Lecture 7

Example: *Unreachable* symbols

Example: *Unreachable* symbols

1) $S \rightarrow a\ S\ b$ **3)** $A \rightarrow b\ S$

2) $S \rightarrow c$ **4)** $A \rightarrow a$

Example: *Unreachable* symbols

1) $S \rightarrow a\,S\,b$ 3) $A \rightarrow b\,S$

2) $S \rightarrow c$ 4) $A \rightarrow a$

Example: *Unreachable* symbols

1) $S \rightarrow a\ S\ b$ 3) $A \rightarrow b\ S$

2) $S \rightarrow c$ 4) $A \rightarrow a$

1) $S \rightarrow a\,S\,b$ 3) $A \rightarrow b\,S$

2) $S \rightarrow c$ 4) $A \rightarrow a$

Example: *Unreachable* symbols

1) $S \rightarrow a\ S\ b$

2) $S \rightarrow c$

Reachable symbols property

Reachable symbols property

If character *A* on the left side of the production is reachable:

Reachable symbols property

If character *A* on the left side of the production is reachable:

$$A \rightarrow \alpha_1 \,/\, \alpha_2 \,/\, ... \,|\, \alpha_n$$

Reachable symbols property

If character *A* on the left side of the production is reachable:

$$A \rightarrow \alpha_1 \,/\, \alpha_2 \,/\, ... \,|\, \alpha_n$$

Then all terminal and nonterminal symbols in strings
$\alpha_1, \alpha_2, ...$ and α_n on the right side of the production are reachable

Reachable symbols property

If character *A* on the left side of the production is reachable:

$$A \rightarrow \alpha_1 \mid \alpha_2 \mid ... \mid \alpha_n$$

Then all terminal and nonterminal symbols in strings
$\alpha_1, \alpha_2, ...$ and α_n on the right side of the production are reachable

$$S \stackrel{*}{\Rightarrow} \beta A \gamma$$

Reachable symbols property

If character *A* on the left side of the production is reachable:

$$A \rightarrow \alpha_1 \,/\, \alpha_2 \,/\, ... \mid \alpha_n$$

Then all terminal and nonterminal symbols in strings
$\alpha_1, \alpha_2, ...$ and α_n on the right side of the production are reachable

$$S \overset{*}{\Rightarrow} \beta A \gamma$$

$$A \Rightarrow \alpha_i$$

Reachable symbols property

If character *A* on the left side of the production is reachable:

$$A \rightarrow \alpha_1 \,/\, \alpha_2 \,/\, ... \,|\, \alpha_n$$

Then all terminal and nonterminal symbols in strings
$\alpha_1, \alpha_2, ...$ and α_n on the right side of the production are reachable

$$S \overset{*}{\Rightarrow} \beta A \gamma$$

$$A \Rightarrow \alpha_i$$

$$S \overset{*}{\Rightarrow} \beta A \gamma \Rightarrow \beta \alpha_i \gamma$$

Algorithm for computing the set of *reachable* symbols

Algorithm for computing the set of *reachable* symbols

CurrentReachableSymbolsList = $\varnothing$;

Algorithm for computing the set of *reachable* symbols

CurrentReachableSymbolsList = ∅;
NewReachableSymbolsList = {*S* | *S* is the start nonterminal symbol};

Algorithm for computing the set of *reachable* symbols

CurrentReachableSymbolsList = ∅;
NewReachableSymbolsList = {*S | S* is the start nonterminal symbol};

while (*CurrentReachableSymbolsList* != *NewReachableSymbolsList*)

Algorithm for computing the set of *reachable* symbols

CurrentReachableSymbolsList = $\varnothing$;
NewReachableSymbolsList = {*S | S* is the start nonterminal symbol};

while (*CurrentReachableSymbolsList != NewReachableSymbolsList*)
{

Algorithm for computing the set of *reachable* symbols

CurrentReachableSymbolsList = $\varnothing$;
NewReachableSymbolsList = {*S* | *S* is the start nonterminal symbol};

<u>while</u> (*CurrentReachableSymbolsList* != *NewReachableSymbolsList*)
{

       CurrentReachableSymbolsList = *NewReachableSymbolsList*;

Algorithm for computing the set of *reachable* symbols

CurrentReachableSymbolsList = $\varnothing$;
NewReachableSymbolsList = {**S | S is the start nonterminal symbol**};

<u>while</u> (*CurrentReachableSymbolsList* != *NewReachableSymbolsList*)
{

      CurrentReachableSymbolsList = *NewReachableSymbolsList*;
      NewReachableSymbolsList =

Algorithm for computing the set of *reachable* symbols

CurrentReachableSymbolsList = $\varnothing$;
NewReachableSymbolsList = {**S | S is the start nonterminal symbol**};

<u>while</u> (*CurrentReachableSymbolsList != NewReachableSymbolsList*)
{

    CurrentReachableSymbolsList = NewReachableSymbolsList;
    NewReachableSymbolsList =
        CurrentReachableSymbolsList

Algorithm for computing the set of *reachable* symbols

CurrentReachableSymbolsList = $\varnothing$;
NewReachableSymbolsList = {S | S is the start nonterminal symbol};

<u>while</u> (*CurrentReachableSymbolsList != NewReachableSymbolsList*)
{

      CurrentReachableSymbolsList = NewReachableSymbolsList;
      NewReachableSymbolsList =
           CurrentReachableSymbolsList
               $\cup$ *{X | X is in string α_i, $A \rightarrow \alpha_i$ i $A \in$*
CurrentReachableSymbolsList};

Algorithm for computing the set of *reachable* symbols

CurrentReachableSymbolsList = ∅;
NewReachableSymbolsList = {*S | S* **is the start nonterminal symbol**};

<u>while</u> (*CurrentReachableSymbolsList* != *NewReachableSymbolsList*)
{

        CurrentReachableSymbolsList = *NewReachableSymbolsList*;
        NewReachableSymbolsList =
             CurrentReachableSymbolsList
                ∪ {*X | X* **is in string** α_i, *A*→α_i **i** *A*∈
CurrentReachableSymbolsList};
}

Algorithm for computing the set of *reachable* symbols

CurrentReachableSymbolsList = $\varnothing$;
NewReachableSymbolsList = {*S | S* **is the start nonterminal symbol**};

<u>while</u> (*CurrentReachableSymbolsList* != *NewReachableSymbolsList*)
{

       CurrentReachableSymbolsList = *NewReachableSymbolsList*;
       NewReachableSymbolsList =
            CurrentReachableSymbolsList
                $\cup$ {*X* **|** *X* **is in string** α_i, *A*$\rightarrow$$\alpha_i$ **i** *A*$\in$
CurrentReachableSymbolsList};
}

CurrentReachableSymbolsList = *NewReachableSymbolsList*;

Algorithm for computing the set of *reachable* symbols

1) $S \to a\,A\,B$ 4) $A \to e$ 7) $C \to c\,A\,B$ 10) $D \to e\,A$

2) $S \to E$ 5) $B \to b\,E$ 8) $C \to d\,S\,D$ 11) $E \to f\,A$

3) $A \to d\,D\,A$ 6) $B \to f$ 9) $C \to a$ 12) $E \to g$

Algorithm for computing the set of *reachable* symbols

1) $S \rightarrow a A B$ 4) $A \rightarrow e$ 7) $C \rightarrow c A B$ 10) $D \rightarrow e A$

2) $S \rightarrow E$ 5) $B \rightarrow b E$ 8) $C \rightarrow d S D$ 11) $E \rightarrow f A$

3) $A \rightarrow d D A$ 6) $B \rightarrow f$ 9) $C \rightarrow a$ 12) $E \rightarrow g$

ReachableSymbolsList = **{ S }**

Algorithm for computing the set of *reachable* symbols

1) $S \rightarrow a\ A\ B$ 4) $A \rightarrow e$ 7) $C \rightarrow c\ A\ B$ 10) $D \rightarrow e\ A$

2) $S \rightarrow E$ 5) $B \rightarrow b\ E$ 8) $C \rightarrow d\ S\ D$ 11) $E \rightarrow f\ A$

3) $A \rightarrow d\ D\ A$ 6) $B \rightarrow f$ 9) $C \rightarrow a$ 12) $E \rightarrow g$

ReachableSymbolsList = $\{\ S\ \}$

Algorithm for computing the set of *reachable* symbols

1) $S \rightarrow a A B$

2) $S \rightarrow E$

3) $A \rightarrow d D A$

4) $A \rightarrow e$

5) $B \rightarrow b E$

6) $B \rightarrow f$

7) $C \rightarrow c A B$

8) $C \rightarrow d S D$

9) $C \rightarrow a$

10) $D \rightarrow e A$

11) $E \rightarrow f A$

12) $E \rightarrow g$

$$ReachableSymbolsList = \{ S, a, A, B, E \}$$

Algorithm for computing the set of *reachable* symbols

1) $S \rightarrow a\,A\,B$

2) $S \rightarrow E$

3) $A \rightarrow d\,D\,A$

4) $A \rightarrow e$

5) $B \rightarrow b\,E$

6) $B \rightarrow f$

7) $C \rightarrow c\,A\,B$

8) $C \rightarrow d\,S\,D$

9) $C \rightarrow a$

10) $D \rightarrow e\,A$

11) $E \rightarrow f\,A$

12) $E \rightarrow g$

ReachableSymbolsList = $\{\ S,\ a,\ A,\ B,\ E\ \}$

Algorithm for computing the set of *reachable* symbols

1) $S \rightarrow a\,A\,B$ 4) $A \rightarrow e$ 7) $C \rightarrow c\,A\,B$ 10) $D \rightarrow e\,A$

2) $S \rightarrow E$ 5) $B \rightarrow b\,E$ 8) $C \rightarrow d\,S\,D$ 11) $E \rightarrow f\,A$

3) $A \rightarrow d\,D\,A$ 6) $B \rightarrow f$ 9) $C \rightarrow a$ 12) $E \rightarrow g$

ReachableSymbolsList = { S, a, A, B, E, D, b, d, e, f, g }

Algorithm for computing the set of *reachable* symbols

1) $S \rightarrow a\,A\,B$ 4) $A \rightarrow e$ 7) $C \rightarrow c\,A\,B$ 10) $D \rightarrow e\,A$

2) $S \rightarrow E$ 5) $B \rightarrow b\,E$ 8) $C \rightarrow d\,S\,D$ 11) $E \rightarrow f\,A$

3) $A \rightarrow d\,D\,A$ 6) $B \rightarrow f$ 9) $C \rightarrow a$ 12) $E \rightarrow g$

ReachableSymbolsList = $\{\ S,\ a,\ A,\ B,\ E,\ D,\ b,\ d,\ e,\ f,\ g\ \}$

Algorithm for computing the set of *reachable* symbols

1) $S \rightarrow a A B$ 4) $A \rightarrow e$ 7) $C \rightarrow c A B$ 10) $D \rightarrow e A$

2) $S \rightarrow E$ 5) $B \rightarrow b E$ 8) $C \rightarrow d S D$ 11) $E \rightarrow f A$

3) $A \rightarrow d D A$ 6) $B \rightarrow f$ 9) $C \rightarrow a$ 12) $E \rightarrow g$

ReachableSymbolsList = { S, a, A, B, E, D, b, d, e, f, g }

1) $S \rightarrow a A B$ 4) $A \rightarrow e$ 7) $C \rightarrow c A B$ 10) $D \rightarrow e A$

2) $S \rightarrow E$ 5) $B \rightarrow b E$ 8) $C \rightarrow d S D$ 11) $E \rightarrow f A$

3) $A \rightarrow d D A$ 6) $B \rightarrow f$ 9) $C \rightarrow a$ 12) $E \rightarrow g$

Algorithm for computing the set of *reachable* symbols

1) $S \rightarrow a\,A\,B$ 4) $A \rightarrow e$ 7) $C \rightarrow c\,A\,B$ 10) $D \rightarrow e\,A$

2) $S \rightarrow E$ 5) $B \rightarrow b\,E$ 8) $C \rightarrow d\,S\,D$ 11) $E \rightarrow f\,A$

3) $A \rightarrow d\,D\,A$ 6) $B \rightarrow f$ 9) $C \rightarrow a$ 12) $E \rightarrow g$

ReachableSymbolsList $= \{\, S,\, a,\, A,\, B,\, E,\, D,\, b,\, d,\, e,\, f,\, g \,\}$

1) $S \rightarrow a\,A\,B$ 4) $A \rightarrow e$ 10) $D \rightarrow e\,A$

2) $S \rightarrow E$ 5) $B \rightarrow b\,E$ 11) $E \rightarrow f\,A$

3) $A \rightarrow d\,D\,A$ 6) $B \rightarrow f$ 12) $E \rightarrow g$

Eliminating *useless* symbols

Eliminating *useless* symbols

Eliminating *nongenerating* symbols

Eliminating *useless* symbols

Eliminating *nongenerating* symbols

Eliminating *nonreachable* symbols

Eliminating *useless* symbols

Eliminating *useless* symbols

$S \rightarrow A\,B \mid a$

$A \rightarrow a$

Eliminating *useless* symbols

$S \rightarrow A\ B\ |\ a$

$A \rightarrow a$

Eliminating nongenerating symbols

Eliminating *useless* symbols

$S \rightarrow A\ B \mid a$

$A \rightarrow a$

Eliminating nongenerating symbols

Eliminating *useless* symbols

$S \rightarrow A\,B \mid a$

$A \rightarrow a$

Eliminating nongenerating symbols

Eliminating *useless* symbols

$S \rightarrow A\ B\ |\ a$

$A \rightarrow a$

Eliminating nongenerating symbols

$S \rightarrow a$

$A \rightarrow a$

Eliminating *useless* symbols

$$S \rightarrow A\,B \mid a$$

$$A \rightarrow a$$

Eliminating nongenerating symbols

$$S \rightarrow a$$

$$A \rightarrow a$$

Eliminating nonreachable symbols

Eliminating *useless* symbols

$$S \rightarrow A\ B \mid a$$

$$A \rightarrow a$$

Eliminating nongenerating symbols

$$S \rightarrow a$$

$$A \rightarrow a$$

Eliminating nonreachable symbols

Eliminating *useless* symbols

$S \rightarrow A\ B\ |\ a$

$A \rightarrow a$

Eliminating nongenerating symbols

$S \rightarrow a$

$A \rightarrow a$

Eliminating nonreachable symbols

$S \rightarrow a$

Eliminating *useless* symbols

$S \rightarrow A\,B \mid a$

$A \rightarrow a$

Eliminating nongenerating symbols

$S \rightarrow a$

$A \rightarrow a$

Eliminating nonreachable symbols

$S \rightarrow a$

$S \rightarrow A\,B \mid a$

$A \rightarrow a$

Eliminating *useless* symbols

$S \rightarrow A\ B\ |\ a$

$A \rightarrow a$

Eliminating nongenerating symbols

$S \rightarrow a$

$A \rightarrow a$

Eliminating nonreachable symbols

$S \rightarrow a$

$S \rightarrow A\ B\ |\ a$

$A \rightarrow a$

Eliminating nonreachable symbols

Eliminating *useless* symbols

$S \rightarrow A\,B \mid a$

$A \rightarrow a$

Eliminating nongenerating symbols

$S \rightarrow a$

$A \rightarrow a$

Eliminating nonreachable symbols

$S \rightarrow a$

$S \rightarrow A\,B \mid a$

$A \rightarrow a$

Eliminating nonreachable symbols

$S \rightarrow A\,B \mid a$

$A \rightarrow a$

Eliminating *useless* symbols

$S \rightarrow A\ B\ |\ a$

$A \rightarrow a$

Eliminating nongenerating symbols

$S \rightarrow a$

$A \rightarrow a$

Eliminating nonreachable symbols

$S \rightarrow a$

$S \rightarrow A\ B\ |\ a$

$A \rightarrow a$

Eliminating nonreachable symbols

$S \rightarrow A\ B\ |\ a$

$A \rightarrow a$

Eliminating nongenerating symbols

Eliminating *useless* symbols

$$S \rightarrow A\,B \mid a$$
$$A \rightarrow a$$

Eliminating nongenerating symbols

$$S \rightarrow a$$
$$A \rightarrow a$$

Eliminating nonreachable symbols

$$S \rightarrow a$$

$$S \rightarrow A\,B \mid a$$
$$A \rightarrow a$$

Eliminating nonreachable symbols

$$S \rightarrow A\,B \mid a$$
$$A \rightarrow a$$

Eliminating nongenerating symbols

$$S \rightarrow a$$
$$A \rightarrow a$$

Eliminating *useless* symbols

Eliminating *useless* symbols

Eliminating nongenerating symbols $\quad\Rightarrow\ G_1$

Eliminating *useless* symbols

Eliminating nongenerating symbols $\Rightarrow$ G_1
Eliminating nonreachable symbols $\Rightarrow$ G_2

Eliminating *useless* symbols

Eliminating nongenerating symbols $\Rightarrow$ G_1
Eliminating nonreachable symbols $\Rightarrow$ G_2

Grammar G_2

Eliminating *useless* symbols

Eliminating nongenerating symbols $\Rightarrow$ G_1
Eliminating nonreachable symbols $\Rightarrow$ G_2

Grammar G_2

Does not contain nonreachable symbols.
Thus, for any symbol X in G_2 stands:

Eliminating *useless* symbols

Eliminating nongenerating symbols $\Rightarrow$ G_1
Eliminating nonreachable symbols $\Rightarrow$ G_2

Grammar G_2

      Does not contain nonreachable symbols.
      Thus, for any symbol X in G_2 stands:

$$S \overset{*}{\underset{G_2}{\Rightarrow}} \alpha\, X\, \beta$$

Eliminating *useless* symbols

Eliminating nongenerating symbols $\Rightarrow$ G_1
Eliminating nonreachable symbols $\Rightarrow$ G_2

Grammar G_2

       Does not contain nonreachable symbols.
       Thus, for any symbol X in G_2 stands:

$$S \overset{*}{\underset{G_2}{\Rightarrow}} \alpha\, X\, \beta$$

All symbols in grammar G_2 are also symbols in grammar G_1

Eliminating *useless* symbols

Eliminating nongenerating symbols $\Rightarrow G_1$
Eliminating nonreachable symbols $\Rightarrow G_2$

Grammar G_2

      Does not contain nonreachable symbols.
      Thus, for any symbol X in G_2 stands:

$$S \underset{G_2}{\overset{*}{\Rightarrow}} \alpha \, X \, \beta$$

All symbols in grammar G_2 are also symbols in grammar G_1
Grammar G_1 does not contain nongenerating symbols.

Eliminating *useless* symbols

Eliminating nongenerating symbols $\Rightarrow$ **G_1**
Eliminating nonreachable symbols $\Rightarrow$ **G_2**

Grammar G_2

> **Does not contain nonreachable symbols.**
> **Thus, for any symbol X in G_2 stands:**

$$S \overset{*}{\underset{G_2}{\Rightarrow}} \alpha\, X\, \beta$$

All symbols in grammar G_2 are also symbols in grammar G_1
Grammar G_1 does not contain nongenerating symbols.

Hence, every symbol in string derivative $\alpha\, X\, \beta$ is generating:

Eliminating *useless* symbols

Eliminating nongenerating symbols $\Rightarrow G_1$
Eliminating nonreachable symbols $\Rightarrow G_2$

Grammar G_2

      Does not contain nonreachable symbols.
      Thus, for any symbol X in G_2 stands:

$$S \overset{*}{\underset{G_2}{\Rightarrow}} \alpha\, X\, \beta$$

All symbols in grammar G_2 are also symbols in grammar G_1
Grammar G_1 does not contain nongenerating symbols.

Hence, every symbol in string derivative $\alpha\, X\, \beta$ is generating:

$$\alpha X \beta \overset{*}{\underset{G_2}{\Rightarrow}} w$$

Eliminating *useless* symbols

Eliminating nongenerating symbols $\Rightarrow G_1$
Eliminating nonreachable symbols $\Rightarrow G_2$

Grammar G_2

Does not contain nonreachable symbols.
Thus, for any symbol X in G_2 stands:

$$S \overset{*}{\underset{G_2}{\Rightarrow}} \alpha\, X\, \beta$$

All symbols in grammar G_2 are also symbols in grammar G_1
Grammar G_1 does not contain nongenerating symbols.

Hence, every symbol in string derivative $\alpha\, X\, \beta$ is generating:

$$\alpha X \beta \overset{*}{\underset{G_2}{\Rightarrow}} w$$

$$S \overset{*}{\underset{G_2}{\Rightarrow}} \alpha X \beta \overset{*}{\underset{G_2}{\Rightarrow}} w$$

Eliminating *useless* symbols

Eliminating *useless* symbols

1) $S \rightarrow a\ c$ 4) $B \rightarrow a\ S\ A$

2) $S \rightarrow b\ A$ 5) $C \rightarrow b\ C$

3) $A \rightarrow c\ B\ C$ 6) $C \rightarrow d$

Eliminating *useless* symbols

1) $S \rightarrow a\,c$ 4) $B \rightarrow a\,S\,A$
2) $S \rightarrow b\,A$ 5) $C \rightarrow b\,C$
3) $A \rightarrow c\,B\,C$ 6) $C \rightarrow d$

Eliminating nongenerating symbols

1) $S \rightarrow a\,c$ 4) $B \rightarrow a\,S\,A$

2) $S \rightarrow b\,A$ 5) $C \rightarrow b\,C$

3) $A \rightarrow c\,B\,C$ 6) $C \rightarrow d$

Eliminating nongenerating symbols

Eliminating *useless* symbols

1) $S \rightarrow a\,c$ 4) $B \rightarrow a\,S\,A$

2) $S \rightarrow b\,A$ 5) $C \rightarrow b\,C$

3) $A \rightarrow c\,B\,C$ 6) $C \rightarrow d$

Eliminating nongenerating symbols

1) $S \rightarrow a\,c$ 5) $C \rightarrow b\,C$

                   6) $C \rightarrow d$

Eliminating *useless* symbols

1) $S \rightarrow a\,c$ 4) $B \rightarrow a\,S\,A$

2) $S \rightarrow b\,A$ 5) $C \rightarrow b\,C$

3) $A \rightarrow c\,B\,C$ 6) $C \rightarrow d$

Eliminating nongenerating symbols

1) $S \rightarrow a\,c$

5) $C \rightarrow b\,C$

6) $C \rightarrow d$

Eliminating nonreachable symbols

Eliminating *useless* symbols

1) $S \rightarrow a\,c$ 4) $B \rightarrow a\,S\,A$

2) $S \rightarrow b\,A$ 5) $C \rightarrow b\,C$

3) $A \rightarrow c\,B\,C$ 6) $C \rightarrow d$

Eliminating nongenerating symbols

1) $\boxed{S \rightarrow a\,c}$

5) $C \rightarrow b\,C$

6) $C \rightarrow d$

Eliminating nonreachable symbols

Eliminating *useless* symbols

1) $S \rightarrow a\,c$
2) $S \rightarrow b\,A$
3) $A \rightarrow c\,B\,C$

4) $B \rightarrow a\,S\,A$
5) $C \rightarrow b\,C$
6) $C \rightarrow d$

Eliminating nongenerating symbols

1) $S \rightarrow a\,c$

5) $C \rightarrow b\,C$
6) $C \rightarrow d$

Eliminating nonreachable symbols

1) $S \rightarrow a\,c$

Eliminating ε-Productions

Eliminating ε-Productions

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A \rightarrow c\,S$
                                              4) $A \rightarrow \varepsilon$

Eliminating ε-Productions

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A \rightarrow c\,S$
                                                           4) $A \rightarrow \varepsilon$

Eliminating ε-Productions

1) $S \rightarrow a\,A\,S\,A$ **2)** $S \rightarrow b$ **3)** $A \rightarrow c\,S$

4) $A \rightarrow \varepsilon$

1) $S \rightarrow a\,A\,S\,A$ **2)** $S \rightarrow b$

Copyright © 2022 S. Srbljić et al.: Introduction to Theoretical Computer Science

Eliminating ε-Productions

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A \rightarrow c\,S$
                                                            4) $A \rightarrow \varepsilon$

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A_{NO} \rightarrow c\,S$

Eliminating ε-Productions

1) $S \rightarrow a A S A$ 2) $S \rightarrow b$ 3) $A \rightarrow c S$
4) $A \rightarrow \varepsilon$

1) $S \rightarrow a A S A$ 2) $S \rightarrow b$ 3) $A_{NO} \rightarrow c S$
4) $A_{YES} \rightarrow \varepsilon$

Eliminating ε-Productions

1) $S \to a\,A\,S\,A$ 2) $S \to b$ 3) $A \to c\,S$
                                          4) $A \to \varepsilon$

1) $S \to a\,A\,S\,A$ 2) $S \to b$ 3) $A_{NO} \to c\,S$
                                          4) $A_{YES} \to \varepsilon$

3) $A_{NO} \to c\,S$
4) $A_{YES} \to \varepsilon$

Eliminating ε-Productions

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A \rightarrow c\,S$
4) $A \rightarrow \varepsilon$

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A_{NO} \rightarrow c\,S$
4) $A_{YES} \rightarrow \varepsilon$

1a) $S \rightarrow a\,A_{NO}\,S\,A_{NO}$

3) $A_{NO} \rightarrow c\,S$
4) $A_{YES} \rightarrow \varepsilon$

Eliminating ε-Productions

1) $S \to a\,A\,S\,A$ 2) $S \to b$ 3) $A \to c\,S$
4) $A \to \varepsilon$

1) $S \to a\,A\,S\,A$ 2) $S \to b$ 3) $A_{NO} \to c\,S$
4) $A_{YES} \to \varepsilon$

1a) $S \to a\,A_{NO}\,S\,A_{NO}$
1b) $S \to a\,A_{NO}\,S\,A_{YES}$ 3) $A_{NO} \to c\,S$
4) $A_{YES} \to \varepsilon$

Eliminating ε-Productions

1) $S \to a\,A\,S\,A$ 2) $S \to b$ 3) $A \to c\,S$
4) $A \to \varepsilon$

1) $S \to a\,A\,S\,A$ 2) $S \to b$ 3) $A_{NO} \to c\,S$
4) $A_{YES} \to \varepsilon$

1a) $S \to a\,A_{NO}\,S\,A_{NO}$
1b) $S \to a\,A_{NO}\,S\,A_{YES}$ 3) $A_{NO} \to c\,S$
1c) $S \to a\,A_{YES}\,SA_{NO}$ 4) $A_{YES} \to \varepsilon$

Eliminating ε-Productions

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A \rightarrow c\,S$
4) $A \rightarrow \varepsilon$

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A_{NO} \rightarrow c\,S$
4) $A_{YES} \rightarrow \varepsilon$

1a) $S \rightarrow a\,A_{NO}\,S\,A_{NO}$
1b) $S \rightarrow a\,A_{NO}\,S\,A_{YES}$ 3) $A_{NO} \rightarrow c\,S$
1c) $S \rightarrow a\,A_{YES}\,SA_{NO}$ 4) $A_{YES} \rightarrow \varepsilon$
1d) $S \rightarrow a\,A_{YES}\,SA_{YES}$

Eliminating ε-Productions

1) $S \to a A S A$ 2) $S \to b$ 3) $A \to c S$

4) $A \to \varepsilon$

1) $S \to a A S A$ 2) $S \to b$ 3) $A_{NO} \to c S$

4) $A_{YES} \to \varepsilon$

1a) $S \to a A_{NO} S A_{NO}$

1b) $S \to a A_{NO} S A_{YES}$

1c) $S \to a A_{YES} S A_{NO}$ 2) $S \to b$ 3) $A_{NO} \to c S$

1d) $S \to a A_{YES} S A_{YES}$ 4) $A_{YES} \to \varepsilon$

Eliminating ε-Productions

1) $S \to a\,A\,S\,A$ **2)** $S \to b$

3) $A \to c\,S$
4) $A \to \varepsilon$

1) $S \to a\,A\,S\,A$ **2)** $S \to b$

3) $A_{NO} \to c\,S$
4) $A_{YES} \to \varepsilon$

1a) $S \to a\,A_{NO}\,S\,A_{NO}$
1b) $S \to a\,A_{NO}\,S\,A_{YES}$
1c) $S \to a\,A_{YES}\,SA_{NO}$ **2)** $S \to b$
1d) $S \to a\,A_{YES}\,SA_{YES}$

3) $A_{NO} \to c\,S$
4) $A_{YES} \to \varepsilon$

Eliminating ε-Productions

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A \rightarrow c\,S$
                                                              4) $A \rightarrow \varepsilon$

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A_{NO} \rightarrow c\,S$
                                                              4) $A_{YES} \rightarrow \varepsilon$

1a) $S \rightarrow a\,A_{NO}\,S\,A_{NO}$
1b) $S \rightarrow a\,A_{NO}\,S\,A_{YES}$ 2) $S \rightarrow b$ 3) $A_{NO} \rightarrow c\,S$
1c) $S \rightarrow a\,A_{YES}\,SA_{NO}$ 4) $A_{YES} \rightarrow \varepsilon$
1d) $S \rightarrow a\,A_{YES}\,SA_{YES}$

Eliminating ε-Productions

1) $S \to a\,A\,S\,A$ 2) $S \to b$

3) $A \to c\,S$
4) $A \to \varepsilon$

1) $S \to a\,A\,S\,A$ 2) $S \to b$

3) $A_{NO} \to c\,S$
4) $A_{YES} \to \varepsilon$

1a) $S \to a\,A_{NO}\,S\,A_{NO}$
1b) $S \to a\,A_{NO}\,S\,A_{YES}$
1c) $S \to a\,A_{YES}\,SA_{NO}$
1d) $S \to a\,A_{YES}\,SA_{YES}$

2) $S \to b$

3) $A_{NO} \to c\,S$
4) $A_{YES} \to \varepsilon$

Eliminating ε-Productions

1) $S \to a\,A\,S\,A$ 2) $S \to b$ 3) $A \to c\,S$
                                                           4) $A \to \varepsilon$

1) $S \to a\,A\,S\,A$ 2) $S \to b$ 3) $A_{NO} \to c\,S$
                                                            4) $A_{YES} \to \varepsilon$

1*a*) $S \to a\,A_{NO}\,S\,A_{NO}$

1*b*) $S \to a\,A_{NO}\,S\,A_{YES}$

1*c*) $S \to a\,A_{YES}\,SA_{NO}$ 2) $S \to b$ 3) $A_{NO} \to c\,S$

1*d*) $S \to a\,A_{YES}\,SA_{YES}$ 4) $A_{YES} \to \varepsilon$

Eliminating ε-Productions

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A \rightarrow c\,S$
4) $A \rightarrow \varepsilon$

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A_{NO} \rightarrow c\,S$
4) $A_{YES} \rightarrow \varepsilon$

1a) $S \rightarrow a\,A_{NO}\,S\,A_{NO}$
1b) $S \rightarrow a\,A_{NO}\,S\,A_{YES}$ 2) $S \rightarrow b$ 3) $A_{NO} \rightarrow c\,S$
1c) $S \rightarrow a\,A_{YES}\,S A_{NO}$ 4) $A_{YES} \rightarrow \varepsilon$
1d) $S \rightarrow a\,A_{YES}\,S\,A_{YES}$

Eliminating ε-Productions

1) $S \rightarrow a\,A\,S\,A$　　2) $S \rightarrow b$　　3) $A \rightarrow c\,S$
　　　　　　　　　　　　　　　　　　　　4) $A \rightarrow \varepsilon$

1) $S \rightarrow a\,A\,S\,A$　　2) $S \rightarrow b$　　3) $A_{NO} \rightarrow c\,S$
　　　　　　　　　　　　　　　　　　　　4) $A_{YES} \rightarrow \varepsilon$

1a) $S \rightarrow a\,A_{NO}\,S\,A_{NO}$
1b) $S \rightarrow a\,A_{NO}\,S\,A_{YES}$　　2) $S \rightarrow b$　　3) $A_{NO} \rightarrow c\,S$
1c) $S \rightarrow a\,A_{YES}\,S\,A_{NO}$　　　　　　　　　　　　4) $A_{YES} \rightarrow \varepsilon$
1d) $S \rightarrow a\,A_{YES}\,S\,A_{YES}$

Eliminating ε-Productions

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A \rightarrow c\,S$
4) $A \rightarrow \varepsilon$

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A_{NO} \rightarrow c\,S$
4) $A_{YES} \rightarrow \varepsilon$

1a) $S \rightarrow a\,A_{NO}\,S\,A_{NO}$
1b) $S \rightarrow a\,A_{NO}\,S\,A_{YES}$
1c) $S \rightarrow a\,A_{YES}\,S\,A_{NO}$ 2) $S \rightarrow b$ 3) $A_{NO} \rightarrow c\,S$
1d) $S \rightarrow a\,A_{YES}\,S\,A_{YES}$

Eliminating ε-Productions

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A \rightarrow c\,S$
                                                        4) $A \rightarrow \varepsilon$

1) $S \rightarrow a\,A\,S\,A$ 2) $S \rightarrow b$ 3) $A_{NO} \rightarrow c\,S$
                                                        4) $A_{YES} \rightarrow \varepsilon$

1a) $S \rightarrow a\,A\quad S\,A$
1b) $S \rightarrow a\,A\quad S$
1c) $S \rightarrow a\quad S\,A$ 2) $S \rightarrow b$ 3) $A \rightarrow c\,S$
1d) $S \rightarrow a\quad S$

Eliminating ε-Productions

Eliminating ε-Productions

1) **Nullable symbols list is incrementally built by discovering which nonterminal symbols are *nullable*:**

Eliminating ε-Productions

1) Nullable symbols list is incrementally built by discovering which nonterminal symbols are *nullable*:

$$A \overset{*}{\Longrightarrow} \varepsilon$$

Eliminating ε-Productions

1) **Nullable symbols list is incrementally built by discovering which nonterminal symbols are *nullable*:**

$$A \overset{*}{\Rightarrow} \varepsilon$$

a) **Nullable symbols list is initialized with symbols that are on the left side of the ε-productions**

Eliminating ε-Productions

1) Nullable symbols list is incrementally built by discovering which nonterminal symbols are *nullable*:

$$A \overset{*}{\Longrightarrow} \varepsilon$$

a) Nullable symbols list is initialized with symbols that are on the left side of the ε-productions

U listu praznih znakova stave se lijeve strane svih ε-produkcija

Eliminating ε-Productions

1) **Nullable symbols list is incrementally built by discovering which nonterminal symbols are *nullable*:**

$$A \overset{*}{\Rightarrow} \varepsilon$$

a) **Nullable symbols list is initialized with symbols that are on the left side of the ε-productions**

U listu praznih znakova stave se lijeve strane svih ε-produkcija

b) **If there is a production $B \to C_1 C_2 \ldots C_k$, where each C_i is nullable, then B is nullable. Note each C_i is a nonterminal symbol. Each such B is added to the nullable list.**

Eliminating ε-Productions

1) Nullable symbols list is incrementally built by discovering which nonterminal symbols are *nullable*:

$$A \overset{*}{\Rightarrow} \varepsilon$$

a) Nullable symbols list is initialized with symbols that are on the left side of the ε-productions

U listu praznih znakova stave se lijeve strane svih ε-produkcija

b) If there is a production $B \to C_1 C_2 \ldots C_k$, where each C_i is nullable, then B is nullable. Note each C_i is a nonterminal symbol. Each such B is added to the nullable list.

c) The algorithm continues until there are no symbols that can be added to the nullable list.

Eliminating ε-Productions

1) Discovering all nullable nonterminal symbols:

$$A \overset{*}{\Longrightarrow} \varepsilon$$

Eliminating ε-Productions

1) Discovering all nullable nonterminal symbols:

$$A \overset{*}{\Longrightarrow} \varepsilon$$

2) Grammar production:

Eliminating ε-Productions

1) **Discovering all nullable nonterminal symbols:**

$$A \overset{*}{\Rightarrow} \varepsilon$$

2) **Grammar production:**

$$A \rightarrow X_1\, X_2\, \dots\, X_n$$

Eliminating ε-Productions

1) Discovering all nullable nonterminal symbols:

$$A \overset{*}{\Longrightarrow} \varepsilon$$

2) Grammar production:

$$A \to X_1 \ X_2 \ ... \ X_n$$

is replaced by production that have the following form:

Eliminating ε-Productions

1) Discovering all nullable nonterminal symbols:

$$A \overset{*}{\Rightarrow} \varepsilon$$

2) Grammar production:

$$A \to X_1 \, X_2 \, \dots \, X_n$$

is replaced by production that have the following form:

$$A \to \xi_1 \, \xi_2 \, \dots \, \xi_n$$

Eliminating ε-Productions

1) **Discovering all nullable nonterminal symbols:**

$$A \overset{*}{\Longrightarrow} \varepsilon$$

2) **Grammar production:**

$$A \rightarrow X_1 X_2 \ldots X_n$$

is replaced by production that have the following form:

$$A \rightarrow \xi_1 \xi_2 \ldots \xi_n$$

where symbol ξ_i becomes the following:

Eliminating ε-Productions

1) **Discovering all nullable nonterminal symbols:**

$$A \overset{*}{\Longrightarrow} \varepsilon$$

2) **Grammar production:**

$$A \to X_1 \, X_2 \, \ldots \, X_n$$

is replaced by production that have the following form:

$$A \to \xi_1 \, \xi_2 \, \ldots \, \xi_n$$

where symbol ξ_i becomes the following:

If X_i is not a nullable nonterminal symbol,

Eliminating ε-Productions

1) **Discovering all nullable nonterminal symbols:**

$$A \overset{*}{\Rightarrow} \varepsilon$$

2) **Grammar production:**

$$A \rightarrow X_1 \, X_2 \, \dots \, X_n$$

is replaced by production that have the following form:

$$A \rightarrow \xi_1 \, \xi_2 \, \dots \, \xi_n$$

where symbol ξ_i becomes the following:

If X_i is not a nullable nonterminal symbol,
then symbol ξ_i is equal X_i

Eliminating ε-Productions

1) **Discovering all nullable nonterminal symbols:**

$$A \overset{*}{\Longrightarrow} \varepsilon$$

2) **Grammar production:**

$$A \to X_1 \, X_2 \, ... \, X_n$$

is replaced by production that have the following form:

$$A \to \xi_1 \, \xi_2 \, ... \, \xi_n$$

where symbol ξ_i becomes the following:

**If X_i is not a nullable nonterminal symbol,
then symbol ξ_i is equal X_i**

If X_i is a nullable nonterminal symbol,

Copyright © 2022 S. Srbljić et al.: Introduction to Theoretical Computer Science

Eliminating ε-Productions

1) **Discovering all nullable nonterminal symbols:**

$$A \overset{*}{\Rightarrow} \varepsilon$$

2) **Grammar production:**

$$A \rightarrow X_1 \, X_2 \, \dots \, X_n$$

is replaced by production that have the following form:

$$A \rightarrow \xi_1 \, \xi_2 \, \dots \, \xi_n$$

where symbol ξ_i becomes the following:

**If X_i is not a nullable nonterminal symbol,
then symbol ξ_i is equal X_i**

**If X_i is a nullable nonterminal symbol,
then symbol ξ_i can ε or X_i**

Eliminating *unit* productions

Eliminating *unit* productions

$$A \rightarrow B$$

Eliminating *unit* productions

$$A \rightarrow B$$

1) Add all nonunit productions to the new production set P_1

Eliminating *unit* productions

$$A \rightarrow B$$

1) Add all nonunit productions to the new production set P_1

2) For each pair *(A, B)* such that

Eliminating *unit* productions

$$A \rightarrow B$$

1) Add all nonunit productions to the new production set P_1

2) For each pair *(A, B)* such that $A \overset{*}{\Rightarrow} B$

Eliminating *unit* productions

$$A \rightarrow B$$

1) Add all nonunit productions to the new production set P_1

2) For each pair *(A, B)* such that $A \overset{*}{\Rightarrow} B$

       Add to P_1 all the productions $A \rightarrow \alpha$, where $B \rightarrow \alpha$ is a nonunit production in P

Chomsky Normal Form

Noam Chomsky, 2015
Izvor: *https://en.wikipedia.org/wiki/Noam_Chomsky*

Chomsky Normal Form

- **Chomsky Normal Form**

Noam Chomsky, 2015
Izvor: *https://en.wikipedia.org/wiki/Noam_Chomsky*

Chomsky Normal Form

- **Chomsky Normal Form**
    - All production are in one of two simple forms, either:

Noam Chomsky, 2015
Izvor: *https://en.wikipedia.org/wiki/Noam_Chomsky*

Chomsky Normal Form

- **Chomsky Normal Form**
    - All production are in one of two simple forms, either:

    1. $A \rightarrow BC$

Noam Chomsky, 2015
Izvor: *https://en.wikipedia.org/wiki/Noam_Chomsky*

Chomsky Normal Form

- **Chomsky Normal Form**
  - All production are in one of two simple forms, either:

    1. $A \rightarrow BC$
    2. $A \rightarrow a$

Noam Chomsky, 2015
Izvor: *https://en.wikipedia.org/wiki/Noam_Chomsky*

Chomsky Normal Form

Chomsky Normal Form

1) **Leave the productions that already satisfy the Chomsky normal form in the production set *P*:**

Chomsky Normal Form

1) **Leave the productions that already satisfy the Chomsky normal form in the production set *P*:**

$$A \rightarrow BC$$

Chomsky Normal Form

1) **Leave the productions that already satisfy the Chomsky normal form in the production set *P:***

$$A \rightarrow BC$$
$$A \rightarrow a$$

Chomsky Normal Form

1) **Leave the productions that already satisfy the Chomsky normal form in the production set _P:_**

$$A \rightarrow BC$$
$$A \rightarrow a$$

2) **Let there is a production in the following form:**

Chomsky Normal Form

1) **Leave the productions that already satisfy the Chomsky normal form in the production set *P*:**

$$A \rightarrow BC$$
$$A \rightarrow a$$

2) **Let there is a production in the following form:**

$$A \rightarrow X_1 \, X_2 \, \text{- - -} \quad X_i \quad \text{- - -} \, X_m$$

Chomsky Normal Form

1) Leave the productions that already satisfy the Chomsky normal form in the production set *P*:

$$A \rightarrow BC$$
$$A \rightarrow a$$

2) Let there is a production in the following form:

$$A \rightarrow X_1 \, X_2 \; \text{-\,-\,-} \quad X_i \quad \text{-\,-\,-} \; X_m$$

If X_i is a terminal symbol *a*,

Chomsky Normal Form

1) **Leave the productions that already satisfy the Chomsky normal form in the production set *P*:**

$$A \rightarrow BC$$
$$A \rightarrow a$$

2) **Let there is a production in the following form:**

$$A \rightarrow X_1 \, X_2 \text{ - - - } X_i \text{ - - - } X_m$$

If X_i is a terminal symbol *a*,

Chomsky Normal Form

1) **Leave the productions that already satisfy the Chomsky normal form in the production set *P*:**

$$A \rightarrow BC$$
$$A \rightarrow a$$

2) **Let there is a production in the following form:**

$$A \rightarrow X_1 \ X_2 \ \text{---} \quad a \quad \text{---} \ X_m$$

If X_i is a terminal symbol *a*,

Chomsky Normal Form

1) **Leave the productions that already satisfy the Chomsky normal form in the production set *P*:**

$$A \rightarrow BC$$
$$A \rightarrow a$$

2) **Let there is a production in the following form:**

$$A \rightarrow X_1 \ X_2 \ \text{- - -} \quad a \quad \text{- - -} \ X_m$$

If X_i is a terminal symbol *a*,
The following production is added to P:

Chomsky Normal Form

1) **Leave the productions that already satisfy the Chomsky normal form in the production set *P*:**

$$A \rightarrow BC$$
$$A \rightarrow a$$

2) **Let there is a production in the following form:**

$$A \rightarrow X_1 \, X_2 \text{ - - - } a \text{ - - - } X_m$$

If X_i is a terminal symbol *a*,
The following production is added to P:

$$C_a \rightarrow a$$

Chomsky Normal Form

1) **Leave the productions that already satisfy the Chomsky normal form in the production set *P*:**

$$A \rightarrow BC$$
$$A \rightarrow a$$

2) **Let there is a production in the following form:**

$$A \rightarrow X_1 \ X_2 \ \text{- - -} \quad a \quad \text{- - -} \ X_m$$

If X_i is a terminal symbol *a*,
The following production is added to P:

$$C_a \rightarrow a$$

All terminal symbols *a* that appear in any production
$A \rightarrow X_1 \ X_2 \ \text{- - -} \ X_m$ are replaced by nonterminal symbol C_a

Chomsky Normal Form

1) **Leave the productions that already satisfy the Chomsky normal form in the production set *P:***

$$A \rightarrow BC$$
$$A \rightarrow a$$

2) **Let there is a production in the following form:**

$$A \rightarrow X_1 \, X_2 \text{ --- } C_a \text{ --- } X_m$$

If X_i is a terminal symbol a,

The following production is added to P:

$$C_a \rightarrow a$$

All terminal symbols a that appear in any production $A \rightarrow X_1 \, X_2 \text{ --- } X_m$ are replaced by nonterminal symbol C_a

Chomsky Normal Form

Chomsky Normal Form

3) **Productions that have 3 or more symbols on the right side:**

Chomsky Normal Form

3) **Productions that have 3 or more symbols on the right side:**

$$A \rightarrow B_1 \ B_2 \ B_3 \ \text{---} \ B_{m-2} \ B_{m-1} \ B_m$$

Chomsky Normal Form

3) Productions that have 3 or more symbols on the right side:

$$A \rightarrow B_1\ B_2\ B_3\ \text{---}\ B_{m-2}\ B_{m-1}\ B_m$$

are replaced by the following productions:

Chomsky Normal Form

3) Productions that have 3 or more symbols on the right side:

$$A \rightarrow B_1 \, B_2 \, B_3 \; \text{---} \; B_{m-2} \, B_{m-1} \, B_m$$

are replaced by the following productions:

$$A \rightarrow B_1 \, D_1$$

Chomsky Normal Form

3) **Productions that have 3 or more symbols on the right side:**

$$A \rightarrow B_1 \; B_2 \; B_3 \; \text{---} \; B_{m-2} \; B_{m-1} \; B_m$$

are replaced by the following productions:

$$A \rightarrow B_1 \, D_1$$
$$D_1 \rightarrow B_2 \, D_2$$

Chomsky Normal Form

3) **Productions that have 3 or more symbols on the right side:**

$$A \to B_1\ B_2\ B_3\ \text{---}\ B_{m-2}\ B_{m-1}\ B_m$$

are replaced by the following productions:

$$A \to B_1\ D_1$$
$$D_1 \to B_2\ D_2$$
$$D_2 \to B_3\ D_3$$

Chomsky Normal Form

3) Productions that have 3 or more symbols on the right side:

$$A \rightarrow B_1\ B_2\ B_3\ \text{---}\ B_{m-2}\ B_{m-1}\ B_m$$

are replaced by the following productions:

$$
\begin{aligned}
A &\rightarrow B_1\ D_1 \\
D_1 &\rightarrow B_2\ D_2 \\
D_2 &\rightarrow B_3\ D_3 \\
&\text{- - -}
\end{aligned}
$$

Chomsky Normal Form

3) **Productions that have 3 or more symbols on the right side:**

$$A \rightarrow B_1 \; B_2 \; B_3 \; \text{---} \; B_{m-2} \; B_{m-1} \; B_m$$

are replaced by the following productions:

$$A \rightarrow B_1 \, D_1$$
$$D_1 \rightarrow B_2 \, D_2$$
$$D_2 \rightarrow B_3 \, D_3$$
$$\text{- - -}$$
$$D_{m-3} \rightarrow B_{m-2} \, D_{m-2}$$

Chomsky Normal Form

3) Productions that have 3 or more symbols on the right side:

$$A \rightarrow B_1\ B_2\ B_3\ \text{---}\ B_{m-2}\ B_{m-1}\ B_m$$

are replaced by the following productions:

$$
\begin{aligned}
A &\rightarrow B_1\ D_1 \\
D_1 &\rightarrow B_2\ D_2 \\
D_2 &\rightarrow B_3\ D_3 \\
&\quad \text{- - -} \\
D_{m-3} &\rightarrow B_{m-2}\ D_{m-2} \\
D_{m-2} &\rightarrow B_{m-1}\ B_m
\end{aligned}
$$

Chomsky Normal Form

3) **Productions that have 3 or more symbols on the right side:**

$$A \rightarrow B_1\ B_2\ B_3\ \text{---}\ B_{m-2}\ B_{m-1}\ B_m$$

are replaced by the following productions:

$$A \rightarrow B_1\ D_1$$
$$D_1 \rightarrow B_2\ D_2$$
$$D_2 \rightarrow B_3\ D_3$$
$$\text{- - -}$$
$$D_{m-3} \rightarrow B_{m-2}\ D_{m-2}$$
$$D_{m-2} \rightarrow B_{m-1}\ B_m$$

Chomsky Normal Form

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

1) $S \rightarrow b\ A$

2) $S \rightarrow a\ B$

3) $A \rightarrow b\ A\ A$

4) $A \rightarrow a\ S$

5) $A \rightarrow a$

6) $B \rightarrow a\ B\ B$

7) $B \rightarrow b\ S$

8) $B \rightarrow b$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

1) $S \rightarrow b\ A$

2) $S \rightarrow a\ B$

3) $A \rightarrow b\ A\ A$

4) $A \rightarrow a\ S$

5) $A \rightarrow a$

6) $B \rightarrow a\ B\ B$

7) $B \rightarrow b\ S$

8) $B \rightarrow b$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

1) $S \rightarrow b \; A$

2) $S \rightarrow a \; B$

3) $A \rightarrow b \; A \, A$

4) $A \rightarrow a \; S$

5) $A \rightarrow a$

6) $B \rightarrow a \; B \, B$

7) $B \rightarrow b \; S$

8) $B \rightarrow b$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

3) $A \rightarrow b\ A\ A$ 6) $B \rightarrow a\ B\ B$

1) $S \rightarrow b\ A$

4) $A \rightarrow a\ S$ 7) $B \rightarrow b\ S$

2) $S \rightarrow a\ B$

5) $A \rightarrow a$ 8) $B \rightarrow b$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

1) $S \rightarrow b\ A$

2) $S \rightarrow a\ B$

3) $A \rightarrow b\ A\ A$

4) $A \rightarrow a\ S$

5) $A \rightarrow a$

6) $B \rightarrow a\ B\ B$

7) $B \rightarrow b\ S$

8) $B \rightarrow b$

9) $C_a \rightarrow a$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

1) $S \rightarrow b\ A$

2) $S \rightarrow C_a\ B$

3) $A \rightarrow b\ A\ A$

4) $A \rightarrow a\ S$

5) $A \rightarrow a$

6) $B \rightarrow a\ B\ B$

7) $B \rightarrow b\ S$

8) $B \rightarrow b$

9) $C_a \rightarrow a$

Chomsky Normal Form

$$A \to BC$$

$$A \to a$$

1) $S \to b\ A$

2) $S \to C_a\ B$

3) $A \to b\ A\ A$

4) $A \to C_a\ S$

5) $A \to a$

6) $B \to a\ B\ B$

7) $B \to b\ S$

8) $B \to b$

9) $C_a \to a$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

1) $S \rightarrow b\ A$

2) $S \rightarrow C_a\ B$

3) $A \rightarrow b\ A\ A$

4) $A \rightarrow C_a\ S$

5) $A \rightarrow a$

6) $B \rightarrow C_a\ B\ B$

7) $B \rightarrow b\ S$

8) $B \rightarrow b$

9) $C_a \rightarrow a$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

1) $S \rightarrow b\ A$

2) $S \rightarrow C_a\ B$

3) $A \rightarrow b\ A\ A$

4) $A \rightarrow C_a\ S$

5) $A \rightarrow a$

6) $B \rightarrow C_a\ B\ B$

7) $B \rightarrow b\ S$

8) $B \rightarrow b$

9) $C_a \rightarrow a$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

1) $S \rightarrow b\ A$

2) $S \rightarrow C_a\ B$

3) $A \rightarrow b\ A\ A$

4) $A \rightarrow C_a\ S$

5) $A \rightarrow a$

6) $B \rightarrow C_a\ B\ B$

7) $B \rightarrow b\ S$

8) $B \rightarrow b$

9) $C_a \rightarrow a$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

1) $S \rightarrow b \ A$

2) $S \rightarrow C_a \ B$

3) $A \rightarrow b \ A \ A$

4) $A \rightarrow C_a \ S$

5) $A \rightarrow a$

6) $B \rightarrow C_a \ B \ B$

7) $B \rightarrow b \ S$

8) $B \rightarrow b$

9) $C_a \rightarrow a$

10) $C_b \rightarrow b$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

3) $A \rightarrow b \ A \ A$

6) $B \rightarrow C_a \ B \ B$

9) $C_a \rightarrow a$

1) $S \rightarrow C_b \ A$

2) $S \rightarrow C_a \ B$

4) $A \rightarrow C_a \ S$

5) $A \rightarrow a$

7) $B \rightarrow b \ S$

8) $B \rightarrow b$

10) $C_b \rightarrow b$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

3) $A \rightarrow C_b\, A\, A$

6) $B \rightarrow C_a\, B\, B$

9) $C_a \rightarrow a$

1) $S \rightarrow C_b\, A$

4) $A \rightarrow C_a\, S$

7) $B \rightarrow b\, S$

10) $C_b \rightarrow b$

2) $S \rightarrow C_a\, B$

5) $A \rightarrow a$

8) $B \rightarrow b$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

1) $S \rightarrow C_b\, A$

2) $S \rightarrow C_a\, B$

3) $A \rightarrow C_b\, A\, A$

4) $A \rightarrow C_a\, S$

5) $A \rightarrow a$

6) $B \rightarrow C_a\, B\, B$

7) $B \rightarrow C_b\, S$

8) $B \rightarrow b$

9) $C_a \rightarrow a$

10) $C_b \rightarrow b$

Chomsky Normal Form

$$A \to BC$$

$$A \to a$$

3a) $A \to C_b\, D_1$

3b) $D_1 \to A\, A$

3) $A \to C_b\, A\, A$ 6) $B \to C_a\, B\, B$

1) $S \to C_b\, A$

2) $S \to C_a\, B$

4) $A \to C_a\, S$

5) $A \to a$

7) $B \to C_b\, S$

8) $B \to b$

9) $C_a \to a$

10) $C_b \to b$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

3a) $A \rightarrow C_b D_1$

3b) $D_1 \rightarrow A A$

6) $B \rightarrow C_a B B$

9) $C_a \rightarrow a$

1) $S \rightarrow C_b A$

4) $A \rightarrow C_a S$

7) $B \rightarrow C_b S$

10) $C_b \rightarrow b$

2) $S \rightarrow C_a B$

5) $A \rightarrow a$

8) $B \rightarrow b$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

6a) $B \rightarrow C_a \, E_1$

6b) $E_1 \rightarrow B \, B$

3a) $A \rightarrow C_b \, D_1$

3b) $D_1 \rightarrow A \, A$

6) $B \rightarrow C_a \, B \, B$

1) $S \rightarrow C_b \, A$

9) $C_a \rightarrow a$

2) $S \rightarrow C_a \, B$

4) $A \rightarrow C_a \, S$

7) $B \rightarrow C_b \, S$

10) $C_b \rightarrow b$

5) $A \rightarrow a$

8) $B \rightarrow b$

Chomsky Normal Form

$$A \rightarrow BC$$

$$A \rightarrow a$$

3a) $A \rightarrow C_b \, D_1$

3b) $D_1 \rightarrow A \, A$

6a) $B \rightarrow C_a \, E_1$

6b) $E_1 \rightarrow B \, B$

1) $S \rightarrow C_b \, A$

2) $S \rightarrow C_a \, B$

4) $A \rightarrow C_a \, S$

5) $A \rightarrow a$

7) $B \rightarrow C_b \, S$

8) $B \rightarrow b$

9) $C_a \rightarrow a$

10) $C_b \rightarrow b$

Greibach Normal Form

Sheila Greibach
Izvor: *http://www.cs.ucla.edu/sheila-greibach/*

Greibach Normal Form

- **Greibach Normal Form**

Sheila Greibach
Izvor: *http://www.cs.ucla.edu/sheila-greibach/*

Greibach Normal Form

- **Greibach Normal Form**
  - All productions are of the form:

Sheila Greibach
Izvor: *http://www.cs.ucla.edu/sheila-greibach/*

Copyright © 2022 S. Srbljić et al.: Introduction to Theoretical Computer Science

Greibach Normal Form

- **Greibach Normal Form**
  - All productions are of the form:

$$A \rightarrow a\,\alpha \quad , \quad a \in T, \alpha \in V^*$$

Sheila Greibach
Izvor: *http://www.cs.ucla.edu/sheila-greibach/*

Greibach Normal Form

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$$
\begin{aligned}
A &\rightarrow \alpha_1 \\
A &\rightarrow \alpha_2 \\
&\text{---} \\
A &\rightarrow \alpha_r
\end{aligned}
$$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$$A \rightarrow \alpha_1$$
$$A \rightarrow \alpha_2$$
$$\text{---}$$
$$A \rightarrow \alpha_r$$

$$B \rightarrow A\,\gamma$$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$$A \rightarrow \alpha_1$$
$$A \rightarrow \alpha_2$$
$$\text{---}$$
$$A \rightarrow \alpha_r$$

$$B \rightarrow A\,\gamma$$

$$B \rightarrow A\,\gamma$$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$A \rightarrow \alpha_1$
$A \rightarrow \alpha_2$
$---$
$A \rightarrow \alpha_r$

$B \rightarrow A\ \gamma$

$B \rightarrow A\ \gamma$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$$A \rightarrow \alpha_1$$
$$A \rightarrow \alpha_2$$
$$\text{---}$$
$$A \rightarrow \alpha_r$$

$$B \rightarrow A\,\gamma$$

$$B \rightarrow A\,\gamma$$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$$A \rightarrow \alpha_1$$
$$A \rightarrow \alpha_2$$
$$\text{---}$$
$$A \rightarrow \alpha_r$$

$$B \rightarrow A\,\gamma$$

$$B \rightarrow A\,\gamma$$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$$A \rightarrow \alpha_1$$
$$A \rightarrow \alpha_2$$
$$\text{---}$$
$$A \rightarrow \alpha_r$$

$$B \rightarrow A\,\gamma$$

$$B \rightarrow \alpha_1\,\gamma$$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$$A \rightarrow \alpha_1$$
$$A \rightarrow \alpha_2$$
$$---$$
$$A \rightarrow \alpha_r$$

$$B \rightarrow A\, \gamma$$

$$B \rightarrow \alpha_1\, \gamma$$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$$A \rightarrow \alpha_1$$
$$A \rightarrow \alpha_2$$
$$---$$
$$A \rightarrow \alpha_r$$

$$B \rightarrow A\,\gamma$$

$$B \rightarrow \alpha_1\,\gamma$$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$$A \rightarrow \alpha_1$$
$$A \rightarrow \alpha_2$$
$$\text{---}$$
$$A \rightarrow \alpha_r$$

$$B \rightarrow A\, \gamma$$

$$B \rightarrow \alpha_1\, \gamma$$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$$A \rightarrow \alpha_1$$
$$A \rightarrow \alpha_2$$
$$---$$
$$A \rightarrow \alpha_r$$

$$B \rightarrow A \ \gamma$$

$$B \rightarrow \alpha_1 \ \gamma$$

$$B \rightarrow \alpha_2 \ \gamma$$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$$A \rightarrow \alpha_1$$
$$A \rightarrow \alpha_2$$
$$---$$
$$A \rightarrow \alpha_r$$

$$B \rightarrow A \, \gamma$$

$$B \rightarrow \alpha_1 \, \gamma$$

$$B \rightarrow \alpha_2 \, \gamma$$
$$--- \qquad ---$$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

A	$\rightarrow$	α_1
A	$\rightarrow$	α_2

A	$\rightarrow$	α_r

B	$\rightarrow$	$A\,\gamma$

B	$\rightarrow$	$\alpha_1\,\gamma$

B	$\rightarrow$	$\alpha_2\,\gamma$
	---	---
B	$\rightarrow$	$\alpha_r\,\gamma$

Greibach Normal Form

Algorithm for substituting the most left nonterminal symbol

$$A \rightarrow \alpha_1$$
$$A \rightarrow \alpha_2$$
$$\text{---}$$
$$A \rightarrow \alpha_r$$

$$B \rightarrow \alpha_1 \, \gamma$$

$$B \rightarrow \alpha_2 \, \gamma$$
$$\text{---} \quad \text{---}$$
$$B \rightarrow \alpha_r \, \gamma$$

Greibach Normal Form

Greibach Normal Form

Algorithm for eliminating left recursion

Greibach Normal Form

Algorithm for eliminating left recursion

$$D_i \rightarrow D_i \, \alpha_k$$

Greibach Normal Form

Algorithm for eliminating left recursion

$$D_i \rightarrow D_i\ \alpha_k$$

$$D_i \rightarrow \beta_l$$

Greibach Normal Form

Algorithm for eliminating left recursion

$$D_i \quad \rightarrow D_i \; \alpha_k$$

$$D_i \quad \rightarrow \beta_l$$

Greibach Normal Form

Algorithm for eliminating left recursion

$D_i \rightarrow D_i \; \alpha_k$
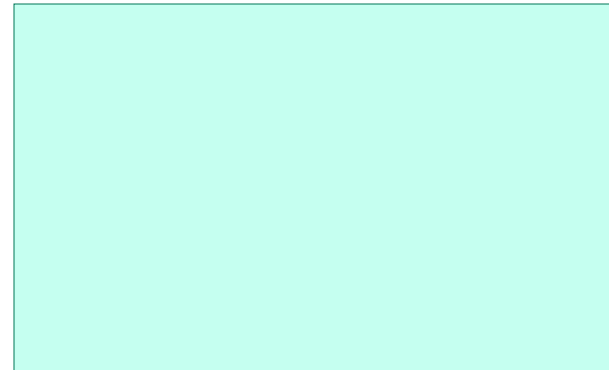
$D_i \rightarrow \beta_l$

$D_i \rightarrow \beta_l$

Greibach Normal Form

Algorithm for eliminating left recursion

$$D_i \rightarrow D_i\ \alpha_k$$

$$D_i \rightarrow \beta_l$$

$$D_i \rightarrow \beta_l$$

$$D_i \rightarrow \beta_l\ C_i$$

Copyright © 2022 S. Srbljić et al.: Introduction to Theoretical Computer Science

Greibach Normal Form

Algorithm for eliminating left recursion

$D_i \rightarrow D_i \; \alpha_k$

$D_i \rightarrow \beta_l$

$D_i \rightarrow \beta_l$

$D_i \rightarrow \beta_l \; C_i$

Greibach Normal Form

Algorithm for eliminating left recursion

$D_i \rightarrow D_i\ \alpha_k$

$D_i \rightarrow \beta_l$

$C_i \rightarrow \alpha_k$

$D_i \rightarrow \beta_l$

$D_i \rightarrow \beta_l\ C_i$

Greibach Normal Form

Algorithm for eliminating left recursion

$D_i \rightarrow D_i\ \alpha_k$

$D_i \rightarrow \beta_l$

$C_i \rightarrow \alpha_k$

$C_i \rightarrow \alpha_k\ C_i$

$D_i \rightarrow \beta_l$

$D_i \rightarrow \beta_l\ C_i$

Greibach Normal Form

Greibach Normal Form

1)   **First, the grammar is put in the Chomsky Normal Form:**

Greibach Normal Form

1)   **First, the grammar is put in the Chomsky Normal Form:**

$$A \rightarrow BC$$

Greibach Normal Form

1) **First, the grammar is put in the Chomsky Normal Form:**

$$A \rightarrow BC$$
$$A \rightarrow a$$

Greibach Normal Form

1) First, the grammar is put in the Chomsky Normal Form:

$$A \rightarrow BC$$
$$A \rightarrow a$$

The set of nonterminal symbols is replaced with the set:

Greibach Normal Form

1)   First, the grammar is put in the Chomsky Normal Form:

$$A \rightarrow BC$$
$$A \rightarrow a$$

The set of nonterminal symbols is replaced with the set:

$$\{ D_1, D_2, ..., D_m \}$$

Greibach Normal Form

1)   **First, the grammar is put in the Chomsky Normal Form:**

$$A \rightarrow BC$$
$$A \rightarrow a$$

The set of nonterminal symbols is replaced with the set:

$$\{ D_1, D_2, ..., D_m \}$$

After the conversion, all the productions are on of the forms:

Greibach Normal Form

1) **First, the grammar is put in the Chomsky Normal Form:**

$$A \rightarrow BC$$
$$A \rightarrow a$$

The set of nonterminal symbols is replaced with the set:

$$\{ D_1, D_2, ..., D_m \}$$

After the conversion, all the productions are on of the forms:

$$D_i \rightarrow D_j D_k$$

Greibach Normal Form

1) First, the grammar is put in the Chomsky Normal Form:

$$A \rightarrow BC$$
$$A \rightarrow a$$

The set of nonterminal symbols is replaced with the set:

$$\{\ D_1,\ D_2,\ ...,\ D_m\ \}$$

After the conversion, all the productions are on of the forms:

$$D_i \rightarrow D_j\,D_k$$

$$D_i \rightarrow a$$

Greibach Normal Form

1) First, the grammar is put in the Chomsky Normal Form:

$$A \rightarrow BC$$
$$A \rightarrow a$$

The set of nonterminal symbols is replaced with the set:

$$\{ D_1, D_2, ..., D_m \}$$

After the conversion, all the productions are on of the forms:

$$D_i \rightarrow D_j D_k$$

$$D_i \rightarrow a$$
Greibach Normal Form

Greibach Normal Form

Greibach Normal Form

2) **Productions of the form:**

Greibach Normal Form

2) **Productions of the form:**

$$D_i \rightarrow D_j\, D_k$$

Greibach Normal Form

2) Productions of the form:

$$D_i \rightarrow D_j\, D_k$$

Are transformed into the following form:

Greibach Normal Form

2) Productions of the form:

$$D_i \rightarrow D_j D_k$$

Are transformed into the following form:

$$D_i \rightarrow D_j \beta \qquad \text{where} \quad j > i$$

Greibach Normal Form

Greibach Normal Form

Nonterminal symbol

Greibach Normal Form

Nonterminal symbol

$$D_1$$

Greibach Normal Form

Nonterminal symbol

$$D_1$$

For the productions of the form

Greibach Normal Form

Nonterminal symbol

$$D_1$$

For the productions of the form

$$D_1 \rightarrow D_j\, D_k$$

Greibach Normal Form

Nonterminal symbol

$$D_1$$

For the productions of the form

$$D_1 \rightarrow D_j\, D_k$$

holds:

Greibach Normal Form

Nonterminal symbol

$$D_1$$

For the productions of the form

$$D_1 \rightarrow D_j D_k$$

holds:

$$j > 1$$

Greibach Normal Form

Nonterminal symbol

$$D_1$$

For the productions of the form

$$D_1 \rightarrow D_j\, D_k$$

holds:

$j > 1$

Production is in a required form

Greibach Normal Form

Nonterminal symbol

$$D_1$$

For the productions of the form

$$D_1 \rightarrow D_j D_k$$

holds:

j > 1

Production is in a required form

j = 1

Greibach Normal Form

Nonterminal symbol

$$D_1$$

For the productions of the form

$$D_1 \rightarrow D_j\, D_k$$

holds:

    j > 1

        Production is in a required form

    j = 1

        Production has a left recursion

Greibach Normal Form

Nonterminal symbol

$$D_1$$

For the productions of the form

$$D_1 \rightarrow D_j \, D_k$$

holds:

        j > 1

            Production is in a required form

        j = 1

            Production has a left recursion

$$D_1 \rightarrow D_1 \, D_k$$

Greibach Normal Form

Nonterminal symbol

$$D_1$$

For the productions of the form

$$D_1 \rightarrow D_j\, D_k$$

holds:

$j > 1$

    Production is in a required form

$j = 1$

    Production has a left recursion

$$D_1 \rightarrow D_1\, D_k$$

    The set of nonterminal symbols is expanded with a new nonterminal symbol C_1

Greibach Normal Form

Greibach Normal Form

Nonterminal symbols

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

For the productions of the form:

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

For the productions of the form:

$$D_i \rightarrow D_j \, \beta$$

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

For the productions of the form:

$$D_i \rightarrow D_j\, \beta$$

where β is a string of

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

For the productions of the form:

$$D_i \rightarrow D_j \, \beta$$

where β is a string of nonterminal symbols

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

For the productions of the form:

$$D_i \rightarrow D_j \, \beta$$

where β is a string of nonterminal symbols

holds:

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

For the productions of the form:

$$D_i \rightarrow D_j\, \beta$$

where β is a string of nonterminal symbols

holds:

$$j < i$$

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

For the productions of the form:

$$D_i \rightarrow D_j\, \beta$$

where β is a string of nonterminal symbols

holds:

j < i

The algorithm for the most left symbol substitution is used

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

For the productions of the form:

$$D_i \rightarrow D_j \, \beta$$

> **where β is a string of nonterminal symbols**

holds:

$j < i$
The algorithm for the most left symbol substitution is used

$j = i$

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

For the productions of the form:

$$D_i \rightarrow D_j \, \beta$$

where β is a string of nonterminal symbols

holds:

j < i
The algorithm for the most left symbol substitution is used

j = i
The algorithm for eliminating left recursion is used

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

For the productions of the form:

$$D_i \rightarrow D_j \, \beta$$

where β is a string of nonterminal symbols

holds:

j < i
The algorithm for the most left symbol substitution is used

j = i
The algorithm for eliminating left recursion is used
The set of nonterminal symbols is expanded with a new nonterminal symbol C_i

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

For the productions of the form:

$$D_i \rightarrow D_j \, \beta$$

where β is a string of nonterminal symbols

holds:

j < i
The algorithm for the most left symbol substitution is used

j = i
The algorithm for eliminating left recursion is used
The set of nonterminal symbols is expanded with a new nonterminal symbol C_i

j > i

Greibach Normal Form

Nonterminal symbols

$$D_2, \quad D_3, \quad ..., \quad D_m$$

For the productions of the form:

$$D_i \rightarrow D_j \, \beta$$

where β is a string of nonterminal symbols

holds:

j < i
The algorithm for the most left symbol substitution is used

j = i
The algorithm for eliminating left recursion is used
The set of nonterminal symbols is expanded with a new nonterminal symbol C_i

j > i
Production is in the required form

Greibach Normal Form

Greibach Normal Form

Once the second step is done, productions are in the following forms:

Greibach Normal Form

Once the second step is done, productions are in the following forms:

$$D_i \rightarrow D_j \beta$$

Greibach Normal Form

Once the second step is done, productions are in the following forms:

$$D_i \rightarrow D_j \beta$$

$j > i$, β **is a string of nonterminal symbols**

Greibach Normal Form

Once the second step is done, productions are in the following forms:

$$D_i \rightarrow D_j \beta$$

$j > i$, β **is a string of nonterminal symbols**

$$D_i \rightarrow a\ \beta$$

Greibach Normal Form

Once the second step is done, productions are in the following forms:

$$D_i \rightarrow D_j \beta$$

$j > i$, β is a string of nonterminal symbols

$$D_i \rightarrow a \, \beta$$

$a \in T$, β is a string of nonterminal symbols

Greibach Normal Form

Once the second step is done, productions are in the following forms:

$$D_i \rightarrow D_j \beta$$

$j > i$, β **is a string of nonterminal symbols**

$$D_i \rightarrow a\, \beta$$

$a \in T$, β **is a string of nonterminal symbols**

$$C_i \rightarrow D_j\, \xi$$

Greibach Normal Form

Once the second step is done, productions are in the following forms:

$$D_i \rightarrow D_j \beta$$

  $j > i$, β **is a string of nonterminal symbols**

$$D_i \rightarrow a\,\beta$$

  $a \in T$, β **is a string of nonterminal symbols**

$$C_i \rightarrow D_j\,\xi$$

  ξ **is a string od nonterminal symbols from the set**
  $\{D_1, D_2, ..., D_m\} \cup \{C_1, C_2, ..., C_{i-1}\}$

Greibach Normal Form

Once the second step is done, productions are in the following forms:

$$D_i \rightarrow D_j \beta$$

$j > i$, β **is a string of nonterminal symbols**

$$D_i \rightarrow a \beta$$

$a \in T$, β **is a string of nonterminal symbols**

$$C_i \rightarrow D_j \xi$$

ξ **is a string od nonterminal symbols from the set**
$\{D_1, D_2, ..., D_m\} \cup \{C_1, C_2, ..., C_{i-1}\}$

Nonterminal symbol D_m

Greibach Normal Form

Once the second step is done, productions are in the following forms:

$$D_i \rightarrow D_j \beta$$

$j > i$, β **is a string of nonterminal symbols**

$$D_i \rightarrow a\, \beta$$

$a \in T$, β **is a string of nonterminal symbols**

$$C_i \rightarrow D_j\, \xi$$

ξ **is a string od nonterminal symbols from the set**
$\{D_1, D_2, ..., D_m\} \cup \{C_1, C_2, ..., C_{i-1}\}$

Nonterminal symbol D_m

It has the greatest index

Greibach Normal Form

Once the second step is done, productions are in the following forms:

$$D_i \rightarrow D_j \beta$$

> $j > i$, β **is a string of nonterminal symbols**

$$D_i \rightarrow a \beta$$

> $a \in T$, β **is a string of nonterminal symbols**

$$C_i \rightarrow D_j \xi$$

> ξ **is a string od nonterminal symbols from the set**
> $\{D_1, D_2, ..., D_m\} \cup \{C_1, C_2, ..., C_{i-1}\}$

Nonterminal symbol D_m

> **It has the greatest index**
> **The right side of its productions start with terminal symbol**

Greibach Normal Form

Once the second step is done, productions are in the following forms:

$$D_i \rightarrow D_j \beta$$

  $j > i$, β is a string of nonterminal symbols

$$D_i \rightarrow a\, \beta$$

  $a \in T$, β is a string of nonterminal symbols

$$C_i \rightarrow D_j\, \xi$$

  ξ is a string od nonterminal symbols from the set
  $\{D_1, D_2, ..., D_m\} \cup \{C_1, C_2, ..., C_{i-1}\}$

Nonterminal symbol D_m

  It has the greatest index
  The right side of its productions start with terminal symbol
  Those productions are already in Greibach Normal Form

Greibach Normal Form

Once the second step is done, productions are in the following forms:

$$D_i \rightarrow D_j \beta$$

$j > i$, β **is a string of nonterminal symbols**

$$D_i \rightarrow a\, \beta$$

$a \in T$, β **is a string of nonterminal symbols**

$$C_i \rightarrow D_j\, \xi$$

ξ **is a string od nonterminal symbols from the set**
$\{D_1, D_2, ..., D_m\} \cup \{C_1, C_2, ..., C_{i-1}\}$

Nonterminal symbol D_m

It has the greatest index
The right side of its productions start with terminal symbol
Those productions are already in Greibach Normal Form

Greibach Normal Form

Greibach Normal Form

3) **For nonterminal symbols**

Greibach Normal Form

3)   **For nonterminal symbols**

$$D_{m-1}, D_{m-2}, D_{m-3}, ..., D_1$$

Greibach Normal Form

3) **For nonterminal symbols**

$$D_{m-1}, D_{m-2}, D_{m-3}, \ldots, D_1$$

Productions of the form:

Greibach Normal Form

3)   **For nonterminal symbols**

$$D_{m-1}, D_{m-2}, \quad D_{m-3}, \quad ..., \quad D_1$$

Productions of the form:

$$D_i \rightarrow D_j \, \beta$$

Greibach Normal Form

3)   **For nonterminal symbols**

$$D_{m-1}, D_{m-2}, D_{m-3}, ..., D_1$$

Productions of the form:

$$D_i \rightarrow D_j \beta$$

are transformed in the following form:

Greibach Normal Form

3)   **For nonterminal symbols**

$$D_{m-1}, D_{m-2}, D_{m-3}, ..., D_1$$

Productions of the form:

$$D_i \rightarrow D_j \beta$$

are transformed in the following form:

$$D_i \rightarrow a \, \alpha \, \beta$$

Greibach Normal Form

3) **For nonterminal symbols**

$$D_{m-1},\ D_{m-2},\ \ D_{m-3},\ \ ...,\ \ D_1$$

Productions of the form:

$$D_i \rightarrow D_j\,\beta$$

are transformed in the following form:

$$D_i \rightarrow a\,\alpha\,\beta$$
$$a \in T$$

Greibach Normal Form

3)　　　**For nonterminal symbols**

$$D_{m-1},\ D_{m-2},\ D_{m-3},\ ...,\ D_1$$

Productions of the form:

$$D_i \rightarrow D_j\,\beta$$

are transformed in the following form:

$$D_i \rightarrow a\,\alpha\,\beta$$

$$a \in T$$

α **i** β **are strings of nonterminal symbols**

Greibach Normal Form

Greibach Normal Form

After the third step productions are in the following forms:

Greibach Normal Form

After the third step productions are in the following forms:

$$D_i \rightarrow a\,\beta$$

Greibach Normal Form

After the third step productions are in the following forms:

$$D_i \rightarrow a \beta$$

$a \in T$, β **is a string of nonterminal symbols**

Greibach Normal Form

After the third step productions are in the following forms:

$$D_i \rightarrow a\ \beta$$

$a \in T$, β **is a string of nonterminal symbols**

$$C_i \rightarrow D_j\ \xi$$

Greibach Normal Form

After the third step productions are in the following forms:

$$D_i \rightarrow a\,\beta$$

$a \in T$, β **is a string of nonterminal symbols**

$$C_i \rightarrow D_j\,\xi$$

ξ **is a string of nonterminal symbols**
$\{D_1, D_2, ..., D_m\} \cup \{C_1, C_2, ..., C_{i-1}\}$

Copyright © 2022 S. Srbljić et al.: Introduction to Theoretical Computer Science

Greibach Normal Form

Greibach Normal Form

4) **Productions that start with C_i are transformed:**

Greibach Normal Form

4) **Productions that start with C_i are transformed:**

Greibach Normal Form

4) Productions that start with C_i are transformed:

$$C_i \rightarrow D_j\, \xi$$

Greibach Normal Form

4) Productions that start with C_i are transformed:

$$C_i \rightarrow D_j\, \xi$$

$$D_i \rightarrow a\, \beta$$

4) Productions that start with C_i are transformed:

$$C_i \rightarrow D_j\, \xi$$

$$D_i \rightarrow a\, \beta$$

$a \in T$, β **is a string of nonterminal symbols**

Greibach Normal Form

4) Productions that start with C_i are transformed:

$$C_i \rightarrow D_j\,\xi$$

$$D_i \rightarrow a\,\beta$$
$a \in T$, β **is a string of nonterminal symbols**

$$C_i \rightarrow a\,\beta$$

Greibach Normal Form

4) Productions that start with C_i are transformed:

$$C_i \rightarrow D_j\, \xi$$

$$D_i \rightarrow a\, \beta$$

$a \in T$, β **is a string of nonterminal symbols**

$$C_i \rightarrow a\, \beta$$

$a \in T$, β **is a string of nonterminal symbols**

Greibach Normal Form

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

2) $A \rightarrow B\ S$ 4) $B \rightarrow S\ A$

1) $S \rightarrow A\ B$

3) $A \rightarrow b$ 5) $B \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $S \rightarrow A\ B$

2) $A \rightarrow B\ S$

3) $A \rightarrow b$

4) $B \rightarrow S\ A$

5) $B \rightarrow a$

S is replaced with D_1

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

2) $A \rightarrow B\ S$ **4)** $B \rightarrow S\ A$

1) $D_1 \rightarrow A\ B$

3) $A \rightarrow b$ **5)** $B \rightarrow a$

S is replaced with D_1

Greibach Normal Form

$$A \rightarrow a \, \alpha$$

1) $D_1 \rightarrow A \; B$

2) $A \rightarrow B \; D_1$ **4)** $B \rightarrow S \; A$

3) $A \rightarrow b$ **5)** $B \rightarrow a$

S is replaced with D_1

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

2) $A \rightarrow B\ D_1$ 4) $B \rightarrow D_1\ A$

1) $D_1 \rightarrow A\ B$

3) $A \rightarrow b$ 5) $B \rightarrow a$

S is replaced with D_1

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

2) $A \rightarrow B\ D_1$ 4) $B \rightarrow D_1\ A$

1) $D_1 \rightarrow A\ B$

3) $A \rightarrow b$ 5) $B \rightarrow a$

S is replaced with D_1

A is replaced with D_2

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\ B$

2) $A \rightarrow B\ D_1$

3) $A \rightarrow b$

4) $B \rightarrow D_1\ A$

5) $B \rightarrow a$

S is replaced with D_1

A is replaced with D_2

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

1) $D_1 \rightarrow D_2\ B$

2) $D_2 \rightarrow B\ D_1$

3) $A \rightarrow b$

4) $B \rightarrow D_1\ A$

5) $B \rightarrow a$

S is replaced with D_1

A is replaced with D_2

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

2) $D_2 \rightarrow B\ D_1$

1) $D_1 \rightarrow D_2\ B$

3) $D_2 \rightarrow b$

4) $B \rightarrow D_1\ A$

5) $B \rightarrow a$

S is replaced with D_1

A is replaced with D_2

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\ B$

2) $D_2 \rightarrow B\ D_1$

3) $D_2 \rightarrow b$

4) $B \rightarrow D_1\ D_2$

5) $B \rightarrow a$

S is replaced with D_1

A is replaced with D_2

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\ B$

2) $D_2 \rightarrow B\ D_1$

3) $D_2 \rightarrow b$

4) $B \rightarrow D_1\ D_2$

5) $B \rightarrow a$

S is replaced with D_1
A is replaced with D_2
B is replaced with D_3

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\ D_3$

2) $D_2 \rightarrow B\ D_1$ 4) $B \rightarrow D_1\ D_2$

3) $D_2 \rightarrow b$ 5) $B \rightarrow a$

S is replaced with D_1

A is replaced with D_2

B is replaced with D_3

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

1) $D_1 \rightarrow D_2\ D_3$

2) $D_2 \rightarrow D_3\ D_1$

3) $D_2 \rightarrow b$

4) $B \rightarrow D_1\ D_2$

5) $B \rightarrow a$

S is replaced with D_1

A is replaced with D_2

B is replaced with D_3

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\ D_3$

2) $D_2 \rightarrow D_3\ D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_1\ D_2$

5) $B \rightarrow a$

S is replaced with D_1	
A is replaced with D_2	
B is replaced with D_3	

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

1) $D_1 \rightarrow D_2\ D_3$

2) $D_2 \rightarrow D_3\ D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_1\ D_2$

5) $D_3 \rightarrow a$

S is replaced with D_1
A is replaced with D_2
B is replaced with D_3

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

1) $D_1 \rightarrow D_2\, D_3$

2) $D_2 \rightarrow D_3\, D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_1\, D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

1) $D_1 \rightarrow D_2\, D_3$

2) $D_2 \rightarrow D_3\, D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_1\, D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \to a\, \alpha$$

1) $D_1 \to D_2\, D_3$

2) $D_2 \to D_3\, D_1$

3) $D_2 \to b$

4) $D_3 \to D_1\, D_2$

5) $D_3 \to a$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\,D_3$

$$1 < 2$$

2) $D_2 \rightarrow D_3\,D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_1\,D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a \, \alpha$$

$$2 < 3$$

1) $D_1 \rightarrow D_2 \, D_3$

2) $D_2 \rightarrow D_3 \, D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_1 \, D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

$$3 > 1$$

1) $D_1 \rightarrow D_2\ D_3$

2) $D_2 \rightarrow D_3\ D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_1\ D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\,D_3$

2) $D_2 \rightarrow D_3\,D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_1\,D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\,D_3$

2) $D_2 \rightarrow D_3\,D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_1\,D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\,D_3$

2) $D_2 \rightarrow D_3\,D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_1\,D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

1) $D_1 \rightarrow D_2\ D_3$

2) $D_2 \rightarrow D_3\ D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_1\ D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

1) $D_1 \rightarrow D_2\, D_3$

2) $D_2 \rightarrow D_3\, D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_2\, D_3\, D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

$$3 > 2$$

1) $D_1 \rightarrow D_2\,D_3$

2) $D_2 \rightarrow D_3\,D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_2\,D_3\,D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

2) $D_2 \rightarrow D_3\, D_1$

1) $D_1 \rightarrow D_2\, D_3$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_2\, D_3\, D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\,D_3$

2) $D_2 \rightarrow D_3\,D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_2\,D_3\,D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

1) $D_1 \rightarrow D_2\, D_3$

2) $D_2 \rightarrow D_3\, D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_2\, D_3\, D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a \, \alpha$$

1) $D_1 \rightarrow D_2 \, D_3$

2) $D_2 \rightarrow D_3 \, D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_2 \, D_3 \, D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

1) $D_1 \rightarrow D_2\ D_3$

2) $D_2 \rightarrow D_3\ D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_3\ D_1\ D_3\ D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

1) $D_1 \rightarrow D_2\, D_3$

2) $D_2 \rightarrow D_3\, D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow D_3\, D_1\, D_3\, D_2$

5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

4) $D_3 \rightarrow b\,D_3\,D_2$

2) $D_2 \rightarrow D_3\,D_1$ 4) $D_3 \rightarrow D_3\,D_1\,D_3\,D_2$

1) $D_1 \rightarrow D_2\,D_3$

3) $D_2 \rightarrow b$ 5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

4) $D_3 \rightarrow b\,D_3\,D_2$

2) $D_2 \rightarrow D_3\,D_1$ 　　4) $D_3 \rightarrow D_3\,D_1\,D_3\,D_2$ 　**3 = 3**

1) $D_1 \rightarrow D_2\,D_3$

3) $D_2 \rightarrow b$ 　　　5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

4) $D_3 \rightarrow b\ D_3\ D_2$

2) $D_2 \rightarrow D_3\ D_1$ 4) $D_3 \rightarrow D_3\ D_1\ D_3\ D_2$

1) $D_1 \rightarrow D_2\ D_3$

3) $D_2 \rightarrow b$ 5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

4) $D_3 \rightarrow b\,D_3\,D_2$

2) $D_2 \rightarrow D_3\,D_1$ **4)** $D_3 \rightarrow D_3\,D_1\,D_3\,D_2$

1) $D_1 \rightarrow D_2\,D_3$

3) $D_2 \rightarrow b$ **5)** $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

4) $D_3 \rightarrow b\, D_3\, D_2$

2) $D_2 \rightarrow D_3\, D_1$ 4) $D_3 \rightarrow D_3\, D_1\, D_3\, D_2$

1) $D_1 \rightarrow D_2\, D_3$

3) $D_2 \rightarrow b$ 5) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

4) $D_3 \rightarrow b\,D_3\,D_2$

2) $D_2 \rightarrow D_3\,D_1$ 4) $D_3 \rightarrow D_3\,D_1\,D_3\,D_2$

1) $D_1 \rightarrow D_2\,D_3$

3) $D_2 \rightarrow b$ 5) $D_3 \rightarrow a$

4) $C_3 \rightarrow$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\,D_3$

2) $D_2 \rightarrow D_3\,D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow b\,D_3\,D_2$

4) $D_3 \rightarrow D_3\,D_1\,D_3\,D_2$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

4) $D_3 \rightarrow b\ D_3\ D_2$

2) $D_2 \rightarrow D_3\ D_1$ 4) $D_3 \rightarrow D_3\ D_1\ D_3\ D_2$

1) $D_1 \rightarrow D_2\ D_3$

3) $D_2 \rightarrow b$ 5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

1) $D_1 \rightarrow D_2\ D_3$

2) $D_2 \rightarrow D_3\ D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow b\ D_3\ D_2$

4) $D_3 \rightarrow D_3\ D_1\ D_3\ D_2$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

4) $D_3 \rightarrow b\ D_3\ D_2$

2) $D_2 \rightarrow D_3\ D_1$

4) $D_3 \rightarrow D_3\ D_1\ D_3\ D_2$

1) $D_1 \rightarrow D_2\ D_3$

3) $D_2 \rightarrow b$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

4) $D_3 \rightarrow b\ D_3\ D_2$

2) $D_2 \rightarrow D_3\ D_1$ 4) $D_3 \rightarrow D_3\ D_1\ D_3\ D_2$

1) $D_1 \rightarrow D_2\ D_3$

3) $D_2 \rightarrow b$ 5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

4) $D_3 \rightarrow b\,D_3\,D_2$

2) $D_2 \rightarrow D_3\,D_1$ 4) $D_3 \rightarrow D_3\,D_1\,D_3\,D_2$

1) $D_1 \rightarrow D_2\,D_3$

3) $D_2 \rightarrow b$ 5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

4) $D_3 \rightarrow b\,D_3\,D_2$

2) $D_2 \rightarrow D_3\,D_1$ 4) $D_3 \rightarrow D_3\,D_1\,D_3\,D_2$

1) $D_1 \rightarrow D_2\,D_3$

3) $D_2 \rightarrow b$ 5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

4) $D_3 \rightarrow b\,D_3\,D_2$

Greibach Normal Form

$$A \rightarrow a \ \alpha$$

1) $D_1 \rightarrow D_2 \ D_3$

2) $D_2 \rightarrow D_3 \ D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow b \ D_3 \ D_2$

4) $D_3 \rightarrow D_3 \ D_1 \ D_3 \ D_2$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1 \ D_3 \ D_2$

4) $C_3 \rightarrow D_1 \ D_3 \ D_2 \ C_3$

4) $D_3 \rightarrow b \ D_3 \ D_2 \ C_3$

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

4) $D_3 \rightarrow b\, D_3\, D_2$

2) $D_2 \rightarrow D_3\, D_1$ 4) $D_3 \rightarrow D_3\, D_1\, D_3\, D_2$

1) $D_1 \rightarrow D_2\, D_3$

3) $D_2 \rightarrow b$ 5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\, D_3\, D_2$

4) $C_3 \rightarrow D_1\, D_3\, D_2\, C_3$

4) $D_3 \rightarrow b\, D_3\, D_2\, C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\, D_3$

2) $D_2 \rightarrow D_3\, D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow b\, D_3\, D_2$

4) $D_3 \rightarrow D_3\, D_1\, D_3\, D_2$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\, D_3\, D_2$

4) $C_3 \rightarrow D_1\, D_3\, D_2\, C_3$

4) $D_3 \rightarrow b\, D_3\, D_2\, C_3$

4) $D_3 \rightarrow a$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

1) $D_1 \rightarrow D_2\ D_3$

2) $D_2 \rightarrow D_3\ D_1$

3) $D_2 \rightarrow b$

4) $D_3 \rightarrow b\ D_3\ D_2$

4) $D_3 \rightarrow D_3\ D_1\ D_3\ D_2$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$

4) $D_3 \rightarrow a\ C_3$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

4) $D_3 \rightarrow b\ D_3\ D_2$

2) $D_2 \rightarrow D_3\ D_1$

1) $D_1 \rightarrow D_2\ D_3$

3) $D_2 \rightarrow b$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$

4) $D_3 \rightarrow a\ C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

4) $D_3 \rightarrow b\,D_3\,D_2$

2) $D_2 \rightarrow D_3\,D_1$

1) $D_1 \rightarrow D_2\,D_3$

3) $D_2 \rightarrow b$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$

4) $D_3 \rightarrow a\,C_3$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

4) $D_3 \rightarrow b\ D_3\ D_2$

2) $D_2 \rightarrow D_3\ D_1$

1) $D_1 \rightarrow D_2\ D_3$

3) $D_2 \rightarrow b$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

2) $D_2 \rightarrow$

4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$

4) $D_3 \rightarrow a\ C_3$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

4) $D_3 \rightarrow b\ D_3\ D_2$

2) $D_2 \rightarrow D_3\ D_1$

1) $D_1 \rightarrow D_2\ D_3$

3) $D_2 \rightarrow b$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$

4) $D_3 \rightarrow a\ C_3$

2) $D_2 \rightarrow$

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

4) $D_3 \rightarrow b\, D_3\, D_2$

2) $D_2 \rightarrow D_3\, D_1$

1) $D_1 \rightarrow D_2\, D_3$

3) $D_2 \rightarrow b$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\, D_3\, D_2$

2) $D_2 \rightarrow$

4) $C_3 \rightarrow D_1\, D_3\, D_2\, C_3$

4) $D_3 \rightarrow b\, D_3\, D_2\, C_3$

4) $D_3 \rightarrow a\, C_3$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

4) $D_3 \rightarrow b\ D_3\ D_2$

2) $D_2 \rightarrow D_3\ D_1$

1) $D_1 \rightarrow D_2\ D_3$

3) $D_2 \rightarrow b$

5) $D_3 \rightarrow a$

2) $D_2 \rightarrow b\ D_3\ D_2$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$

4) $D_3 \rightarrow a\ C_3$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

4) $D_3 \rightarrow b\ D_3\ D_2$

2) $D_2 \rightarrow D_3\ \boxed{D_1}$

1) $D_1 \rightarrow D_2\ D_3$

3) $D_2 \rightarrow b$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

2) $D_2 \rightarrow b\ D_3\ D_2$

4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$

4) $D_3 \rightarrow a\ C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

4) $D_3 \rightarrow b\,D_3\,D_2$

2) $D_2 \rightarrow D_3\,D_1$

1) $D_1 \rightarrow D_2\,D_3$

3) $D_2 \rightarrow b$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

2) $D_2 \rightarrow b\,D_3\,D_2\,D_1$

4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$

4) $D_3 \rightarrow a\,C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

4) $D_3 \rightarrow b\,D_3\,D_2$

2) $D_2 \rightarrow D_3\,D_1$

1) $D_1 \rightarrow D_2\,D_3$

3) $D_2 \rightarrow b$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

2) $D_2 \rightarrow b\,D_3\,D_2\,D_1$

4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

2) $D_2 \rightarrow a\,D_1$

4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$

4) $D_3 \rightarrow a\,C_3$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

4) $D_3 \rightarrow b\ D_3\ D_2$

2) $D_2 \rightarrow D_3\ D_1$

1) $D_1 \rightarrow D_2\ D_3$

3) $D_2 \rightarrow b$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

2) $D_2 \rightarrow b\ D_3\ D_2\ D_1$

4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

2) $D_2 \rightarrow a\ D_1$

4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$

2) $D_2 \rightarrow b\ D_3\ D_2\ C_3\ D_1$

4) $D_3 \rightarrow a\ C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

4) $D_3 \rightarrow b\,D_3\,D_2$

2) $D_2 \rightarrow D_3\,D_1$

1) $D_1 \rightarrow D_2\,D_3$

3) $D_2 \rightarrow b$

5) $D_3 \rightarrow a$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

2) $D_2 \rightarrow b\,D_3\,D_2\,D_1$

4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

2) $D_2 \rightarrow a\,D_1$

4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$

2) $D_2 \rightarrow b\,D_3\,D_2\,C_3\,D_1$

4) $D_3 \rightarrow a\,C_3$

2) $D_2 \rightarrow a\,C_3\,D_1$

Greibach Normal Form

$$A \to a\, \alpha$$

4) $D_3 \to b\, D_3\, D_2$

1) $D_1 \to D_2\, D_3$

3) $D_2 \to b$

5) $D_3 \to a$

4) $C_3 \to D_1\, D_3\, D_2$

2) $D_2 \to b\, D_3\, D_2\, D_1$

4) $C_3 \to D_1\, D_3\, D_2\, C_3$

2) $D_2 \to a\, D_1$

4) $D_3 \to b\, D_3\, D_2\, C_3$

2) $D_2 \to b\, D_3\, D_2\, C_3\, D_1$

4) $D_3 \to a\, C_3$

2) $D_2 \to a\, C_3\, D_1$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\,D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\,D_3\,D_2\,D_1$
2) $D_2 \rightarrow a\,D_1$
2) $D_2 \rightarrow b\,D_3\,D_2\,C_3\,D_1$
2) $D_2 \rightarrow a\,C_3\,D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\,D_3\,D_2$
4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$
4) $D_3 \rightarrow a\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2$
4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

1) $D_1 \rightarrow D_2\ D_3$

1) $D_1 \rightarrow b\ D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\ D_3\ D_2\ D_1$
2) $D_2 \rightarrow a\ D_1$
2) $D_2 \rightarrow b\ D_3\ D_2\ C_3\ D_1$
2) $D_2 \rightarrow a\ C_3\ D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\ D_3\ D_2$
4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$
4) $D_3 \rightarrow a\ C_3$

4) $C_3 \rightarrow D_1\ D_3\ D_2$
4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

1) $D_1 \rightarrow D_2\ D_3$

1) $D_1 \rightarrow b\ D_3$
1) $D_1 \rightarrow b\ D_3\ D_2\ D_1\ D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\ D_3\ D_2\ D_1$
2) $D_2 \rightarrow a\ D_1$
2) $D_2 \rightarrow b\ D_3\ D_2\ C_3\ D_1$
2) $D_2 \rightarrow a\ C_3\ D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\ D_3\ D_2$
4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$
4) $D_3 \rightarrow a\ C_3$

4) $C_3 \rightarrow D_1\ D_3\ D_2$
4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

Copyright © 2022 S. Srbljić et al.: Introduction to Theoretical Computer Science

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

1) $D_1 \rightarrow D_2\ D_3$

1) $D_1 \rightarrow b\ D_3$
1) $D_1 \rightarrow b\ D_3\ D_2\ D_1\ D_3$
1) $D_1 \rightarrow a\ D_1\ D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\ D_3\ D_2\ D_1$
2) $D_2 \rightarrow a\ D_1$
2) $D_2 \rightarrow b\ D_3\ D_2\ C_3\ D_1$
2) $D_2 \rightarrow a\ C_3\ D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\ D_3\ D_2$
4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$
4) $D_3 \rightarrow a\ C_3$

4) $C_3 \rightarrow D_1\ D_3\ D_2$
4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

1) $D_1 \rightarrow D_2\, D_3$

1) $D_1 \rightarrow b\, D_3$
1) $D_1 \rightarrow b\, D_3\, D_2\, D_1\, D_3$
1) $D_1 \rightarrow a\, D_1\, D_3$
1) $D_1 \rightarrow b\, D_3\, D_2\, C_3\, D_1\, D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\, D_3\, D_2\, D_1$
2) $D_2 \rightarrow a\, D_1$
2) $D_2 \rightarrow b\, D_3\, D_2\, C_3\, D_1$
2) $D_2 \rightarrow a\, C_3\, D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\, D_3\, D_2$
4) $D_3 \rightarrow b\, D_3\, D_2\, C_3$
4) $D_3 \rightarrow a\, C_3$

4) $C_3 \rightarrow D_1\, D_3\, D_2$
4) $C_3 \rightarrow D_1\, D_3\, D_2\, C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow D_2\,D_3$

1) $D_1 \rightarrow b\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,D_1\,D_3$
1) $D_1 \rightarrow a\,D_1\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3$
1) $D_1 \rightarrow a\,C_3\,D_1\,D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\,D_3\,D_2\,D_1$
2) $D_2 \rightarrow a\,D_1$
2) $D_2 \rightarrow b\,D_3\,D_2\,C_3\,D_1$
2) $D_2 \rightarrow a\,C_3\,D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\,D_3\,D_2$
4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$
4) $D_3 \rightarrow a\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2$
4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

$$3)\ D_2 \rightarrow b$$

$$5)\ D_3 \rightarrow a$$

1) $D_1 \rightarrow b\ D_3$
1) $D_1 \rightarrow b\ D_3\ D_2\ D_1\ D_3$
1) $D_1 \rightarrow a\ D_1\ D_3$
1) $D_1 \rightarrow b\ D_3\ D_2\ C_3\ D_1\ D_3$
1) $D_1 \rightarrow a\ C_3\ D_1\ D_3$

2) $D_2 \rightarrow b\ D_3\ D_2\ D_1$
2) $D_2 \rightarrow a\ D_1$
2) $D_2 \rightarrow b\ D_3\ D_2\ C_3\ D_1$
2) $D_2 \rightarrow a\ C_3\ D_1$

4) $D_3 \rightarrow b\ D_3\ D_2$
4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$
4) $D_3 \rightarrow a\ C_3$

4) $C_3 \rightarrow D_1\ D_3\ D_2$
4) $C_3 \rightarrow D_1\ D_3\ D_2\ C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow b\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,D_1\,D_3$
1) $D_1 \rightarrow a\,D_1\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3$
1) $D_1 \rightarrow a\,C_3\,D_1\,D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\,D_3\,D_2\,D_1$
2) $D_2 \rightarrow a\,D_1$
2) $D_2 \rightarrow b\,D_3\,D_2\,C_3\,D_1$
2) $D_2 \rightarrow a\,C_3\,D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\,D_3\,D_2$
4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$
4) $D_3 \rightarrow a\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow b\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,D_1\,D_3$
1) $D_1 \rightarrow a\,D_1\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3$
1) $D_1 \rightarrow a\,C_3\,D_1\,D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\,D_3\,D_2\,D_1$
2) $D_2 \rightarrow a\,D_1$
2) $D_2 \rightarrow b\,D_3\,D_2\,C_3\,D_1$
2) $D_2 \rightarrow a\,C_3\,D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\,D_3\,D_2$
4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$
4) $D_3 \rightarrow a\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

4) $C_3 \rightarrow b\,D_3\,D_3\,D_2\,C_3$

Greibach Normal Form

$$A \rightarrow a \, \alpha$$

1) $D_1 \rightarrow b \, D_3$
1) $D_1 \rightarrow b \, D_3 \, D_2 \, D_1 \, D_3$
1) $D_1 \rightarrow a \, D_1 \, D_3$
1) $D_1 \rightarrow b \, D_3 \, D_2 \, C_3 \, D_1 \, D_3$
1) $D_1 \rightarrow a \, C_3 \, D_1 \, D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b \, D_3 \, D_2 \, D_1$
2) $D_2 \rightarrow a \, D_1$
2) $D_2 \rightarrow b \, D_3 \, D_2 \, C_3 \, D_1$
2) $D_2 \rightarrow a \, C_3 \, D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b \, D_3 \, D_2$
4) $D_3 \rightarrow b \, D_3 \, D_2 \, C_3$
4) $D_3 \rightarrow a \, C_3$

4) $C_3 \rightarrow D_1 \, D_3 \, D_2 \, C_3$

4) $C_3 \rightarrow D_1 \, D_3 \, D_2$

4) $C_3 \rightarrow b \, D_3 \, D_3 \, D_2 \, C_3$
4) $C_3 \rightarrow b \, D_3 \, D_2 \, D_1 \, D_3 \, D_3 \, D_2 \, C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow b\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,D_1\,D_3$
1) $D_1 \rightarrow a\,D_1\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3$
1) $D_1 \rightarrow a\,C_3\,D_1\,D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\,D_3\,D_2\,D_1$
2) $D_2 \rightarrow a\,D_1$
2) $D_2 \rightarrow b\,D_3\,D_2\,C_3\,D_1$
2) $D_2 \rightarrow a\,C_3\,D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\,D_3\,D_2$
4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$
4) $D_3 \rightarrow a\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

4) $C_3 \rightarrow b\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow b\,D_3\,D_2\,D_1\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow a\,D_1\,D_3\,D_3\,D_2\,C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow b\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,D_1\,D_3$
1) $D_1 \rightarrow a\,D_1\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3$
1) $D_1 \rightarrow a\,C_3\,D_1\,D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\,D_3\,D_2\,D_1$
2) $D_2 \rightarrow a\,D_1$
2) $D_2 \rightarrow b\,D_3\,D_2\,C_3\,D_1$
2) $D_2 \rightarrow a\,C_3\,D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\,D_3\,D_2$
4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$
4) $D_3 \rightarrow a\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

4) $C_3 \rightarrow b\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow b\,D_3\,D_2\,D_1\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow a\,D_1\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3\,D_3\,D_2\,C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow b\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,D_1\,D_3$
1) $D_1 \rightarrow a\,D_1\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3$
1) $D_1 \rightarrow a\,C_3\,D_1\,D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\,D_3\,D_2\,D_1$
2) $D_2 \rightarrow a\,D_1$
2) $D_2 \rightarrow b\,D_3\,D_2\,C_3\,D_1$
2) $D_2 \rightarrow a\,C_3\,D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\,D_3\,D_2$
4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$
4) $D_3 \rightarrow a\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

4) $C_3 \rightarrow b\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow b\,D_3\,D_2\,D_1\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow a\,D_1\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow a\,C_3\,D_1\,D_3\,D_3\,D_2\,C_3$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow b\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,D_1\,D_3$
1) $D_1 \rightarrow a\,D_1\,D_3$
1) $D_1 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3$
1) $D_1 \rightarrow a\,C_3\,D_1\,D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\,D_3\,D_2\,D_1$
2) $D_2 \rightarrow a\,D_1$
2) $D_2 \rightarrow b\,D_3\,D_2\,C_3\,D_1$
2) $D_2 \rightarrow a\,C_3\,D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\,D_3\,D_2$
4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$
4) $D_3 \rightarrow a\,C_3$

4) $C_3 \rightarrow D_1\,D_3\,D_2$

4) $C_3 \rightarrow b\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow b\,D_3\,D_2\,D_1\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow a\,D_1\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3\,D_3\,D_2\,C_3$
4) $C_3 \rightarrow a\,C_3\,D_1\,D_3\,D_3\,D_2\,C_3$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

1) $D_1 \rightarrow b\ D_3$
1) $D_1 \rightarrow b\ D_3\ D_2\ D_1\ D_3$
1) $D_1 \rightarrow a\ D_1\ D_3$
1) $D_1 \rightarrow b\ D_3\ D_2\ C_3\ D_1\ D_3$
1) $D_1 \rightarrow a\ C_3\ D_1\ D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\ D_3\ D_2\ D_1$
2) $D_2 \rightarrow a\ D_1$
2) $D_2 \rightarrow b\ D_3\ D_2\ C_3\ D_1$
2) $D_2 \rightarrow a\ C_3\ D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\ D_3\ D_2$
4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$
4) $D_3 \rightarrow a\ C_3$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow b\ D_3\ D_3\ D_2$

4) $C_3 \rightarrow b\ D_3\ D_3\ D_2\ C_3$
4) $C_3 \rightarrow b\ D_3\ D_2\ D_1\ D_3\ D_3\ D_2\ C_3$
4) $C_3 \rightarrow a\ D_1\ D_3\ D_3\ D_2\ C_3$
4) $C_3 \rightarrow b\ D_3\ D_2\ C_3\ D_1\ D_3\ D_3\ D_2\ C_3$
4) $C_3 \rightarrow a\ C_3\ D_1\ D_3\ D_3\ D_2\ C_3$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

1) $D_1 \rightarrow b\ D_3$
1) $D_1 \rightarrow b\ D_3\ D_2\ D_1\ D_3$
1) $D_1 \rightarrow a\ D_1\ D_3$
1) $D_1 \rightarrow b\ D_3\ D_2\ C_3\ D_1\ D_3$
1) $D_1 \rightarrow a\ C_3\ D_1\ D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\ D_3\ D_2\ D_1$
2) $D_2 \rightarrow a\ D_1$
2) $D_2 \rightarrow b\ D_3\ D_2\ C_3\ D_1$
2) $D_2 \rightarrow a\ C_3\ D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\ D_3\ D_2$
4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$
4) $D_3 \rightarrow a\ C_3$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow b\ D_3\ D_3\ D_2\ C_3$
4) $C_3 \rightarrow b\ D_3\ D_2\ D_1\ D_3\ D_3\ D_2\ C_3$
4) $C_3 \rightarrow a\ D_1\ D_3\ D_3\ D_2\ C_3$
4) $C_3 \rightarrow b\ D_3\ D_2\ C_3\ D_1\ D_3\ D_3\ D_2\ C_3$
4) $C_3 \rightarrow a\ C_3\ D_1\ D_3\ D_3\ D_2\ C_3$

4) $C_3 \rightarrow b\ D_3\ D_3\ D_2$
4) $C_3 \rightarrow b\ D_3\ D_2\ D_1\ D_3\ D_3\ D_2$

Greibach Normal Form

$$A \rightarrow a\, \alpha$$

1) $D_1 \rightarrow b\, D_3$
1) $D_1 \rightarrow b\, D_3\, D_2\, D_1\, D_3$
1) $D_1 \rightarrow a\, D_1\, D_3$
1) $D_1 \rightarrow b\, D_3\, D_2\, C_3\, D_1\, D_3$
1) $D_1 \rightarrow a\, C_3\, D_1\, D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\, D_3\, D_2\, D_1$
2) $D_2 \rightarrow a\, D_1$
2) $D_2 \rightarrow b\, D_3\, D_2\, C_3\, D_1$
2) $D_2 \rightarrow a\, C_3\, D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\, D_3\, D_2$
4) $D_3 \rightarrow b\, D_3\, D_2\, C_3$
4) $D_3 \rightarrow a\, C_3$

4) $C_3 \rightarrow D_1\, D_3\, D_2$

4) $C_3 \rightarrow b\, D_3\, D_3\, D_2\, C_3$
4) $C_3 \rightarrow b\, D_3\, D_2\, D_1\, D_3\, D_3\, D_2\, C_3$
4) $C_3 \rightarrow a\, D_1\, D_3\, D_3\, D_2\, C_3$
4) $C_3 \rightarrow b\, D_3\, D_2\, C_3\, D_1\, D_3\, D_3\, D_2\, C_3$
4) $C_3 \rightarrow a\, C_3\, D_1\, D_3\, D_3\, D_2\, C_3$

4) $C_3 \rightarrow b\, D_3\, D_3\, D_2$
4) $C_3 \rightarrow b\, D_3\, D_2\, D_1\, D_3\, D_3\, D_2$
4) $C_3 \rightarrow a\, D_1\, D_3\, D_3\, D_2$

Greibach Normal Form

$$A \rightarrow a\ \alpha$$

1) $D_1 \rightarrow b\ D_3$
1) $D_1 \rightarrow b\ D_3\ D_2\ D_1\ D_3$
1) $D_1 \rightarrow a\ D_1\ D_3$
1) $D_1 \rightarrow b\ D_3\ D_2\ C_3\ D_1\ D_3$
1) $D_1 \rightarrow a\ C_3\ D_1\ D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\ D_3\ D_2\ D_1$
2) $D_2 \rightarrow a\ D_1$
2) $D_2 \rightarrow b\ D_3\ D_2\ C_3\ D_1$
2) $D_2 \rightarrow a\ C_3\ D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\ D_3\ D_2$
4) $D_3 \rightarrow b\ D_3\ D_2\ C_3$
4) $D_3 \rightarrow a\ C_3$

4) $C_3 \rightarrow D_1\ D_3\ D_2$

4) $C_3 \rightarrow b\ D_3\ D_3\ D_2\ C_3$
4) $C_3 \rightarrow b\ D_3\ D_2\ D_1\ D_3\ D_3\ D_2\ C_3$
4) $C_3 \rightarrow a\ D_1\ D_3\ D_3\ D_2\ C_3$
4) $C_3 \rightarrow b\ D_3\ D_2\ C_3\ D_1\ D_3\ D_3\ D_2\ C_3$
4) $C_3 \rightarrow a\ C_3\ D_1\ D_3\ D_3\ D_2\ C_3$

4) $C_3 \rightarrow b\ D_3\ D_3\ D_2$
4) $C_3 \rightarrow b\ D_3\ D_2\ D_1\ D_3\ D_3\ D_2$
4) $C_3 \rightarrow a\ D_1\ D_3\ D_3\ D_2$
4) $C_3 \rightarrow b\ D_3\ D_2\ C_3\ D_1\ D_3\ D_3\ D_2$

Greibach Normal Form

$$A \rightarrow a \, \alpha$$

1) $D_1 \rightarrow b \, D_3$
1) $D_1 \rightarrow b \, D_3 \, D_2 \, D_1 \, D_3$
1) $D_1 \rightarrow a \, D_1 \, D_3$
1) $D_1 \rightarrow b \, D_3 \, D_2 \, C_3 \, D_1 \, D_3$
1) $D_1 \rightarrow a \, C_3 \, D_1 \, D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b \, D_3 \, D_2 \, D_1$
2) $D_2 \rightarrow a \, D_1$
2) $D_2 \rightarrow b \, D_3 \, D_2 \, C_3 \, D_1$
2) $D_2 \rightarrow a \, C_3 \, D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b \, D_3 \, D_2$
4) $D_3 \rightarrow b \, D_3 \, D_2 \, C_3$
4) $D_3 \rightarrow a \, C_3$

4) $C_3 \rightarrow D_1 \, D_3 \, D_2$

4) $C_3 \rightarrow b \, D_3 \, D_3 \, D_2 \, C_3$
4) $C_3 \rightarrow b \, D_3 \, D_2 \, D_1 \, D_3 \, D_3 \, D_2 \, C_3$
4) $C_3 \rightarrow a \, D_1 \, D_3 \, D_3 \, D_2 \, C_3$
4) $C_3 \rightarrow b \, D_3 \, D_2 \, C_3 \, D_1 \, D_3 \, D_3 \, D_2 \, C_3$
4) $C_3 \rightarrow a \, C_3 \, D_1 \, D_3 \, D_3 \, D_2 \, C_3$

4) $C_3 \rightarrow b \, D_3 \, D_3 \, D_2$
4) $C_3 \rightarrow b \, D_3 \, D_2 \, D_1 \, D_3 \, D_3 \, D_2$
4) $C_3 \rightarrow a \, D_1 \, D_3 \, D_3 \, D_2$
4) $C_3 \rightarrow b \, D_3 \, D_2 \, C_3 \, D_1 \, D_3 \, D_3 \, D_2$
4) $C_3 \rightarrow a \, C_3 \, D_1 \, D_3 \, D_3 \, D_2$

Greibach Normal Form

$$A \rightarrow a\,\alpha$$

1) $D_1 \rightarrow b\,D_3$

1) $D_1 \rightarrow b\,D_3\,D_2\,D_1\,D_3$

1) $D_1 \rightarrow a\,D_1\,D_3$

1) $D_1 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3$

1) $D_1 \rightarrow a\,C_3\,D_1\,D_3$

3) $D_2 \rightarrow b$

2) $D_2 \rightarrow b\,D_3\,D_2\,D_1$

2) $D_2 \rightarrow a\,D_1$

2) $D_2 \rightarrow b\,D_3\,D_2\,C_3\,D_1$

2) $D_2 \rightarrow a\,C_3\,D_1$

5) $D_3 \rightarrow a$

4) $D_3 \rightarrow b\,D_3\,D_2$

4) $D_3 \rightarrow b\,D_3\,D_2\,C_3$

4) $D_3 \rightarrow a\,C_3$

4) $C_3 \rightarrow b\,D_3\,D_3\,D_2\,C_3$

4) $C_3 \rightarrow b\,D_3\,D_2\,D_1\,D_3\,D_3\,D_2\,C_3$

4) $C_3 \rightarrow a\,D_1\,D_3\,D_3\,D_2\,C_3$

4) $C_3 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3\,D_3\,D_2\,C_3$

4) $C_3 \rightarrow a\,C_3\,D_1\,D_3\,D_3\,D_2\,C_3$

4) $C_3 \rightarrow b\,D_3\,D_3\,D_2$

4) $C_3 \rightarrow b\,D_3\,D_2\,D_1\,D_3\,D_3\,D_2$

4) $C_3 \rightarrow a\,D_1\,D_3\,D_3\,D_2$

4) $C_3 \rightarrow b\,D_3\,D_2\,C_3\,D_1\,D_3\,D_3\,D_2$

4) $C_3 \rightarrow a\,C_3\,D_1\,D_3\,D_3\,D_2$