Introduction to Theoretical Computer Science

https://www.fer.unizg.hr/en/course/ittcs_b

ittcs@fer.hr

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University of Zagreb

Faculty of Electrical Engineering and Computing

Introduction

```
import java.awt.*;public class Example{public static void main(String args[ ]){}}
```

```
import java.awt.*;

public class Example
{
        public static void main(String args[ ])
        {
        }
}
```

Introduction

```
import java.awt.*;public class Example{public static void main(String args[ ]){}}
```

```
import java.awt.*;public class Example{ public static void main(String args[ ]){int
a=2;int b=3;int c;c=a+b;}}
```

```
import java.awt.*;

public class Example
{
        public static void main(String args[ ])
        {
                int a=2;
                int b=3;
                int c;

                c=a+b;
        }
}
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Introduction

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```
import java.awt.*;

public class Example
{
        public static void main(String args[ ])
        {
                int a=2;
                int b=3;
                int c;

                c=a-b;
        }
}
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Introduction

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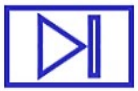


```
import java.awt.*;

public class Example
{
        public static void main(String args[ ])
        {
                int a=2;
                int b=3;
                int c;

                c(((((=a-b;
        }
}
```

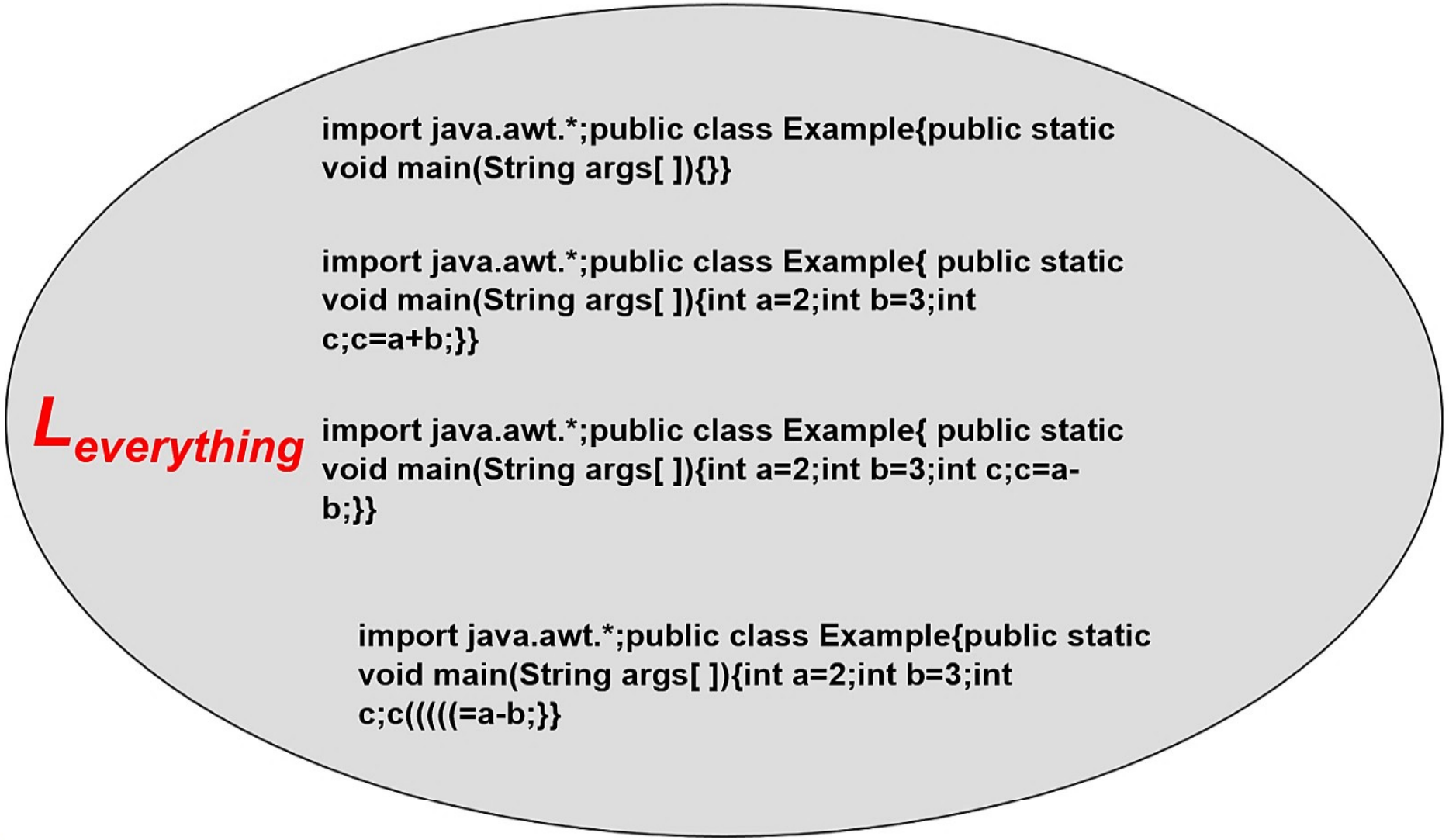
String

```
import java.awt.*;public
class Example{public
static void main(String
args[ ]){}}
```

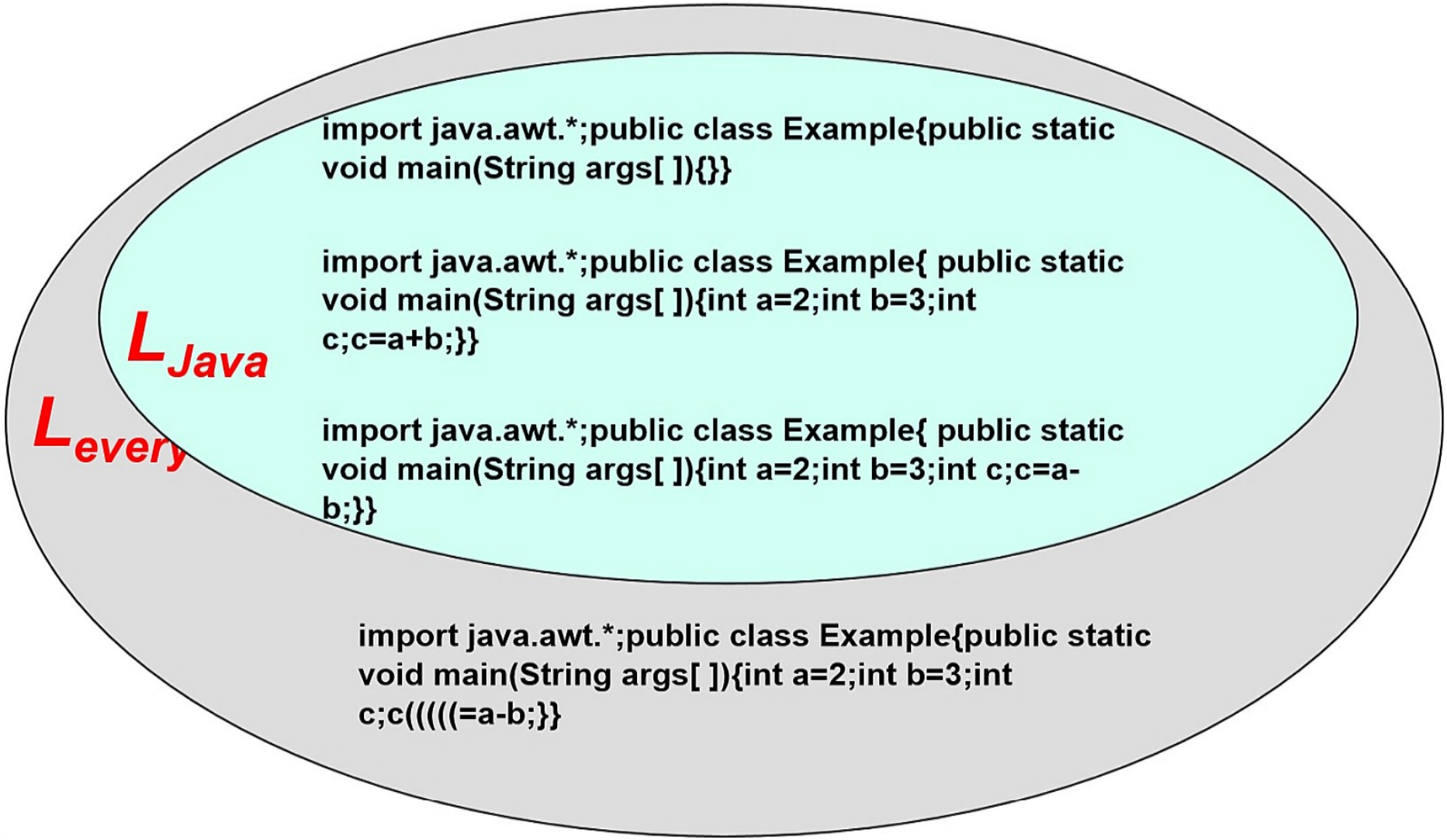
Alphabet

{a, b, c, … , y, z,
A, B, C, … , Y, Z,
0, 1, 2, … , 8, 9,
(,), ;, :, ….}

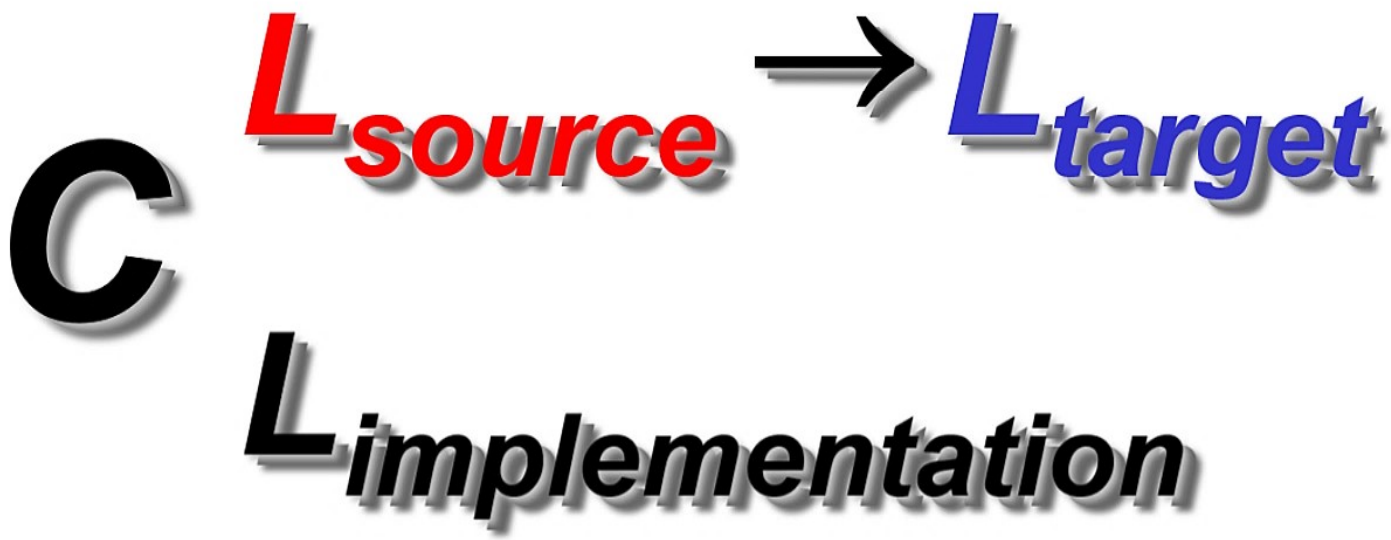
LANGUAGE - *L*



LANGUAGE - *L*



COMPILER



Lecture 1

NOTATIONS, SYMBOLS, and CONCEPTS Used Throughout the Course

- Symbol and alphabet
- String
- Language
- Set, finite set, infinite set
- Graph
- Directed graph
- Tree
- Mathematical induction, inductive proof
- Relations

Staff and Workload

Lecturers

Prof. Siniša Srbljić, Ph.D.
Assoc. Prof. Dejan Škvorc, Ph.D.
Assoc. Prof. Marin Šilić, Ph.D.
Asst. Prof. Ante Đerek, Ph.D.
Assoc. Prof. Zoran Kalafatić, Ph.D.
Assoc. Prof. Goran Delač, Ph.D.
Assoc. Prof. Klemo Vladimir, Ph.D.
Adrian Satja Kurdija, Ph.D.

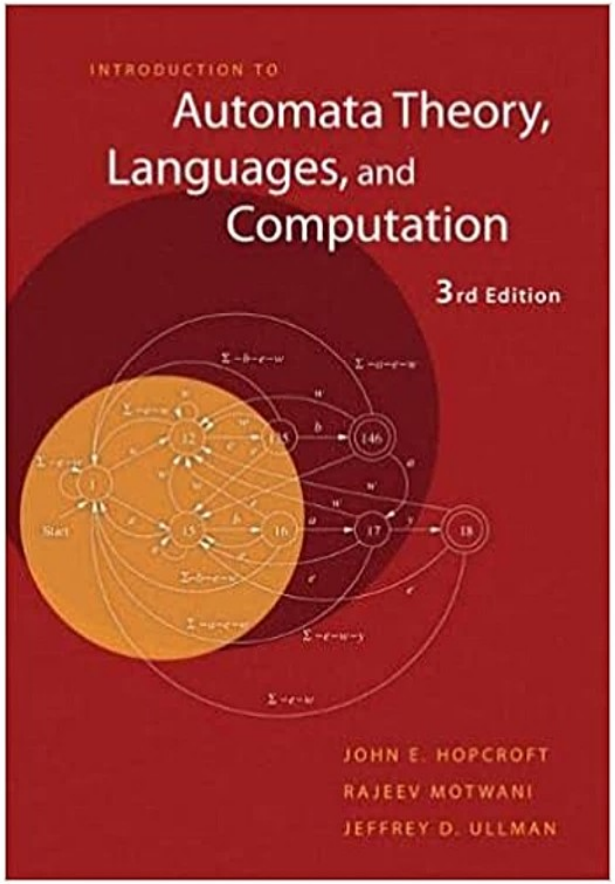
Assistants

Lucija Šikić, mag. ing.
Helena Čeović, mag. ing.
Ivan Mikulić, mag. ing.
Davor Vukadin, mag. ing.
Marin Vlaić, mag. ing.
Petar Paradžik, mag. math. et inf.

ECTS: 5

Form of teaching	Workload
Lectures	60
Labs	10

Literature



J. E. Hopcroft, R. Motwani, J. D. Ullman

Introduction to Automata Theory, Languages, and Computation

Pearson, 3rd edition, 2006

**** additional literature specified on the course website***

Lecture Schedule

	LISTOPAD 2021.					STUDENI 2021.					PROSINAC 2021.				
Tjedan		1	2	3	4	5	6	7	8a	8b		9	10	11	
Po		4	11	18	25	1	8	15	22	29		6	13	20	27
Ut		5	12	19	26	2	9	16	23	30		7	14	21	28
Sr		6	13	20	27	3	10	17	24		1	8	15	22	29
Če		7	14	21	28	4	11	18	25		2	9	16	23	30
Pe	1	8	15	22	29	5	12	19	26		3	10	17	24	31
Su	2	9	16	23	30	6	13	20	27		4	11	18	25	
Ne	3	10	17	24	31	7	14	21	28		5	12	19	26	

	SIJEČANJ 2022.						VELJAČA 2022.					OŽUJAK 2022.				
Tjedan		12	13	14	15a		15b					1	2	3	4	5
Po		3	10	17	24	31		7	14	21	28		7	14	21	28
Ut		4	11	18	25		1	8	15	22		1	8	15	22	29
Sr		5	12	19	26		2	9	16	23		2	9	16	23	30
Če		6	13	20	27		3	10	17	24		3	10	17	24	31
Pe		7	14	21	28		4	11	18	25		4	11	18	25	
Su	1	8	15	22	29		5	12	19	26		5	12	19	26	
Ne	2	9	16	23	30		6	13	20	27		6	13	20	27	

	TRAVANJ 2022.					SVIBANJ 2022.					LIPANJ 2022.				
Tjedan	5	6	7	8a	8b	9	10	11	12	13	13	14	15a	15b	15c
Po		4	11	18	25	2	9	16	23	30		6	13	20	27
Ut		5	12	19	26	3	10	17	24	31		7	14	21	28
Sr		6	13	20	27	4	11	18	25		1	8	15	22	29
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Pe	1	8	15	22	29	6	13	20	27		3	10	17	24	
Su	2	9	16	23	30	7	14	21	28		4	11	18	25	
Ne	3	10	17	24		1	8	15	22	29	5	12	19	26	

	SRPANJ 2022.					KOLOVOZ 2022.					RUJAN 2022.				
Tjedan	15c														
Po		4	11	18	25	1	8	15	22	29		5	12	19	26
Ut		5	12	19	26	2	9	16	23	30		6	13	20	27
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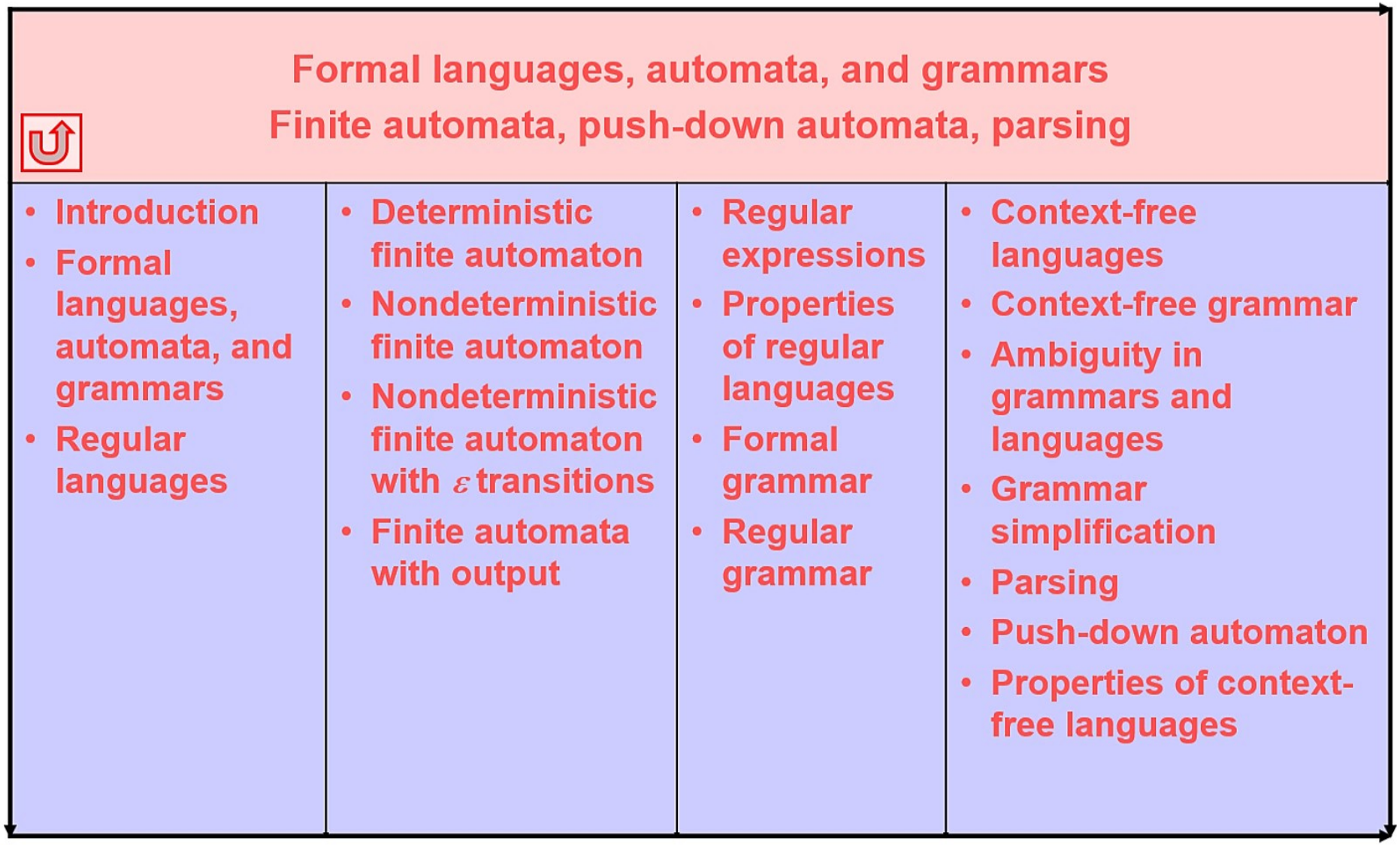
Pe	1	8	15	22	29	5	12	19	26		2	9	16	23	30
Su	2	9	16	23	30	6	13	20	27		3	10	17	24	
Ne	3	10	17	24	31	7	14	21	28		4	11	18	25	

Lecture Schedule (tentative)

LECTURE 1 and 2	LECTURE 3 and 4	LECTURE 5 and 6	LECTURE 7 and 8
LECTURE 9 and 10	EXERCISES	EXERCISES	
MIDTERM EXAM Apr 18, 2022 – Apr 29, 2022			

LECTURE 11 and 12	LECTURE 13 and 14	LECTURE 15 and 16
LECTURE 17 and 18	LECTURE 19	EXERCISES
FINAL EXAM Jun 13, 2022 – Jul 01, 2022		

Topics: Lectures 1 – 10



Topics: Lectures 11 – 19

Turing machines, linear bounded automata
Chomsky hierarchy of languages, grammars, and automata
Computational complexity of languages

• Recursively enumerable languages • Turing machine • Language generator • Unrestricted grammar • Prioperties of recursive and recursively enumerable languages	• Context-sensitive languages • Context-sensitive grammar • Linear bounded automaton • Properties of context-sensitive languages	• Chomsky hierarchy of languages, grammars, and automata • Space and time complexity of languages • Complexity classes of languages • Polynomial complexity classes of languages

Computer Science Curricula 2013

Curriculum Guidelines for Undergraduate Degree Programs in Computer Science

Final Report
(December 20, 2013)

The Joint Task Force on Computing Curricula
Association for Computing Machinery
IEEE Computer Society

18 Knowledge Areas

AL	**Algorithms and Complexity**
AR	Architecture and Organization
CN	Computational Science
DS	Discrete Structures
GV	Graphics and Visualization
HCI	Human-Computer Interaction
IAS	Information Assurance and Security
IM	Information Management
IS	Intelligent Systems
NC	Networking and Communication
OS	Operating Systems
PBD	Platform-based Development
PD	Parallel and Distributed Computing
PL	Programming Languages
SDF	Software Development Fundamentals
SE	Software Engineering
SF	**Systems Fundamentals**
SP	Social Issues and Professional Practice

AL/Basic Automata Computability and Complexity

Topics:

Finite state machines
Regular expressions
The halting problem
Context-free grammars
Introduction to the P and NP classes and the P vs. NP problem
Introduction to the NP complete class and exemplary NP-complete problems

AL/Advanced Computational Complexity

Topics:

Review of the classes P and NP; introduce P-space and EXP
Polynomial hierarchy
NP-completeness (Cook's theorem)
Classic NP-complete problems
Reduction techniques

AL/Advanced Automata Theory and Computability

Topics:

- Sets and languages
  - Regular languages
  - Review of deterministic finite automata (DFAs)
  - Nondeterministic finite automata (NFAs)
  - Equivalence of DFAs and NFAs
  - Review of regular expressions; their equivalence to finite automata
  - Closure properties
  - Proving languages non-regular, via the pumping lemma or alternative means
- Context-free languages
  - Push-down automata (PDAs)
  - Relationship of PDAs and context-free grammars
  - Properties of context-free languages
- Turing machines, or an equivalent formal model of universal computation
- Nondeterministic Turing machines
- Chomsky hierarchy
- The Church-Turing thesis
- Computability
- Rice's Theorem
- Examples of uncomputable functions
- Implications of uncomputability

Assessment

1) Continuous assessment

Theory + problem solving

- **Midterm exam** — **35 points**
- **Final exam** — **35 points**

- **Total theory + problem solving** — **70 points**

- **Bonus**
  - Active class participation — + ∝ points

Labs

- **Programming-based exercises** — **15+15 points**
  Finite automata, push-down automata, parsing

- **Total labs** — **30 points**

Assessment

1) Continuous assessment

Labs **30 points**

- **Minimum to pass** **15 points**
  - Mutually independent programming-based assignments
    — 2 assignments
  - Task descriptions and due dates are to be announced on the course website
    — visit the website regularly
  - Solving at home individually
  - Demonstration to teaching assistants using your own equipment
    — algorithmic correctness
    — understanding how the program works
    — plagiarism
  - **No overdue submissions**

Assessment

1) Continuous assessment

Total (theory + problem solving + labs)

- 100 points

Thresholds

- minimum to pass
  - theory + problem solving — 35 / 70 points (50 %)
  - labs — 15 / 30 points (50 %)

Grading

- sufficient (2) — 50 points
- good (3) — 63 points
- very good (4) — 75 points
- excellent (5) — 88 points

Assessment

2) Exam

Written exam

- 100 points
- Can be replaced by/combined with oral exam

Qualifying threshold

- lab threshold 15 / 30 points (50 %)
- DO NOT CONTRIBUTE TO THE EXAM POINTS

Grading

- sufficient (2) 50 points
- good (3) 63 points
- very good (4) 75 points
- excellent (5) 88 points

Lecture 1

1 INTRODUCTION

1.1 NOTATIONS, SYMBOLS, AND CONCEPTS USED THROUGHOUT THE COURSE

1.2 EXAMPLE OF A FORMAL LANGUAGE, AUTOMATON, AND GRAMMAR

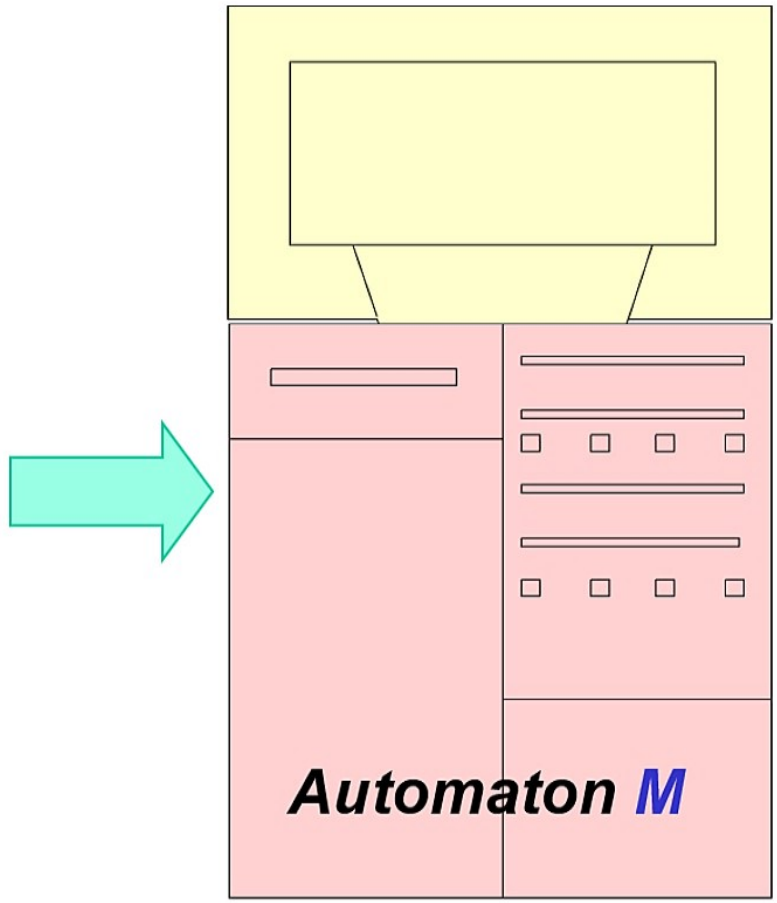
Source Language – FORMAL AUTOMATON

Σ^* *set of all strings over the alphabet* Σ

Language L for which holds the following:

$L = L(M)$
and
$L \subseteq \Sigma^*$

W



Source Language – FORMAL AUTOMATON

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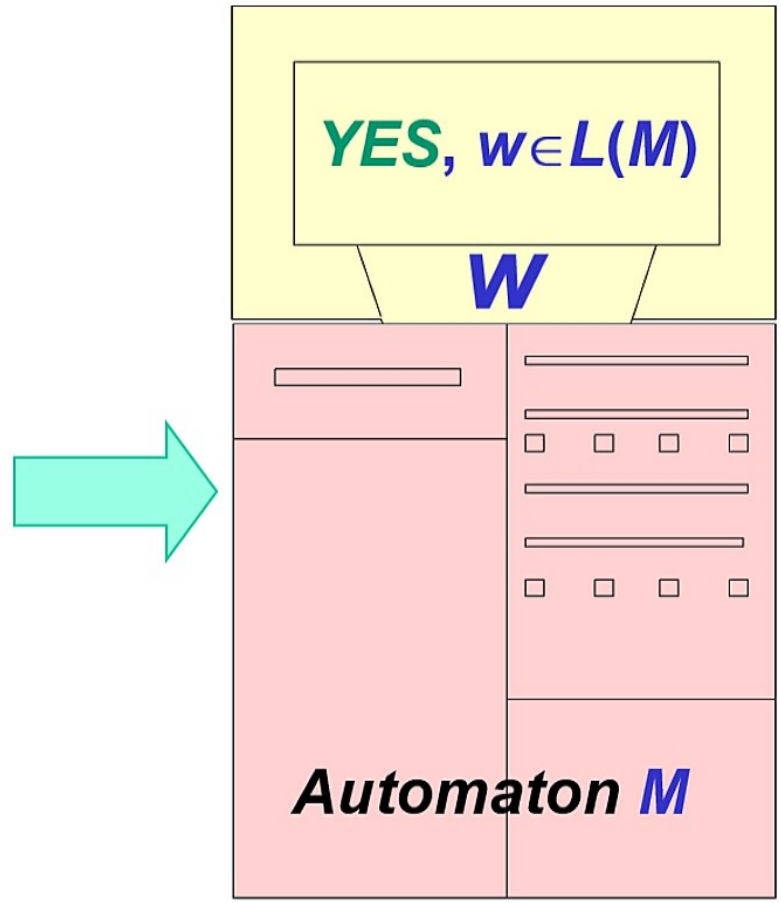

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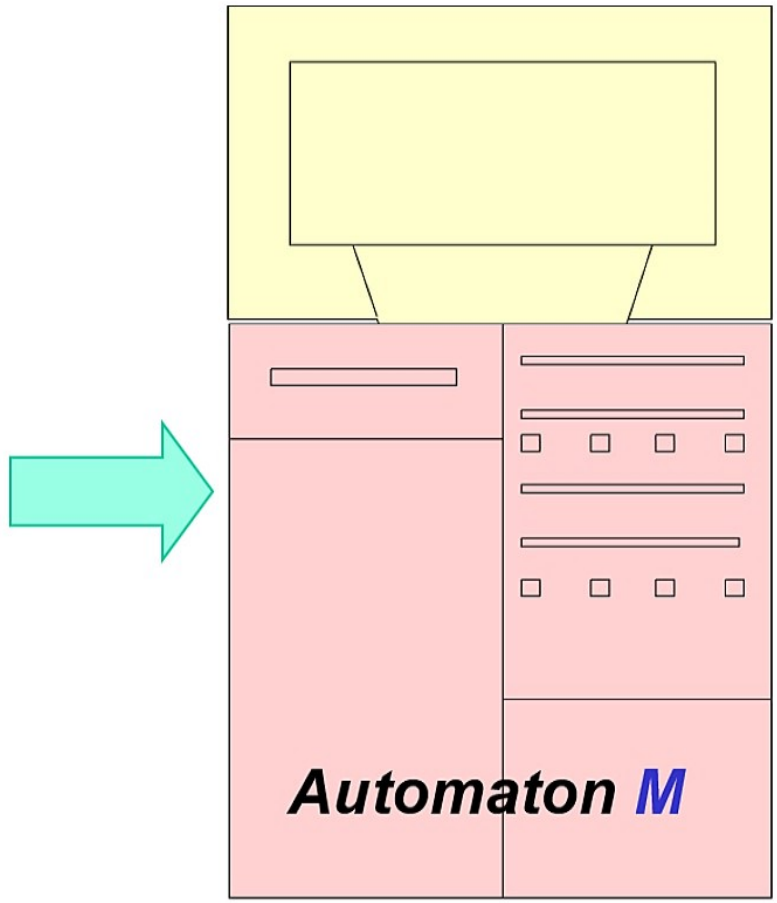

Source Language – FORMAL AUTOMATON

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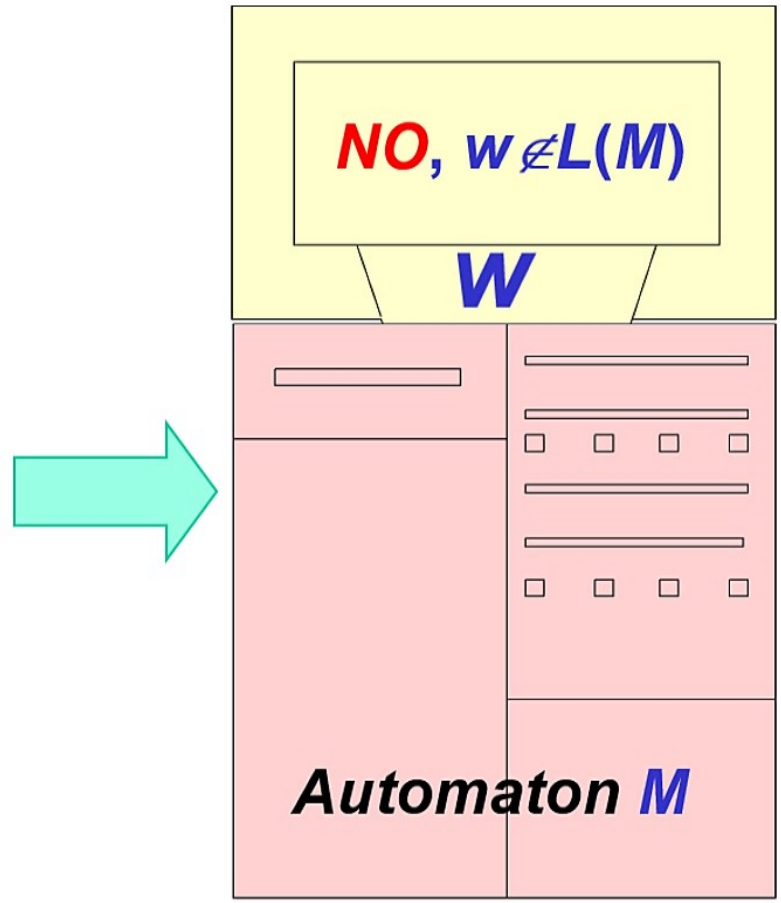
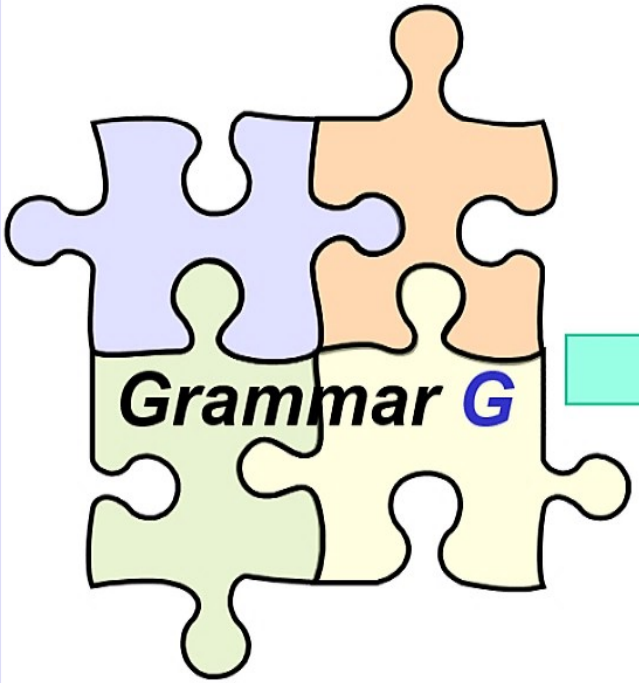


Target Language – FORMAL GRAMMAR



Σ^* *set of all strings over the alphabet* Σ

Language L for which holds the following:

$L = L(G)$

and

$L \subseteq \Sigma^*$

$w_3 \quad w_2 \quad w_1$

Example of a FORMAL LANGUAGE

D^* set of all strings over the alphabet

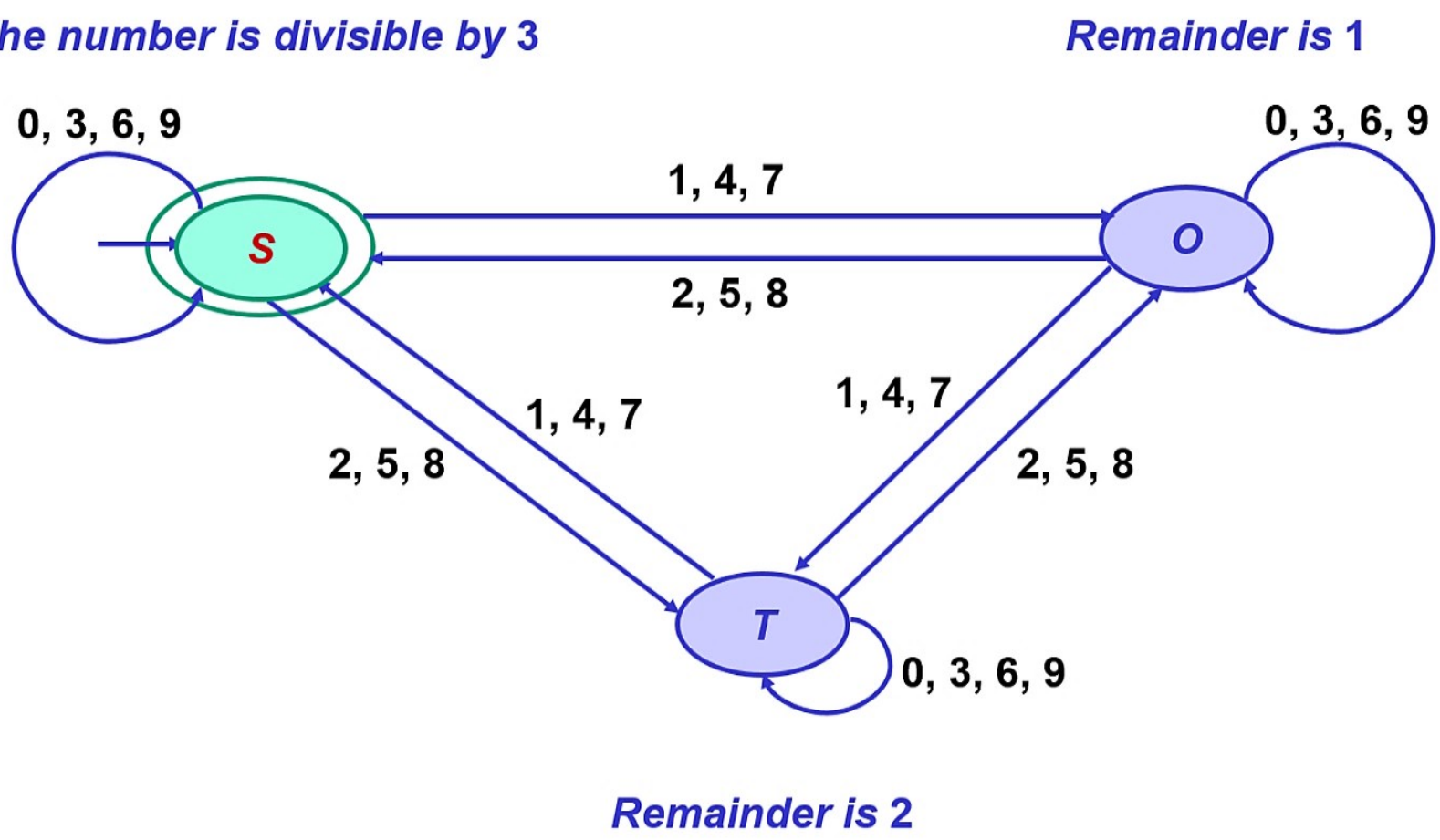
$D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$D^* = \{\varepsilon, 0, 1, 2, 3, 4, 5, 6, \ldots, 124, 125, 126, \ldots\}$

Language $L \subset D^*$

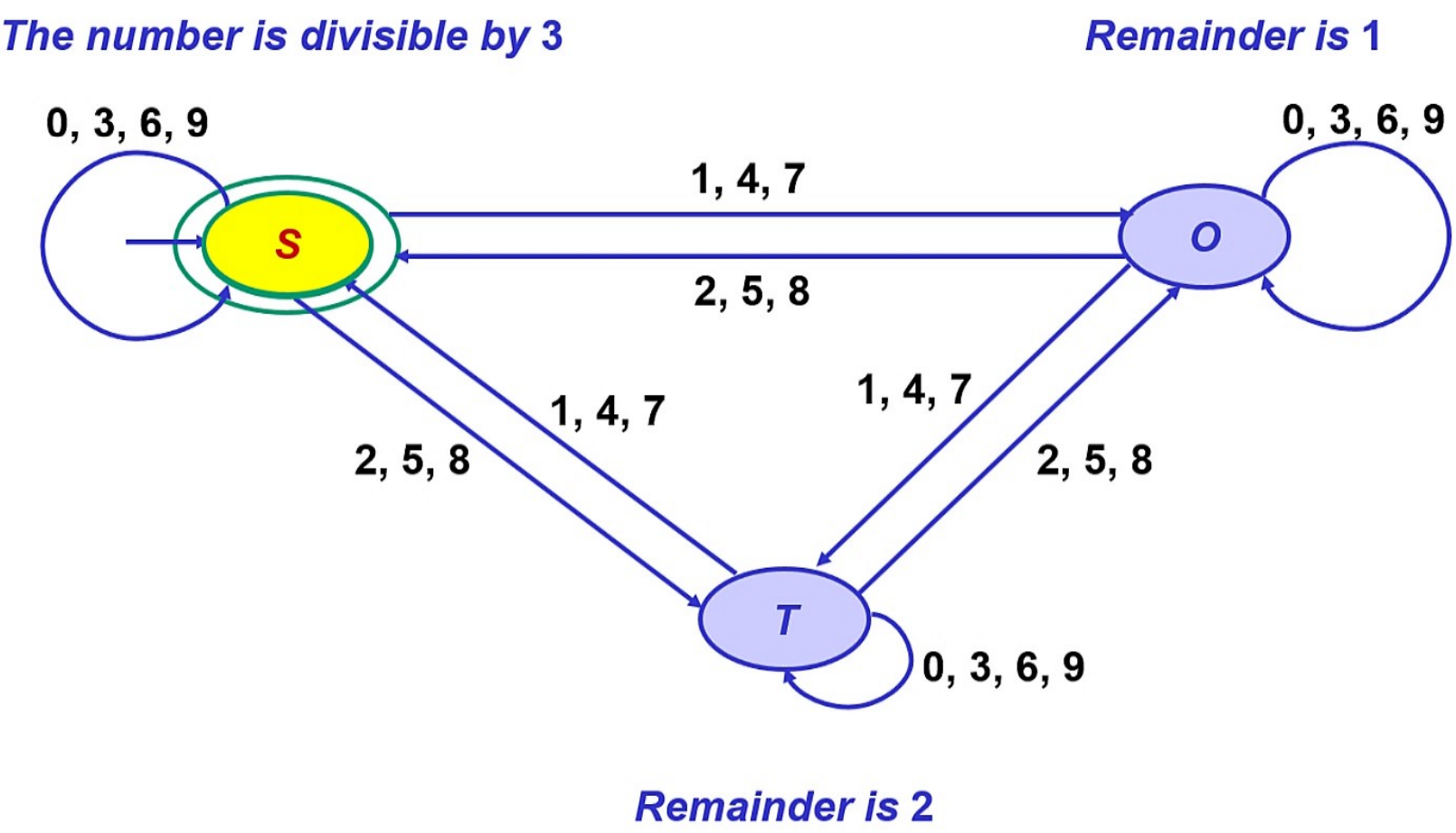
$L = \{\varepsilon, 0, 3, 6, 9, 12, 15, \ldots, 123, \ldots\}$

Example of an AUTOMATON



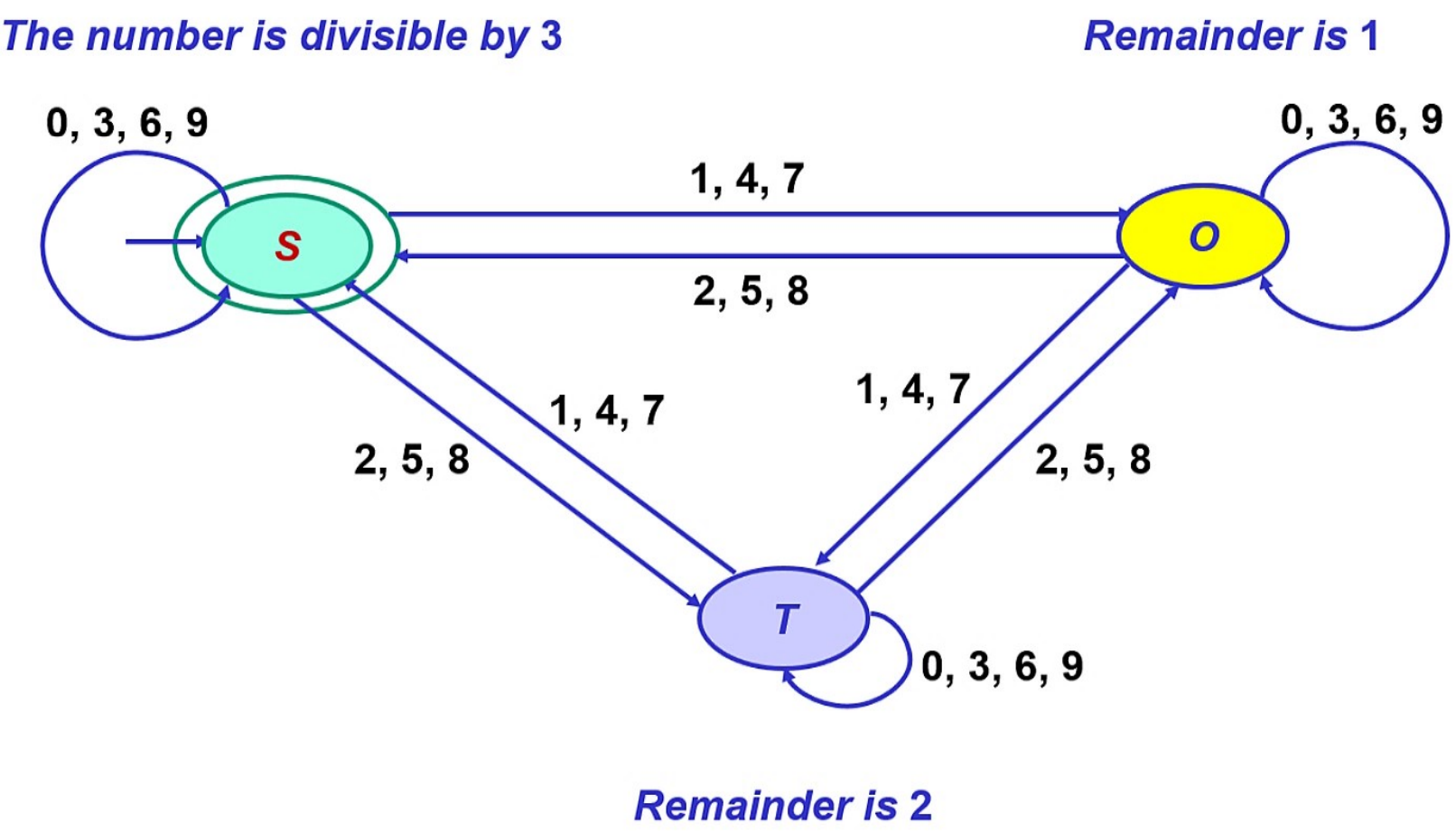
Example of an AUTOMATON

1	4	0	4



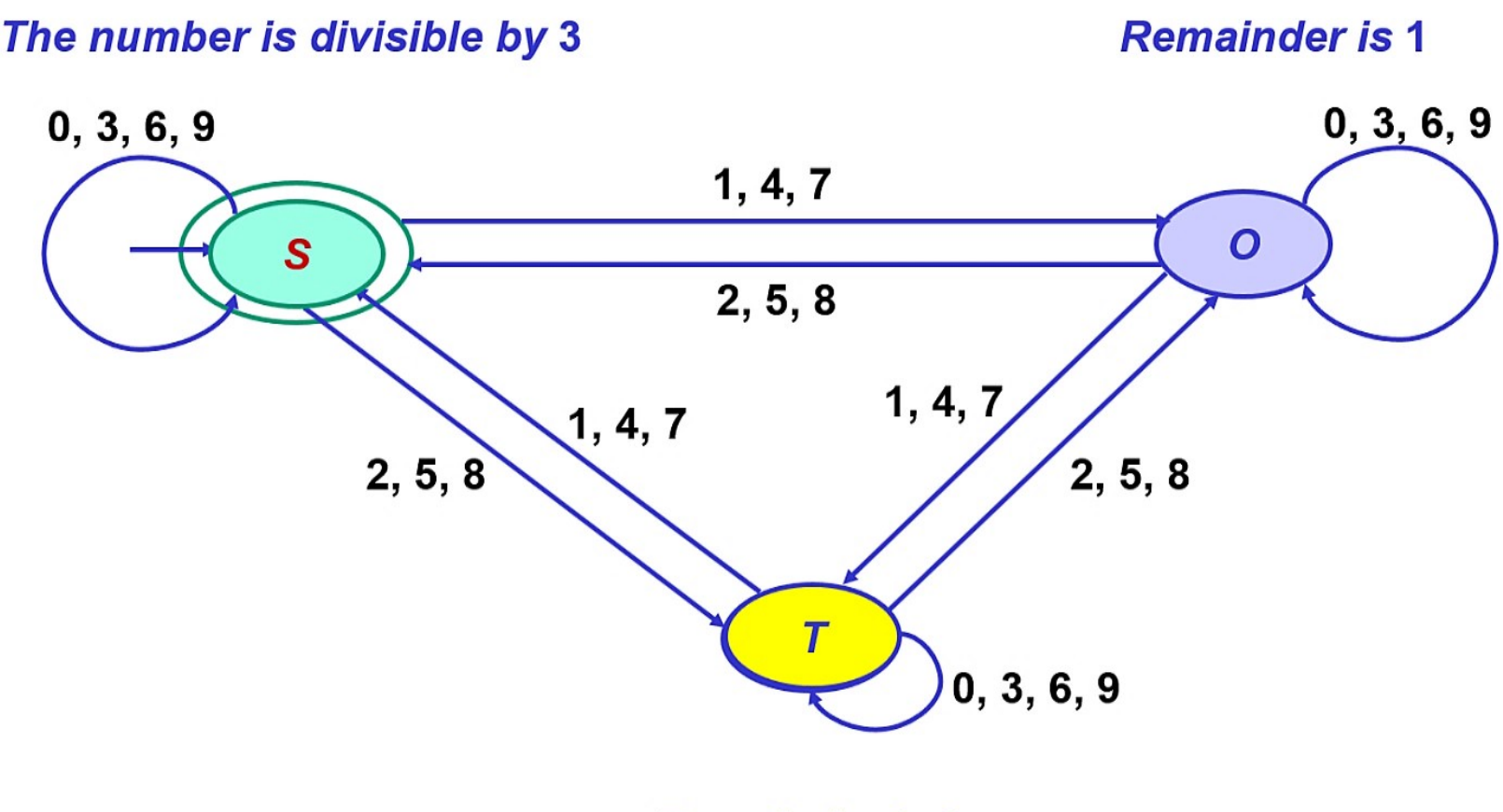
Example of an AUTOMATON

4	0	4



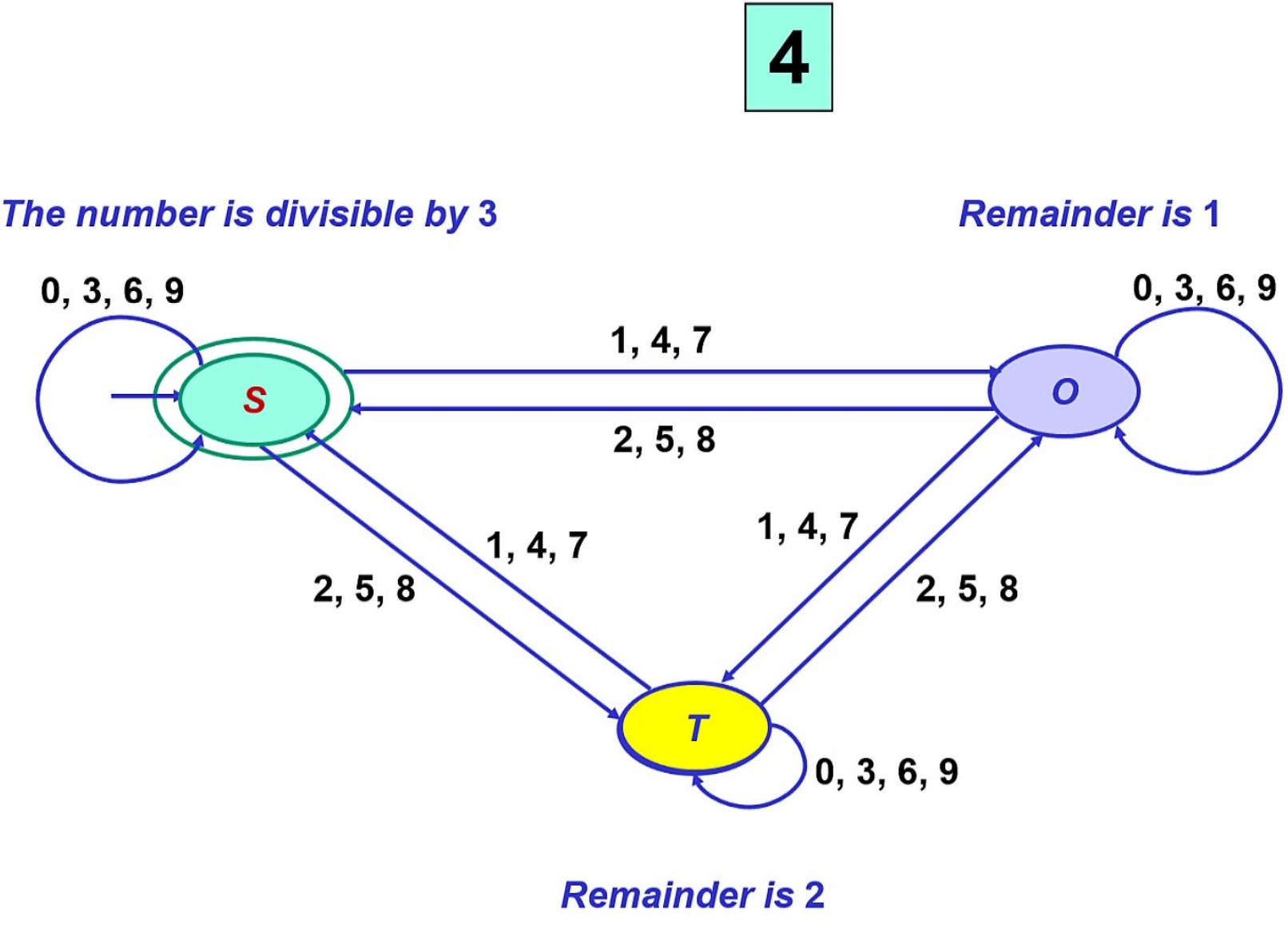
Example of an AUTOMATON

0	4

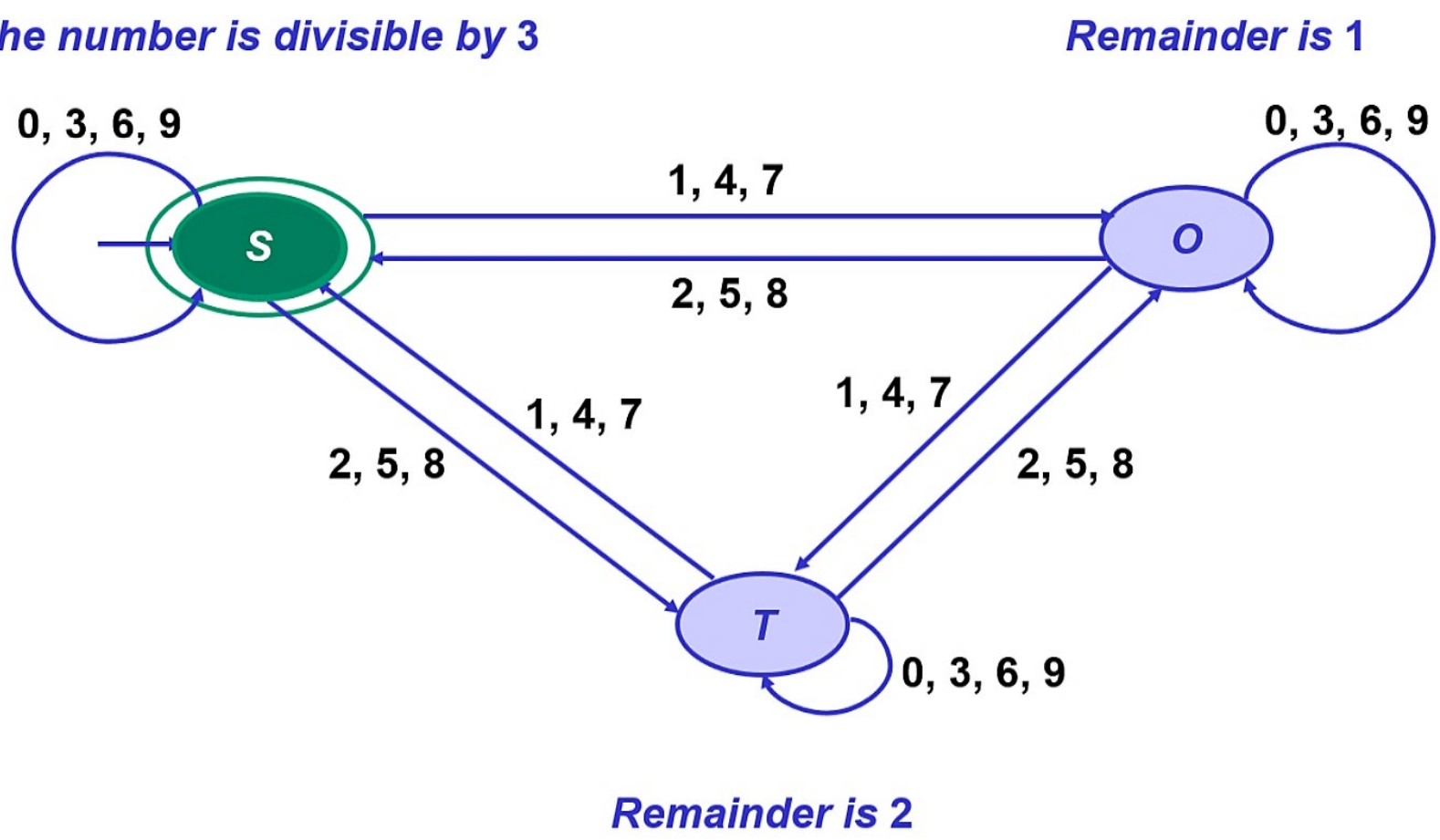


Example of an AUTOMATON

4

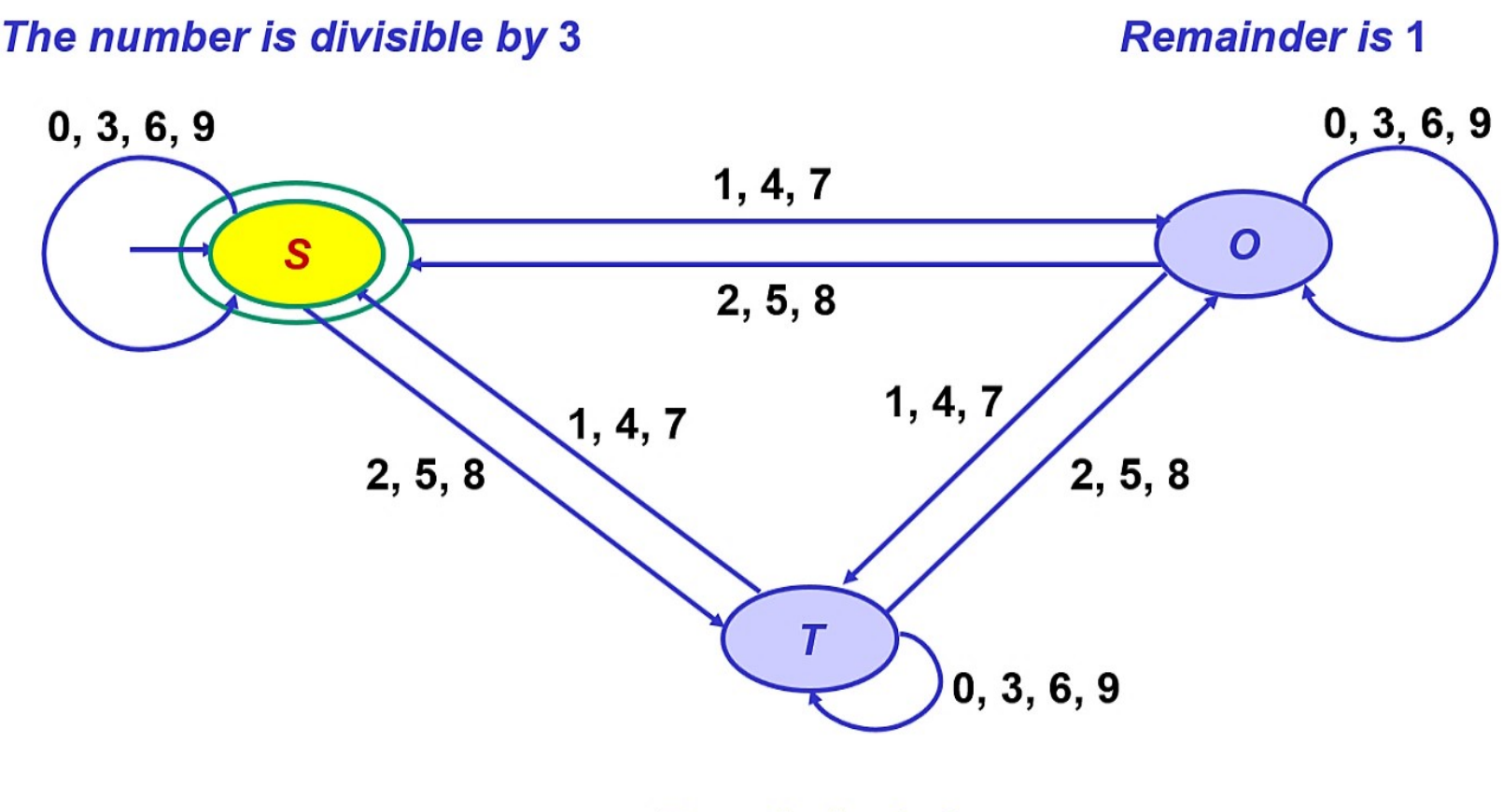


Example of an AUTOMATON



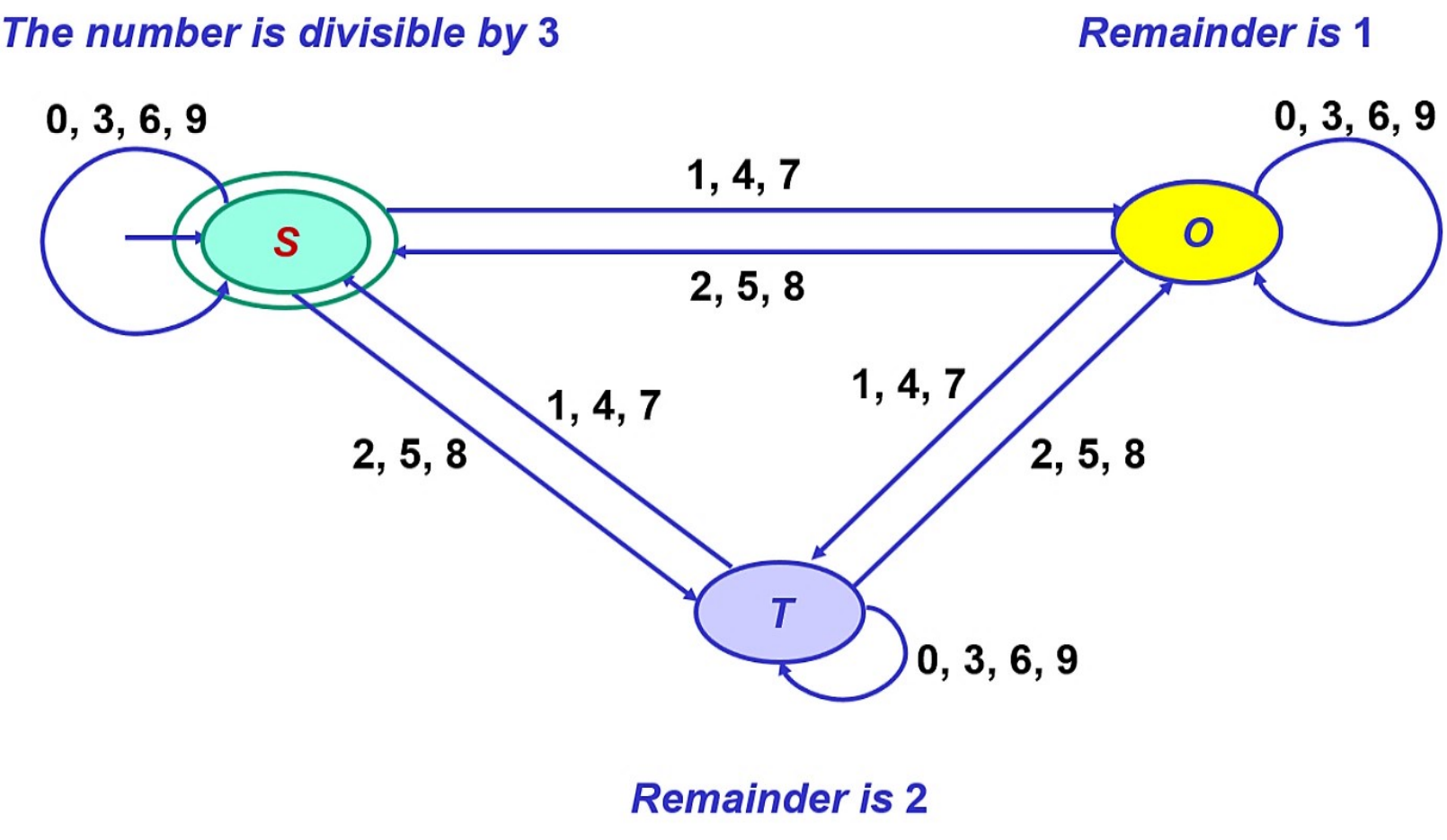
Example of an AUTOMATON

4	0	0



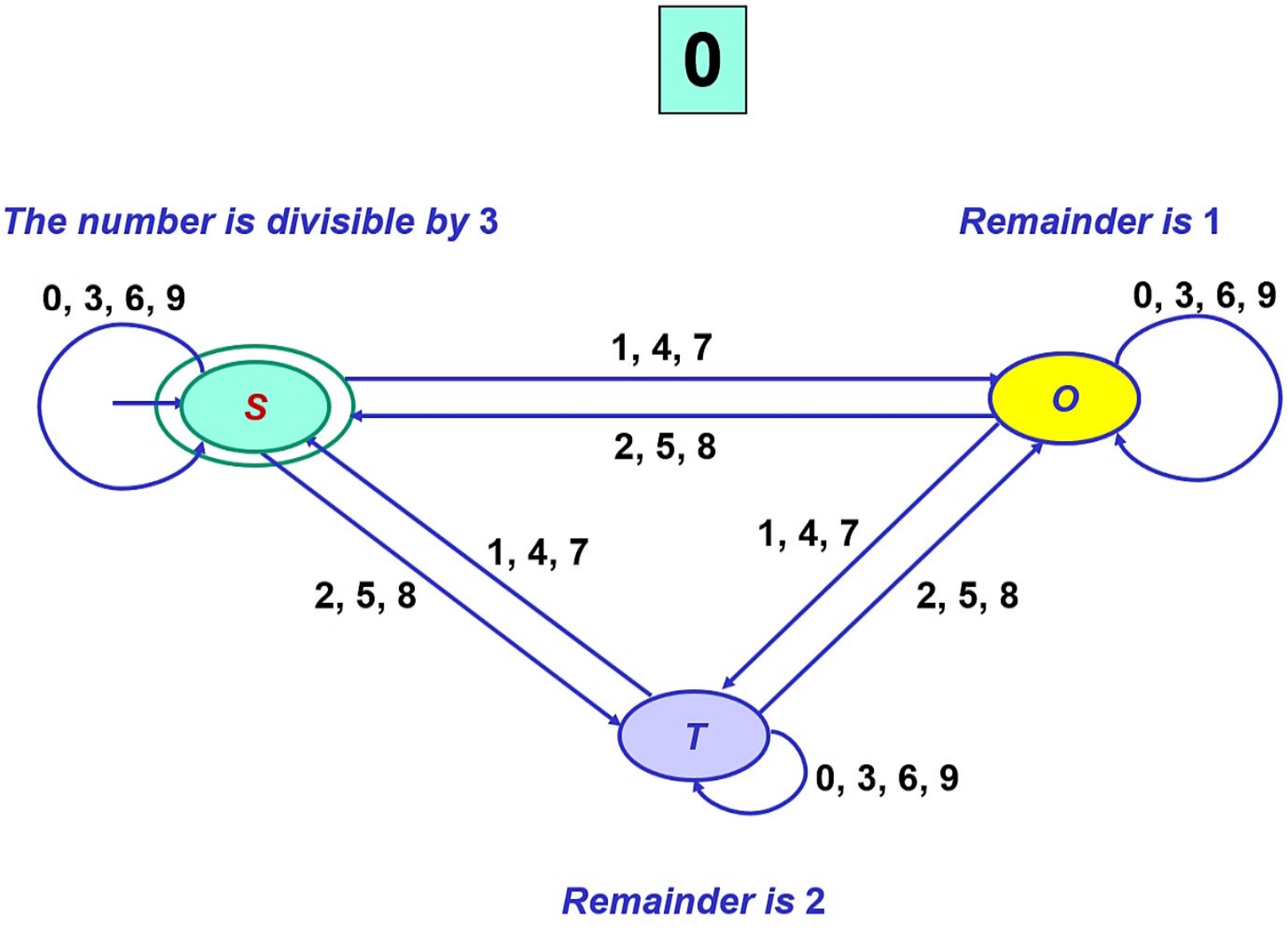
Example of an AUTOMATON

0	0



Example of an AUTOMATON

0



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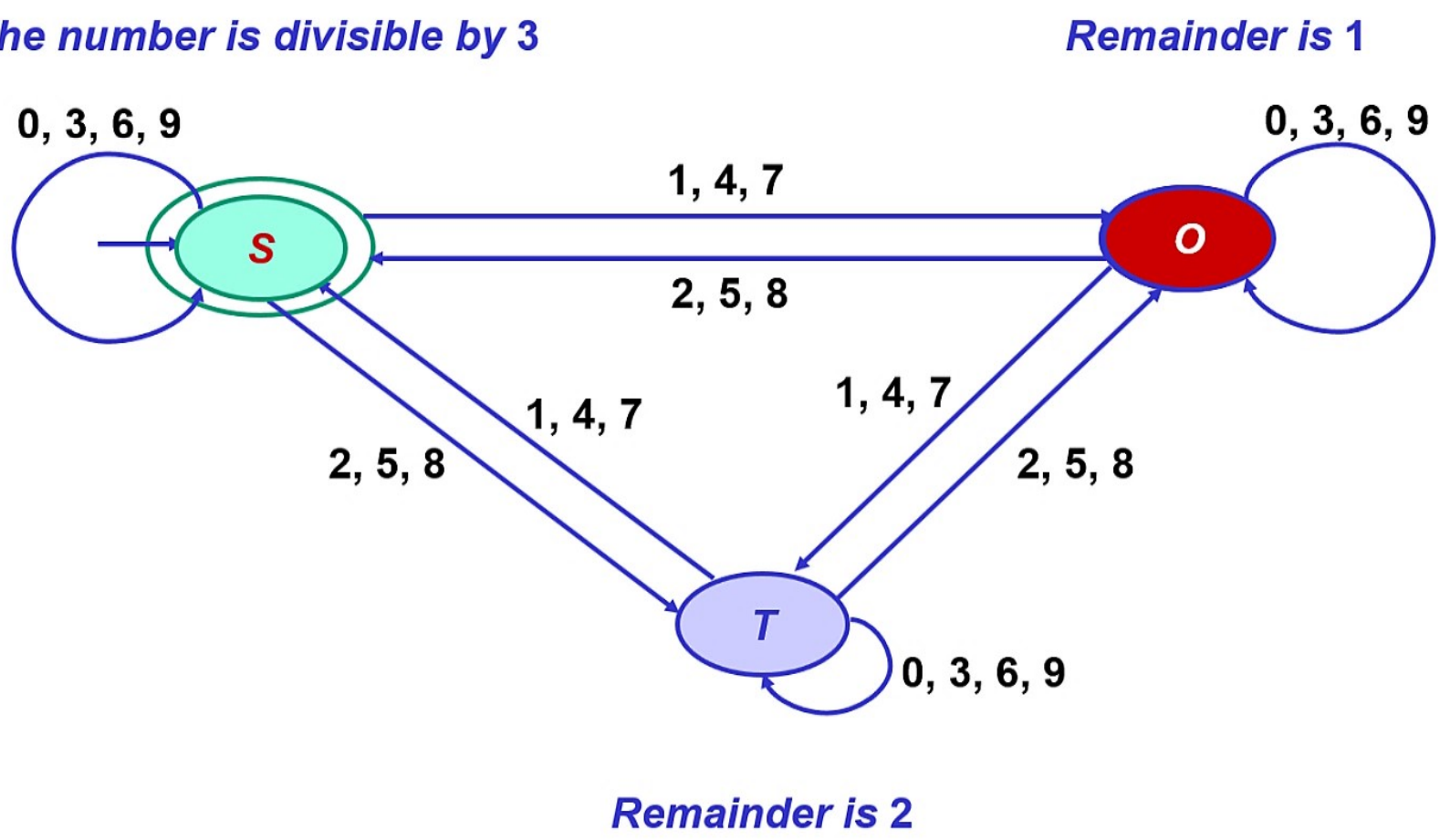


Example of an AUTOMATON

	0	1	2	3	4	5	6	7	8	9	
S	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	1
O	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	0
T	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	0

Example of an AUTOMATON

1	4	0	4

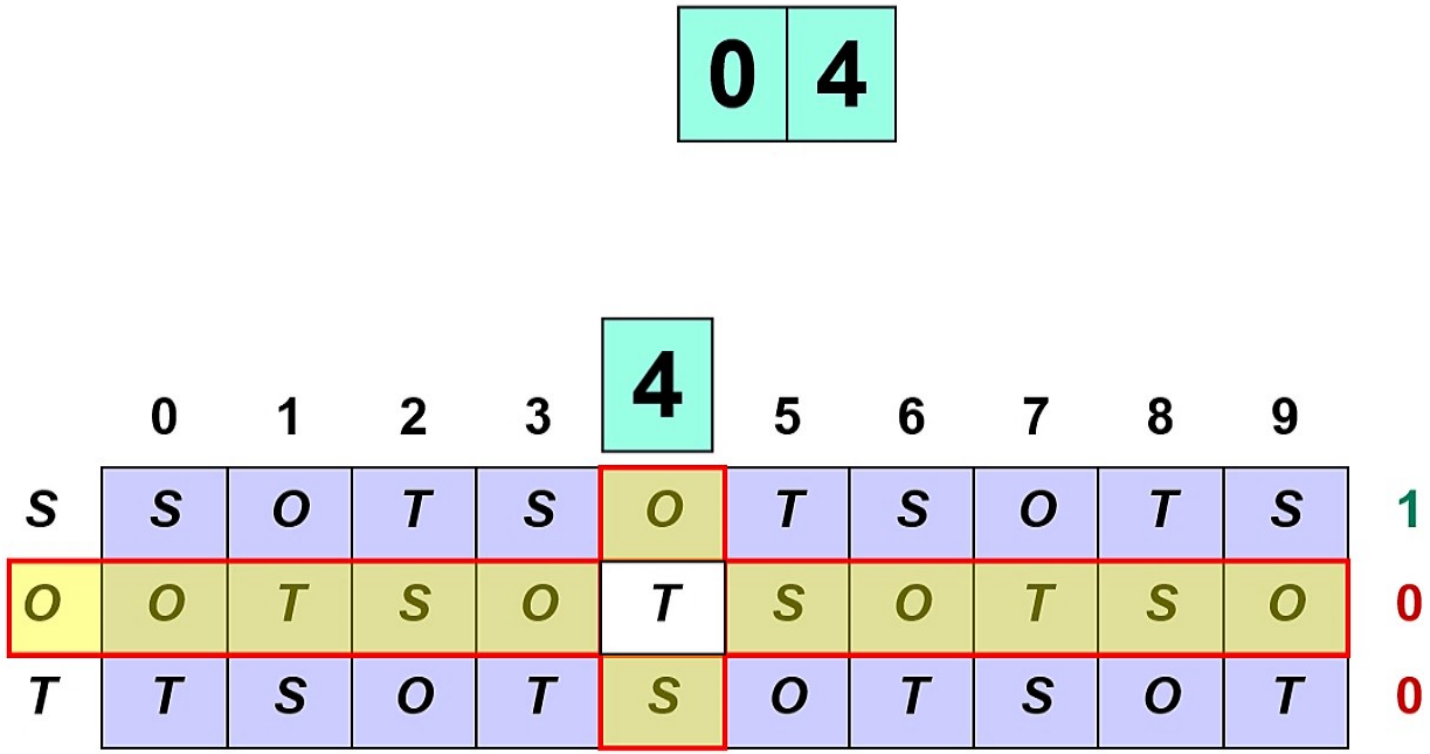
	0	1	2	3	4	5	6	7	8	9	
S	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	1
O	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	0
T	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	0

Example of an AUTOMATON

4	0	4

	0	1	2	3	4	5	6	7	8	9	
S	S	O	T	S	O	T	S	O	T	S	1
O	O	T	S	O	T	S	O	T	S	O	0
T	T	S	O	T	S	O	T	S	O	T	0

Example of an AUTOMATON

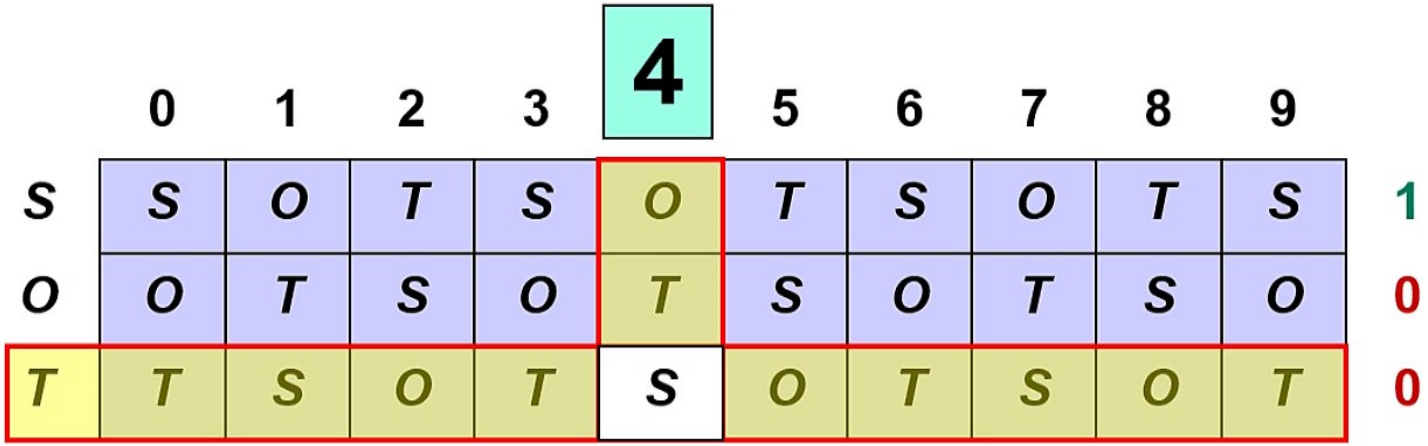


Example of an AUTOMATON

4

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
S	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	**1**
O	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	**0**
T	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	**0**

Example of an AUTOMATON

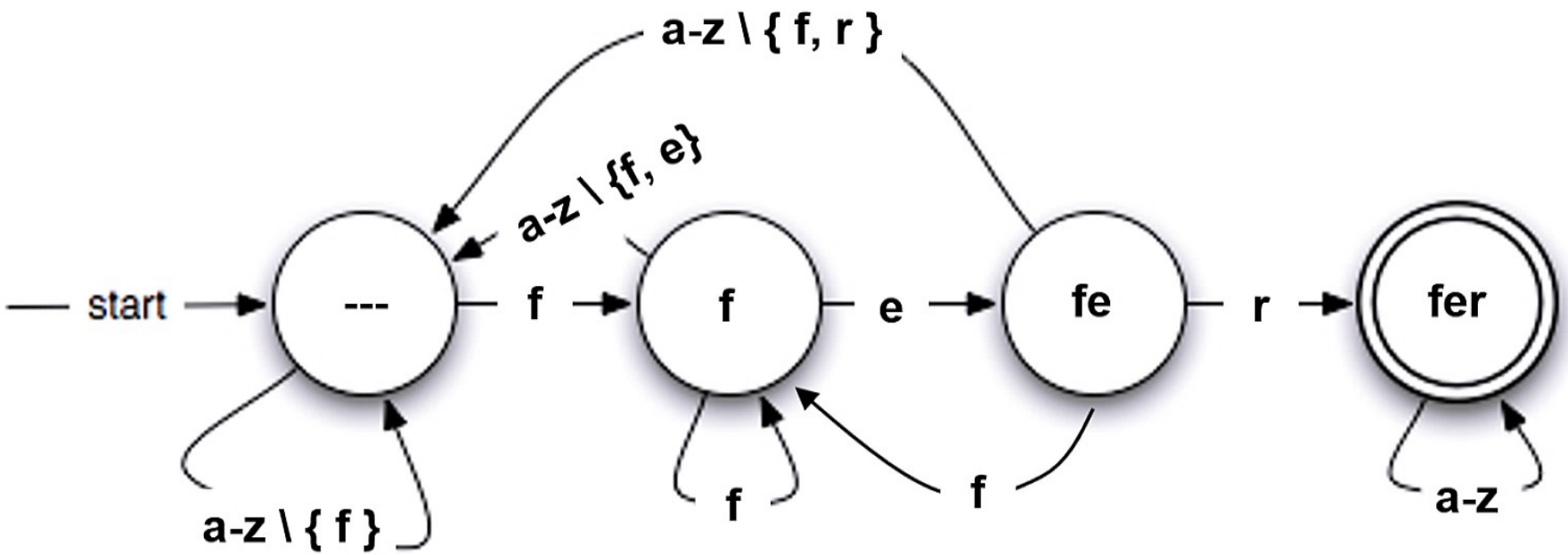


Example of an AUTOMATON

	0	1	2	3	4	5	6	7	8	9	
S	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	1
O	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	0
T	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	0

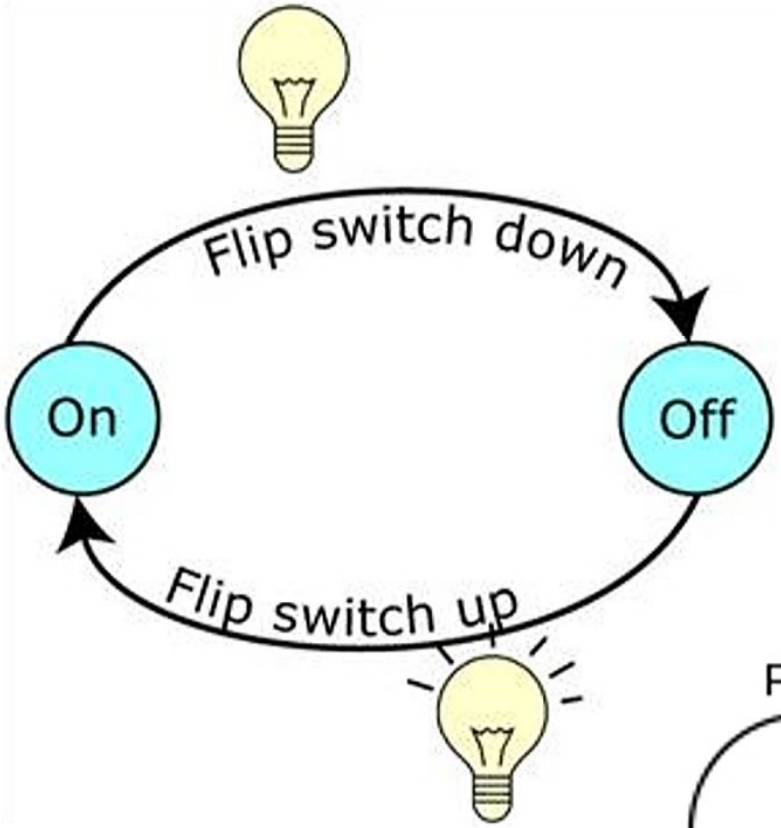
Application of AUTOMATA

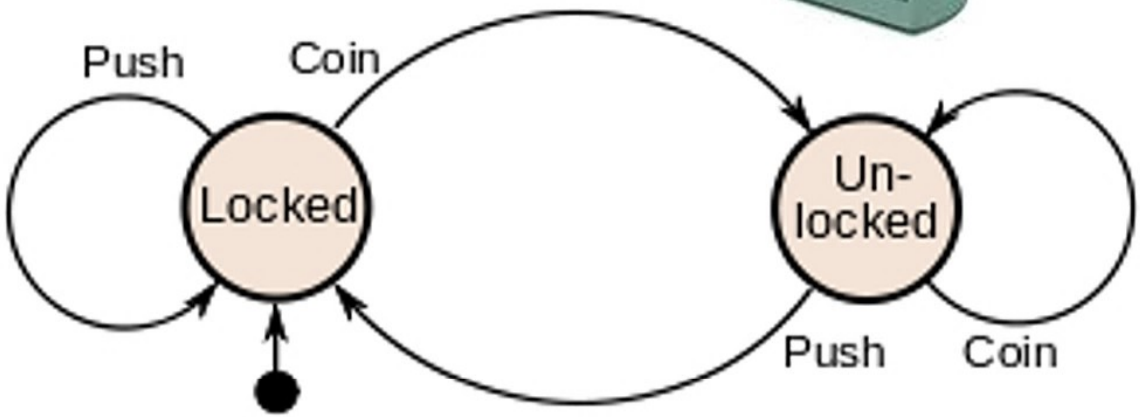
- Text analysis
  - Substring search
    - `regex` library



Application of AUTOMATA

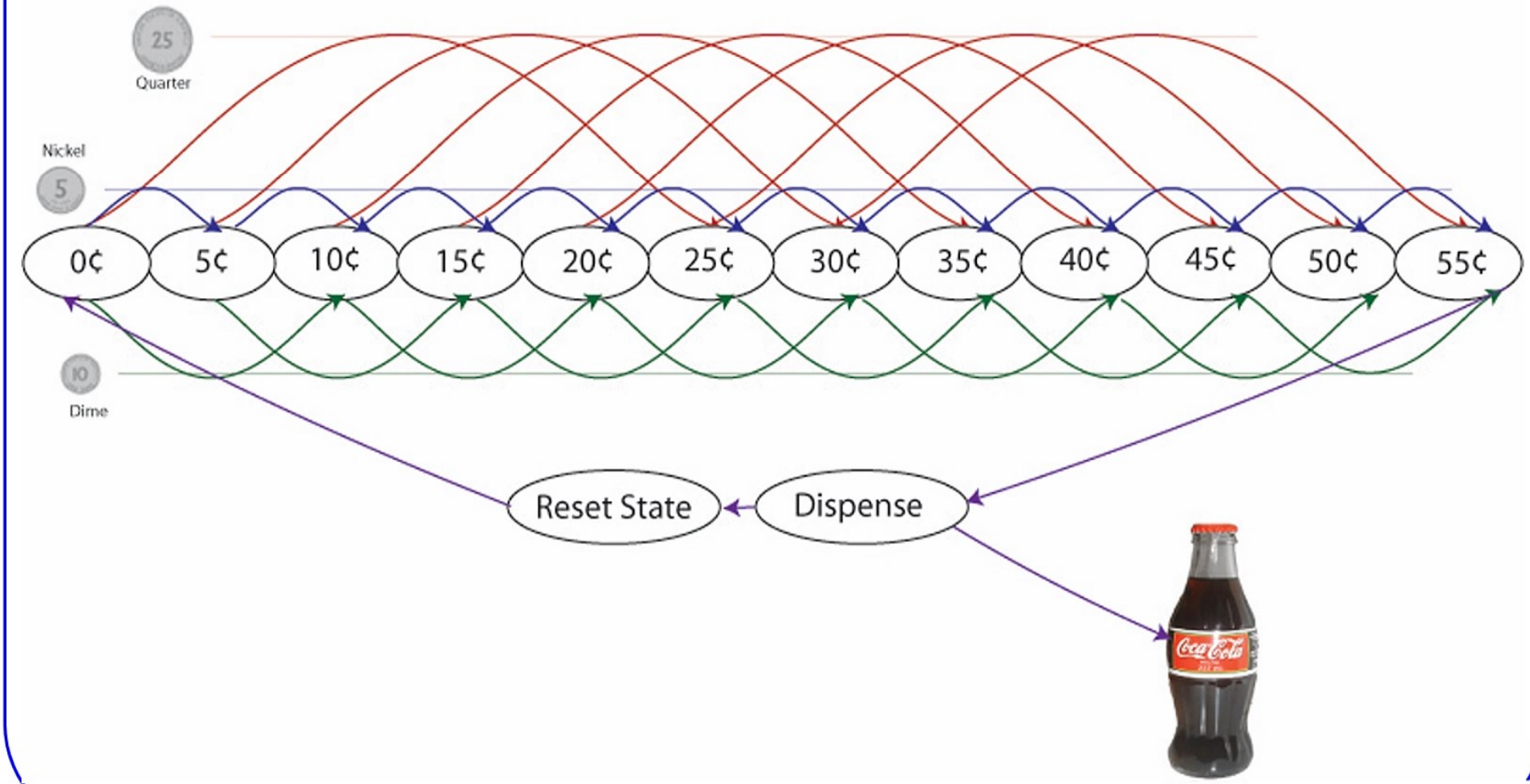
- **Modelling of hardware and technical systems**





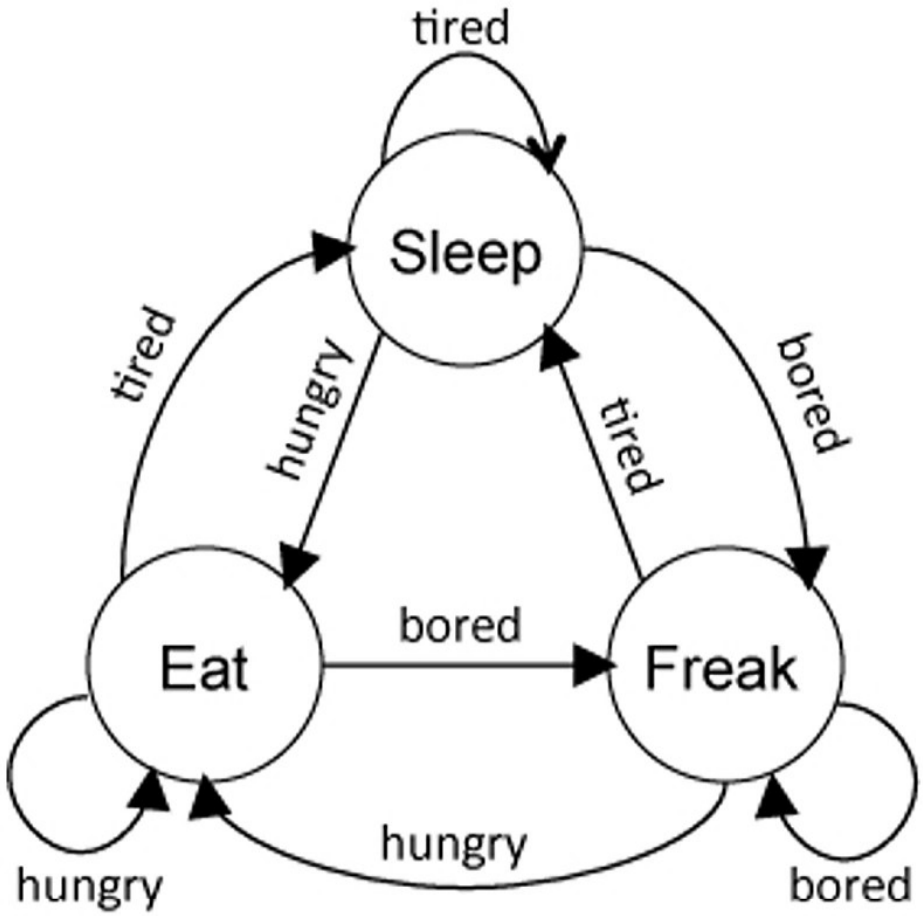
Application of AUTOMATA

- **Modelling of hardware and technical systems**



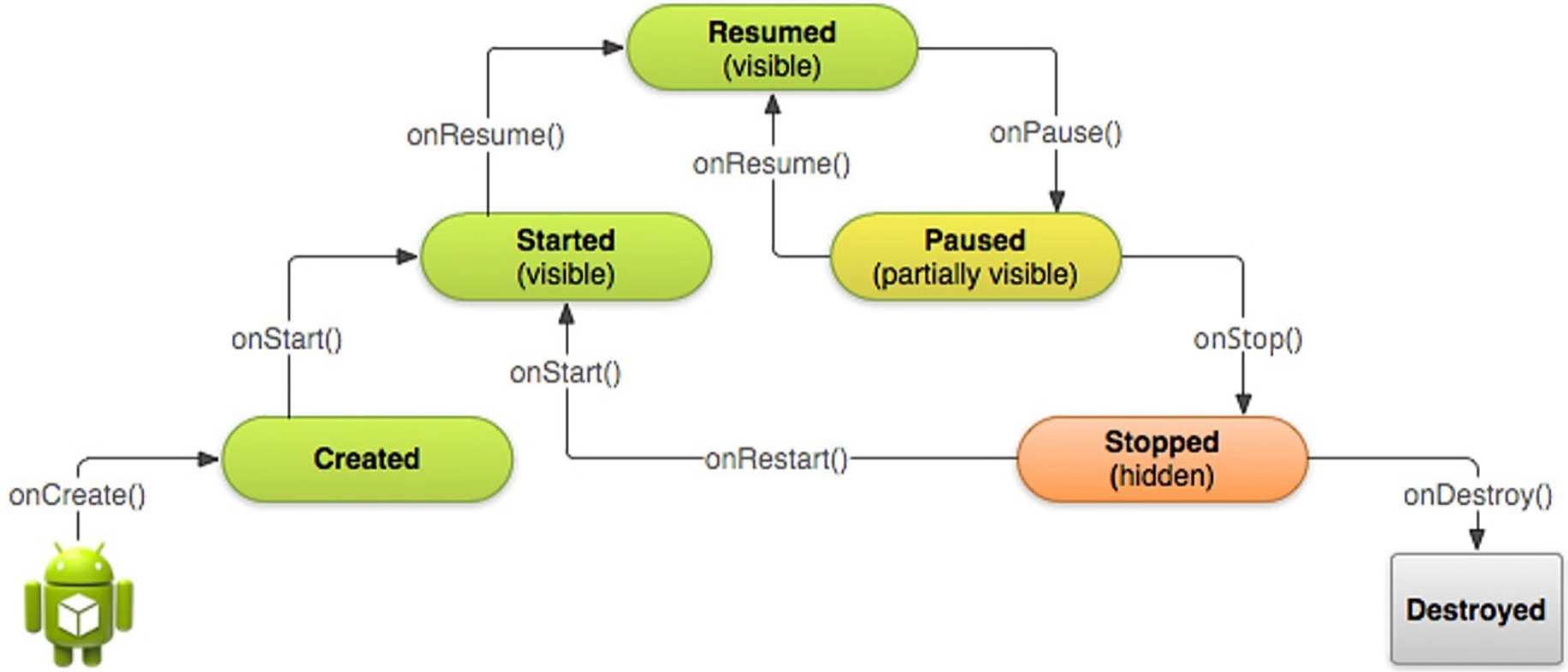
Application of AUTOMATA

- **Modelling of biological organisms and social systems**



Application of AUTOMATA

- **Software development**
  - **Android OS**
    - `Activity` **object lifecycle**



Example of a GRAMMAR

<S> → 0 <S>
<S> → 3 <S>
<S> → 6 <S>
<S> → 9 <S>
<S> → ε

Example of a GRAMMAR

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<S> → 3 <S>
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Example of a GRAMMAR

$\langle S \rangle \rightarrow 0 \langle S \rangle$
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$\langle S \rangle \rightarrow 6 \langle S \rangle$
$\langle S \rangle \rightarrow 9 \langle S \rangle$
$\langle S \rangle \rightarrow \varepsilon$

$\langle S \rangle \Rightarrow 6 \langle S \rangle$

Example of a GRAMMAR

$\langle S \rangle \rightarrow 0 \langle S \rangle$

$\langle S \rangle \rightarrow 3 \langle S \rangle$

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$\langle S \rangle \Rightarrow 6 \langle S \rangle \Rightarrow 6\ 3 \langle S \rangle$

Example of a GRAMMAR

<S> → 0 <S>
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<S> → ε

<S> ⇒ 6 <S> ⇒ 6 3 <S>

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<S> → ε

<S> ⇒ 6 <S> ⇒ 6 3 <S> ⇒ 6 3 6 <S>

Example of a GRAMMAR

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$\langle S \rangle \rightarrow 6 \langle S \rangle$

$\langle S \rangle \rightarrow 9 \langle S \rangle$

$\langle S \rangle \rightarrow \varepsilon$

$\langle S \rangle \Rightarrow 6 \langle S \rangle \Rightarrow 6\ 3 \langle S \rangle \Rightarrow 6\ 3\ 6 \langle S \rangle$

Example of a GRAMMAR

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$\langle S \rangle \Rightarrow 6 \langle S \rangle \Rightarrow 6\ 3 \langle S \rangle \Rightarrow 6\ 3\ 6 \langle S \rangle \Rightarrow 6\ 3\ 6\ 0 \langle S \rangle$

Example of a GRAMMAR

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<S> ⇒ 6 <S> ⇒ 6 3 <S> ⇒ 6 3 6 <S> ⇒ 6 3 6 0 <S>

Example of a GRAMMAR

<S> → 0 <S>
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<S> ⇒ 6 <S> ⇒ 6 3 <S> ⇒ 6 3 6 <S> ⇒ 6 3 6 0 <S> ⇒ 6 3 6 0

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$\langle S \rangle \Rightarrow 6 \langle S \rangle \Rightarrow 6\ 3 \langle S \rangle \Rightarrow 6\ 3\ 6 \langle S \rangle \Rightarrow 6\ 3\ 6\ 0 \langle S \rangle \Rightarrow 6\ 3\ 6\ 0$

$\langle S \rangle \rightarrow 1 \langle O \rangle \mid 4 \langle O \rangle \mid 7 \langle O \rangle$

$\langle O \rangle \rightarrow 0 \langle O \rangle \mid 3 \langle O \rangle \mid 6 \langle O \rangle \mid 9 \langle O \rangle$
$\langle O \rangle \rightarrow 2 \langle S \rangle \mid 5 \langle S \rangle \mid 8 \langle S \rangle$
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1 4 0 4

Example of a GRAMMAR

$\langle S \rangle \rightarrow 0 \langle S \rangle$
$\langle S \rangle \rightarrow 3 \langle S \rangle$
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$\langle S \rangle \rightarrow 9 \langle S \rangle$
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$\langle O \rangle \rightarrow 2 \langle S \rangle \mid 5 \langle S \rangle \mid 8 \langle S \rangle$
$\langle O \rangle \rightarrow 1 \langle T \rangle \mid 4 \langle T \rangle \mid 7 \langle T \rangle$

$\langle S \rangle \rightarrow 2 \langle T \rangle \mid 5 \langle T \rangle \mid 8 \langle T \rangle$

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$\langle T \rangle \rightarrow 1 \langle S \rangle \mid 4 \langle S \rangle \mid 7 \langle S \rangle$
$\langle T \rangle \rightarrow 2 \langle O \rangle \mid 5 \langle O \rangle \mid 8 \langle O \rangle$

1 4 0 4

$\langle S \rangle \Rightarrow 1 \langle O \rangle$

Example of a GRAMMAR

$\langle S \rangle \rightarrow 0 \langle S \rangle$
$\langle S \rangle \rightarrow 3 \langle S \rangle$
$\langle S \rangle \rightarrow 6 \langle S \rangle$
$\langle S \rangle \rightarrow 9 \langle S \rangle$
$\langle S \rangle \rightarrow \varepsilon$

$\langle S \rangle \Rightarrow 6 \langle S \rangle \Rightarrow 6\ 3 \langle S \rangle \Rightarrow 6\ 3\ 6 \langle S \rangle \Rightarrow 6\ 3\ 6\ 0 \langle S \rangle \Rightarrow 6\ 3\ 6\ 0$

$\langle S \rangle \rightarrow 1 \langle O \rangle \mid 4 \langle O \rangle \mid 7 \langle O \rangle$

$\langle O \rangle \rightarrow 0 \langle O \rangle \mid 3 \langle O \rangle \mid 6 \langle O \rangle \mid 9 \langle O \rangle$
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$\langle S \rangle \rightarrow 2 \langle T \rangle \mid 5 \langle T \rangle \mid 8 \langle T \rangle$

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$\langle T \rangle \rightarrow 1 \langle S \rangle \mid 4 \langle S \rangle \mid 7 \langle S \rangle$
$\langle T \rangle \rightarrow 2 \langle O \rangle \mid 5 \langle O \rangle \mid 8 \langle O \rangle$

1 4 0 4

$\langle S \rangle \Rightarrow 1 \langle O \rangle$

Example of a GRAMMAR

$\langle S \rangle \rightarrow 0 \langle S \rangle$
$\langle S \rangle \rightarrow 3 \langle S \rangle$
$\langle S \rangle \rightarrow 6 \langle S \rangle$
$\langle S \rangle \rightarrow 9 \langle S \rangle$
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$\langle S \rangle \Rightarrow 6 \langle S \rangle \Rightarrow 6\ 3 \langle S \rangle \Rightarrow 6\ 3\ 6 \langle S \rangle \Rightarrow 6\ 3\ 6\ 0 \langle S \rangle \Rightarrow 6\ 3\ 6\ 0$

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1 4 0 4

$\langle S \rangle \Rightarrow 1 \langle O \rangle \Rightarrow 1\ 4 \langle T \rangle$

Example of a GRAMMAR

$\langle S \rangle \rightarrow 0 \langle S \rangle$
$\langle S \rangle \rightarrow 3 \langle S \rangle$
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$\langle S \rangle \Rightarrow 6 \langle S \rangle \Rightarrow 6\ 3 \langle S \rangle \Rightarrow 6\ 3\ 6 \langle S \rangle \Rightarrow 6\ 3\ 6\ 0 \langle S \rangle \Rightarrow 6\ 3\ 6\ 0$

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1 4 0 4

$\langle S \rangle \Rightarrow 1 \langle O \rangle \Rightarrow 1\ 4 \langle T \rangle$

Example of a GRAMMAR

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$\langle T \rangle \rightarrow 2 \langle O \rangle \mid 5 \langle O \rangle \mid 8 \langle O \rangle$

1 4 0 4

$\langle S \rangle \Rightarrow 1 \langle O \rangle \Rightarrow 1\ 4 \langle T \rangle \Rightarrow 1\ 4\ 0 \langle T \rangle$

Example of a GRAMMAR

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$\langle S \rangle \rightarrow 3 \langle S \rangle$
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1 4 0 4

$\langle S \rangle \Rightarrow 1 \langle O \rangle \Rightarrow 1\ 4 \langle T \rangle \Rightarrow 1\ 4\ 0 \langle T \rangle$

Example of a GRAMMAR

$\langle S \rangle \rightarrow 0 \langle S \rangle$
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$\langle T \rangle \rightarrow 0 \langle T \rangle \mid 3 \langle T \rangle \mid 6 \langle T \rangle \mid 9 \langle T \rangle$
$\langle T \rangle \rightarrow 1 \langle S \rangle \mid 4 \langle S \rangle \mid 7 \langle S \rangle$
$\langle T \rangle \rightarrow 2 \langle O \rangle \mid 5 \langle O \rangle \mid 8 \langle O \rangle$

1 4 0 4

$\langle S \rangle \Rightarrow 1 \langle O \rangle \Rightarrow 1\ 4 \langle T \rangle \Rightarrow 1\ 4\ 0 \langle T \rangle \Rightarrow 1\ 4\ 0\ 4 \langle S \rangle$

Example of a GRAMMAR

$\langle S \rangle \rightarrow 0 \langle S \rangle$
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$\langle T \rangle \rightarrow 2 \langle O \rangle \mid 5 \langle O \rangle \mid 8 \langle O \rangle$

1 4 0 4

$\langle S \rangle \Rightarrow 1 \langle O \rangle \Rightarrow 1\ 4 \langle T \rangle \Rightarrow 1\ 4\ 0 \langle T \rangle \Rightarrow 1\ 4\ 0\ 4 \langle S \rangle$

Example of a GRAMMAR

$\langle S \rangle \rightarrow 0 \langle S \rangle$
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$\langle S \rangle \rightarrow 9 \langle S \rangle$
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$\langle S \rangle \rightarrow 2 \langle T \rangle \mid 5 \langle T \rangle \mid 8 \langle T \rangle$

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1 4 0 4

$\langle S \rangle \Rightarrow 1 \langle O \rangle \Rightarrow 1\ 4 \langle T \rangle \Rightarrow 1\ 4\ 0 \langle T \rangle \Rightarrow 1\ 4\ 0\ 4 \langle S \rangle \Rightarrow 1\ 4\ 0\ 4$

Example of a GRAMMAR

$\langle S \rangle \rightarrow 0 \langle S \rangle$
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1 4 0 4

$\langle S \rangle \Rightarrow 1 \langle O \rangle \Rightarrow 1\ 4 \langle T \rangle \Rightarrow 1\ 4\ 0 \langle T \rangle \Rightarrow 1\ 4\ 0\ 4 \langle S \rangle \Rightarrow 1\ 4\ 0\ 4$

4 0 0

$\langle S \rangle \Rightarrow 4 \langle O \rangle \Rightarrow 4\ 0 \langle O \rangle \Rightarrow 4\ 0\ 0 \langle O \rangle$

COMPILER

- **Source program analysis**
  - to simplify the acceptance of a source program
- **Target program synthesis**
  - to simplify the generation of a target program

COMPILER

- **Source program analysis**

- two steps of translation

1. during the lexical analysis

$$element = array[i][j + 7 + 24];$$
$$V \leftarrow V[V][V + K + K];$$

2. during the syntax analysis (parsing), semantic analysis, and high-level intermediate code generation

$$element = array[i][j + 31];$$

COMPILER

- **Target program synthesis**
  - three steps of translation

1. Translation of the high- to medium-level intermediate code

$$v1 \leftarrow i*100$$
$$v2 \leftarrow j+31$$
$$v3 \leftarrow v1+v2$$
$$v4 \leftarrow 4*v3$$
$$v5 \leftarrow AddrArray$$
$$v6 \leftarrow v5+v4$$
$$element \leftarrow @v6$$

COMPILER

2. Translation of the medium- to low-level intermediate code

r1 ← [*Start*]

r2 ← r1∗100

r3 ← [*Start*+4]

r4 ← r3+31

r5 ← r4+r2

r6 ← 4∗r5

r7 ← *Start*+8

r8 ← [r7+r6]

COMPILER

3. Translation of the low-level intermediate code to the target code (usually, the machine code)