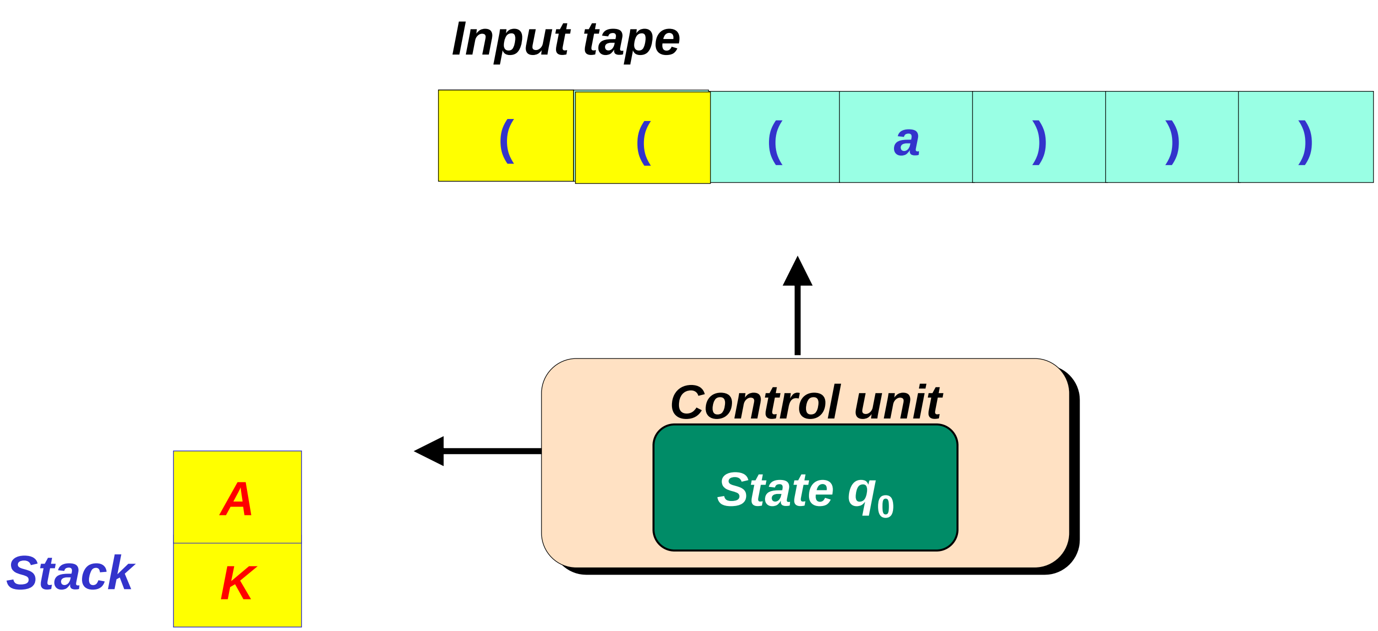

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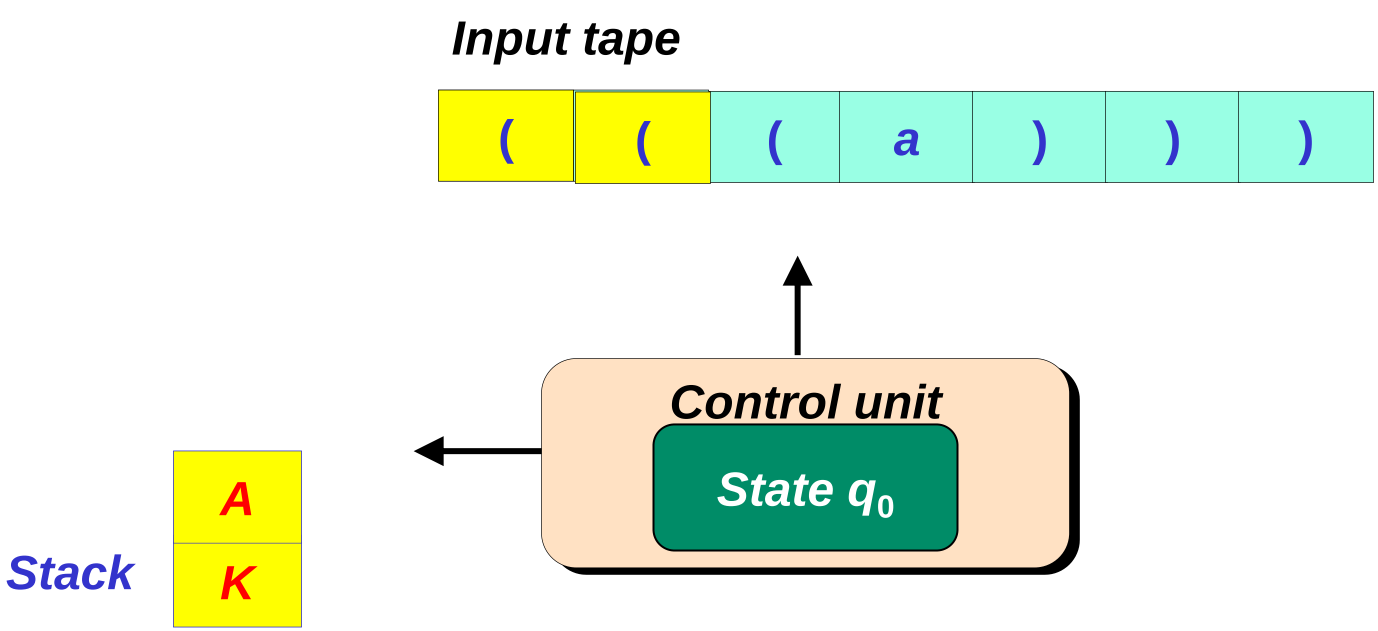

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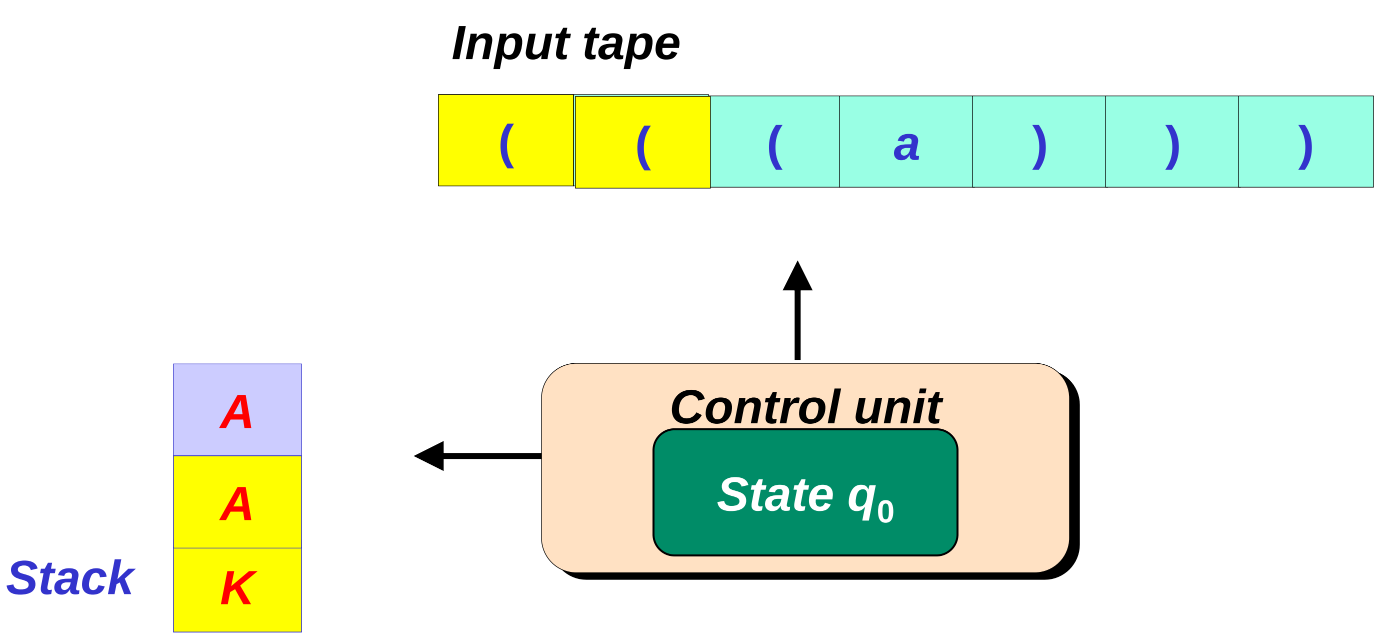

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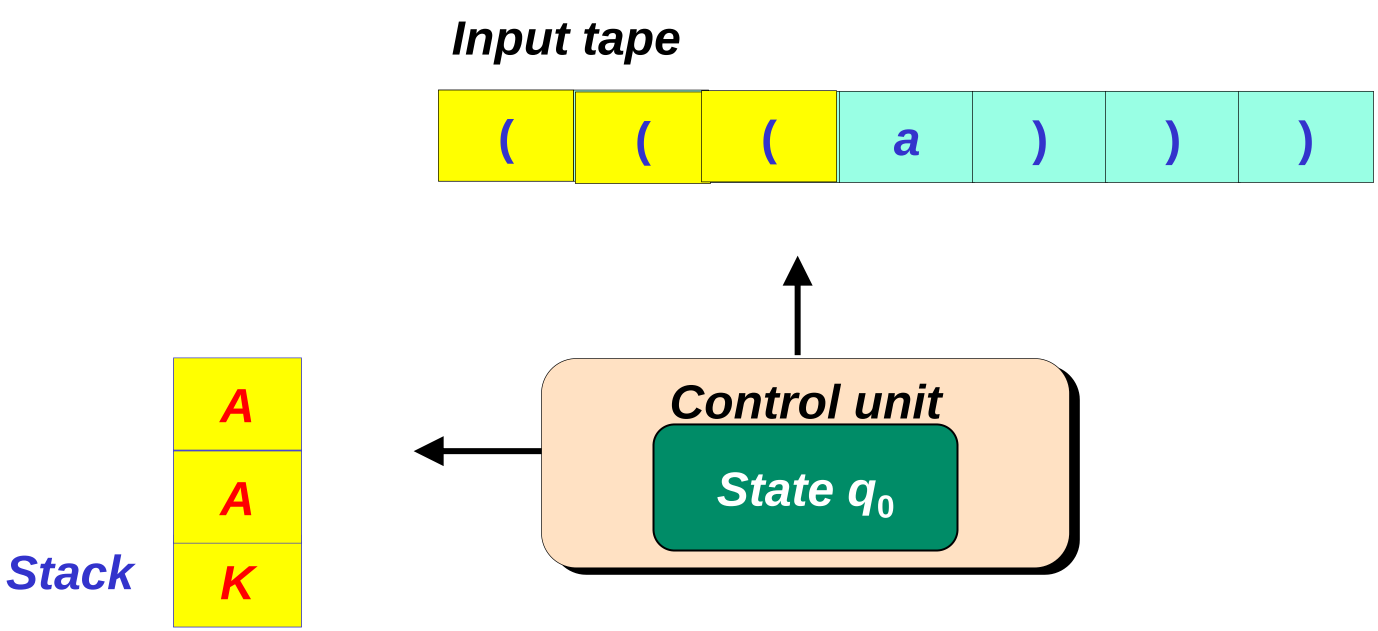
3)	q_0	a	A	q_1	A	Move to right
4)	q_1	)	A	q_1	ε	Move to right
5)	q_1	ε	K	q_1	ε	Remain at position



$N(M) = \{\ (^n\, a\ )^n \ \mid n \geq 1\}$

Transition	State	Input	Stack	New state	New stack top	Read-only head
1)	q_0	(	K	q_0	AK	Move to right
2)	q_0	(	A	q_0	AA	Move to right
3)	q_0	a	A	q_1	A	Move to right
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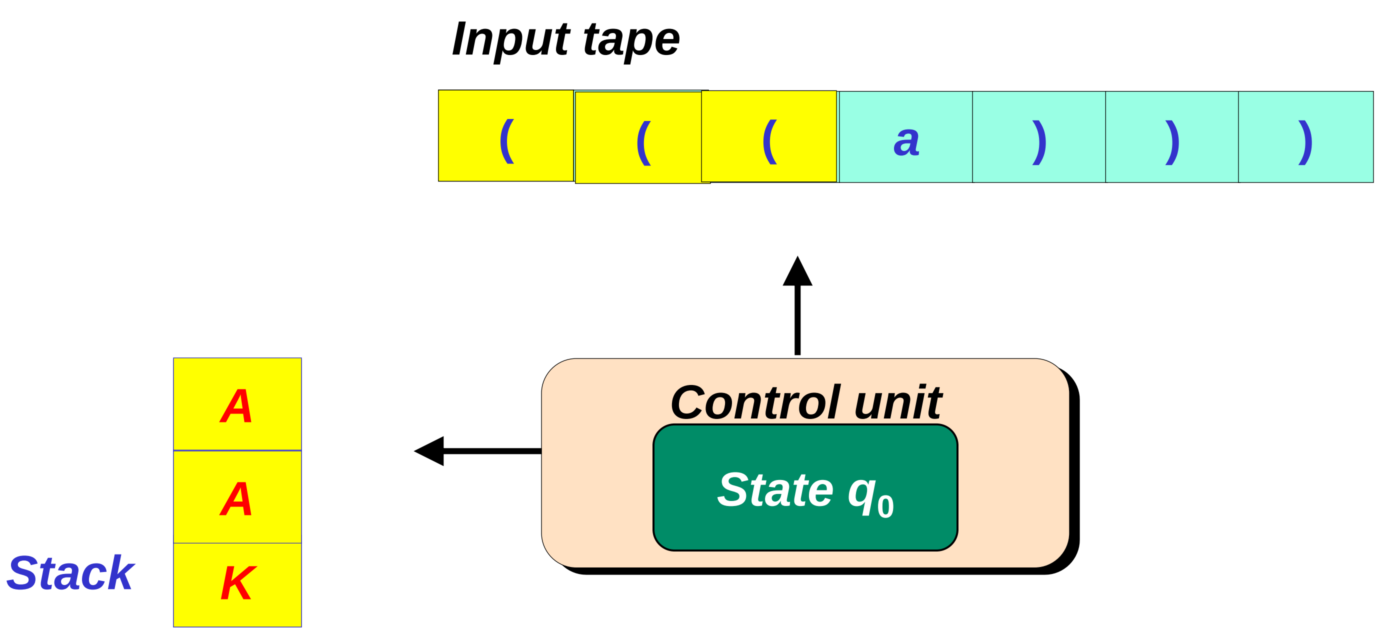

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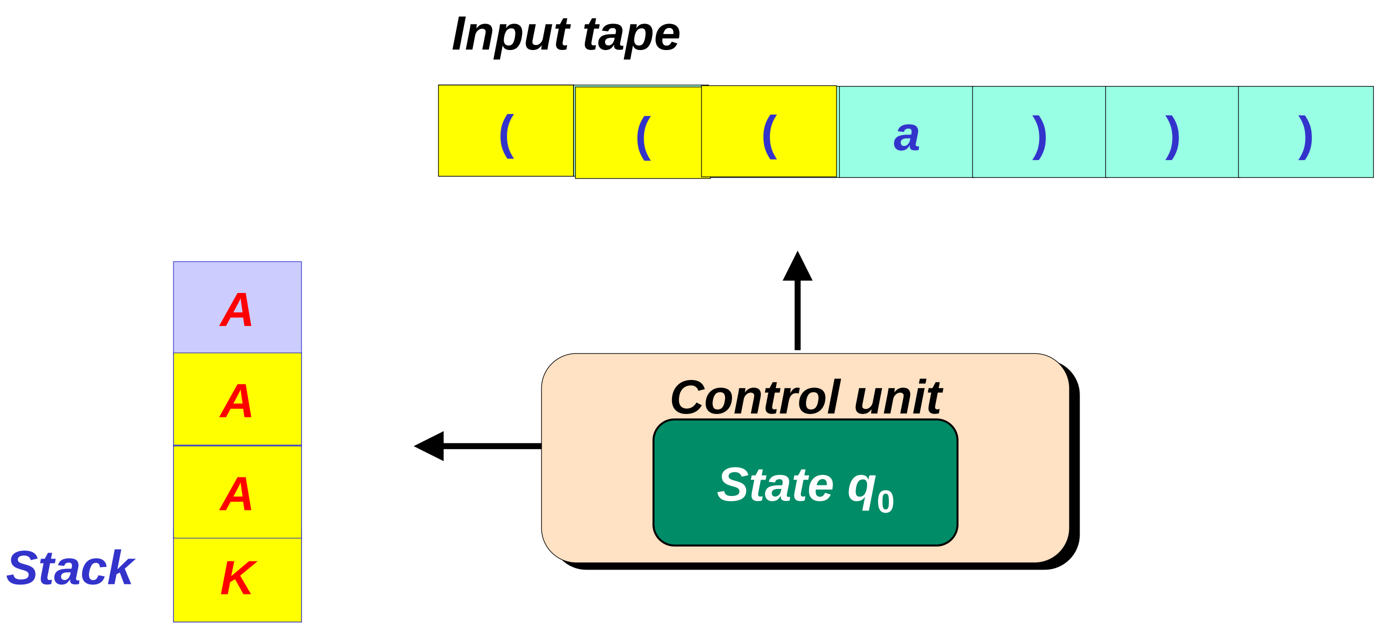

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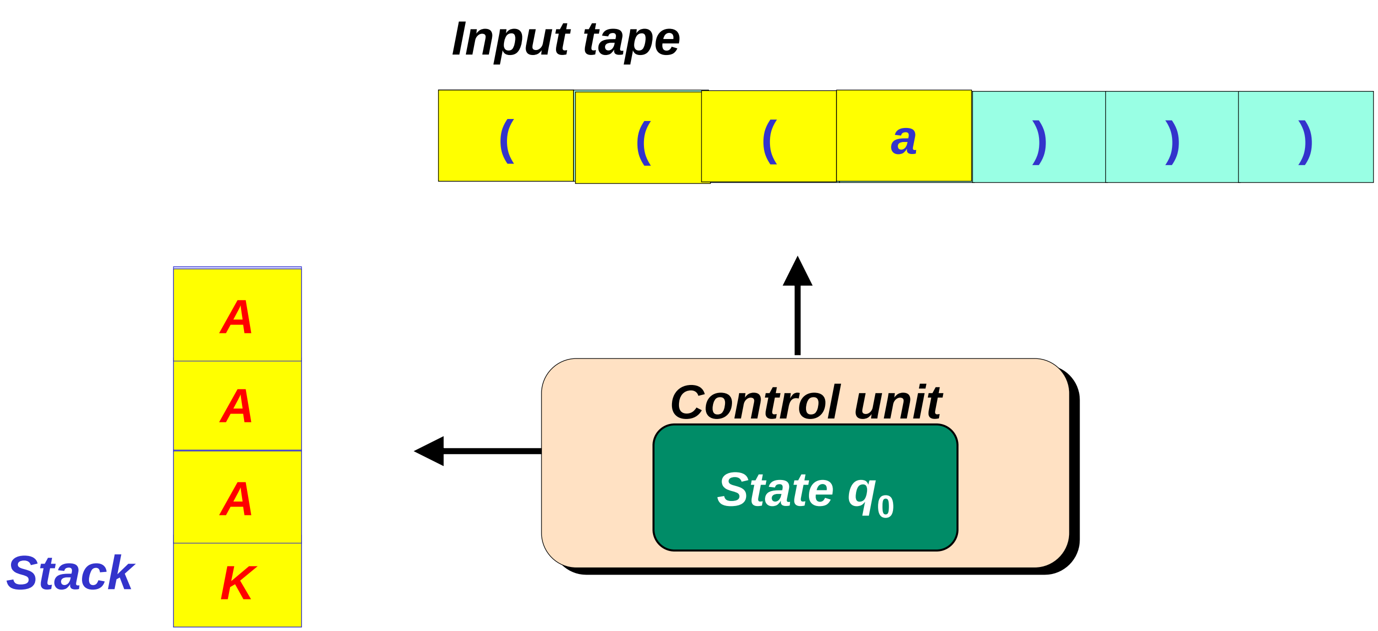

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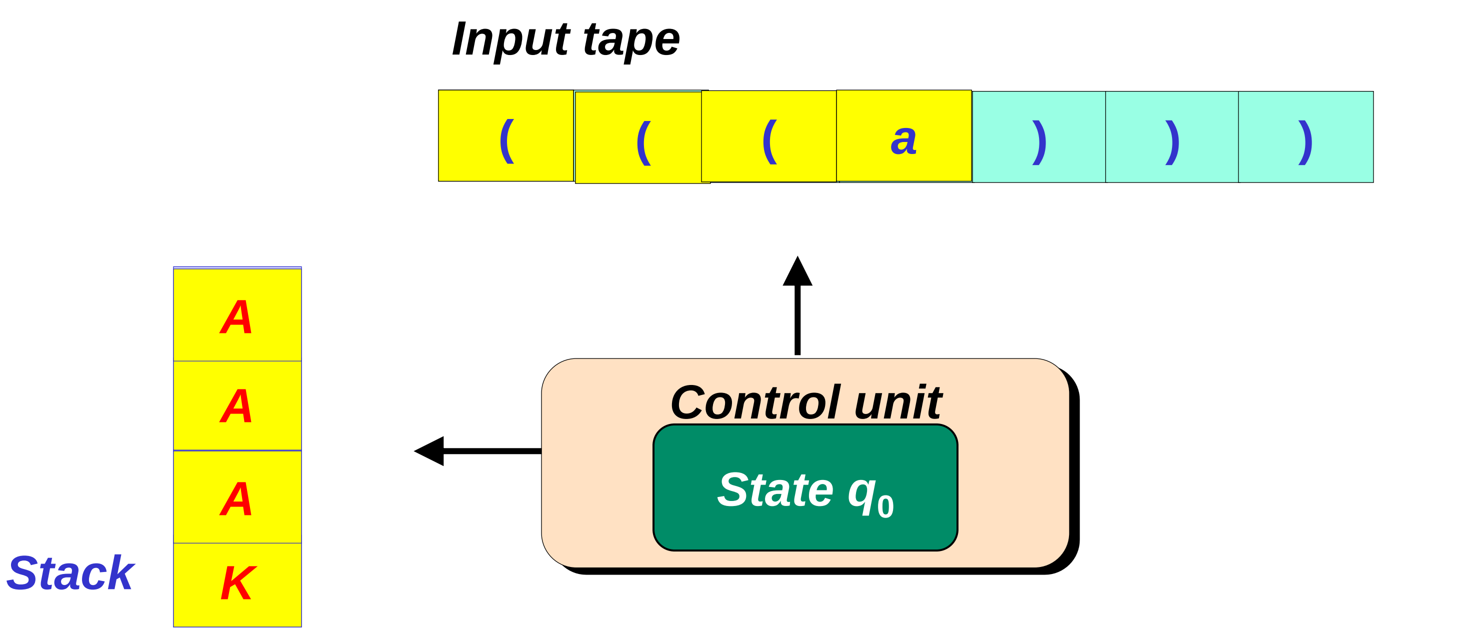

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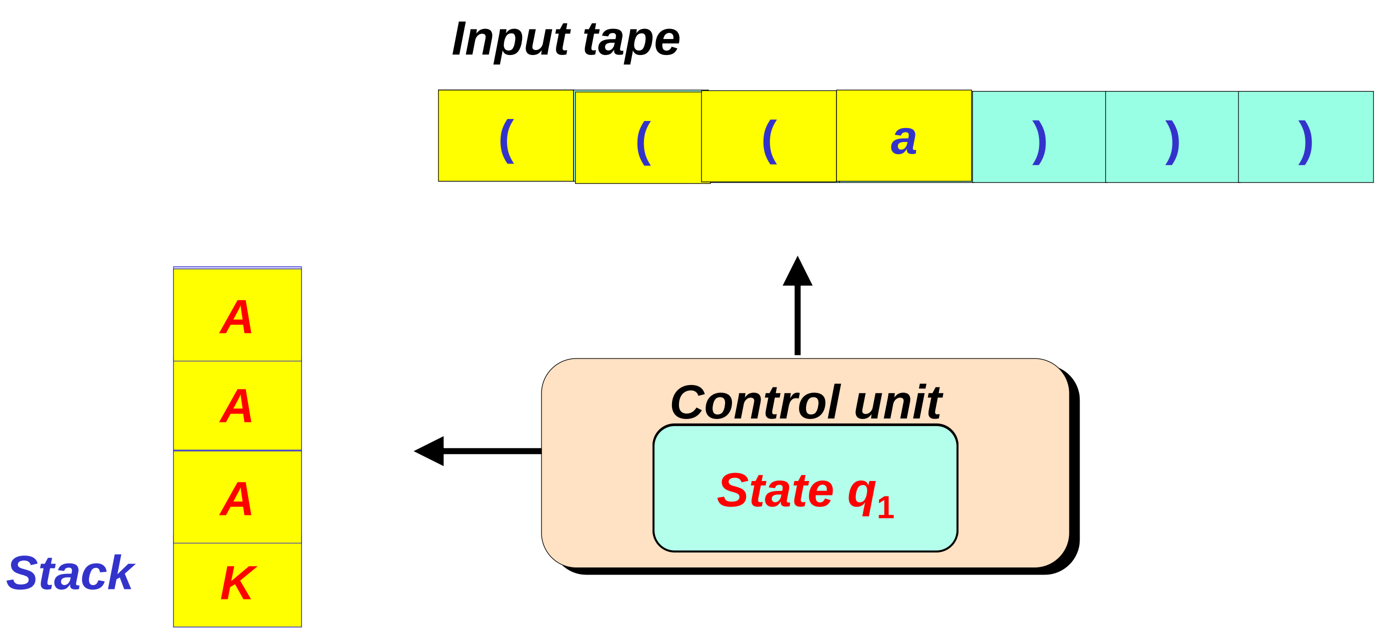

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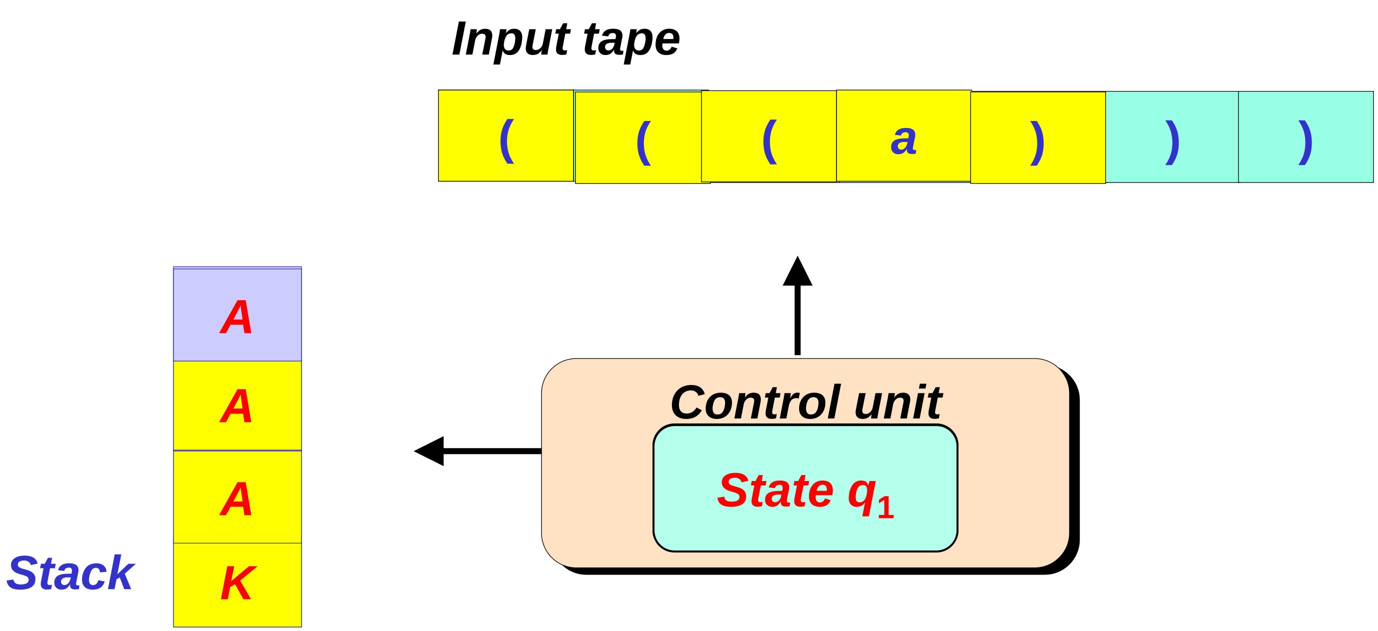

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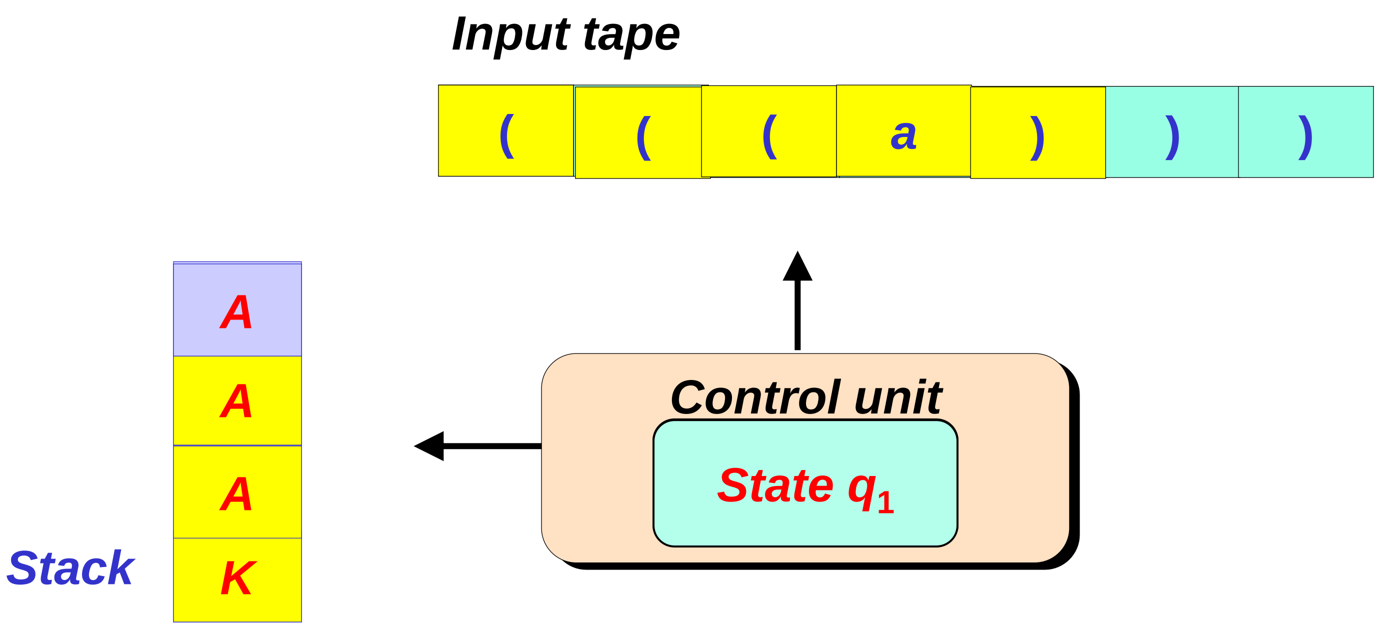

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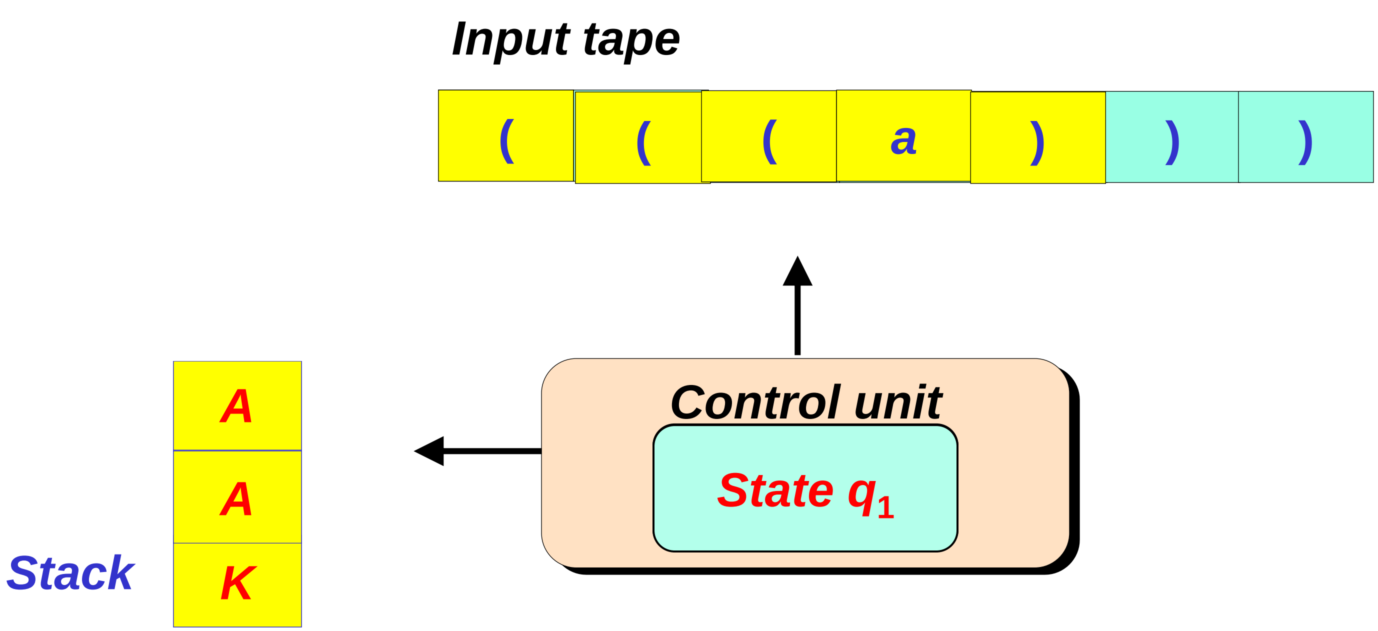

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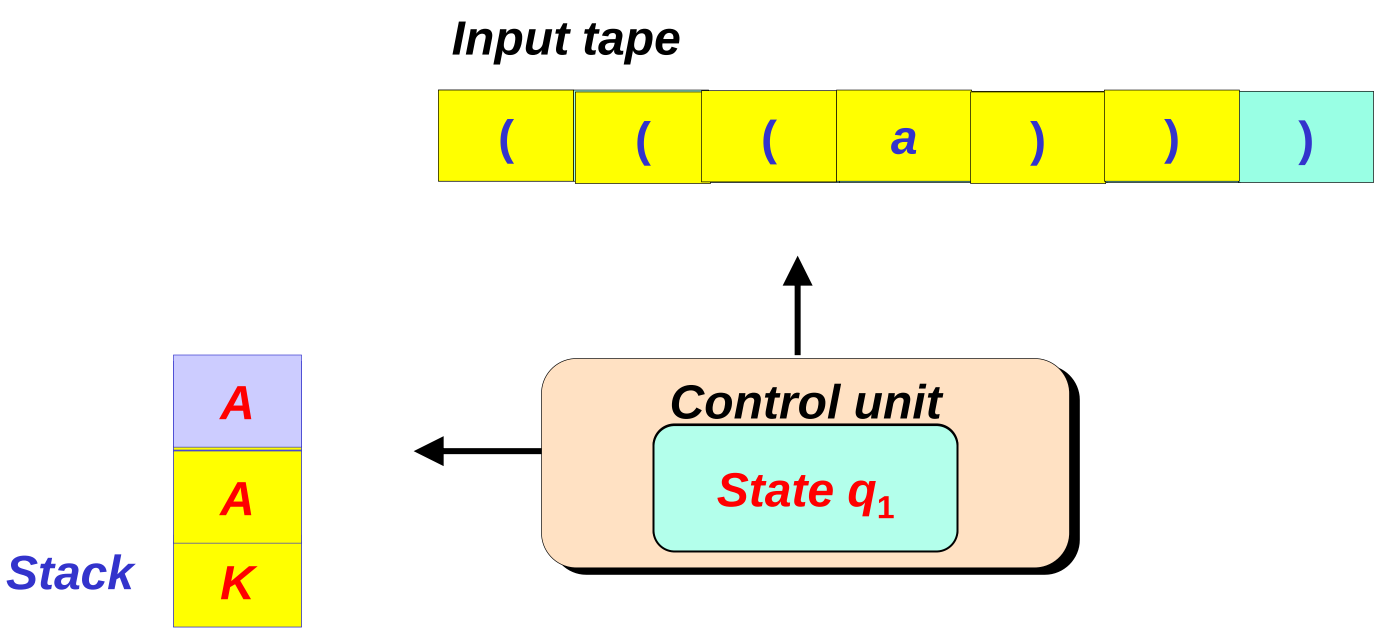

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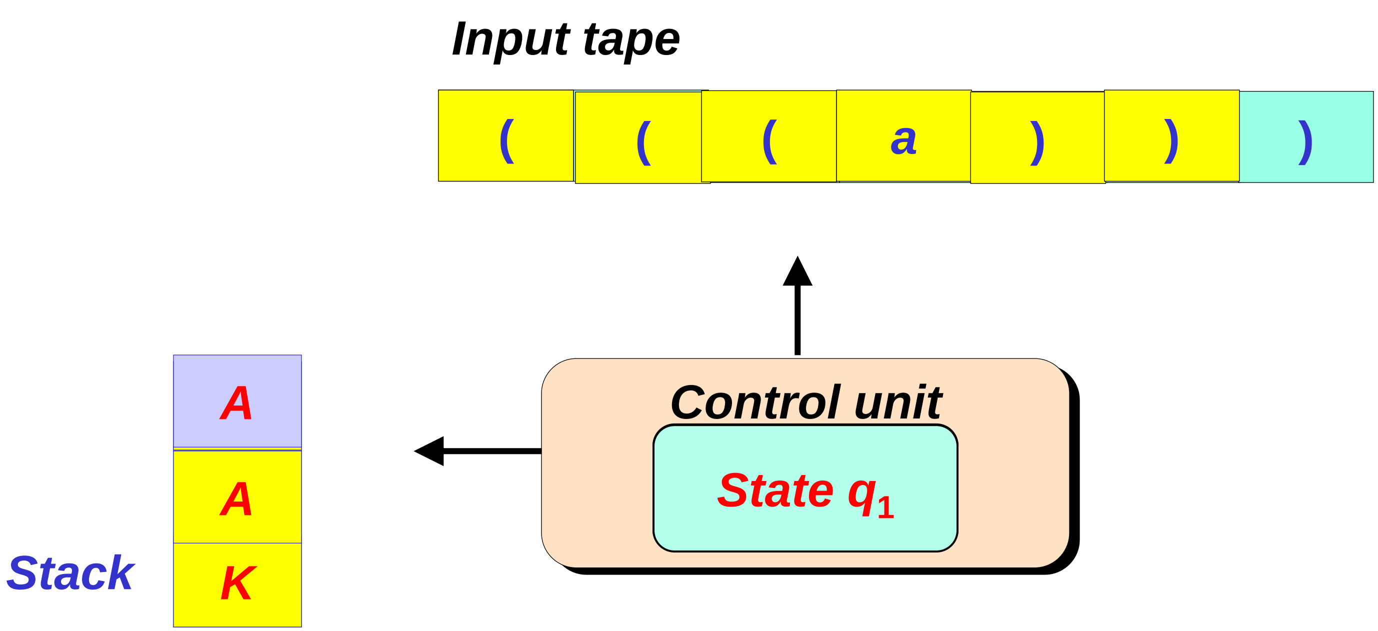

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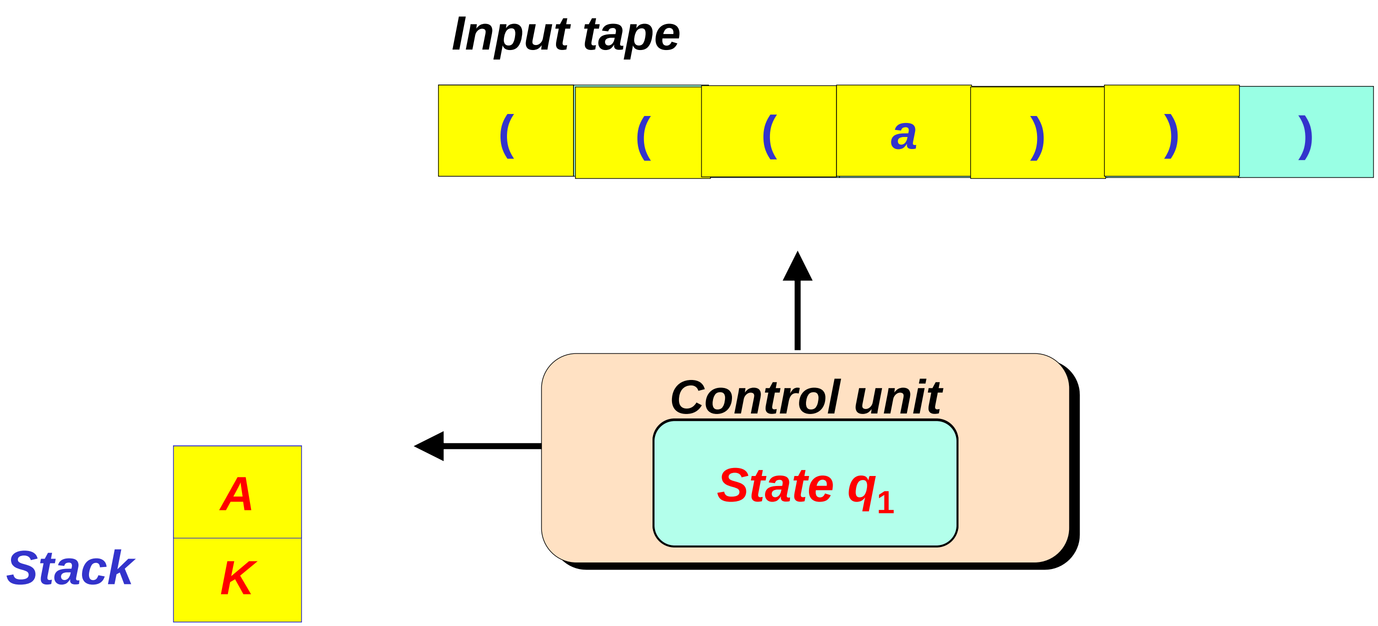

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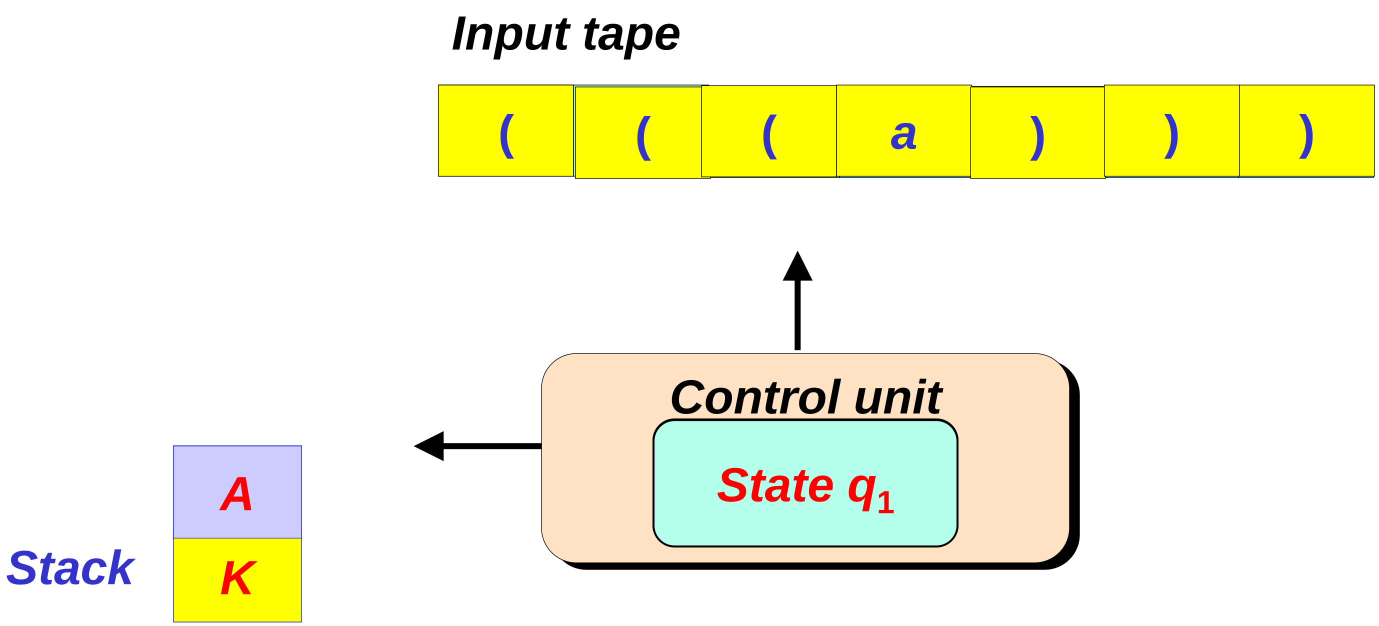

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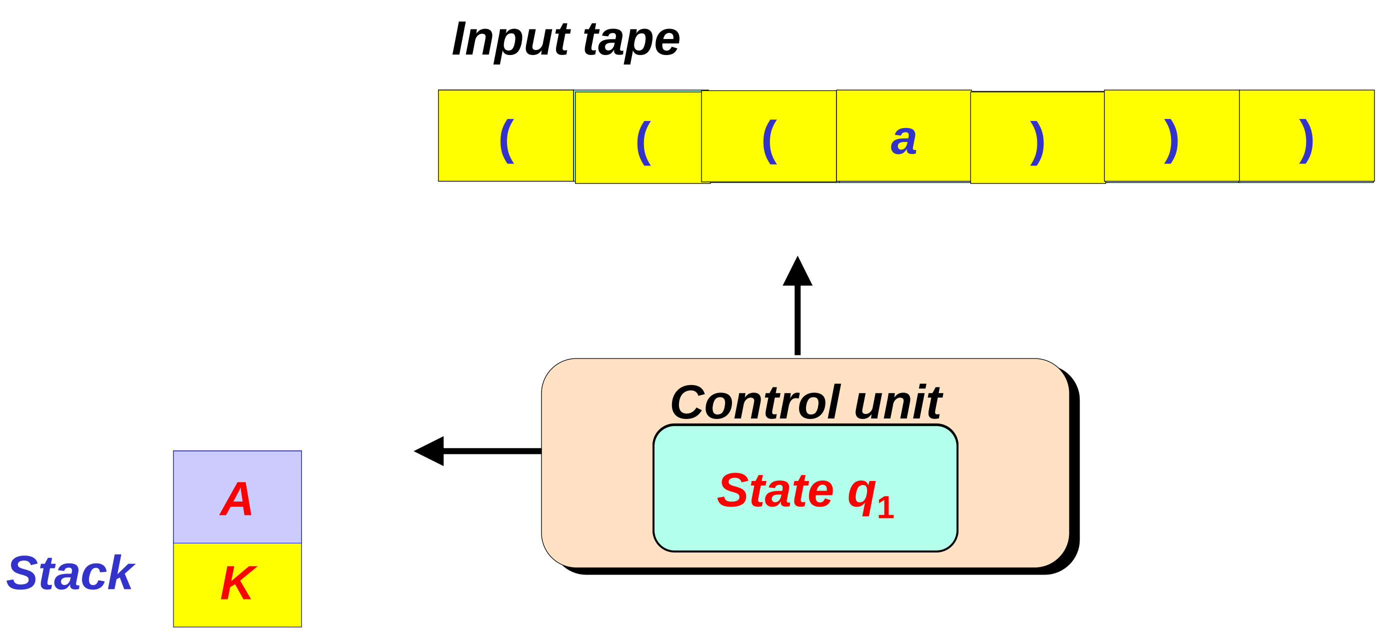

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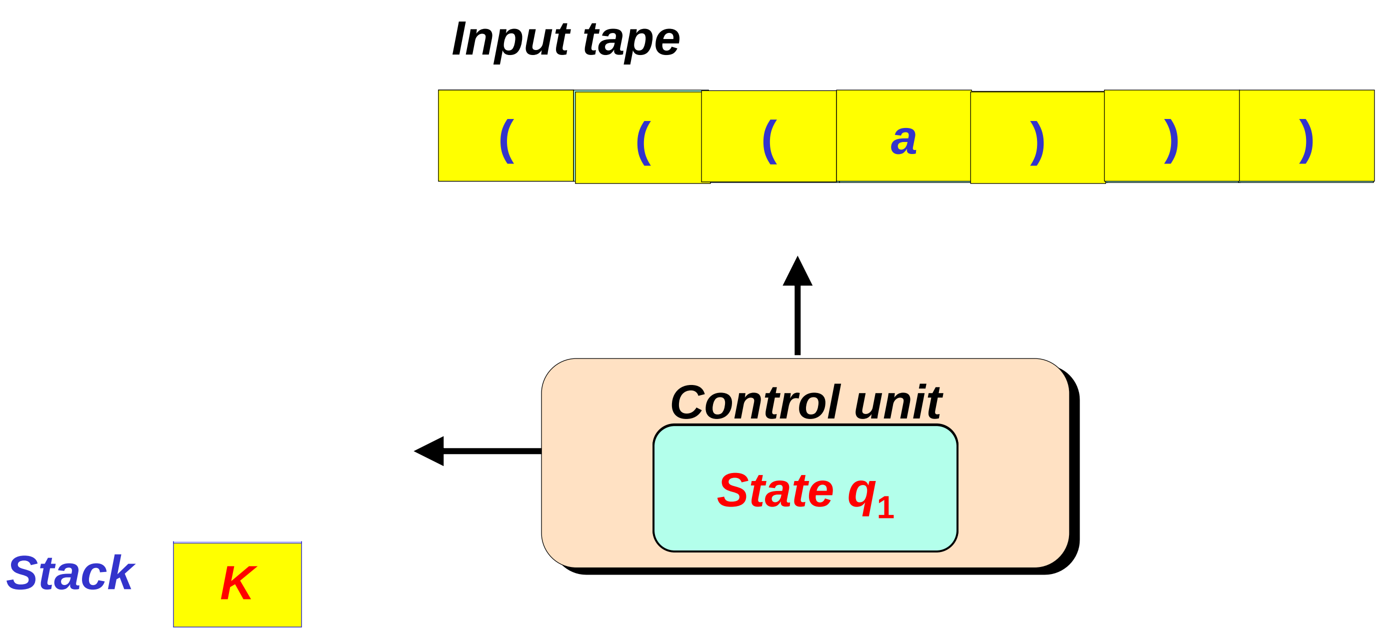

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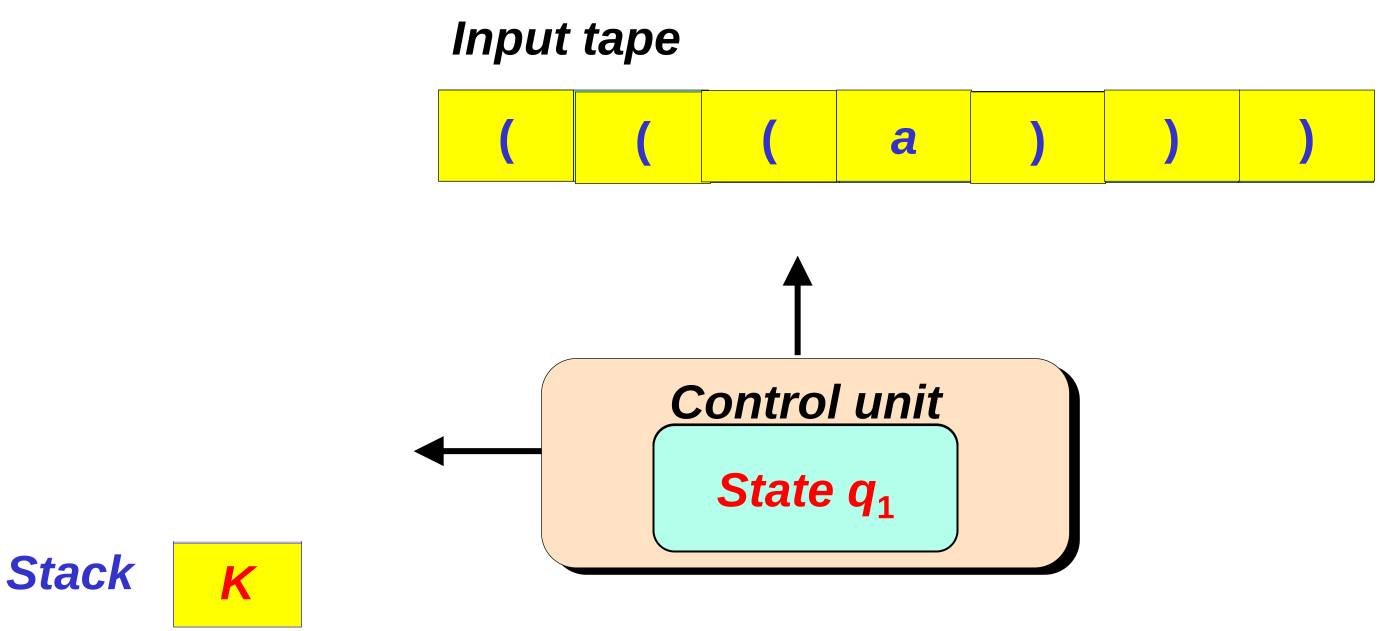

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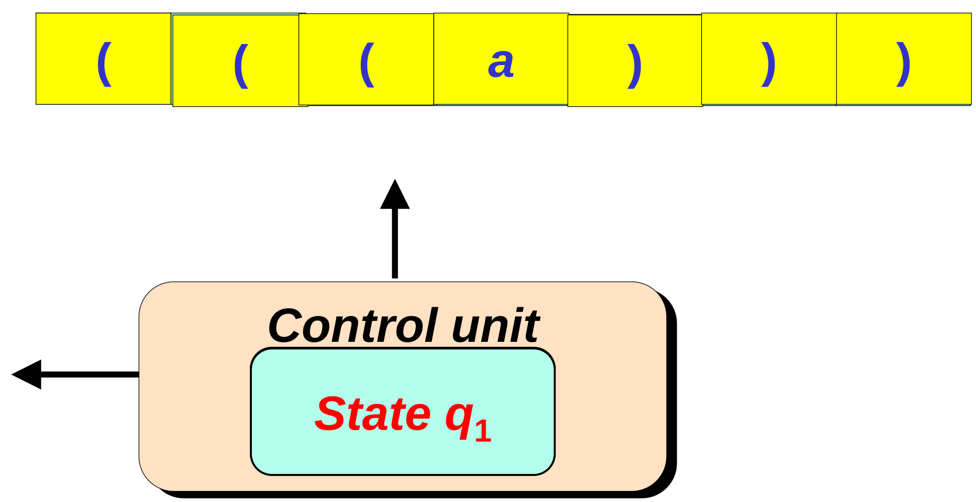
Input tape



Stack

Lecture 9

PA Definition

PA Definition

$$pa = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

PA Definition

$$pa = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

Q - finite set of states

PA Definition

$$pa = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

Q	- finite set of states
Σ	- finite set of input symbols

PA Definition

$$pa = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$$

Q	- finite set of states
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Q	- finite set of states
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Q	- finite set of states
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$$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \rightarrow Q \times \Gamma^*$$

PA Definition

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Σ - finite set of input symbols

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$$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \rightarrow Q \times \Gamma^*$$

$q_0 \in Q$ - start state

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$F \subseteq Q$	- set of accepting states

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$$\delta(q, a, Z) = \{ (p_1, \gamma_1), (p_2, \gamma_2), \ldots, (p_m, \gamma_m) \}$$

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$a \in \Sigma$

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$$p_1 \in Q$$

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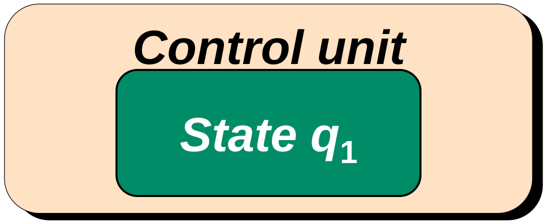

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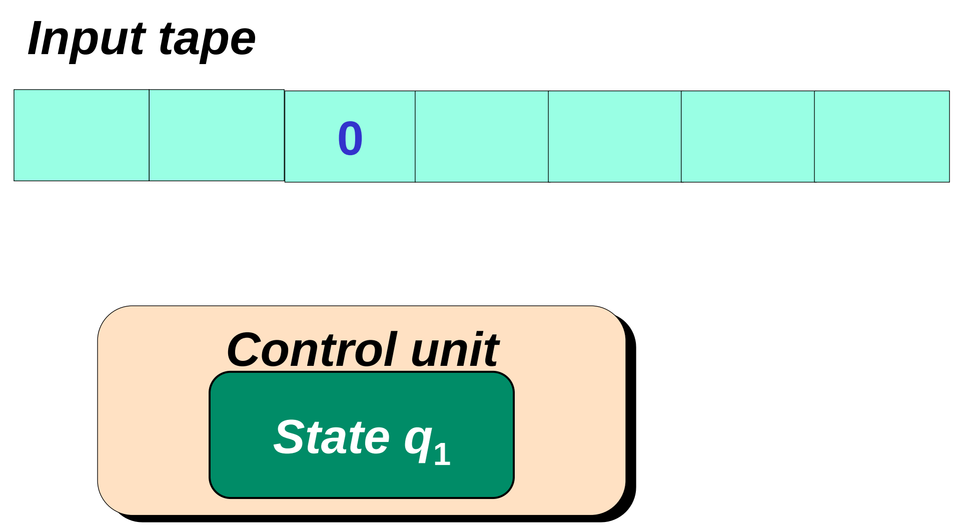

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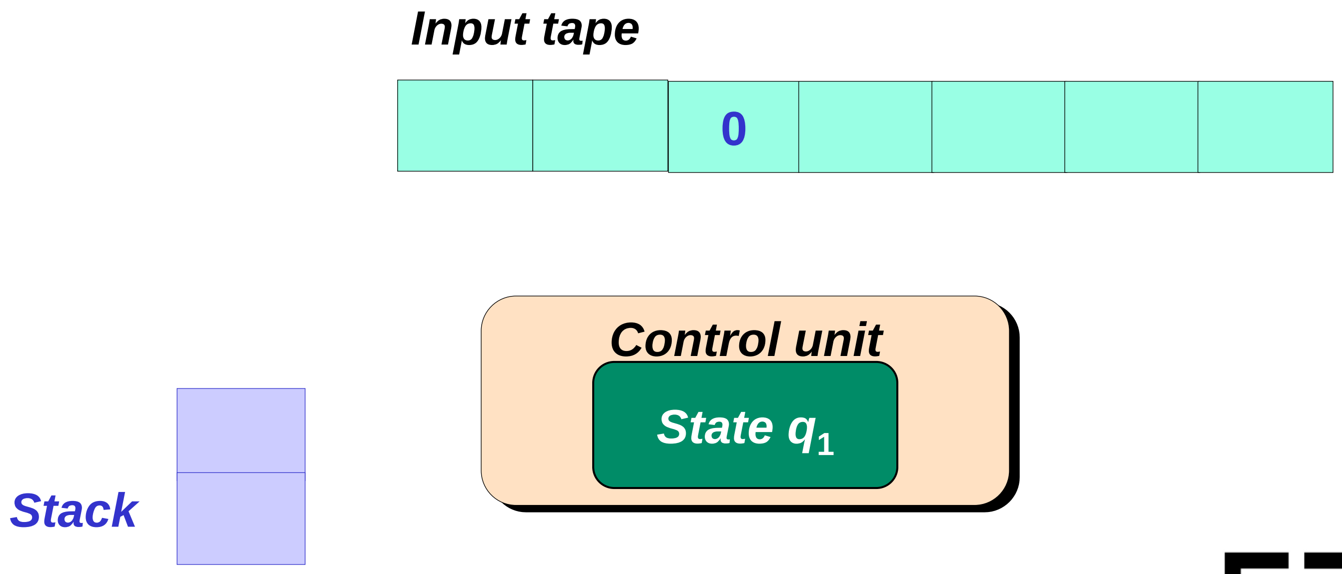

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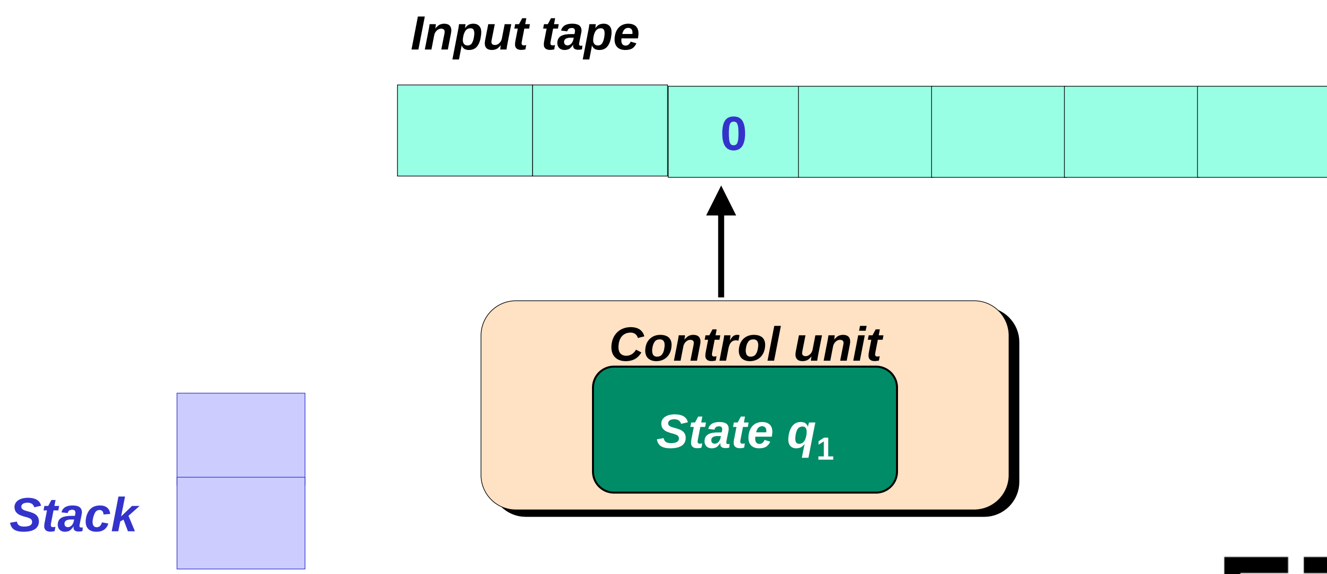

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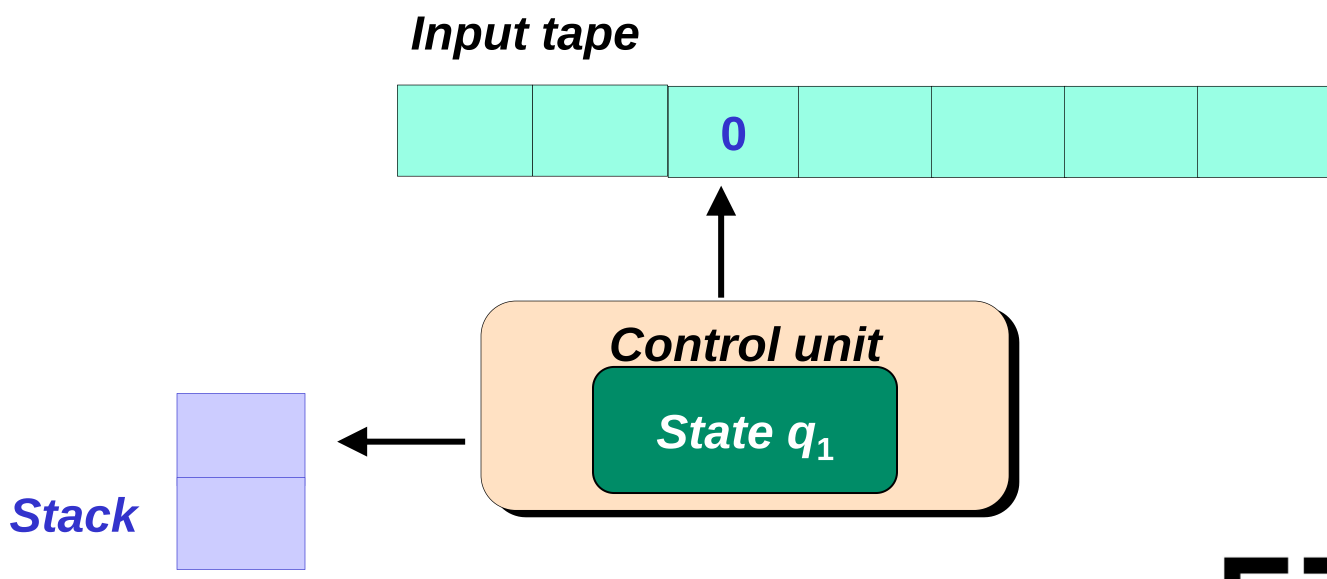

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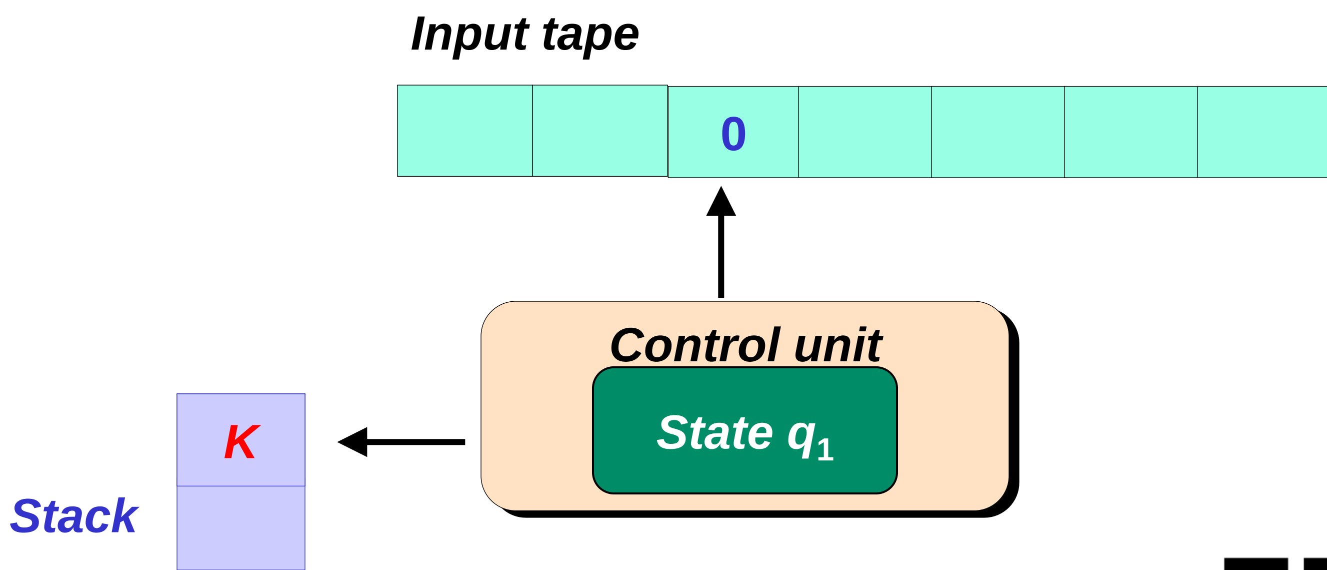

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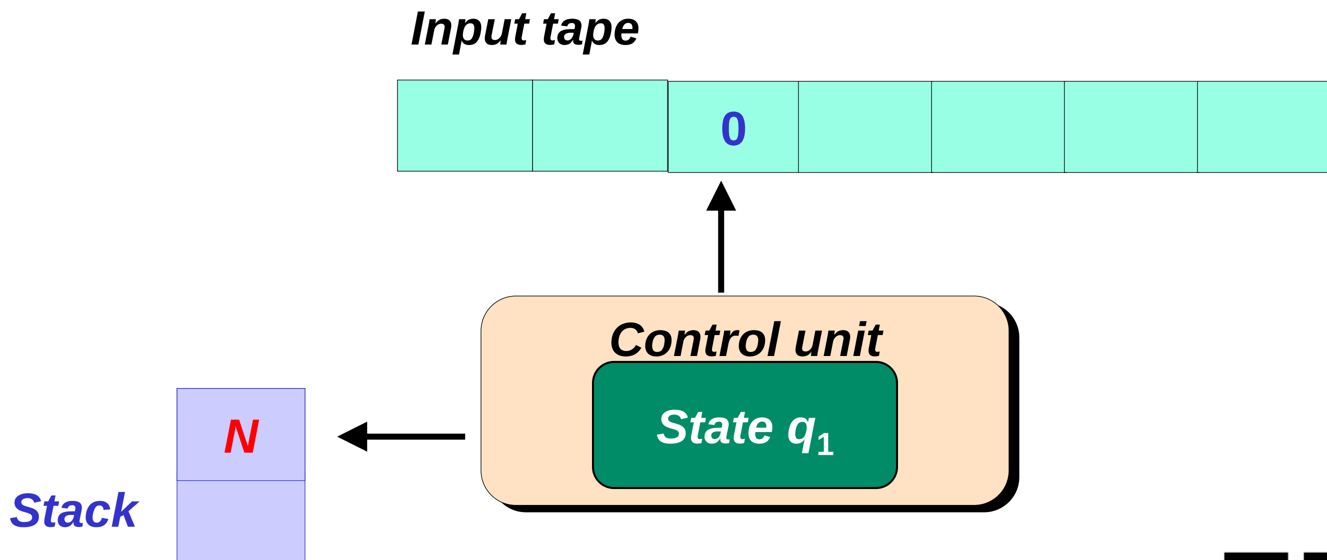

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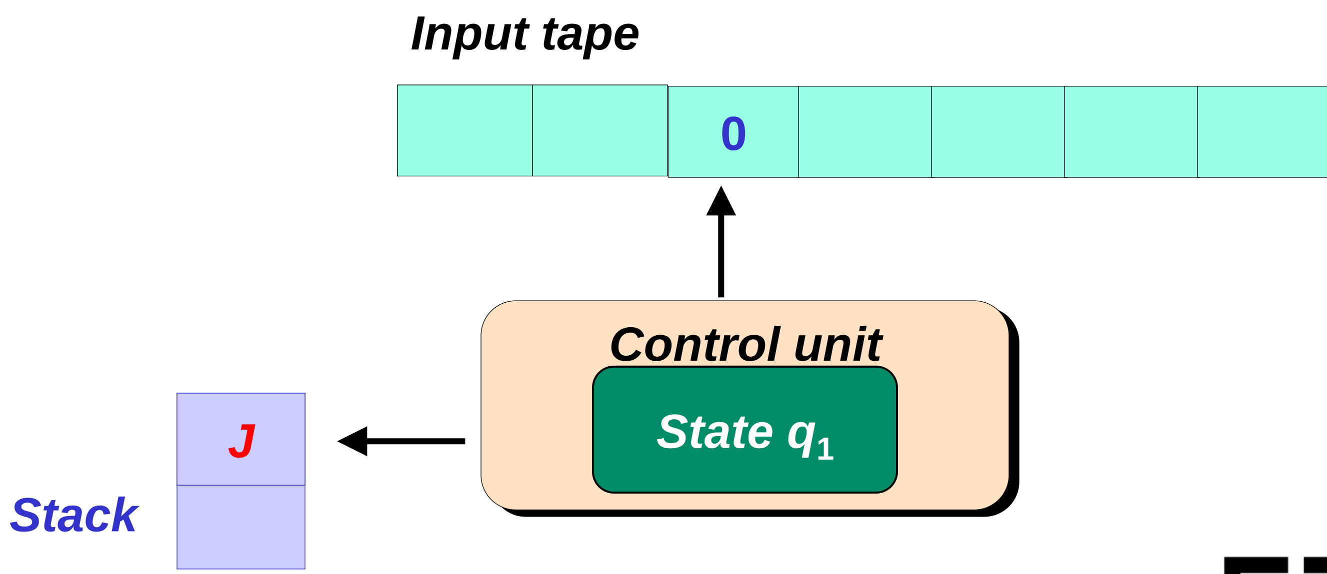

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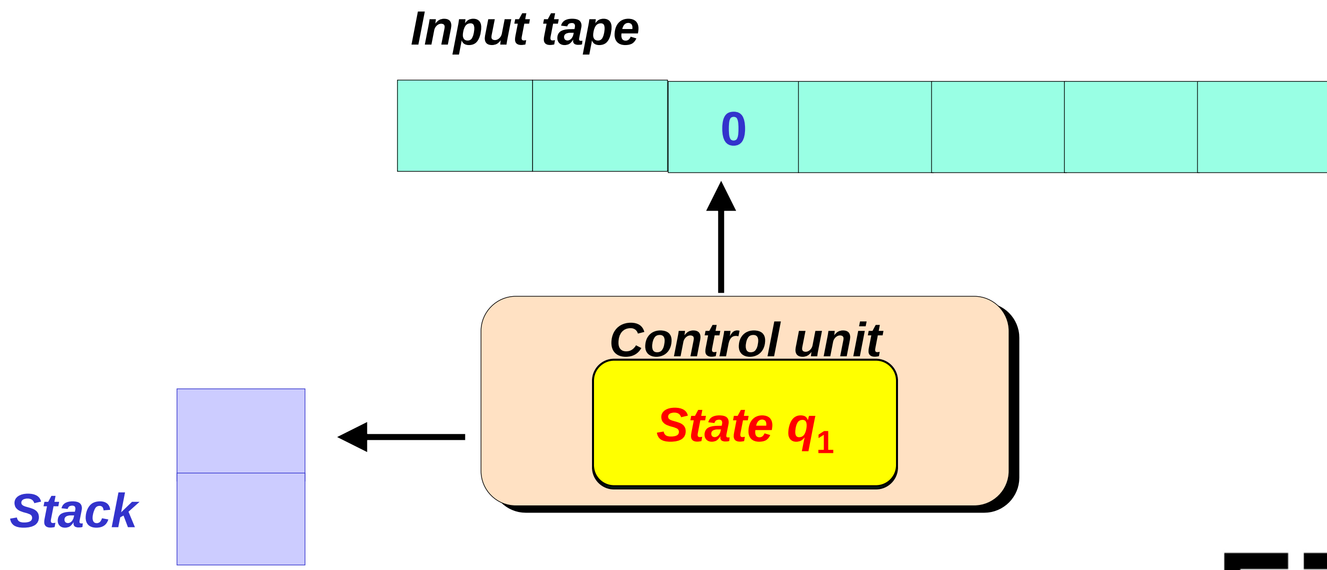

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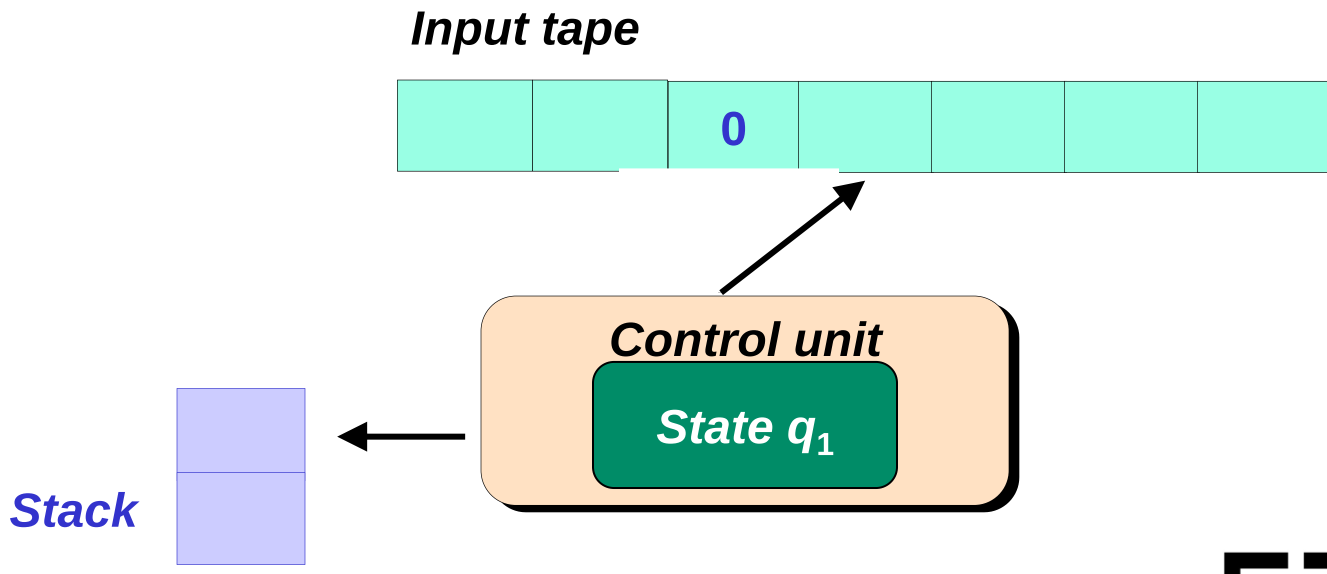

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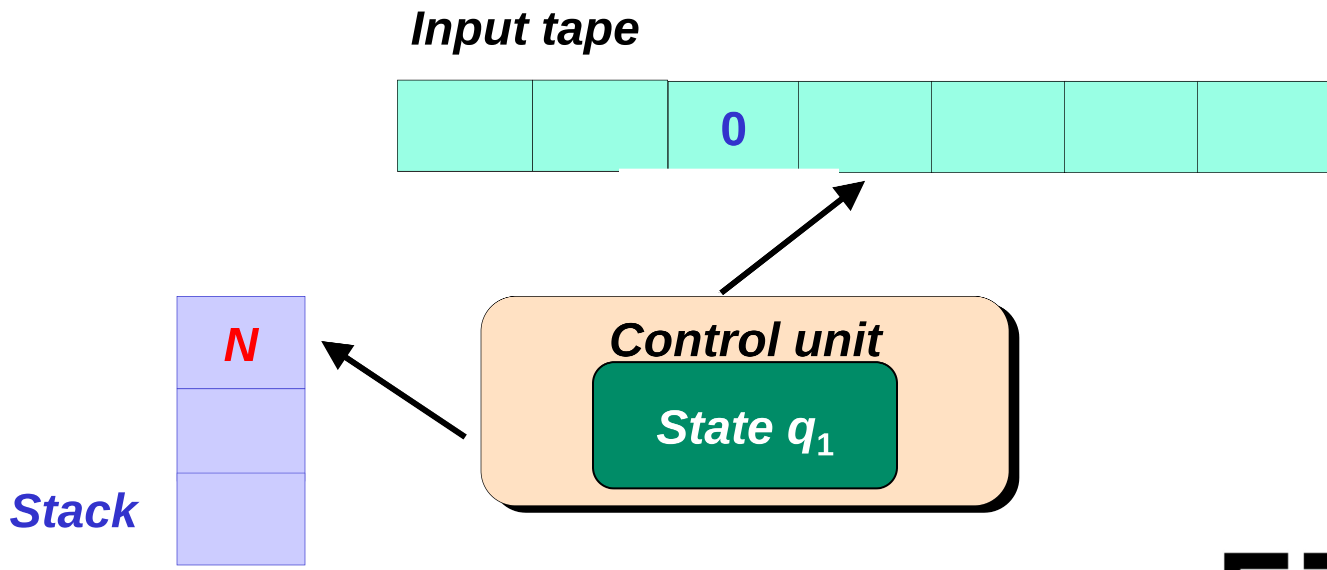

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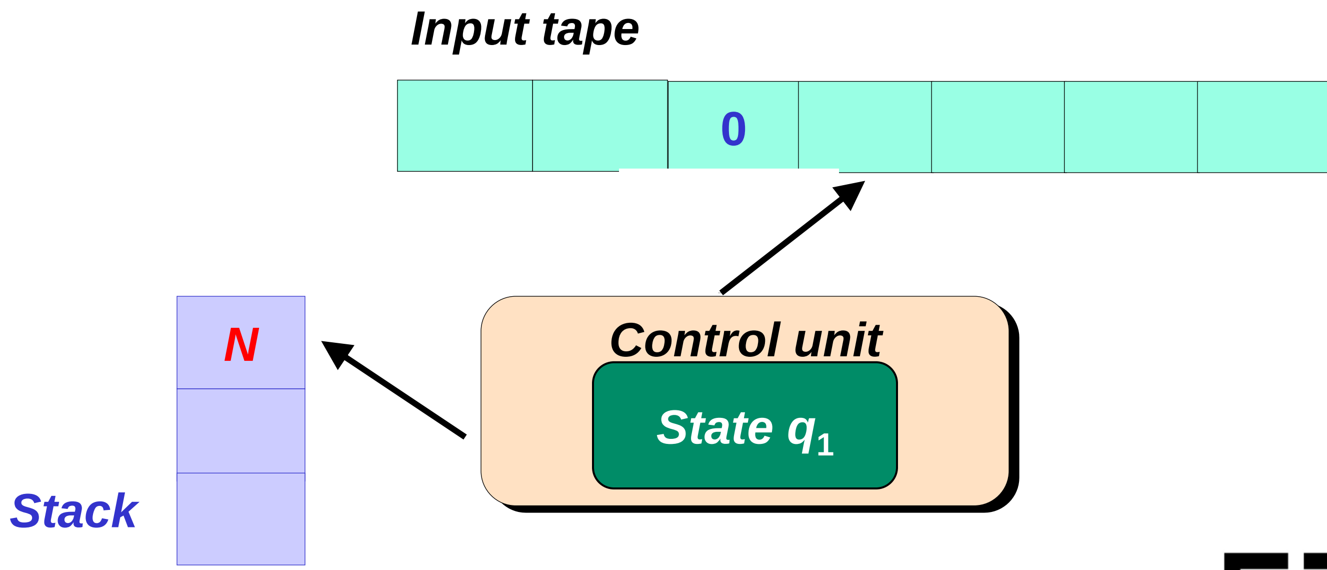

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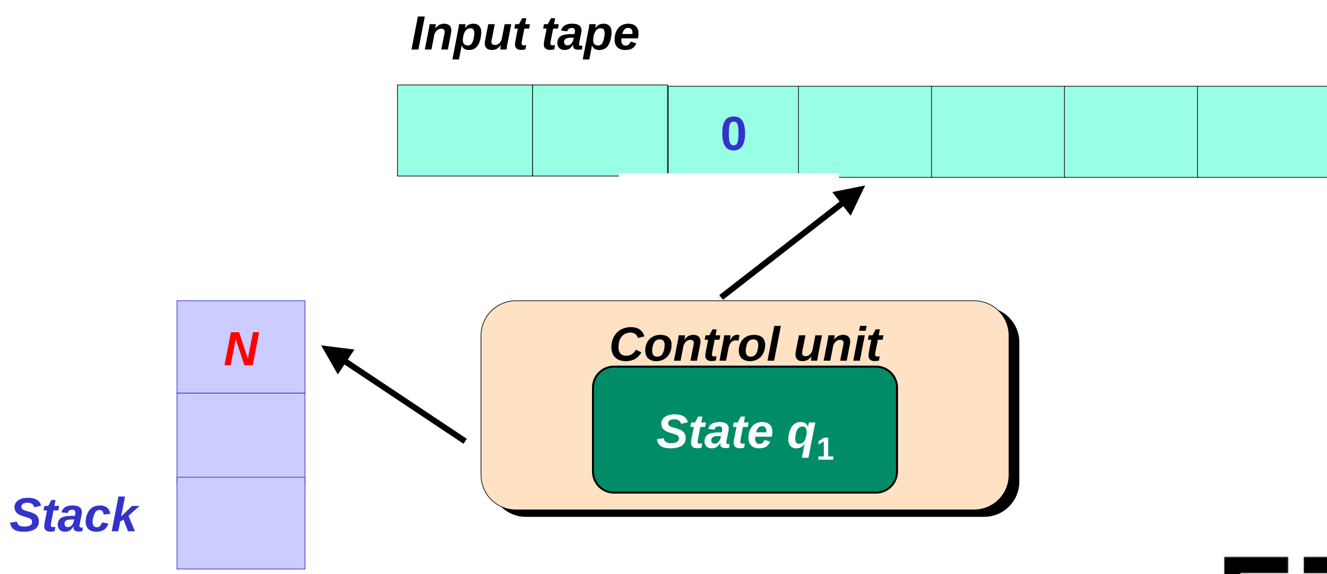

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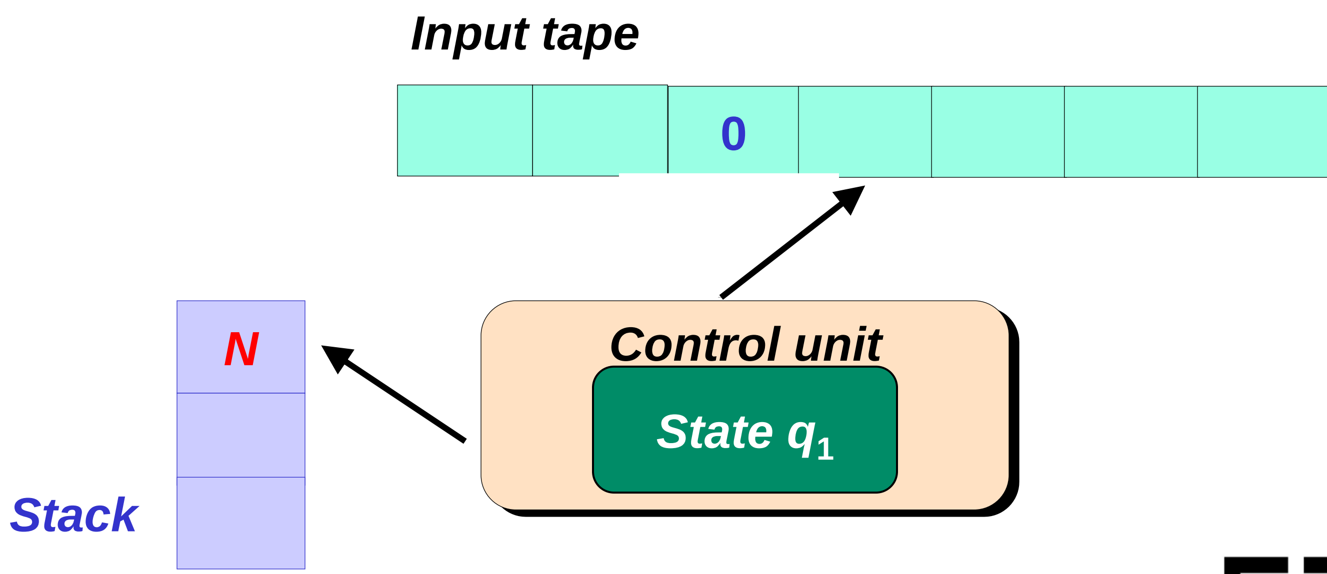

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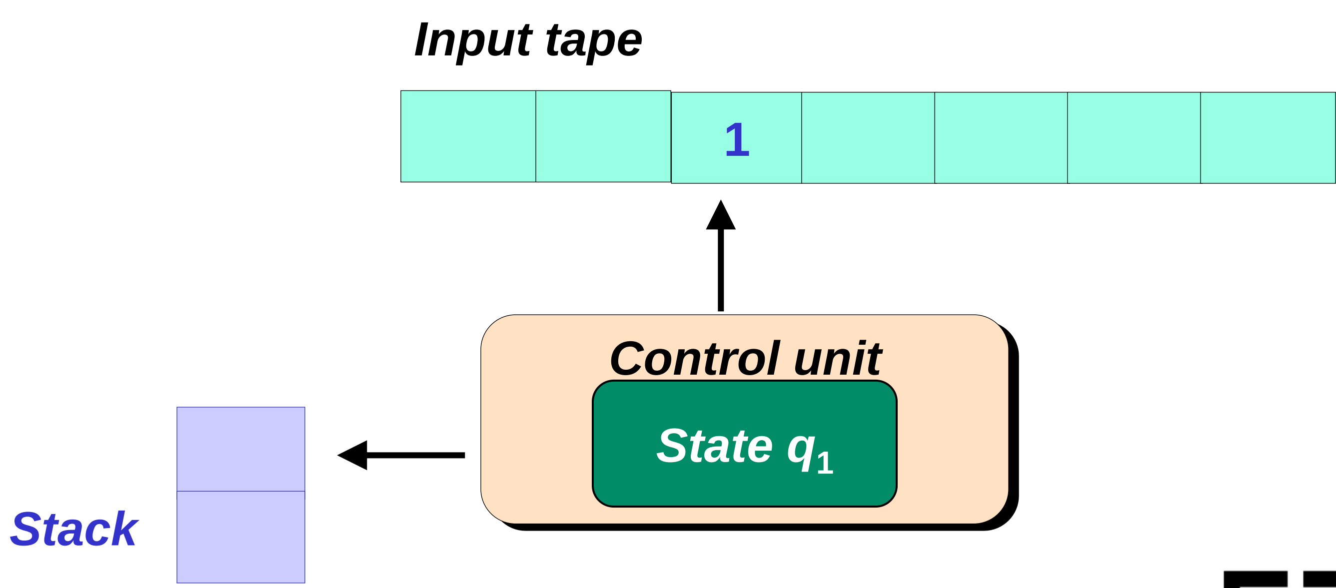

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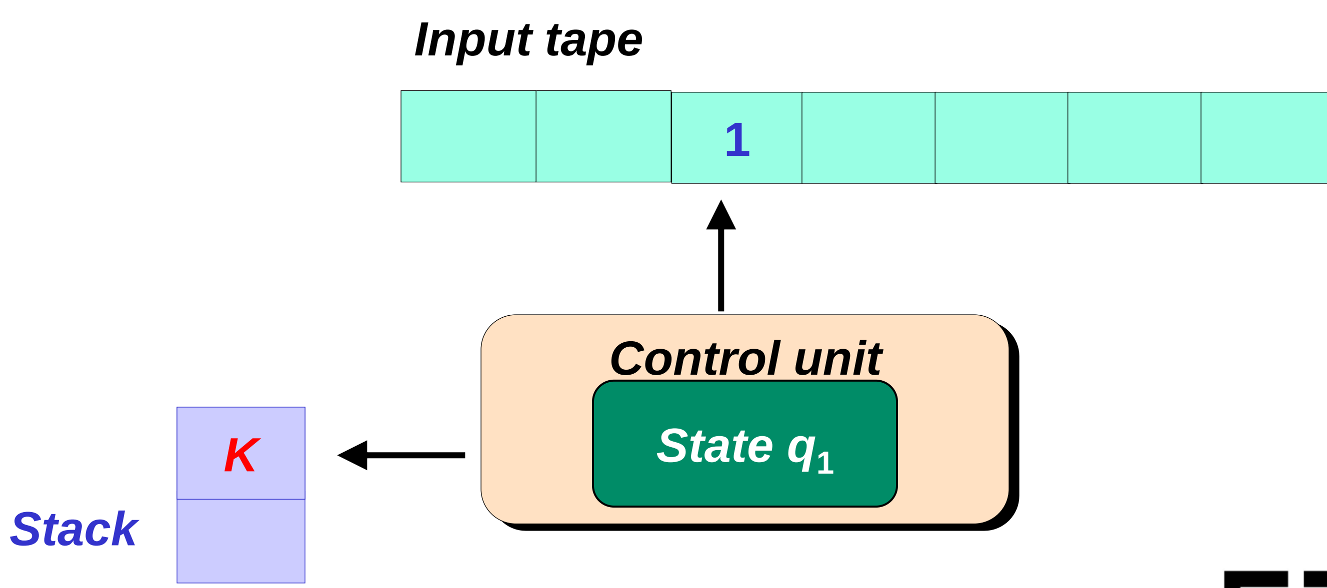

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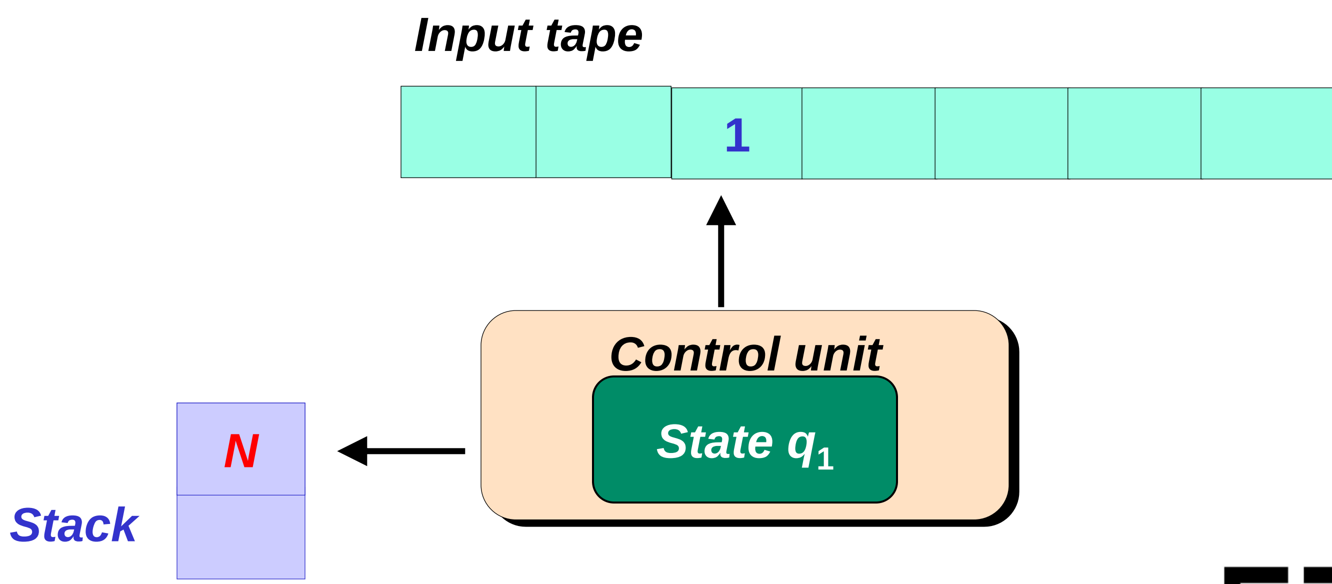

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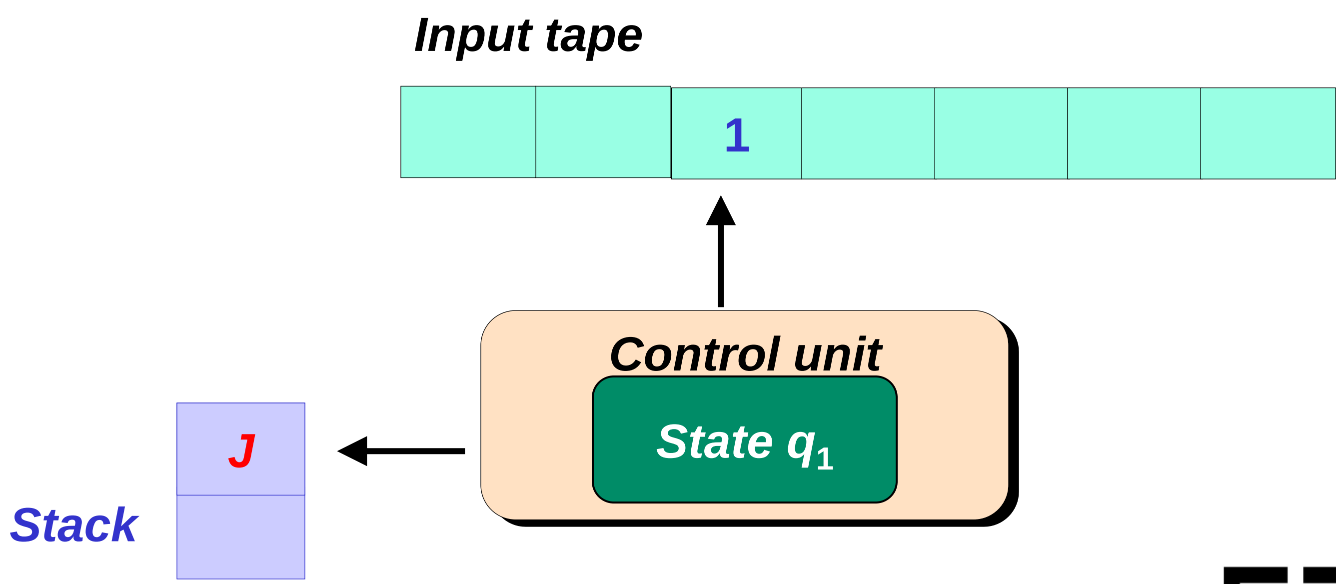

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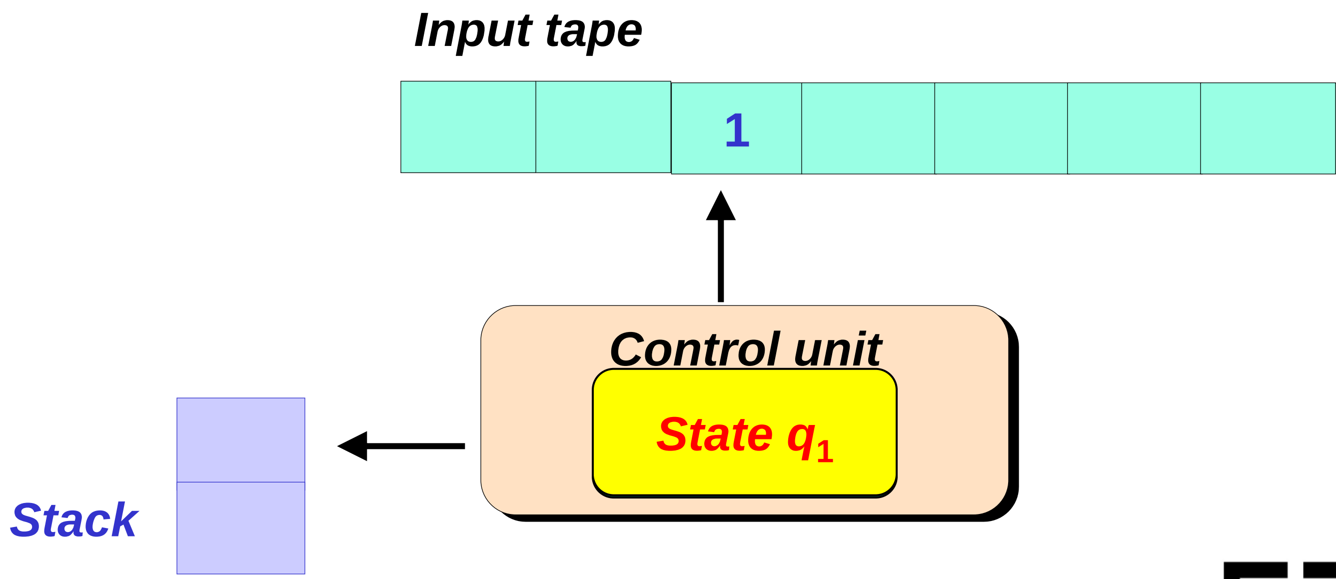

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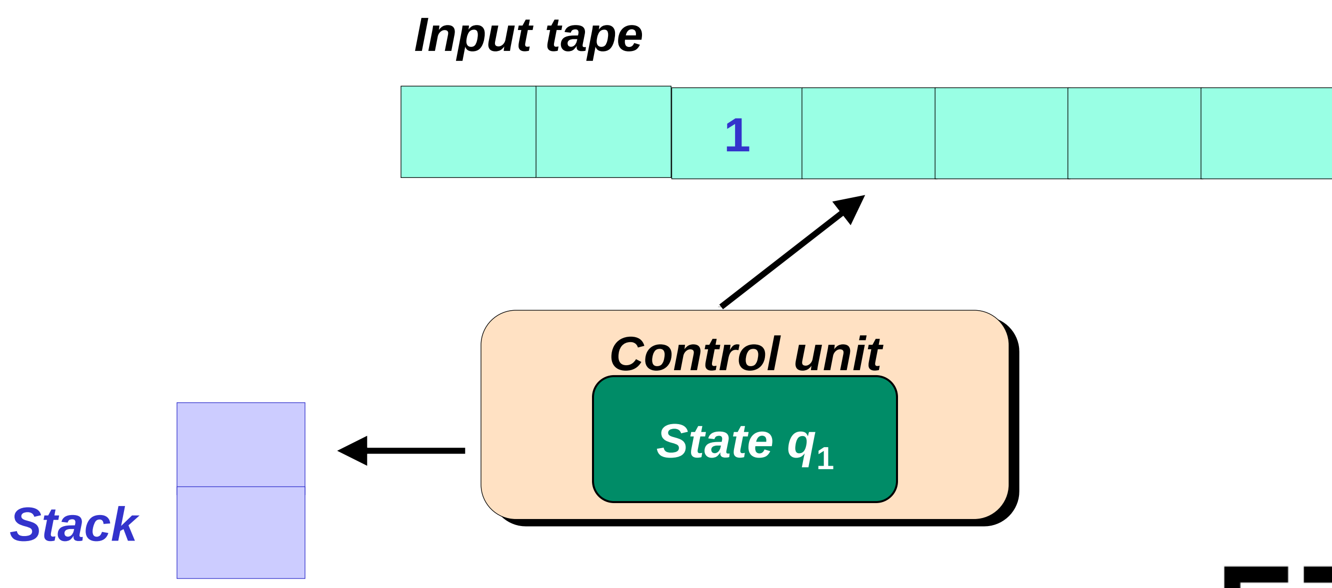

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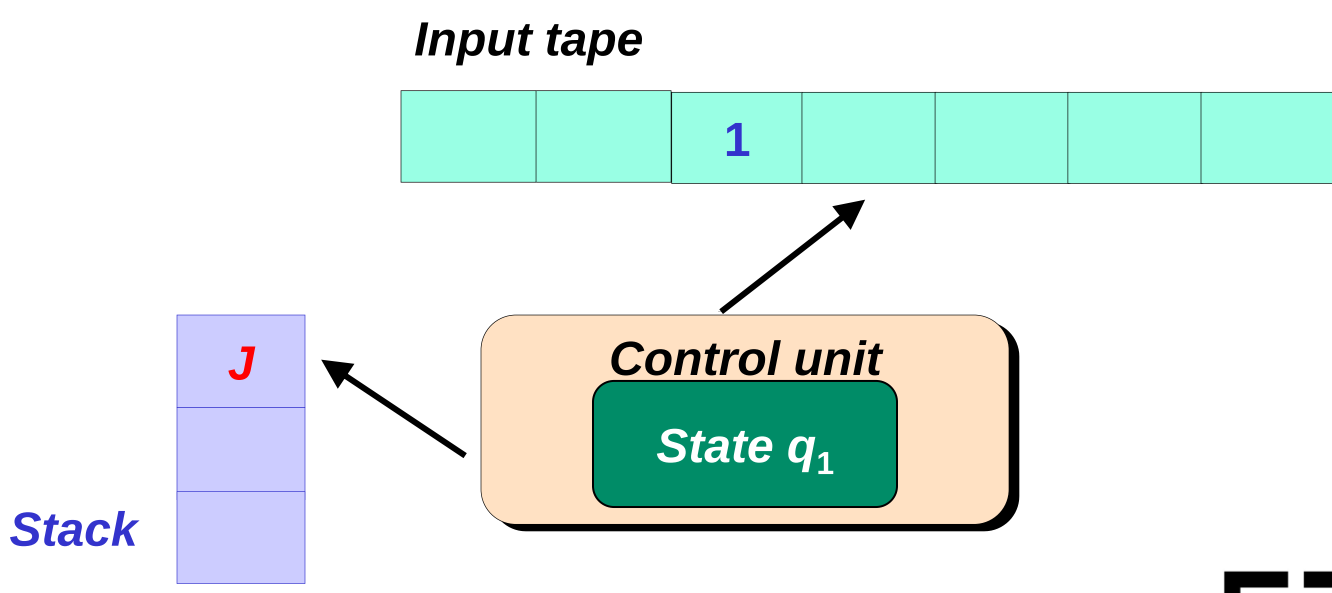

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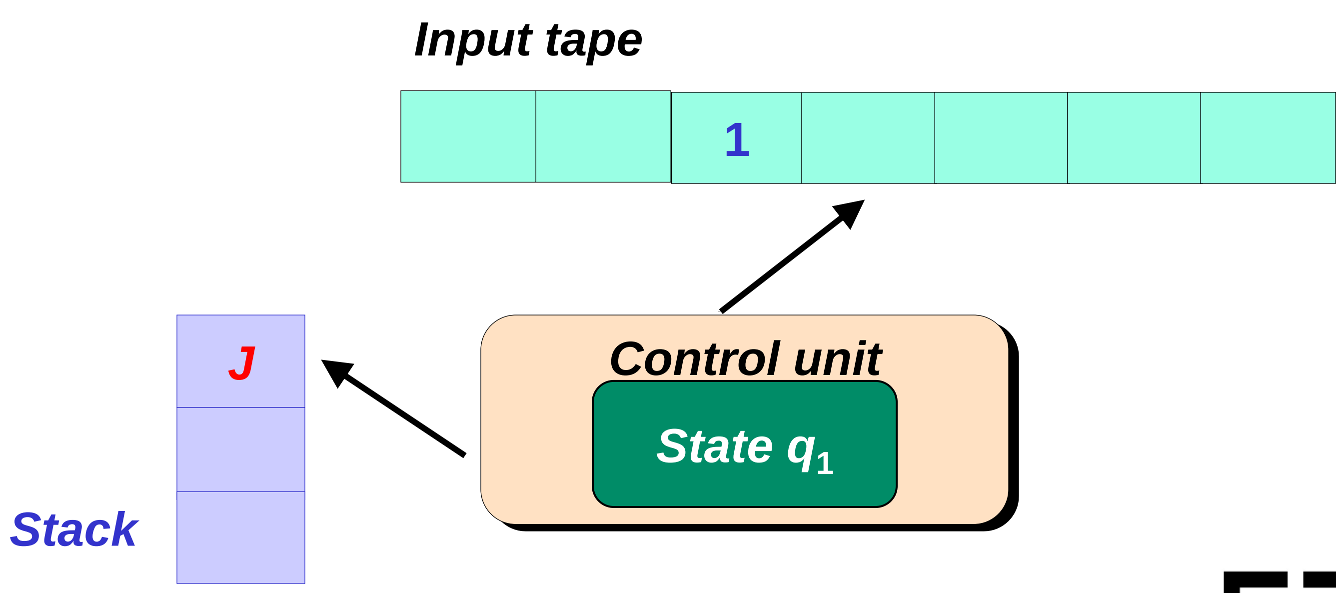

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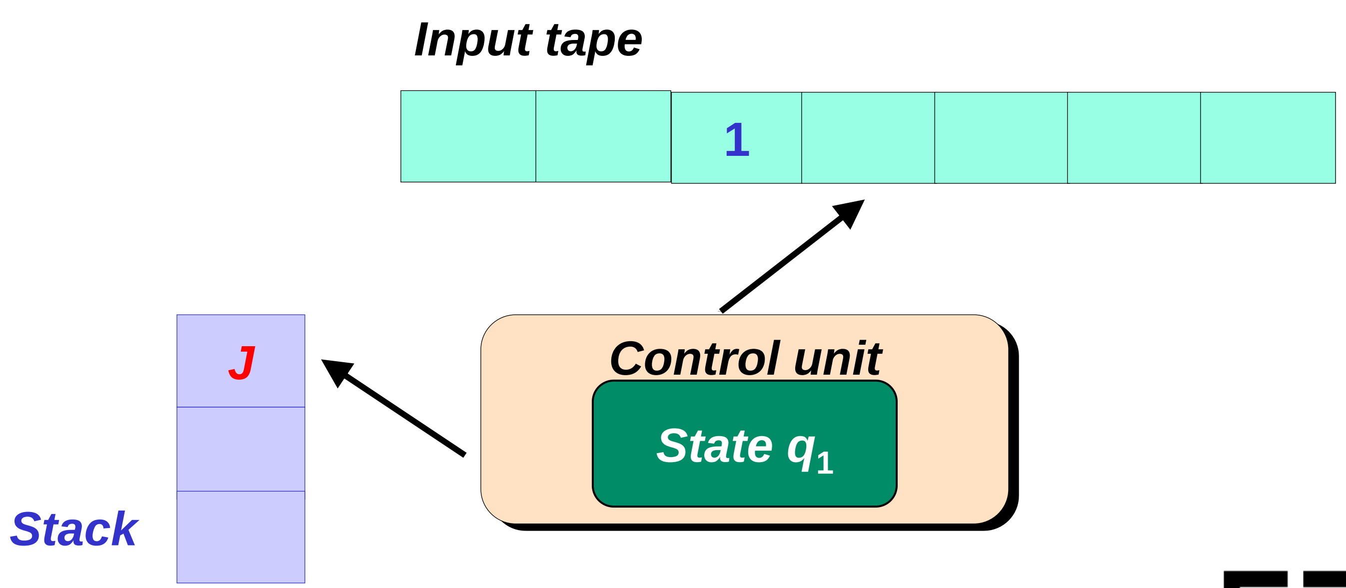

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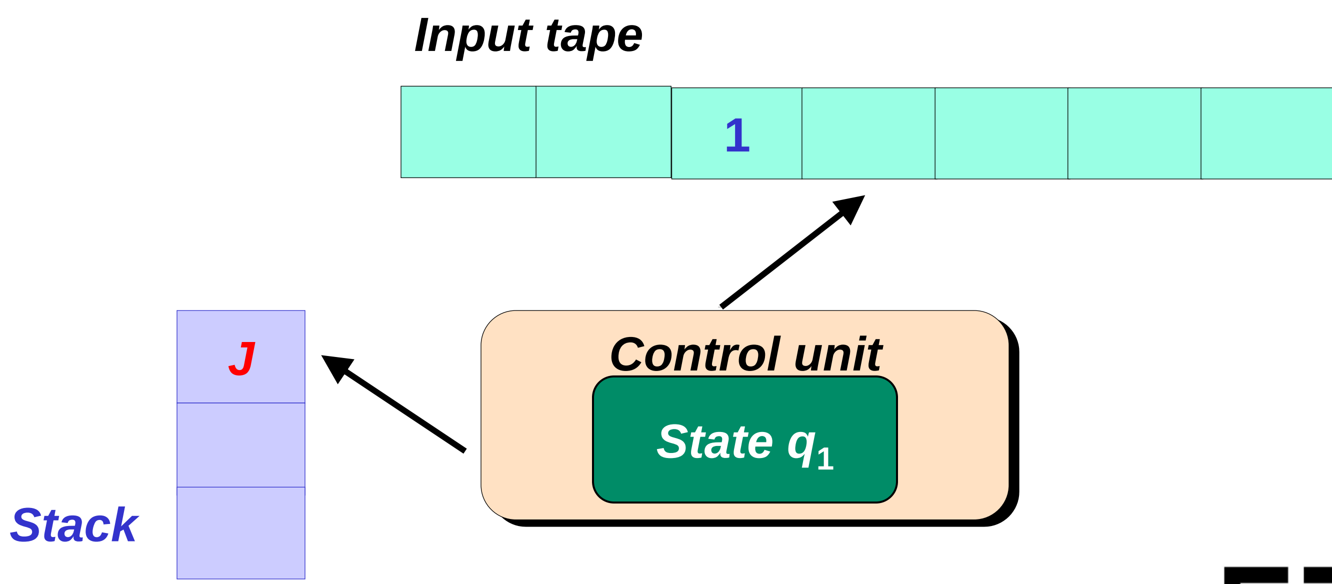

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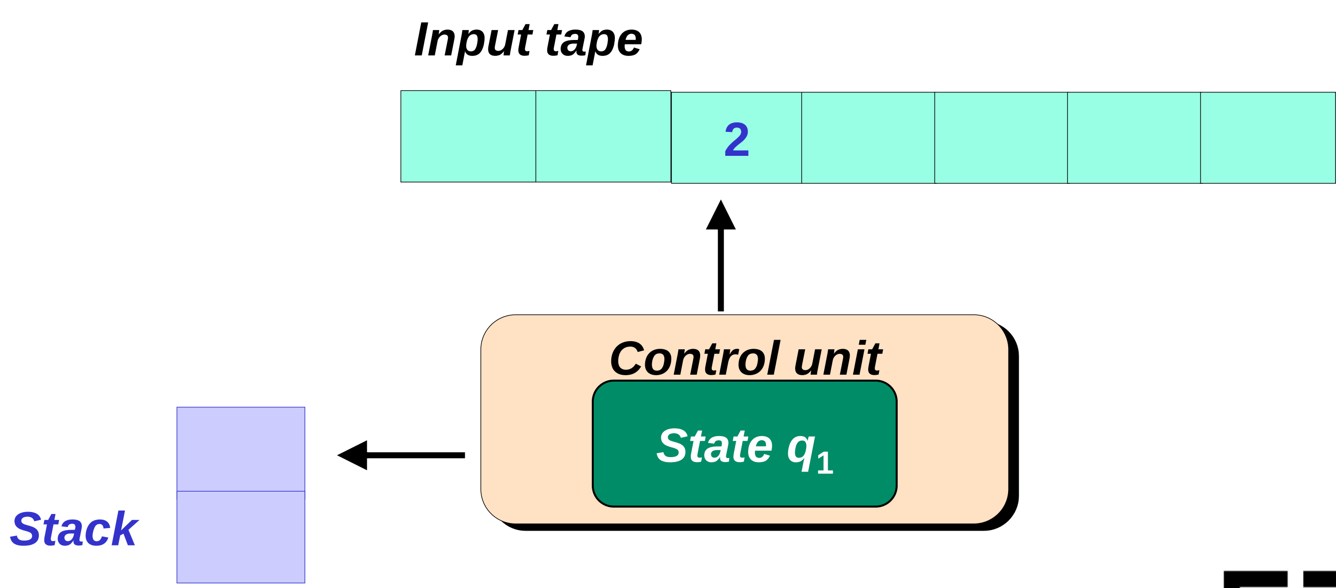

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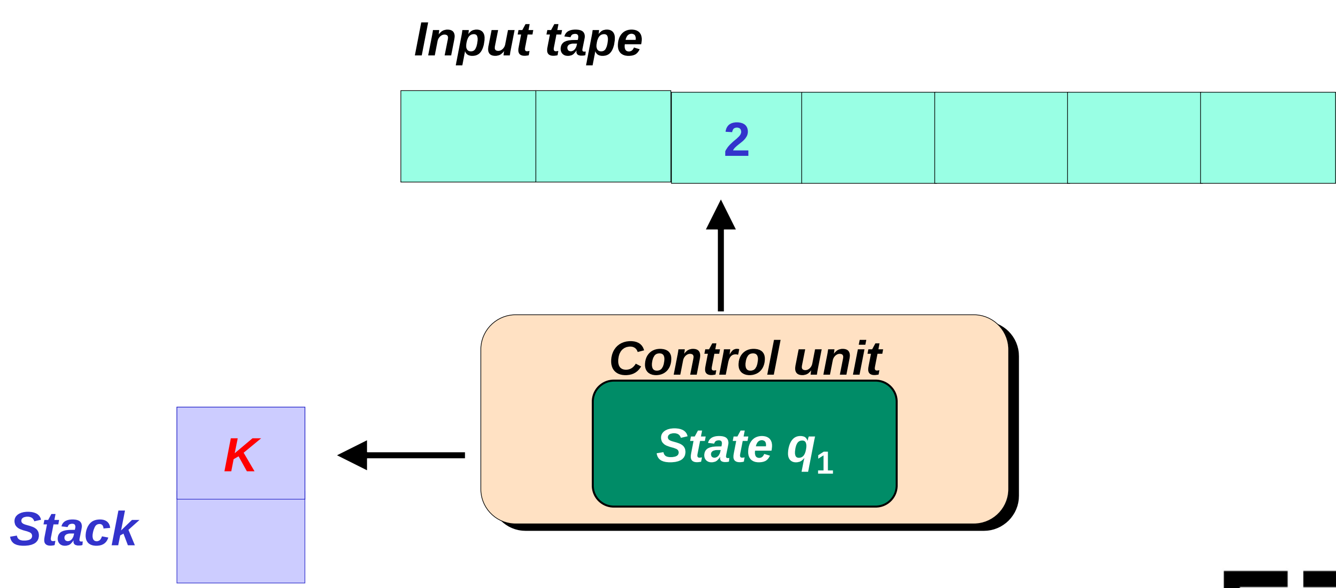

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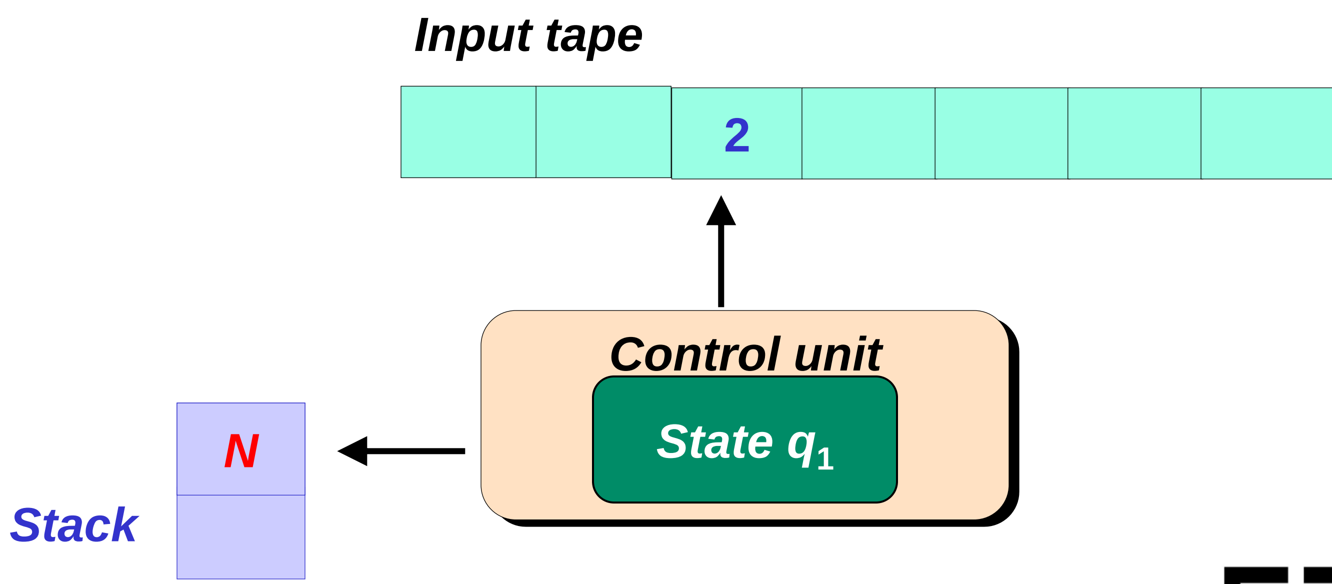

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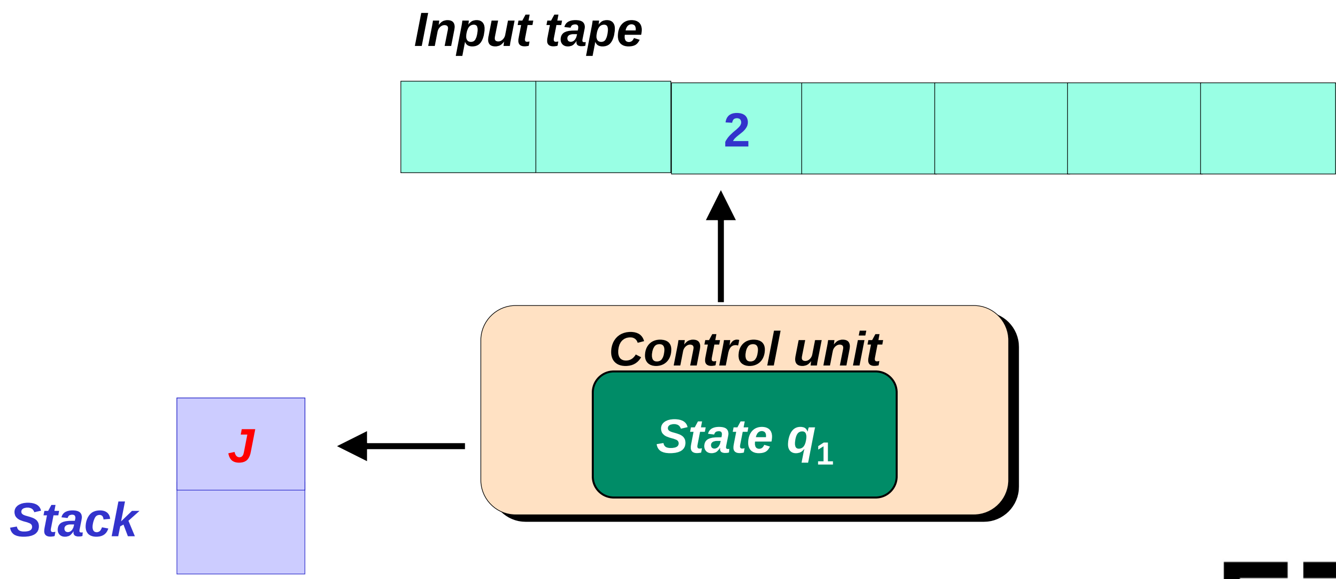

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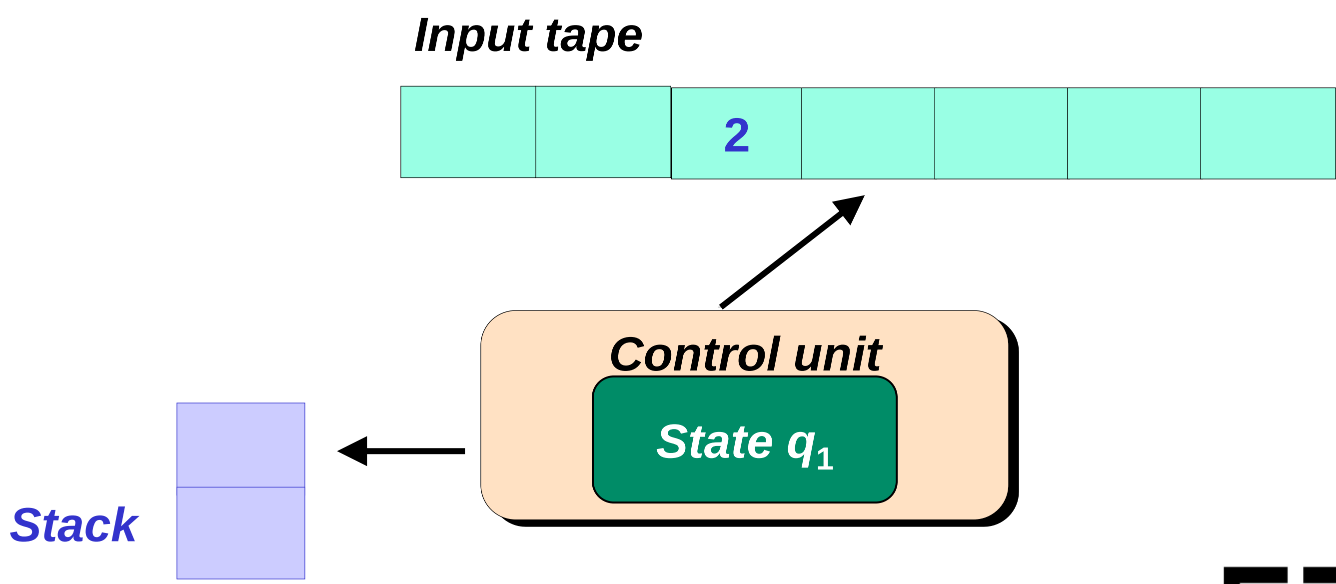

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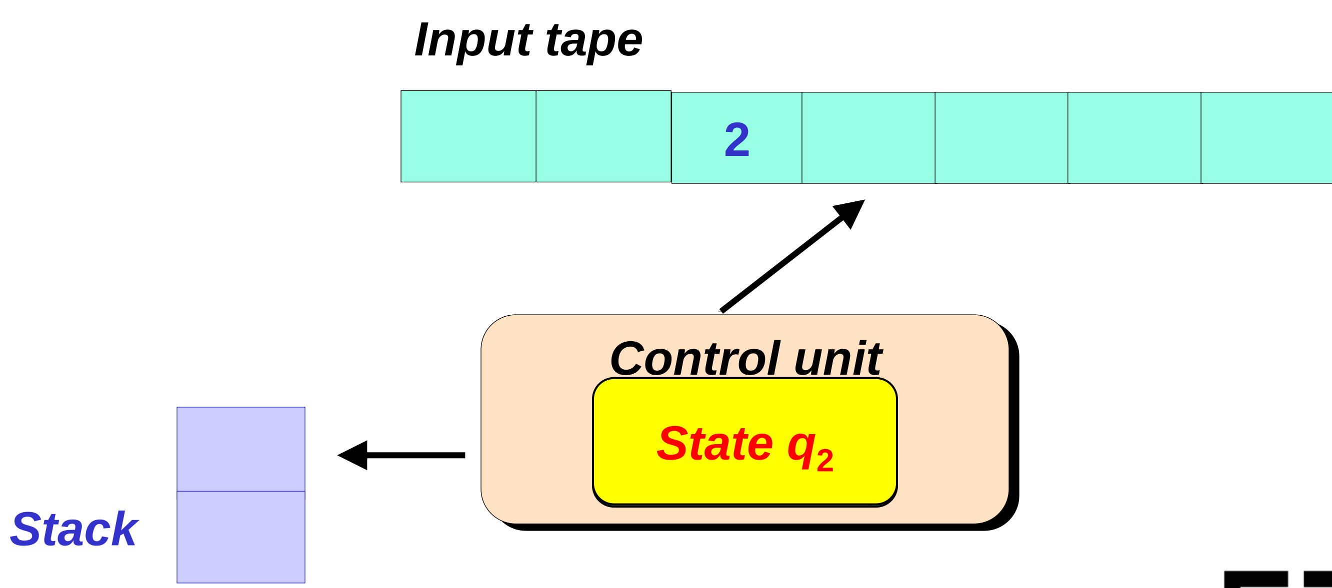

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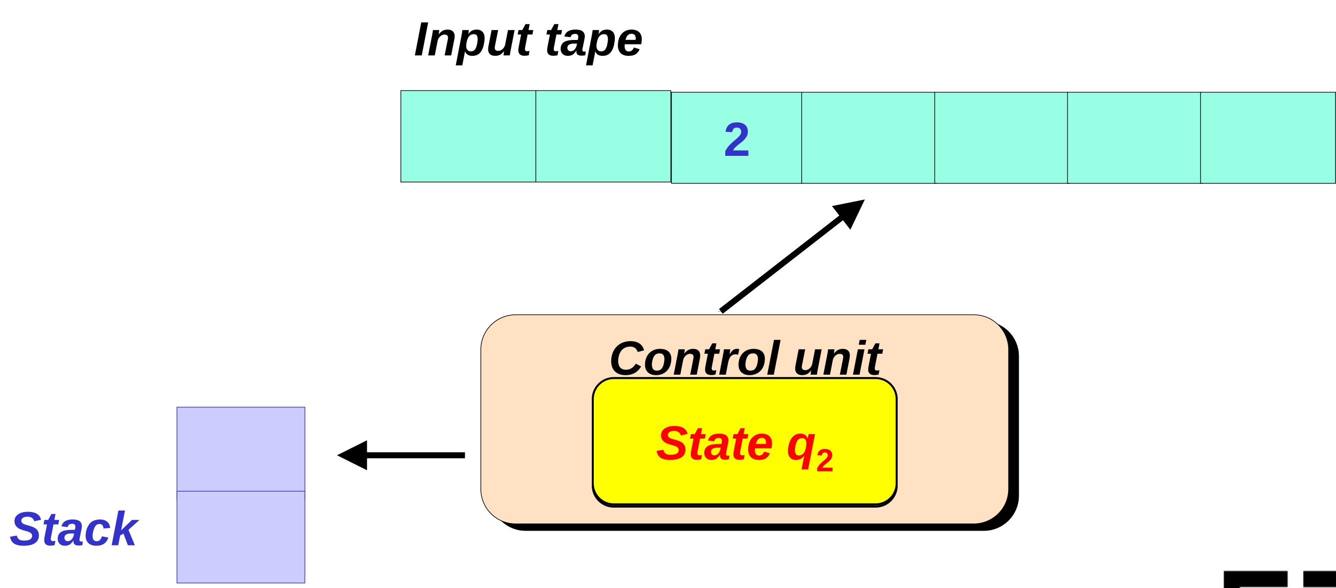

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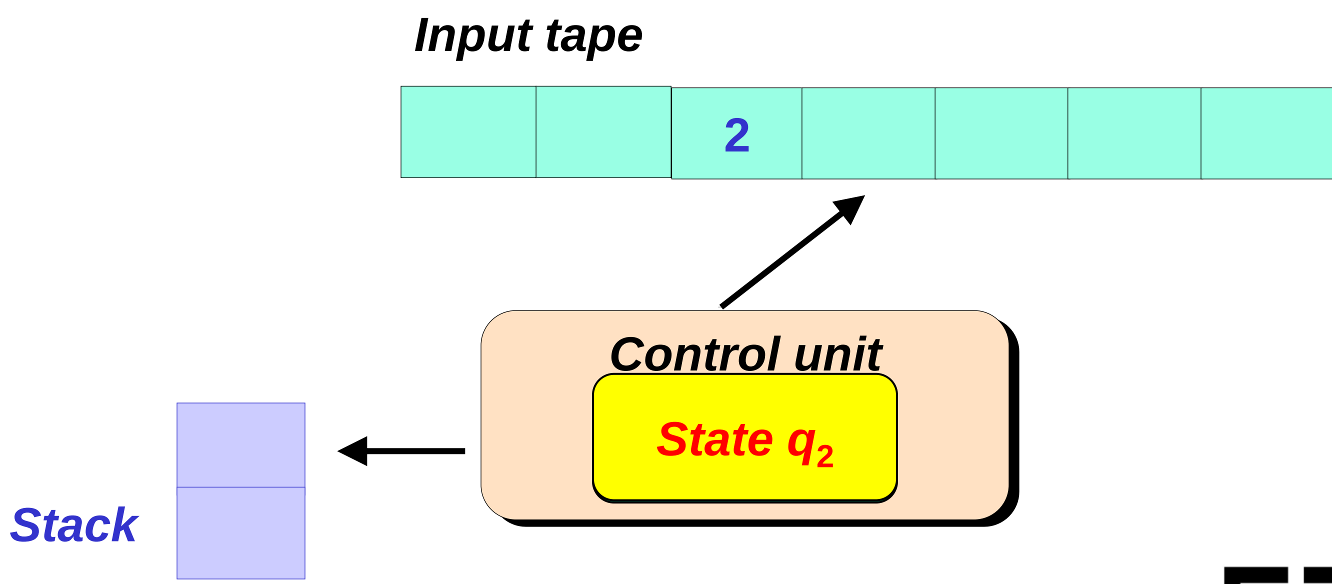

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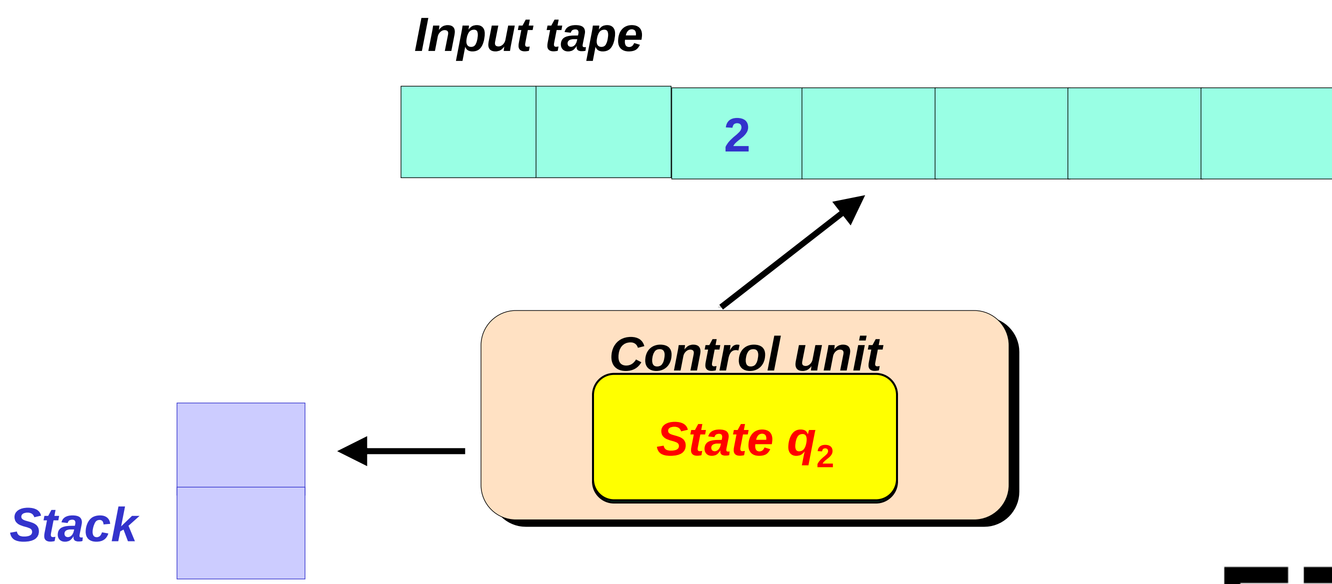

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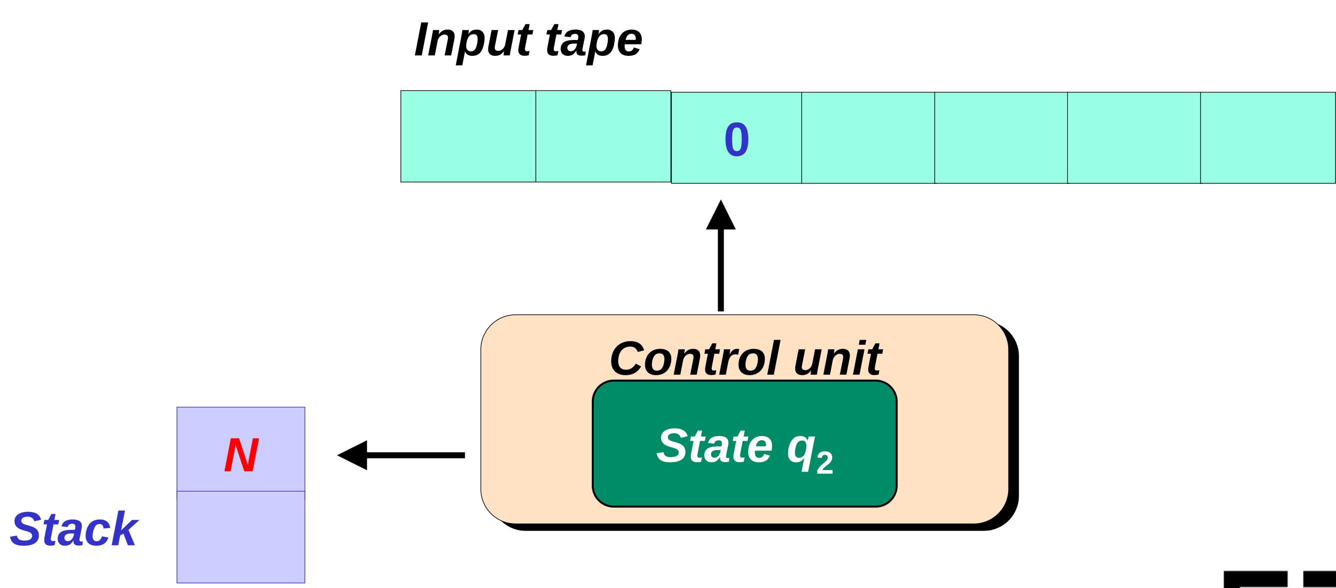

PA Definition

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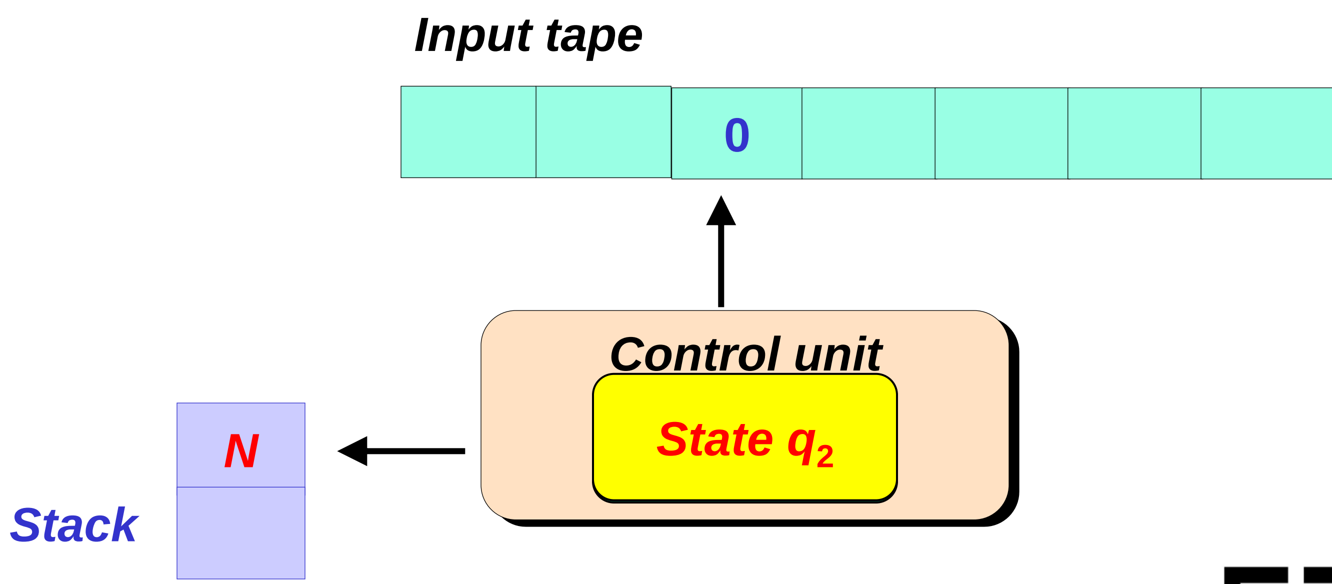

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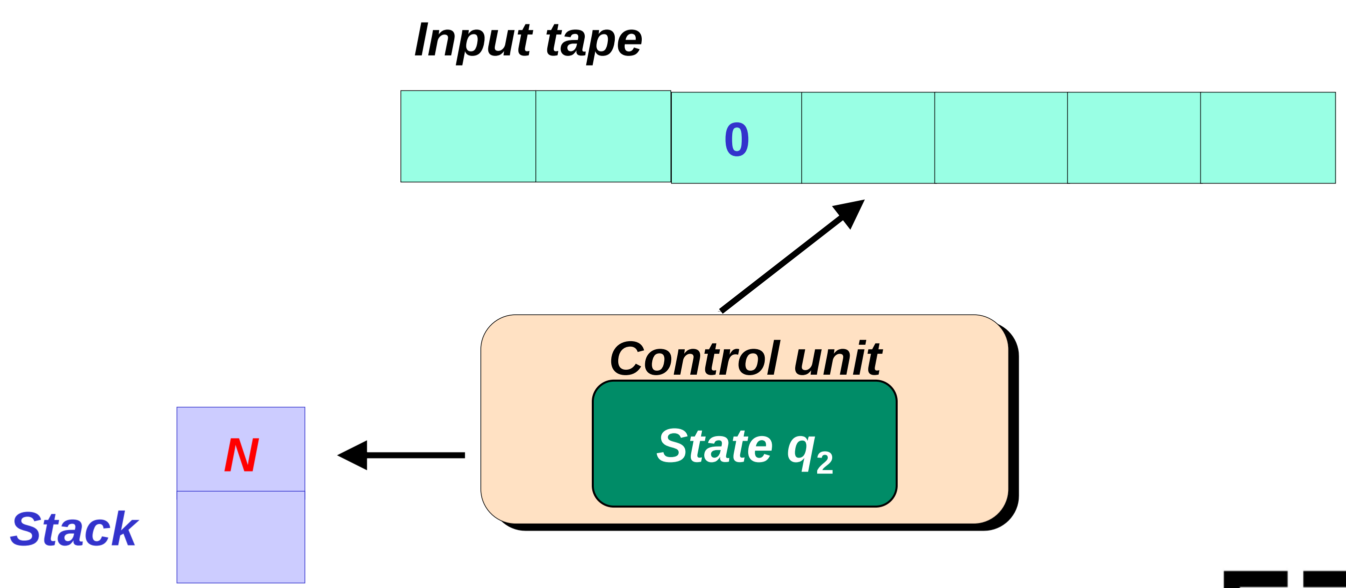

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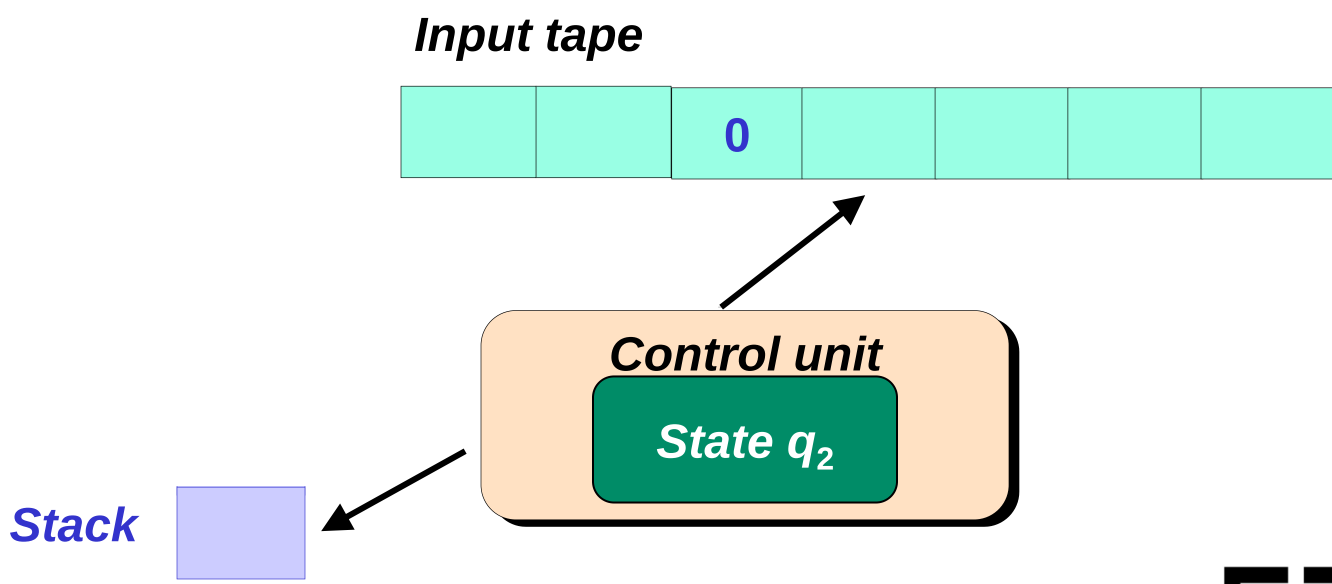

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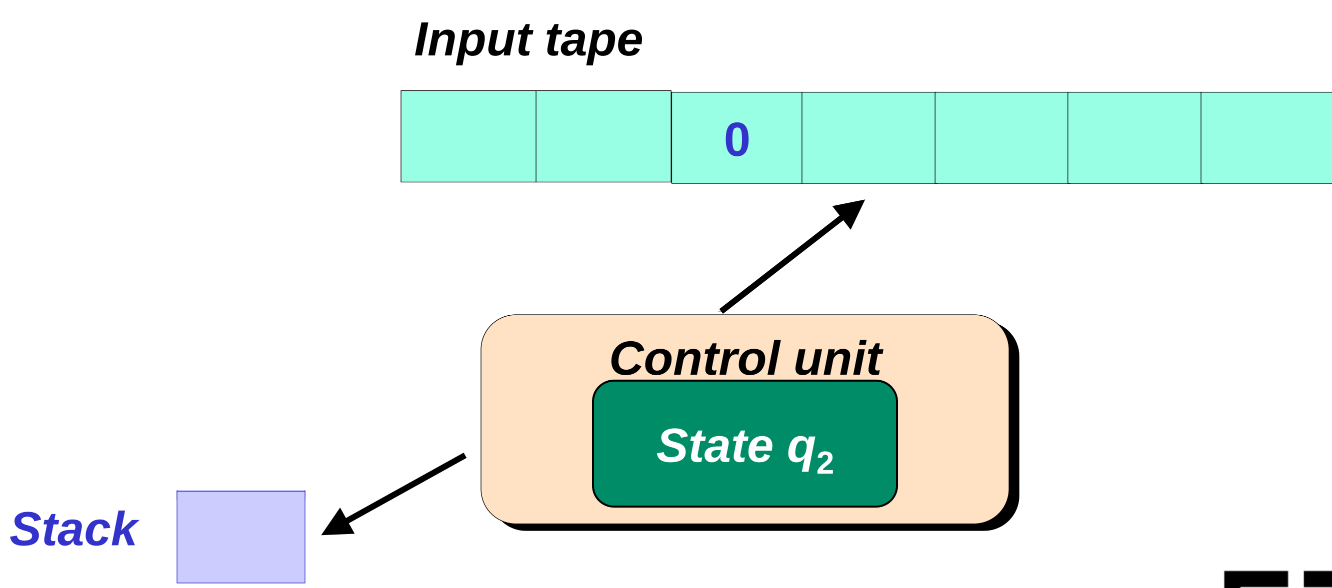

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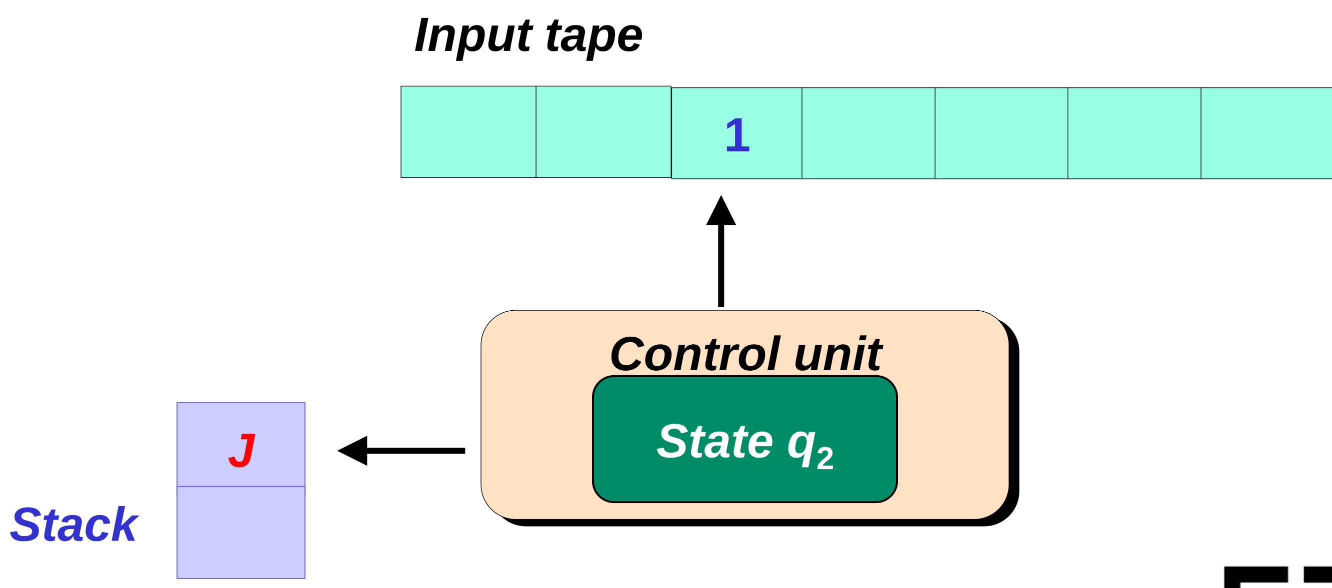

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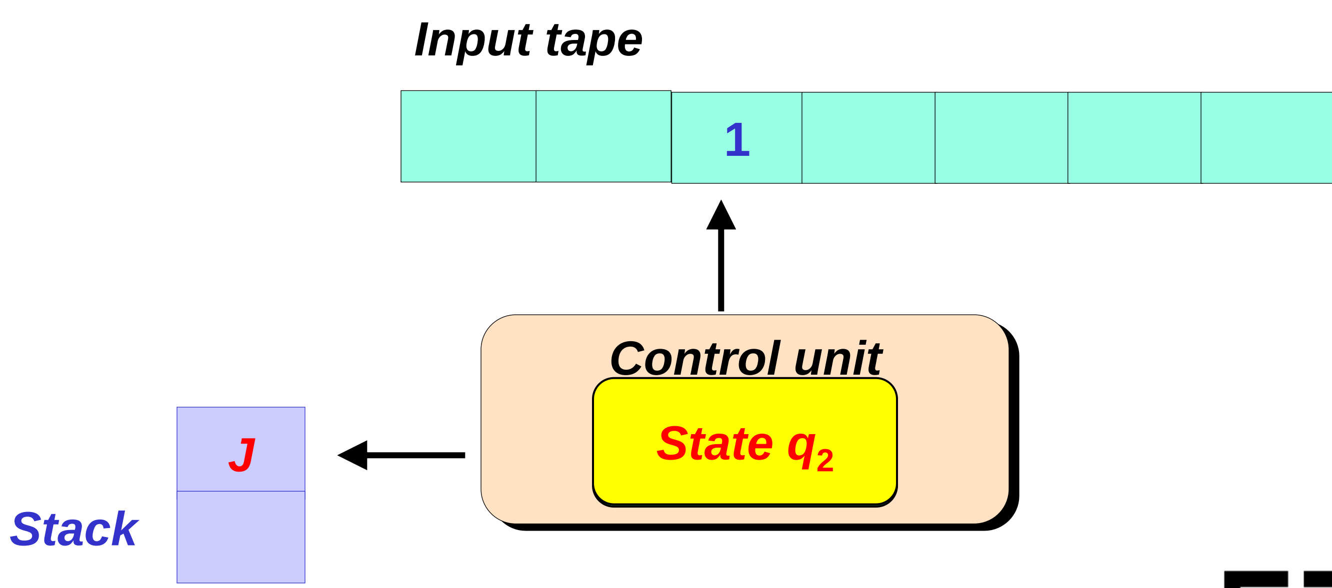

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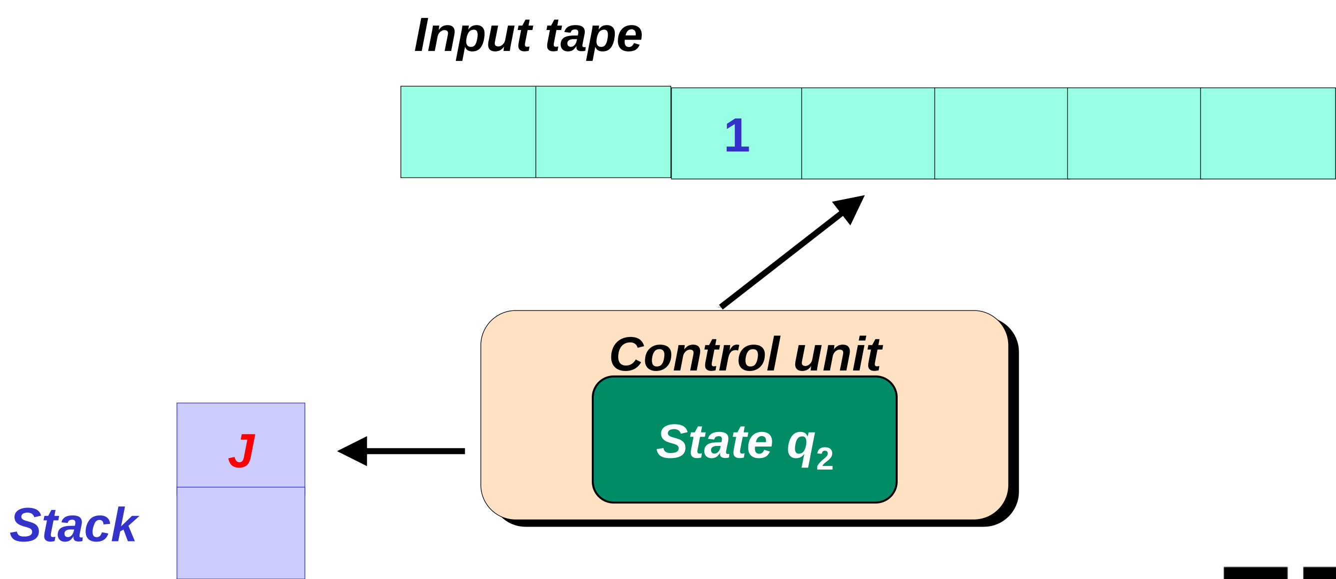

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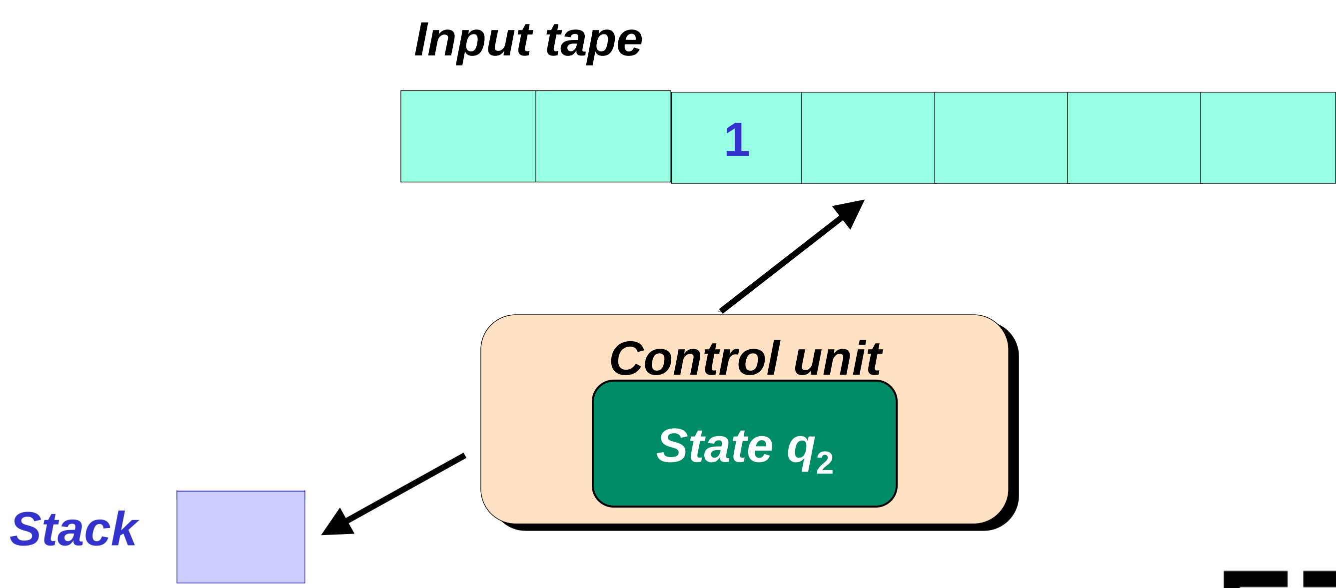

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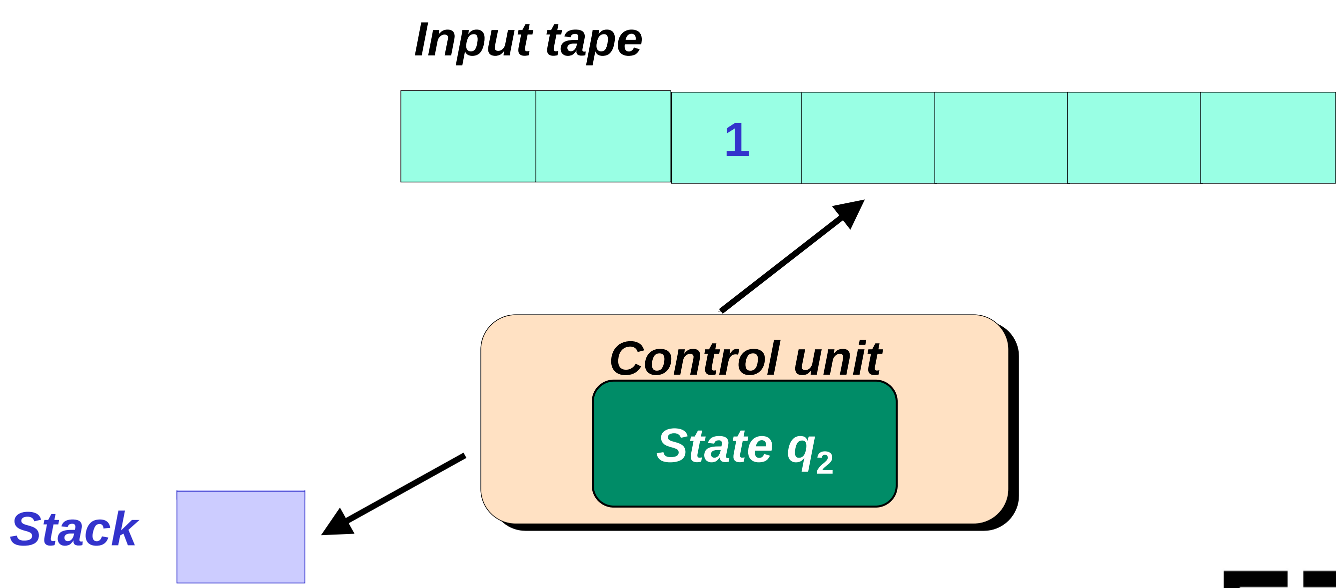

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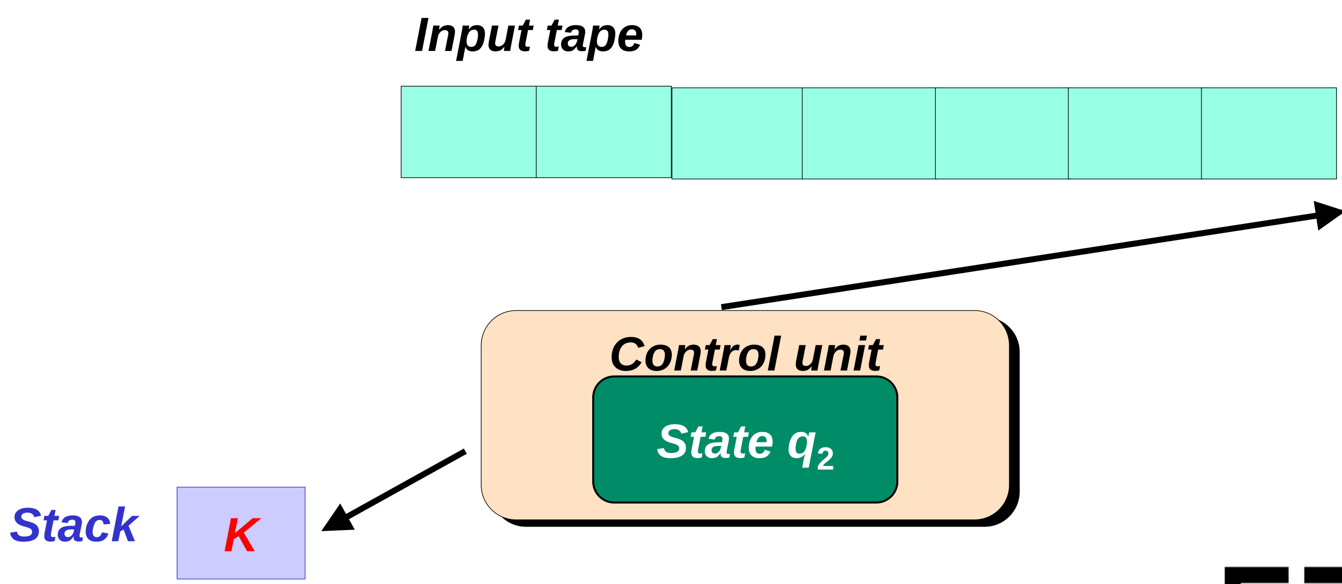

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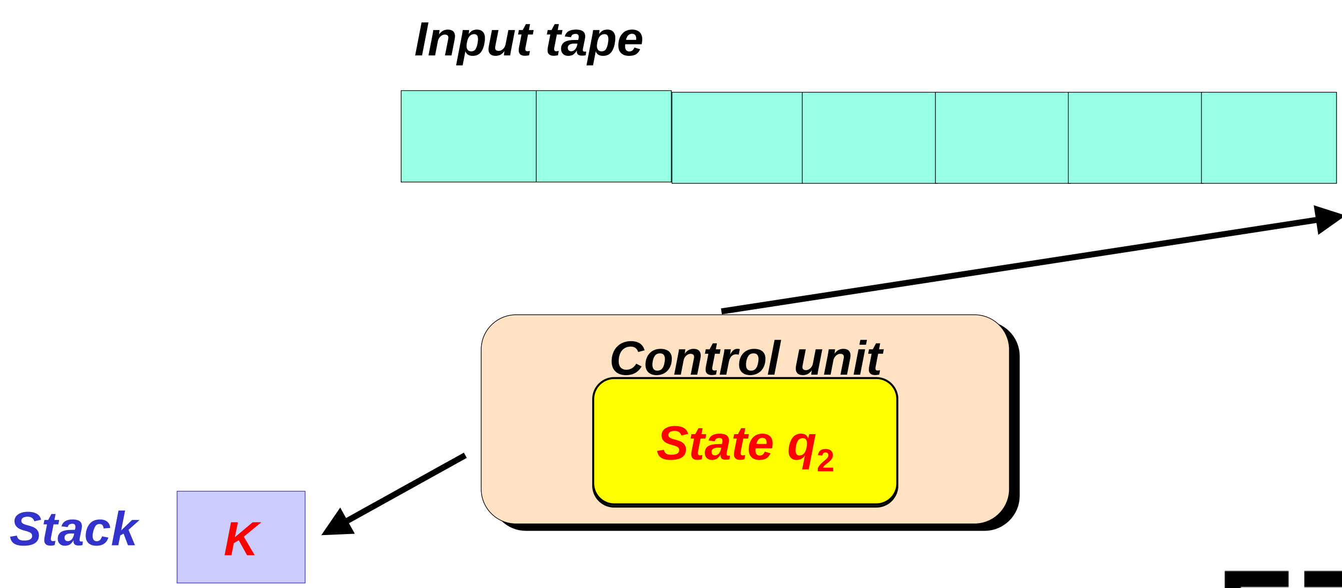

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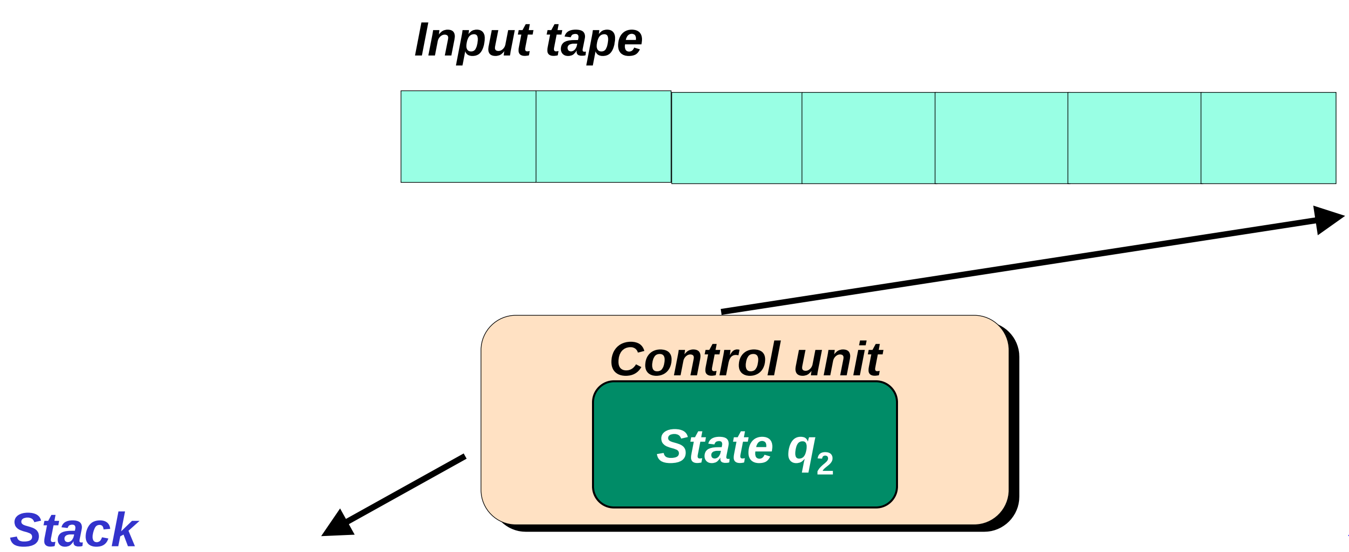

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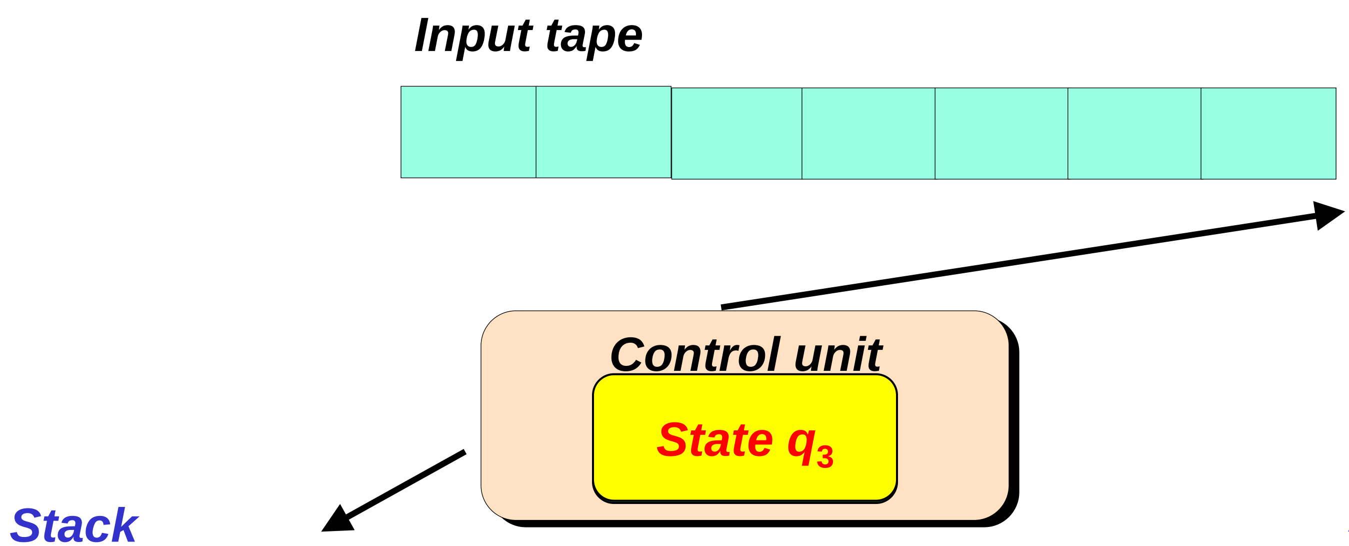

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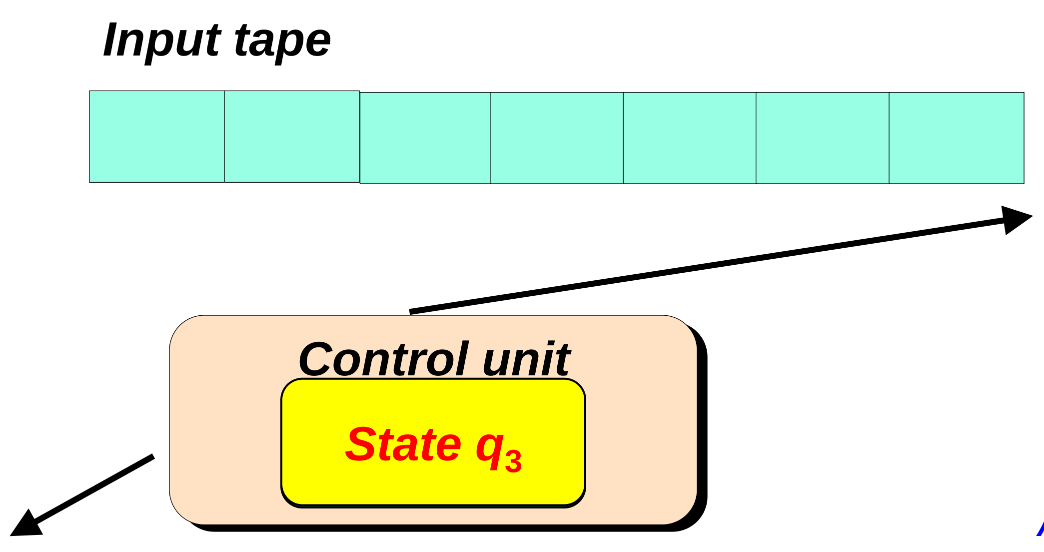

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State	*Remaining input*	*Stack*	*Transition*

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
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11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	*Remaining input*	*Stack*	*Transition*
q_1	0012100	K	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

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11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	K	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
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State	*Remaining input*	*Stack*	*Transition*
q_1	0012100	K	

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3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	K	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	Remaining input	Stack	Transition
q_1	0012100	K	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	Remaining input	Stack	Transition
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1			

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	*Remaining input*	*Stack*	*Transition*
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100		

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$ 2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$ 4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$ 6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$ 10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$ 11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$ 12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	*K*	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	*NK*	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	*Remaining input*	*Stack*	*Transition*
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	Remaining input	Stack	Transition
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	*Remaining input*	*Stack*	*Transition*
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	$\delta(q_1, 0, N) = \{(q_1, NN)\}$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	NNK	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	NNK	$\delta(q_1, 1, N) = \{(q_1, JN)\}$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$

8) $\delta(q_1, 2, N) = \{(q_2, N)\}$

9) $\delta(q_1, 2, J) = \{(q_2, J)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	*K*	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	*NK*	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	*NNK*	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
q_1	2100	*JNNK*	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	NNK	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
q_1	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	NNK	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
q_1	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$
q_2	100	$JNNK$	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	*Remaining input*	*Stack*	*Transition*
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	NNK	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
q_1	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$
q_2	100	$JNNK$	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	*K*	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	*NK*	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	*NNK*	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
q_1	2100	*JNNK*	$\delta(q_1, 2, J) = \{(q_2, J)\}$
q_2	100	*JNNK*	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
q_2	00	*NNK*	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	NNK	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
q_1	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$
q_2	100	$JNNK$	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
q_2	00	NNK	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	*Remaining input*	*Stack*	*Transition*
q_1	0012100	*K*	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	*NK*	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	*NNK*	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
q_1	2100	*JNNK*	$\delta(q_1, 2, J) = \{(q_2, J)\}$
q_2	100	*JNNK*	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
q_2	00	*NNK*	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
q_2	0	*NK*	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	***Remaining input***	***Stack***	***Transition***
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	NNK	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
q_1	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$
q_2	100	$JNNK$	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
q_2	00	NNK	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
q_2	0	NK	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	*Remaining input*	*Stack*	*Transition*
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	NNK	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
q_1	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$
q_2	100	$JNNK$	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
q_2	00	NNK	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
q_2	0	NK	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
q_2		K	

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	*Remaining input*	*Stack*	*Transition*
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	NNK	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
q_1	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$
q_2	100	$JNNK$	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
q_2	00	NNK	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
q_2	0	NK	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
q_2		K	$\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	*Remaining input*	*Stack*	*Transition*
q_1	0012100	K	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	NK	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	NNK	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
q_1	2100	$JNNK$	$\delta(q_1, 2, J) = \{(q_2, J)\}$
q_2	100	$JNNK$	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
q_2	00	NNK	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
q_2	0	NK	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
q_2		K	$\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$
q_3			

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$
8) $\delta(q_1, 2, N) = \{(q_2, N)\}$
9) $\delta(q_1, 2, J) = \{(q_2, J)\}$
10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

State	*Remaining input*	*Stack*	*Transition*
q_1	0012100	*K*	$\delta(q_1, 0, K) = \{(q_1, NK)\}$
q_1	012100	*NK*	$\delta(q_1, 0, N) = \{(q_1, NN)\}$
q_1	12100	*NNK*	$\delta(q_1, 1, N) = \{(q_1, JN)\}$
q_1	2100	*JNNK*	$\delta(q_1, 2, J) = \{(q_2, J)\}$
q_2	100	*JNNK*	$\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
q_2	00	*NNK*	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
q_2	0	*NK*	$\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
q_2		*K*	$\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$
q_3			$q_3 \in F$ and string is not accepted

Instantaneous Descriptions of PA

Instantaneous Descriptions of PA

State	*Remaining input*	*Stack*
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		

Instantaneous Descriptions of PA

State	*Remaining input*	*Stack*
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		

q

Instantaneous Descriptions of PA

State	*Remaining input*	*Stack*
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		
q	w	

Instantaneous Descriptions of PA

State	*Remaining input*	*Stack*
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		
q	w	γ

Instantaneous Descriptions of PA

State	*Remaining input*	*Stack*
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		

$$(q, w, \gamma)$$

Instantaneous Descriptions of PA

State	*Remaining input*	*Stack*
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		

$$(q, w, \gamma)$$

$(q, aw, Z\alpha)$

Instantaneous Descriptions of PA

State	Remaining input	Stack
q_1	0012100	K
q_1	012100	NK
q_1	12100	NNK
q_1	2100	$JNNK$
q_2	100	$JNNK$
q_2	00	NNK
q_2	0	NK
q_2		K
q_3		

$$(q, w, \gamma)$$

$$(q, aw, Z\alpha) \underset{M}{\succ}$$

Instantaneous Descriptions of PA

State	*Remaining input*	*Stack*
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		

$$(q, w, \gamma)$$

$$(q, aw, Z\alpha) \underset{M}{\succ} (p, w, \beta\alpha)$$

Instantaneous Descriptions of PA

State	***Remaining input***	***Stack***
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		

$$(q, w, \gamma)$$

$$(q, aw, Z\alpha) \underset{M}{\succ} (p, w, \beta\alpha)$$

if and only if $(p, \beta) \in \delta(q, a, Z)$

Instantaneous Descriptions of PA

State	Remaining input	Stack
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		

Instantaneous Descriptions of PA

State	*Remaining input*	*Stack*
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		

$$(q_1, 0012100, K) \succ (q_1, 012100, NK) \succ (q_1, 12100, NNK) \succ (q_1, 2100, JNNK) \succ$$

Instantaneous Descriptions of PA

State	*Remaining input*	*Stack*
q_1	0012100	K
q_1	012100	NK
q_1	12100	NNK
q_1	2100	$JNNK$
q_2	100	$JNNK$
q_2	00	NNK
q_2	0	NK
q_2		K
q_3		

$(q_1, 0012100, K) \succ (q_1, 012100, NK) \succ (q_1, 12100, NNK) \succ (q_1, 2100, JNNK) \succ$
$(q_2, 100, JNNK) \succ (q_2, 00, NNK) \succ (q_2, 0, NK) \succ (q_2, \varepsilon, K) \succ (q_3, \varepsilon, \varepsilon)$

Instantaneous Descriptions of PA

State	***Remaining input***	***Stack***
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		

$(q_1, 0012100, K) \succ (q_1, 012100, NK) \succ (q_1, 12100, NNK) \succ (q_1, 2100, JNNK) \succ$
$(q_2, 100, JNNK) \succ (q_2, 00, NNK) \succ (q_2, 0, NK) \succ (q_2, \varepsilon, K) \succ (q_3, \varepsilon, \varepsilon)$

$(q_1, 0012100, K)$

Instantaneous Descriptions of PA

State	*Remaining input*	*Stack*
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		

$(q_1, 0012100, K) \succ (q_1, 012100, NK) \succ (q_1, 12100, NNK) \succ (q_1, 2100, JNNK) \succ$
$(q_2, 100, JNNK) \succ (q_2, 00, NNK) \succ (q_2, 0, NK) \succ (q_2, \varepsilon, K) \succ (q_3, \varepsilon, \varepsilon)$

$$(q_1, 0012100, K) \overset{*}{\underset{M}{\succ}}$$

Instantaneous Descriptions of PA

State	*Remaining input*	*Stack*
q_1	0012100	K
q_1	012100	NK
q_1	12100	NNK
q_1	2100	$JNNK$
q_2	100	$JNNK$
q_2	00	NNK
q_2	0	NK
q_2		K
q_3		

$$(q_1, 0012100, K) \succ (q_1, 012100, NK) \succ (q_1, 12100, NNK) \succ (q_1, 2100, JNNK) \succ$$
$$(q_2, 100, JNNK) \succ (q_2, 00, NNK) \succ (q_2, 0, NK) \succ (q_2, \varepsilon, K) \succ (q_3, \varepsilon, \varepsilon)$$

$$(q_1, 0012100, K) \overset{*}{\underset{M}{\succ}} (q_3, \varepsilon, \varepsilon)$$

Instantaneous Descriptions of PA

State	Remaining input	Stack
q_1	0012100	*K*
q_1	012100	*NK*
q_1	12100	*NNK*
q_1	2100	*JNNK*
q_2	100	*JNNK*
q_2	00	*NNK*
q_2	0	*NK*
q_2		*K*
q_3		

$$(q_1, 0012100, K) \succ (q_1, 012100, NK) \succ (q_1, 12100, NNK) \succ (q_1, 2100, JNNK) \succ$$
$$(q_2, 100, JNNK) \succ (q_2, 00, NNK) \succ (q_2, 0, NK) \succ (q_2, \varepsilon, K) \succ (q_3, \varepsilon, \varepsilon)$$

$$(q_1, 0012100, K) \underset{M}{\overset{*}{\succ}} (q_3, \varepsilon, \varepsilon)$$

$$(q_1, 0012100, K) \underset{M}{\overset{8}{\succ}} (q_3, \varepsilon, \varepsilon)$$

Language Acceptance By PA

Language Acceptance By PA

PA M **= (Q, Σ, Γ, δ, q_0, Z_0, F)**

Language Acceptance By PA

PA M **= (*Q*, Σ, Γ, δ, q_0, Z_0, *F*)**

acceptance by final state:

Language Acceptance By PA

PA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

acceptance by final state:

$L(M) = \{ w \mid (q_0, w, Z_0)$

Language Acceptance By PA

PA M $= (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

acceptance by final state:

$L(M) = \{ w \mid (q_0, w, Z_0) \overset{*}{\underset{M}{\succ}}$

Language Acceptance By PA

$PA\ M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

acceptance by final state:

$$L(M) = \{\ w \mid (q_0, w, Z_0) \overset{*}{\underset{M}{\succ}} (p, \varepsilon, \gamma) \text{ for some state } p \in F \text{ i } \gamma \in \Gamma^*\ \}$$

Language Acceptance By PA

PA M = $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

acceptance by final state:

$$L(M) = \{\, w \mid (q_0, w, Z_0) \overset{*}{\underset{M}{\succ}} (p, \varepsilon, \gamma) \text{ for some state } p \in F \text{ i } \gamma \in \Gamma^* \,\}$$

PA M = $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

Language Acceptance By PA

PA M **= (*Q*, Σ, Γ, δ, *q*$_0$, *Z*$_0$, *F*)**

acceptance by final state:

$$L(M) = \{\ w \mid (q_0, w, Z_0) \overset{*}{\underset{M}{\succ}} (p, \varepsilon, \gamma) \text{ for some state } p \in F \text{ i } \gamma \in \Gamma^* \}$$

PA M **= (*Q*, Σ, Γ, δ, *q*$_0$, *Z*$_0$, *F*)**

acceptance by empty stack:

Language Acceptance By PA

PA M = (Q, Σ, Γ, δ, q_0, Z_0, F)

acceptance by final state:

$L(M) = \{\ w \mid (q_0, w, Z_0) \overset{*}{\underset{M}{\succ}} (p, \varepsilon, \gamma)$ **for some state** $p \in F$ **i** $\gamma \in \Gamma^*\ \}$

PA M = (Q, Σ, Γ, δ, q_0, Z_0, F)

acceptance by empty stack:

$N(M) = \{\ w \mid (q_0, w, Z_0)$

Language Acceptance By PA

PA M **=** $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

acceptance by final state:

$$L(M) = \{\ w \mid (q_0, w, Z_0) \underset{M}{\overset{*}{\succ}} (p, \varepsilon, \gamma) \text{ for some state } p \in F \text{ i } \gamma \in \Gamma^* \ \}$$

PA M **=** $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

acceptance by empty stack:

$$N(M) = \{\ w \mid (q_0, w, Z_0) \underset{M}{\overset{*}{\succ}}$$

Language Acceptance By PA

PA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

acceptance by final state:

$$L(M) = \{\, w \mid (q_0, w, Z_0) \overset{*}{\underset{M}{\succ}} (p, \varepsilon, \gamma) \text{ for some state } p \in F \text{ i } \gamma \in \Gamma^* \,\}$$

PA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

acceptance by empty stack:

$$N(M) = \{\, w \mid (q_0, w, Z_0) \overset{*}{\underset{M}{\succ}} (p, \varepsilon, \varepsilon) \text{ for some state } p \in Q \,\}$$

$L(M_1) = \{\ w\ 2\ w^R \mid w \in (0+1)^*\ \}$

PA $M_1 = (\ \{q_1, q_2, q_3\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{\ q_3\ \}\ )$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$

8) $\delta(q_1, 2, N) = \{(q_2, N)\}$

9) $\delta(q_1, 2, J) = \{(q_2, J)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

$L(M_1) = \{\ w\ \boxed{2}\ w^R \mid w \in (0+1)^*\ \}$

PA $M_1 = (\ \{q_1, q_2, q_3\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{\ q_3\ \}\ )$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$

8) $\delta(q_1, 2, N) = \{(q_2, N)\}$

9) $\delta(q_1, 2, J) = \{(q_2, J)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

$L(M_1) = \{\ w\ \boxed{2}\ w^R \mid w \in (0+1)^* \}$

$\text{PA } M_1 = (\{q_1, q_2, q_3\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{\boxed{q_3}\})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$

8) $\delta(q_1, 2, N) = \{(q_2, N)\}$

9) $\delta(q_1, 2, J) = \{(q_2, J)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

$L(M_1) = \{ \; w \; \boxed{2} \; w^R \mid w \in (0+1)^* \}$

PA $M_1 = (\{q_1, q_2, \boxed{q_3}\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{ \boxed{q_3} \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$

8) $\delta(q_1, 2, N) = \{(q_2, N)\}$

9) $\delta(q_1, 2, J) = \{(q_2, J)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

$L(M_1) = \{\ w\ \boxed{2}\ w^R \mid w \in (0+1)^* \}$

PA $M_1 = (\{q_1, q_2, \boxed{q_3}\ \}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{\ \boxed{q_3}\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$

8) $\delta(q_1, 2, N) = \{(q_2, N)\}$

9) $\delta(q_1, 2, J) = \{(q_2, J)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

$L(M_1) = \{ \ w \ \boxed{2} \ w^R \mid w \in (0+1)^* \}$

PA $M_1 = (\{q_1, q_2 \boxed{q_3} \}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{ \boxed{q_3} \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$

8) $\delta(q_1, 2, N) = \{(q_2, N)\}$

9) $\delta(q_1, 2, J) = \{(q_2, J)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

$L(M_1) = \{\ w\ \boxed{2}\ w^R \mid w \in (0+1)^* \}$

$\text{PA } M_1 = (\{q_1, q_2, \boxed{q_3}\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{\boxed{q_3}\})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$

8) $\delta(q_1, 2, N) = \{(q_2, N)\}$

9) $\delta(q_1, 2, J) = \{(q_2, J)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

$L(M_1) = \{\ w\ \boxed{2}\ w^R \mid w \in (0+1)^*\ \}$

PA $M_1 = (\ \{q_1, q_2, \boxed{q_3}\ \}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{\ \boxed{q_3}\ \}\ )$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\boxed{\delta(q_1, 1, J) = \{(q_1, JJ)\}}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$

8) $\delta(q_1, 2, N) = \{(q_2, N)\}$

9) $\delta(q_1, 2, J) = \{(q_2, J)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

$L(M_1) = \{\ w\ \boxed{2}\ w^R \mid w \in (0+1)^* \}$

PA $M_1 = (\{q_1, q_2\ \boxed{q_3}\ \}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{\ \boxed{q_3}\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$

8) $\delta(q_1, 2, N) = \{(q_2, N)\}$

9) $\delta(q_1, 2, J) = \{(q_2, J)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

$L(M_1) = \{ \; w \; \boxed{2} \; w^R \mid w \in (0+1)^* \}$

PA $M_1 = (\{q_1, q_2, \boxed{q_3} \}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{ \boxed{q_3} \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$

8) $\delta(q_1, 2, N) = \{(q_2, N)\}$

9) $\delta(q_1, 2, J) = \{(q_2, J)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_3, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

$L(M_1) = \{\ w\ \boxed{2}\ w^R \mid w \in (0+1)^* \}$

PA $M_1 = (\{q_1, q_2, \boxed{q_3}\}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{\boxed{q_3}\})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

7) $\delta(q_1, 2, K) = \{(q_2, K)\}$

8) $\delta(q_1, 2, N) = \{(q_2, N)\}$

9) $\delta(q_1, 2, J) = \{(q_2, J)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{\boxed{q_2}\ \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$L(M_1) = \{ \; w \quad w^R \mid w \in (0+1)^* \}$

PA $M_1 = (\{q_1, q_2, q_3 \}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{ q_3 \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$L(M_1) = \{ \ w \quad w^R \mid w \in (0+1)^* \}$

PA $M_1 = (\{q_1, q_2, q_3 \}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{ q_3 \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$L(M_1) = \{ \quad w \qquad w^R \mid w \in (0+1)^* \}$

PA $M_1 = (\{q_1, q_2, q_3 \}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{ q_3 \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$L(M_1) = \{\ w \quad w^R \mid w \in (0+1)^*\ \}$

PA $M_1 = (\ \{q_1, q_2, q_3\ \}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{\ q_3\ \}\ )$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$L(M_1) = \{ \; w \quad w^R \mid w \in (0+1)^* \}$

PA $M_1 = (\{q_1, q_2, q_3 \}, \{0, 1, 2\}, \{N, J, K\}, \delta, q_1, K, \{ q_3 \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$

3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$

5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$

4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$

6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$

11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$

12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

$(q_1, 01100, NK)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

$(q_1, 01100, NK)$ $(q_2, 001100, \varepsilon)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

(q_1, 001100, K)

(q_1, 01100, NK) (q_2, 001100, ε)

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

$(q_1, 01100, NK)$ $(q_2, 001100, \varepsilon)$

$(q_1, 1100, NNK)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

$(q_1, 01100, NK)$ $\quad$ $(q_2, 001100, \varepsilon)$

$(q_1, 1100, NNK)$ $\quad$ $(q_2, 1100, K)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

$(q_1, 01100, NK)$ $(q_2, 001100, \varepsilon)$

$(q_1, 1100, NNK)$ $(q_2, 1100, K)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

$(q_1, 01100, NK)$ $(q_2, 001100, \varepsilon)$

$(q_1, 1100, NNK)$ $(q_2, 1100, K)$

$(q_2, 1100, \varepsilon)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

$(q_1, 01100, NK)$ $(q_2, 001100, \varepsilon)$

$(q_1, 1100, NNK)$ $(q_2, 1100, K)$

$(q_2, 1100, \varepsilon)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$$\begin{tikzcd}
(q_1, 001100, K) \arrow[d] \arrow[dr] & & \\
(q_1, 01100, NK) \arrow[d] \arrow[dr] & (q_2, 001100, \varepsilon) & \\
(q_1, 1100, NNK) \arrow[d] & (q_2, 1100, K) \arrow[dr] & \\
(q_1, 100, JNNK) & & (q_2, 1100, \varepsilon)
\end{tikzcd}$$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
11) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

$(q_1, 01100, NK)$ $\quad$ $(q_2, 001100, \varepsilon)$

$(q_1, 1100, NNK)$ $\quad$ $(q_2, 1100, K)$

$(q_1, 100, JNNK)$ $\quad$ $(q_2, 1100, \varepsilon)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$

10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
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12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

$(q_1, 01100, NK)$ $\quad$ $(q_2, 001100, \varepsilon)$

$(q_1, 1100, NNK)$ $\quad$ $(q_2, 1100, K)$

$(q_1, 100, JNNK)$ $\quad$ $(q_2, 1100, \varepsilon)$

$(q_1, 00, JJNNK)$

$(q_1, 0, NJJNNK)$

$(q_1, \varepsilon, NNJJNNK)$ $\quad$ $(q_1, \varepsilon, JJNNK)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

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10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
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12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

$(q_1, 01100, NK)$ $(q_2, 001100, \varepsilon)$

$(q_1, 1100, NNK)$ $(q_2, 1100, K)$

$(q_1, 100, JNNK)$ $(q_2, 1100, \varepsilon)$

$(q_1, 00, JJNNK)$ $(q_2, 00, NNK)$

$(q_1, 0, NJJNNK)$

$(q_1, \varepsilon, NNJJNNK)$ $(q_1, \varepsilon, JJNNK)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

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10) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
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12) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism

$(q_1, 001100, K)$

$(q_1, 01100, NK)$ $(q_2, 001100, \varepsilon)$

$(q_1, 1100, NNK)$ $(q_2, 1100, K)$

$(q_1, 100, JNNK)$ $(q_2, 1100, \varepsilon)$

$(q_1, 00, JJNNK)$ $(q_2, 00, NNK)$

$(q_1, 0, NJJNNK)$ $(q_2, 0, NK)$

$(q_1, \varepsilon, NNJJNNK)$ $(q_1, \varepsilon, JJNNK)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
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Nondeterminism

$(q_1, 001100, K)$

$(q_1, 01100, NK)$ $(q_2, 001100, \varepsilon)$

$(q_1, 1100, NNK)$ $(q_2, 1100, K)$

$(q_1, 100, JNNK)$ $(q_2, 1100, \varepsilon)$

$(q_1, 00, JJNNK)$ $(q_2, 00, NNK)$

$(q_1, 0, NJJNNK)$ $(q_2, 0, NK)$

$(q_1, \varepsilon, NNJJNNK)$ $(q_1, \varepsilon, JJNNK)$ (q_2, ε, K)

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$

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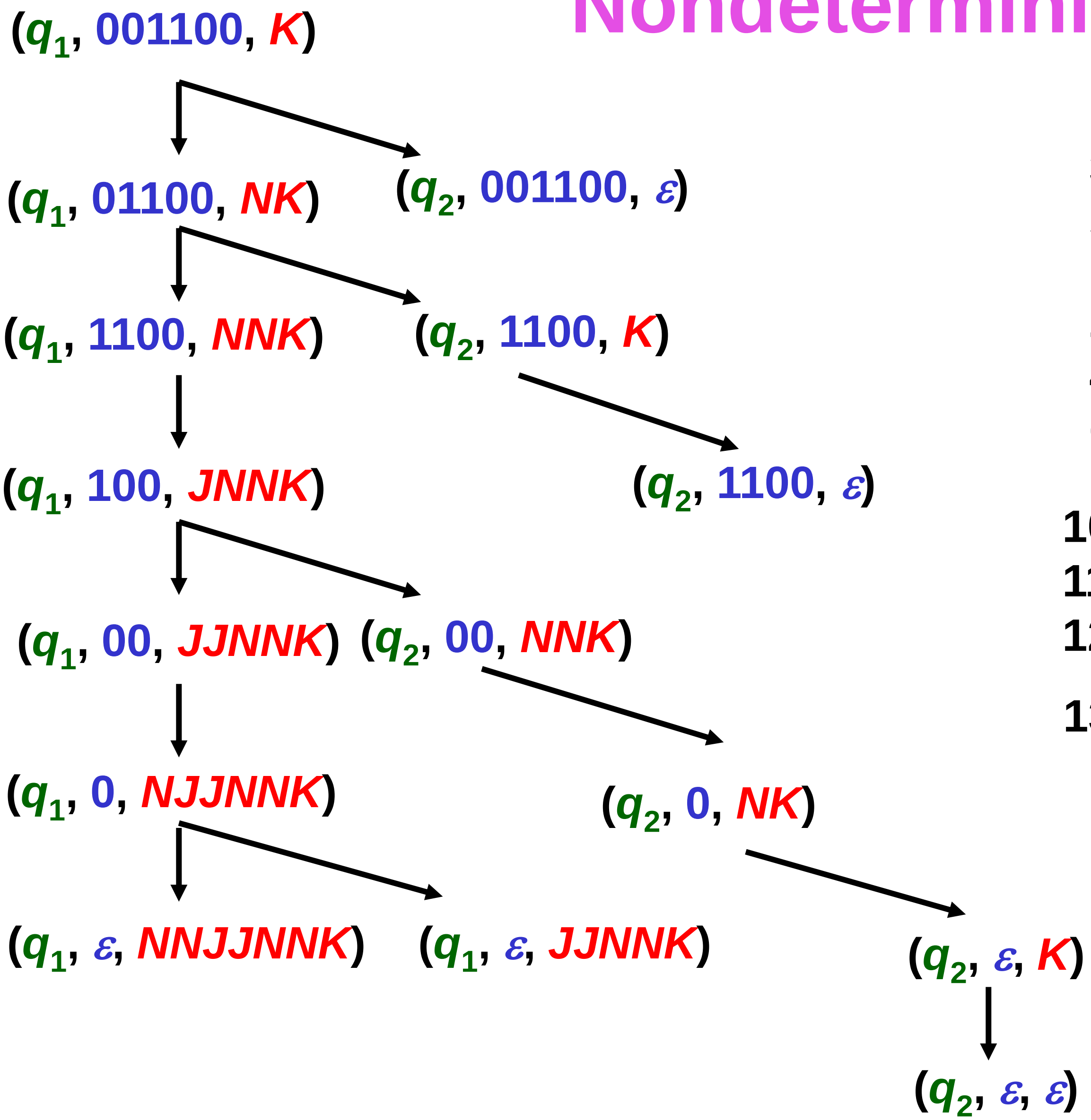
13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Nondeterminism



1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
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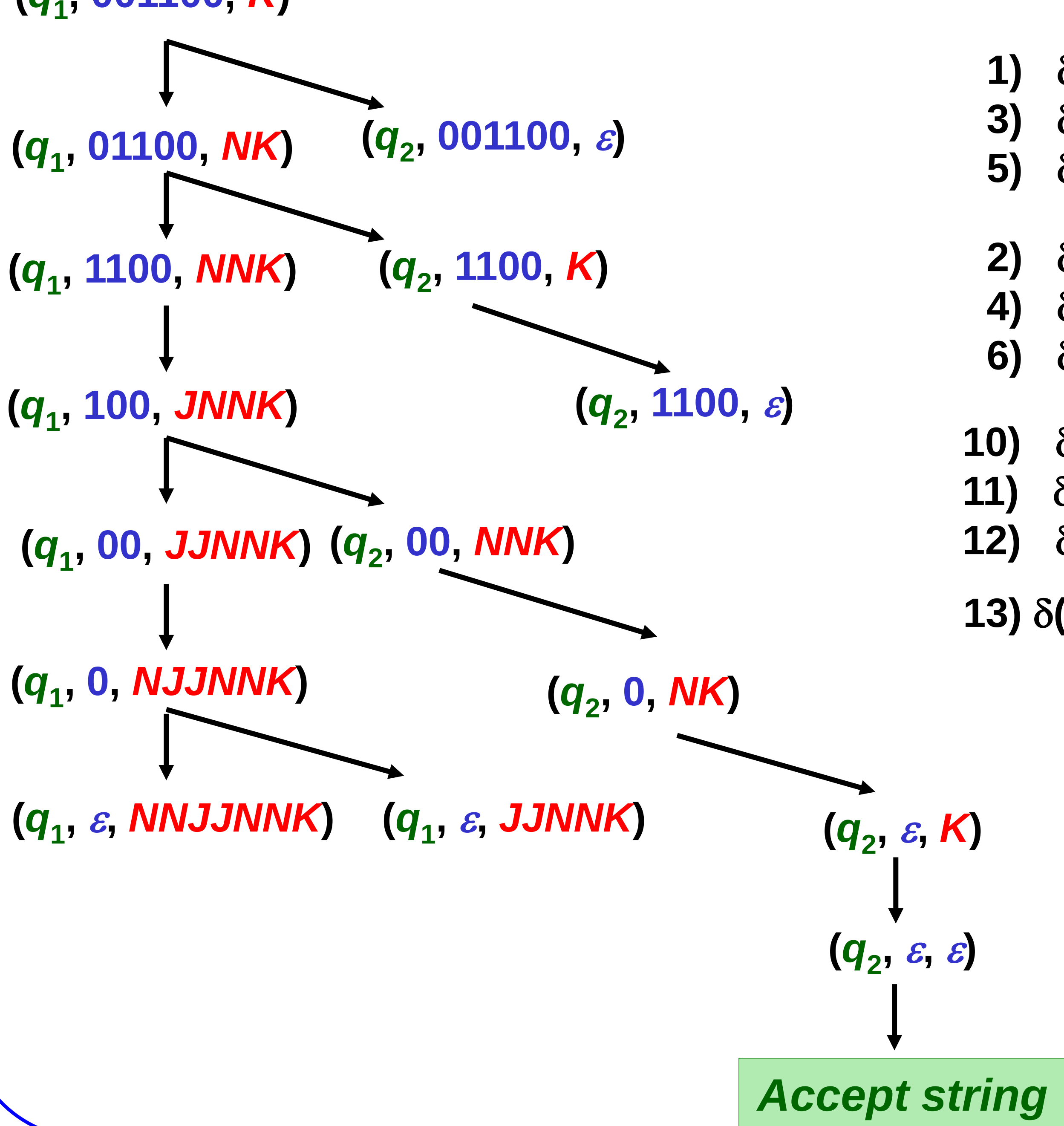
Nondeterminism



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3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
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13) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$

Deterministic PA (DPA)

Deterministic PA (DPA)

Deterministic DPA $M=(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$:

Deterministic PA (DPA)

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Deterministic PA (DPA)

Deterministic DPA $M=(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$:

If $(\ \delta(q, \varepsilon, Z) \neq \emptyset\ )$

Deterministic PA (DPA)

Deterministic DPA $M=(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$:

If $(\ \delta(q, \varepsilon, Z) \neq \varnothing\ )$

then

Deterministic PA (DPA)

Deterministic DPA $M=(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$:

If ($\delta(q, \varepsilon, Z) \neq \emptyset$)

then

$$\delta(q, a, Z) = \emptyset, \; \forall a \in \Sigma$$

Deterministic PA (DPA)

Deterministic DPA $M=(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$:

If $(\ \delta(q, \varepsilon, Z) \neq \varnothing\ )$

then

$$\delta(q, a, Z) = \varnothing, \ \forall a \in \Sigma$$

Deterministic PA (DPA)

Deterministic DPA $M=(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$:

If $(\delta(q, \varepsilon, Z) \neq \varnothing)$

then

$$\delta(q, a, Z) = \varnothing, \forall a \in \Sigma$$

There is at most one element in set $\delta(q, a, Z)$

$\forall q \in Q$

Deterministic PA (DPA)

Deterministic DPA $M=(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$:

If $(\ \delta(q, \varepsilon, Z) \neq \emptyset\ )$ **then** $\delta(q, a, Z) = \emptyset,\ \forall a \in \Sigma$
There is at most one element in set $\delta(q, a, Z)$ $\forall q \in Q$ $\forall Z \in \Gamma$

Deterministic PA (DPA)

Deterministic DPA $M=(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$:

If $(\ \delta(q, \varepsilon, Z) \neq \emptyset\ )$

then

$$\delta(q, a, Z) = \emptyset,\ \forall a \in \Sigma$$

There is at most one element in set $\delta(q, a, Z)$

$$\forall q \in Q$$

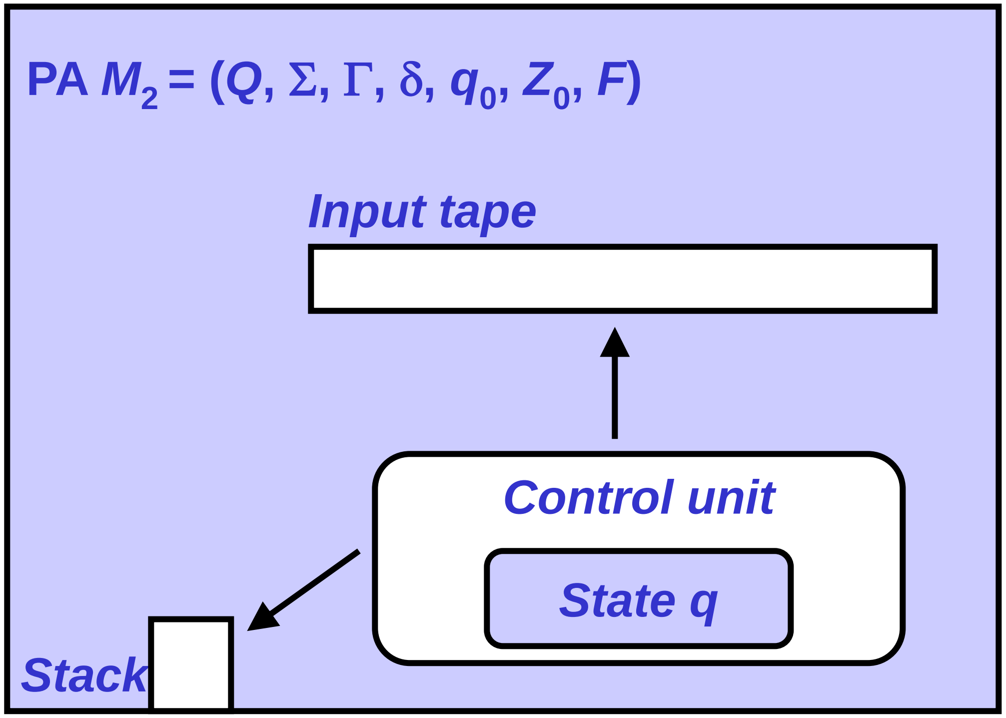
$$\forall Z \in \Gamma$$

$$\forall a \in (\Sigma \cup \{\varepsilon\})$$

Lecture 9

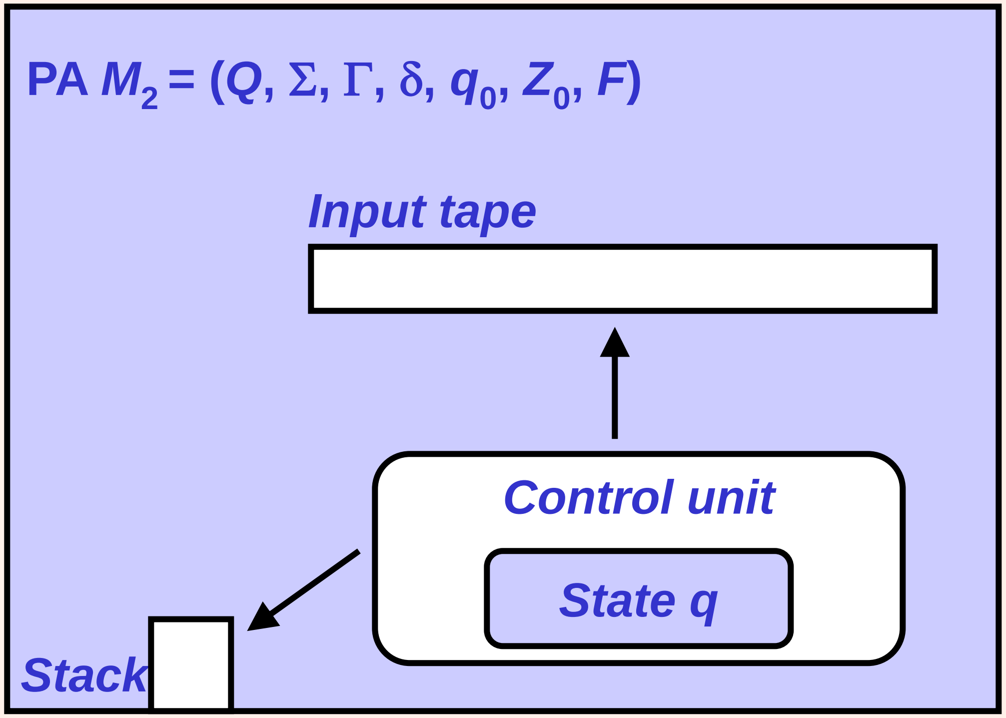
Constructing a PA that accepts by empty stack from a PA that accepts by final state

Constructing a PA that accepts by empty stack from a PA that accepts by final state



Constructing a PA that accepts by empty stack from a PA that accepts by final state

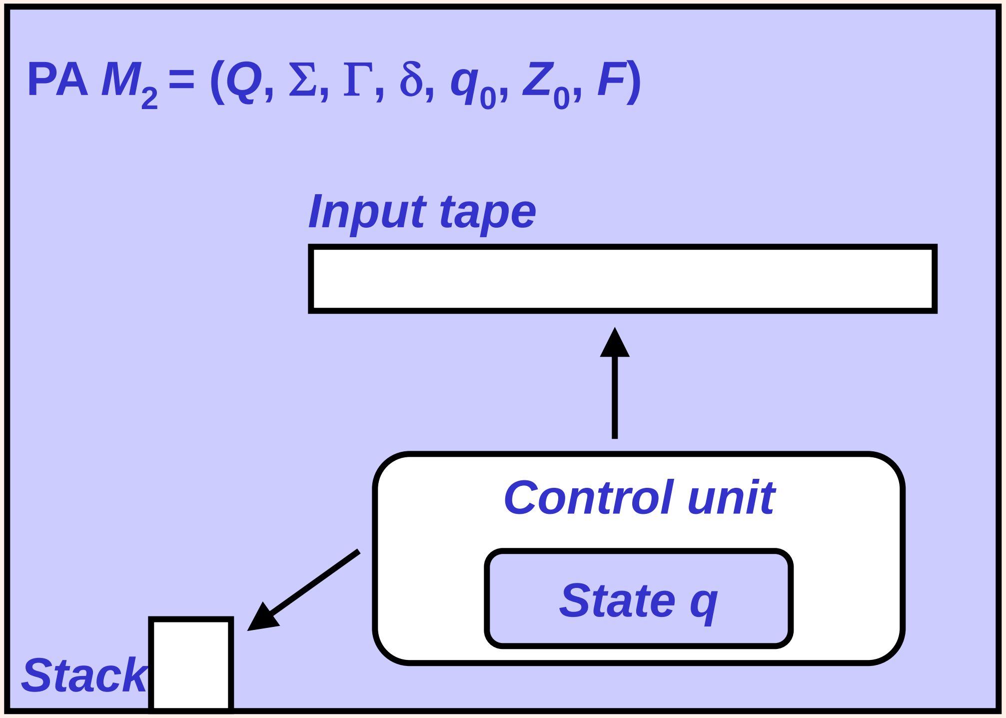
PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \varnothing)$



Constructing a PA that accepts by empty stack from a PA that accepts by final state

PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \varnothing)$

1) set $\delta'(q, a, Z)$ = set $\delta(q, a, Z)$

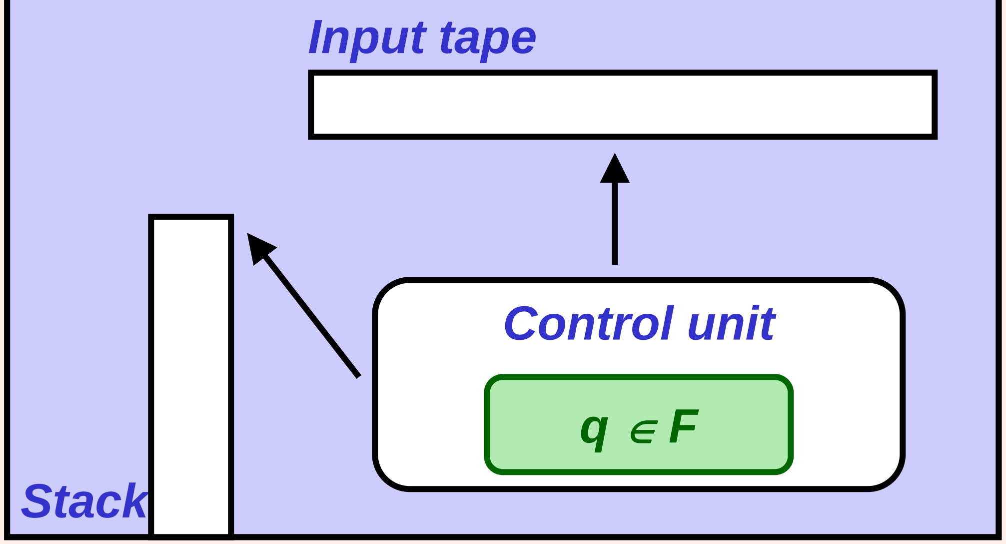


Constructing a PA that accepts by empty stack from a PA that accepts by final state

PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \emptyset)$

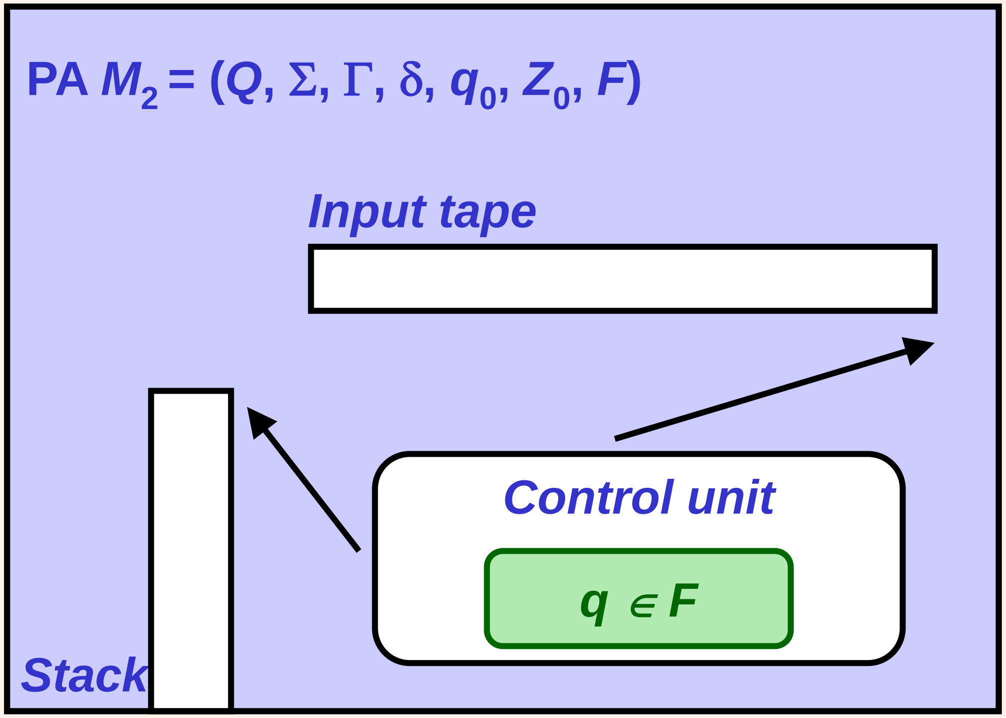
1) set $\delta'(q, a, Z)$ = set $\delta(q, a, Z)$

PA $M_2 = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$



Constructing a PA that accepts by empty stack from a PA that accepts by final state

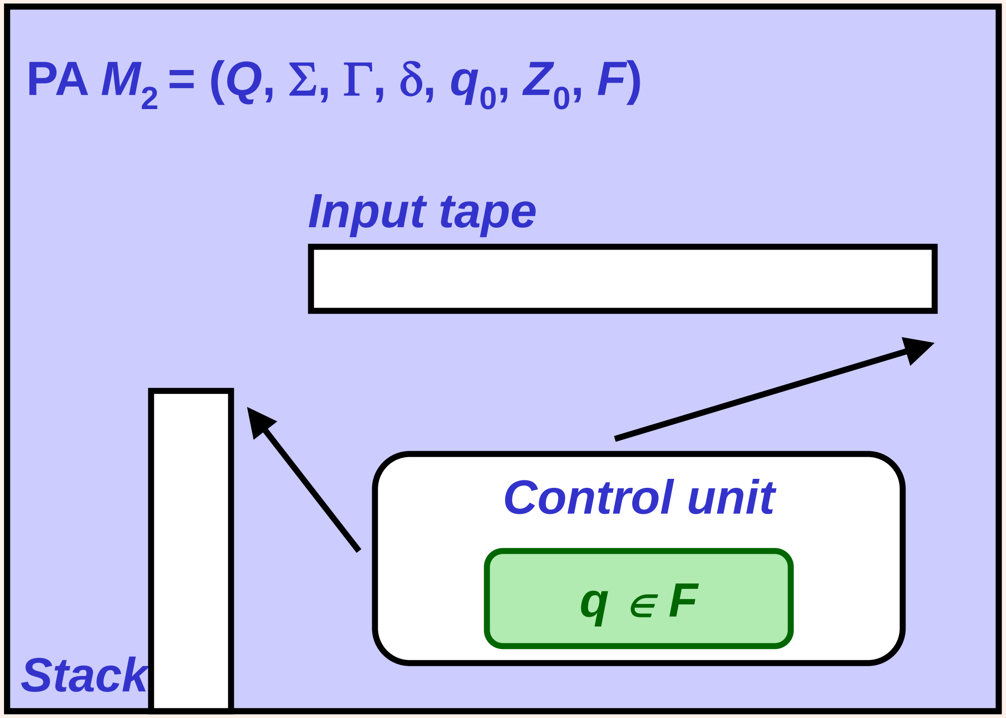
PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \varnothing)$



1) set $\delta'(q, a, Z)$ = set $\delta(q, a, Z)$

Constructing a PA that accepts by empty stack from a PA that accepts by final state

PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \varnothing)$



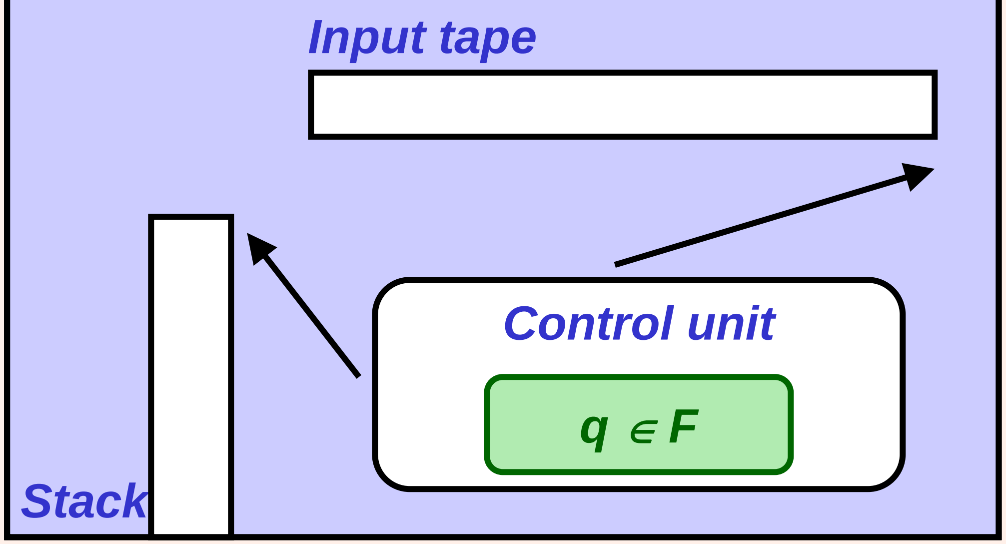
1) set $\delta'(q, a, Z)$ = set $\delta(q, a, Z)$

2) $q \in F$

Constructing a PA that accepts by empty stack from a PA that accepts by final state

PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \varnothing)$

PA $M_2 = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$



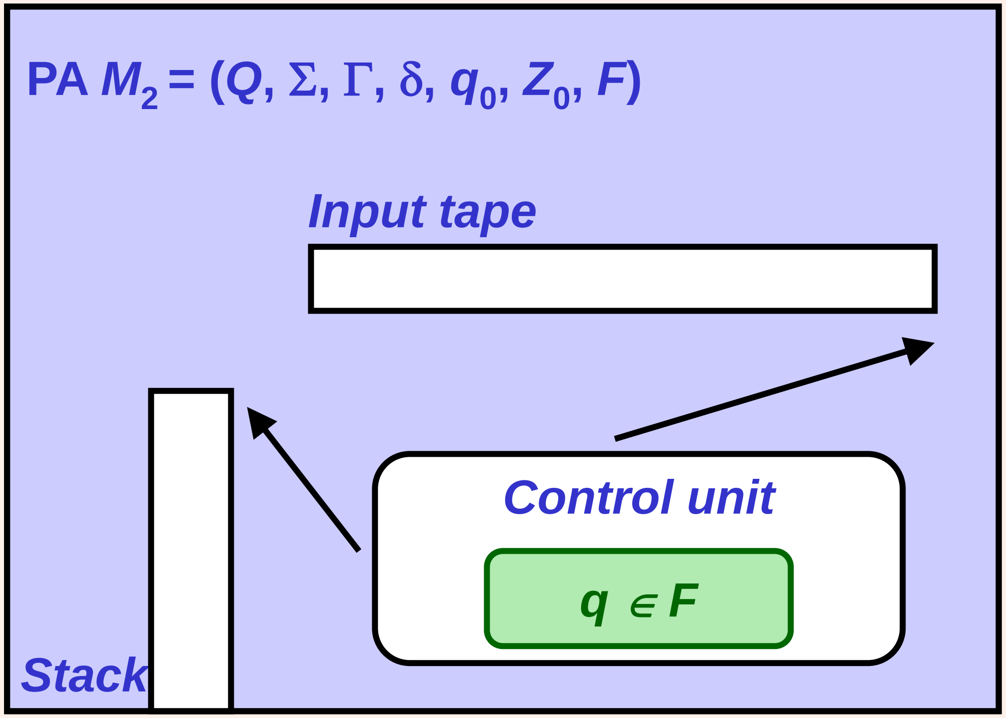
1) set $\delta'(q, a, Z)$ = set $\delta(q, a, Z)$

2) $q \in F$

$Q' = Q \cup \{ q_e \}$

Constructing a PA that accepts by empty stack from a PA that accepts by final state

PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \varnothing)$



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2) $q \in F$

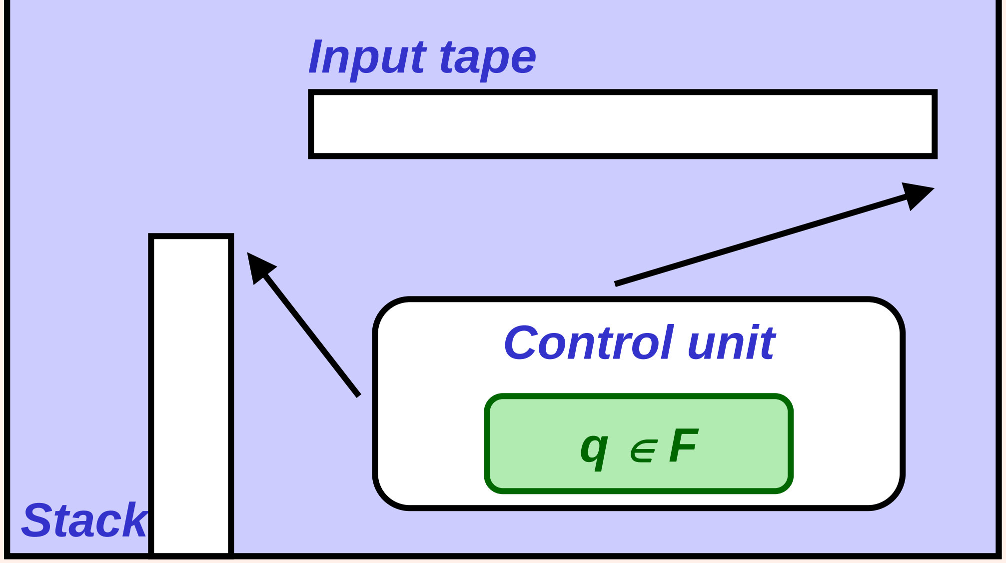
$Q' = Q \cup \{ q_e \}$

$\delta'(q, \varepsilon, Z) = \delta'(q, \varepsilon, Z) \cup (q_e, \varepsilon)$

Constructing a PA that accepts by empty stack from a PA that accepts by final state

PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \varnothing)$

PA $M_2 = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$



1) set $\delta'(q, a, Z)$ = set $\delta(q, a, Z)$

2) $q \in F$

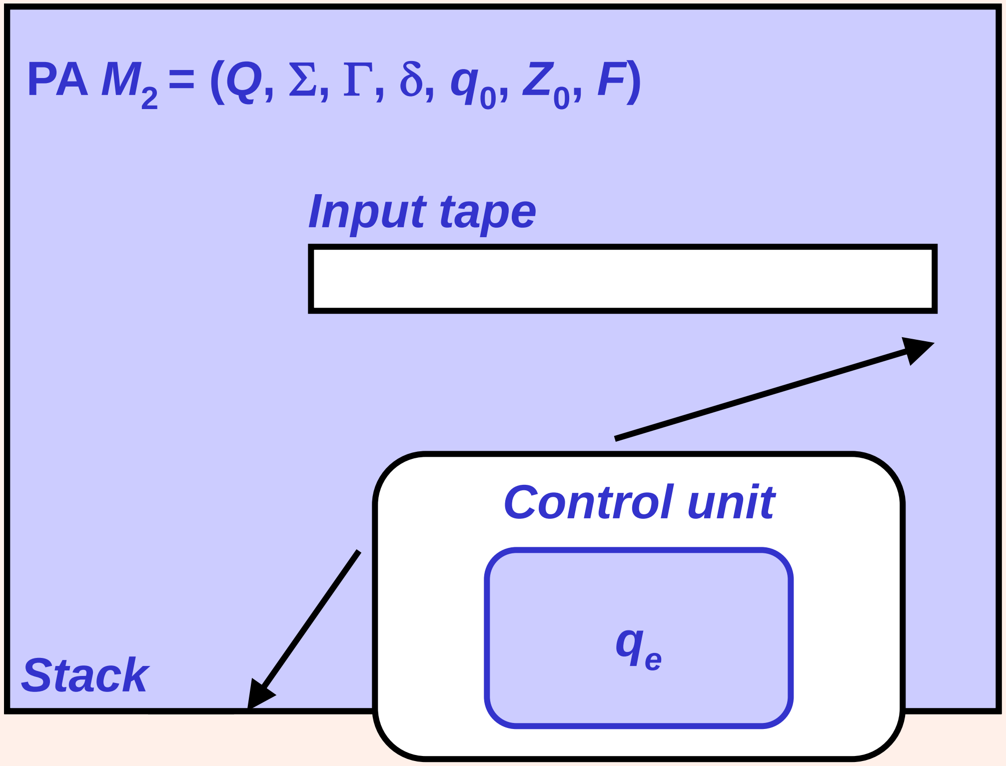
$Q' = Q \cup \{ q_e \}$

$\delta'(q, \varepsilon, Z) = \delta'(q, \varepsilon, Z) \cup (q_e, \varepsilon)$

$(q_e, \varepsilon) \in \delta'(q_e, \varepsilon, Z)$

Constructing a PA that accepts by empty stack from a PA that accepts by final state

PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \emptyset)$



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$\delta'(q, \varepsilon, Z) = \delta'(q, \varepsilon, Z) \cup (q_e, \varepsilon)$

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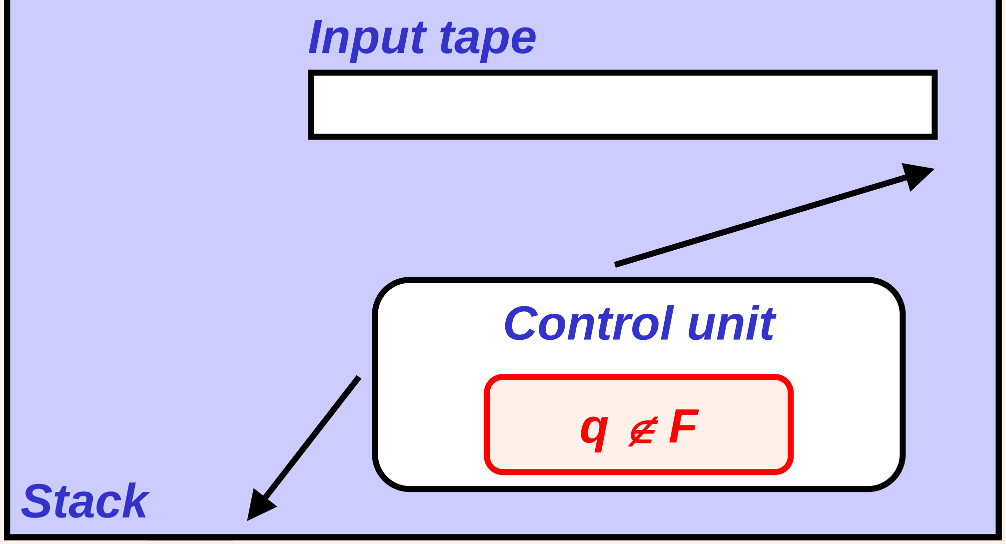
Constructing a PA that accepts by empty stack from a PA that accepts by final state

PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \varnothing)$

PA $M_2 = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$



1) set $\delta'(q, a, Z)$ = set $\delta(q, a, Z)$

2) $q \in F$

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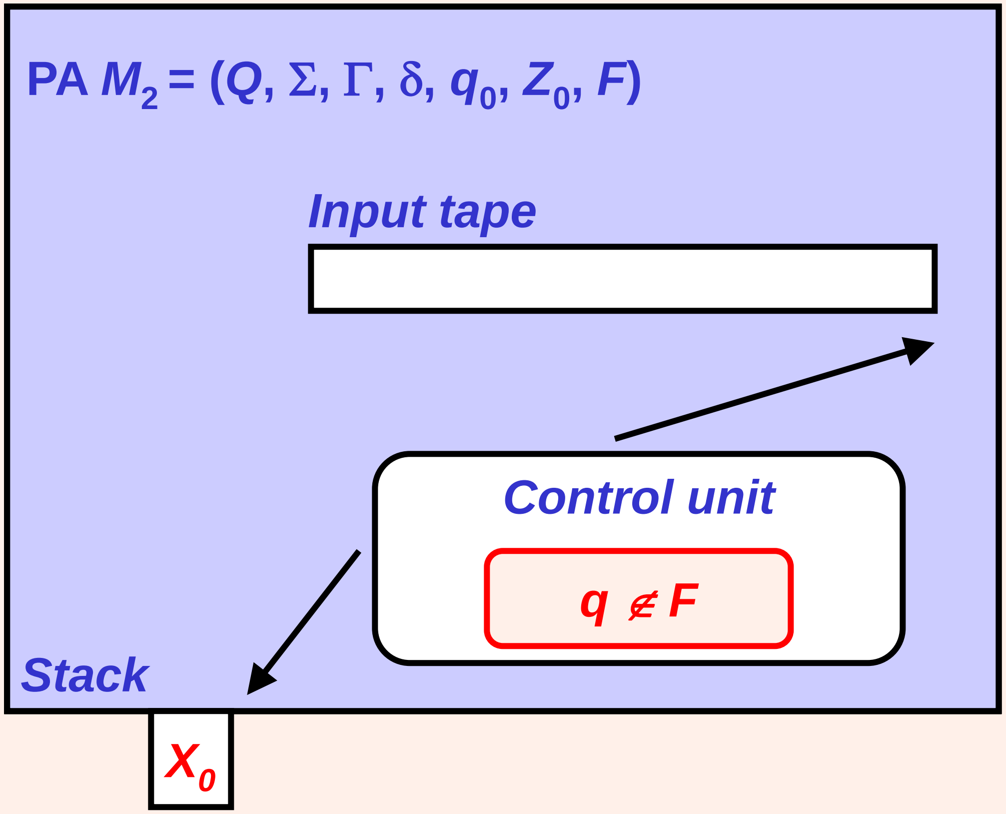
$\delta'(q, \varepsilon, Z) = \delta'(q, \varepsilon, Z) \cup (q_e, \varepsilon)$

$(q_e, \varepsilon) \in \delta'(q_e, \varepsilon, Z)$

Constructing a PA that accepts by empty stack from a PA that accepts by final state

PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \varnothing)$



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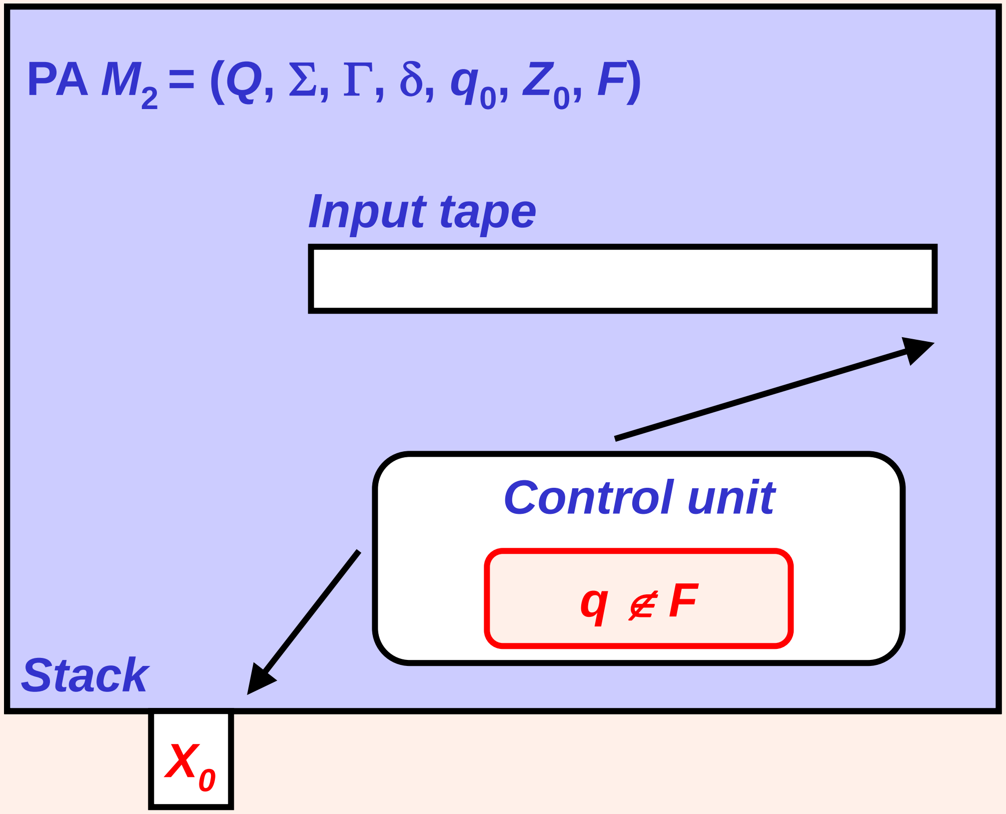
Constructing a PA that accepts by empty stack from a PA that accepts by final state

PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \varnothing)$



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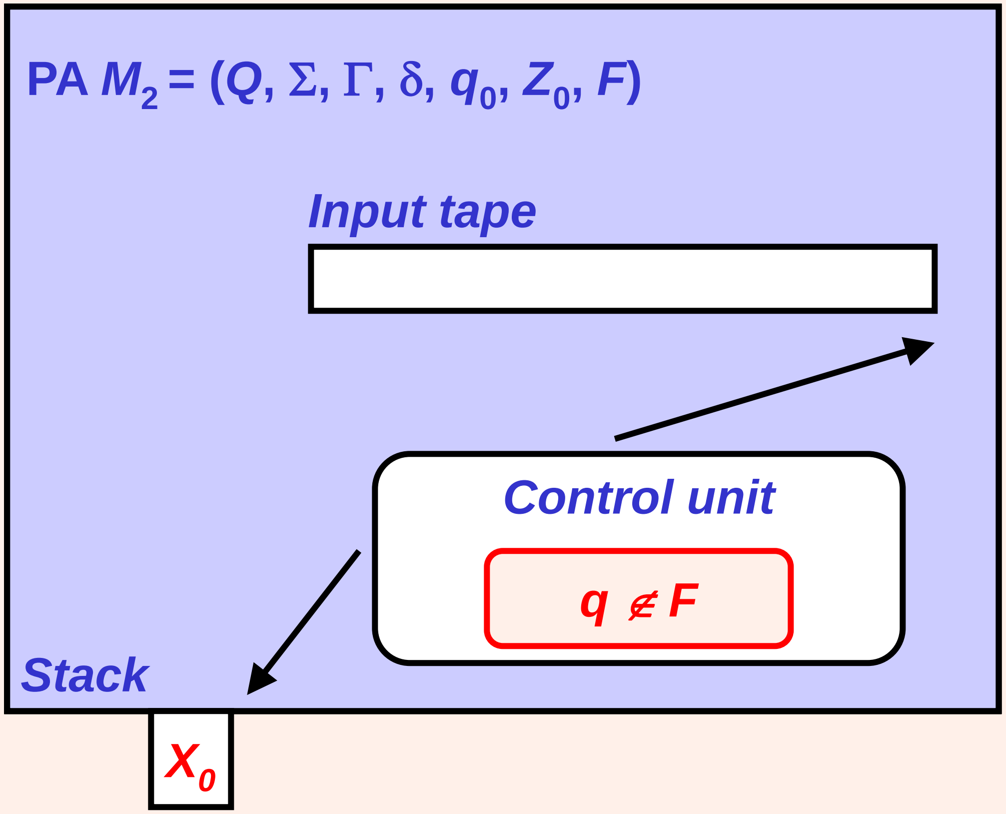
$(q_e, \varepsilon) \in \delta'(q_e, \varepsilon, Z)$

3) $Z_0' = X_0$

Constructing a PA that accepts by empty stack from a PA that accepts by final state

PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \varnothing)$



1) set $\delta'(q, a, Z)$ = set $\delta(q, a, Z)$

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$\delta'(q, \varepsilon, Z) = \delta'(q, \varepsilon, Z) \cup (q_e, \varepsilon)$

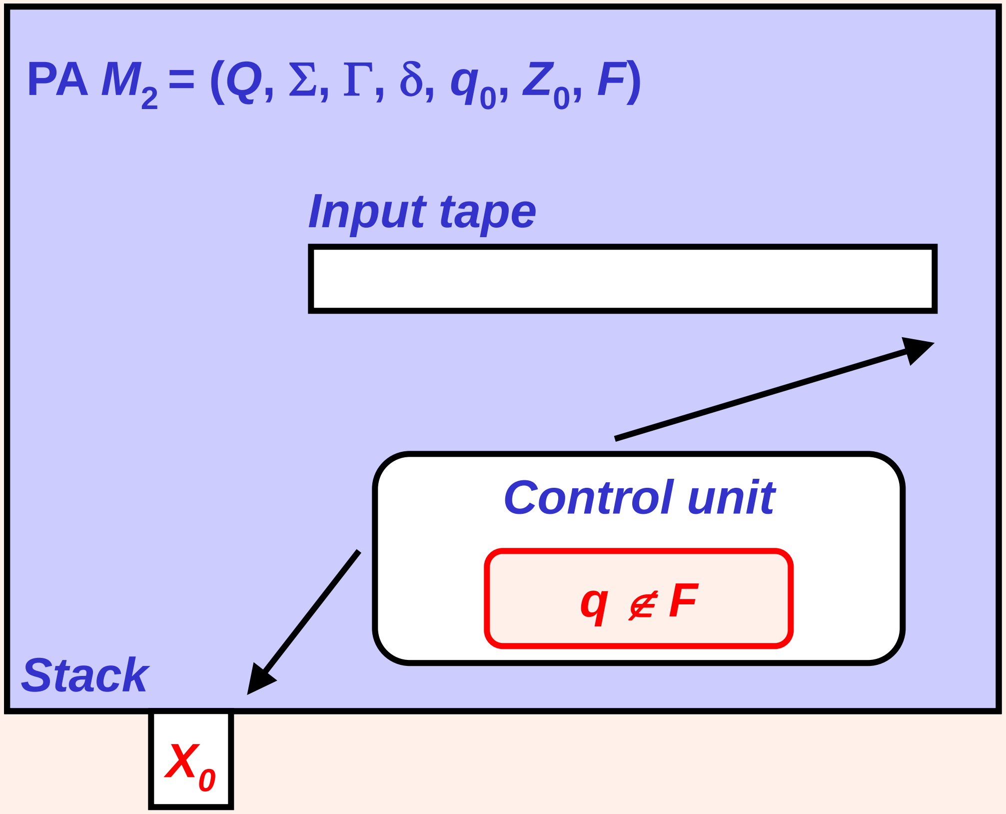
$(q_e, \varepsilon) \in \delta'(q_e, \varepsilon, Z)$

3) $Z_0' = X_0$

$\Gamma' = \Gamma \cup \{X_0\}$

Constructing a PA that accepts by empty stack from a PA that accepts by final state

PA $M_1 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', \emptyset)$



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Constructing a PA that accepts by empty stack from a PA that accepts by final state

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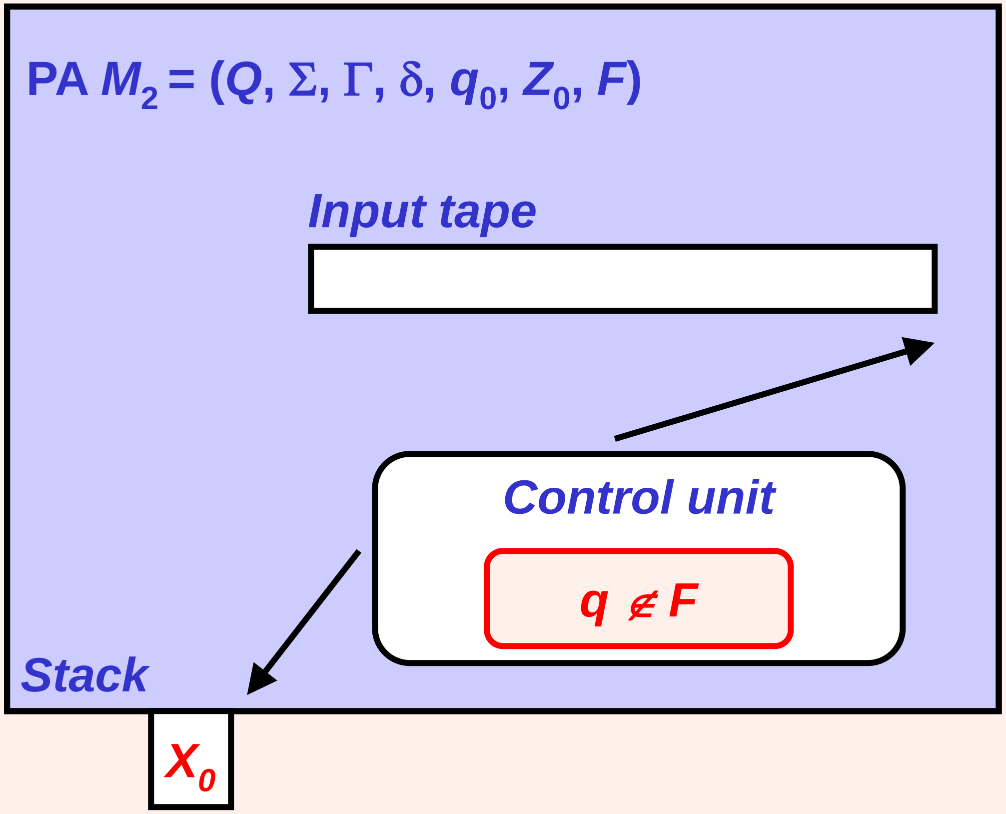

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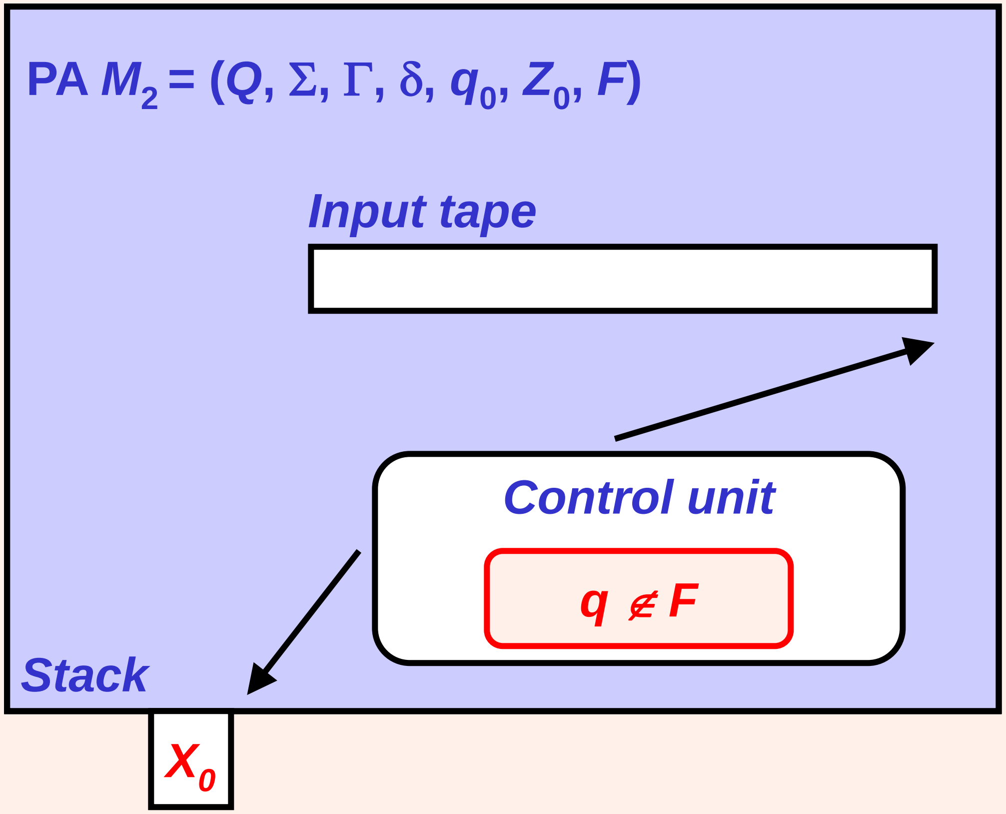

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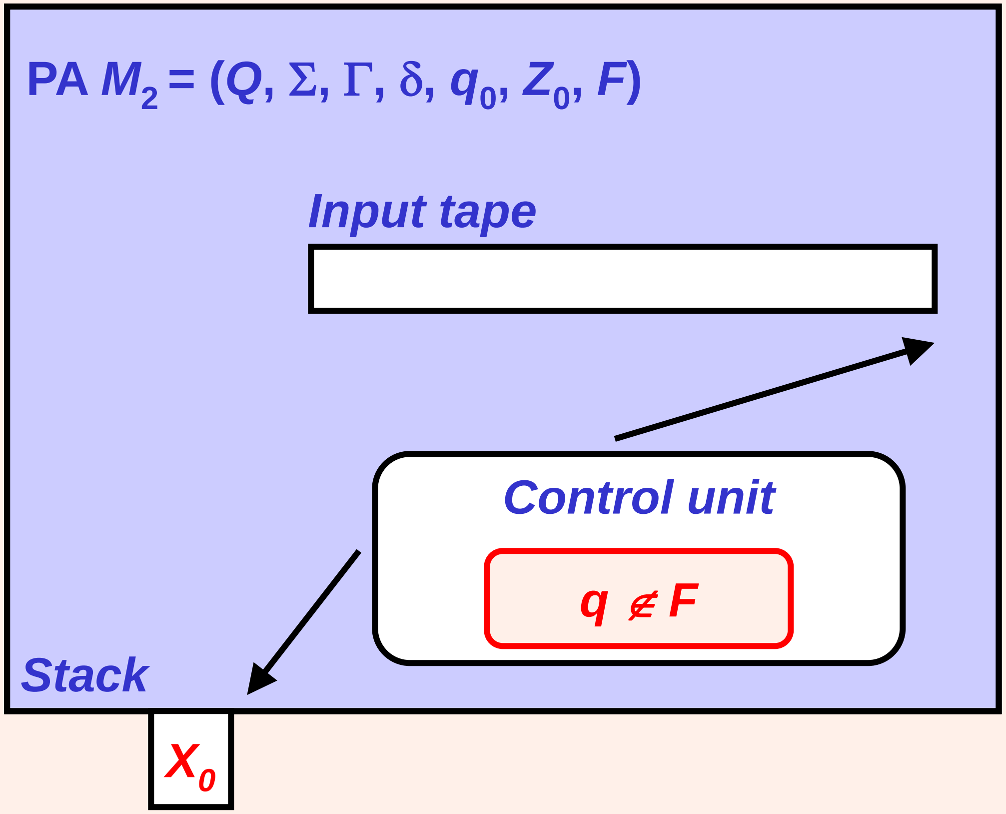

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(q_0, x, Z_0)

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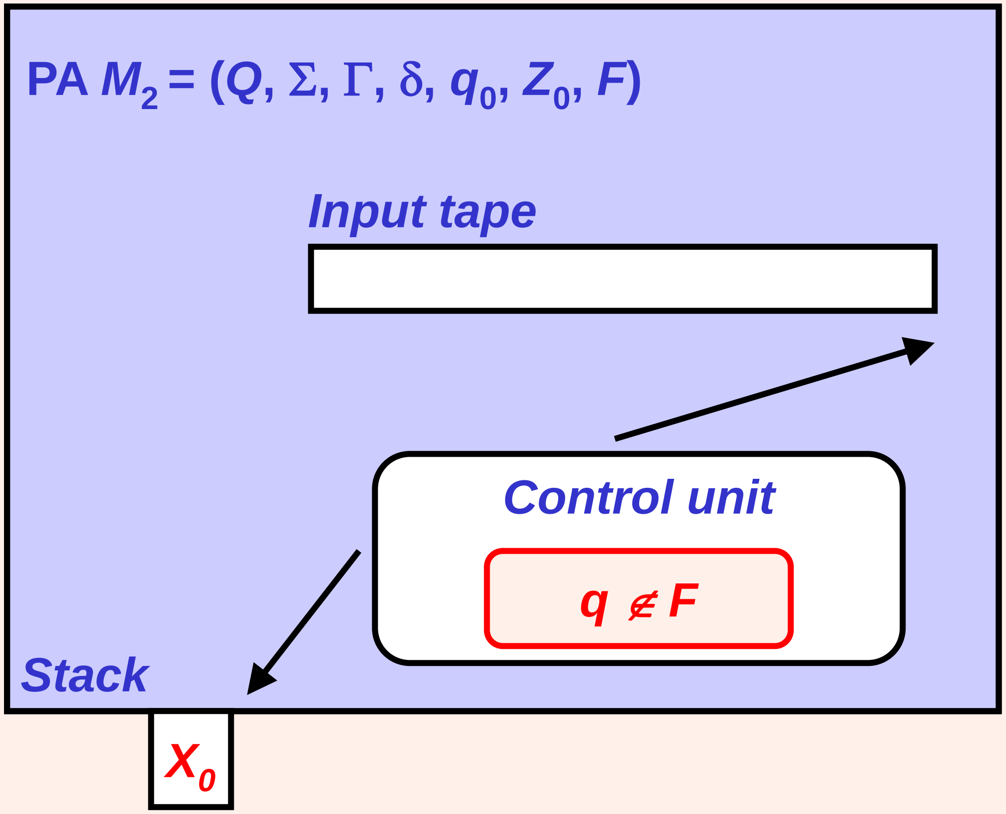

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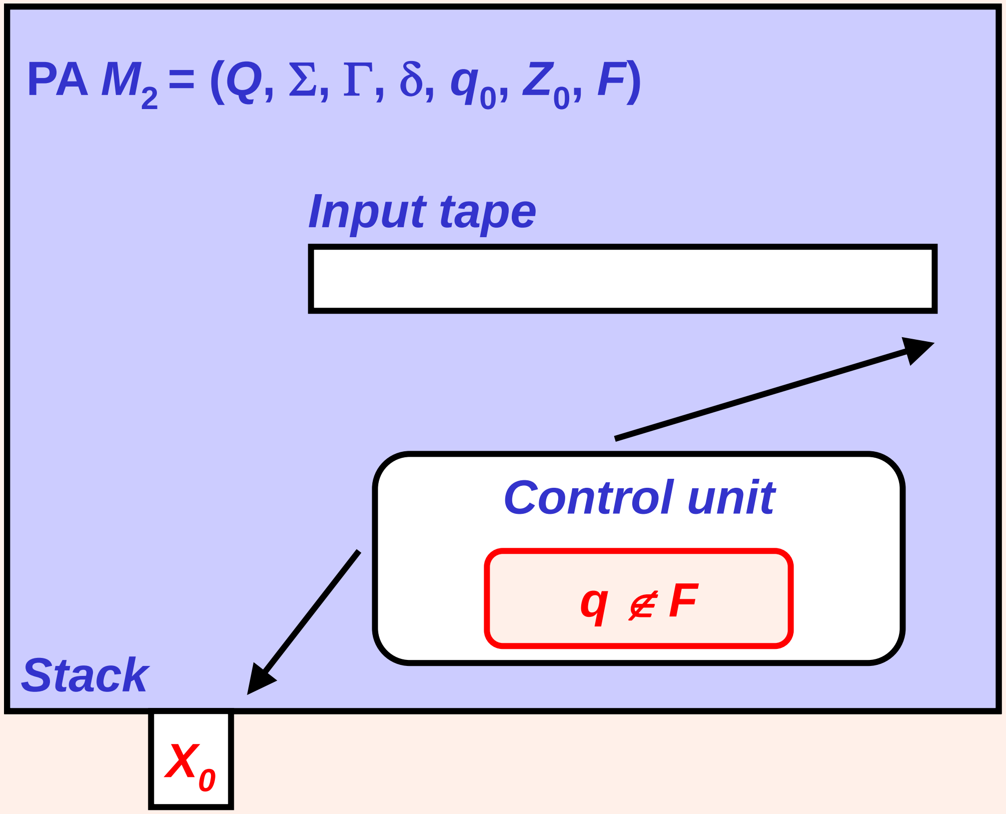

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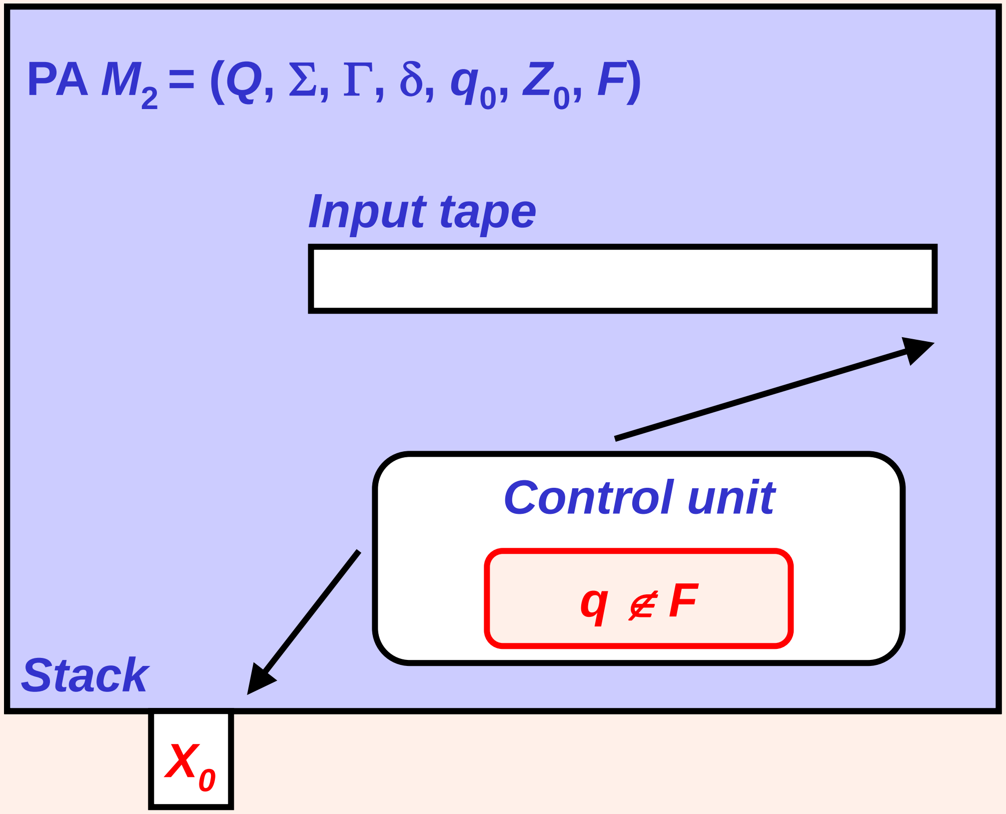

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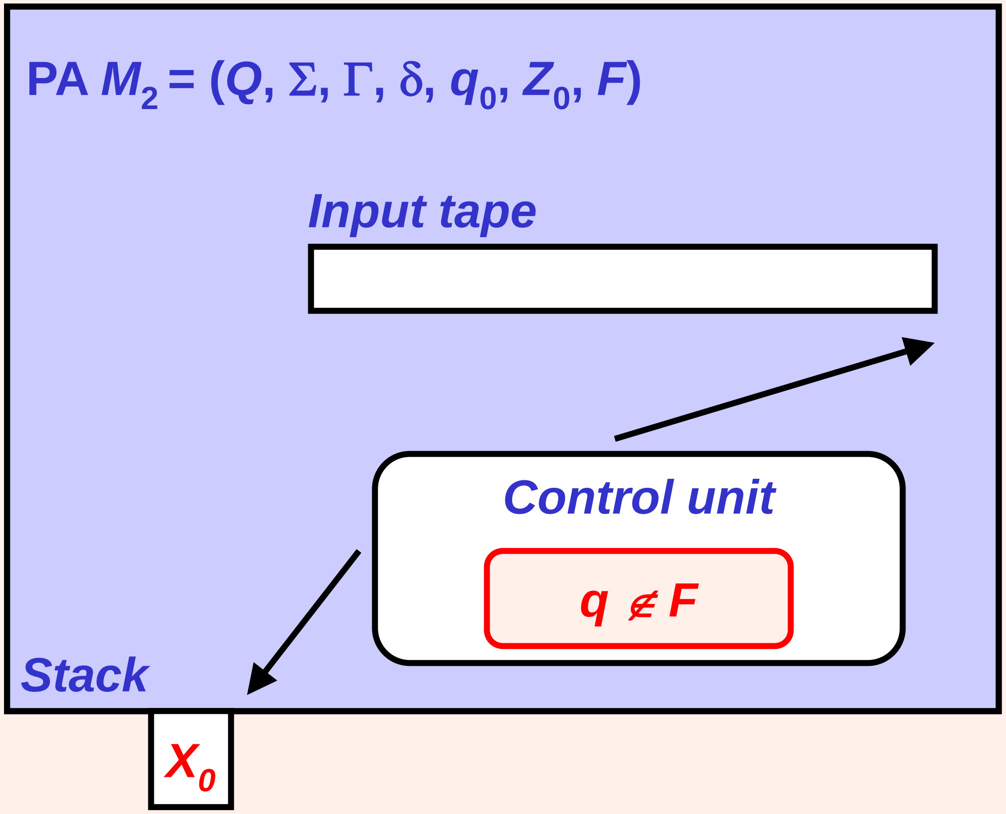

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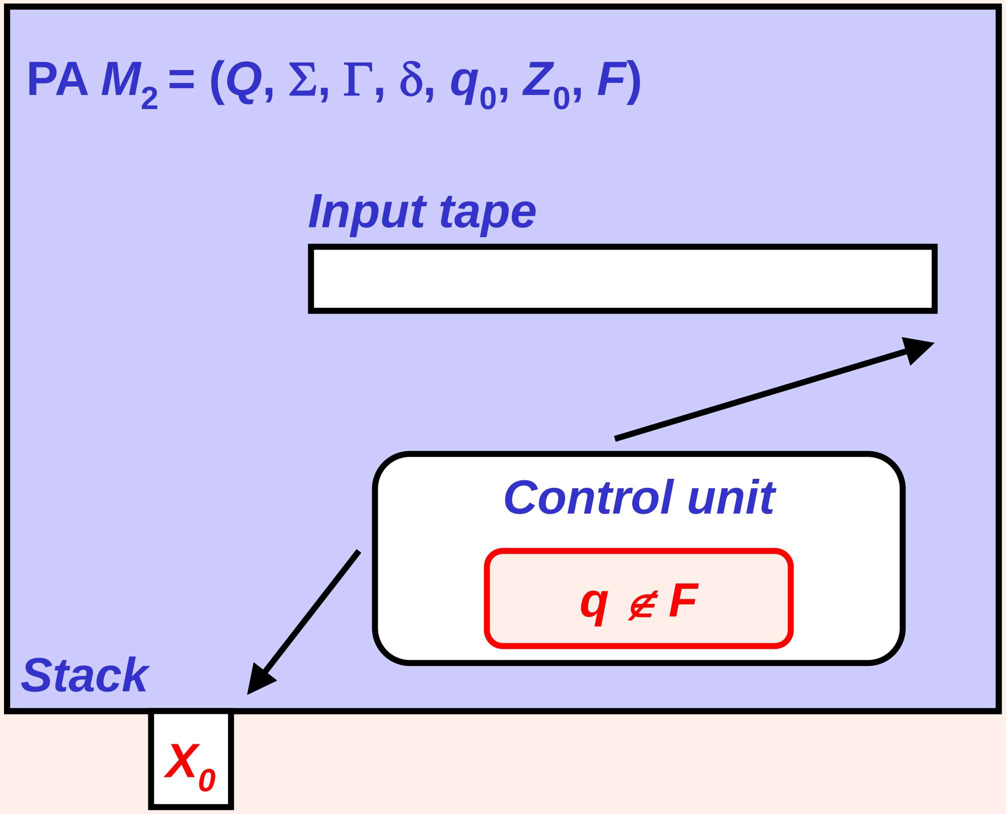

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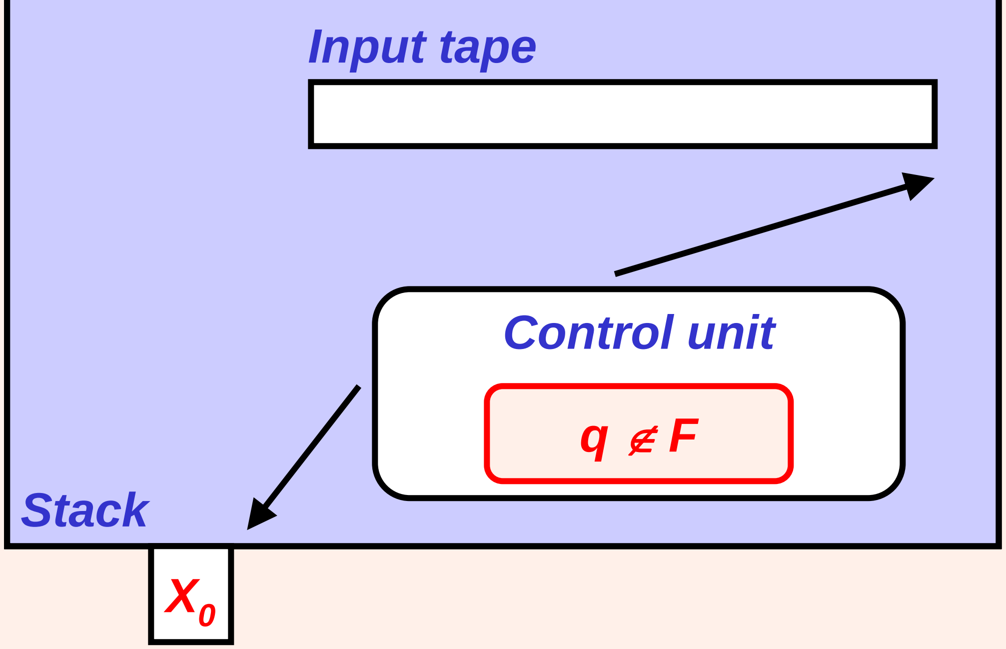

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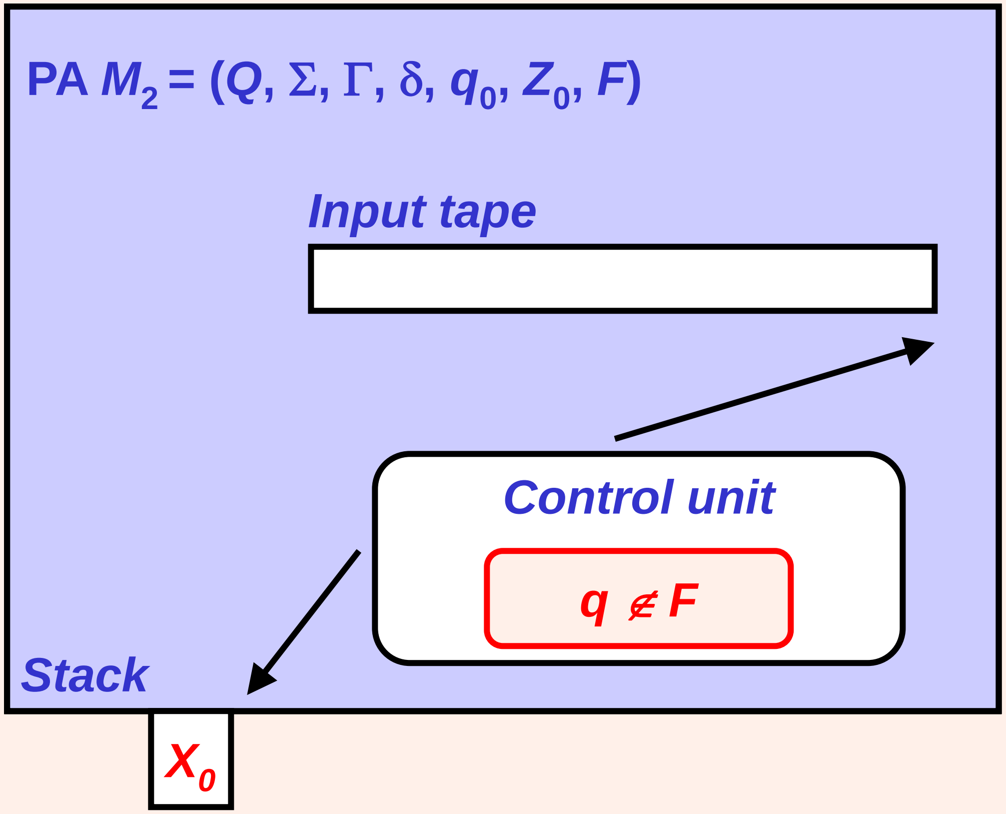

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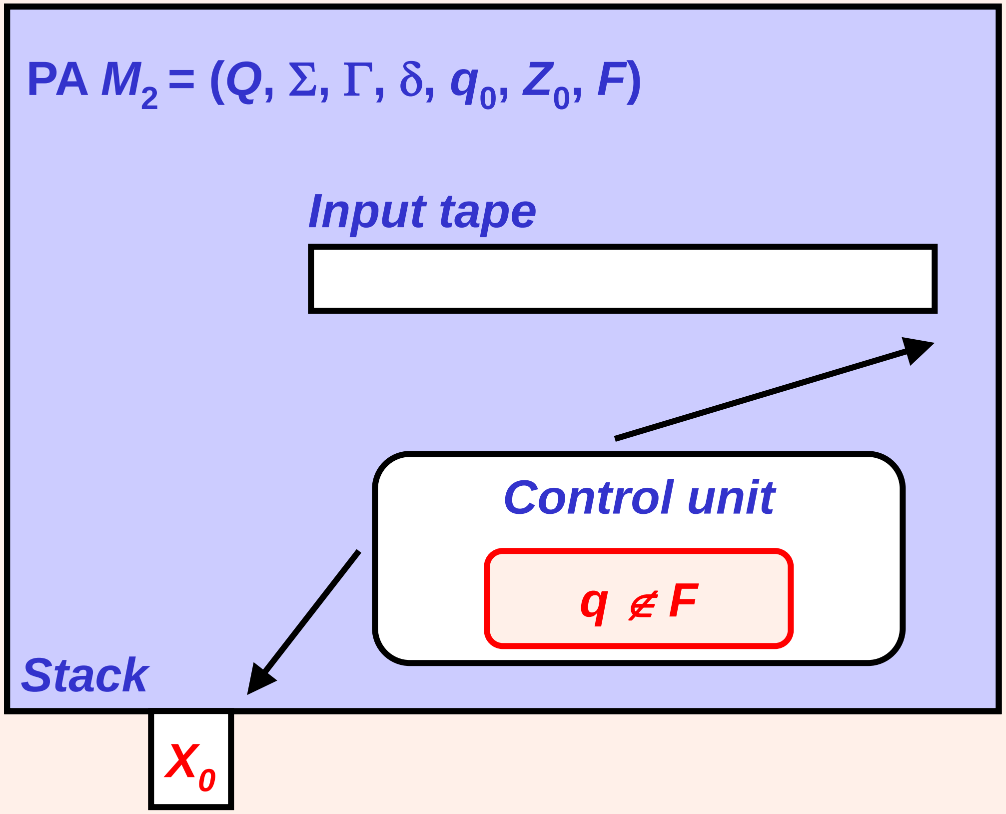

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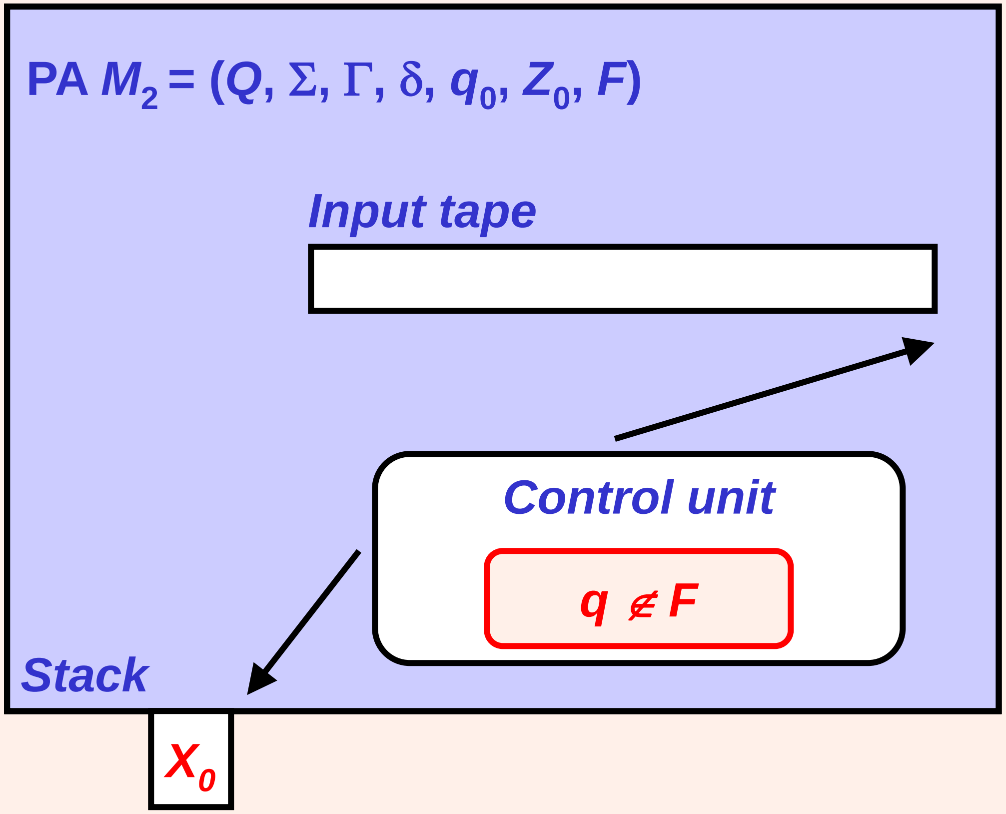

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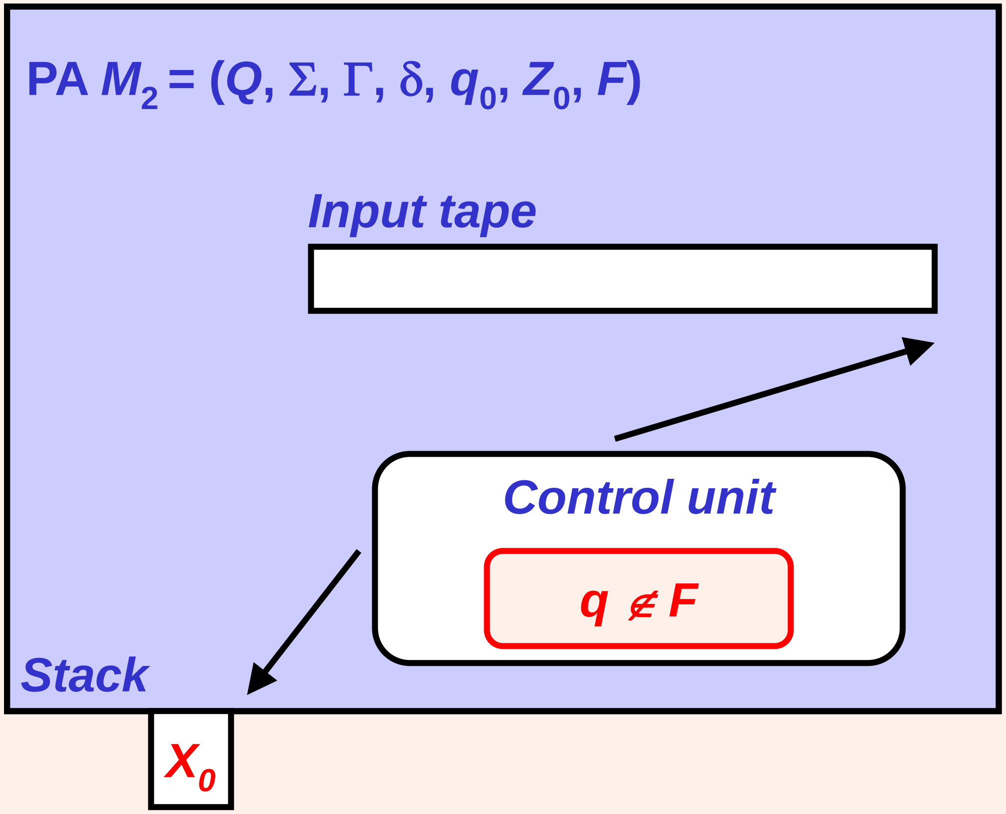

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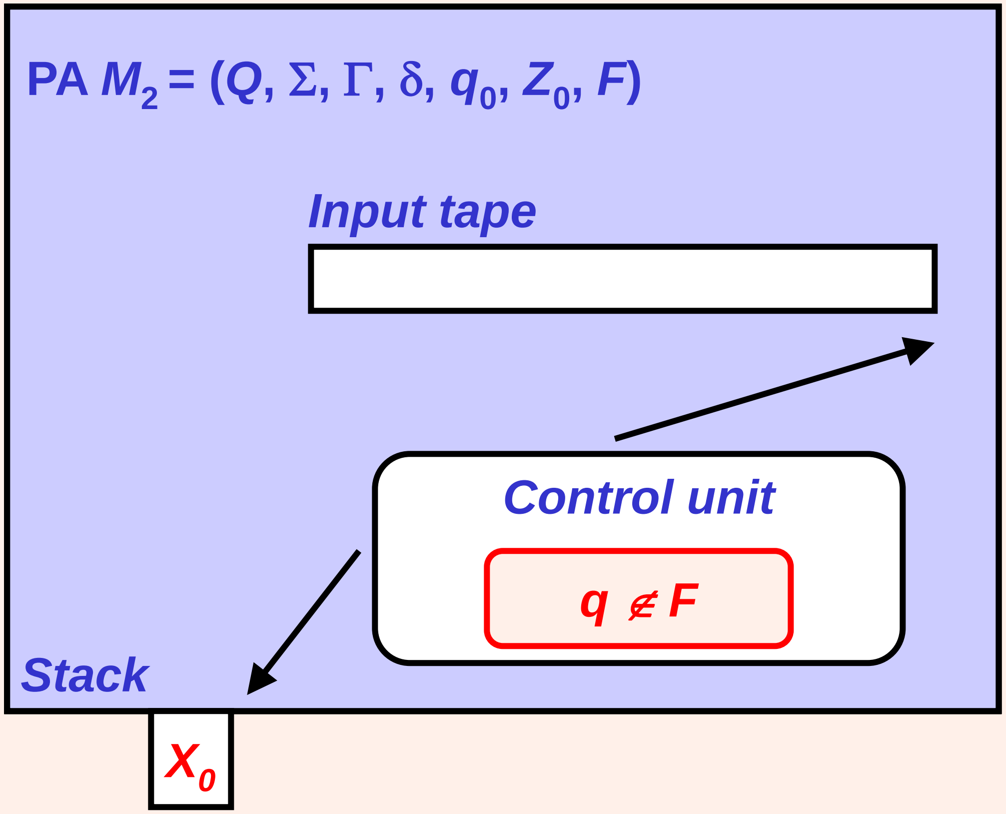

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$(q_1, 00011, K) \succ (q_1, 0011, NK) \succ (q_1, 011, NNK)$
$\succ (q_1, 11, NNNK) \succ (q_2, 1, NNK)\ \ (q_2, \varepsilon, NK)$

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$(q_1, 00011, K) \succ (q_1, 0011, NK) \succ (q_1, 011, NNK)$
$\succ (q_1, 11, NNNK) \succ (q_2, 1, NNK)\ \ (q_2, \varepsilon, NK)$

$(q_0', 00011, X_0) \succ (q_1, 00011, KX_0) \succ (q_1, 0011, NKX_0) \succ (q_1, 011, NNKX_0) \succ$
$(q_1, 11, NNNKX_0) \succ (q_2, 1, NNKX_0) \succ (q_2, \varepsilon, NKX_0) \succ$
$(q_e, \varepsilon, KX_0) \succ (q_e, \varepsilon, X_0) \succ (q_e, \varepsilon, \varepsilon)$

Constructing a PA that accepts by final state from a PA that accepts by empty stack

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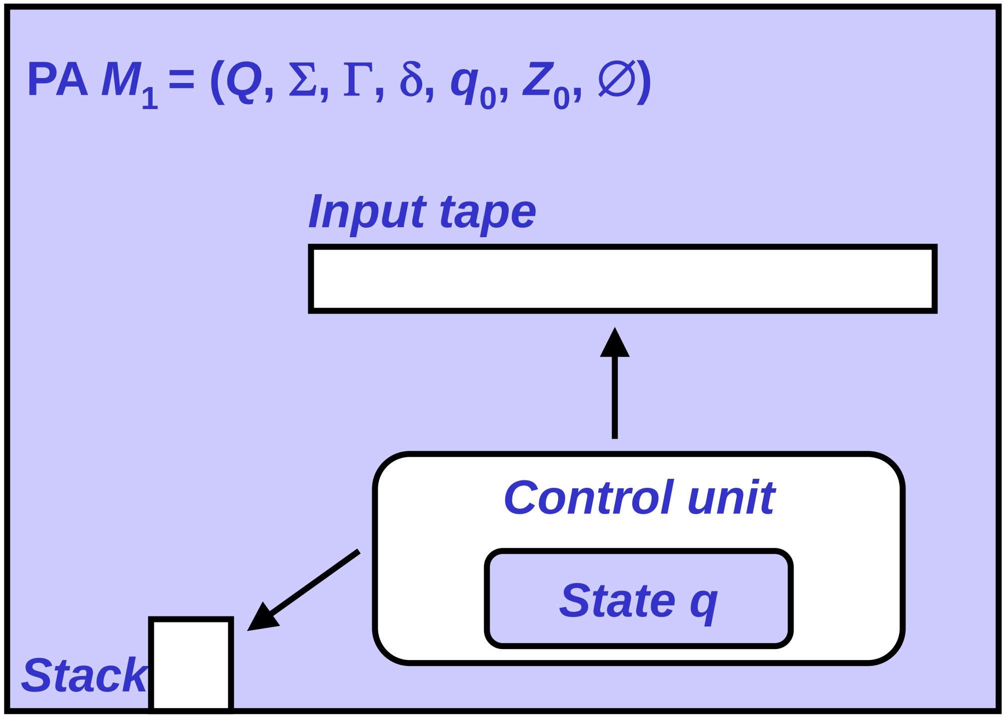

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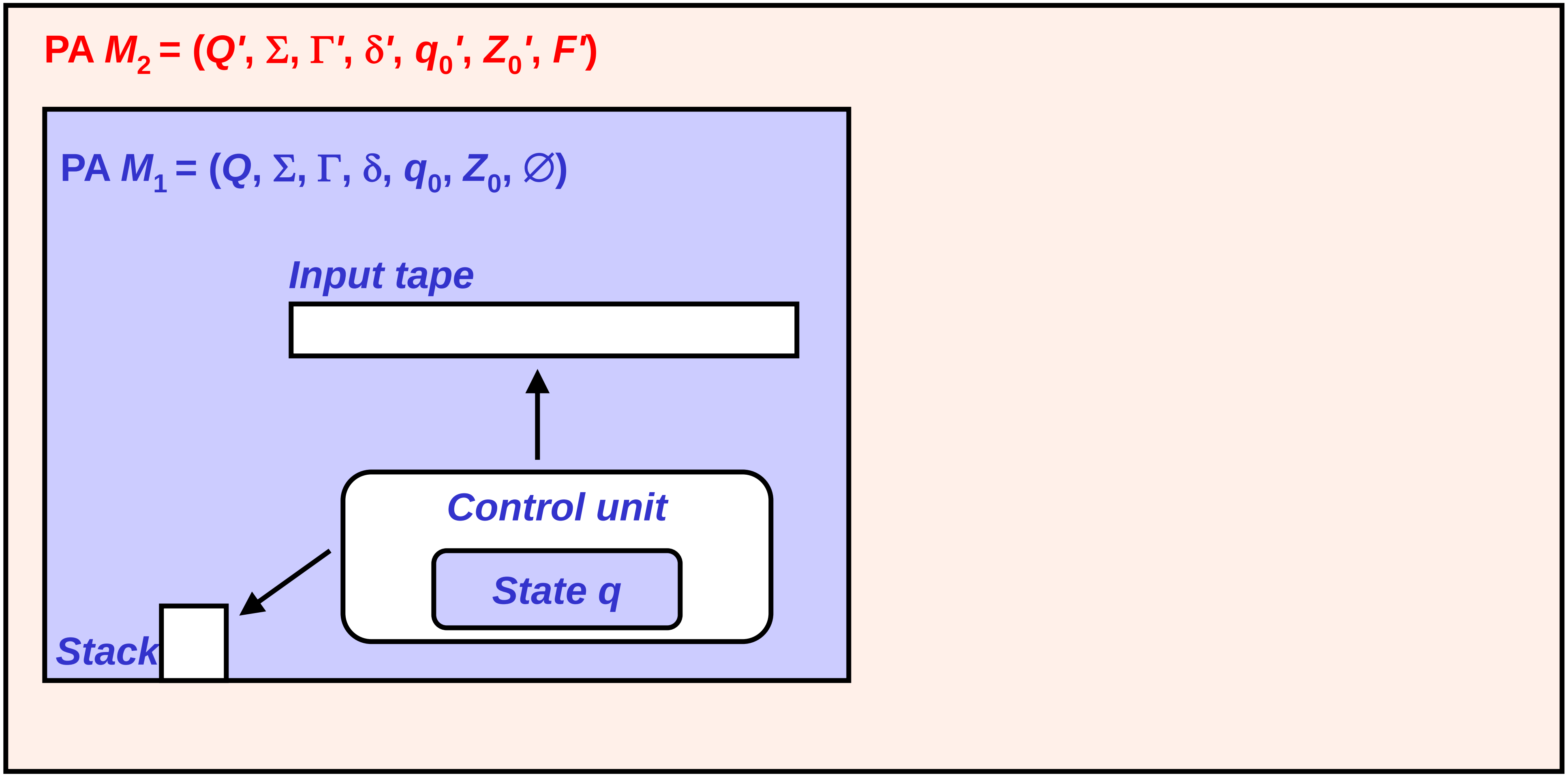
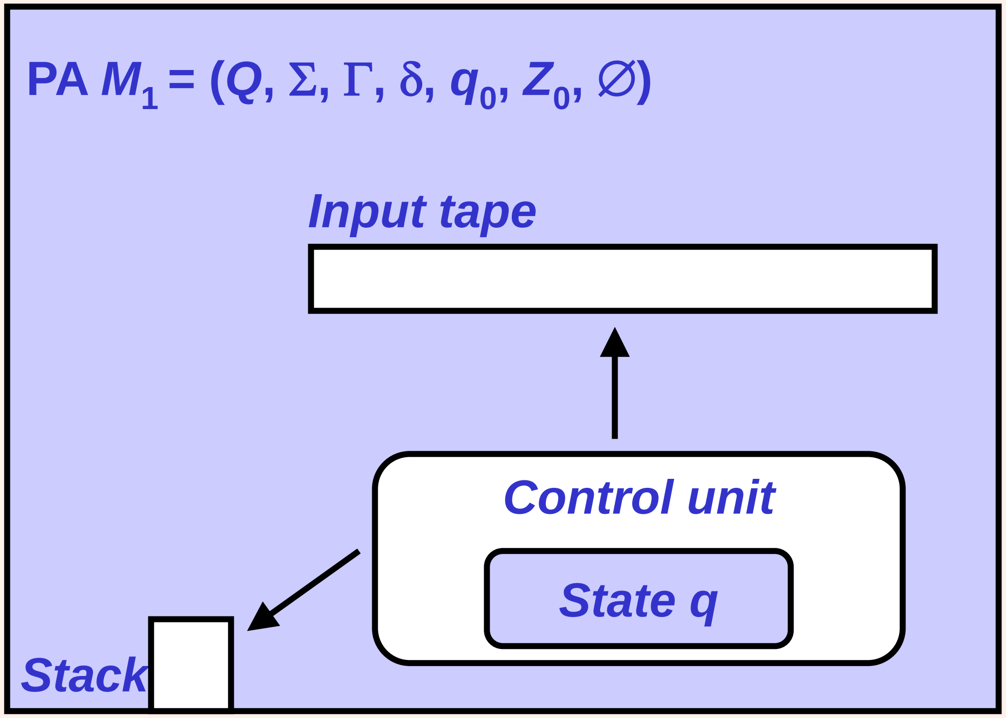


Constructing a PA that accepts by final state from a PA that accepts by empty stack

PA $M_2 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', F')$



1) set $\delta'(q, a, Z)$ = set $\delta(q, a, Z)$

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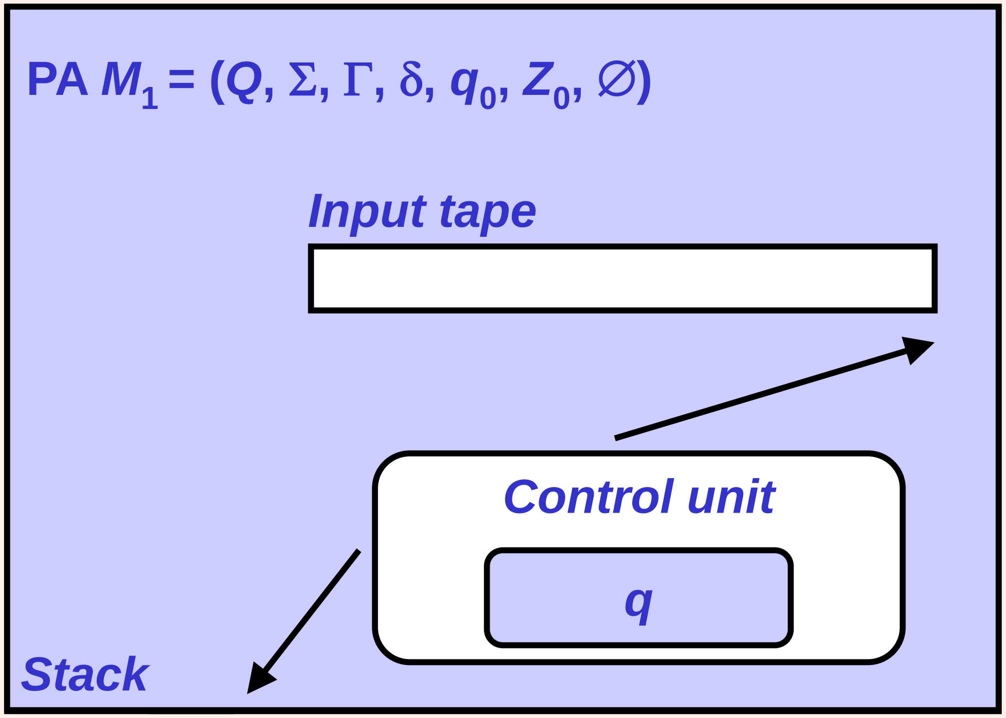

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Constructing a PA that accepts by final state from a PA that accepts by empty stack

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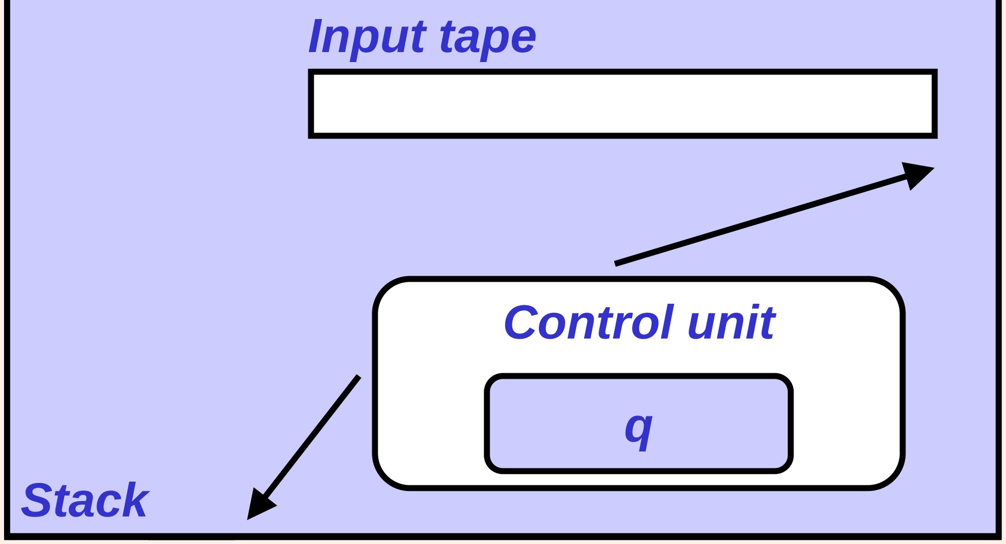
PA $M_1 = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, \varnothing)$



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2) $Q' = Q' \cup \{ q_f \}$

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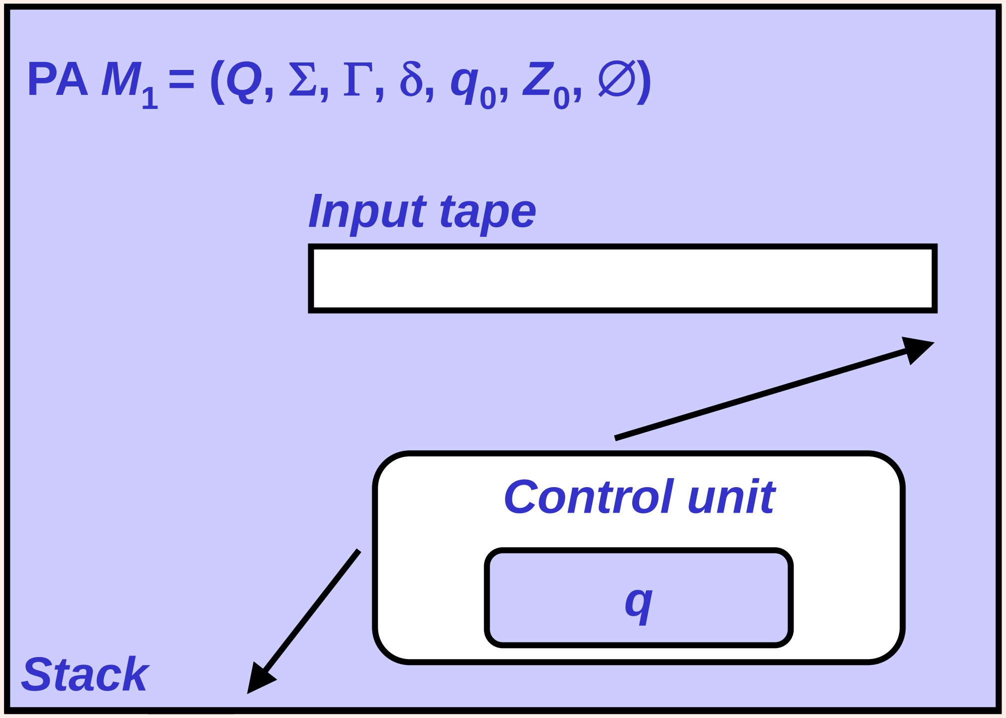

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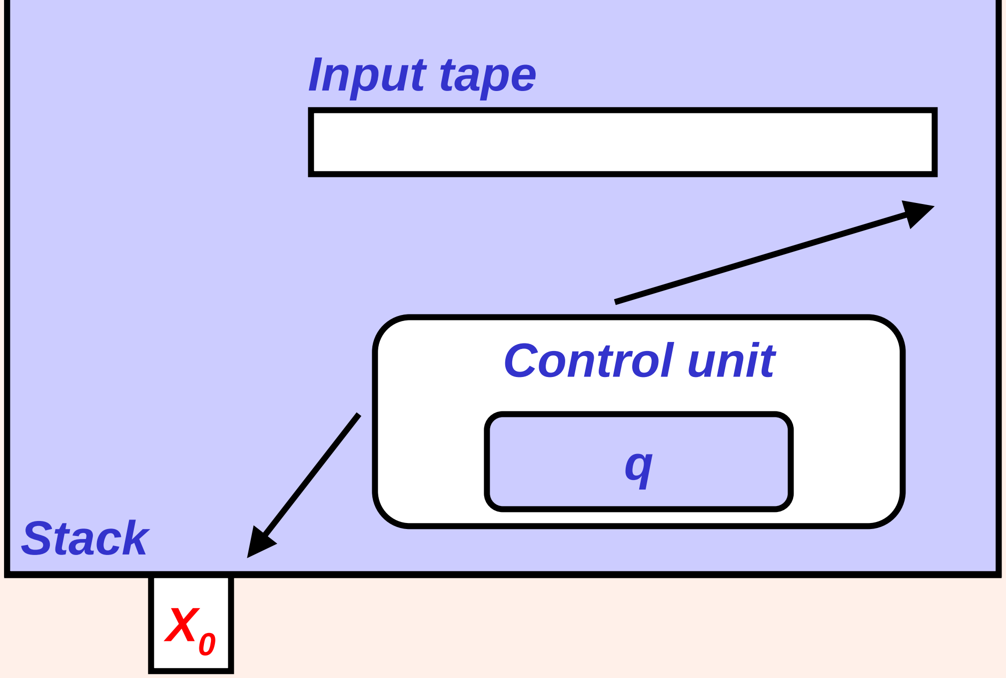

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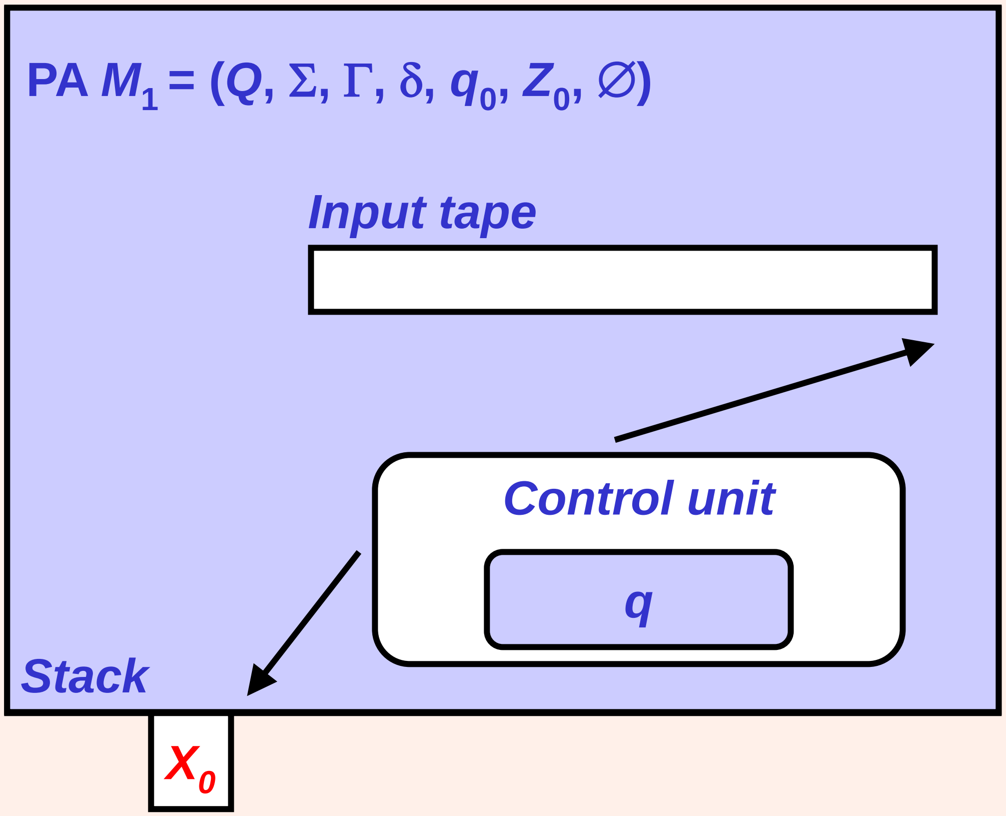

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2) $Q' = Q' \cup \{ q_f \}$

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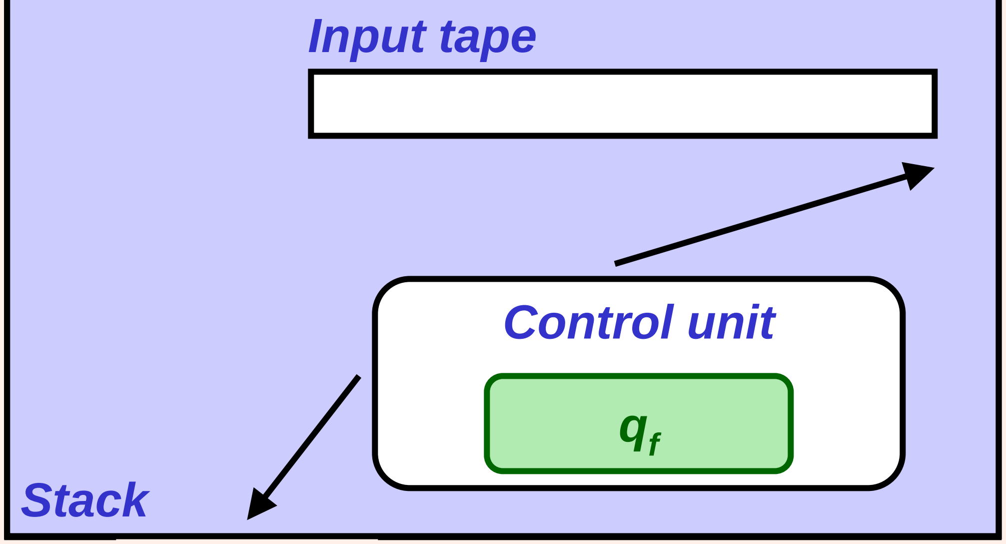
   $(q_f, \varepsilon) \in \delta'(q, \varepsilon, X_0)$

Constructing a PA that accepts by final state from a PA that accepts by empty stack

PA $M_2 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', F')$

PA $M_1 = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, \emptyset)$



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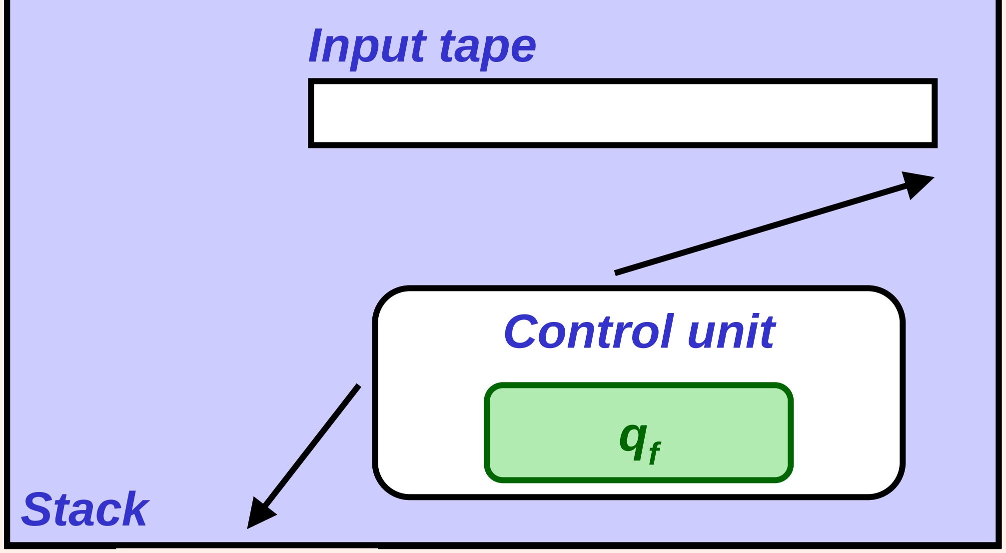

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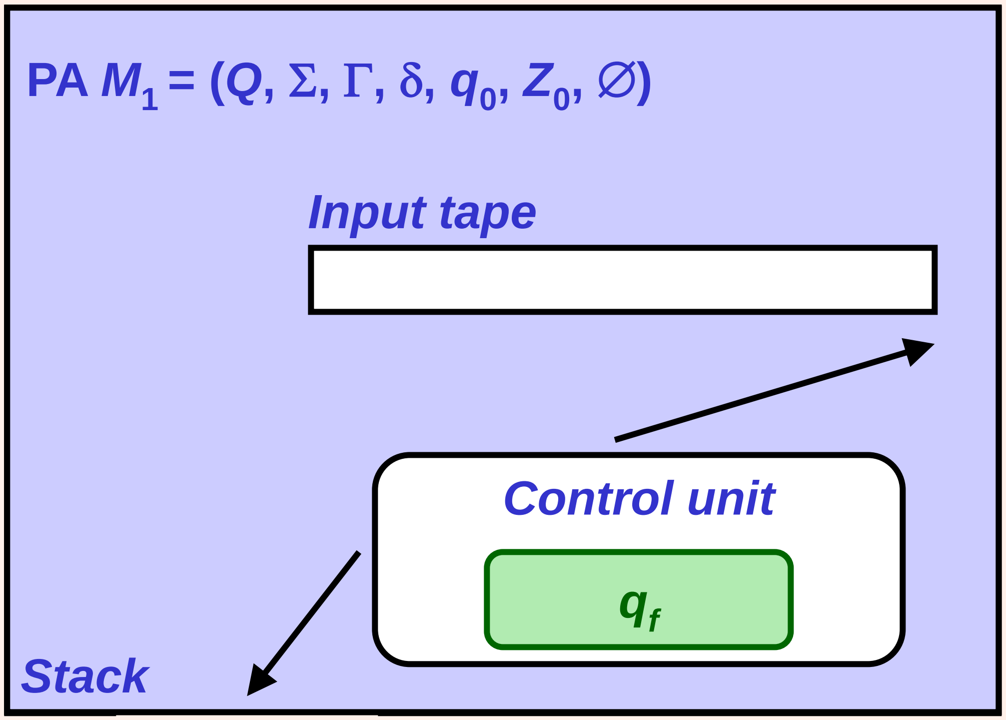

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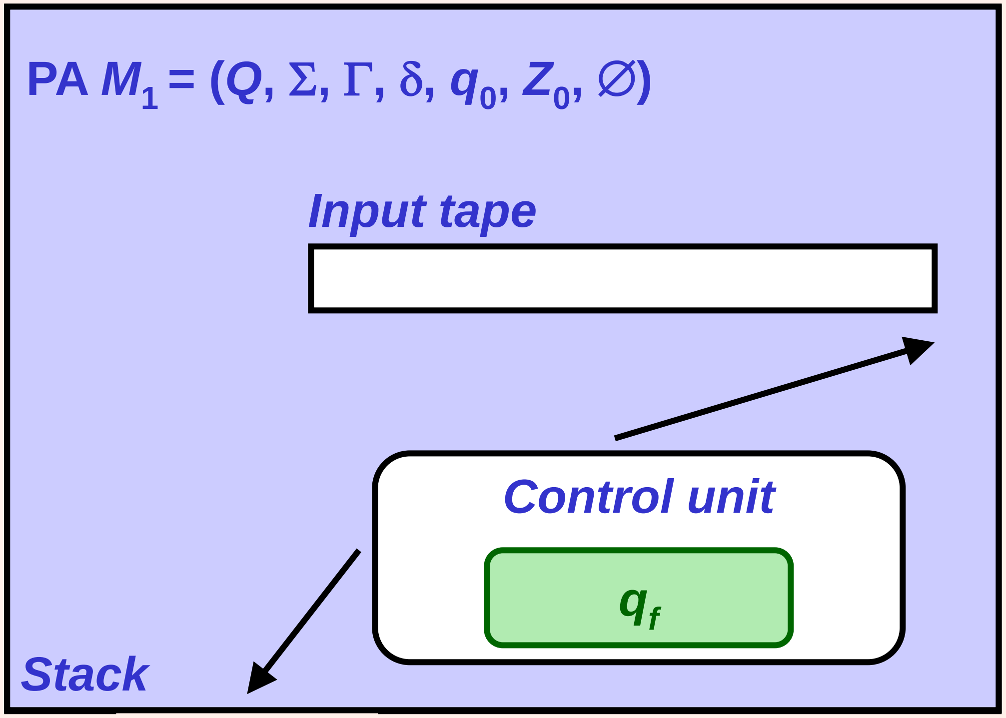

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Constructing a PA that accepts by final state from a PA that accepts by empty stack

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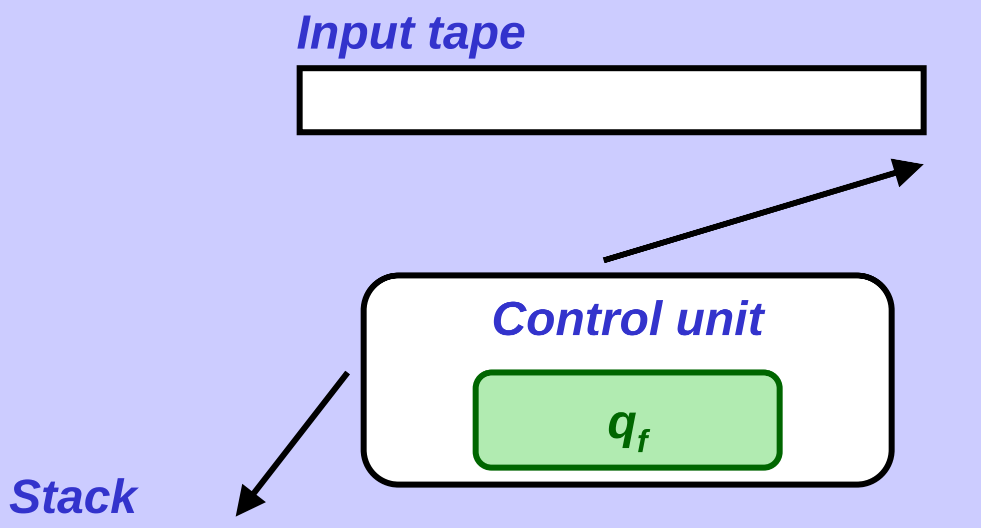

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Constructing a PA that accepts by final state from a PA that accepts by empty stack

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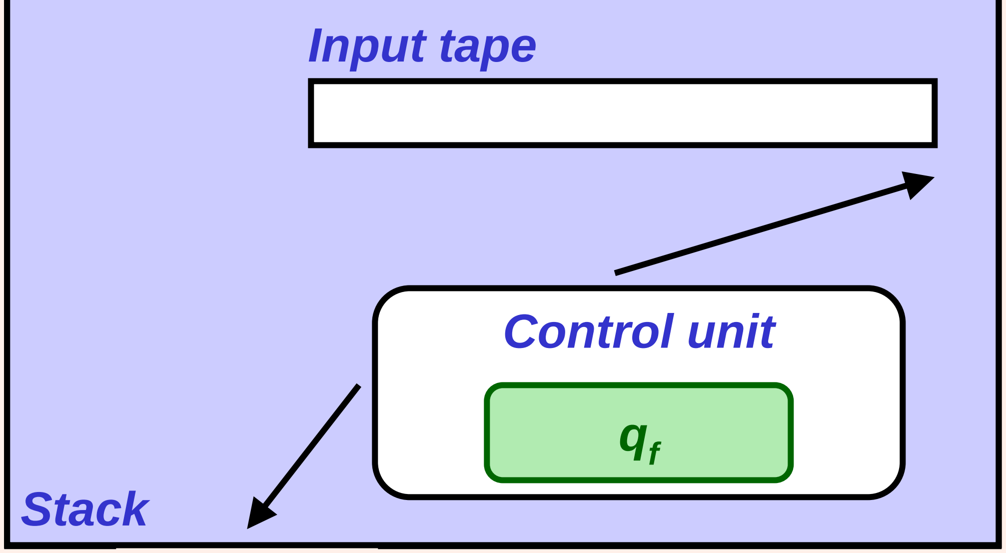

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(q_0, x, Z_0)

Constructing a PA that accepts by final state from a PA that accepts by empty stack

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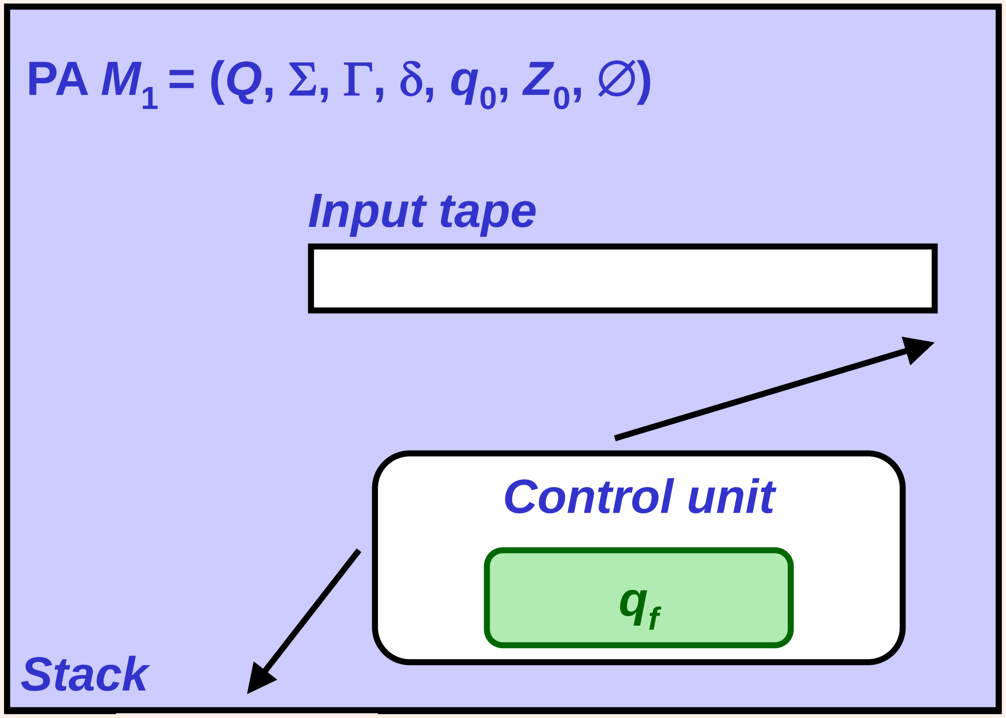

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$(q_0, x, Z_0) \underset{M_1}{\succ}$

Constructing a PA that accepts by final state from a PA that accepts by empty stack

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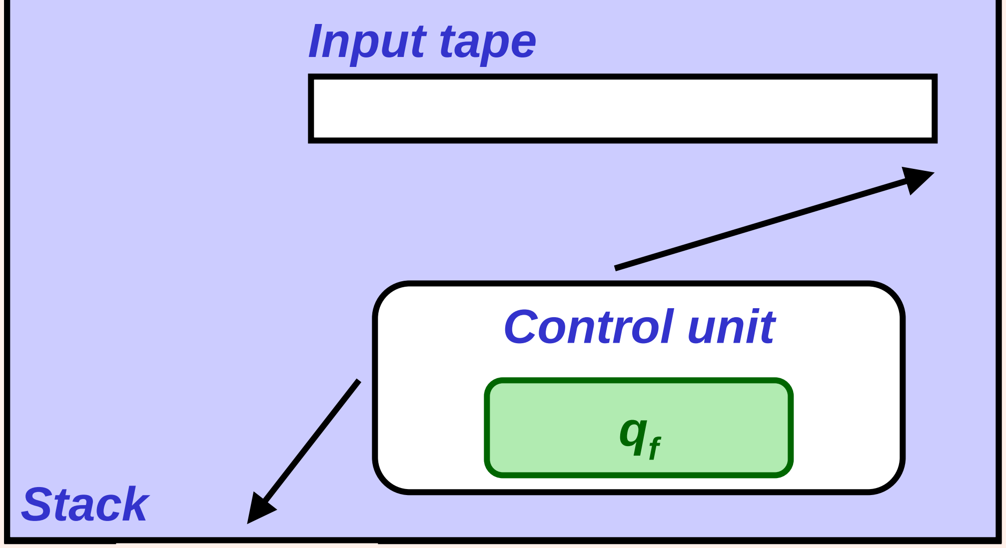

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$\delta'(q_0', \varepsilon, X_0) = \{(q_0, Z_0 X_0)\}$

$$(q_0, x, Z_0) \underset{M_1}{\succ} (q, \varepsilon, \varepsilon)$$

Constructing a PA that accepts by final state from a PA that accepts by empty stack

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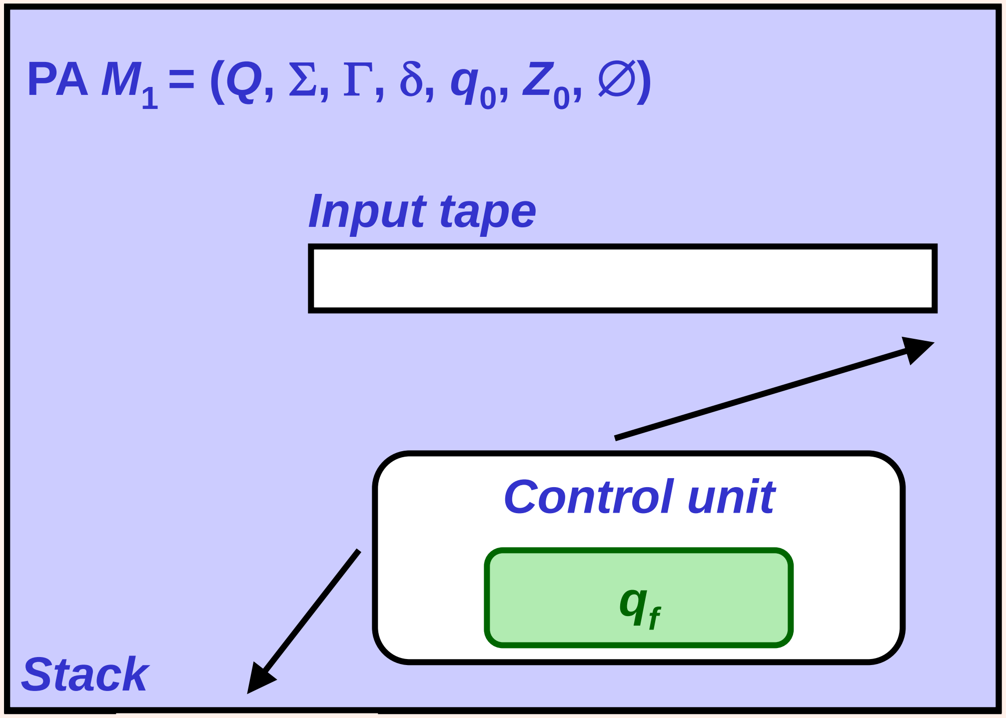

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(q_0', x, X_0)

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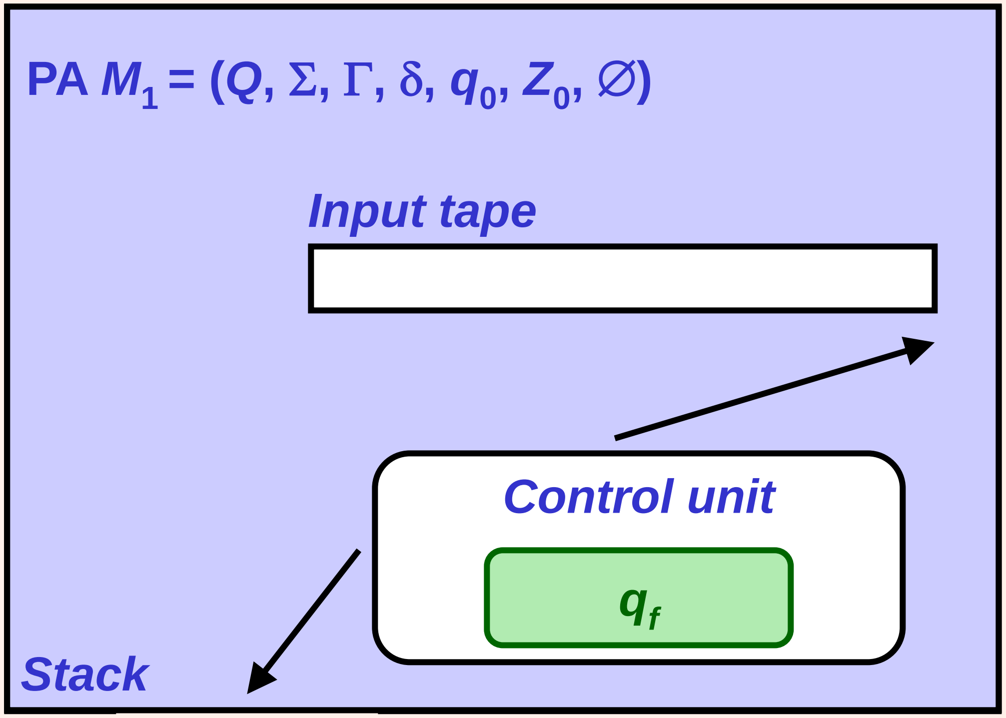

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$$(q_0, x, Z_0) \underset{M_1}{\succ} (q, \varepsilon, \varepsilon)$$

$$(q_0', x, X_0) \underset{M_2}{\succ}$$

Constructing a PA that accepts by final state from a PA that accepts by empty stack

PA $M_2 = (Q', \Sigma, \Gamma', \delta', q_0', Z_0', F')$

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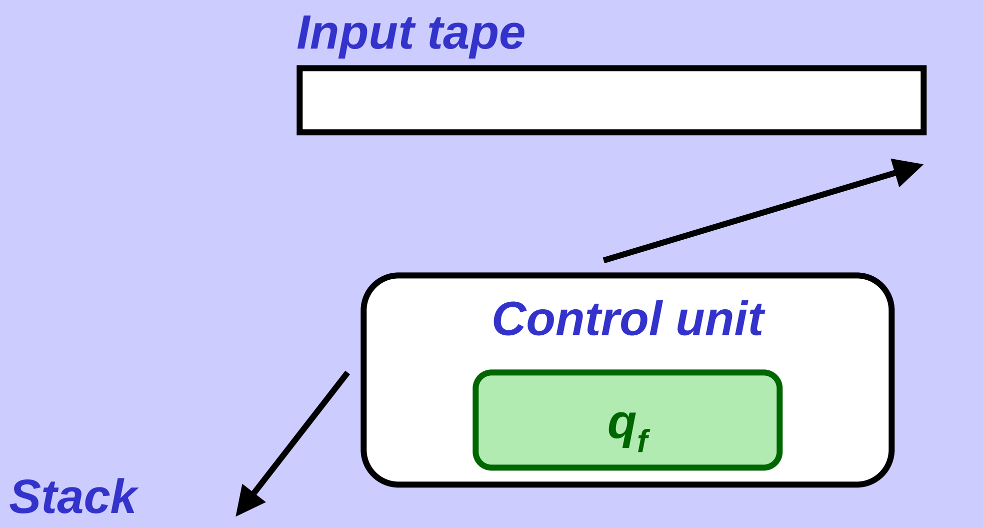

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$$(q_0, x, Z_0) \underset{M_1}{\succ} (q, \varepsilon, \varepsilon)$$

$$(q_0', x, X_0) \underset{M_2}{\succ} (q_0, x, Z_0 X_0)$$

Constructing a PA that accepts by final state from a PA that accepts by empty stack

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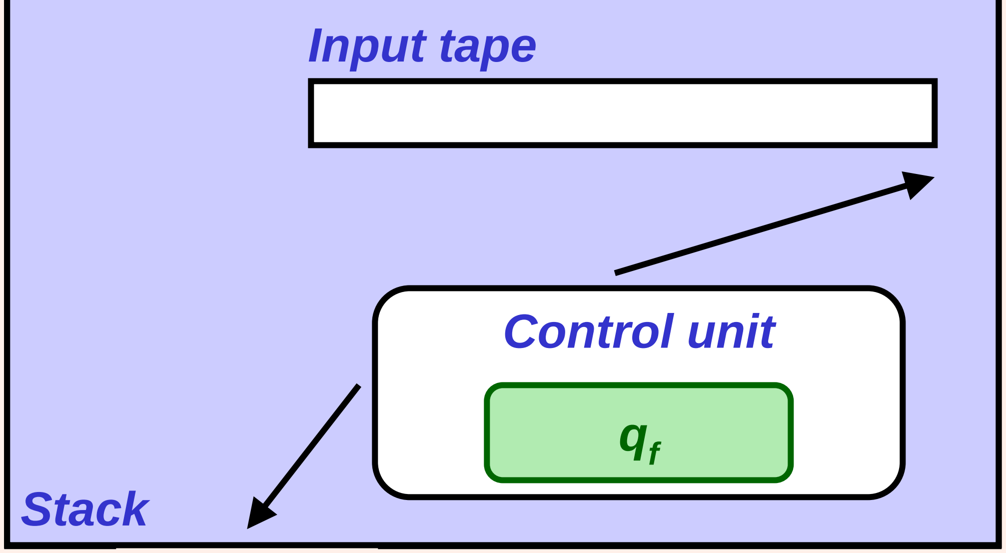

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Constructing a PA that accepts by final state from a PA that accepts by empty stack

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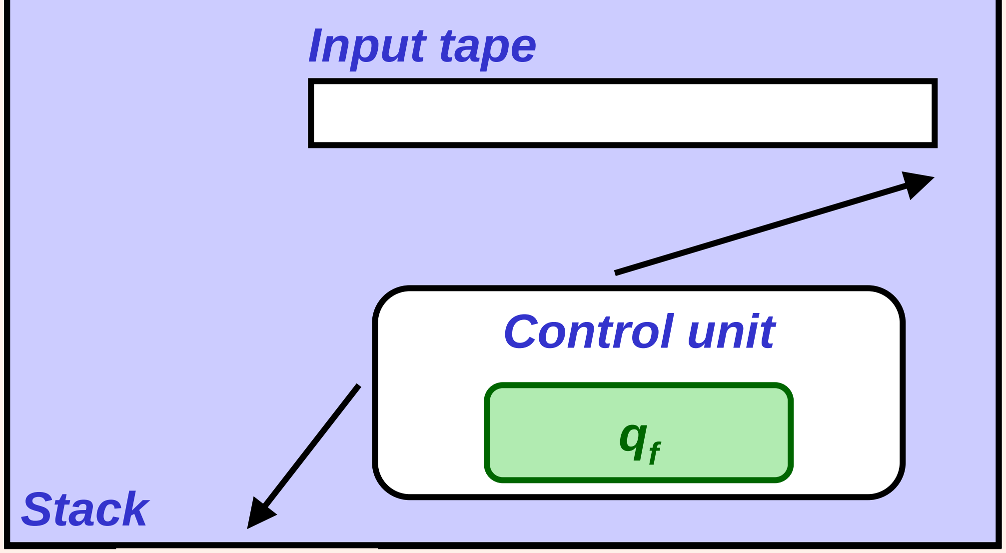

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$\delta'(q_0', \varepsilon, X_0) = \{(q_0, Z_0 X_0)\}$

$$(q_0, x, Z_0) \underset{M_1}{\succ} (q, \varepsilon, \varepsilon)$$

$$(q_0', x, X_0) \underset{M_2}{\succ} (q_0, x, Z_0 X_0) \underset{M_2}{\succ} (q, \varepsilon, \varepsilon X_0)$$

Constructing a PA that accepts by final state from a PA that accepts by empty stack

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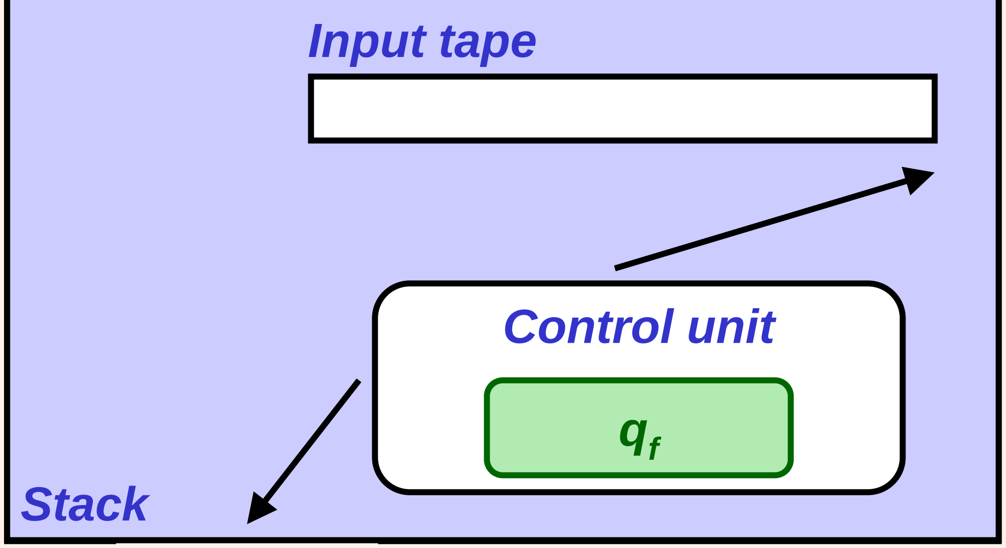

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$\delta'(q_0', \varepsilon, X_0) = \{(q_0, Z_0 X_0)\}$

$$(q_0, x, Z_0) \underset{M_1}{\succ} (q, \varepsilon, \varepsilon)$$

$$(q_0', x, X_0) \underset{M_2}{\succ} (q_0, x, Z_0 X_0) \underset{M_2}{\succ} (q, \varepsilon, \varepsilon X_0) \underset{M_2}{\succ}$$

Constructing a PA that accepts by final state from a PA that accepts by empty stack

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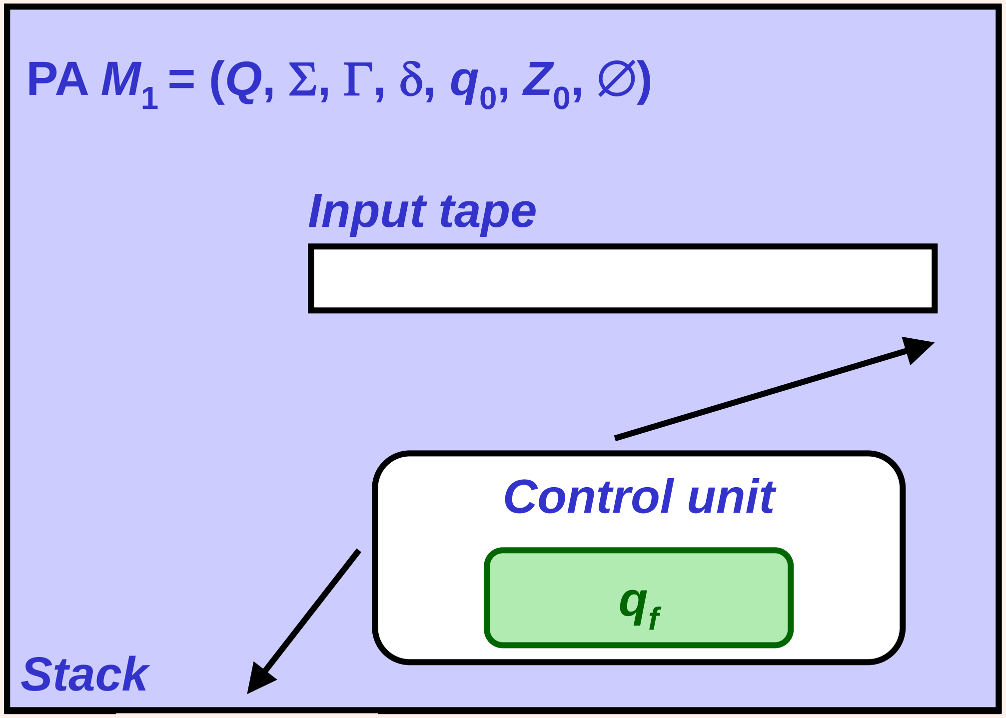

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   $\delta'(q_0', \varepsilon, X_0) = \{(q_0, Z_0 X_0)\}$

$$(q_0, x, Z_0) \underset{M_1}{\succ} (q, \varepsilon, \varepsilon)$$

$$(q_0', x, X_0) \underset{M_2}{\succ} (q_0, x, Z_0 X_0) \underset{M_2}{\succ} (q, \varepsilon, \varepsilon X_0) \underset{M_2}{\succ} (q_f, \varepsilon, \varepsilon)$$

PA $M_1 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \emptyset)$

PA $M_1 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \varnothing)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$
7) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
8) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
9) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$
10) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

PA $M_2 = (\{q_1, q_2, q_0', q_f\}, \{0, 1\}, \{N, J, K, X_0\}, \delta', q_0', X_0, \{q_f\})$

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9) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$
10) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

0) $\delta'(q_0', \varepsilon, X_0) = \{(q_1, KX_0)\}$

PA $M_2 = (\{q_1, q_2, q_0', q_f\}, \{0, 1\}, \{N, J, K, X_0\}, \delta', q_0', X_0, \{q_f\})$

PA $M_1 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \varnothing)$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 1, K) = \{(q_1, JK)\}$
3) $\delta(q_1, 0, N) = \{(q_1, NN), (q_2, \varepsilon)\}$
4) $\delta(q_1, 1, N) = \{(q_1, JN)\}$
5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$
7) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
8) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
9) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$
10) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

0) $\delta'(q_0', \varepsilon, X_0) = \{(q_1, KX_0)\}$

11) $\delta'(q_1, \varepsilon, X_0) = \{(q_f, \varepsilon)\}$
12) $\delta'(q_2, \varepsilon, X_0) = \{(q_f, \varepsilon)\}$

PA $M_2 = (\{q_1, q_2, q_0', q_f\}, \{0, 1\}, \{N, J, K, X_0\}, \delta', q_0', X_0, \{q_f\})$

PA $M_1 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \varnothing)$

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7) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
8) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
9) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$
10) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

0) $\delta'(q_0', \varepsilon, X_0) = \{(q_1, KX_0)\}$

11) $\delta'(q_1, \varepsilon, X_0) = \{(q_f, \varepsilon)\}$
12) $\delta'(q_2, \varepsilon, X_0) = \{(q_f, \varepsilon)\}$

$(q_1, 001100, K) \succ (q_1, 01100, NK) \succ (q_1, 1100, NNK) \succ$
$(q_1, 100, JNNK) \succ (q_2, 00, NNK) \succ (q_2, 0, NK) \succ (q_2, \varepsilon, K) \succ (q_2, \varepsilon, \varepsilon)$

PA $M_2 = (\{q_1, q_2, q_0', q_f\}, \{0, 1\}, \{N, J, K, X_0\}, \delta', q_0', X_0, \{q_f\})$

PA $M_1 = (\{q_1, q_2\}, \{0, 1\}, \{N, J, K\}, \delta, q_1, K, \varnothing)$

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5) $\delta(q_1, 0, J) = \{(q_1, NJ)\}$
6) $\delta(q_1, 1, J) = \{(q_1, JJ), (q_2, \varepsilon)\}$
7) $\delta(q_2, 0, N) = \{(q_2, \varepsilon)\}$
8) $\delta(q_2, 1, J) = \{(q_2, \varepsilon)\}$
9) $\delta(q_1, \varepsilon, K) = \{(q_2, \varepsilon)\}$
10) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

0) $\delta'(q_0', \varepsilon, X_0) = \{(q_1, KX_0)\}$

11) $\delta'(q_1, \varepsilon, X_0) = \{(q_f, \varepsilon)\}$
12) $\delta'(q_2, \varepsilon, X_0) = \{(q_f, \varepsilon)\}$

$(q_1, 001100, K) \succ (q_1, 01100, NK) \succ (q_1, 1100, NNK) \succ$
$(q_1, 100, JNNK) \succ (q_2, 00, NNK) \succ (q_2, 0, NK) \succ (q_2, \varepsilon, K) \succ (q_2, \varepsilon, \varepsilon)$

$(q_0', 001100, X_0) \succ (q_1, 001100, KX_0) \succ (q_1, 01100, NKX_0) \succ (q_1, 1100, NNKX_0) \succ$
$(q_1, 100, JNNKX_0) \succ (q_2, 00, NNKX_0) \succ (q_2, 0, NKX_0) \succ (q_2, \varepsilon, KX_0) \succ (q_2, \varepsilon, \varepsilon X_0)$
$\succ (q_f, \varepsilon, \varepsilon)$