Lecture 10

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Constructing a PA That Accepts By Empty Stack Given a Context-Free Grammar

Constructing a PA That Accepts By Empty Stack Given a Context-Free Grammar

$G = (V, T, P, S)$

Constructing a PA That Accepts By Empty Stack Given a Context-Free Grammar

$G = (V, T, P, S)$

Greibach normal form:

$A \to a\,\beta$

$A \in V$, $a \in T$, $\beta \in V^*$

Constructing a PA That Accepts By Empty Stack Given a Context-Free Grammar

PA $M = (\{q\}, \Sigma, \Gamma, \delta, q, S, \emptyset)$

$G = (V, T, P, S)$

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***a*) PA *M* has only one state *q* which is also its start state**

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***a*) PA *M* has only one state *q* which is also its start state**

***b*) $\Sigma = T$**

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***a*) PA *M* has only one state *q* which is also its start state**

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***c*) $\Gamma = V$**

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***a*) PA *M* has only one state *q* which is also its start state**

***b*) $\Sigma = T$**

***c*) $\Gamma = V$**

d*) *Start stack symbol* = *S

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---

a) PA *M* has only one state *q* which is also its start state

b) $\Sigma = T$

c) $\Gamma = V$

d) *Start stack symbol* = *S*

e) $F = \varnothing$

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---

a) **PA *M* has only one state *q* which is also its start state**

b) $\Sigma = T$

c) $\Gamma = V$

d) ***Start stack symbol*** = S

e) $F = \varnothing$

f) **PA *M* accepts by empty stack**

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Transition function:

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$(q, \gamma) \in \delta(q, a, A)$ **if and only if** $A \to a\ \gamma$

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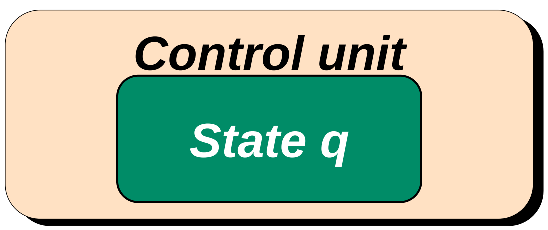

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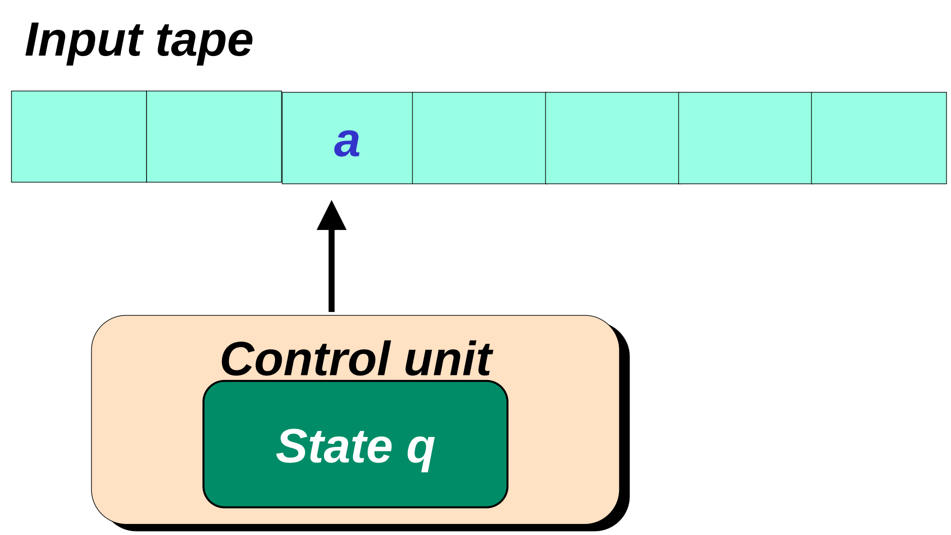

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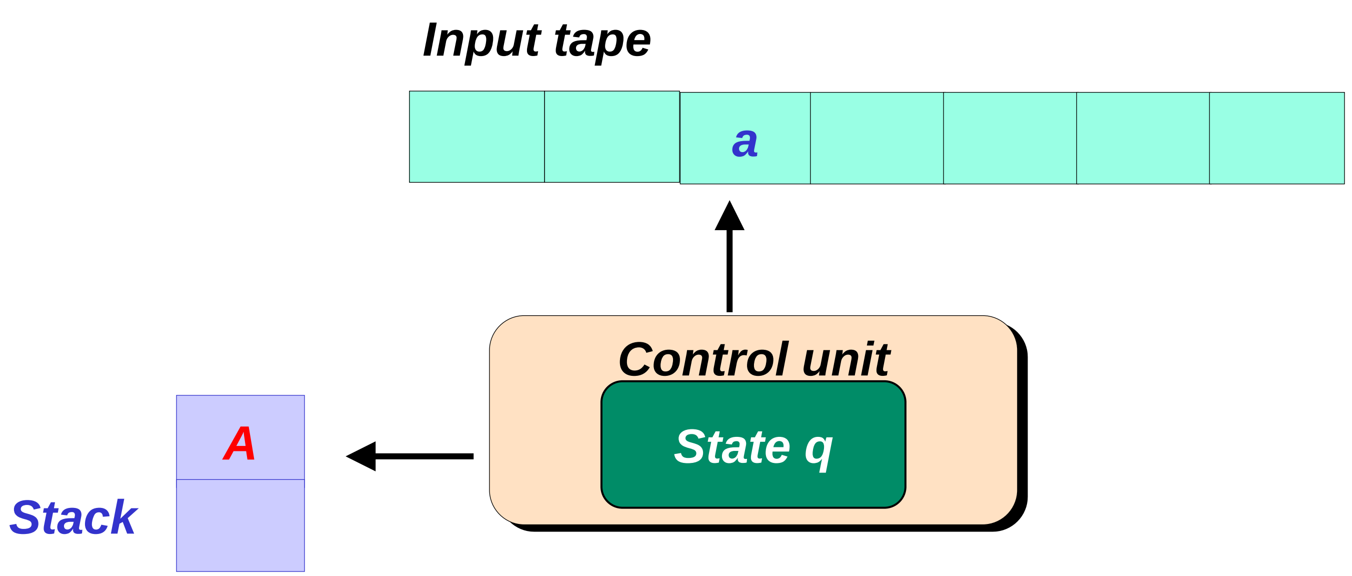

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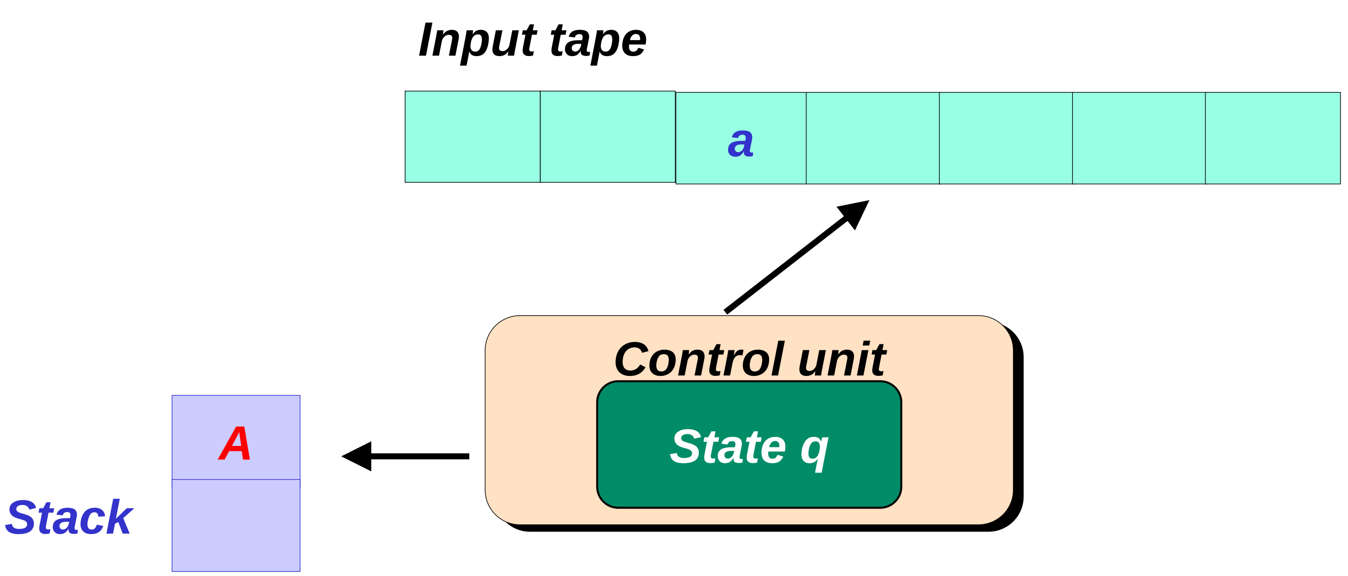

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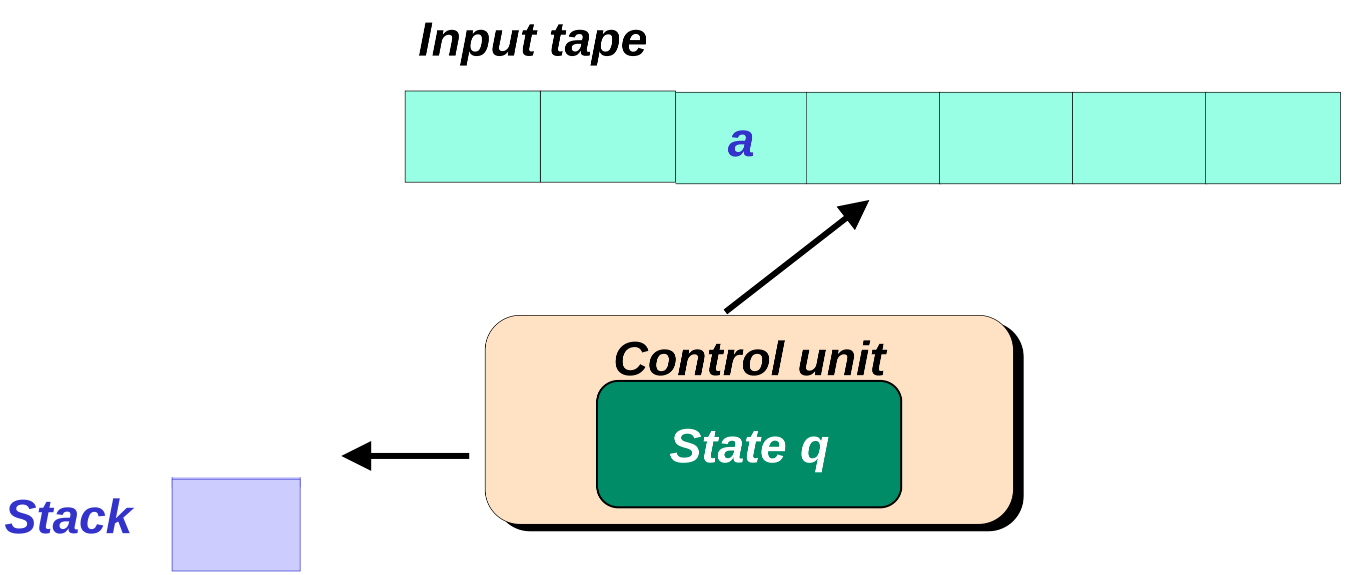

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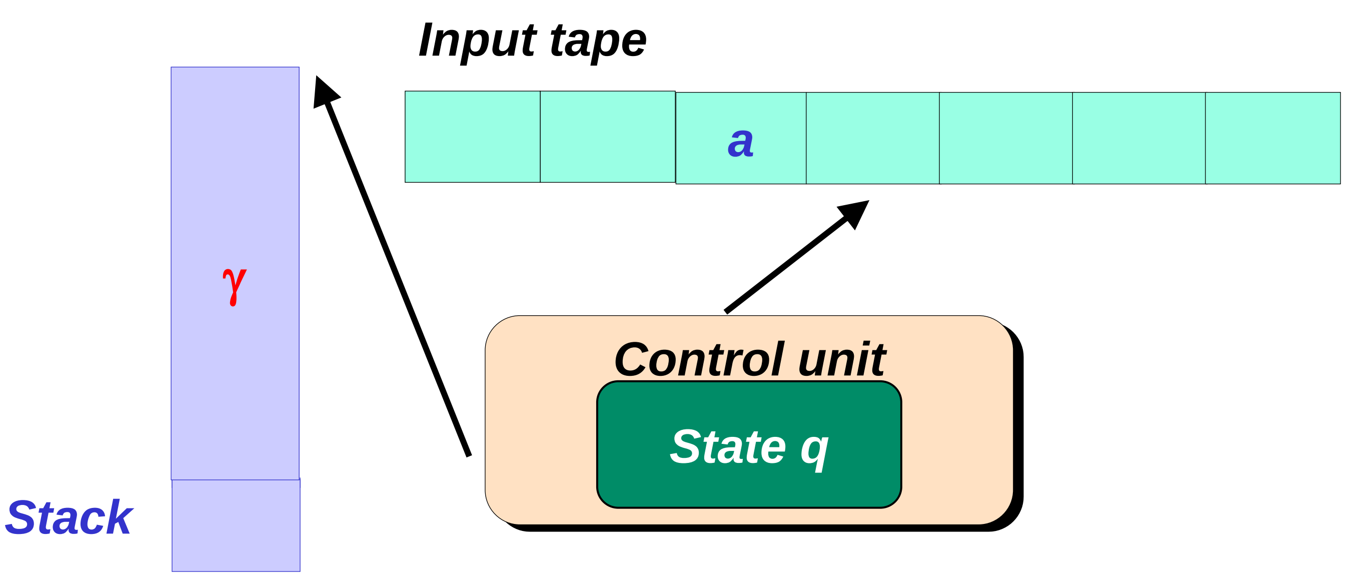

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(q, x, S)

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$$\boxed{(q, x, S)} \ \underset{M}{\succ}$$

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$(q, x, S) \underset{M}{\succ} (q, \varepsilon, \alpha)$

S

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Greibach normal form:

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$$A \in V,\ a \in T,\ \beta \in V^*$$

$$(q, x, S) \underset{M}{\succ} (q, \varepsilon, \alpha)$$

$$S \underset{G}{\Rightarrow}$$

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Greibach normal form:

$A \rightarrow a\ \beta$

$A \in V,\ a \in T,\ \beta \in V^*$

$$(q, x, S) \underset{M}{\succ} (q, \varepsilon, \alpha)$$

$$S \underset{G}{\Rightarrow} x\ \alpha$$

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$A \in V,\ a \in T,\ \beta \in V^*$

$$\boxed{(q, x, S)} \underset{M}{\succ} \boxed{(q, \varepsilon, \varepsilon)}$$

$$\boxed{S} \underset{G}{\Rightarrow} \boxed{x\,\alpha}$$

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$$\boxed{(q, x, S)} \ \underset{M}{\succ}\ \boxed{(q, \varepsilon, \varepsilon)}$$

$$\boxed{S} \ \underset{G}{\Rightarrow}\ \boxed{x}$$

Constructing a PA That Accepts By Empty Stack Given a Context-Free Grammar

Constructing a PA That Accepts By Empty Stack Given a Context-Free Grammar

$$G = (\{S, A\}, \{a, b\},$$
$$\{S \rightarrow aAA\ ,\ A \rightarrow aS \mid bS \mid a\}, S)$$

Constructing a PA That Accepts By Empty Stack Given a Context-Free Grammar

$M = (\{q\}, \{a, b\}, \{S, A\}, \delta, q, S, \emptyset)$

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$G = (\{S, A\}, \{a, b\},$
$\{S \to aAA\ ,\ A \to aS \mid bS \mid a\}, S)$

$S \to aAA$

Constructing a PA That Accepts By Empty Stack Given a Context-Free Grammar

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$G = (\{S, A\}, \{a, b\}, \{S \rightarrow aAA\ ,\ A \rightarrow aS \mid bS \mid a\}, S)$

$\delta(q, a, S) = \{(q, AA)\}$ $S \rightarrow aAA$

Constructing a PA That Accepts By Empty Stack Given a Context-Free Grammar

$M = (\{q\}, \{a, b\}, \{S, A\}, \delta, q, S, \varnothing)$

$G = (\{S, A\}, \{a, b\}, \{S \to aAA\ , A \to aS \mid bS \mid a\}, S)$

$\delta(q, a, S) = \{(q, AA)\}$ — $S \to aAA$

$A \to aS,\ \ A \to a$

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$G = (\{S, A\}, \{a, b\}, \{S \rightarrow aAA , A \rightarrow aS \mid bS \mid a\}, S)$

$\delta(q, a, S) = \{(q, AA)\}$ $\quad S \rightarrow aAA$

$\delta(q, a, A) = \{(q, S), (q, \varepsilon)\}$ $\quad A \rightarrow aS,\ A \rightarrow a$

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$\delta(q, a, S) = \{(q, AA)\}$ — $S \to aAA$

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$A \to bS$

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$\delta(q, a, S) = \{(q, AA)\}$ — $S \to aAA$

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$\delta(q, b, A) = \{(q, S)\}$ — $A \to bS$

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$\delta(q, a, S) = \{(q, AA)\}$ — $S \to aAA$

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$\delta(q, b, A) = \{(q, S)\}$ — $A \to bS$

$S \Rightarrow aAA \Rightarrow abSA \Rightarrow abaAAA \Rightarrow$

$abaaAA \Rightarrow abaaaA \Rightarrow abaaaa$

Constructing a PA That Accepts By Empty Stack Given a Context-Free Grammar

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$G = (\{S, A\}, \{a, b\}, \{S \rightarrow aAA\ ,\ A \rightarrow aS \mid bS \mid a\}, S)$

$\delta(q, a, S) = \{(q, AA)\}$ $\quad S \rightarrow aAA$

$\delta(q, a, A) = \{(q, S), (q, \varepsilon)\}$ $\quad A \rightarrow aS,\ A \rightarrow a$

$\delta(q, b, A) = \{(q, S)\}$ $\quad A \rightarrow bS$

$$S \Rightarrow aAA \Rightarrow abSA \Rightarrow abaAAA \Rightarrow abaaAA \Rightarrow abaaaA \Rightarrow abaaaa$$

$$(q, abaaaa, S) \succ (q, baaaa, AA) \succ (q, aaaa, SA) \succ (q, aaa, AAA) \succ (q, aa, AA) \succ (q, a, A) \succ (q, \varepsilon, \varepsilon)$$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

PA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G = (V, T, P, S)$

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Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G = (V, T, P, S)$

PA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$[q, A, p] \in V$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G = (V, T, P, S)$

PA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$[q, A, p] \in V$

$q \in Q$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G = (V, T, P, S)$

PA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$$[q, A, p] \in V$$

$$p \in Q$$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

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PA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$[q, A, p] \in V$

$A \in \Gamma$

Constructing a Context-Free Grammar
Given a PA That Accepts By Empty Stack

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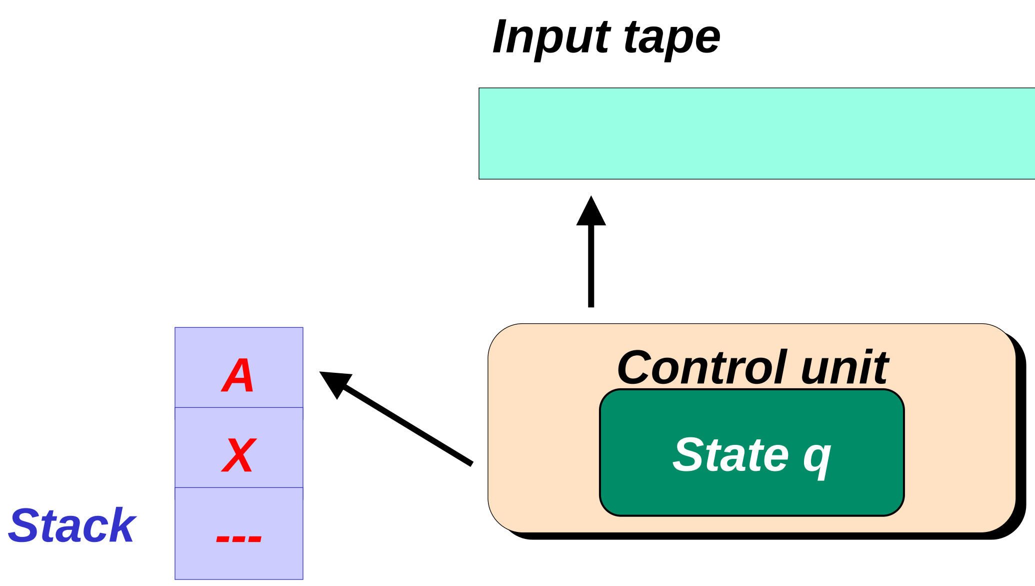

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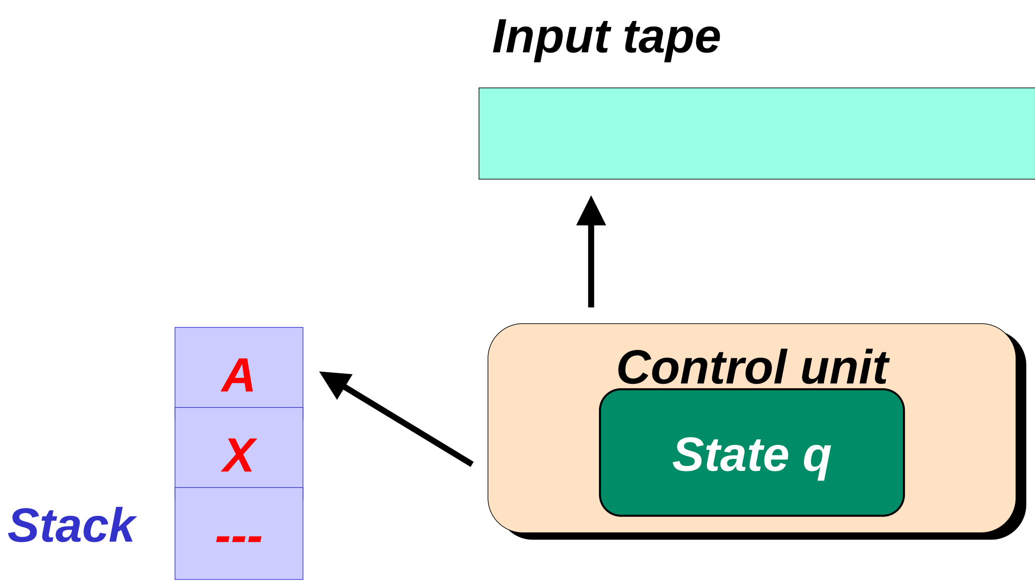

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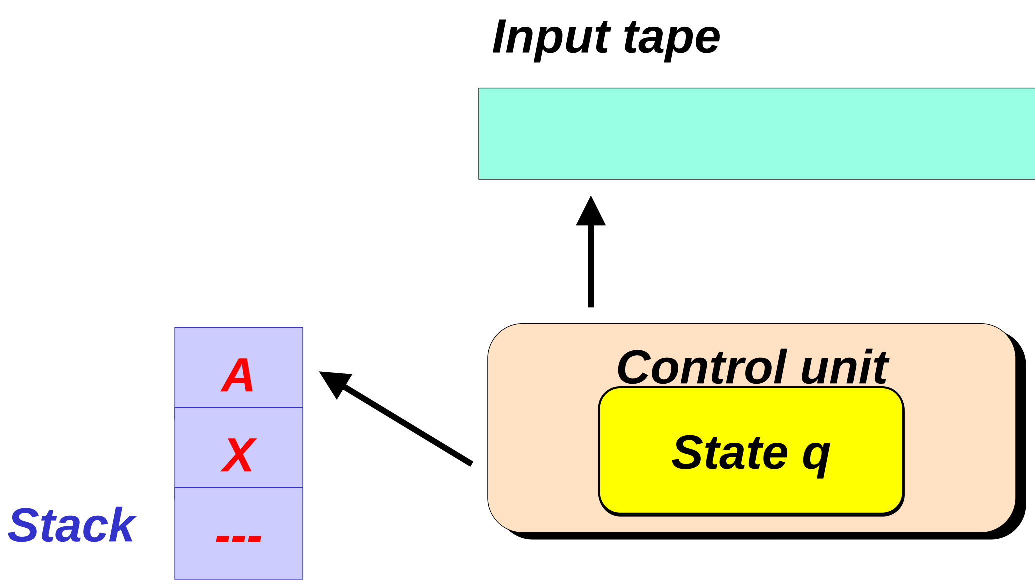

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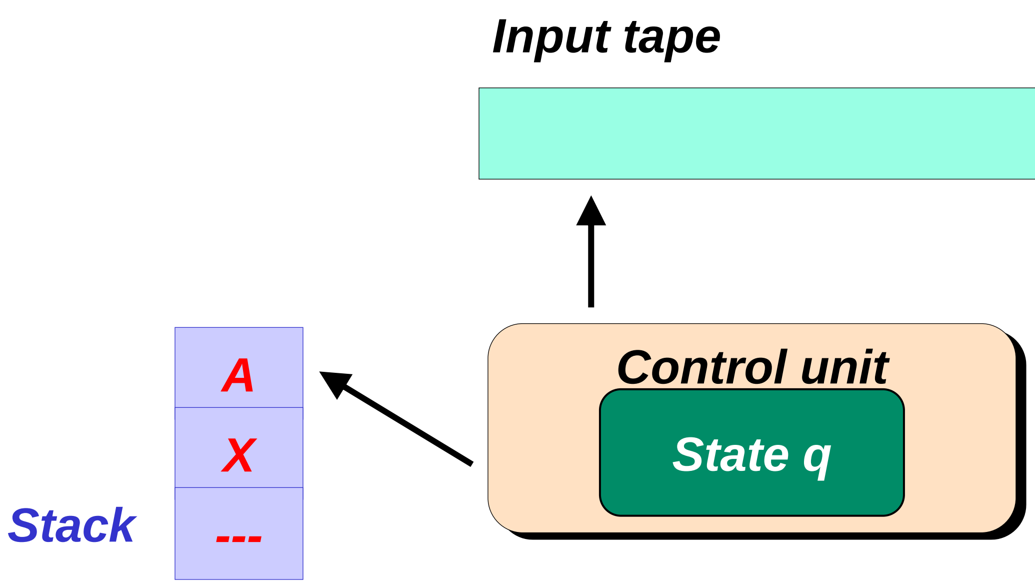

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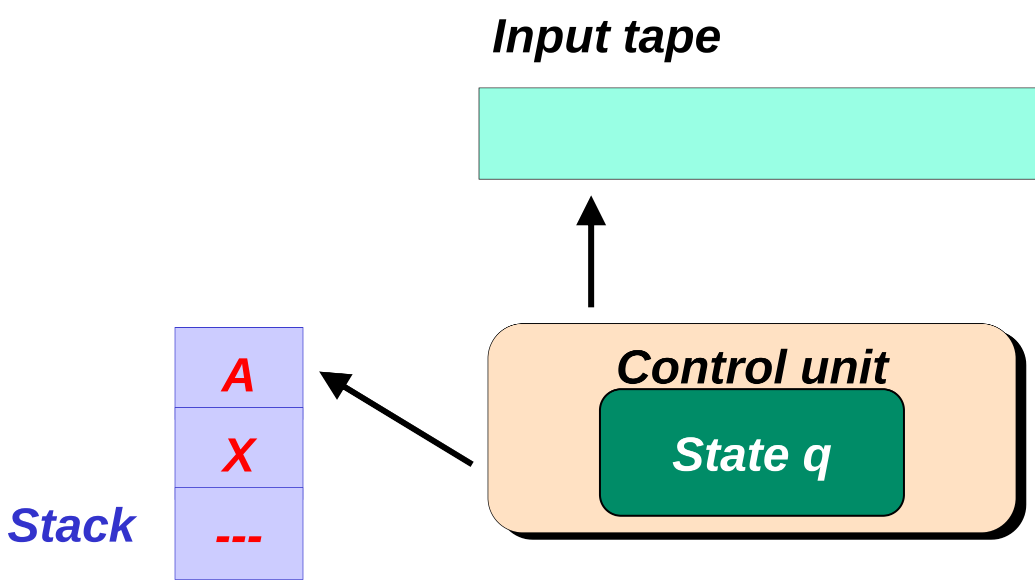

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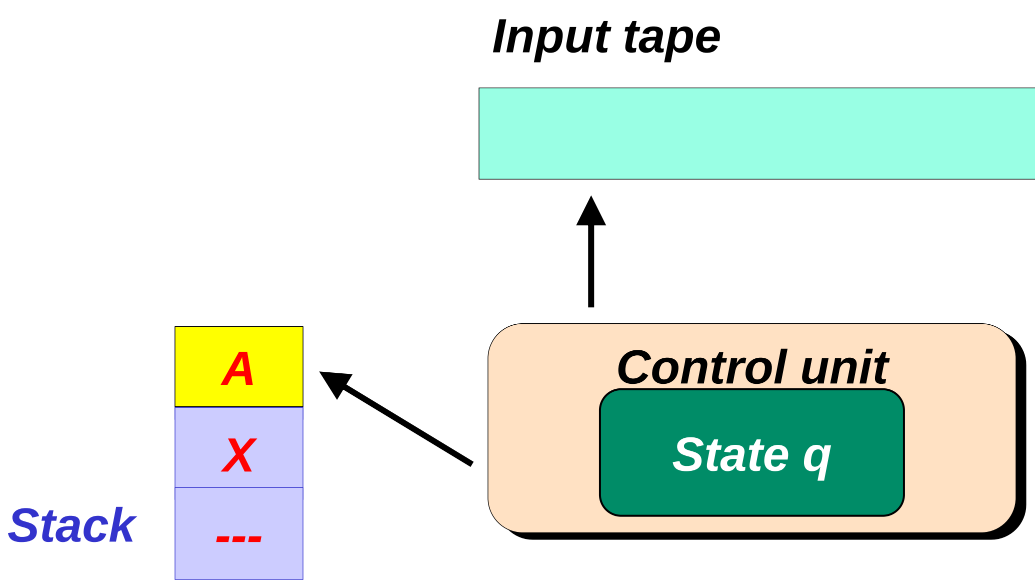

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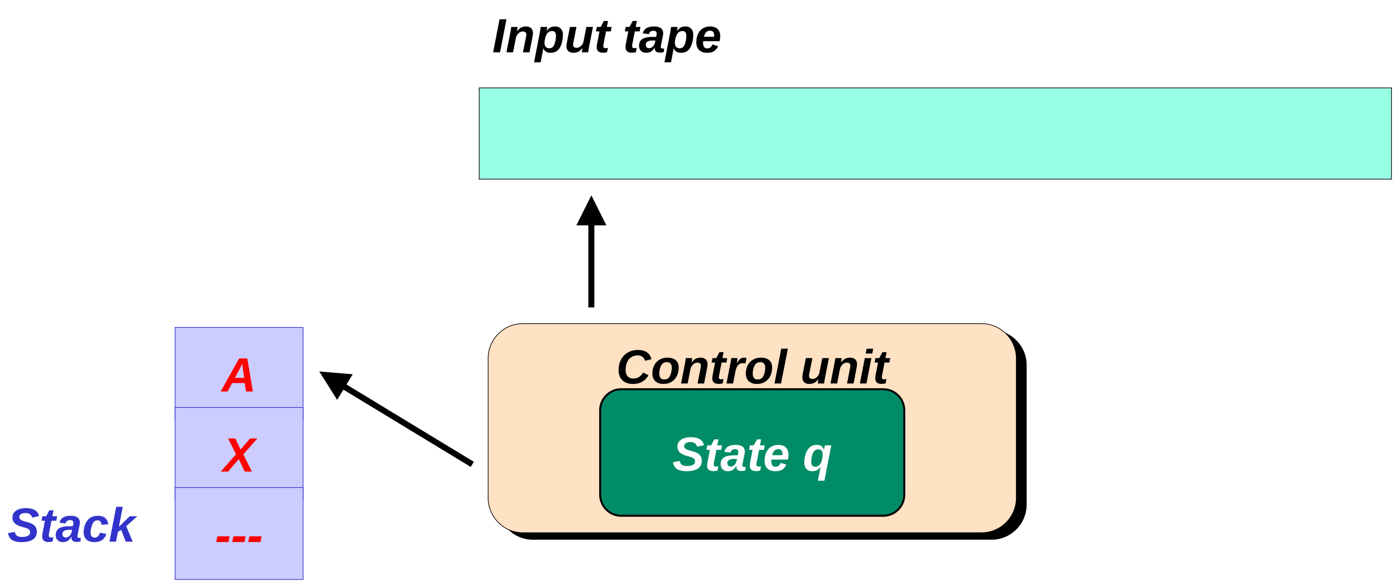

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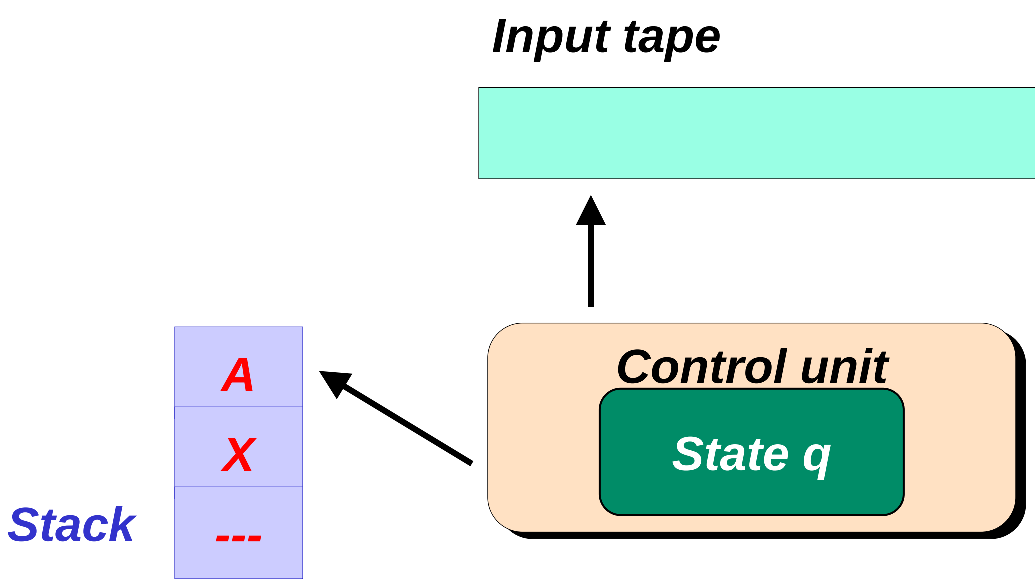

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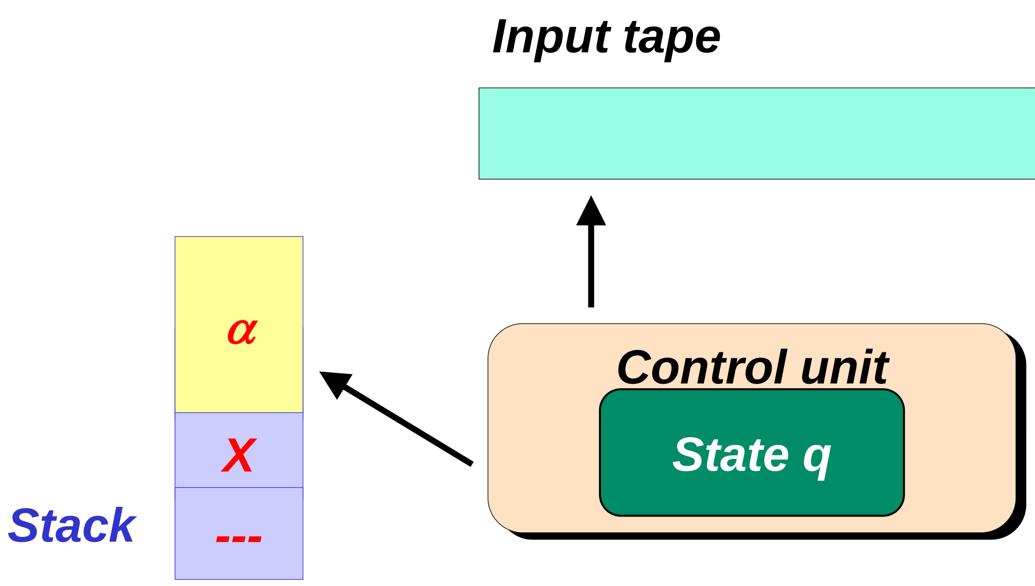

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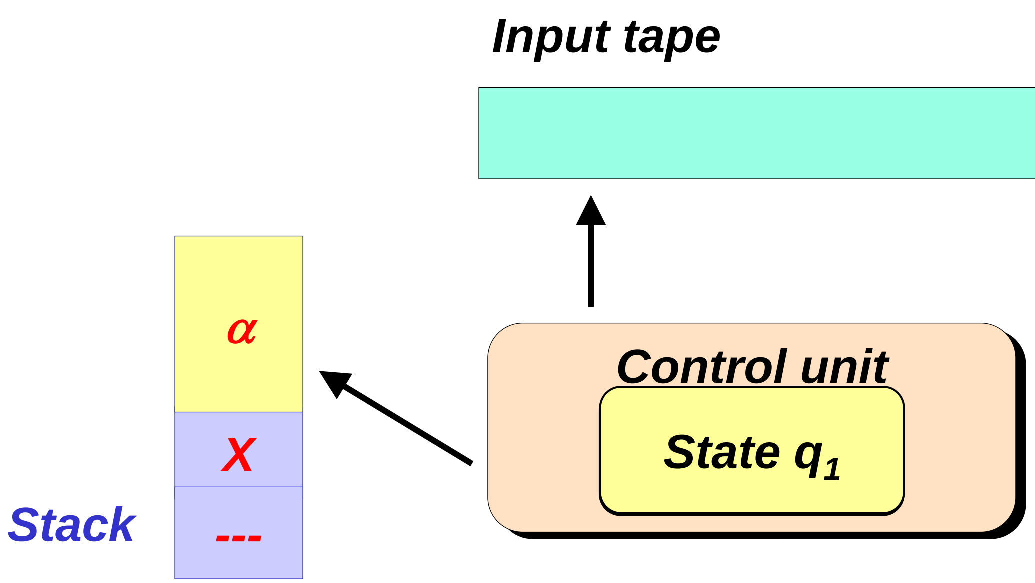

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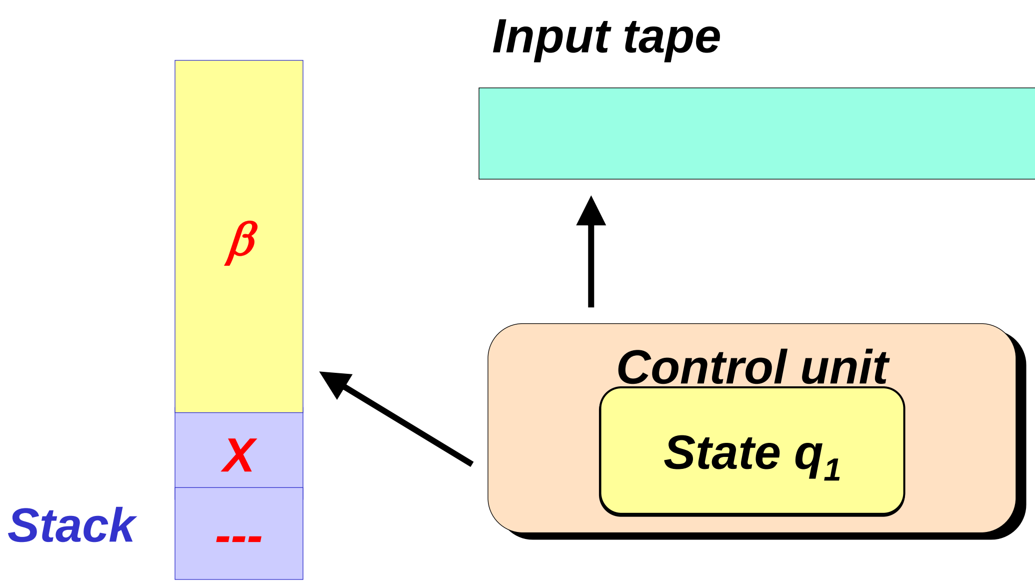

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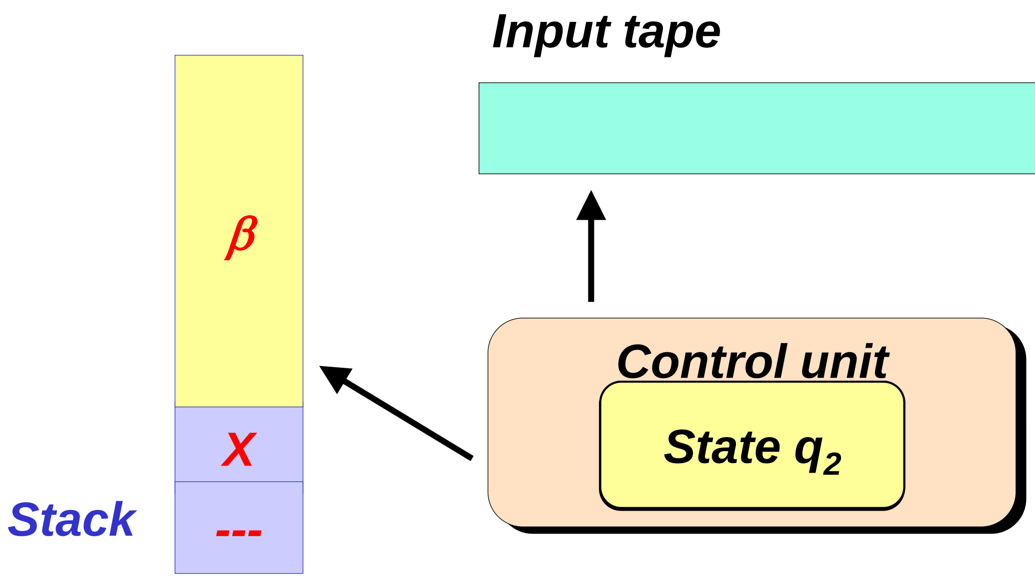

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

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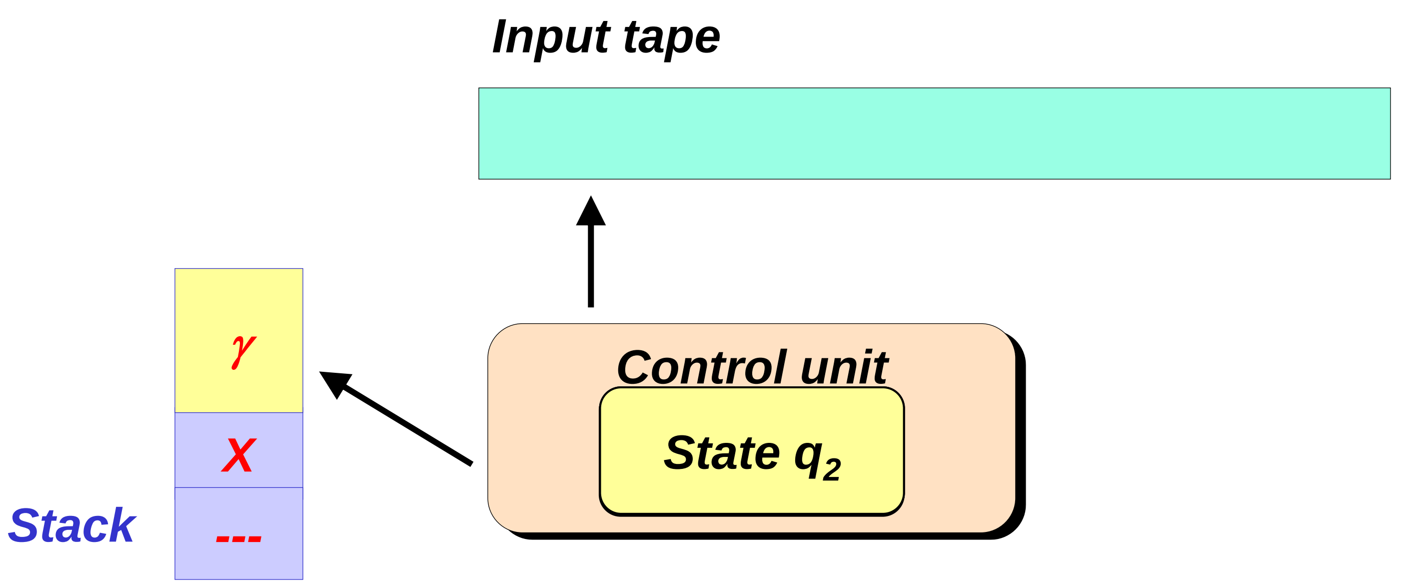

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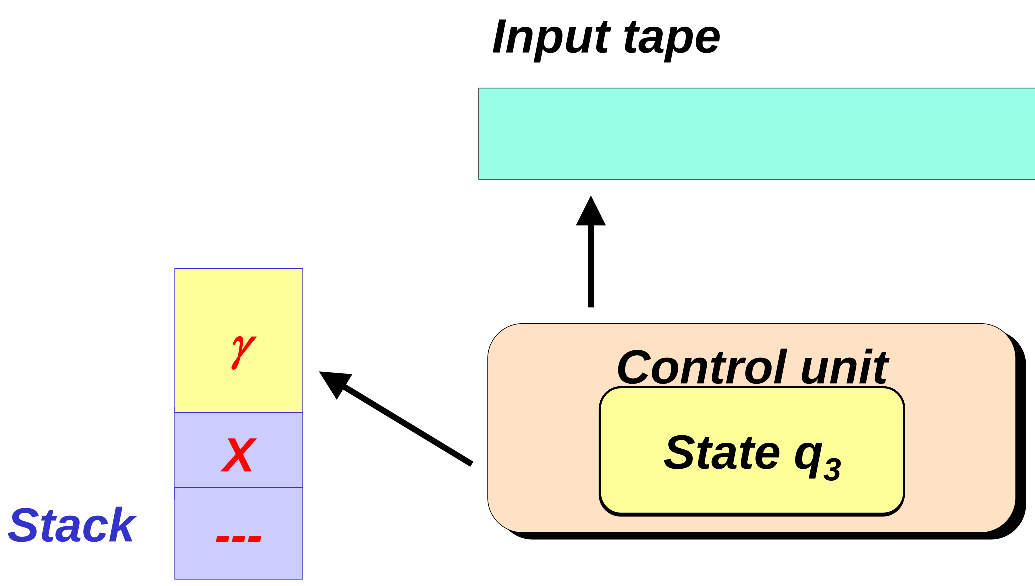

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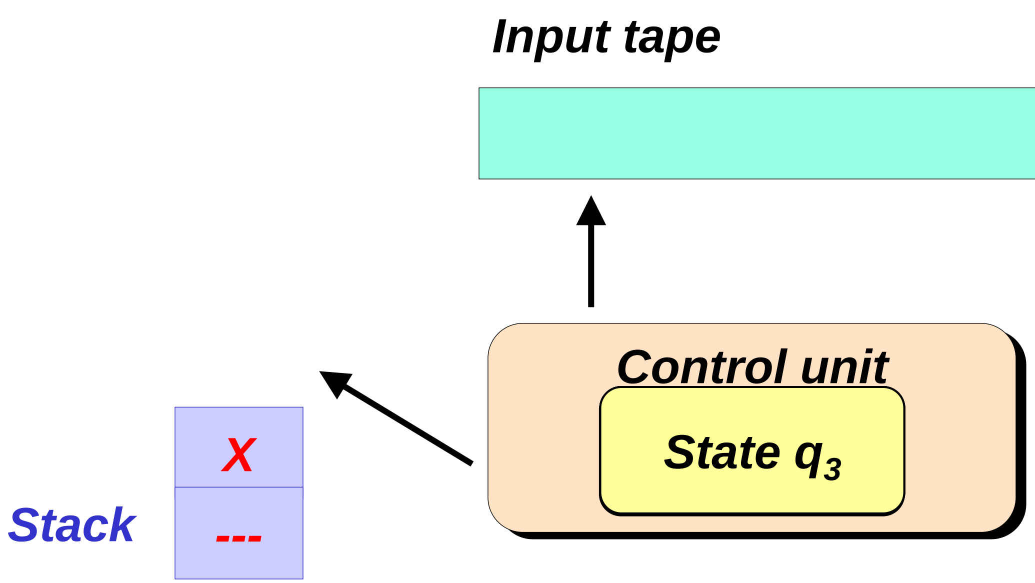

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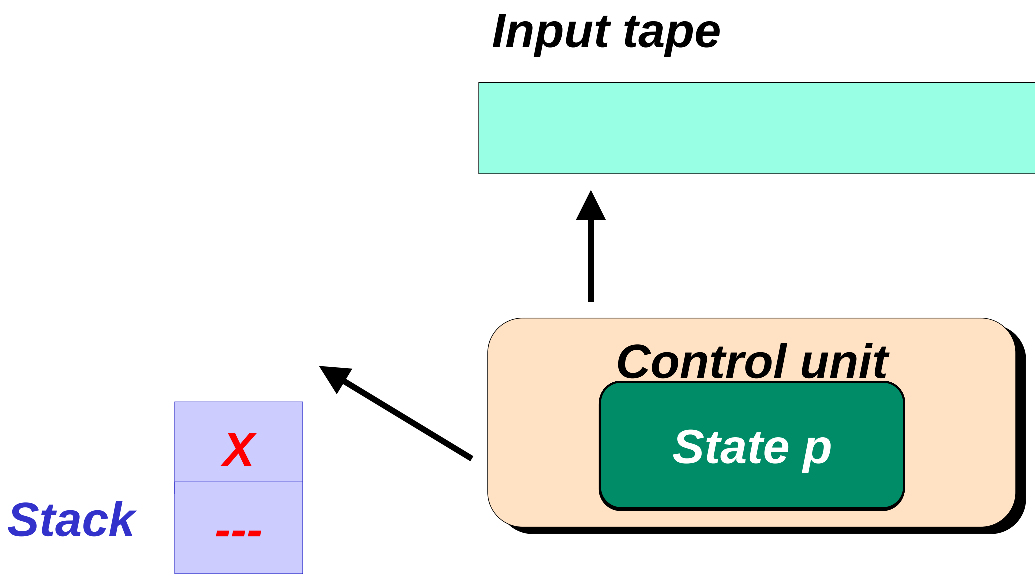

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- - -

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$$S \rightarrow [q_0, Z_0, q_0]$$

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- - -

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- - -

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- - - $\qquad$ - - -

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- - -   - - -

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- - -

$$q \underset{M}{\succ} q_1 \underset{M}{\succ} q_n \underset{M}{\succ} q_n \quad \overset{*}{\underset{M}{\succ}} \quad q_n \underset{M}{\succ} q_n \underset{M}{\succ} q_{n\text{-}1}$$

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- - -

$$q \underset{M}{\succ} q_1 \underset{M}{\succ} q_n \underset{M}{\succ} q_n \quad \overset{*}{\underset{M}{\succ}} \quad q_n \underset{M}{\succ} q_n \underset{M}{\succ} q_{n-1}$$

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$$[q, A, q_1] \rightarrow a\ [q_1, B_1, q_1]\ [q_1, B_2, q_1] - - - [q_1, B_{m-1}, q_1]\ [q_1, B_m, q_1]$$

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Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G = (V, T, P, S)$

PA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$$(q_1, B_1B_2 - - - B_m) \in \delta(q, a, A)$$

$$[q, A, q_{m+1}] \rightarrow a\, [q_1, B_1, q_2]\, [q_2, B_2, q_3]\,]\, - - -\, [q_{m-1}, B_{m-1}, q_m]\, [q_m, B_m, q_{m+1}]$$

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$$[q, A, q_{m+1}] \to a\, [q_1, B_1, q_2]\, [q_2, B_2, q_3]\,] - - - [q_{m-1}, B_{m-1}, q_m]\, [q_m, B_m, q_{m+1}]$$

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$$(q, x, A) \quad \overset{*}{\underset{M}{\succ}}$$

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$$(\, q_1, \varepsilon) \in \delta(\, q, a, A \,)$$

$$[q, A, q_1] \rightarrow a$$

$$(\, q, x, A \,) \overset{*}{\underset{M}{\succ}} (\, p, \varepsilon, \varepsilon \,)$$

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$$[q, A, q_1] \to a$$

$$[\, q, A, p\,] \overset{*}{\underset{G}{\Rightarrow}} \qquad (q, x, A) \overset{*}{\underset{M}{\succ}} (p, \varepsilon, \varepsilon)$$

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$$(q_1, \varepsilon) \in \delta(q, a, A)$$

$$[q, A, q_1] \rightarrow a$$

$$[q, A, p] \overset{*}{\underset{G}{\Rightarrow}} x \qquad (q, x, A) \overset{*}{\underset{M}{\succ}} (p, \varepsilon, \varepsilon)$$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

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$V = \{\ S, [q_1, N, q_1], [q_1, N, q_2], [q_2, N, q_1], [q_2, N, q_2],$
$[q_1, K, q_1], [q_1, K, q_2], [q_2, K, q_1], [q_2, K, q_2]\ \}$

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ii) $[q_1, K, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

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2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$
4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

---

i) $S \to [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \to 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \to 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \to 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \to 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

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1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
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---

i) $S \rightarrow [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

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---

i) $S \to [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \to 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

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iv) $[q_1, N, q_1] \to 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \to 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \to 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

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---

i) $S \rightarrow [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

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Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

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---

i) $S \to [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \to 0\,[q_1, N, q_1]\,[q_1, K, q_1] \mid 0\,[q_1, N, q_2]\,[q_2, K, q_1]$

iii) $[q_1, K, q_2] \to 0\,[q_1, N, q_1]\,[q_1, K, q_2] \mid 0\,[q_1, N, q_2]\,[q_2, K, q_2]$

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$[q_1, N, q_2] \to 0\,[q_1, N, q_1]\,[q_1, N, q_2] \mid 0\,[q_1, N, q_2]\,[q_2, N, q_2]$

v) $[q_1, N, q_2] \to 1$

$[q_2, K, q_2] \to \varepsilon$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

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---

i) $S \to [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \to 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \to 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \to 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \to 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \to 1$

$[q_2, K, q_2] \to \varepsilon$

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---

i) $S \rightarrow [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

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6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

i) $S \rightarrow [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

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---

i) $S \rightarrow [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

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---

i) $S \to [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \to 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \to 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \to 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \to 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \to 1$

$[q_2, K, q_2] \to \varepsilon$

$[q_2, N, q_2] \to \varepsilon$

$[q_2, N, q_2] \to 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

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---

i) $S \to [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \to 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \to 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \to 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \to 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \to 1$

$[q_2, K, q_2] \to \varepsilon$

$[q_2, N, q_2] \to \varepsilon$

$[q_2, N, q_2] \to 1$

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---

i) $S \to [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \to 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \to 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \to 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \to 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \to 1$

$[q_2, K, q_2] \to \varepsilon$

$[q_2, N, q_2] \to \varepsilon$

$[q_2, N, q_2] \to 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

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6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

i) $S \rightarrow [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

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---

i) $S \rightarrow [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow 1$

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PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$
4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

---

i) $S \to [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \to 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \to 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \to 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \to 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \to 1$

$[q_2, K, q_2] \to \varepsilon$

$[q_2, N, q_2] \to \varepsilon$

$[q_2, N, q_2] \to 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$
4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

---

i) $S \to [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \to 0\ [q_1, N, q_1]\ [q_1, K, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_1]$

iii) $[q_1, K, q_2] \to 0\ [q_1, N, q_1]\ [q_1, K, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \to 0\ [q_1, N, q_1]\ [q_1, N, q_1] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_1]$

$[q_1, N, q_2] \to 0\ [q_1, N, q_1]\ [q_1, N, q_2] \mid 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \to 1$

$[q_2, K, q_2] \to \varepsilon$

$[q_2, N, q_2] \to \varepsilon$

$[q_2, N, q_2] \to 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

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3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$
4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

---

i) $S \rightarrow [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \rightarrow 0\,[q_1, N, q_1]\,[q_1, K, q_1] \mid 0\,[q_1, N, q_2]\,[q_2, K, q_1]$

iii) $[q_1, K, q_2] \rightarrow 0\,[q_1, N, q_1]\,[q_1, K, q_2] \mid 0\,[q_1, N, q_2]\,[q_2, K, q_2]$

iv) $[q_1, N, q_1] \rightarrow 0\,[q_1, N, q_1]\,[q_1, N, q_1] \mid 0\,[q_1, N, q_2]\,[q_2, N, q_1]$

$[q_1, N, q_2] \rightarrow 0\,[q_1, N, q_1]\,[q_1, N, q_2] \mid 0\,[q_1, N, q_2]\,[q_2, N, q_2]$

v) $[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
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3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$
4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

---

i) $S \to [q_1, K, q_1] \mid [q_1, K, q_2]$

ii) $[q_1, K, q_1] \to 0\,[q_1, N, q_1]\,[q_1, K, q_1]$

iii) $[q_1, K, q_2] \to 0\,[q_1, N, q_1]\,[q_1, K, q_2] \mid 0\,[q_1, N, q_2]\,[q_2, K, q_2]$

iv) $[q_1, N, q_1] \to 0\,[q_1, N, q_1]\,[q_1, N, q_1]$

$[q_1, N, q_2] \to 0\,[q_1, N, q_1]\,[q_1, N, q_2] \mid 0\,[q_1, N, q_2]\,[q_2, N, q_2]$

v) $[q_1, N, q_2] \to 1$

$[q_2, K, q_2] \to \varepsilon$

$[q_2, N, q_2] \to \varepsilon$

$[q_2, N, q_2] \to 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

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2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$
4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

---

i) $S \to [q_1, K, q_1]\ |\ [q_1, K, q_2]$

ii) $[q_1, K, q_1] \to 0\ [q_1, N, q_1]\ [q_1, K, q_1]$

iii) $[q_1, K, q_2] \to 0\ [q_1, N, q_1]\ [q_1, K, q_2]\ |\ 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \to 0\ [q_1, N, q_1]\ [q_1, N, q_1]$

$[q_1, N, q_2] \to 0\ [q_1, N, q_1]\ [q_1, N, q_2]\ |\ 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \to 1$

$[q_2, K, q_2] \to \varepsilon$

$[q_2, N, q_2] \to \varepsilon$

$[q_2, N, q_2] \to 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$
4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

---

i) $S \to [q_1, K, q_1]\ |\ [q_1, K, q_2]$

ii) $[q_1, K, q_1] \to 0\ [q_1, N, q_1]\ [q_1, K, q_1]$

iii) $[q_1, K, q_2] \to 0\ [q_1, N, q_1]\ [q_1, K, q_2]\ |\ 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \to 0\ [q_1, N, q_1]\ [q_1, N, q_1]$

$[q_1, N, q_2] \to 0\ [q_1, N, q_1]\ [q_1, N, q_2]\ |\ 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \to 1$

$[q_2, K, q_2] \to \varepsilon$

$[q_2, N, q_2] \to \varepsilon$

$[q_2, N, q_2] \to 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$	4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$	5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$	6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

---

i) $S \rightarrow [q_1, K, q_1] \mid [q_1, K, q_2]$

iii) $[q_1, K, q_2] \rightarrow 0\, [q_1, N, q_1]\, [q_1, K, q_2] \mid 0\, [q_1, N, q_2]\, [q_2, K, q_2]$

iv) $[q_1, N, q_1] \rightarrow 0\, [q_1, N, q_1]\, [q_1, N, q_1]$

$[q_1, N, q_2] \rightarrow 0\, [q_1, N, q_1]\, [q_1, N, q_2] \mid 0\, [q_1, N, q_2]\, [q_2, N, q_2]$

v) $[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$
4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

---

i) $S \to [q_1, K, q_2]$

iii) $[q_1, K, q_2] \to 0\ [q_1, N, q_1]\ [q_1, K, q_2]\ \mid\ 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \to 0\ [q_1, N, q_1]\ [q_1, N, q_1]$

$[q_1, N, q_2] \to 0\ [q_1, N, q_1]\ [q_1, N, q_2]\ \mid\ 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \to 1$

$[q_2, K, q_2] \to \varepsilon$

$[q_2, N, q_2] \to \varepsilon$

$[q_2, N, q_2] \to 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$	4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$	5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$	6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

---

i) $S \rightarrow [q_1, K, q_2]$

iii) $[q_1, K, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_2] \ \mid\ 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_1]$

$[q_1, N, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_2] \ \mid\ 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G = (V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$
2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$
4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

---

i) $S \rightarrow [q_1, K, q_2]$

iii) $[q_1, K, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, K, q_2]\ \mid\ 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

iv) $[q_1, N, q_1] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_1]$

$[q_1, N, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_2]\ \mid\ 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$	4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$	5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$	6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

i) $S \rightarrow [q_1, K, q_2]$

iii) $[q_1, K, q_2] \rightarrow 0\,[q_1, N, q_1]\,[q_1, K, q_2] \mid 0\,[q_1, N, q_2]\,[q_2, K, q_2]$

$[q_1, N, q_2] \rightarrow 0\,[q_1, N, q_1]\,[q_1, N, q_2] \mid 0\,[q_1, N, q_2]\,[q_2, N, q_2]$

v) $[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$	4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$	5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$	6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

i) $S \rightarrow [q_1, K, q_2]$

iii) $[q_1, K, q_2] \rightarrow 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

$[q_1, N, q_2] \rightarrow 0\ [q_1, N, q_1]\ [q_1, N, q_2]\ \mid\ 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

v) $[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

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2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$	5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$	6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

---

i) $S \rightarrow [q_1, K, q_2]$

iii) $[q_1, K, q_2] \rightarrow 0\,[q_1, N, q_2]\,[q_2, K, q_2]$

$[q_1, N, q_2] \rightarrow 0\,[q_1, N, q_2]\,[q_2, N, q_2]$

v) $[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

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2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$	5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$	6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

$S \rightarrow [q_1, K, q_2]$

$[q_1, K, q_2] \rightarrow 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

$[q_1, N, q_2] \rightarrow 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

$[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow 1$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

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PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

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2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$	5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$	6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

$S \rightarrow [q_1, K, q_2]$

$[q_1, K, q_2] \rightarrow 0\ [q_1, N, q_2]\ [q_2, K, q_2]$

$[q_1, N, q_2] \rightarrow 0\ [q_1, N, q_2]\ [q_2, N, q_2]$

$[q_1, N, q_2] \rightarrow 1$

$[q_2, K, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow \varepsilon$

$[q_2, N, q_2] \rightarrow 1$

$(q_1, 00011, K) \succ (q_1, 0011, NK) \succ (q_1, 011, NNK) \succ (q_1, 11, NNNK) \succ$
$(q_2, 1, NNK) \succ (q_2, \varepsilon, NK) \succ (q_2, \varepsilon, K) \succ (q_2, \varepsilon, \varepsilon)$

Constructing a Context-Free Grammar Given a PA That Accepts By Empty Stack

$G=(V, \{0, 1\}, P, S)$

PA $M = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta, q_1, K, \{\ \})$

1) $\delta(q_1, 0, K) = \{(q_1, NK)\}$	4) $\delta(q_2, 1, N) = \{(q_2, \varepsilon)\}$
2) $\delta(q_1, 0, N) = \{(q_1, NN)\}$	5) $\delta(q_2, \varepsilon, N) = \{(q_2, \varepsilon)\}$
3) $\delta(q_1, 1, N) = \{(q_2, \varepsilon)\}$	6) $\delta(q_2, \varepsilon, K) = \{(q_2, \varepsilon)\}$

$$S \to [q_1, K, q_2]$$
$$[q_1, K, q_2] \to 0\,[q_1, N, q_2]\,[q_2, K, q_2]$$
$$[q_1, N, q_2] \to 0\,[q_1, N, q_2]\,[q_2, N, q_2]$$

$$[q_1, N, q_2] \to 1$$
$$[q_2, K, q_2] \to \varepsilon$$
$$[q_2, N, q_2] \to \varepsilon$$
$$[q_2, N, q_2] \to 1$$

$$(q_1, 00011, K) \succ (q_1, 0011, NK) \succ (q_1, 011, NNK) \succ (q_1, 11, NNNK) \succ$$
$$(q_2, 1, NNK) \succ (q_2, \varepsilon, NK) \succ (q_2, \varepsilon, K) \succ (q_2, \varepsilon, \varepsilon)$$

$$S \Rightarrow [q_1,K,q_2] \Rightarrow 0[q_1,N,q_2][q_2,K,q_2] \Rightarrow 00[q_1,N,q_2][q_2,N,q_2][q_2,K,q_2] \Rightarrow$$
$$000[q_1,N,q_2][q_2,N,q_2][q_2,N,q_2][q_2,K,q_2] \Rightarrow 0001[q_2,N,q_2][q_2,N,q_2][q_2,K,q_2] \Rightarrow$$
$$00011[q_2,N,q_2][q_2,K,q_2] \Rightarrow 00011\varepsilon[q_2,K,q_2] \Rightarrow 00011\varepsilon\varepsilon \Rightarrow 00011$$

Lecture 10

Properties of Context-Free Languages

Properties of Context-Free Languages

Languages that are not context-free

Properties of Context-Free Languages

Languages that are not context-free

It is not possible to define them using a context-free grammar

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$$L = \{ a^i b^i c^i \mid i \geq 1 \}$$

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2^{Σ^*} set of all languages
given an input symbol set Σ

Properties of Context-Free Languages

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$$L = \{ a^i b^i c^i \mid i \geq 1\}$$

2^{Σ^*} set of all languages given an input symbol set Σ

Context-free languages

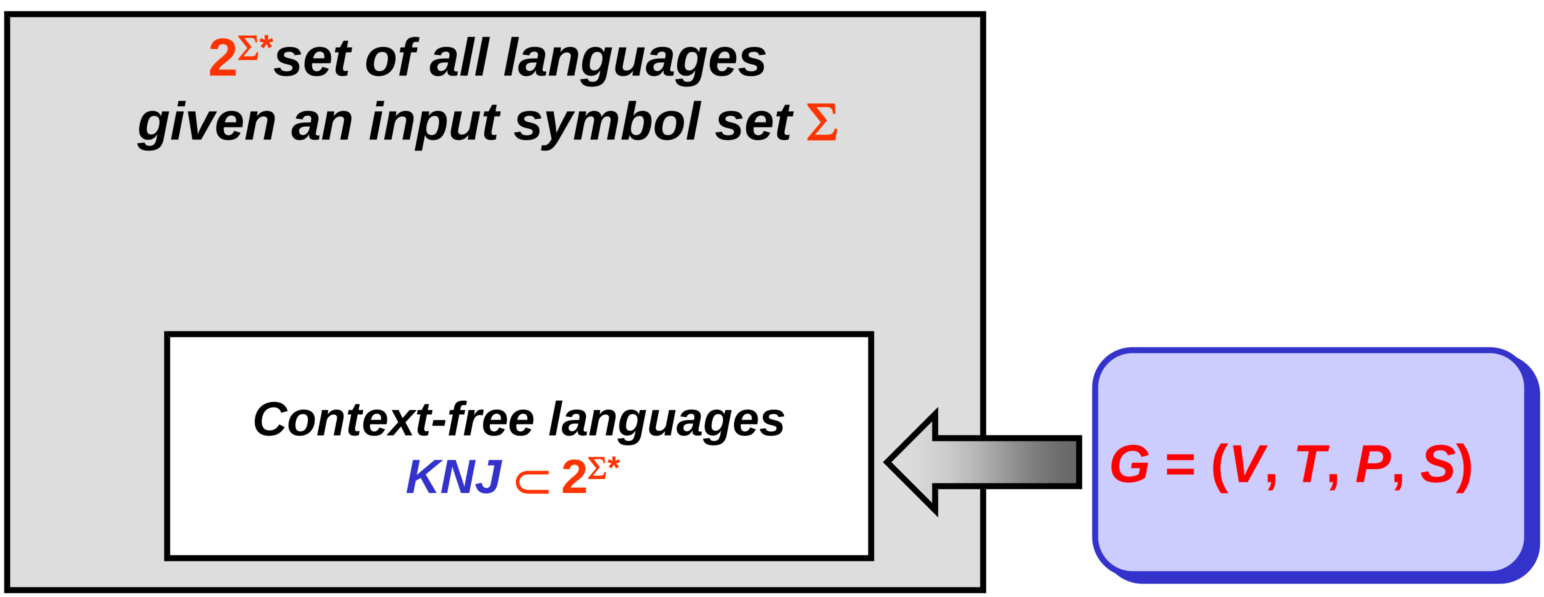
$KNJ \subset 2^{\Sigma^*}$

Properties of Context-Free Languages

Languages that are not context-free

It is not possible to define them using a context-free grammar

$$L = \{\, a^i\, b^i\, c^i \mid i \geq 1 \}$$



Properties of Context-Free Languages

Properties of Context-Free Languages

Finite automata

Properties of Context-Free Languages

Finite automata

Regular languages*: *REG

Properties of Context-Free Languages

Deterministic pushdown automata

Finite automata

Regular languages: *REG*

Properties of Context-Free Languages

Deterministic pushdown automata

Finite automata

Deterministic context-free languages : *DCFL*

Language L_3: { $w2w^R$ | $w \in (0+1)^*$ i $|w|>1$ }

$L_3 \in DKNJ$ i $L_3 \notin REG$

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Properties of Context-Free Languages

Nondeterministic pushdown automata

Deterministic pushdown automata

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Properties of Context-Free Languages

Nondeterministic pushdown automata

Deterministic pushdown automata

Finite automata

Nondeterministic context-free languages: *NCFL*

Language L_2: { ww^R | $w \in (0+1)^*$ i $|w|>1$ }

$L_2 \in$ *NKNJ* i $L_2 \notin$ *DKNJ*

Deterministic context-free languages : *DCFL*

Language L_3: { $w2w^R$ | $w \in (0+1)^*$ i $|w|>1$ }

$L_3 \in$ *DKNJ* i $L_3 \notin$ *REG*

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Properties of Context-Free Languages

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- **Union**

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  - $G_1 = (V_1, T_1, P_1, S_1)$, $G_2 = (V_2, T_2, P_2, S_2)$

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Properties of Context-Free Languages

Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

1) **Proof** $(L(G_1) \cup L(G_2)) \subseteq L(G_3)$

Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

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Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

1) Proof $(L(G_1) \cup L(G_2)) \subseteq L(G_3)$

(*a*)

2) Proof $L(G_3) \subseteq (L(G_1) \cup L(G_2))$

Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

1) **Proof** $(L(G_1) \cup L(G_2)) \subseteq L(G_3)$

(*a*) S_3

2) **Proof** $L(G_3) \subseteq (L(G_1) \cup L(G_2))$

Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

1) **Proof** $(L(G_1) \cup L(G_2)) \subseteq L(G_3)$

(*a*) $S_3 \underset{G_3}{\Rightarrow}$

2) **Proof** $L(G_3) \subseteq (L(G_1) \cup L(G_2))$

Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

1) Proof $(L(G_1) \cup L(G_2)) \subseteq L(G_3)$

(*a*) $S_3 \underset{G_3}{\Rightarrow} S_1$

2) Proof $L(G_3) \subseteq (L(G_1) \cup L(G_2))$

Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

1) Proof $(L(G_1) \cup L(G_2)) \subseteq L(G_3)$

$$(a) \quad S_3 \underset{G_3}{\Rightarrow} S_1 \underset{G_1}{\overset{*}{\Rightarrow}}$$

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Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

1) **Proof** $(L(G_1) \cup L(G_2)) \subseteq L(G_3)$

$$(a) \quad S_3 \underset{G_3}{\Rightarrow} S_1 \overset{*}{\underset{G_1}{\Rightarrow}} w$$

2) **Proof** $L(G_3) \subseteq (L(G_1) \cup L(G_2))$

Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

1) Proof $(L(G_1) \cup L(G_2)) \subseteq L(G_3)$

(*a*) $S_3 \underset{G_3}{\Rightarrow} S_1 \overset{*}{\underset{G_1}{\Rightarrow}} w$

(*b*)

2) Proof $L(G_3) \subseteq (L(G_1) \cup L(G_2))$

Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

1) **Proof** $(L(G_1) \cup L(G_2)) \subseteq L(G_3)$

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Properties of Context-Free Languages

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Properties of Context-Free Languages

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Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

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Properties of Context-Free Languages

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2) Proof $L(G_3) \subseteq (L(G_1) \cup L(G_2))$

$V_1 \cap V_2 = \emptyset$

Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

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2) Proof $L(G_3) \subseteq (L(G_1) \cup L(G_2))$

$$V_1 \cap V_2 = \varnothing$$

In step (a) only the productions of grammar G_1 are used

Properties of Context-Free Languages

Proof $L(G_3) = L(G_1) \cup L(G_2)$

1) Proof $(L(G_1) \cup L(G_2)) \subseteq L(G_3)$

$$(a) \quad S_3 \underset{G_3}{\Rightarrow} S_1 \overset{*}{\underset{G_1}{\Rightarrow}} w$$

$$(b) \quad S_3 \underset{G_3}{\Rightarrow} S_2 \overset{*}{\underset{G_2}{\Rightarrow}} w$$

2) Proof $L(G_3) \subseteq (L(G_1) \cup L(G_2))$

$$V_1 \cap V_2 = \varnothing$$

In step (*a*) only the productions of grammar G_1 are used

In step (*b*) only the productions of grammar G_2 are used

Properties of Context-Free Languages

Properties of Context-Free Languages

- **Concatenation**

Properties of Context-Free Languages

- **Concatenation**
  - $G_1 = (V_1, T_1, P_1, S_1)$, $G_2 = (V_2, T_2, P_2, S_2)$

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- $L(G_4) = L(G_1)\, L(G_2)$

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S_4

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$$S_4 \underset{G_4}{\Rightarrow}$$

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$$S_4 \underset{G_4}{\Rightarrow} S_1 S_2$$

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Properties of Context-Free Languages

- **Concatenation**
  - $G_1 = (V_1, T_1, P_1, S_1)$, $G_2 = (V_2, T_2, P_2, S_2)$
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$$S_4 \underset{G_4}{\Rightarrow} S_1\, S_2 \overset{*}{\underset{G_1}{\Rightarrow}} w_1\, S_2 \overset{*}{\underset{G_2}{\Rightarrow}}$$

Properties of Context-Free Languages

- **Concatenation**
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$$S_4 \underset{G_4}{\Rightarrow} S_1\, S_2 \underset{G_1}{\overset{*}{\Rightarrow}} w_1\, S_2 \underset{G_2}{\overset{*}{\Rightarrow}} w_1\, w_2$$

Properties of Context-Free Languages

Properties of Context-Free Languages

- **Kleen operator (closure) L^***

Properties of Context-Free Languages

- **Kleen operator (closure) L^***
  - $G_1 = (V_1, T_1, P_1, S_1)$

Properties of Context-Free Languages

- **Kleen operator (closure) L^***
  - $G_1 = (V_1, T_1, P_1, S_1)$

- $L(G_5) = L(G_1)^*$

Properties of Context-Free Languages

- Kleen operator (closure) L^*
  - $G_1 = (V_1, T_1, P_1, S_1)$

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  - $G_5 = (V_5, T_5, P_5, S_5)$

Properties of Context-Free Languages

- Kleen operator (closure) L^*
  - $G_1 = (V_1, T_1, P_1, S_1)$

- $L(G_5) = L(G_1)^*$
  - $G_5 = (V_5, T_5, P_5, S_5)$

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Properties of Context-Free Languages

- Kleen operator (closure) L^*
  - $G_1 = (V_1, T_1, P_1, S_1)$
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  - $S_2 \to 0\, S_2\, 0 \mid 2\, S_2\, 2 \mid \varepsilon$

---

1) $V' = \{S\} \cup \{S_1\} \cup \{S_2\} = \{\, S, S_1, S_2 \,\}$
2) $T' = T_1 \cup T_2 - - - T_k$
3) $S' = S$
4) Set of productions $P' = P_1 \cup P_2 - - - P_k$

   modified productions of grammar G are added:

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Properties of Context-Free Languages

- $S \rightarrow aSbS \mid bSaS \mid \varepsilon$

  - $a \in L(G)$ **are replaced by strings in** $L_1 = \{ 0^n 1^n \mid n \geq 1 \}$
  - $S_1 \rightarrow 0\ S_1\ 1 \mid 0\ 1$

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Properties of Context-Free Languages

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b

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$$L_1 = \{\, a^i b^i c^j \mid i \geq 1 \text{ i } j \geq 1 \,\}$$

$$L = L_1 \cap L_2$$

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Intersection of CFLs is *not* a CFL

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$$

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  - $L_1 \cap L_2 = (L^c_1 \cup L^c_2)^c$

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    - **—intersection of CFL's appears to be a CFL**

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Properties of Context-Free Languages

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- **Intersection of CFL with a regular language**

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- **Intersection of CFL with a regular language**
  - **L_1 accepts PA $M_1 = (Q_1, \Sigma, \Gamma, \delta_1, q_0, Z_1, F_1)$**

Properties of Context-Free Languages

- **Intersection of CFL with a regular language**
  - L_1 **accepts** **PA** M_1 $= (Q_1, \Sigma, \Gamma, \delta_1, q_0, Z_1, F_1)$
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DKA M_2 $- \delta_2(p, a) = p'$

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DKA M_2 - $\delta_2(p, a) = p'$

PA M_1 - $(q', \gamma) \in \delta_1(q, a, X)$

If a is ε, then $p' = p$

Properties of Context-Free Languages

Properties of Context-Free Languages

PA *M‘*

Properties of Context-Free Languages

PA *M'*

$$([\,p_0, q_0\,], w, Z_0)$$

Properties of Context-Free Languages

PA *M'*

$$([\,p_0, q_0\,], w, Z_0) \;\underset{M'}{\overset{*}{\succ}}$$

Properties of Context-Free Languages

PA *M'*

$$([p_0, q_0], w, Z_0) \underset{M'}{\overset{*}{\succ}} ([p, q], \varepsilon, \gamma)$$

Properties of Context-Free Languages

PA *M'*

$$([\,p_0,\, q_0\,],\, w,\, Z_0) \;\overset{*}{\underset{M'}{\succ}}\; ([p,\, q],\, \varepsilon,\, \gamma\,)$$

PA M_1

Properties of Context-Free Languages

PA *M'*

$$([p_0, q_0], w, Z_0) \overset{*}{\underset{M'}{\succ}} ([p, q], \varepsilon, \gamma)$$

PA M_1

$$(q_0, w, Z_0)$$

Properties of Context-Free Languages

PA *M'*

$$([\,p_0, q_0\,], w, Z_0) \ \overset{*}{\underset{M'}{\succ}} \ ([p, q], \varepsilon, \gamma)$$

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Properties of Context-Free Languages

PA M'

$$([p_0, q_0], w, Z_0) \underset{M'}{\overset{*}{\succ}} ([p, q], \varepsilon, \gamma)$$

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$$(q_0, w, Z_0) \underset{M_1}{\overset{*}{\succ}} (q, \varepsilon, \gamma)$$

Properties of Context-Free Languages

PA *M'*

$$([p_0, q_0], w, Z_0) \overset{*}{\underset{M'}{\succ}} ([p, q], \varepsilon, \gamma)$$

PA M_1

$$(q_0, w, Z_0) \overset{*}{\underset{M_1}{\succ}} (q, \varepsilon, \gamma)$$

DKA M_2

Properties of Context-Free Languages

PA M'

$$([\,p_0, q_0\,], w, Z_0) \ \underset{M'}{\overset{*}{\succ}} \ (\,[p, q], \varepsilon, \gamma\,)$$

PA M_1

$$(q_0, w, Z_0) \ \underset{M_1}{\overset{*}{\succ}} \ (q, \varepsilon, \gamma)$$

DKA M_2

$$\delta_2(p_0, w) = p$$

Properties of Context-Free Languages

1) $Q' = Q_2 \times Q_1$

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3) $F' = F_2 \times F_1$

Properties of Context-Free Languages

- PA $M_1 = (\{q_1, q_2\}, \{0, 1\}, \{N, K\}, \delta_1, q_1, K, \{q_2\})$

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$\delta_1(q_1, 0, K) = \{(q_1, NK)\}$ $\delta_1(q_1, 0, N) = \{(q_1, NN)\}$
$\delta_1(q_1, 1, N) = \{(q_2, \varepsilon)\}$ $\delta_1(q_2, 1, N) = \{(q_2, \varepsilon)\}$

---

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Properties of Context-Free Languages

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  - accepts language $L(M_1)=\{ 0^n1^m \mid n\geq 1, m\geq 1, m\leq n\}$

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- DKA $M_2 = (\{p_1, p_2, p_3\}, \{0, 1\}, \delta_2, p_1, \{p_3\})$

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2) $q'_0 = [\, p_0, q_0 \,]$
3) $F' = F_2 \times F_1$

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- **PA *M'***

Properties of Context-Free Languages

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- **PA *M'***
  - **accepts language $L_3 = L(M_1) \cap L(M_2) = \{0^n1^m \mid n \geq 2, m \geq 2, m \leq n\}$**

Properties of Context-Free Languages

Properties of Context-Free Languages

***String* 00011**

Properties of Context-Free Languages

***String* 00011**

DKA M_2

Properties of Context-Free Languages

String **00011**

DKA M_2

$\delta(p_1, 00011) = \delta(p_1, 0011) = \delta(p_1, 011) = \delta(p_1, 11) =$
$\delta(p_2, 1) = p_3$ i $p_3 \in F_2$

Properties of Context-Free Languages

***String* 00011**

DKA M_2

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PA M_1

Properties of Context-Free Languages

String **00011**

DKA M_2

$\delta(p_1, 00011) = \delta(p_1, 0011) = \delta(p_1, 011) = \delta(p_1, 11) =$
$\delta(p_2, 1) = p_3$ i $p_3 \in F_2$

PA M_1

$(q_1, 00011, K) \succ (q_1, 0011, NK) \succ (q_1, 011, NNK) \succ$
$(q_1, 11, NNNK) \succ (q_2, 1, NNK) \succ (q_2, \varepsilon, NKX_0)$ i $q_2 \in F_1$

Properties of Context-Free Languages

***String* 00011**

DKA M_2

$\delta(p_1, 00011) = \delta(p_1, 0011) = \delta(p_1, 011) = \delta(p_1, 11) =$
$\delta(p_2, 1) = p_3$ i $p_3 \in F_2$

PA M_1

$(q_1, 00011, K) \succ (q_1, 0011, NK) \succ (q_1, 011, NNK) \succ$
$(q_1, 11, NNNK) \succ (q_2, 1, NNK) \succ (q_2, \varepsilon, NKX_0)$ i $q_2 \in F_1$

PA *M'*

Properties of Context-Free Languages

String **00011**

DKA M_2

$\delta(p_1, 00011) = \delta(p_1, 0011) = \delta(p_1, 011) = \delta(p_1, 11) =$
$\delta(p_2, 1) = p_3$ i $p_3 \in F_2$

PA M_1

$(q_1, 00011, K) \succ (q_1, 0011, NK) \succ (q_1, 011, NNK) \succ$
$(q_1, 11, NNNK) \succ (q_2, 1, NNK) \succ (q_2, \varepsilon, NKX_0)$ i $q_2 \in F_1$

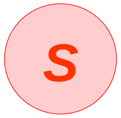
PA M'

$([p_1, q_1], 00011, K) \succ ([p_1, q_1], 0011, NK) \succ ([p_1, q_1], 011, NNK) \succ$
$([p_1, q_1], 11, NNNK) \succ ([p_2, q_2], 1, NNK) \succ$
$([p_3, q_2], \varepsilon, NK)$ i $[p_3, q_2] \in F'$

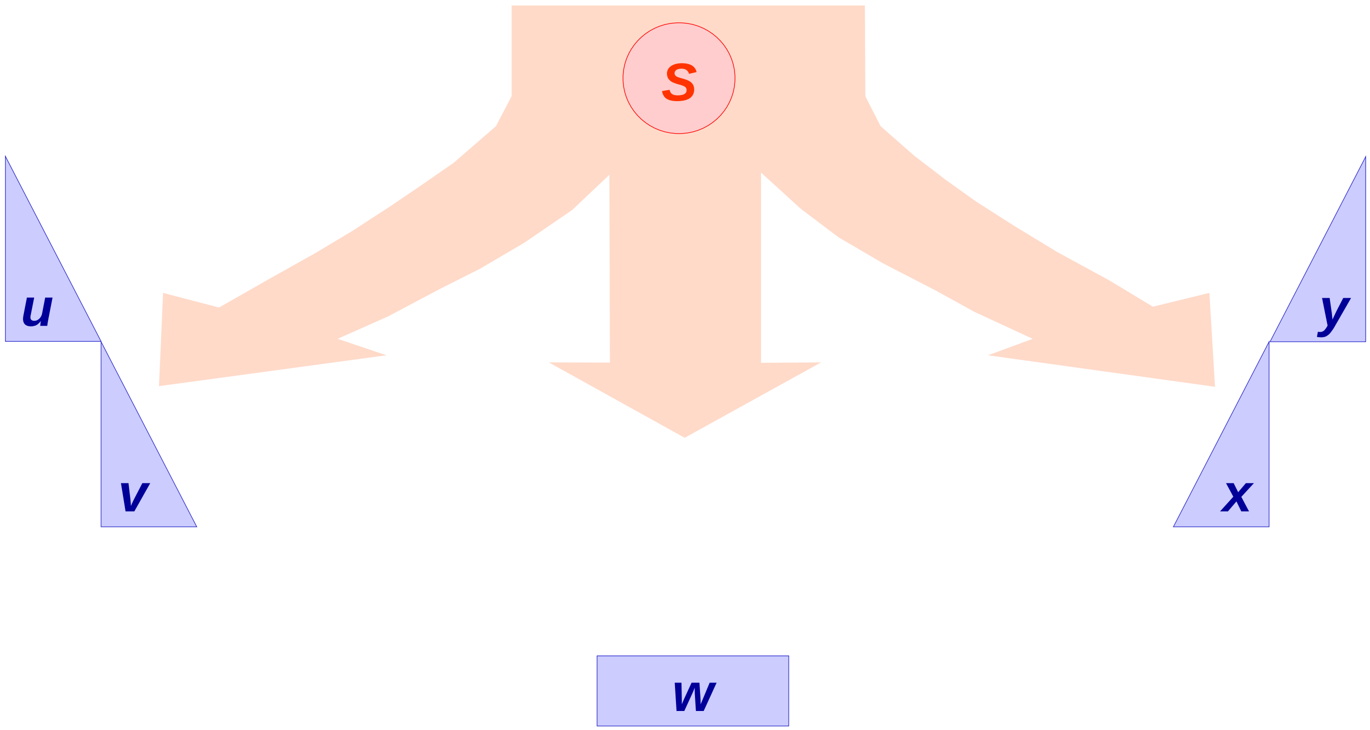
Lecture 10

Pumping Lemma

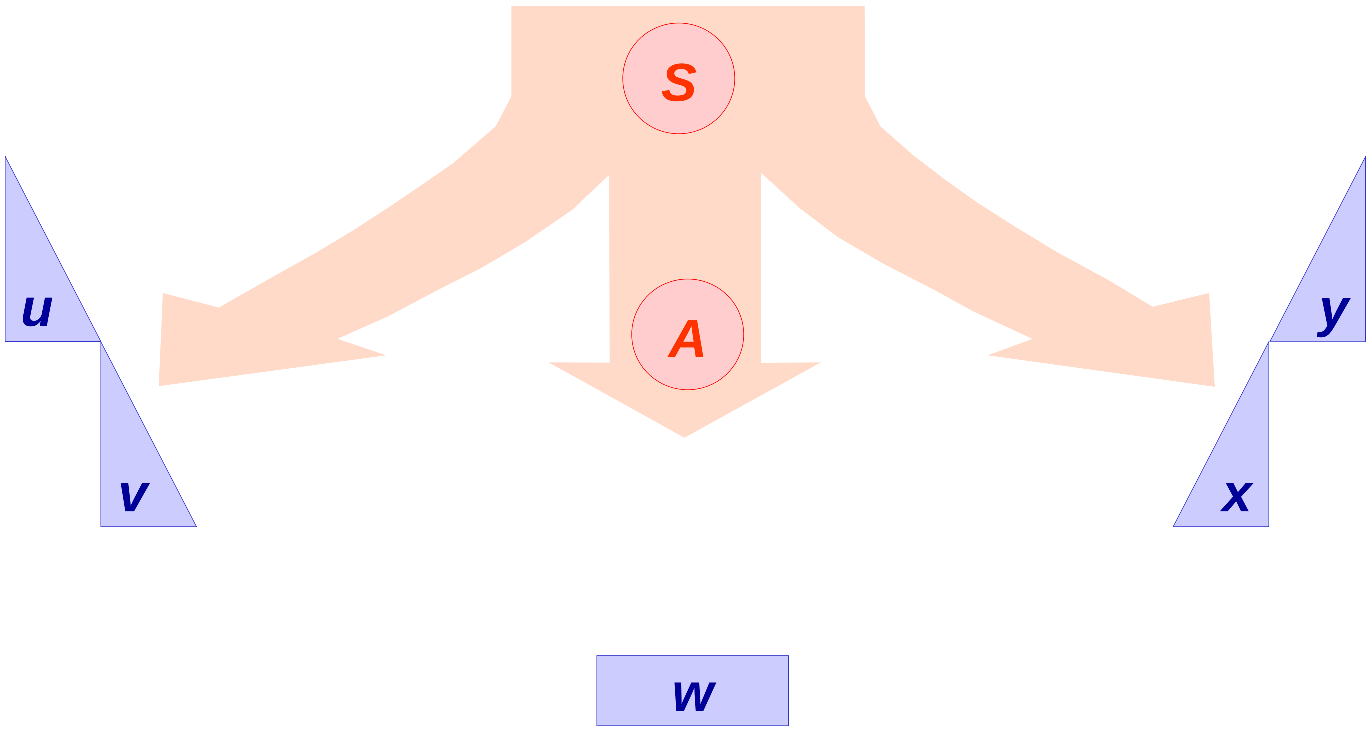
Pumping Lemma



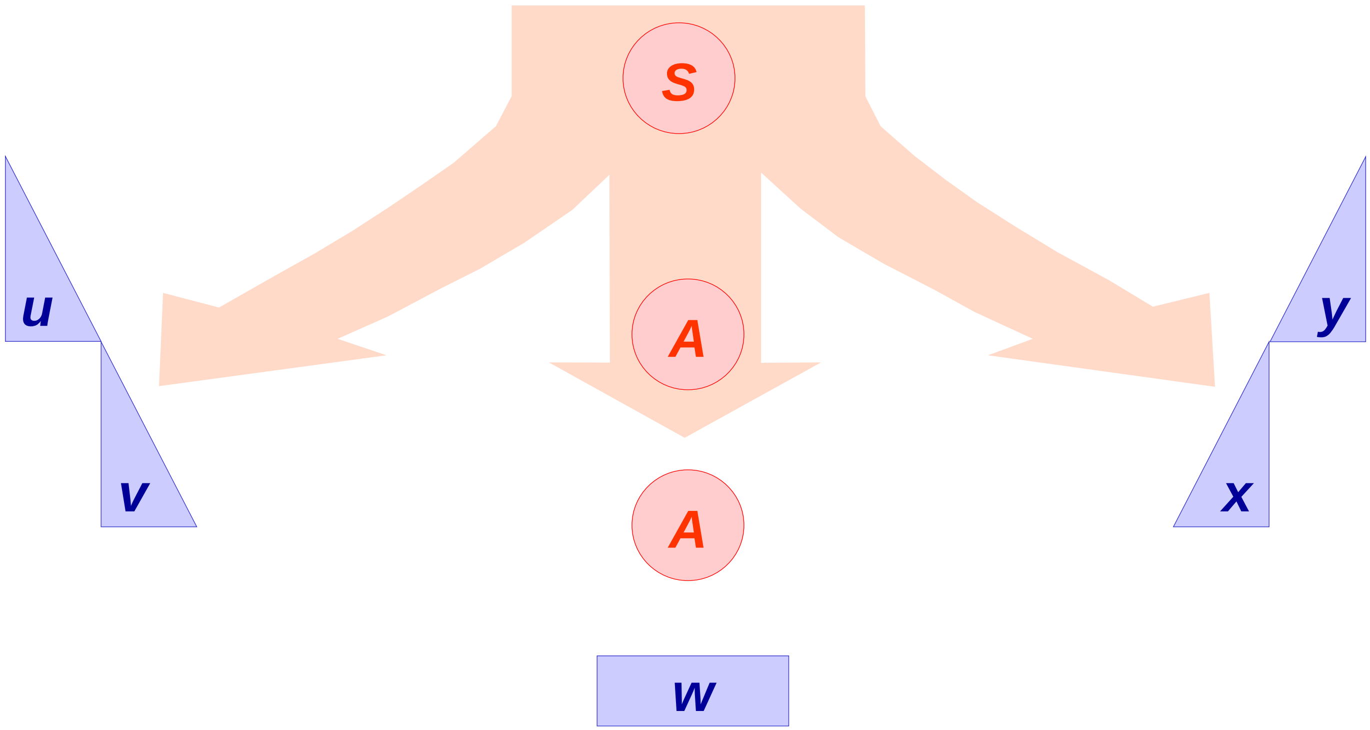
Pumping Lemma



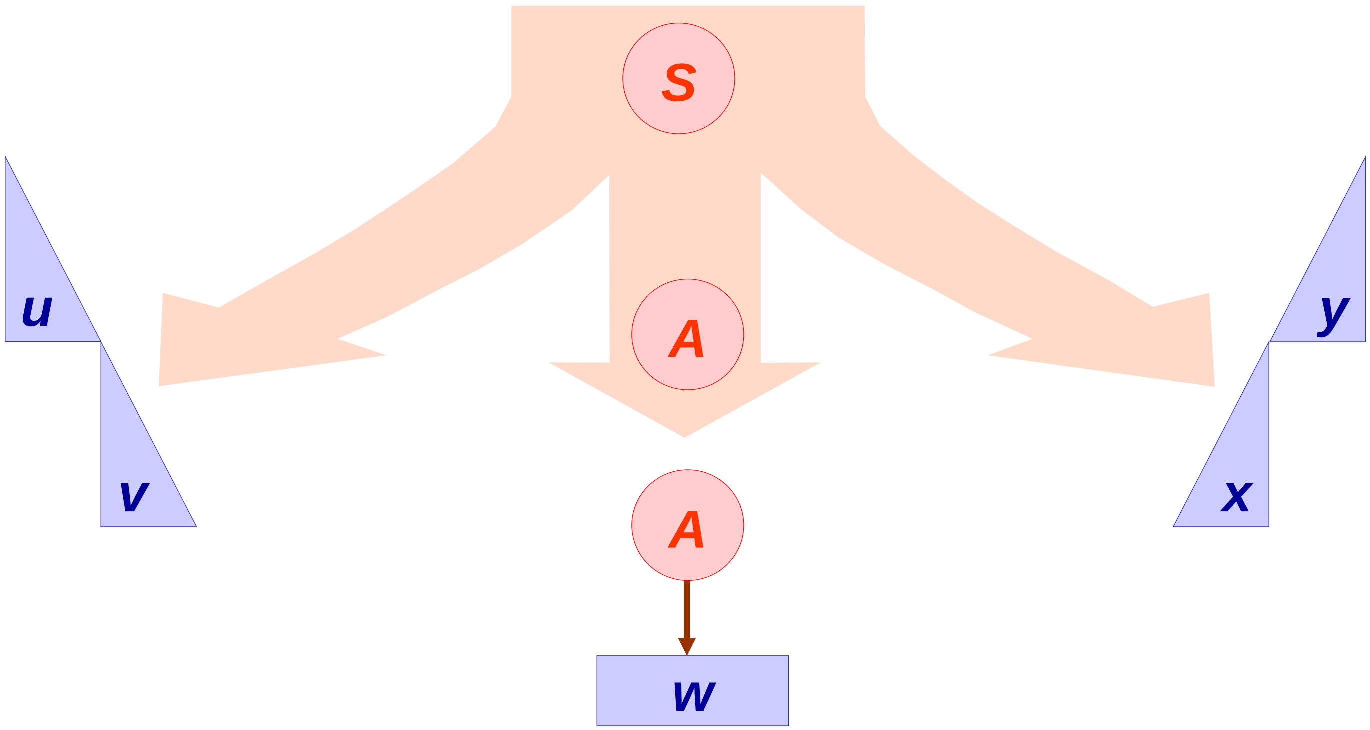
Pumping Lemma



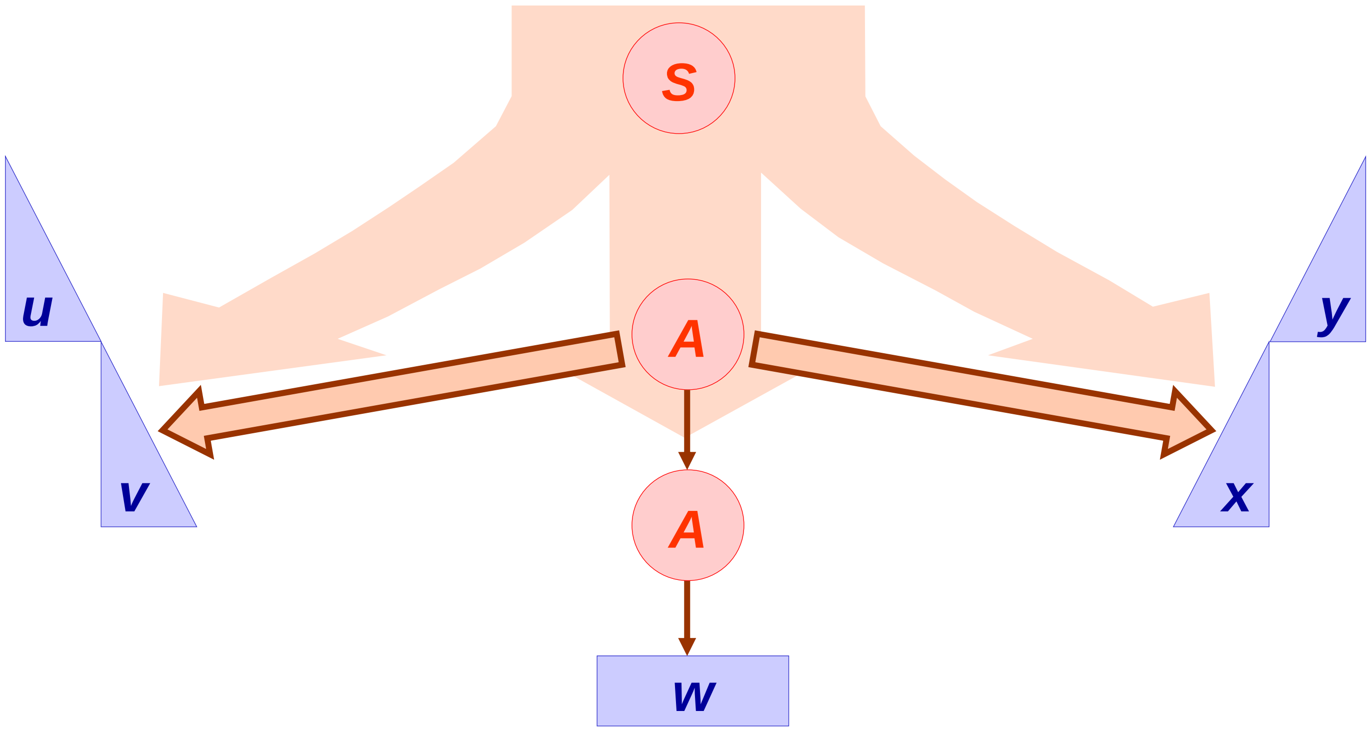
Pumping Lemma



Pumping Lemma

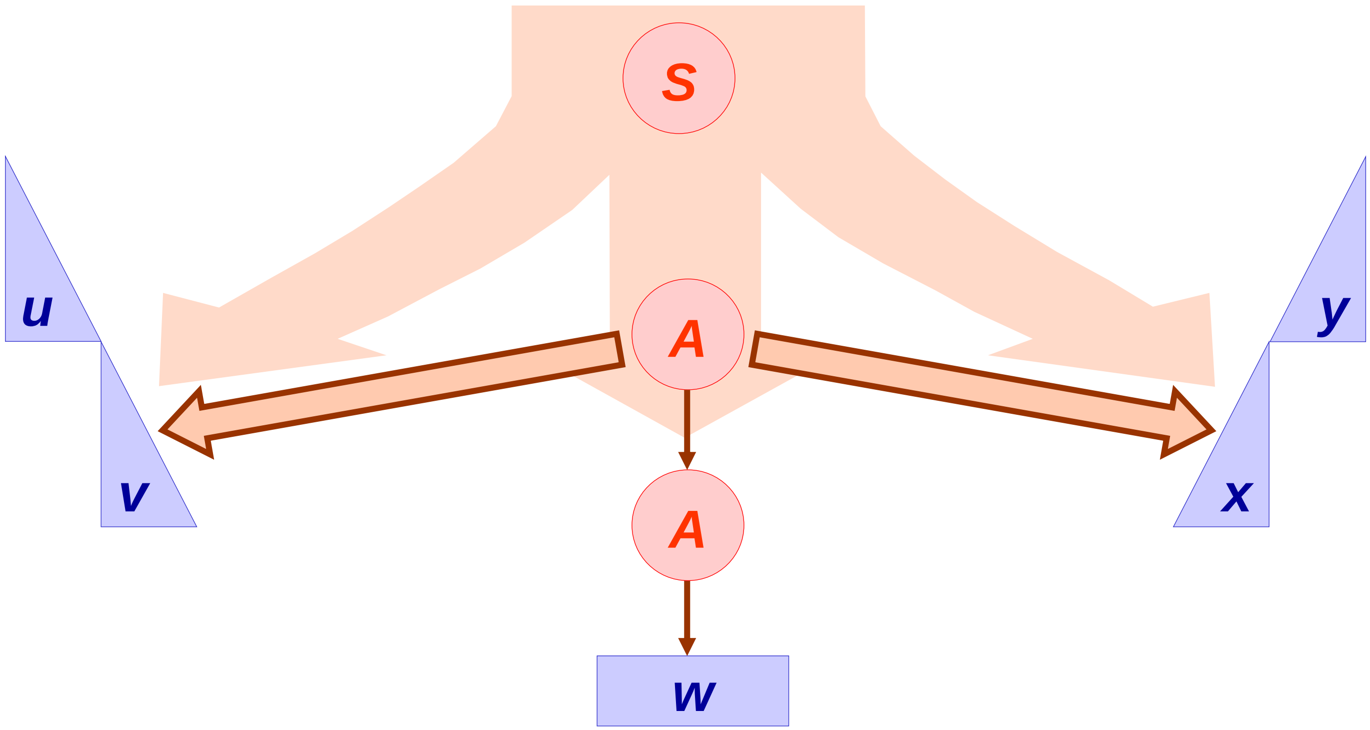


Pumping Lemma



Pumping Lemma

Number of parse tree inner nodes > Number of grammar variables



Pumping Lemma

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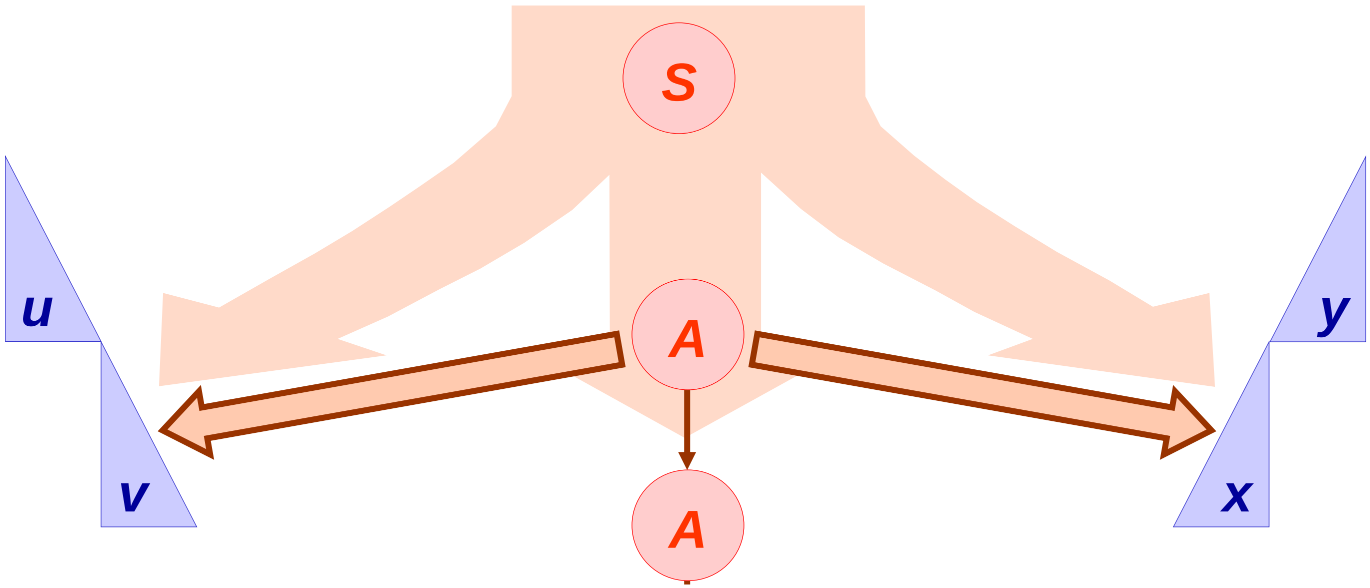

Pumping Lemma

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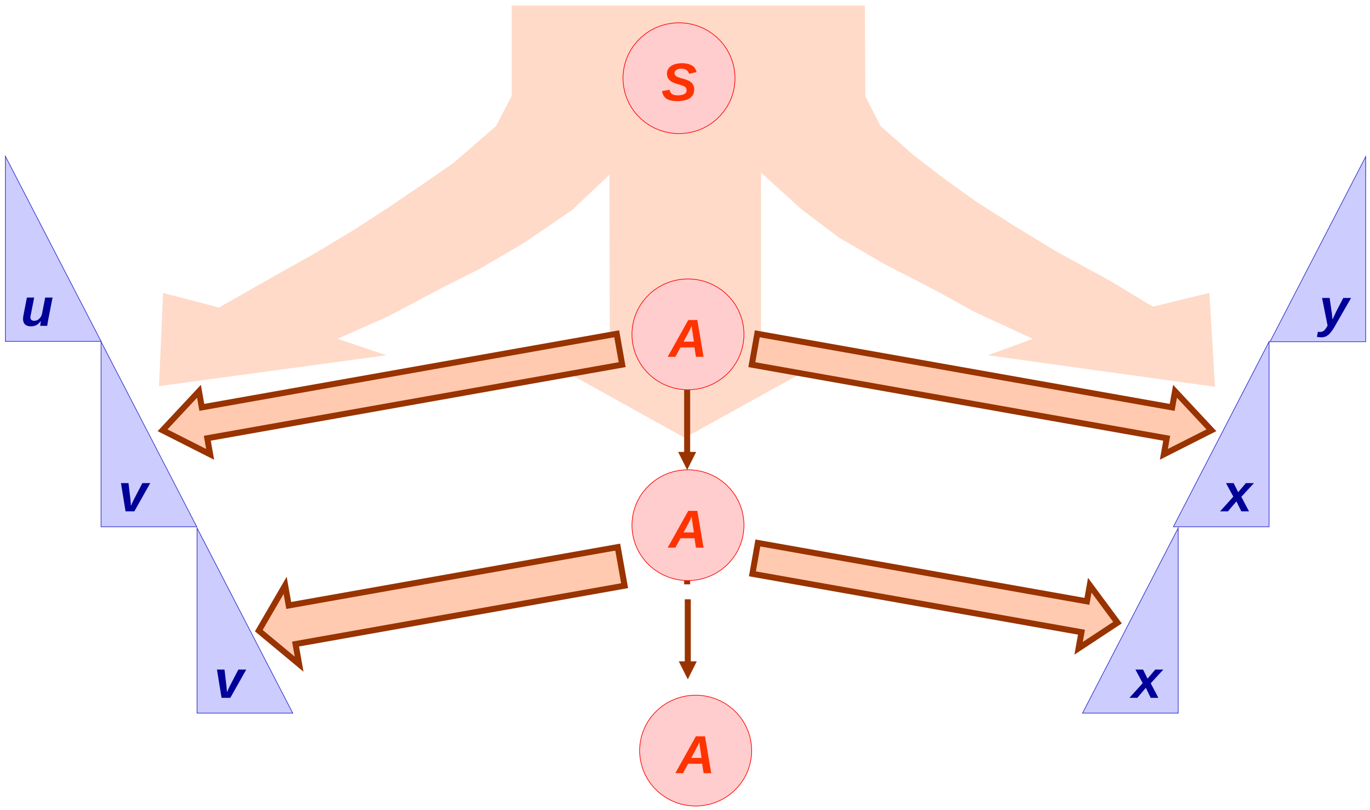

Pumping Lemma

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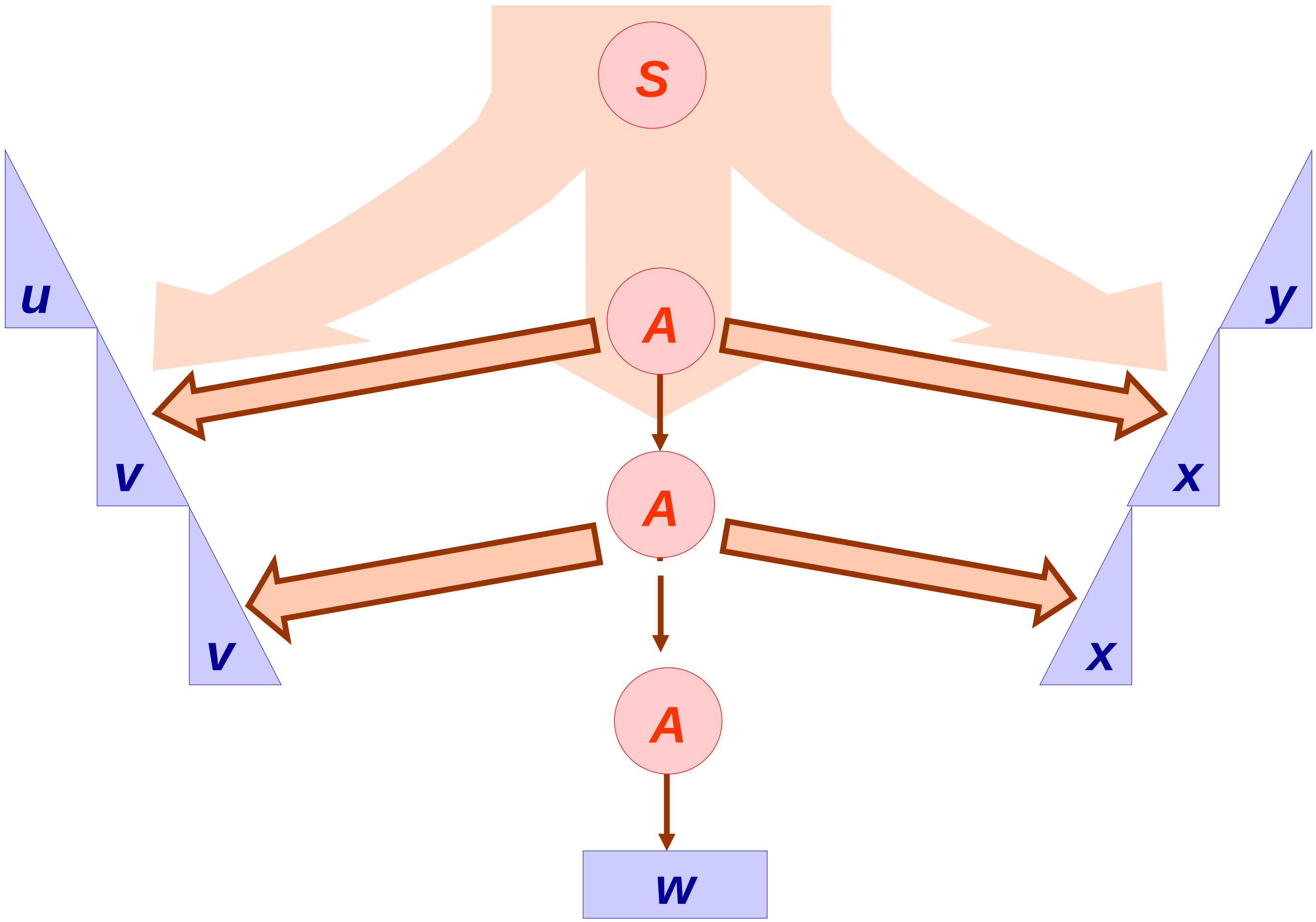

Pumping Lemma

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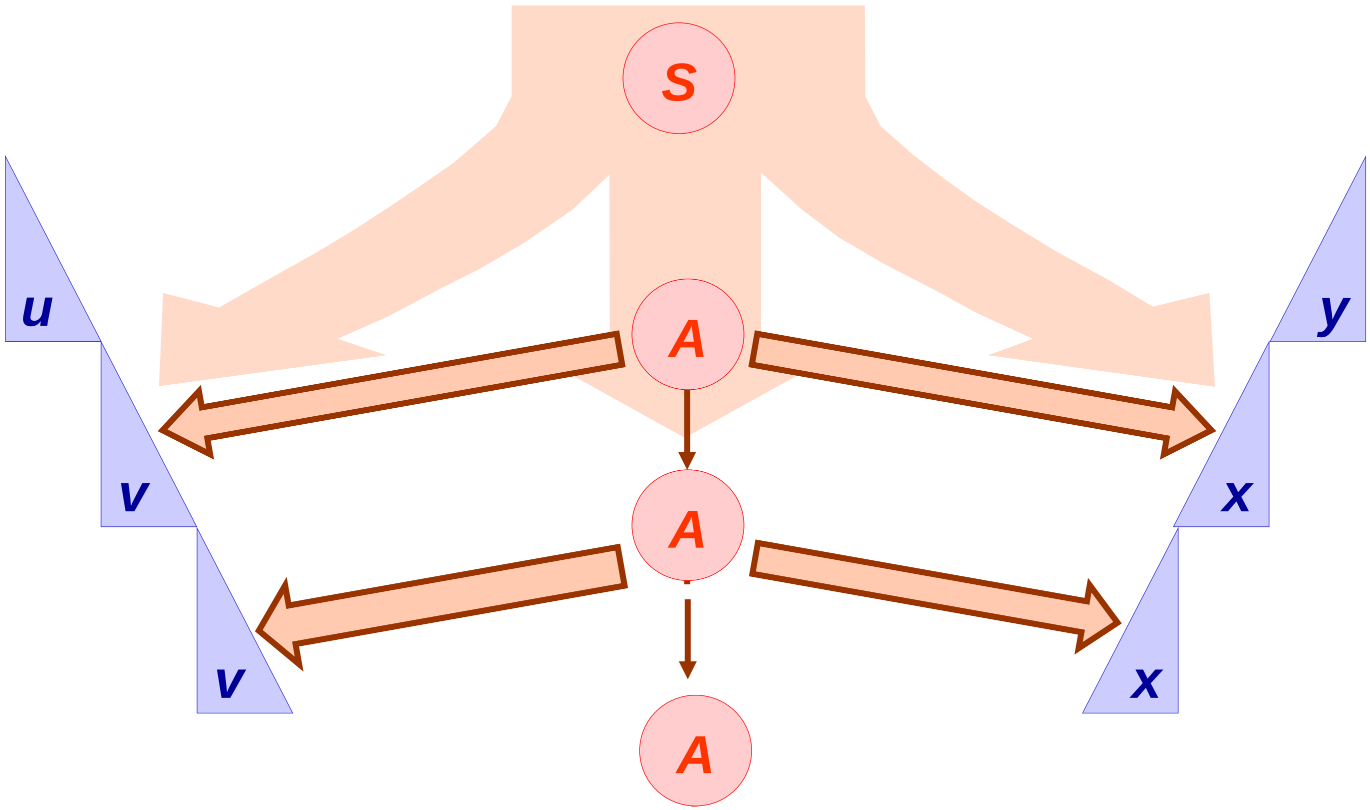
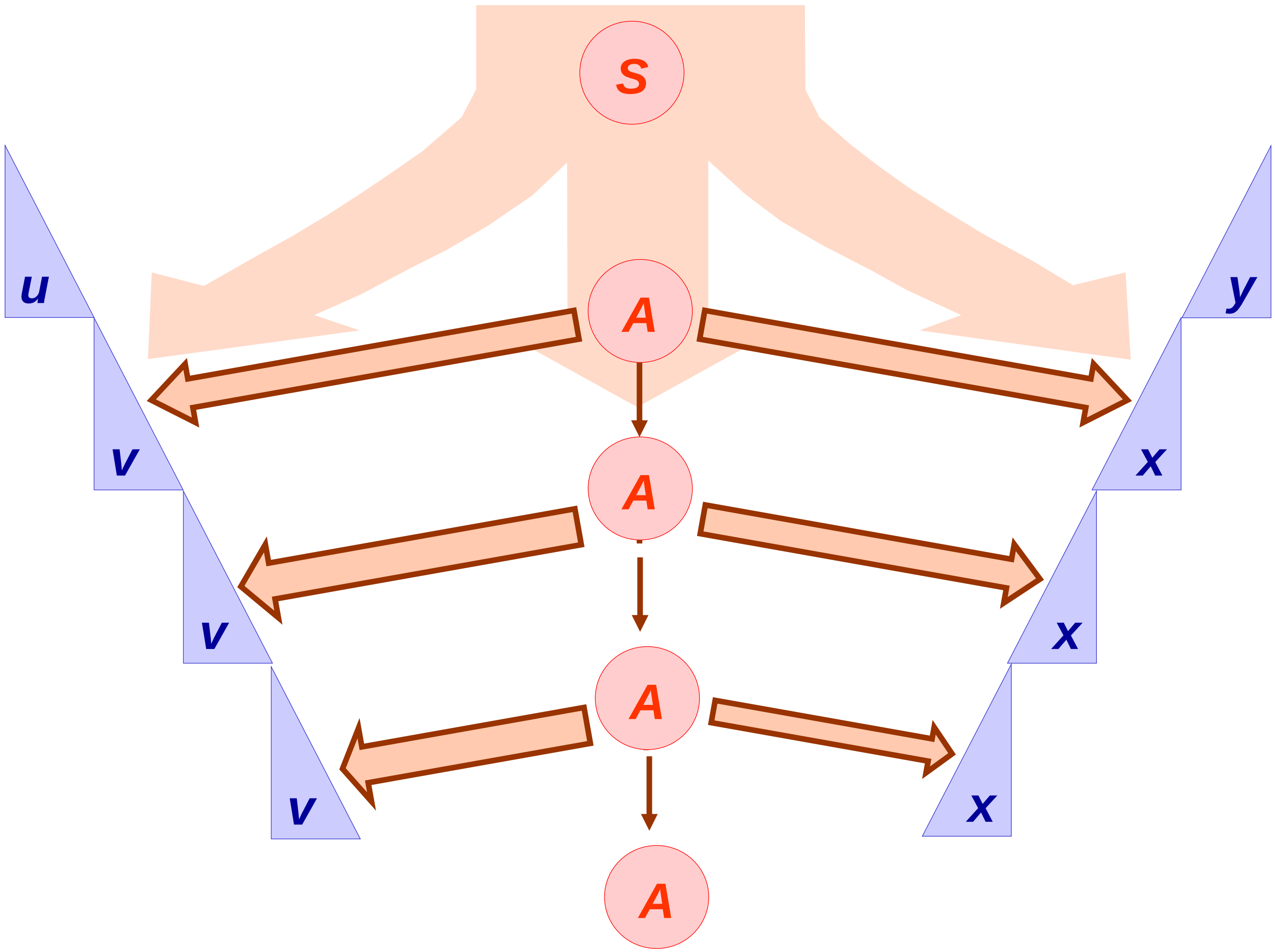


Pumping Lemma

Number of parse tree inner nodes > Number of grammar variables



Pumping Lemma

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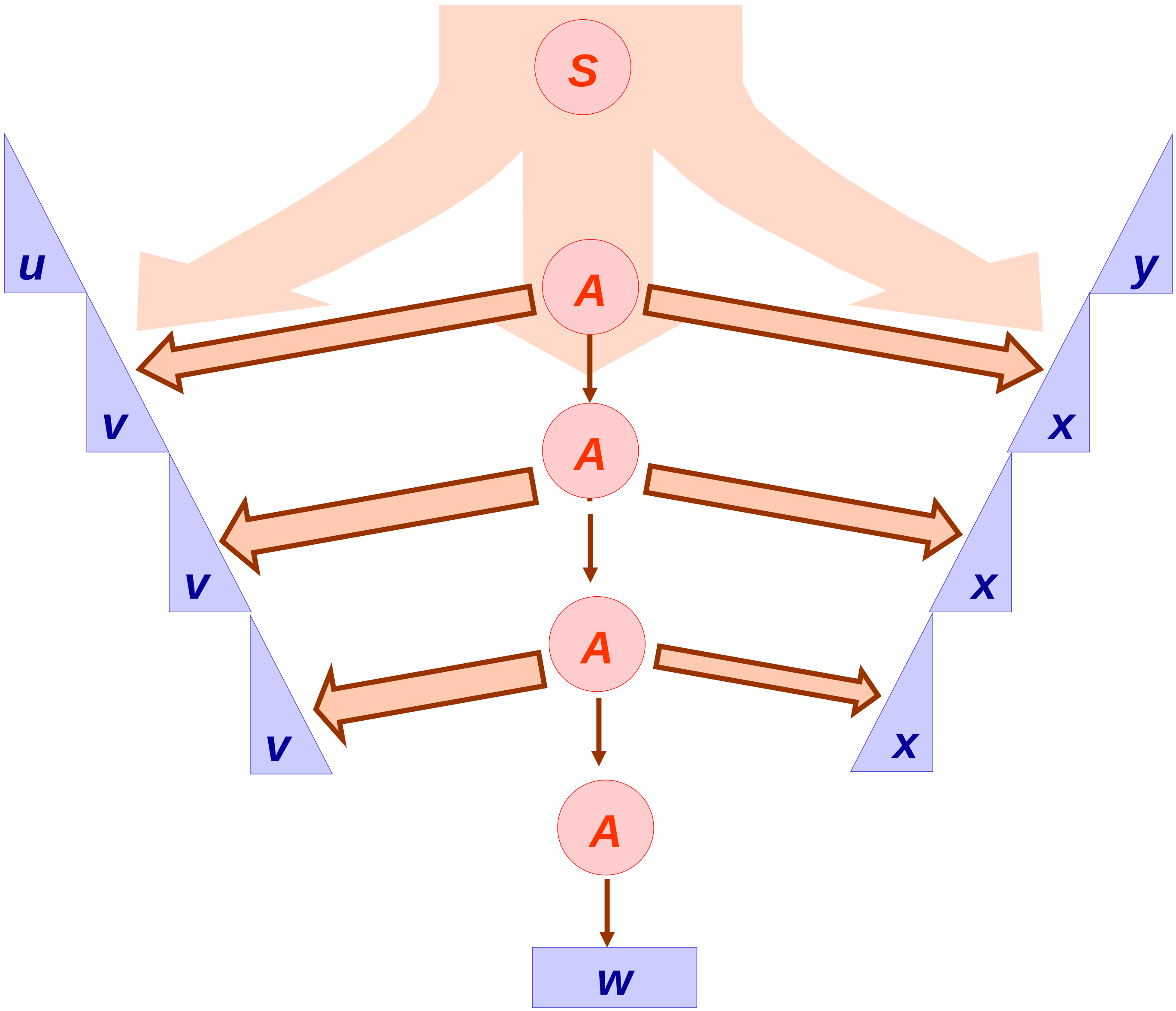

Pumping Lemma

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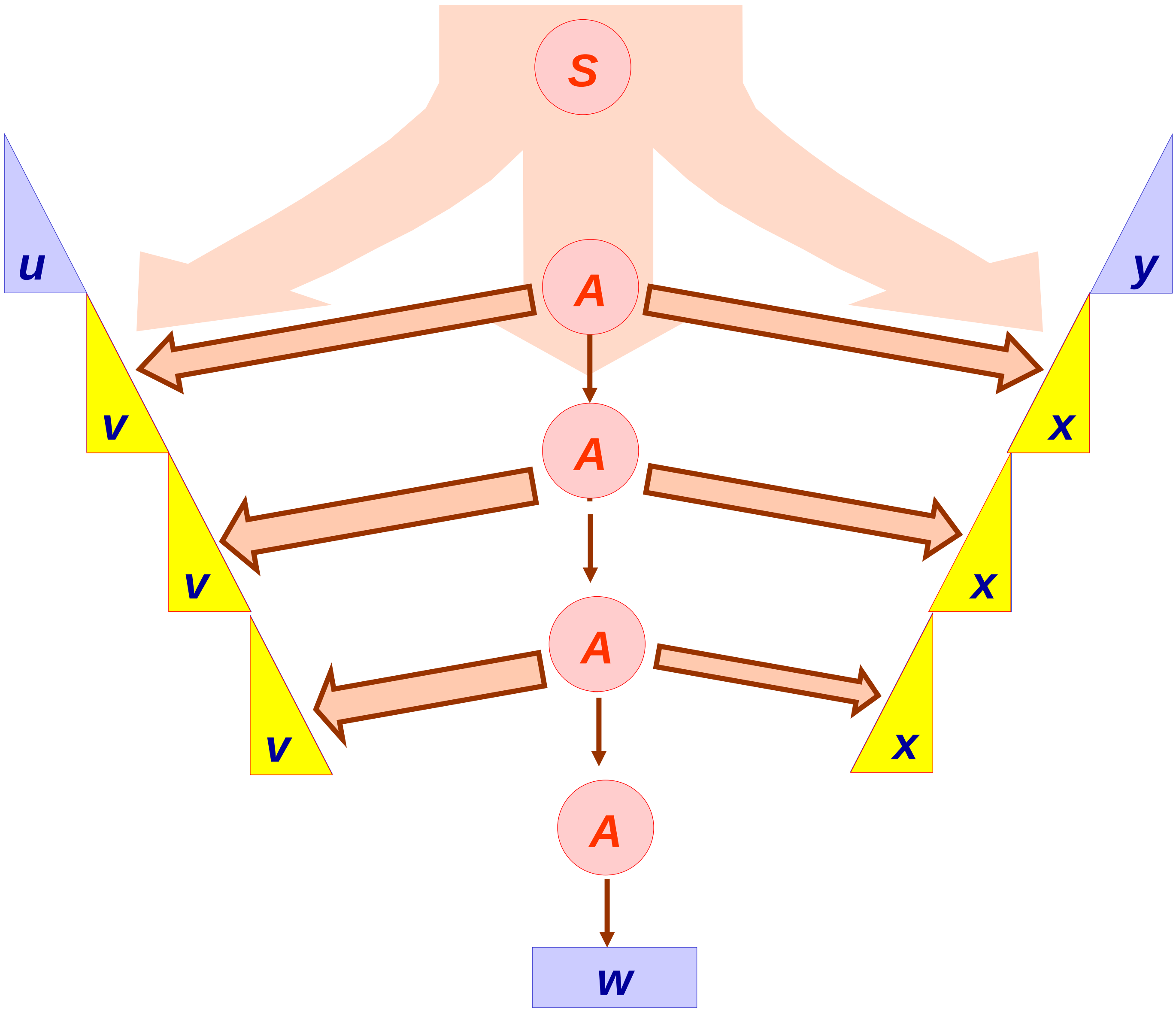

Pumping Lemma

Pumping Lemma

$$A \overset{*}{\underset{G}{\Rightarrow}} w \qquad\qquad A \overset{*}{\underset{G}{\Rightarrow}} v\,\underline{A}\,x$$

Pumping Lemma

$$A \overset{*}{\underset{G}{\Rightarrow}} w \qquad\qquad A \overset{*}{\underset{G}{\Rightarrow}} v\,\underline{A}\,x$$

S

Pumping Lemma

$$A \overset{*}{\underset{G}{\Rightarrow}} w \qquad\qquad A \overset{*}{\underset{G}{\Rightarrow}} v\,\underline{A}\,x$$

$$S \overset{*}{\underset{G}{\Rightarrow}}$$

Pumping Lemma

$$A \overset{*}{\underset{G}{\Rightarrow}} w \qquad\qquad A \overset{*}{\underset{G}{\Rightarrow}} v\,\underline{A}\,x$$

$$S \overset{*}{\underset{G}{\Rightarrow}} u\,\underline{A}\,y$$

Pumping Lemma

$$A \overset{*}{\underset{G}{\Rightarrow}} w \qquad\qquad A \overset{*}{\underset{G}{\Rightarrow}} v\,\underline{A}\,x$$

$$S \overset{*}{\underset{G}{\Rightarrow}} u\,\underline{A}\,y \overset{*}{\underset{G}{\Rightarrow}}$$

Pumping Lemma

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Pumping Lemma

$$A \overset{*}{\underset{G}{\Rightarrow}} w \qquad\qquad A \overset{*}{\underset{G}{\Rightarrow}} v\,\underline{A}\,x$$

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Pumping Lemma

$A \overset{*}{\underset{G}{\Rightarrow}} w$ $\qquad$ $A \overset{*}{\underset{G}{\Rightarrow}} v \underline{A} x$

$S \overset{*}{\underset{G}{\Rightarrow}} u \underline{A} y \overset{*}{\underset{G}{\Rightarrow}}$ $\qquad$ $\boxed{u w y}$

$S \overset{*}{\underset{G}{\Rightarrow}} u \underline{A} y \overset{*}{\underset{G}{\Rightarrow}} u \underline{v A x} y \overset{*}{\underset{G}{\Rightarrow}}$

Pumping Lemma

$$A \overset{*}{\underset{G}{\Rightarrow}} w \qquad\qquad A \overset{*}{\underset{G}{\Rightarrow}} v\,\underline{A}\,x$$

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$$S \overset{*}{\underset{G}{\Rightarrow}} u\,\underline{A}\,y \overset{*}{\underset{G}{\Rightarrow}} u\,\underline{v\,A\,x}\,y \overset{*}{\underset{G}{\Rightarrow}} u\,\underline{v^i\,A\,x^i}\,y$$

Pumping Lemma

$$A \overset{*}{\underset{G}{\Rightarrow}} w \qquad\qquad A \overset{*}{\underset{G}{\Rightarrow}} v\,\underline{A}\,x$$

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$$S \overset{*}{\underset{G}{\Rightarrow}} u\,\underline{A}\,y \overset{*}{\underset{G}{\Rightarrow}} u\,\underline{v\,A\,x}\,y \overset{*}{\underset{G}{\Rightarrow}} \boxed{u\,\underline{v}\,w\,\underline{x}\,y}$$

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Pumping Lemma

Pumping Lemma

Proof goal: **Language** $L = \{ a^i b^i c^i \mid i \geq 1\}$ **–** ***is not a CFL***

Pumping Lemma

Proof goal: Language $L = \{ a^i b^i c^i \mid i \geq 1\}$ – ***is not a CFL***

Assumption: ***L*** **is a context-free language**

Pumping Lemma

Proof goal: Language $L = \{ a^i b^i c^i \mid i \geq 1\}$ – *is not a CFL*

Assumption: *L* is a context-free language

Constant: *n*

Pumping Lemma

Proof goal: Language $L = \{ a^i b^i c^i \mid i \geq 1\}$ – *is not a CFL*

Assumption: L is a context-free language

Constant: n

String: $z = a^n b^n c^n$, $|z| \geq n$

Pumping Lemma

Proof goal: Language $L = \{ a^i b^i c^i \mid i \geq 1\}$ – *is not a CFL*

Assumption: L is a context-free language

Constant: n

String: $z = a^n b^n c^n$, $|z| \geq n$

z is broken into: $u\ v\ w\ x\ y$

Pumping Lemma

Proof goal: Language $L = \{ a^i b^i c^i \mid i \geq 1\}$ – *is not a CFL*

Assumption: *L* is a context-free language

Constant: n

String: $z = a^n b^n c^n$, $|z| \geq n$

***z* is broken into:** $u\ v\ w\ x\ y$

It is necessary to determine: substrings *v* and *x*

Pumping Lemma

Proof goal: Language $L = \{ a^i b^i c^i \mid i \geq 1 \}$ – *is not a CFL*

Assumption: L is a context-free language

Constant: n

String: $z = a^n b^n c^n$, $|z| \geq n$

z is broken into: $u\ v\ w\ x\ y$

It is necessary to determine: substrings v and x

in the string $a^n b^n c^n$

Pumping Lemma

Proof goal: Language $L = \{ a^i b^i c^i \mid i \geq 1\}$ – *is not a CFL*

Assumption: L is a context-free language

Constant: n

String: $z = a^n b^n c^n$, $|z| \geq n$

z **is broken into:** $u\ v\ w\ x\ y$

It is necessary to determine: substrings v and x

in the string $a^n b^n c^n$

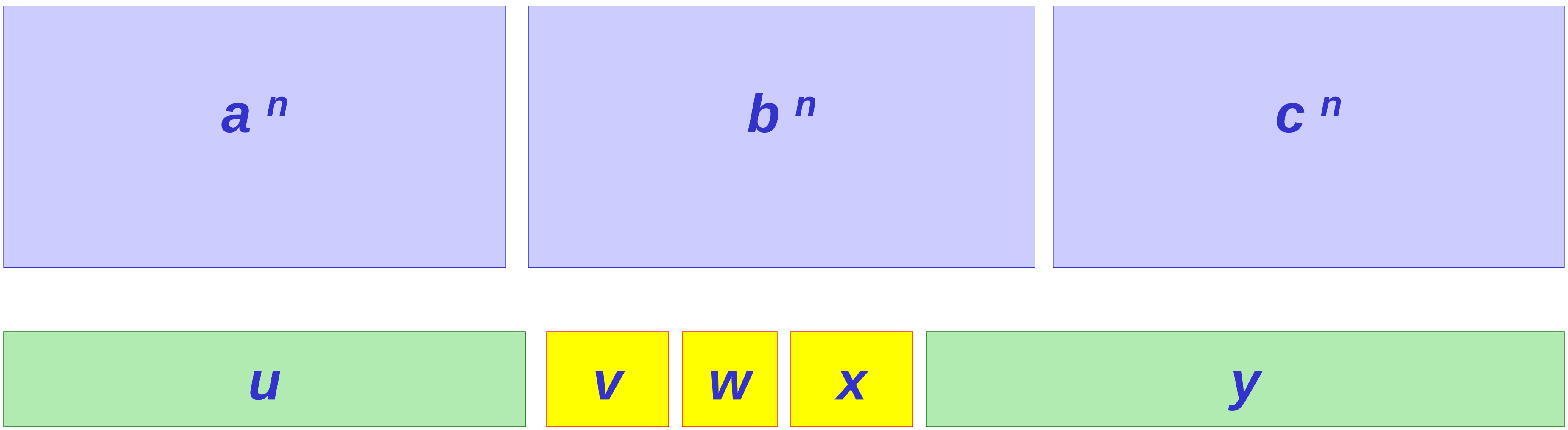
that can be repeated (pumped) any number of times

Pumping Lemma

Pumping Lemma

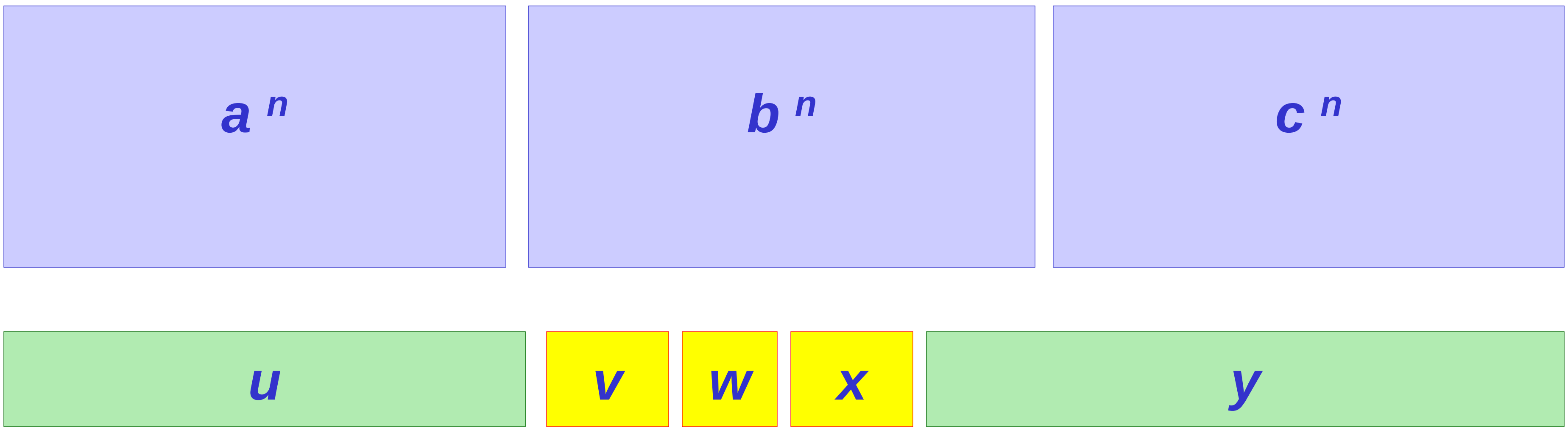
a^n	b^n	c^n

Pumping Lemma



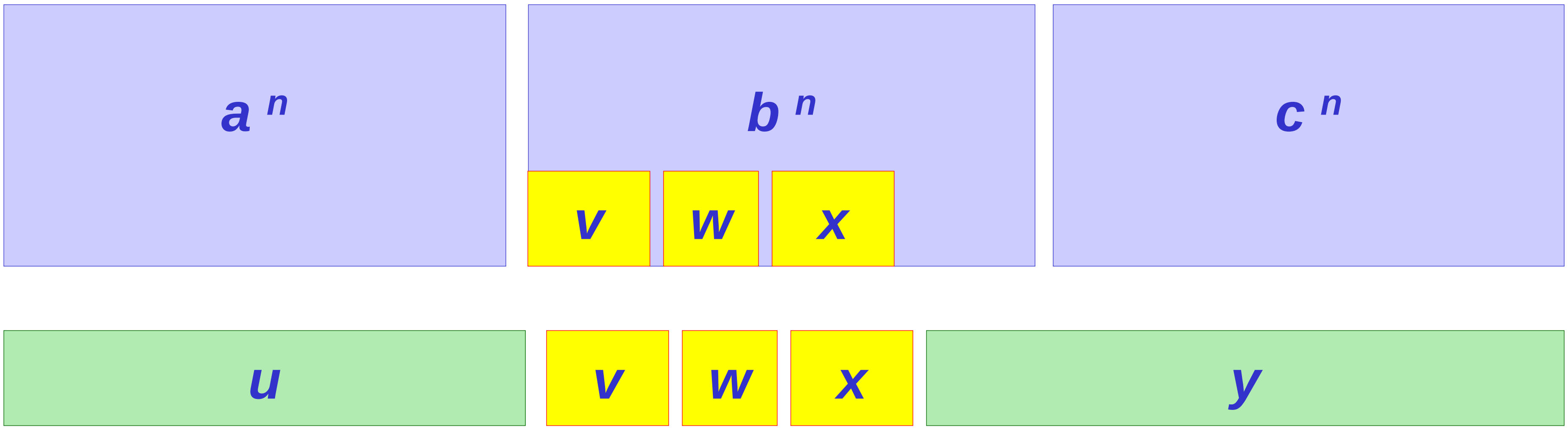
Pumping Lemma

$$|\, v\, w\, x \,| \leq n$$



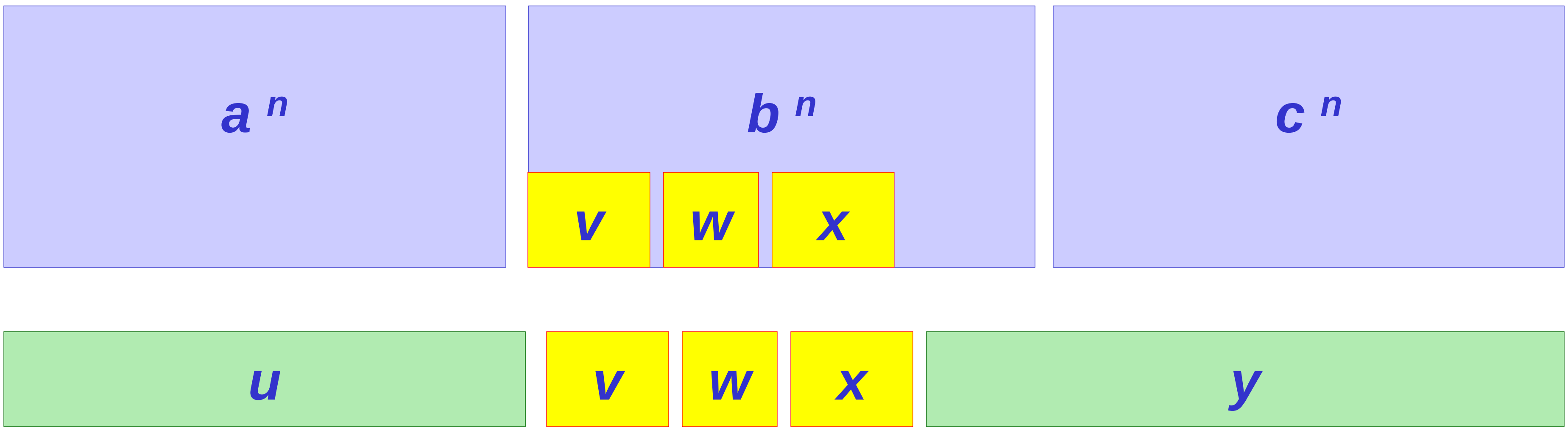
Pumping Lemma

$$| v\, w\, x | \leq n$$



Pumping Lemma

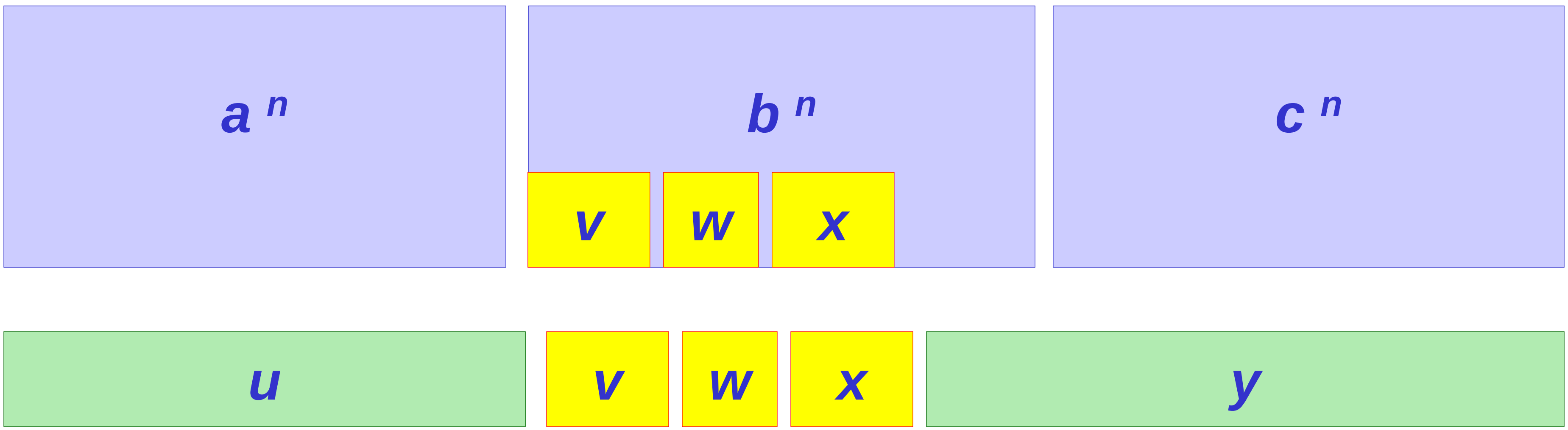
$$| v\,w\,x | \leq n$$



Strings *v* i *x* contain exclusively symbols *b*

Pumping Lemma

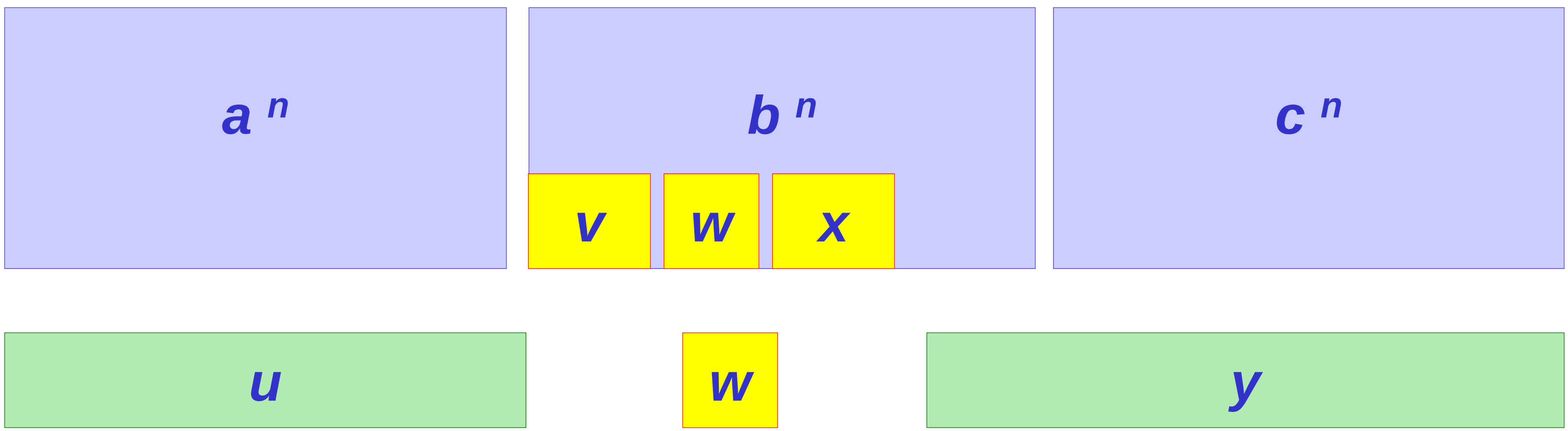
$$| vwx | \leq n$$



Strings v **i** x **contain exclusively symbols** b
String $u\,w\,y$

Pumping Lemma

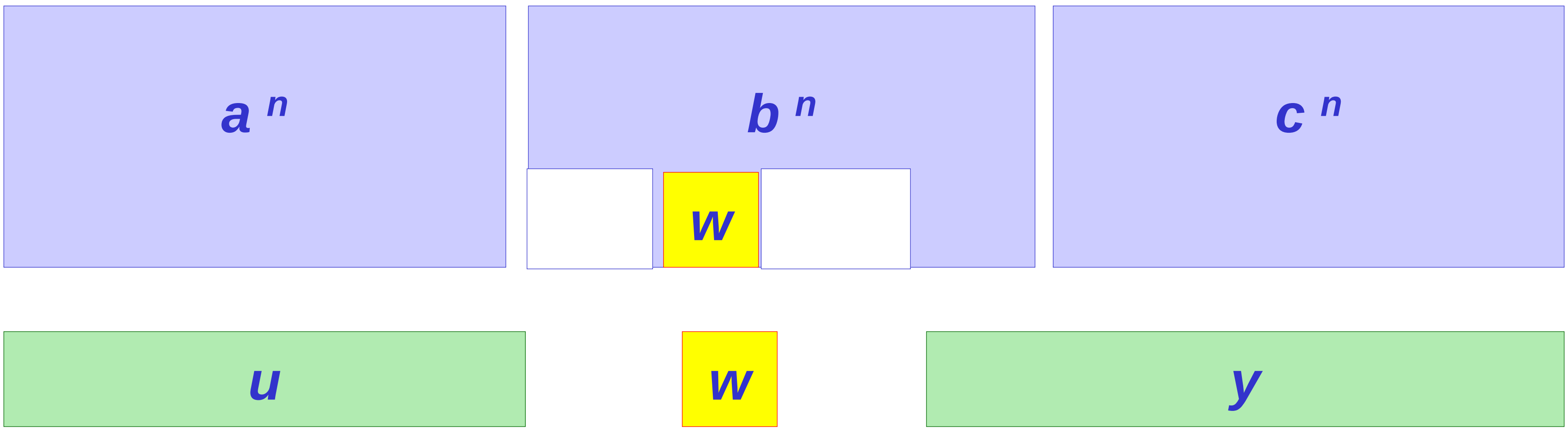
$$|\,v\,w\,x\,| \leq n$$



Strings v **i** x **contain exclusively symbols** b
String $u\,w\,y$

Pumping Lemma

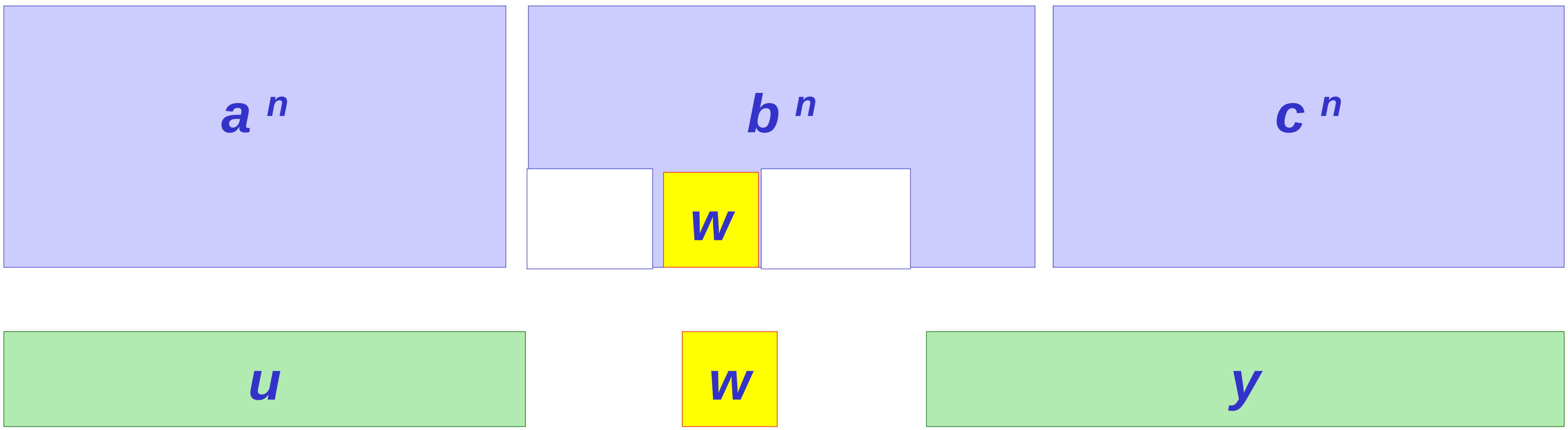
$| v\, w\, x | \leq n$



Strings ***v*** **i** ***x*** **contain exclusively symbols** ***b***
String ***u w y***

Pumping Lemma

$$| v\, w\, x | \leq n$$



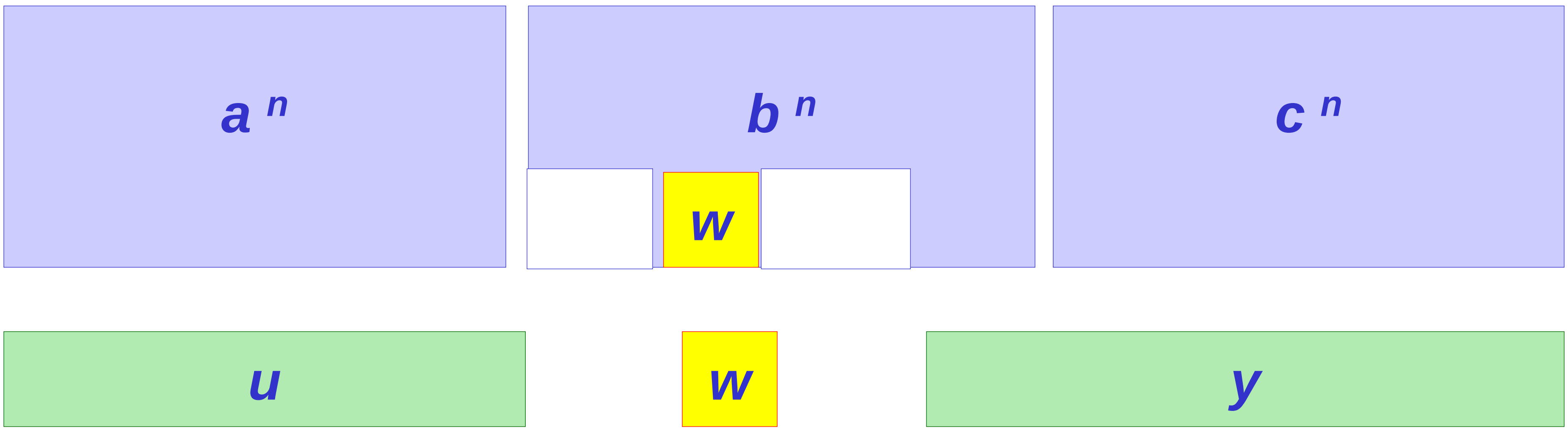
Strings *v* i *x* contain exclusively symbols *b*

String *u w y*

n* symbols *a

Pumping Lemma

$$|\, v\, w\, x \,| \leq n$$



Strings v **i** x **contain exclusively symbols** b

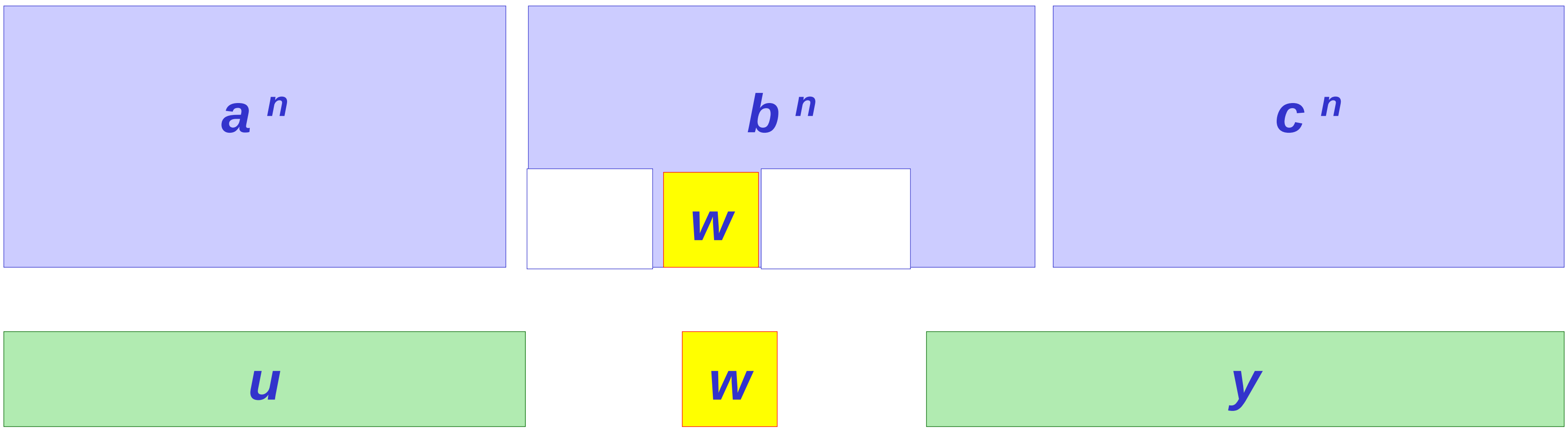
String $u\, w\, y$

n **symbols** a

n **symbols** c

Pumping Lemma

$$| \, v \, w \, x \, | \leq n$$



Strings v **i** x **contain exclusively symbols** b

String $u \, w \, y$

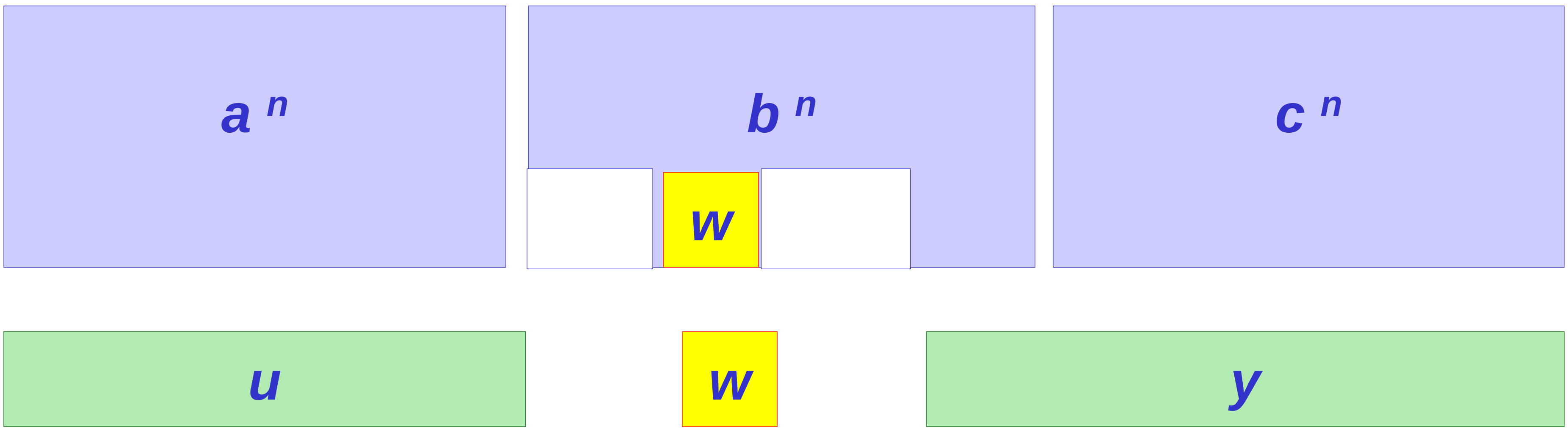
n **symbols** a

n **symbols** c

less then n **symbols** b**, because** $| \, v \, x \, | \geq 1$

Pumping Lemma

$$| \, v \, w \, x \, | \leq n$$



Strings v **i** x **contain exclusively symbols** b

String $u \, w \, y$

n **symbols** a

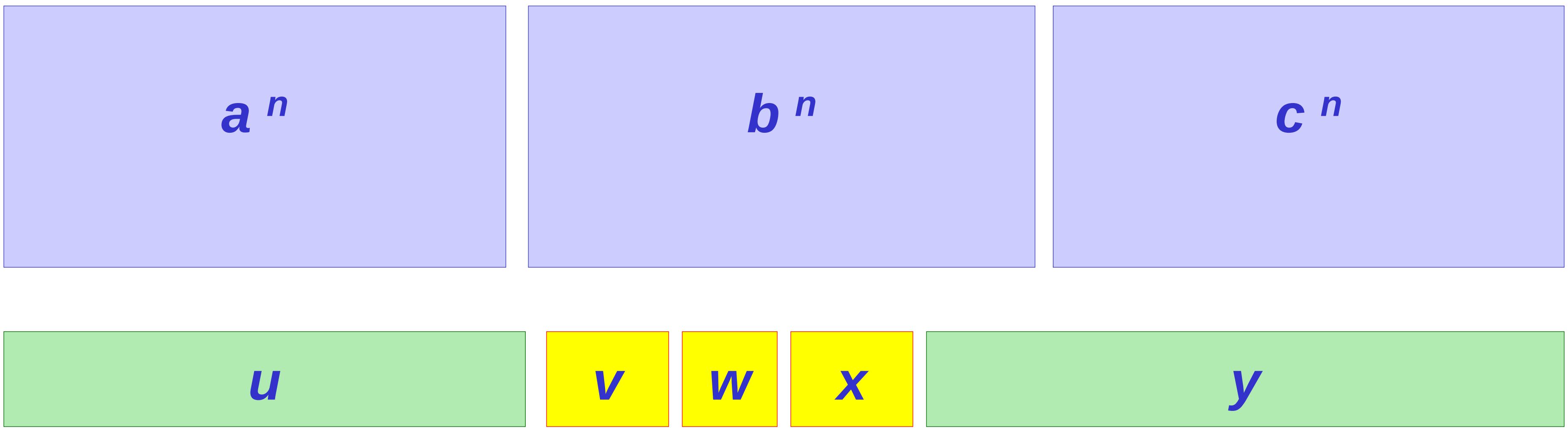
n **symbols** c

less then n **symbols** b**, because** $| \, v \, x \, | \geq 1$

String $u \, w \, y$ *does not have form* $a^j \, b^j \, c^j$

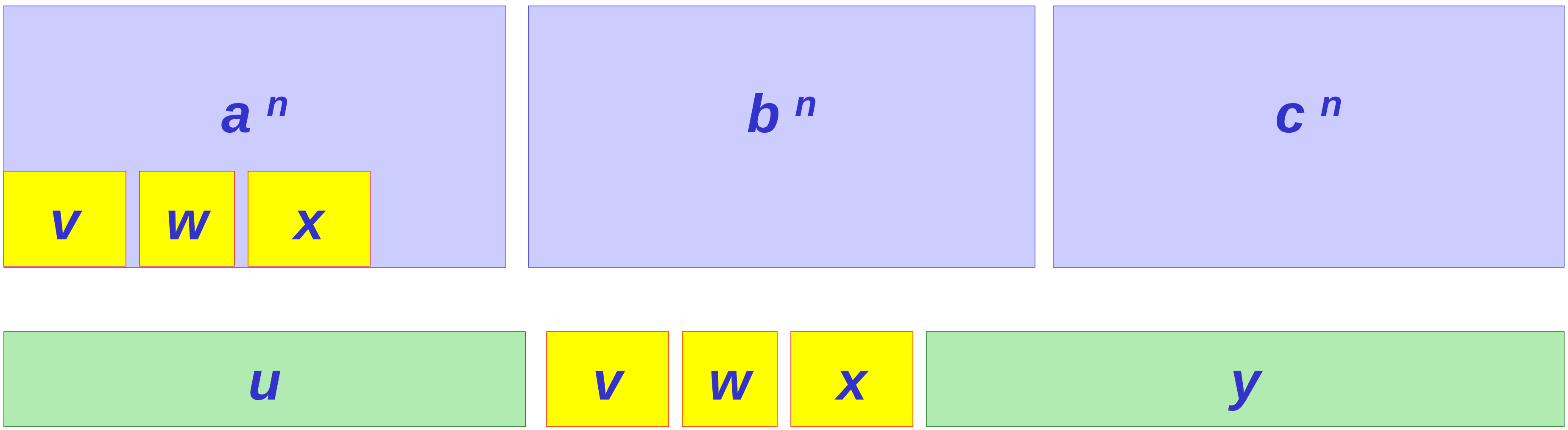
Pumping Lemma

$$|\, v\, w\, x\, | \leq n$$



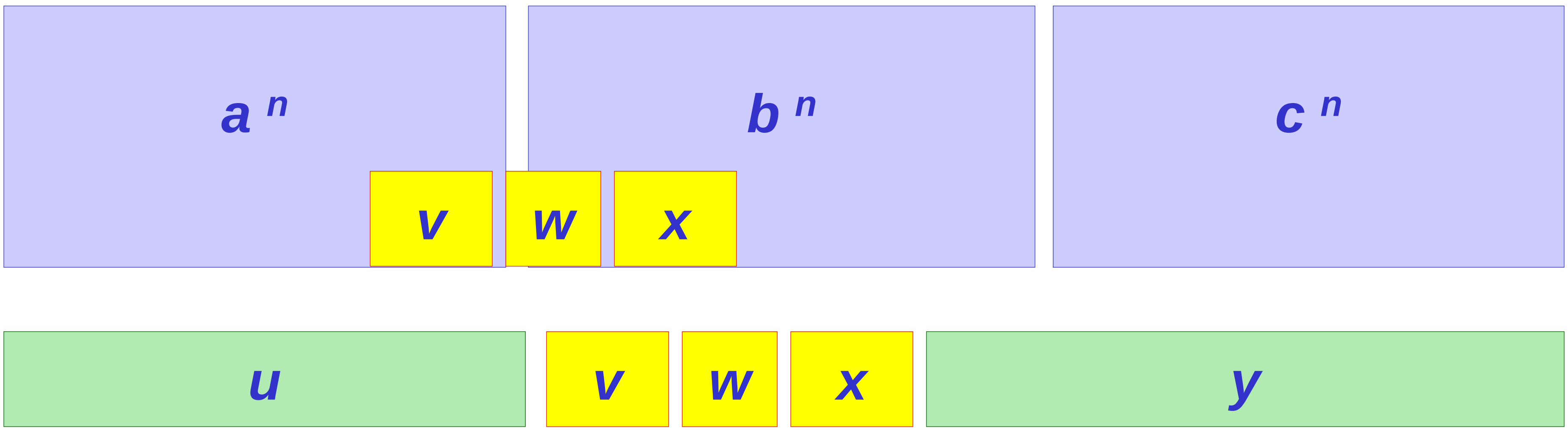
Pumping Lemma

$$|vwx| \leq n$$



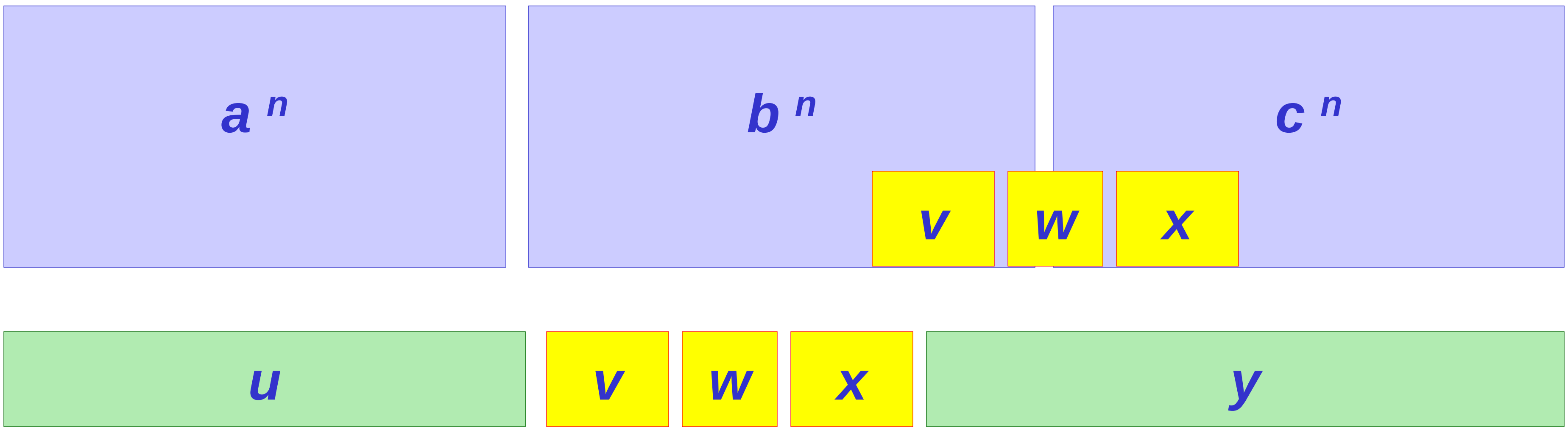
Pumping Lemma

$$| v\, w\, x | \leq n$$



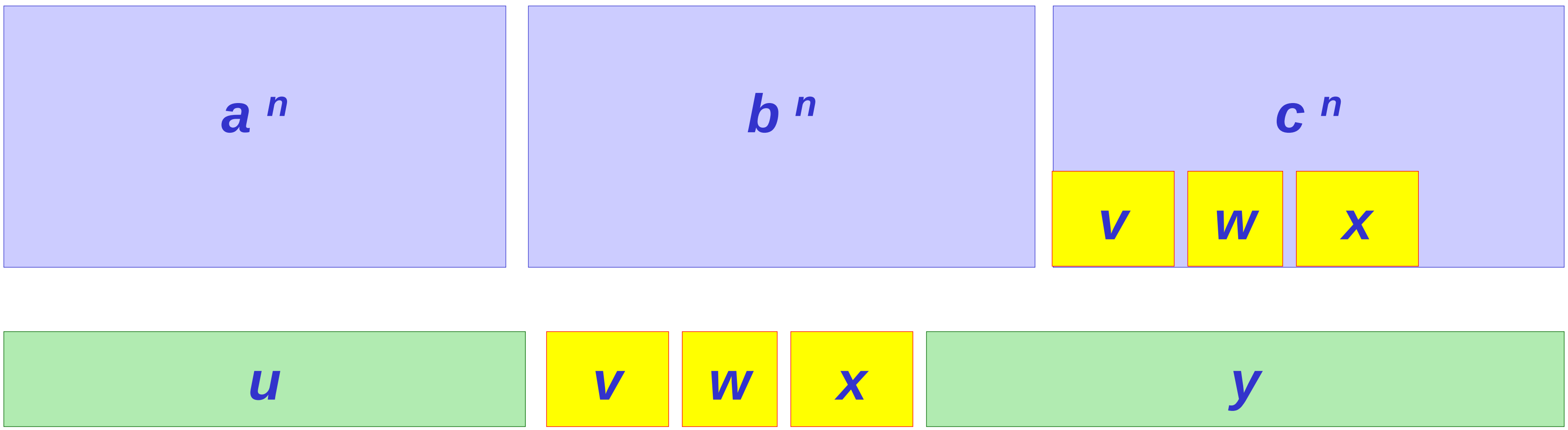
Pumping Lemma

$$|\, v\, w\, x \,| \leq n$$



Pumping Lemma

$$| v w x | \leq n$$



Pumping Lemma

$$| v w x | \leq n$$

a^n	b^n	c^n

u	v	w	x	y