Introduction to Artificial Intelligence

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Exercises, v3

6 Automated reasoning

1 (T) Refutation resolution is a sound and complete inference rule for FOL. **What does that mean?**

- A The procedure derives the NIL clause if and only if the goal is the logical consequence of the premises
- B Whenever a set of premises is consistent, the resolution rule derives the NIL clause
- C If the formula is not a logical consequence, then we cannot prove it with refutation resolution
- D The procedure terminates in a finite number of steps with a decision whether the formula is a logical consequence of the premise or not

2 (T) The resolution method can easily be automated because it relies on a single inference rule. However, we also require a proof method to be sound and preferably complete. Resolution in PL and FOL is both, but only under certain conditions. **Under which conditions is resolution both sound and complete in FOL?**

- A It is complete when run as refutation resolution, if no variable appears in more than a single clause and is never substituted by a term than contains it, while it is sound if factorization is carried out in every step
- B It is sound if all existentially quantified variables are skolemized and if the proof is done with the set-of-support strategy, and complete if run as refutation resolution over standardized and factorized clauses
- C It is unconditionally complete, but sound only when run as refutation resolution over clauses that share no common variables, and if resolving all combinations of original and factor clauses
- D It is unconditionally sound, but complete only when run as refutation resolution on standardized clauses, provided all combinations of original and factor clauses are resolved under a complete proof strategy

3 (P) Consider the following two premises:

$$\forall x \text{KNOWS}(x, \textit{Elizabeth})$$
$$\forall x \big(\text{KNOWS}(\textit{John}, x) \rightarrow \text{HATES}(\textit{John}, x)\big)$$

We wish to use resolution to prove that, since everybody knows Elizabeth, and John hates everybody he knows, he also hates Elizabeth. **What is this concrete example showcasing?**

- A That refutation resolution is not sound if we allow for a variable to be substituted by a term that contain that very variable
- B That refutation resolution is not complete if we don't do standardization
- C That direct resolution without standardization cannot prove all that can be proven using direct resolution with standardization
- D That direct resolution is incomplete, whereas refutation resolution is complete

4 (P) The MGU algorithm is being applied to the following pair of atoms: $P(f(a,x), x, g(y), f(z,a))$ and $P(y, g(z), w, f(b,a))$. **What is the substitution returned by the MGU algorithm?**

- A $\{g(b)/x, f(a, g(b))/y, b/z, g(f(a, g(b)))/w\}$
- B $\{g(b)/y, f(a, g(a))/x, b/z, x/w\}$
- C $\{g(f(a, b))/x, f(a, g(a))/x, b/z, g(g(b))/w\}$
- D The algorithm returns an error

5 (P) We are given clauses $\{P(g(y), x), \neg Q(x, b)\}$ and $\{Q(f(x), y)\}$. **What is their resolvent?**

A $P(g(y), f(z))$ B $P(g(y), x)$ C $P(g(y), b)$ D $P(g(y), y)$

6 (P) Let $T(x)$ stand for "x is a town", $I(x, y)$ for "x is in y", and $P(x)$ for "x is a post office". Write down the sentence *"There is a post office in every town"* as a FOL formula. **What is the clausal form of this formula?**

- A $\big(\neg T(x) \vee I(f(x), x)\big) \wedge \big(\neg T(x') \vee P(f(x'))\big)$
- B $\big(\neg T(x) \vee P(y)\big) \wedge \big(\neg T(x') \vee I(y', x')\big)$
- C $T(x'') \wedge P(f(x')) \wedge I(f(x), x)$
- D $\neg T(x) \vee \neg P(f(x)) \vee I(f(x), x)$

7 (C) We're given the following two premises:

$$\exists x\Big(\exists y R(y) \vee \forall z \neg Q(x, z)\Big) \rightarrow P(a),\ \exists x \forall y\Big(Q(x, y) \rightarrow R(y)\Big)$$

Which of the following formulas can be derived from these premises using refutation resolution?

A $\exists x P(x)$ B $\exists x \forall y P(y)$ C $\exists x P(x) \wedge P(b)$ D $\forall x Q(a, x)$

8 (C) Consider the following premises: *"Lovers are all those and only those who adore somebody"*, (2) *"Everybody adores lovers"*, and (3) *"Jane adores Tom"*. Formalize these premises using $L(x)$ for "x is a lover" and $A(x, y)$ for "x adores y". Then, using refutation resolution with a set-of-support strategy, prove *"Everybody adores everybody"*. **Is the goal provable and, if so, what is the least number of resolution steps needed to prove it?**

A Provable in 3 steps B Provable in 4 steps C Provable in 5 steps D Not provable