

### Finite state automaton



Finite state automaton



Deterministic pushdown automaton

Finite state automaton



### Deterministic pushdown automaton

Finite state automaton

Deterministic context-free languages: DCFL Language  $L_3$ : {  $w2w^R \mid w \in (0+1)^* i \mid w \mid >1$  }  $L_3 \in DCFL i L_3 \notin RL$ 



### Non-deterministic pushdown automaton

Deterministic pushdown automaton

Finite state automaton

Deterministic context-free languages: DCFL Language  $L_3$ : {  $w2w^R \mid w \in (0+1)^* i \mid w \mid >1$  }  $L_3 \in DCFL i L_3 \notin RL$ 



#### Non-deterministic pushdown automaton

Deterministic pushdown automaton

Finite state automaton

```
Non-deterministic context-free languages: NDCFL Language L_2: { ww^R \mid w \in (0+1)^* i \mid w \mid >1 }
L_2 \in NDCFL \ i \ L_2 \notin DCFL
```

Deterministic context-free languages: DCFL Language  $L_3$ : {  $w2w^R \mid w \in (0+1)^* i \mid w \mid >1$  }  $L_3 \in DCFL i L_3 \notin RL$ 



### **Turing machine**

Non-deterministic pushdown automaton

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Non-deterministic context-free languages: NDCFL Language  $L_2$ : {  $ww^R \mid w \in (0+1)^* i \mid w \mid >1$  }  $L_2 \in NDCFL \ i \ L_2 \notin DCFL$ 

Deterministic context-free languages: DCFL Language  $L_3$ : {  $w2w^R \mid w \in (0+1)^* i \mid w \mid >1$  }  $L_3 \in DCFL i L_3 \notin RL$ 



### **Turing machine**

Non-deterministic pushdown automaton

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### Recursively enumerable languages

Non-deterministic context-free languages: NDCFL

Language  $L_2$ : {  $ww^R \mid w \in (0+1)^* i \mid w \mid >1$  }

<u>L∝∈ NDCFL i L∝∉ DCFL</u>

Deterministic context-free languages: DCFL

Language  $L_3$ : {  $w2w^R \mid w \in (0+1)^* i \mid w \mid >1$  }

 $L_3 \in DCFL i L_3 \notin RL$ 



### Lecture 11.

4 RECURSIVELY ENUMERABLE LANGUAGES	126
4.1 TURING MACHINE (TM)	126
4.1.1 Simple Turing machine	126
4.1.2 Programming techniques for Turing machines	133



### **Lecture outline**

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Recursively enumerable language



- Recursively enumerable language
  - Turing machine



- Recursively enumerable language
  - Turing machine
  - Most general computational model



### **Alan Turing**

- Born on 23 June 1912 in London
- Studied computer science, mathematics, philosophy, physics and biology
- As a student, in 1936, he published the work "On Computable Numbers" in which he presented the concept of the so-called Turing machine





### **Alan Turing**

- During the Second World War he worked as a cryptanalyst in the UK
- After the war, Turing worked at the National Physical Laboratory in London on the ACE (Automatic Computing Engine) project
- He later worked at the University of Manchester on the MADM (Manchester Automatic Digital Machine) project





### **Alan Turing**

- In 1951 he became a member of the Royal Society
- Died on 7 June 1954 in Wilmslow as a result of cyanide poisoning (alleged suicide)





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**Finite automaton** 



**Finite automaton** 

Finite control



**Finite automaton** 

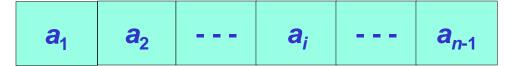
Finite control

State q<sub>0</sub>



### **Finite automaton**

### **Tape**

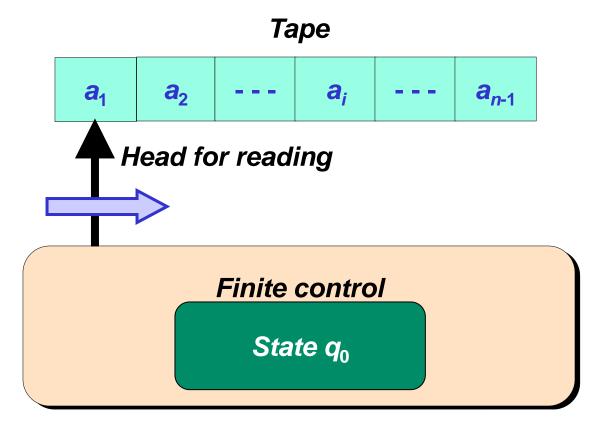


Finite control

State q<sub>0</sub>

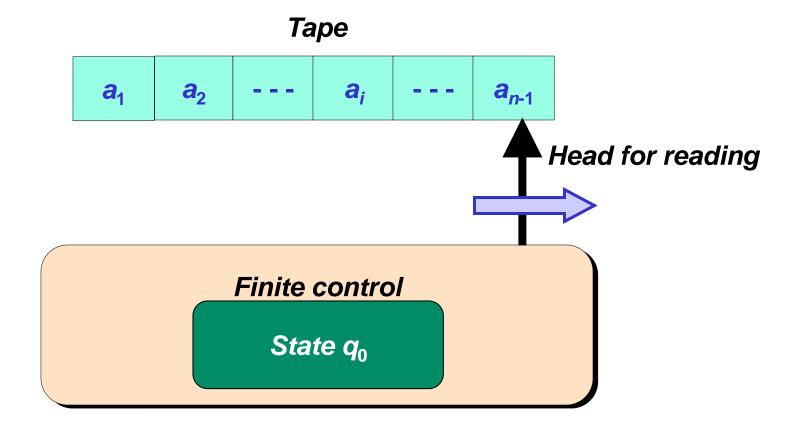


**Finite automaton** 

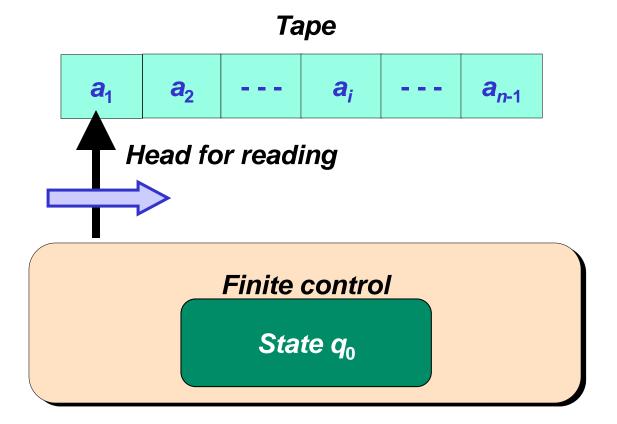




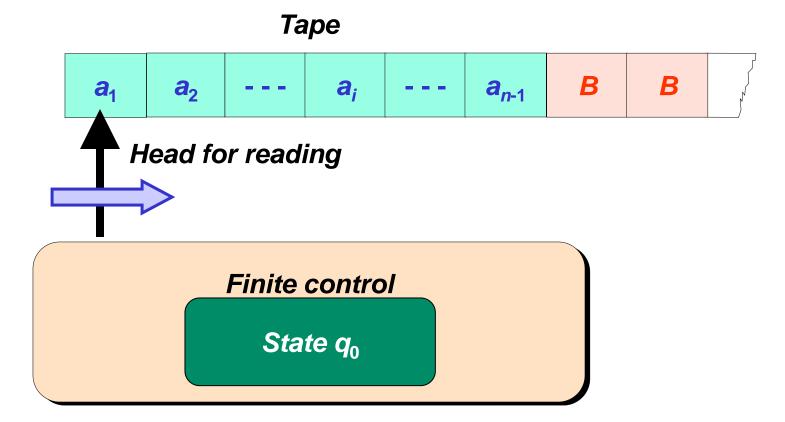
**Finite automaton** 



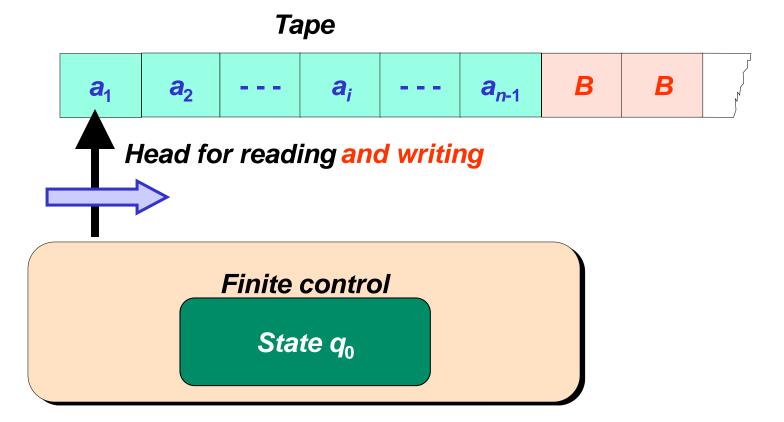




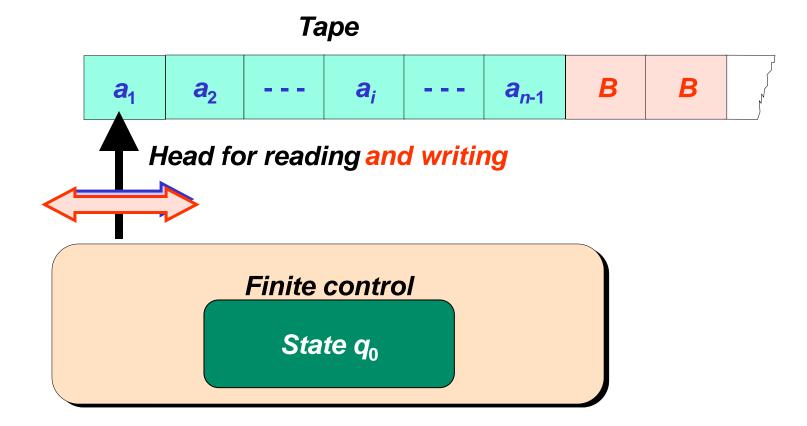




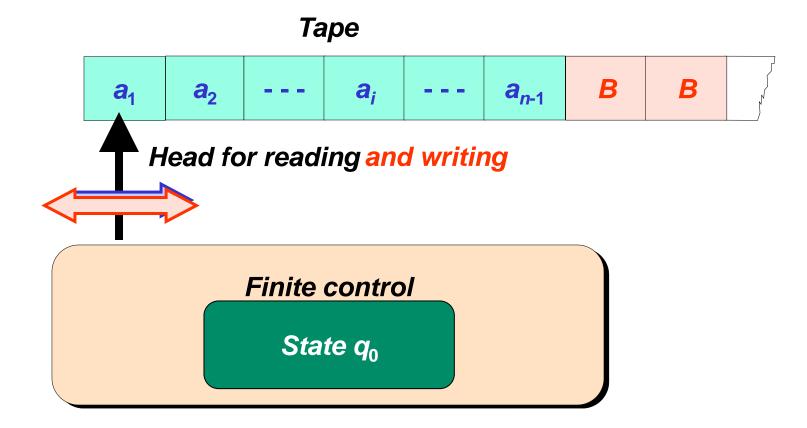
















Decision



- Decision
  - Input



- Decision
  - Input
    - 1. State



- Decision
  - Input
    - 1. State
    - 2. Tape symbol



- Decision
  - Input
    - 1. State
    - 2. Tape symbol
  - Output



- Decision
  - Input
    - 1. State
    - 2. Tape symbol
  - Output
    - 1. New state



- Decision
  - Input
    - 1. State
    - 2. Tape symbol
  - Output
    - 1. New state
    - 2. Symbol written on the tape



#### Decision

- Input
  - 1. State
  - 2. Tape symbol
- Output
  - 1. New state
  - 2. Symbol written on the tape
  - 3. Head movement direction (left or right)





$$tm = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$



$$tm = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

Q

- the finite set of states



$$tm = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

Q

Γ

- the finite set of states
- the complete set of tape symbols



$$tm = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

Q

Γ

**B**∈Γ

- the finite set of states
- the complete set of tape symbols
- the *blank* tape symbol



$$tm = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

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- the finite set of states
- the complete set of tape symbols
- the *blank* tape symbol
- the finite set of input symbols



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- the complete set of tape symbols
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- the finite set of input symbols
- the transition function



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$$\delta: \mathbf{Q} \times \Gamma \rightarrow \mathbf{Q} \times \Gamma \times \{\mathbf{L}, \mathbf{R}\}\$$



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$$q_0 \in Q$$

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- the start state



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 $F \subset Q$ 

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- the set of accepting states



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$$\delta(q, V) = (p, Z, W)$$



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$$Z \in \Gamma$$



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$$W \in \{L, R\}$$



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TM 
$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, X, Y, B\}, \delta, q_0, B, \{q_4\})$$



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2. 
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 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
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6. 
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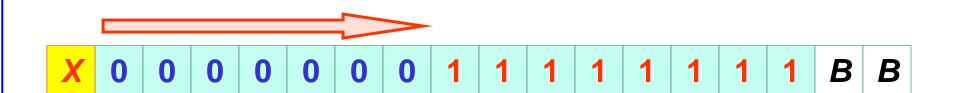
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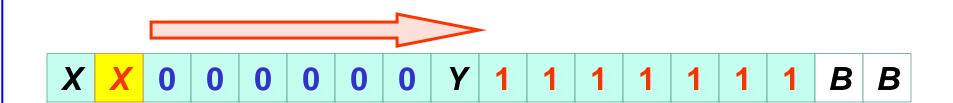
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2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

$$(\alpha, X) = (\alpha, X, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

6.  $\delta(q_0, Y) = (q_3, Y, R)$ 

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



TM 
$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, X, Y, B\}, \delta, q_0, B, \{q_4\})$$

1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$
  
7.  $\delta(q_0, Y) = (q_0, Y, R)$ 

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



TM 
$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, X, Y, B\}, \delta, q_0, B, \{q_4\})$$

1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

5 
$$\delta(q_2, X) = (q_0, X, R)$$
 8  $\delta(q_2, Y) = (q_2, Y, L)$ 

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

8. 
$$\delta(q_2, Y) = (q_2, Y, L)$$

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 







TM 
$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, X, Y, B\}, \delta, q_0, B, \{q_4\})$$

1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



TM 
$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, X, Y, B\}, \delta, q_0, B, \{q_4\})$$

1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

8. 
$$\delta(q_2, Y) = (q_2, Y, L)$$

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



TM 
$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, X, Y, B\}, \delta, q_0, B, \{q_4\})$$

1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

$$6 \delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

8. 
$$\delta(q_2, Y) = (q_2, Y, L)$$

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



TM 
$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, X, Y, B\}, \delta, q_0, B, \{q_4\})$$

1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

$$5 \delta(q_0,X)=(q_0,X,R)$$

$$6 \frac{\delta(q_0, Y) = (q_3, Y, R)}{\delta(q_0, Y)}$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 







TM 
$$M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{0, 1, X, Y, B\}, \delta, q_0, B, \{q_4\})$$

1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

$$5 \delta(q_0, X) = (q_0, X, R)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 







1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$q_0 \mid \mathbf{0}$$

0

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$q_0$$
 0

0

1

1

В

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$q_0 \mid \mathbf{0}$$

0

1

1

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

71



1

1

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$X q_1$$



B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

 $q_1$ 







1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

 $q_1$ 

•



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$X q_2$$

 $\gamma_2$ 



1

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$X q_2$$

**1**<sub>2</sub>



1

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$\mathbf{q}_{2} \mid \mathbf{X}$$

0



1

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

5 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

8. 
$$\delta(q_2, Y) = (q_2, Y, L)$$

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$q_2 X$$



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$X q_0$$





B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$X q_0$$

70





B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



 $q_1$ 



1

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

71



B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

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6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

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5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



 $q_1$ 



B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

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6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

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 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 





 $q_1$ 





1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 







B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 











1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 









1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$  3.  $\delta(q_2,0)=(q_2,0,L)$  5.  $\delta(q_2,X)=(q_0,X,R)$  8.  $\delta(q_2,Y)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

8. 
$$\delta(q_2, Y) = (q_2, Y, L)$$

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$X q_2$$









1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



70



B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

$$6 \delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



**7**0 \







1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 





 $q_3$ 



B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



 $q_3$ 



B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 













1. 
$$\delta(q_0,0)=(q_1,X,R)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 











1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 







$$q_4$$



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$q_0 \mid \mathbf{0}$$

0

B

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

 $q_1$ 



1

B

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

71

В



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$X q_2$$

 $\gamma_2$ 



B

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
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$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$q_2 X$$







1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

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$$\delta(q_2, X) = (q_0, X, R)$$
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9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$X q_0$$

0





B

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

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 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



71



B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

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 4.  $\delta(q_1,1)=(q_2,Y,L)$ 

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 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

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 $q_1 \mid B$ 





1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
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$$\delta(q_2,0)=(q_2,0,L)$$

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 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$q_0 \mid \mathbf{0}$$

1

1

B

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

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$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

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B

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

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9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 

$$q_2 X$$







1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
 4.  $\delta(q_1,1)=(q_2,Y,L)$  7.  $\delta(q_1,Y)=(q_1,Y,R)$ 

3. 
$$\delta(q_2,0)=(q_2,0,L)$$

6. 
$$\delta(q_0, Y) = (q_3, Y, R)$$

7. 
$$\delta(q_1, Y) = (q_1, Y, R)$$

5. 
$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



B

B

B



1. 
$$\delta(q_0,0)=(q_1,X,R)$$

2. 
$$\delta(q_1,0)=(q_1,0,R)$$
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$$\delta(q_2, X) = (q_0, X, R)$$
 8.  $\delta(q_2, Y) = (q_2, Y, L)$ 

9. 
$$\delta(q_3, Y) = (q_3, Y, R)$$
 10.  $\delta(q_3, B) = (q_4, B, R)$ 



73

B

B

B







TM configuration:

 $\alpha_1$ 











$$\alpha_1$$

$$\alpha_2$$

$$\delta(q, X_i) = (p, Y, L):$$



TM configuration:

$$\alpha_1$$

$$\alpha_2$$

$$\delta(q, X_i) = (p, Y, L):$$



TM configuration:

$$\alpha_1$$

$$\alpha_2$$

$$\delta(q, X_i) = (p, Y, L):$$



TM configuration:

$$\alpha_1$$

$$\alpha_2$$

$$\delta(q, X_i) = (p, Y, L):$$

$$X_1 X_2 - - - X_{i-2}$$



TM configuration:

$$\alpha_1$$

$$\alpha_2$$

$$\delta(q, X_i) = (p, Y, L):$$

$$X_1 X_2 - - - X_{i-2} p X_{i-1}$$



TM configuration:

$$\alpha_1$$

$$\alpha_2$$

$$\delta(q, X_i) = (p, Y, L):$$

$$X_1 X_2 - - - X_{i-2} p X_{i-1}$$

$$\delta(q, X_i) = (p, Y, R):$$



TM configuration:

$$\alpha_1$$

$$\alpha_2$$

$$\delta(q, X_i) = (p, Y, L):$$

$$X_1 X_2 - - - X_{i-2} p X_{i-1}$$

$$\delta(q, X_i) = (p, Y, R):$$



TM configuration:

$$\alpha_1$$

$$\alpha_2$$

$$\delta(q, X_i) = (p, Y, L):$$

$$X_1 X_2 - - - X_{i-2} p X_{i-1}$$

$$\delta(q, X_i) = (p, Y, R):$$



TM configuration:

$$\alpha_1$$

$$\alpha_2$$

$$\delta(q, X_i) = (p, Y, L):$$

$$X_1 X_2 - - - X_{i-2} p X_{i-1}$$

$$\delta(q, X_i) = (p, Y, R):$$

$$X_1 X_2 - - - X_{i-2} X_{i-1} q X_i X_{i+1} - - - X_n \rightarrow X_1 X_2 - - - X_{i-2} X_{i-1} q X_i X_{i+1} - - - X_n$$

$$X_{i+1} - - - X_n$$



TM configuration:

$$\alpha_1$$

$$\alpha_2$$

$$\delta(q, X_i) = (p, Y, L):$$

$$X_1 X_2 - - - X_{i-2} p X_{i-1}$$

$$\delta(q, X_i) = (p, Y, R):$$

$$X_1 X_2 - - - X_{i-2} X_{i-1} q X_i X_{i+1} - - - X_n \rightarrow X_1 X_2 - - - X_{i-2} X_{i-1} \mid Y \mid_{\mathcal{D}} X_{i+1} - - - X_n$$

$$X_1 X_2 - - - X_{i-2} X_{i-1}$$







```
q_0 \ 0 \ 0 \ 1 \ 1 \rightarrow X \ q_1 \ 0 \ 1 \ 1 \rightarrow X \ 0 \ q_1 \ 1 \ 1 \rightarrow X \ q_2 \ 0 \ Y \ 1 \rightarrow q_2 \ X \ 0 \ Y \ 1 \rightarrow X \ Q_0 \ 0 \ Y \ 1 \rightarrow X \ X \ Y \ Q_1 \ 1 \rightarrow X \ X \ Q_2 \ Y \ Y \rightarrow X \ X \ Y \ Q_3 \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \rightarrow X \ X \ Y \ Y \ Q_4 \ Y \ Y \ Y \ Q_4 \ Y \ Y \ Q_5 \ Y \ Y \ Y \ Q_6 \ Y \ Y \ Y \ Q_7 \ Q_7 \ Y \ Q_
```

 $q_0 0 0 1 1$ 



```
q_0 \ 0 \ 0 \ 1 \ 1 \rightarrow X \ q_1 \ 0 \ 1 \ 1 \rightarrow X \ 0 \ q_1 \ 1 \ 1 \rightarrow X \ q_2 \ 0 \ Y \ 1 \rightarrow q_2 \ X \ 0 \ Y \ 1 \rightarrow X \ Q_0 \ 0 \ Y \ 1 \rightarrow X \ X \ Y \ Q_1 \ 1 \rightarrow X \ X \ Q_2 \ Y \ Y \rightarrow X \ X \ Y \ Q_3 \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \rightarrow X \ X \ Y \ Y \ Q_4 \ Y \ Y \ Y \ Q_4 \ Y \ Y \ Q_5 \ Y \ Y \ Y \ Q_6 \ Y \ Y \ Y \ Q_7 \ Q_7 \ Y \ Q_
```

$$q_0 0 0 1 1$$
 \*



```
q_0 \ 0 \ 0 \ 1 \ 1 \rightarrow X \ q_1 \ 0 \ 1 \ 1 \rightarrow X \ 0 \ q_1 \ 1 \ 1 \rightarrow X \ q_2 \ 0 \ Y \ 1 \rightarrow q_2 \ X \ 0 \ Y \ 1 \rightarrow X \ Q_0 \ 0 \ Y \ 1 \rightarrow X \ X \ Y \ Q_1 \ 1 \rightarrow X \ X \ Q_2 \ Y \ Y \rightarrow X \ X \ Y \ Q_3 \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \ P \ Q_4
```

$$q_0 0 0 1 1 \qquad \stackrel{*}{\succ} \quad X X Y Y B q_4$$



```
q_0 \ 0 \ 0 \ 1 \ 1 \rightarrow X \ q_1 \ 0 \ 1 \ 1 \rightarrow X \ 0 \ q_1 \ 1 \ 1 \rightarrow X \ q_2 \ 0 \ Y \ 1 \rightarrow q_2 \ X \ 0 \ Y \ 1 \rightarrow X \ Q_0 \ 0 \ Y \ 1 \rightarrow X \ X \ Y \ Q_1 \ 1 \rightarrow X \ X \ Q_2 \ Y \ Y \rightarrow X \ X \ Y \ Q_3 \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \ P \ Q_4
```

```
q_0 \ 0 \ 0 \ 1 \ 1 
x \ x \ x \ y \ y \ B \ q_4
q_0 \ 0 \ 0 \ 1 \ 1
```



```
q_0 \ 0 \ 0 \ 1 \ 1 \rightarrow X \ q_1 \ 0 \ 1 \ 1 \rightarrow X \ 0 \ q_1 \ 1 \ 1 \rightarrow X \ q_2 \ 0 \ Y \ 1 \rightarrow q_2 \ X \ 0 \ Y \ 1 \rightarrow X \ Q_0 \ 0 \ Y \ 1 \rightarrow X \ X \ Y \ Q_1 \ 1 \rightarrow X \ X \ Q_2 \ Y \ Y \rightarrow X \ X \ Y \ Q_3 \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \ P \ Q_4
```

```
q_0 \ 0 \ 0 \ 1 \ 1 \ \stackrel{*}{\succ} \ X X Y Y B q_4
q_0 \ 0 \ 0 \ 1 \ 1 \ \stackrel{13}{\succ}
```



```
q_0 \ 0 \ 0 \ 1 \ 1 \rightarrow X \ q_1 \ 0 \ 1 \ 1 \rightarrow X \ 0 \ q_1 \ 1 \ 1 \rightarrow X \ q_2 \ 0 \ Y \ 1 \rightarrow q_2 \ X \ 0 \ Y \ 1 \rightarrow X \ Q_0 \ 0 \ Y \ 1 \rightarrow X \ X \ Y \ Q_1 \ 1 \rightarrow X \ X \ Q_2 \ Y \ Y \rightarrow X \ X \ Y \ Q_3 \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \ P \ Q_4
```



```
q_0 \ 0 \ 0 \ 1 \ 1 \rightarrow X \ q_1 \ 0 \ 1 \ 1 \rightarrow X \ 0 \ q_1 \ 1 \ 1 \rightarrow X \ q_2 \ 0 \ Y \ 1 \rightarrow q_2 \ X \ 0 \ Y \ 1 \rightarrow X \ Q_0 \ 0 \ Y \ 1 \rightarrow X \ X \ Y \ Q_1 \ 1 \rightarrow X \ X \ Q_2 \ Y \ Y \rightarrow X \ X \ Y \ Q_3 \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \ Y \ Q_4 \ Y \ Y \ Q_4 \ Y \ Y \ Q_5 \ Y \ Y \ Q_6 \ Y \ Y \ Q_7 \ Y
```

```
q_0 \ 0 \ 0 \ 1 \ 1 \ \stackrel{*}{\succ} \ XXYYB \ q_4
q_0 \ 0 \ 0 \ 1 \ 1 \ \stackrel{13}{\succ} \ XXYYB \ q_4
q_0 \ 0 \ 0 \ 1 \ 1
```



```
q_0 \ 0 \ 0 \ 1 \ 1 \rightarrow X \ q_1 \ 0 \ 1 \ 1 \rightarrow X \ 0 \ q_1 \ 1 \ 1 \rightarrow X \ q_2 \ 0 \ Y \ 1 \rightarrow q_2 \ X \ 0 \ Y \ 1 \rightarrow X \ Q_0 \ 0 \ Y \ 1 \rightarrow X \ X \ Y \ Q_1 \ 1 \rightarrow X \ X \ Q_2 \ Y \ Y \rightarrow X \ X \ Y \ Q_3 \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \ Y \ Q_4 \ Y \ Y \ Q_4 \ Y \ Y \ Q_5 \ Y \ Y \ Q_6 \ Y \ Y \ Q_7 \ Y
```

$$q_0 \ 0 \ 0 \ 1 \ 1 \ X \ X \ Y \ Y \ B \ q_4$$

$$q_0 \ 0 \ 0 \ 1 \ 1 \ X \ X \ Y \ Y \ B \ q_4$$

$$q_0 \ 0 \ 0 \ 1 \ 1 \ X \ X \ Y \ Y \ B \ q_4$$



```
q_0 \ 0 \ 0 \ 1 \ 1 \rightarrow X \ q_1 \ 0 \ 1 \ 1 \rightarrow X \ 0 \ q_1 \ 1 \ 1 \rightarrow X \ q_2 \ 0 \ Y \ 1 \rightarrow q_2 \ X \ 0 \ Y \ 1 \rightarrow X \ Q_0 \ 0 \ Y \ 1 \rightarrow X \ X \ Y \ Q_1 \ 1 \rightarrow X \ X \ Q_2 \ Y \ Y \rightarrow X \ X \ Y \ Q_3 \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \rightarrow X \ X \ Y \ Y \ Q_3 \ Y \ Y \ P \ Q_4
```







$$L(M) = \{ w \mid w \in \Sigma^*$$



$$L(M) = \{ w \mid w \in \Sigma^* \}$$
and  $q_0 w$ 



$$L(M) = \{ w \mid w \in \Sigma^* \}$$
and  $q_0 w \stackrel{*}{\succ}$ 



$$L(M) = \{ w \mid w \in \Sigma^*$$
 and  $q_0 w \not\succeq \alpha_1 p \alpha_2$ ,



```
L(M) = \{ w \mid w \in \Sigma^* \}
and q_0 w \not\succeq \alpha_1 p \alpha_2,
p \in F, \quad \alpha_1 \text{ and } \alpha_2 \in \Gamma^* \}
```





Recursively enumerable languages



- Recursively enumerable languages
  - Enumerable
    - —It is possible to construct a TM that outputs (enumerates) all strings from the language



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    - —It is possible to construct a TM that outputs (enumerates) all strings from the language
  - String  $w \in L(M)$ 
    - —TM *M* stops and accepts a string *w*



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  - String w is not in the language L(M)
    - —It is possible that TM M never stops



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  - String w∈ L(M)
    - —TM *M* stops and accepts a string *w*
  - String w is not in the language L(M)
    - —It is possible that TM M never stops
- Recursive languages
  - —There exists a TM that halts on every given input





• Integer  $i \ge 0$ 



- Integer  $i \ge 0$ 
  - coded with a string 0<sup>i</sup>



- Integer  $i \ge 0$ 
  - coded with a string 0<sup>i</sup>
- Integer function over k arguments i1, i2, ..., ik



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  - coded with a string 0<sup>i1</sup> 1 0<sup>i2</sup> 1 --- 1 0<sup>ik</sup>



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  - coded with a string 0<sup>i1</sup> 1 0<sup>i2</sup> 1 --- 1 0<sup>ik</sup>
- TM halts and the tape contains  $0^m$



- Integer  $i \ge 0$ 
  - coded with a string 0<sup>i</sup>
- Integer function over k arguments i1, i2, ..., ik
  - coded with a string 0<sup>i1</sup> 1 0<sup>i2</sup> 1 --- 1 0<sup>ik</sup>
- TM halts and the tape contains  $0^m$ 
  - function result  $f(i_1, i_2, ..., i_k) = m$





- Partially recursive functions
  - —TM might not halt



- Partially recursive functions
  - —TM might not halt
- Total recursive functions
  - —TM always halts for every input





Function m÷n



- Function m÷n
  - $m \ge n$ 
    - —Result of the function  $m \div n$  is subtraction m-n



- Function m÷n
  - $m \ge n$ 
    - —Result of the function  $m \div n$  is subtraction m-n
  - m < n</li>
    - —Result of the function  $m \div n$  is 0





 $m \div n$ 



 $m \div n$ 



 $m \div n$ 

0<sup>m</sup> 1 0<sup>n</sup>



$$m \div n$$

0<sup>m</sup> 1 0<sup>n</sup>

$$\delta(q_0, 0) = (q_1, B, R)$$



$$m \div n$$

$$\delta(q_0, 0) = (q_1, B, R)$$



$$m \div n$$

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$$m \div n$$

BB 0<sup>m-2</sup> 11 0<sup>n-1</sup>

0<sup>m</sup> 1 0<sup>n</sup>

$$\delta(q_0, 0) = (q_1, B, R)$$

B 0<sup>m-1</sup> 1 0<sup>n</sup>

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#### B 0<sup>m-1</sup> 1 1 0<sup>n-1</sup>

$$\delta(q_3, 1) = (q_3, 1, L)$$

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$$\delta(q_3, 0) = (q_3, 0, L)$$

$$\delta(q_3, B) = (q_0, B, R)$$

$$B^{n+1} 0^{m-n-1} 1^{n+1}$$

$$\delta(\mathbf{q_2},\,\mathbf{B})=(\mathbf{q_4},\,\mathbf{B},\,\mathbf{L})$$



$$m \div n$$

$$\delta(q_0, 0) = (q_1, B, R)$$

$$\delta(q_1, 0) = (q_1, 0, R)$$

$$\delta(q_1, 1) = (q_2, 1, R)$$

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$$B^{n+1} 0^{m-n-1} 1^{n+1}$$

$$\delta(\mathbf{q}_2, \mathbf{B}) = (\mathbf{q}_4, \mathbf{B}, \mathbf{L})$$

$$\delta(q_4, 1) = (q_4, B, L)$$



$$m \div n$$

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$$\delta(q_3, B) = (q_0, B, R)$$

$$\delta(\mathbf{q}_2, \mathbf{B}) = (\mathbf{q}_4, \mathbf{B}, \mathbf{L})$$

$$\delta(q_{\scriptscriptstyle A},\,1)=(q_{\scriptscriptstyle A},\,B,\,L)$$

$$\delta(q_4, 0) = (q_4, 0, L)$$



$$m \div n$$

0<sup>m</sup> 1 0<sup>n</sup>

$$\delta(q_0, 0) = (q_1, B, R)$$

B 0<sup>m-1</sup> 1 0<sup>n</sup>

$$\delta(q_1, 0) = (q_1, 0, R)$$

$$\delta(q_1, 1) = (q_2, 1, R)$$

$$\delta(q_2, 0) = (q_3, 1, L)$$

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B 0<sup>m-1</sup> 1 1 0<sup>n-1</sup>

$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, 0) = (q_3, 0, L)$$

$$\delta(q_3, B) = (q_0, B, R)$$

$$B^{n+1}$$
  $0^{m-n-1}$   $1^{n+1}$ 

$$\delta(\mathbf{q}_2, \mathbf{B}) = (\mathbf{q}_4, \mathbf{B}, \mathbf{L})$$

$$\delta(q_4, 1) = (q_4, B, L)$$

$$\delta(q_{\scriptscriptstyle A},\,0)=(q_{\scriptscriptstyle A},\,0,\,L)$$

$$\delta(q_4, B) = (q_6, 0, R)$$



$$m \div n$$

0<sup>m</sup> 1 0<sup>n</sup>

$$\delta(q_0, 0) = (q_1, B, R)$$

B 0<sup>m-1</sup> 1 0<sup>n</sup>

$$\delta(q_1, 0) = (q_1, 0, R)$$

$$\delta(q_1, 1) = (q_2, 1, R)$$

$$\delta(q_2, 0) = (q_3, 1, L)$$

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B 0<sup>m-1</sup> 1 1 0<sup>n-1</sup>

$$\delta(q_3, 1) = (q_3, 1, L)$$

$$\delta(q_3, 0) = (q_3, 0, L)$$

$$\delta(q_3, B) = (q_0, B, R)$$

$$\delta(q_2, B) = (q_4, B, L)$$

$$\delta(q_4, 1) = (q_4, B, L)$$

$$\delta(q_4, 0) = (q_4, 0, L)$$

$$\delta(q_4, B) = (q_6, 0, R)$$



$$m \div n$$

$$\delta(q_0, 0) = (q_1, B, R)$$

$$\delta(q_1, 0) = (q_1, 0, R)$$

$$\delta(q_1, 1) = (q_2, 1, R)$$

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$$m \le n$$



$$m \div n$$

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$$m \le n$$



$$m \div n$$

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$$m \le n$$



$$m \div n$$

$$\delta(q_0, 0) = (q_1, B, R)$$

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$$\delta(q_3, 0) = (q_3, 0, L)$$

$$\delta(q_3, B) = (q_0, B, R)$$

$$m \le n$$

$$\delta(q_0, 1) = (q_5, B, R)$$



$$m \div n$$

$$\delta(q_0, 0) = (q_1, B, R)$$

$$\delta(q_1, 0) = (q_1, 0, R)$$

$$\delta(q_1, 1) = (q_2, 1, R)$$

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$$\delta(q_2, 0) = (q_3, 1, L)$$

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$$\delta(q_3, 0) = (q_3, 0, L)$$

$$\delta(q_3, B) = (q_0, B, R)$$

$$m \le n$$

$$\delta(q_0, 1) = (q_5, B, R)$$

$$\delta(q_5, 1) = (q_5, B, R)$$



$$m \div n$$

$$\delta(q_0, 0) = (q_1, B, R)$$

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$$\delta(q_1, 1) = (q_2, 1, R)$$

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$$m \le n$$

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$$m \div n$$

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$$\delta(q_0, 1) = (q_5, B, R)$$

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$$\delta(q_5, 0) = (q_5, B, R)$$

$$\delta(q_5, B) = (q_6, B, R)$$



$$m \div n$$

0<sup>m</sup> 1 0<sup>n</sup>

$$\delta(q_0, 0) = (q_1, B, R)$$

B 0<sup>m-1</sup> 1 0<sup>n</sup>

$$\delta(q_1, 0) = (q_1, 0, R)$$

$$\delta(q_1, 1) = (q_2, 1, R)$$

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B 0<sup>m-1</sup> 1 1 0<sup>n-1</sup>

$$\delta(q_3, 1) = (q_3, 1, L)$$

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$$m \le n$$

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```
q_00010 \succ Bq_1010 \succ B0q_110 \succ B01q_20 \succ B0q_311 \succ Bq_3011 \succ Paq_3011 \sim Paq_3011
```



```
q_00010 \rightarrow Bq_1010 \rightarrow B0q_110 \rightarrow B01q_20 \rightarrow B0q_311 \rightarrow Bq_3011 \rightarrow Bq_3011 \rightarrow Bq_0011 \rightarrow Bq_111 \rightarrow Bq_111 \rightarrow Bq_21 \rightarrow Bq_11q_2 \rightarrow Bq_11 \rightarrow Bq_21 \rightarrow Bq_3 \rightarrow Bq_4 \rightarrow Bq_5
```

$$2 \div 1 = 1$$



$$q_00010$$
  $\succ$   $Bq_1010$   $\succ$   $B0q_110$   $\succ$   $B01q_20$   $\succ$   $B0q_311$   $\succ$   $Bq_3011$   $\succ$   $Paq_3011$   $\sim$   $Paq_3011$ 

$$2 \div 1 = 1$$



$$2 \div 1$$
 $q_00010 \qquad \succ \qquad Bq_1010 \qquad \succ \qquad B0q_110 \qquad \succ \qquad B01q_20 \qquad \succ \\ B0q_311 \qquad \succ \qquad Bq_3011 \qquad \succ \qquad q_3B011 \qquad \succ \qquad Bq_0011 \qquad \succ$ 

$$BBq_111 \rightarrow BB1q_21 \rightarrow BB11q_2 \rightarrow BB1q_41 \rightarrow$$

$$BBq_41 \rightarrow Bq_4 \rightarrow B0q_6$$

$$2 \div 1 = 1$$

$$q_00100 \rightarrow Bq_1100 \rightarrow B1q_200 \rightarrow Bq_3110 \rightarrow q_3B110 \rightarrow Bq_0110 \rightarrow BBBq_50 \rightarrow BBBBq_6$$



#### **Computing functions with Turing machines**



 $1 \div 2 = 0$ 

#### **Lecture outline**

4 RECURSIVELY ENUMERABLE LANGUAGES	126
4.1 TURING MACHINE (TM)	126
4.1.1 Simple Turing machine	126
4.1.2 Programming techniques for Turing machines	133







States with multiple components



- States with multiple components
  - $[q_1, q_2, ..., q_n]$



- States with multiple components
  - $[q_1, q_2, ..., q_n]$
  - Control components
    - —Define state/position of TM



- States with multiple components
  - $[q_1, q_2, ..., q_n]$
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  - Storage components
    - -Hold a finite amount of data



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- Tape symbols with multiple components
  - $[a_1, a_2, ..., a_n]$

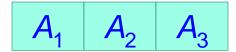


- States with multiple components
  - $[q_1, q_2, ..., q_n]$
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  - $[q_1, q_2, ..., q_n]$
  - Control components
    - —Define state/position of TM
  - Storage components
    - —Hold a finite amount of data

- Tape symbols with multiple components
  - $[a_1, a_2, ..., a_n]$
  - Multiple tracks on the tape

$A_1$	$A_2$	$A_3$	
$a_1$	$a_1$	$a_1$	
$a_2$	$a_2$	$a_2$	
$a_3$	$a_3$	$a_3$	
			N
a <sub>n</sub>	$a_n$	a <sub>n</sub>	





Language L



- Language L
  - Set of strings where leftmost symbol does not appear elsewhere in the string



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  - Set of strings where leftmost symbol does not appear elsewhere in the string
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  - Two components [q, a]



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  - Two components [q, a]
  - Storage component a
    - —Holds leftmost symbol on the tape



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  - Control component q



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  - Control component q
    - —Two values:  $q_0$  and  $q_1$



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    - —State  $q_0$ : TM M reads leftmost symbol and stores it in the storage component



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- Composite states of the TM M
  - Two components [q, a]
  - Storage component a
    - —Holds leftmost symbol on the tape
  - Control component q
    - —Two values:  $q_0$  and  $q_1$
    - —State  $q_0$ : TM M reads leftmost symbol and stores it in the storage component
    - —State  $q_1$ : TM M reads rest of the string and compares symbols with the symbol stored in the storage component





• Control component:  $q_0$  and  $q_1$ 



- Control component: q<sub>0</sub> and q<sub>1</sub>
- Storage component: 0, 1 and B



- Control component: q<sub>0</sub> and q<sub>1</sub>
- Storage component: 0, 1 and B
  - Set of composite states



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- Storage component: 0, 1 and B
  - Set of composite states

```
Q = \{[q_0, B], [q_0, 0], [q_0, 1], [q_1, B], [q_1, 0], [q_1, 1]\}
```



- Control component: q<sub>0</sub> and q<sub>1</sub>
- Storage component: 0, 1 and B
  - Set of composite states  $Q = \{[q_0, B], [q_0, 0], [q_0, 1], [q_1, B], [q_1, 0], [q_1, 1]\}$
- $TMM = (Q, \{0,1\}, \{0,1,B\}, \delta, [q_0,B], B, \{[q_1,B]\})$



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$$\delta([q_0, B], 0) =$$



- Control component: q<sub>0</sub> and q<sub>1</sub>
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  - Set of composite states  $Q = \{[q_0, B], [q_0, 0], [q_0, 1], [q_1, B], [q_1, 0], [q_1, 1]\}$

• 
$$TMM = (Q, \{0,1\}, \{0,1,B\}, \delta, [q_0,B], B, \{[q_1,B]\})$$

$$\delta([q_0, B], 0) = ([q_1, ], 0, R)$$



- Control component: q<sub>0</sub> and q<sub>1</sub>
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  - Set of composite states  $Q = \{[q_0, B], [q_0, 0], [q_0, 1], [q_1, B], [q_1, 0], [q_1, 1]\}$

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$$\delta([q_0, B], 0) = ([q_1, 0], 0, R)$$
  
 $\delta([q_0, B], 1) = ([q_1, 0], 1, R)$ 



- Control component: q<sub>0</sub> and q<sub>1</sub>
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  - Set of composite states  $Q = \{[q_0, B], [q_0, 0], [q_0, 1], [q_1, B], [q_1, 0], [q_1, 1]\}$
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 $\delta([q_0, B], 1) = ([q_1, 1], 1, R)$ 

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 $\delta([q_0, B], 1) = ([q_1, 1], 1, R)$ 

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- $TMM = (Q, \{0,1\}, \{0,1,B\}, \delta, [q_0,B], B, \{[q_1,B]\})$

$$\delta([q_0, B], 0) = ([q_1, 0], 0, R)$$
  
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$$\delta([q_0, B], 0) = ([q_1, 0], 0, R)$$
  
 $\delta([q_0, B], 1) = ([q_1, 1], 1, R)$   $\delta([q_1, 0], B) = ([q_1, B], B, L)$   
 $\delta([q_1, 1], B) = ([q_1, B], B, L)$ 

$$\delta([q_1, 0], 1) = ([q_1, 0], 1, R)$$
  
 $\delta([q_1, 1], 0) = ([q_1, 1], 0, R)$ 





• TM M



- TM M
  - Shift symbols over n cells using n storage components



- TM M
  - Shift symbols over n cells using n storage components
- 2-cell shift



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  - Composite states: [q, R<sub>1</sub>, R<sub>2</sub>]



- TM M
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- TM M
  - Shift symbols over n cells using n storage components
- 2-cell shift
  - Composite states: [q, R<sub>1</sub>, R<sub>2</sub>]
    - —Control component q is  $q_1$  or  $q_2$
    - —Storage components  $R_1$  and  $R_2$  hold tape symbols  $\Gamma$





$$\delta([q_1, B, B], A_1) = ([q_1, B, B], B, R)$$



$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$



$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

$$\delta([q_1, B, A_1], A_2) = ([q_1, B, A_1], B, R)$$



$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

$$\delta([q_1, B, A_1], A_2) = ([q_1, A_1, A_1], B, R)$$



$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

$$\delta([q_1, B, A_1], A_2) = ([q_1, A_1, A_2], B, R)$$



$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

$$\delta([q_1, B, A_1], A_2) = ([q_1, A_1, A_2], B, R)$$

$$\delta([q_1, A_1, A_2], A_3) = ([q_1, A_1, A_2], A_3, R)$$



$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

$$\delta([q_1, B, A_1], A_2) = ([q_1, A_1, A_2], B, R)$$

$$\delta([q_1, A_1, A_2], A_3) = ([q_1, A_1, A_2], A_1, R)$$



$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

$$\delta([q_1, B, A_1], A_2) = ([q_1, A_1, A_2], B, R)$$

$$\delta([q_1, A_1, A_2], A_3) = ([q_1, A_2, A_2], A_1, R)$$



$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

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$$\delta([q_1, A_1, A_2], A_3) = ([q_1, A_2, A_3], A_1, R)$$

$$\delta([q_1, A_1, A_2], B) = ([q_1, A_1, A_2], B, R)$$



$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

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$$\delta([q_1, A_1, A_2], B) = ([q_1, A_2, B], A_1, R)$$



$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

$$\delta([q_1, B, A_1], A_2) = ([q_1, A_1, A_2], B, R)$$

$$\delta([q_1, A_1, A_2], A_3) = ([q_1, A_2, A_3], A_1, R)$$

$$\delta([q_1, A_1, A_2], B) = ([q_1, A_2, B], A_1, R)$$

$$\delta([q_1, A_1, B], B) = ([q_2, B, B], B, L)$$



$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

$$\delta([q_1, B, A_1], A_2) = ([q_1, A_1, A_2], B, R)$$

$$\delta([q_1, A_1, A_2], A_3) = ([q_1, A_2, A_3], A_1, R)$$

$$\delta([q_1, A_1, A_2], B) = ([q_1, A_2, B], A_1, R)$$

$$\delta([q_1, A_1, B], B) = ([q_2, B, B], A_1, L)$$



$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

$$\delta([q_1, B, A_1], A_2) = ([q_1, A_1, A_2], B, R)$$

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$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

$$\delta([q_1, B, A_1], A_2) = ([q_1, A_1, A_2], B, R)$$

$$\delta([q_1, A_1, A_2], A_3) = ([q_1, A_2, A_3], A_1, R)$$

$$\delta([q_1, A_1, A_2], B) = ([q_1, A_2, B], A_1, R)$$

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$$\delta([q_1, B, B], A_1) = ([q_1, B, A_1], B, R)$$

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$$\delta([q_1, A_1, B], B) = ([q_2, B, B], A_1, L)$$

$$\delta([q_2, B, B], A) = ([q_2, B, B], A, L)$$





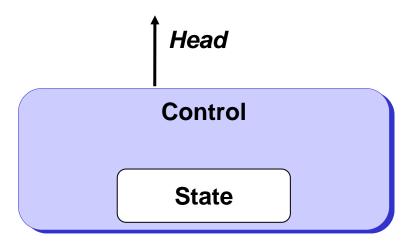
Language L



- Language L
  - Set of all prime numbers



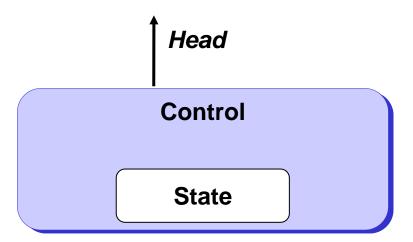
- Language L
  - Set of all prime numbers





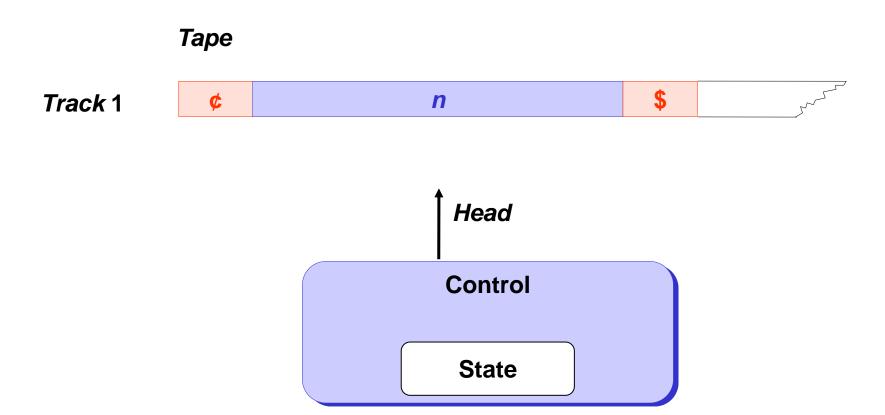
- Language L
  - Set of all prime numbers

**Tape** 





- Language L
  - Set of all prime numbers



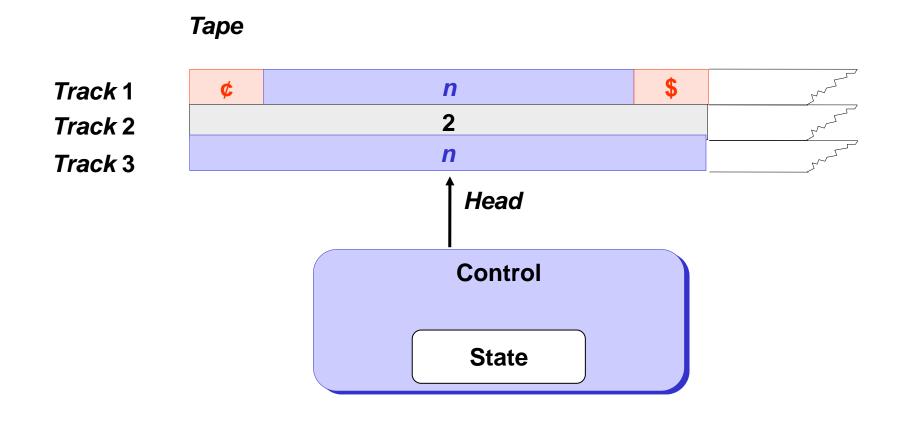


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# Track 1 Track 2 The state Track 1 Control State

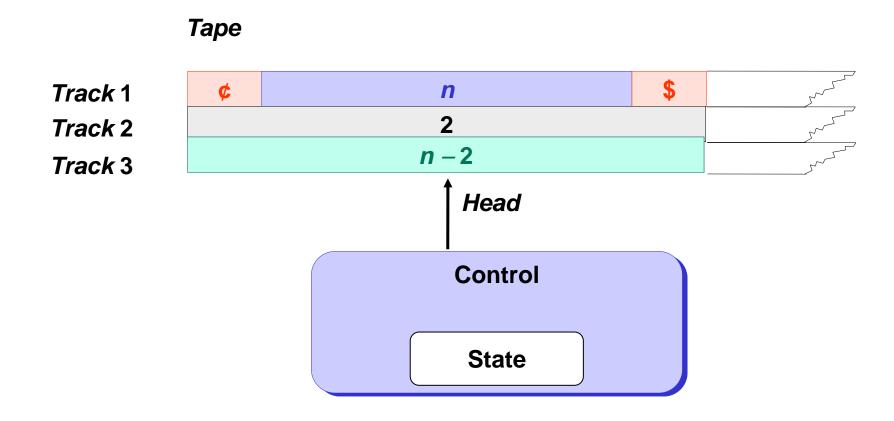


- Language L
  - Set of all prime numbers



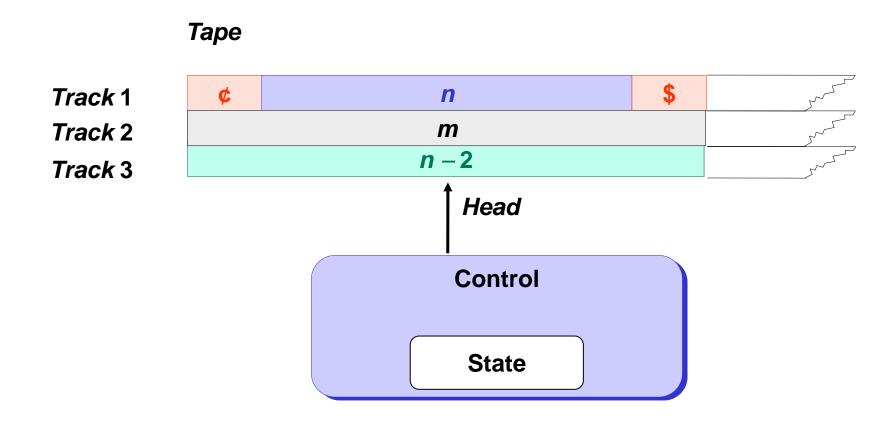


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  - Set of all prime numbers



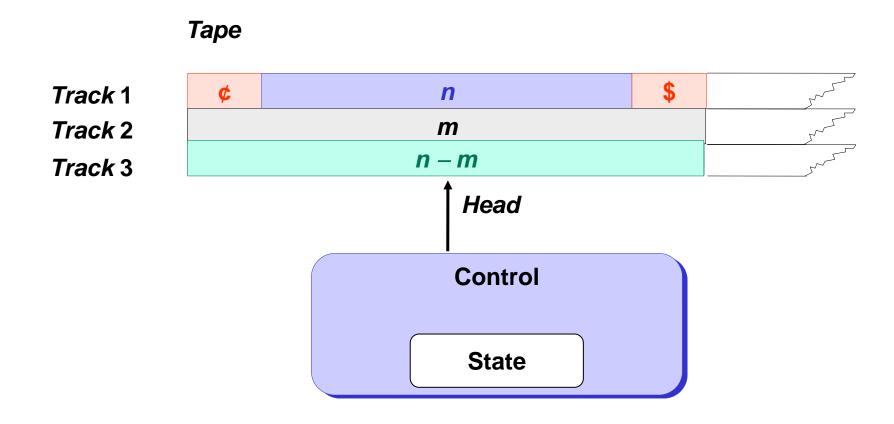


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  - Set of all prime numbers



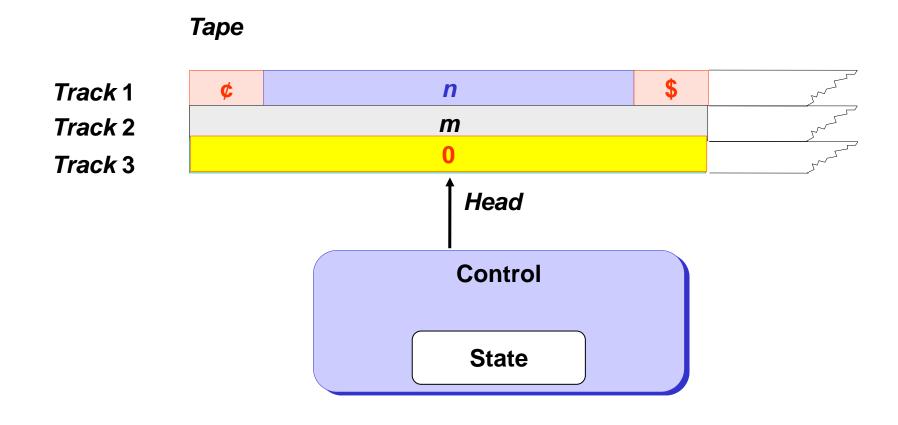


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  - Set of all prime numbers



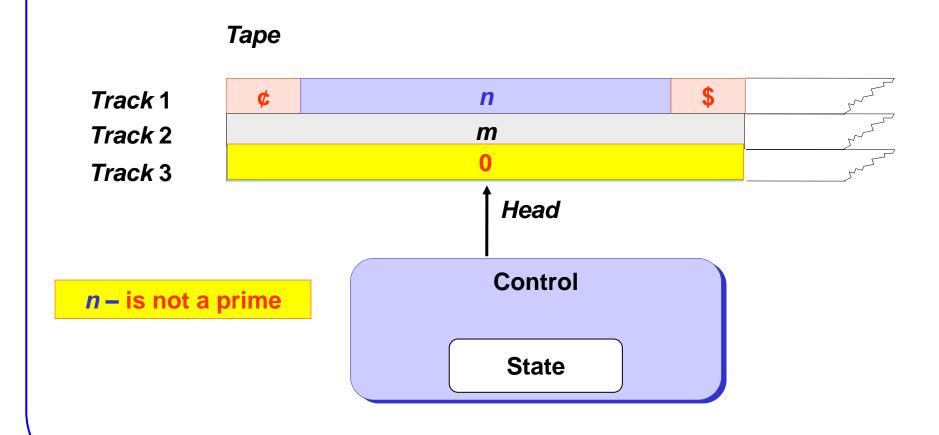


- Language L
  - Set of all prime numbers



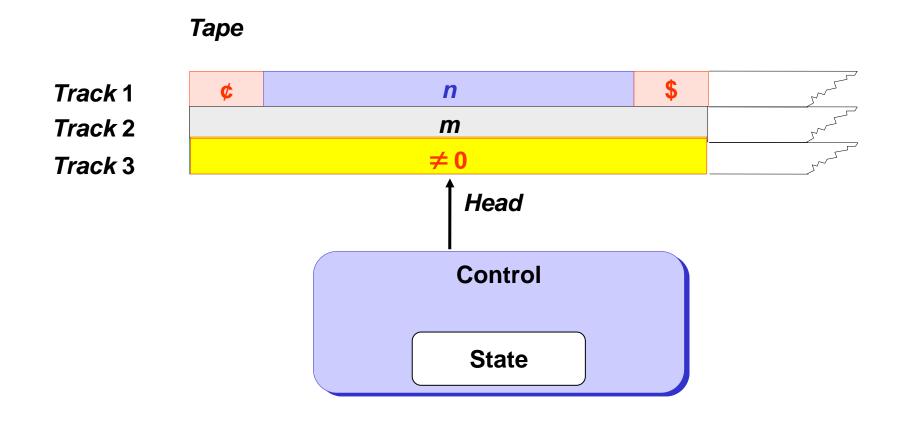


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  - Set of all prime numbers



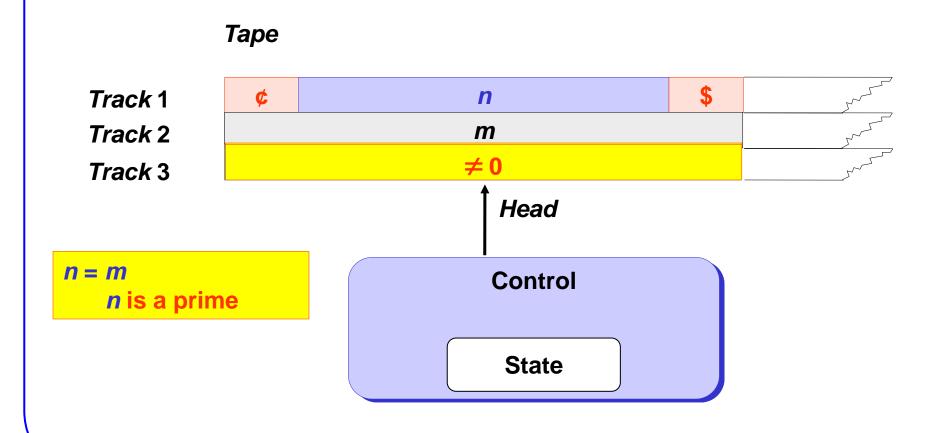


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  - Set of all prime numbers



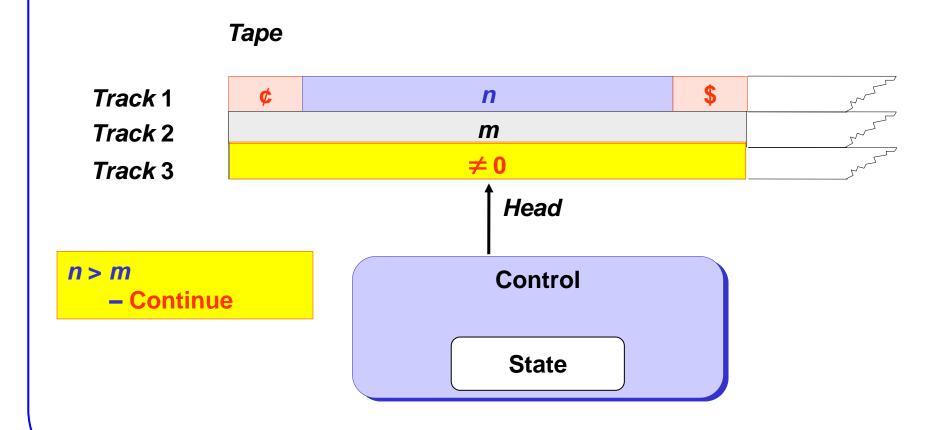


- Language L
  - Set of all prime numbers



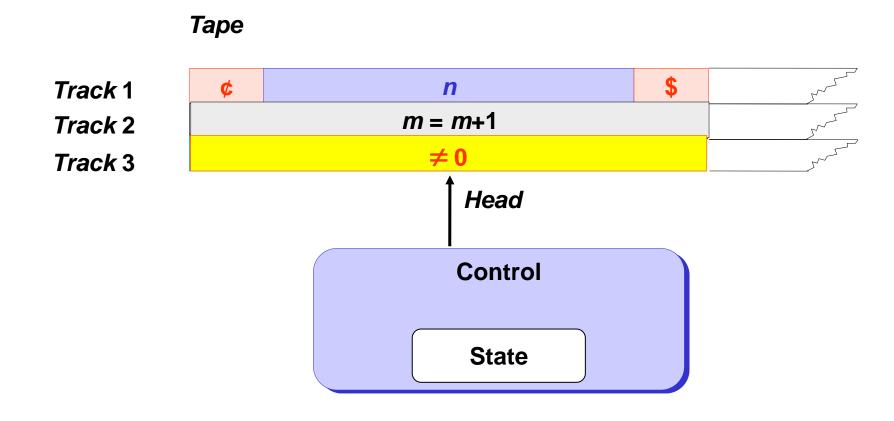


- Language L
  - Set of all prime numbers



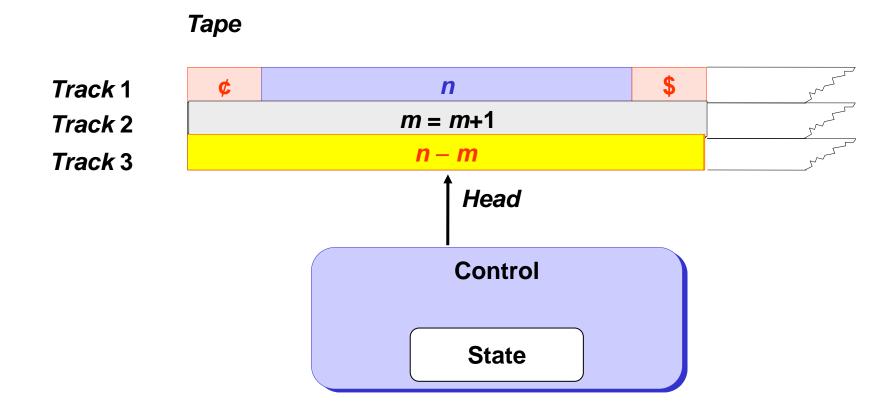


- Language L
  - Set of all prime numbers



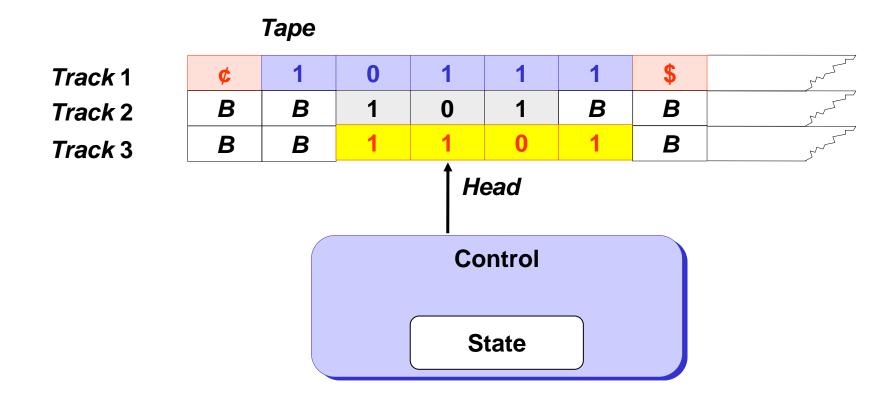


- Language L
  - Set of all prime numbers





- Language L
  - Set of all prime numbers







Repeating substrings



## Repeating substrings

 $\{wcw \mid w \in \Sigma^*\}, \{wcy \mid w, y \in \Sigma^*, w \neq y\}, \{ww^R \mid w \in \Sigma^*\}, w \in (a+b)^+$ 



## Repeating substrings

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#### Repeating substrings

 $\{wcw \mid w \in \Sigma^*\}, \{wcy \mid w, y \in \Sigma^*, w \neq y\}, \{ww^R \mid w \in \Sigma^*\}, w \in (a+b)^+$ 

## **Comparing substrings**

 $\{a^ib^i \mid i\geq 1\}, \{a^ib^jc^k \mid i\neq j \text{ ill } j\neq k\}$ 



## Repeating substrings

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 $\{a^ib^i\mid i\geq 1\}, \{a^ib^jc^k\mid i\neq j \text{ ill } j\neq k\}$ 



## Repeating substrings

$$\{a^ib^i \mid i\geq 1\}, \{a^ib^jc^k \mid i\neq j \text{ ill } j\neq k\}$$

$$Q = [q, d]$$



## Repeating substrings

$$\{a^ib^i \mid i\geq 1\}, \{a^ib^jc^k \mid i\neq j \text{ ill } j\neq k\}$$

$$Q = [q, d]$$
  
  $q \text{ is } q_1, q_2, ... q_9$ 



## Repeating substrings

```
\{a^ib^i\mid i\geq 1\}, \{a^ib^jc^k\mid i\neq j \text{ ill } j\neq k\}
```

```
Q = [q, d]

q \text{ is } q_1, q_2, \dots q_9

d \text{ is symbol } a, b \text{ or blank } B
```



#### Repeating substrings

```
\{a^ib^i \mid i\geq 1\}, \{a^ib^jc^k \mid i\neq j \text{ ill } j\neq k\}
```

```
Q = [q, d]

q \text{ is } q_1, q_2, \dots q_9

d \text{ is symbol } a, b \text{ or blank } B
```

$$\Gamma = [X, d]$$



## Repeating substrings

```
\{a^ib^i \mid i\geq 1\}, \{a^ib^jc^k \mid i\neq j \text{ ill } j\neq k\}
```

```
Q = [q, d]

q \text{ is } q_1, q_2, \dots q_9

d \text{ is symbol } a, b \text{ or blank } B
```

```
\Gamma = [X, d]
X is B or \sqrt{\phantom{a}}
```



## Repeating substrings

```
\{a^ib^i \mid i\geq 1\}, \{a^ib^jc^k \mid i\neq j \text{ ill } j\neq k\}
```

```
Q = [q, d]

q \text{ is } q_1, q_2, \dots q_9

d \text{ is symbol } a, b \text{ or blank } B
```

```
\Gamma = [X, d]
X is B or \sqrt{d}
d is a, b, c or blank B
```









В	В	B	В	В	B	B	В	B	В	В	B	B	В	В	B	B	В	B	
a	a	b	a	a	a	b	b	С	а	а	b	а	а	а	b	b	В	B	



$$\{wcw \mid w \in \Sigma^*\}$$

В	В	В	В	В	В	В	В	В	В	В	B	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, B], [\sqrt{d}, R))$$



$$\{wcw \mid w \in \Sigma^*\}$$

В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	С	а	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, B], [\sqrt{d}, R))$$



$$\{wcw \mid w \in \Sigma^*\}$$

В	В	В	В	В	В	В	В	В	В	В	B	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, B], [\sqrt{d}, R])$$



$$\{wcw \mid w \in \Sigma^*\}$$

В	В	В	В	В	В	В	В	В	В	В	B	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$



$$\{wcw \mid w \in \Sigma^*\}$$

В	В	В	В	В	В	В	В	В	В	В	B	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], \sqrt{d}, R)$$



$$\{wcw \mid w \in \Sigma^*\}$$

1	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$



$$\{wcw \mid w \in \Sigma^*\}$$

1	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$



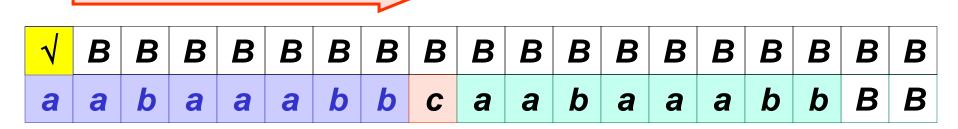
7	B	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	B	В

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$



$$\{wcw \mid w \in \Sigma^*\}$$



$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$



<b>√</b>	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	C	a	a	b	a	a	а	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$



1	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	B	В

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2,\,d],\,[B,\,c])=([q_3,\,d],\,[B,\,c],\,R)$$



<b>√</b>	B	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$



1	В	В	В	В	В	В	В	В	В	В	B	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$



<b>√</b>	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	C	a	a	b	a	а	a	b	b	В	В

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$



$$\{wcw \mid w \in \Sigma^*\}$$



1	В	B	B	B	B	B	B	B	B	B	B	B	B	В	B	B	B	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$



1	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	B	В

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [B, d], L)$$



1	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	B	В

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [B, d], L)$$



<b>√</b>	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$



<b>√</b>	В	В	В	В	В	В	В	В	1	В	В	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	C	а	a	b	a	а	а	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$



<b>√</b>	B	В	В	В	В	В	В	В	1	В	B	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	С	a	а	b	а	a	а	b	b	B	В

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$



<b>√</b>	В	В	В	В	В	В	В	В	1	В	В	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	C	а	a	b	a	а	а	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$





	В	B	В	B	B	B	B	B	<b>V</b>	В	B	В	B	В	B	B	B	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$



1	В	В	В	В	В	В	В	В	1	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	а	a	b	а	a	а	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$



7		В	В	В	В	В	В	В	В	1	В	В	В	В	В	В	В	В	В
a	2	a	b	a	a	a	b	b	С	a	a	b	a	а	a	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$



1	В	В	В	В	В	В	В	В	1	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	В	В

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$



7		В	В	В	В	В	В	В	В	1	В	В	В	В	В	В	В	В	В
a	2	a	b	a	a	a	b	b	С	a	a	b	a	а	a	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$



1	В	В	В	В	В	В	В	В	1	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	a	a	b	а	a	а	b	b	B	В

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$



1	В	В	В	В	В	В	В	В	1	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$



1	В	В	В	В	В	В	В	В	1	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	a	a	b	а	a	a	b	b	В	В

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$



1	В	В	В	В	В	В	В	В	1	В	B	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$





1	В	B	B	B	B	B	В	В	1	В	B	B	В	В	В	В	B	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$



1	В	В	В	В	В	В	В	В	1	В	B	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$



1	В	В	В	В	В	В	В	В	1	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	а	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$



1	В	В	В	В	В	В	В	В	1	В	В	В	В	В	В	В	В	В
a	a	b	a	a	a	b	b	С	а	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt{, d}]) = ([q_1, B], [\sqrt{, d}], R)$$



<b>√</b>																		
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$



1	В	В	В	В	В	В	В	В	1	В	B	В	В	В	В	В	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$



1	1	1	1	1	1	1	1	В	1	1	1	1	1	1	1	1	В	B
a	a	b	a	a	a	b	b	С	a	а	b	а	a	а	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{d}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$





1	1	1	1	1	1	1	1	В	1	1	1	1	1	1	1	1	В	B
a	a	b	а	a	а	b	b	С	a	a	b	a	a	а	b	b	B	В

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{d}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$



1	1	1	1	1	1	1	1	В	1	1	1	1	1	1	1	1	В	B
a	a	b	a	a	a	b	b	C	а	a	b	a	a	a	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{d}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$



1	1	1	1	1	1	1	1	В	1	1	1	1	1	1	1	1	В	B
a	a	b	a	a	a	b	b	С	а	a	b	a	a	а	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{d}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$



1	1	1	1	1	1	1	1	В	1	1	1	1	1	1	1	1	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	а	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{d}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$

$$\delta([q_5, B], [\sqrt, d]) = ([q_7, B], [\sqrt, d], R)$$



1	1	1	1	1	1	1	1	В	1	1	1	1	1	1	1	1	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	а	а	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{d}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$

$$\delta([q_5, B], [\sqrt{, d}]) = ([q_7, B], [\sqrt{, d}], R)$$



1	1	1	1	1	1	1	1	В	1	1	1	1	1	1	1	1	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	а	а	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{d}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$

$$\delta([q_5, B], [\sqrt, d]) = ([q_7, B], [\sqrt, d], R)$$



1	1	1	1	1	1	1	1	В	1	1	1	1	1	1	1	1	В	B
a	a	b	a	a	a	b	b	С	a	а	b	а	a	а	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{d}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$

$$\delta([q_5, B], [\sqrt, d]) = ([q_7, B], [\sqrt, d], R)$$



1	1	1	1	1	1	1	1	В	1	1	1	1	1	1	1	1	В	B
a	a	b	a	a	a	b	b	С	a	а	b	а	a	а	b	b	В	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{d}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

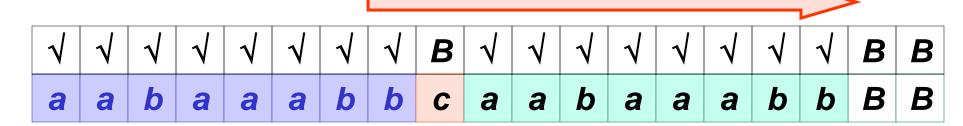
$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$

$$\delta([q_5, B], [\sqrt, d]) = ([q_7, B], [\sqrt, d], R)$$

$$\delta([q_7, B], [B, c]) = ([q_8, B], [B, c], R)$$

$$\delta([q_8, B], [\sqrt, d]) = ([q_8, B], [\sqrt, d], R)$$





$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{d}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$

$$\delta([q_5, B], [\sqrt, d]) = ([q_7, B], [\sqrt, d], R)$$

$$\delta([q_7, B], [B, c]) = ([q_8, B], [B, c], R)$$

$$\delta([q_8, B], [\sqrt, d]) = ([q_8, B], [\sqrt, d], R)$$



1	1	1	1	1	1	1	1	В	1	1	1	1	1	1	1	1	В	B
a	a	b	a	a	a	b	b	С	a	a	b	a	a	а	b	b	B	B

$$\delta([q_1, B], [B, d]) = ([q_2, d], [\sqrt{d}, R])$$

$$\delta([q_2, d], [B, e]) = ([q_2, d], [B, e], R)$$

$$\delta([q_2, d], [B, c]) = ([q_3, d], [B, c], R)$$

$$\delta([q_3, d], [\sqrt{e}]) = ([q_3, d], [\sqrt{e}], R)$$

$$\delta([q_3, d], [B, d]) = ([q_4, B], [\sqrt{d}, d], L)$$

$$\delta([q_4, B], [\sqrt, d]) = ([q_4, B], [\sqrt, d], L)$$

$$\delta([q_4, B], [B, c]) = ([q_5, B], [B, c], L)$$

$$\delta([q_5, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [B, d]) = ([q_6, B], [B, d], L)$$

$$\delta([q_6, B], [\sqrt, d]) = ([q_1, B], [\sqrt, d], R)$$

$$\delta([q_5, B], [\sqrt, d]) = ([q_7, B], [\sqrt, d], R)$$

$$\delta([q_7, B], [B, c]) = ([q_8, B], [B, c], R)$$

$$\delta([q_8, B], [\sqrt, d]) = ([q_8, B], [\sqrt, d], R)$$

$$\delta([q_8, B], [B, B]) = ([q_9, B], [B, B], L)$$

