# Lecture 12.

4.1.3 Extensions to the basic Turing machine 138


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# Extensions to the basic Turing machine

- TM with a two-sided infinite tape
- Multitape TM
- Nondeterministic TM
- TM with a multidimensional input


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# TM with a two-sided infinite tape


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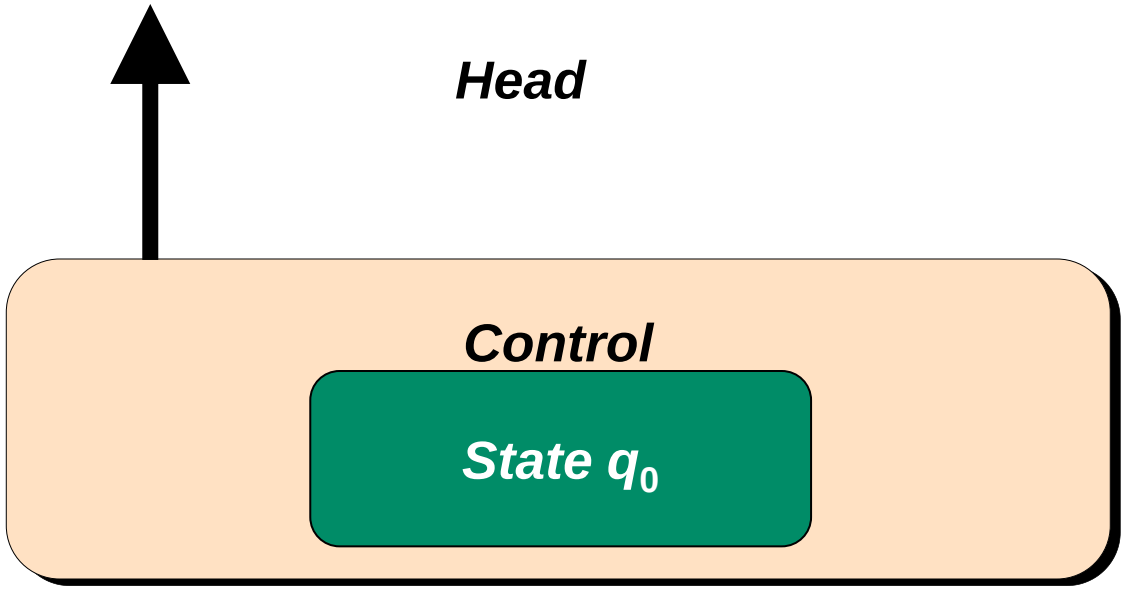
# TM with a two-sided infinite tape





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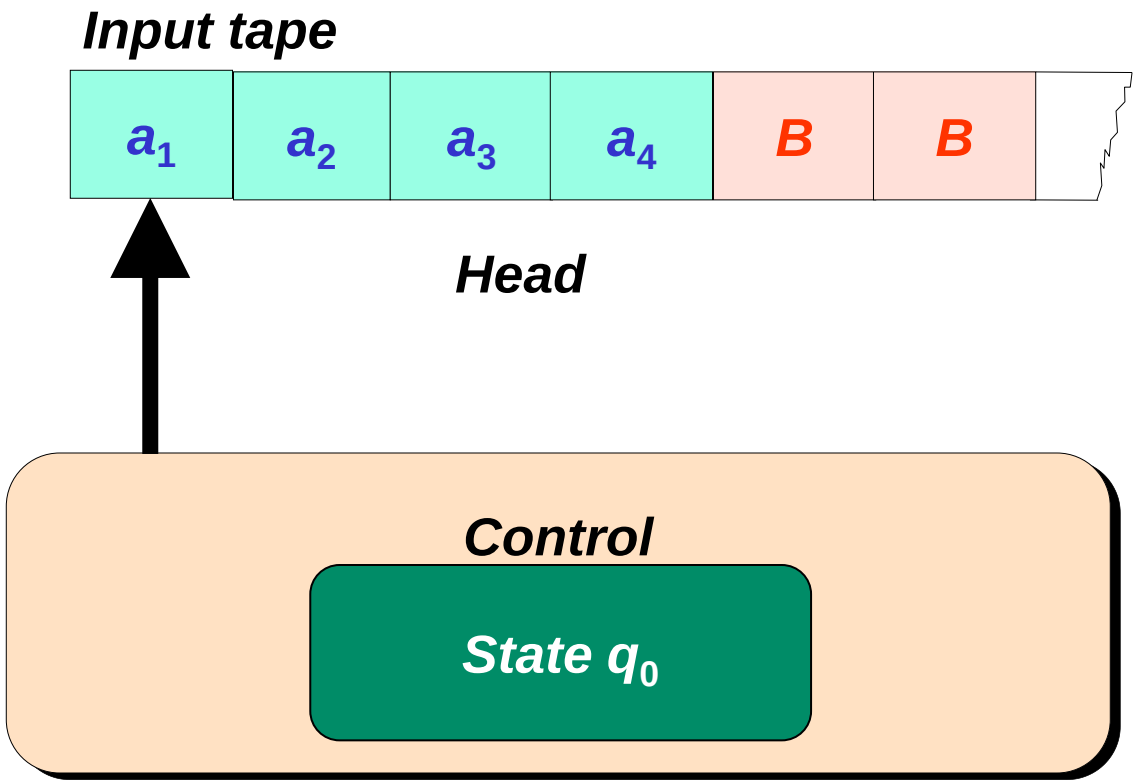



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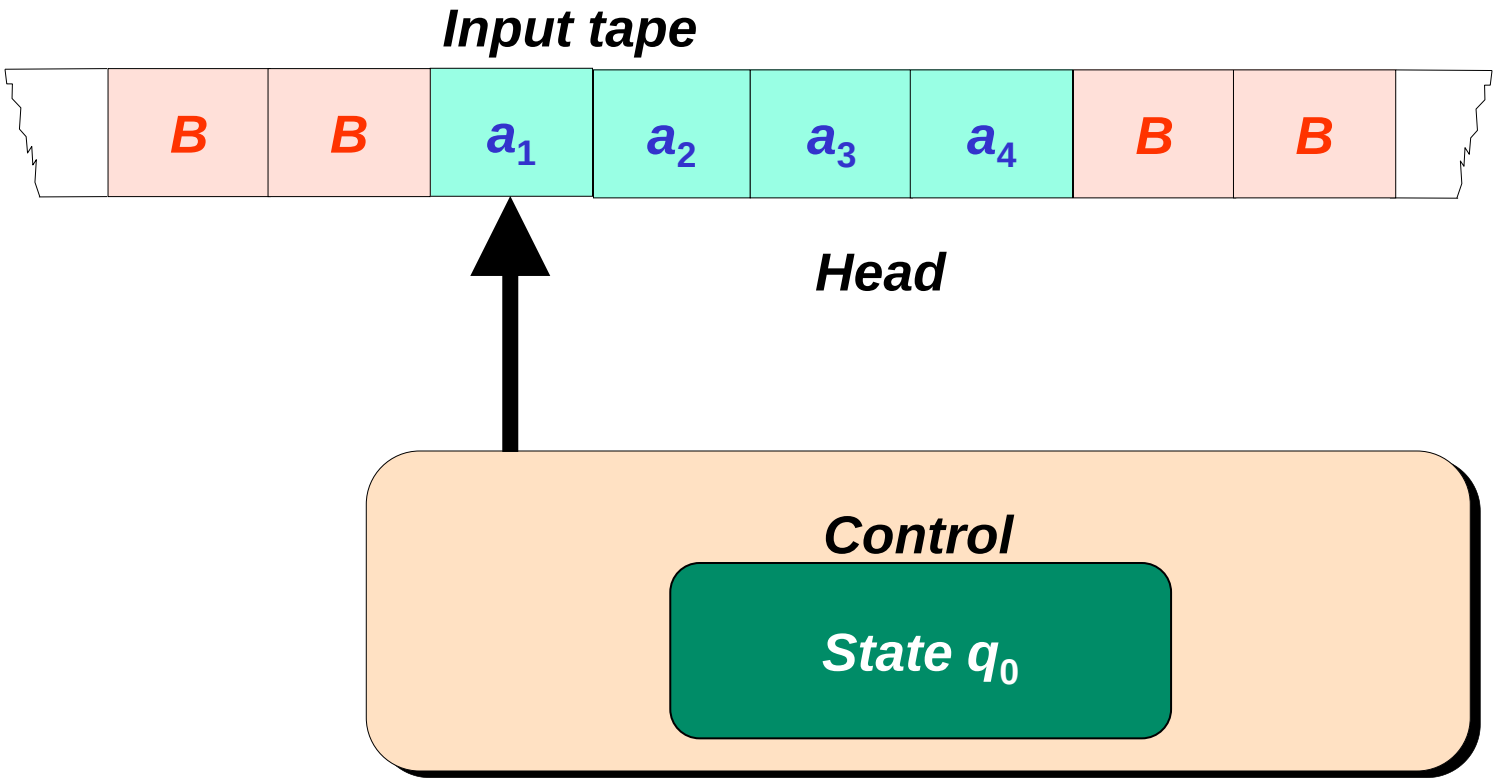



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# TM with a two-sided infinite tape


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# TM with a two-sided infinite tape

**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**


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**Input symbols:** [*a*, *B*]

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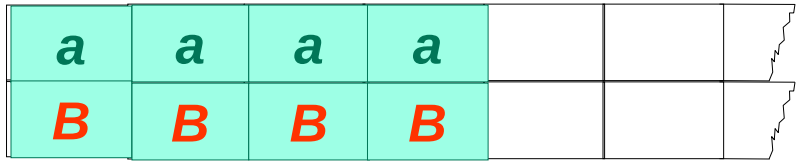



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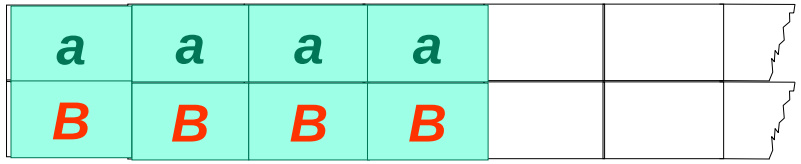




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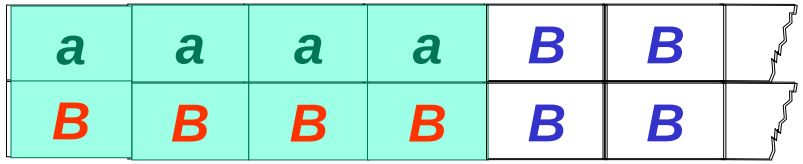




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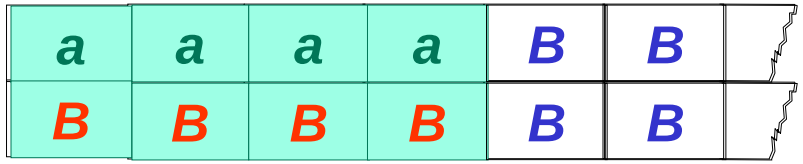


**States have two components:**


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# TM with a two-sided infinite tape

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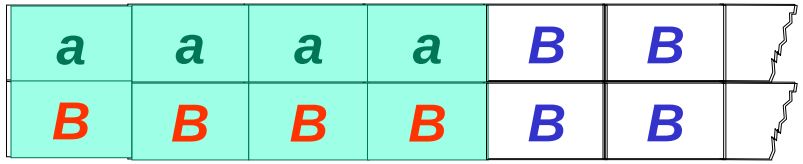


**States have two components:**

**First component: state TM $M_2$**



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# TM with a two-sided infinite tape

**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**

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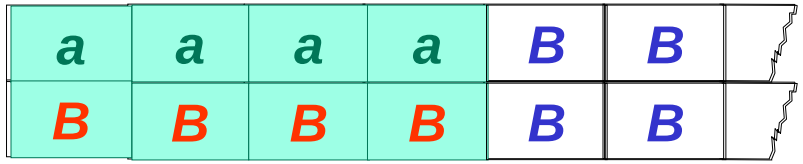


**States have two components:**

**First component: state TM $M_2$**

**Second component: symbol *U* or *D***



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# TM with a two-sided infinite tape

**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**

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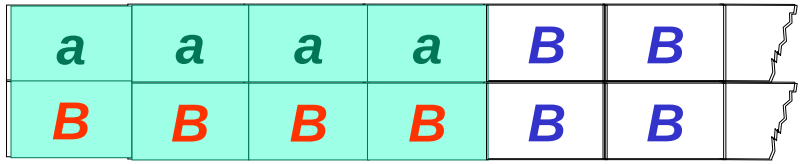
**Input symbols:** $[a, B]$

**Blank cells:** $[B, B]$




**States have two components:**

**First component: state TM $M_2$**

**Second component: symbol $U$ or $D$**

$[q, U]$, $[q, D]$



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# TM with a two-sided infinite tape

**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**

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**TS $M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$**

**Input symbols:** [*a*, *B*]

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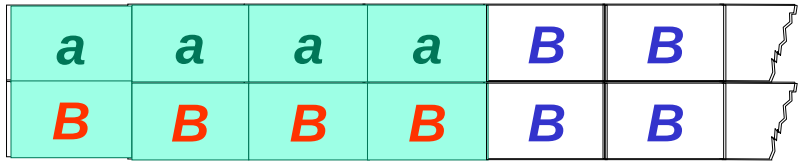


**States have two components:**

**First component: state TM $M_2$**

**Second component: symbol *U* or *D***

[*q*, *U*], [*q*, *D*]

**Tape symbols:** [*X*, *Y*]



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# TM with a two-sided infinite tape

TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$

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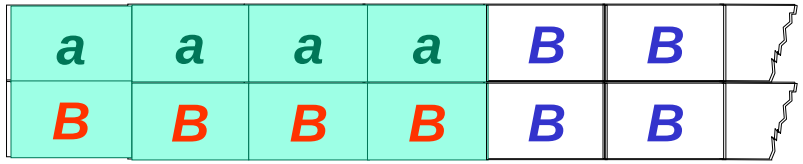


**States have two components:**

**First component: state TM $M_2$**

**Second component: symbol $U$ or $D$**

$[q, U], [q, D]$

**Tape symbols:** $[X, Y]$

$F_1 = \{[q, U], [q, D] \mid q \in F_2\}$



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# TM with a two-sided infinite tape

**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**

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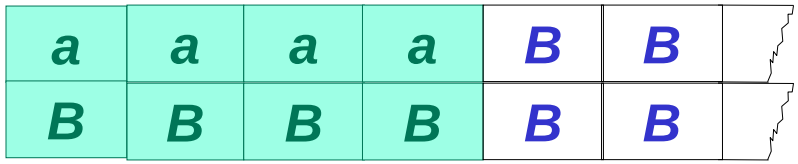



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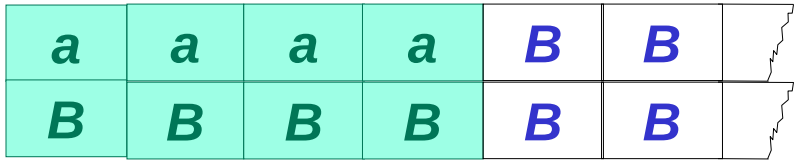


**$q_1$, $q_2$ – Start states TM $M_1$, TM $M_2$**


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# TM with a two-sided infinite tape

**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**

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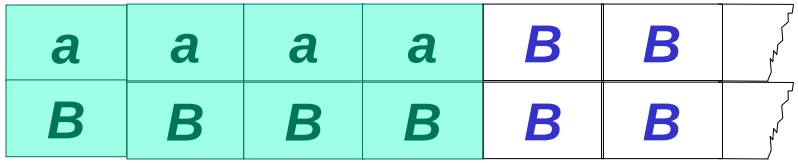


**$q_1$, $q_2$ – Start states TM $M_1$, TM $M_2$**

**If $\delta_2(q_2, a)=(q, X, R)$**


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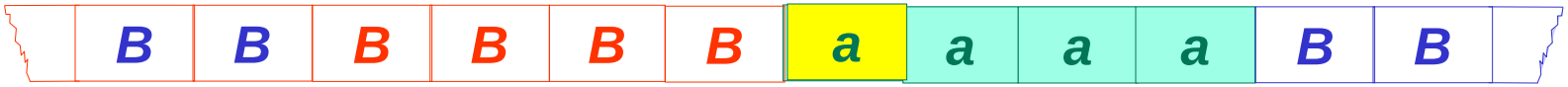
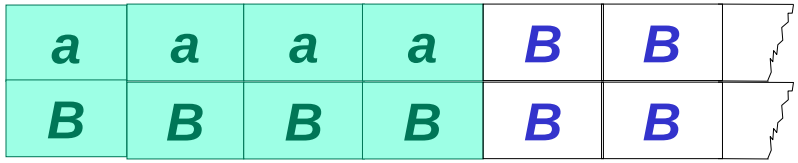
**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**




**TS $M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$**




**$q_1$, $q_2$ – Start states TM $M_1$, TM $M_2$**

**If $\delta_2(q_2, a)=(q, X, R)$**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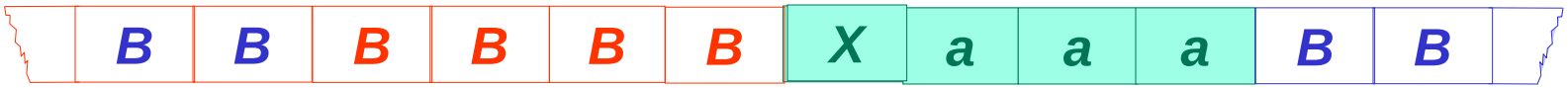


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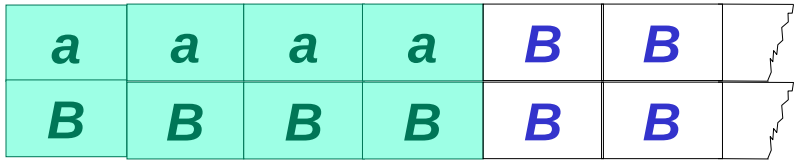


**$q_1$, $q_2$ – Start states TM $M_1$, TM $M_2$**

**If $\delta_2(q_2, a)=(q, X, R)$**


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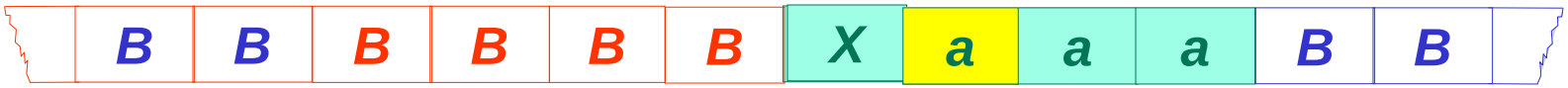
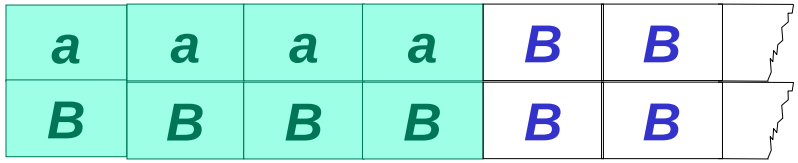



**TS $M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$**




**$q_1$, $q_2$ – Start states TM $M_1$, TM $M_2$**

**If $\delta_2(q_2, a)=(q, X, R)$**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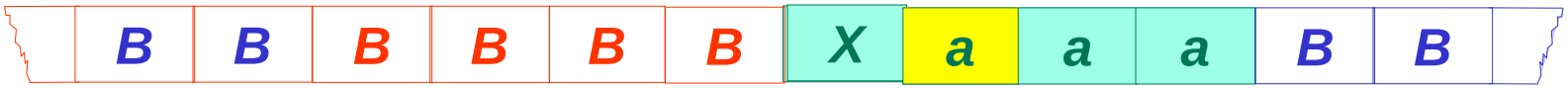


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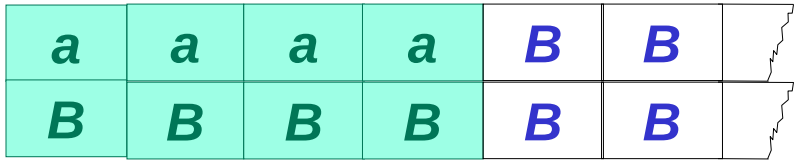


**$q_1$, $q_2$ – Start states TM $M_1$, TM $M_2$**

**If $\delta_2(q_2, a)=(q, X, R)$**

**then**

$$\delta_1(q_1, [a, B]) = ([q, U], [X, ¢], R)$$


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# TM with a two-sided infinite tape

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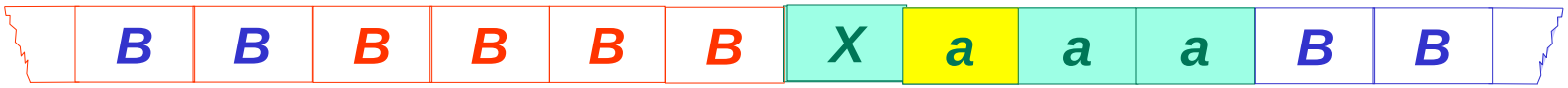


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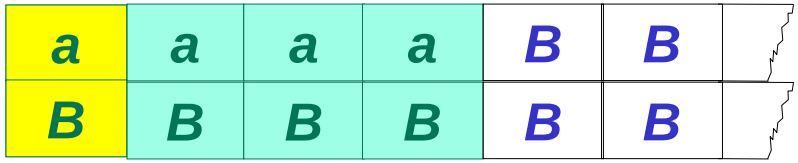


$q_1$, $q_2$ – **Start states** TM $M_1$, TM $M_2$

**If** $\delta_2(q_2, a)=(q, X, R)$

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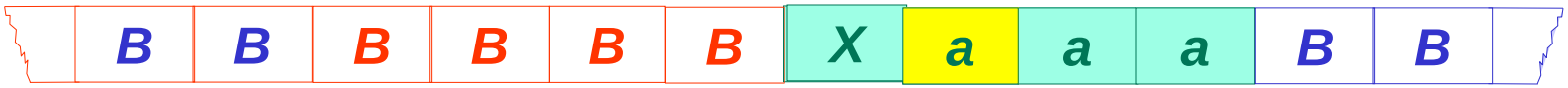


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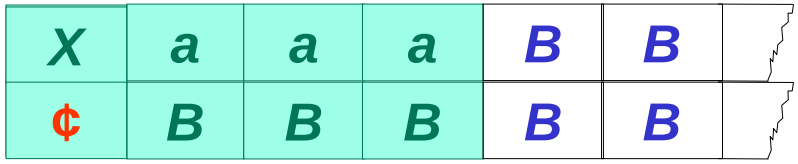


$q_1$, $q_2$ – **Start states** TM $M_1$, TM $M_2$

**If** $\delta_2(q_2, a)=(q, X, R)$

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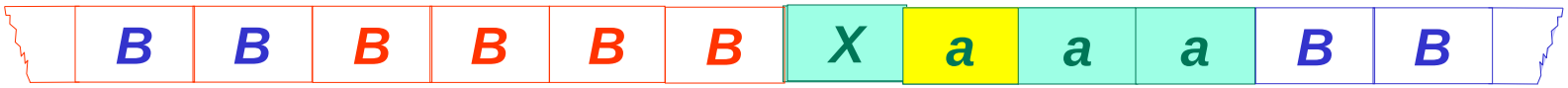


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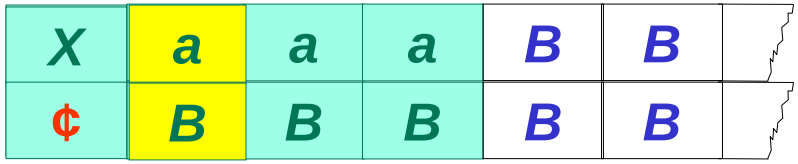


**$q_1$, $q_2$ – Start states TM $M_1$, TM $M_2$**

**If $\delta_2(q_2, a)=(q, X, R)$**

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$$\delta_1(q_1, [a, B]) = ([q, U], [X, ¢], R)$$


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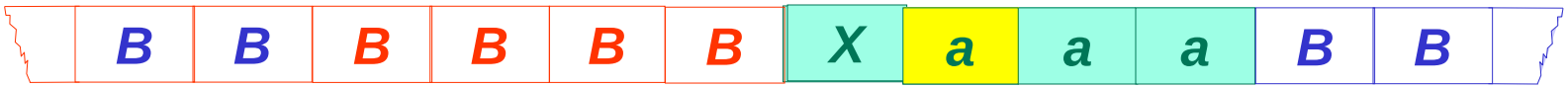
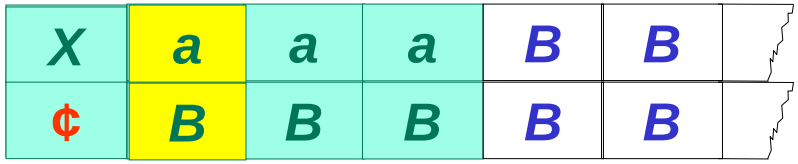
# TM with a two-sided infinite tape

TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$




TS $M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$




$q_1$, $q_2$ – **Start states** TM $M_1$, TM $M_2$

**If** $\delta_2(q_2, a)=(q, X, R)$

**then**

$$\delta_1(q_1, [a, B]) = ([q, U], [X, ¢], R)$$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# TM with a two-sided infinite tape

**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**

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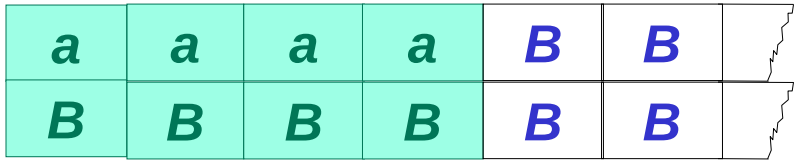
**TM $M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$**





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# TM with a two-sided infinite tape

TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$

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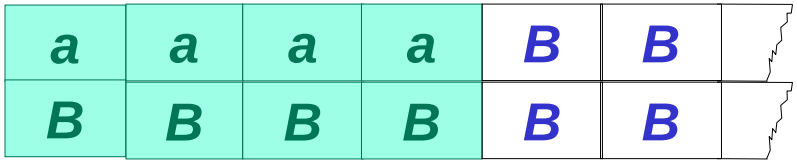


$q_1$, $q_2$ – **Start states TM $M_1$, TM $M_2$**


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# TM with a two-sided infinite tape

**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**

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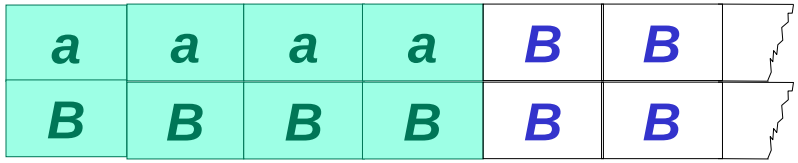


**$q_1$, $q_2$ – Start states TM $M_1$, TM $M_2$**

**If $\delta_2(q_2, a)=(q, X, L)$**


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# TM with a two-sided infinite tape

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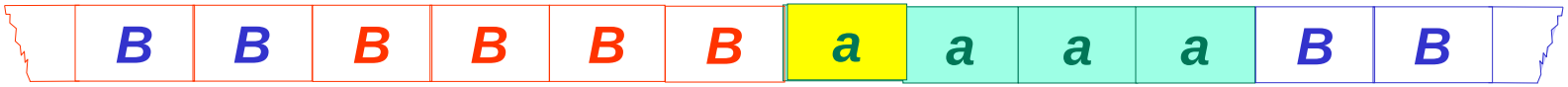
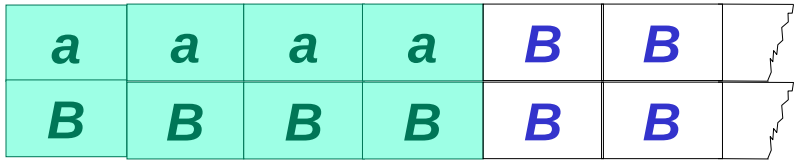



TM $M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$




$q_1$, $q_2$ – **Start states** TM $M_1$, TM $M_2$

**If** $\delta_2(q_2, a)=(q, X, L)$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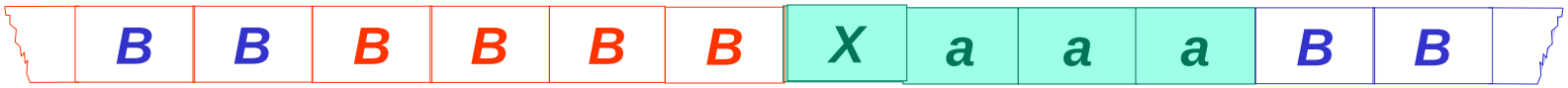
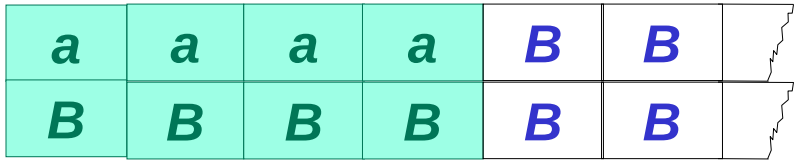



TM $M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$




$q_1$, $q_2$ – **Start states** TM $M_1$, TM $M_2$

**If** $\delta_2(q_2, a)=(q, X, L)$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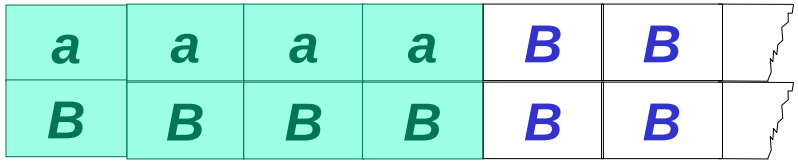


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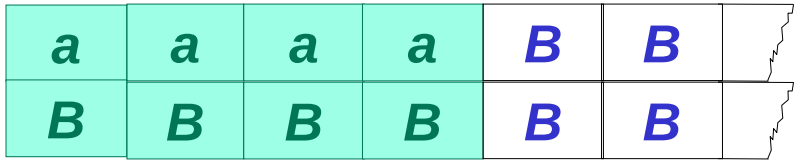


$q_1$, $q_2$ – **Start states TM $M_1$, TM $M_2$**

**If $\delta_2(q_2, a)=(q, X, L)$**

**then**

$$\delta_1(q_1, [a, B]) = ([q, D], [X, ¢], R)$$


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# TM with a two-sided infinite tape

**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**

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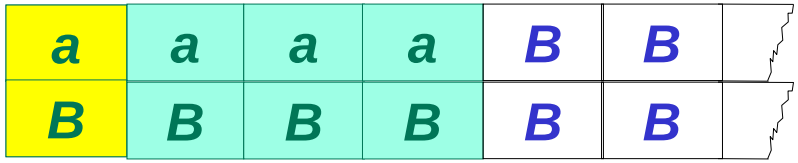
**TM $M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$**




$q_1$, $q_2$ – **Start states TM $M_1$, TM $M_2$**

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# TM with a two-sided infinite tape

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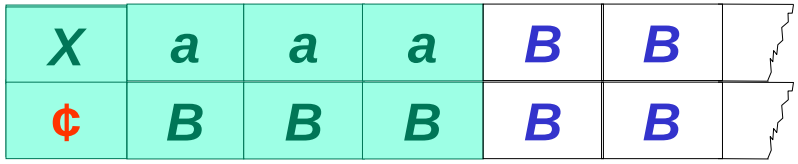


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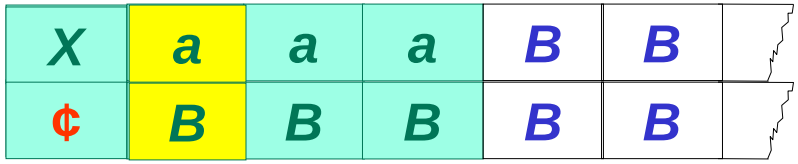
**TM $M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$**

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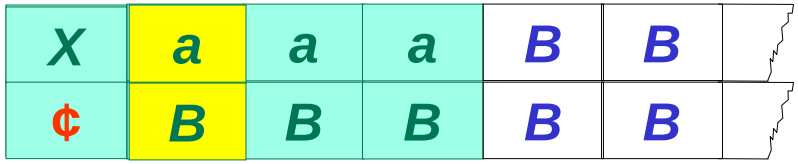


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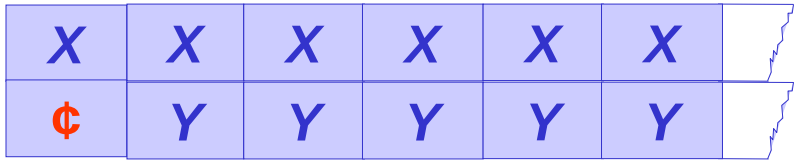



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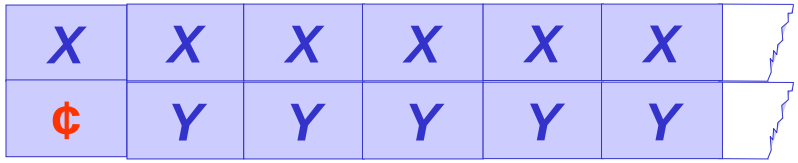


**Ako $\delta_2(q, X)=(p, Z, A)$**


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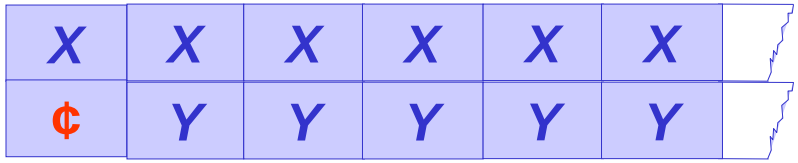


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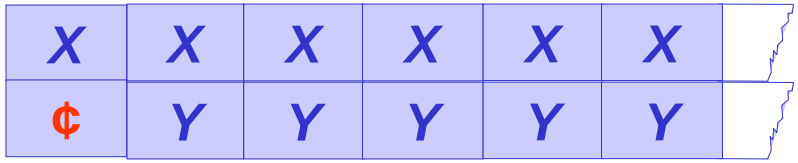


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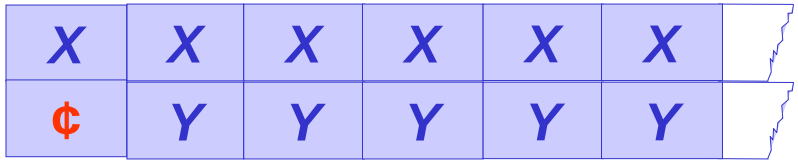


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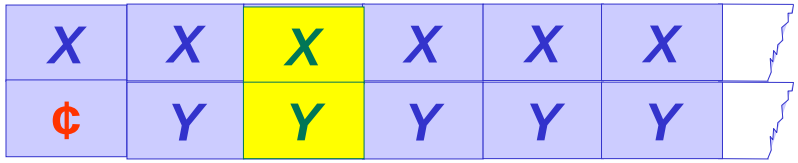


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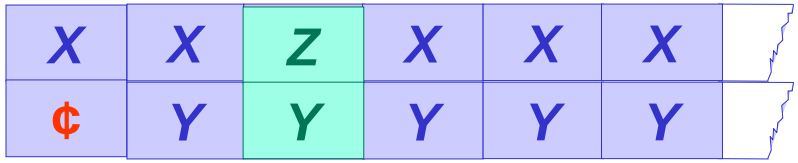


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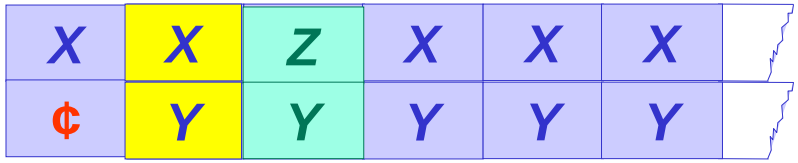


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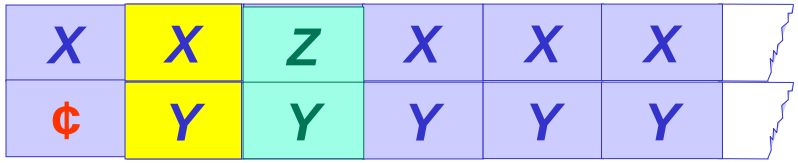


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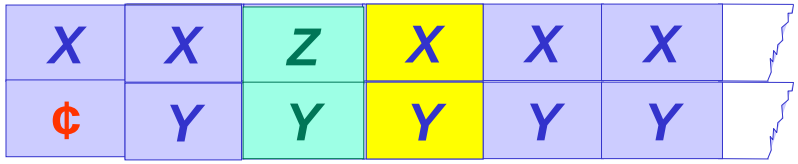


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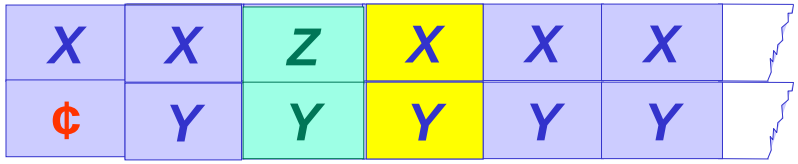


**Ako $\delta_2(q, X)=(p, Z, A)$**

**onda**

$$\delta_1([q, U], [X, Y]) = ([p, U], [Z, Y], A)$$


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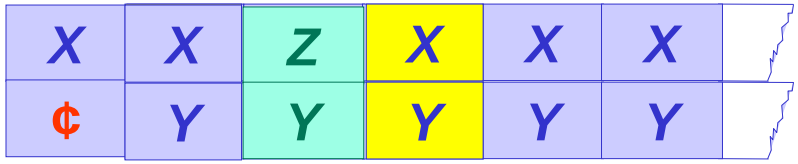


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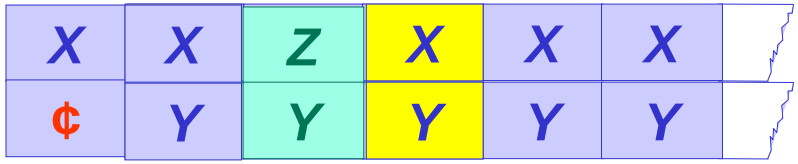


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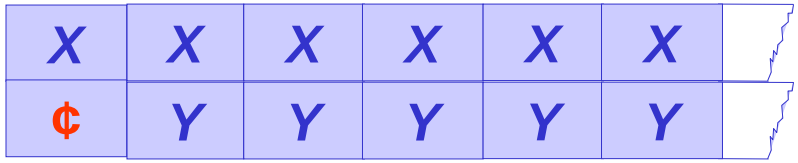



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# TM with a two-sided infinite tape

**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**

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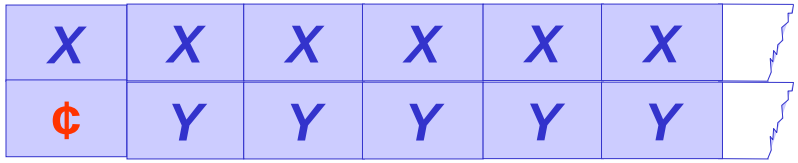
**TS $M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$**




**Ako $\delta_2(q, Y)=(p, Z, A)$**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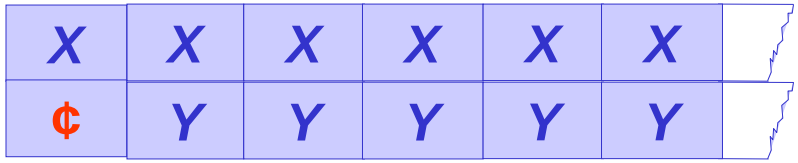


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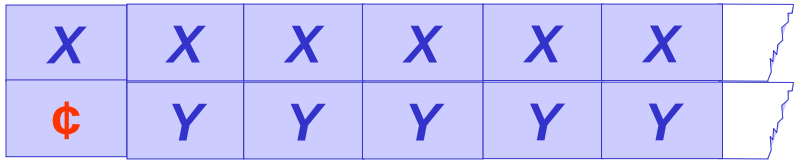


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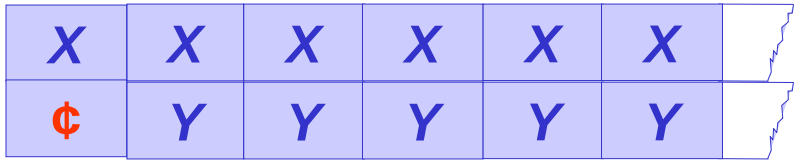


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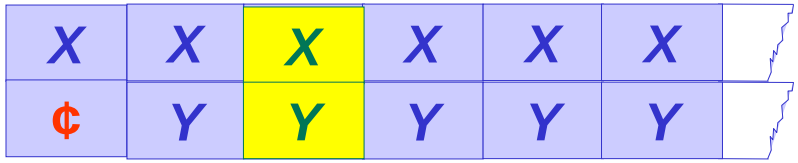


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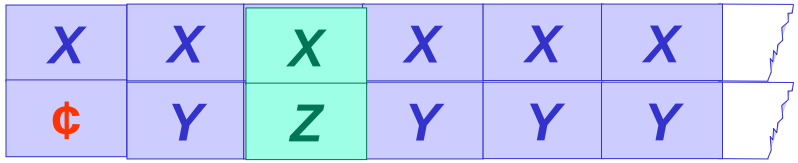


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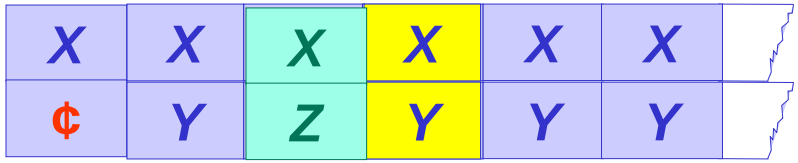


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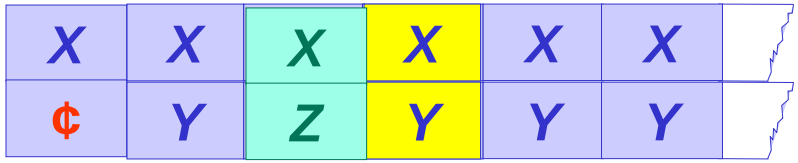


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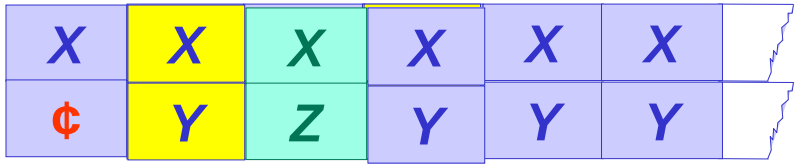


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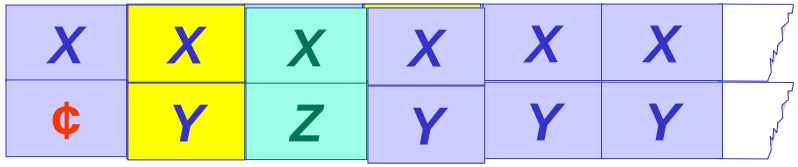


**Ako** $\delta_2(q, Y)=(p, Z, A)$

**onda**

$$\delta_1([q, D], [X, Y]) = ([p, D], [X, Z], -A)$$


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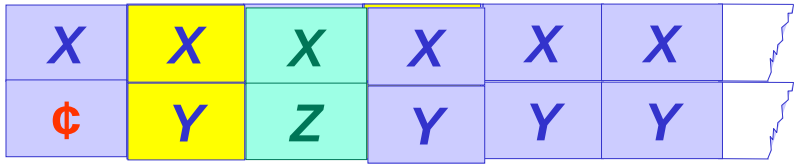


**Ako $\delta_2(q, Y)=(p, Z, A)$**

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$$\delta_1([q, D], [X, Y]) = ([p, D], [X, Z], -A)$$


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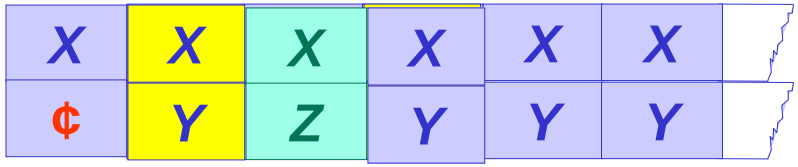


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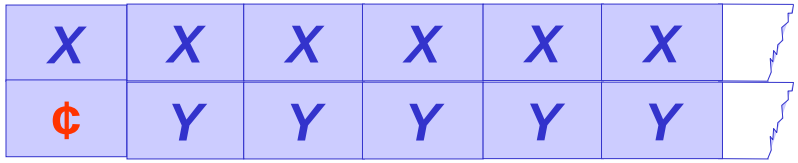



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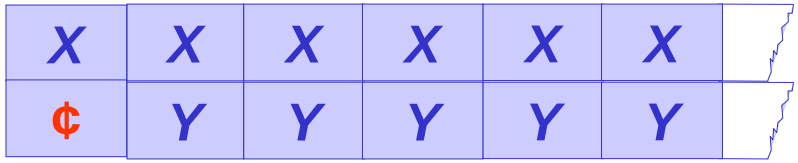


**Ako $\delta_2(q, X)=(p, Z, R)$**


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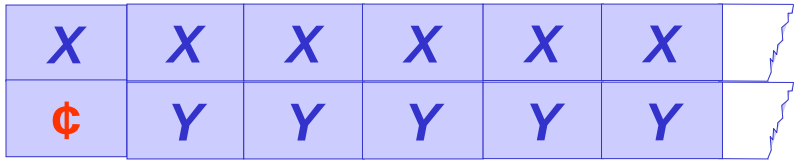


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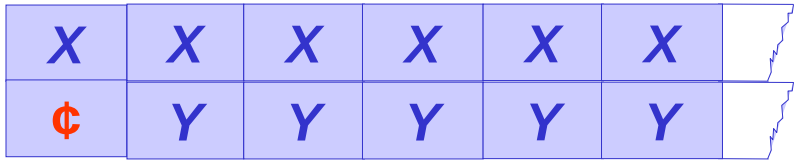


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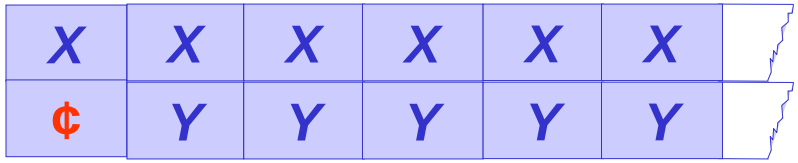


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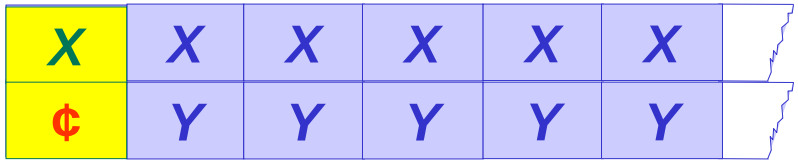


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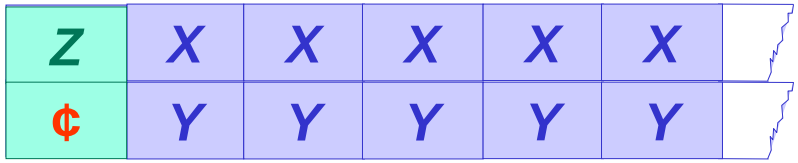


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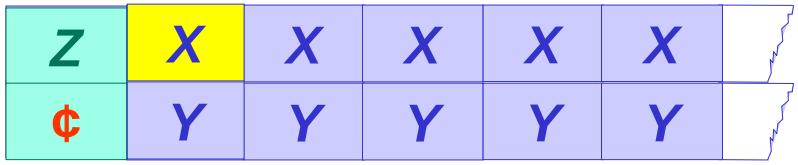


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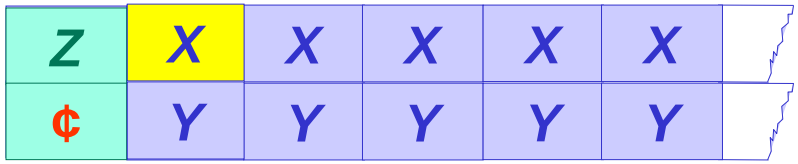


**Ako** $\delta_2(q, X)=(p, Z, R)$

**onda**

$$\delta_1([q, U], [X, ¢]) = ([p, U], [Z, ¢], R)$$


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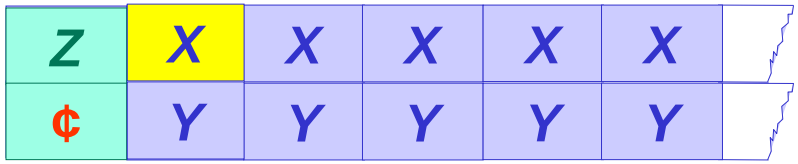


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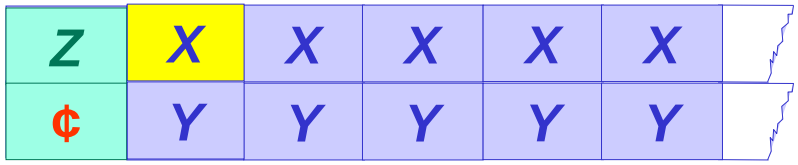


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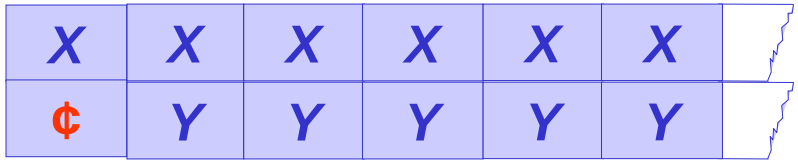



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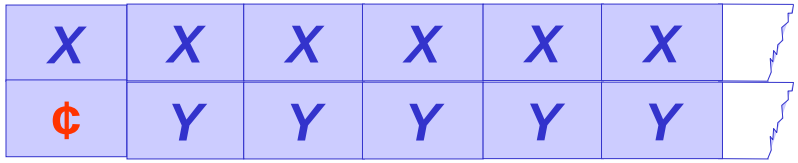


**Ako $\delta_2(q, X)=(p, Z, L)$**


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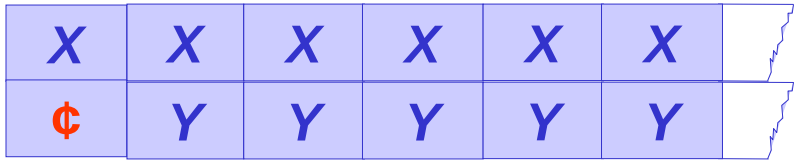


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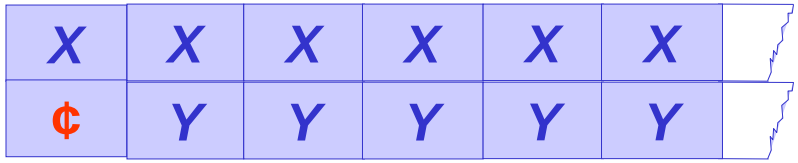


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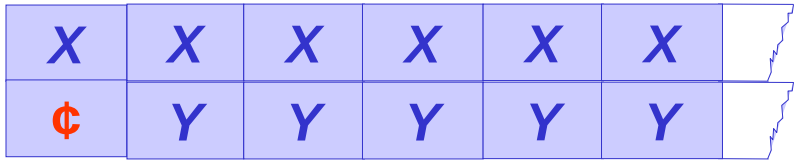


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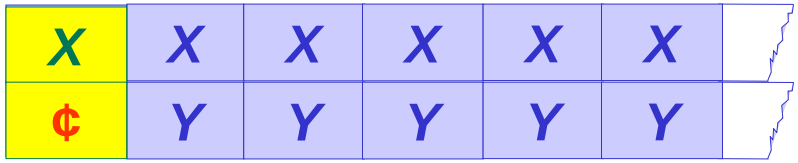


**Ako $\delta_2(q, X)=(p, Z, L)$**


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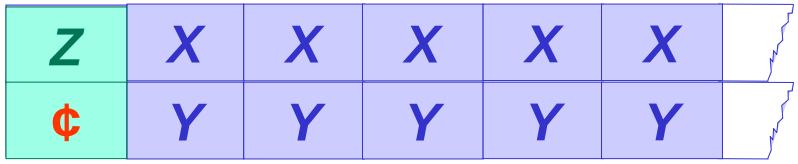
# TM with a two-sided infinite tape

**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**

---

**TS $M_1 = (Q_1, \Sigma_1, \Gamma_1, \delta_1, q_1, B_1, F_1)$**

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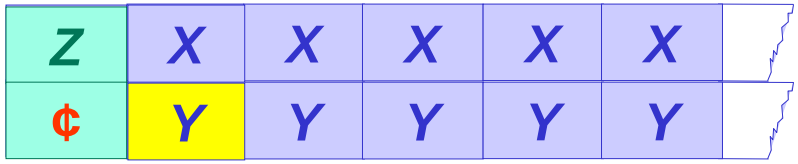


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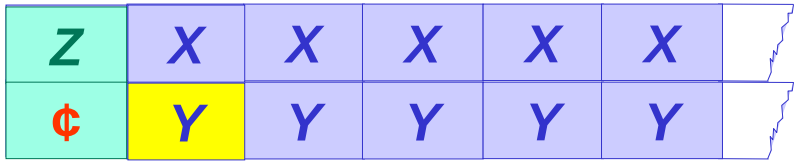


**Ako** $\delta_2(q, X)=(p, Z, L)$

**onda**

$$\delta_1([q, U], [X, ¢]) = ([p, D], [Z, ¢], R)$$


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# TM with a two-sided infinite tape

**TS $M_2 = (Q_2, \Sigma_2, \Gamma_2, \delta_2, q_2, B, F_2)$**

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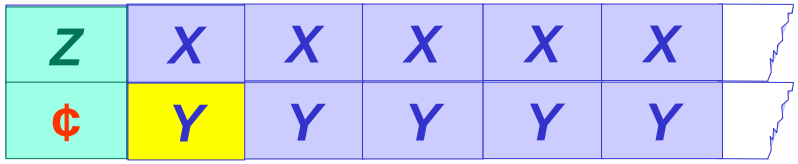


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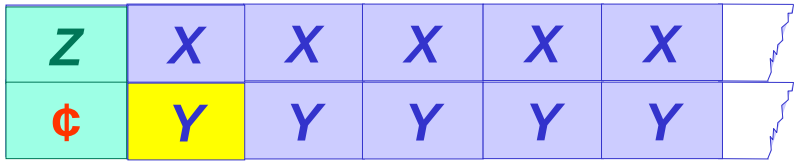


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# Multitape TM


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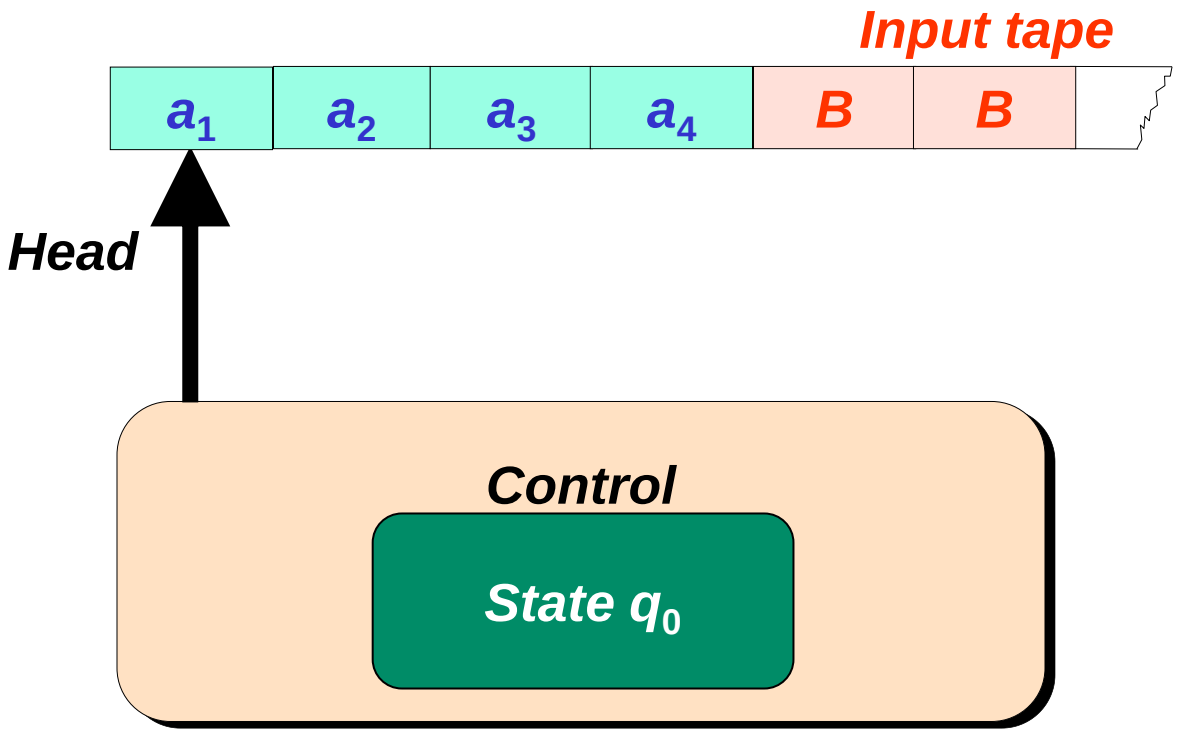
# Multitape TM





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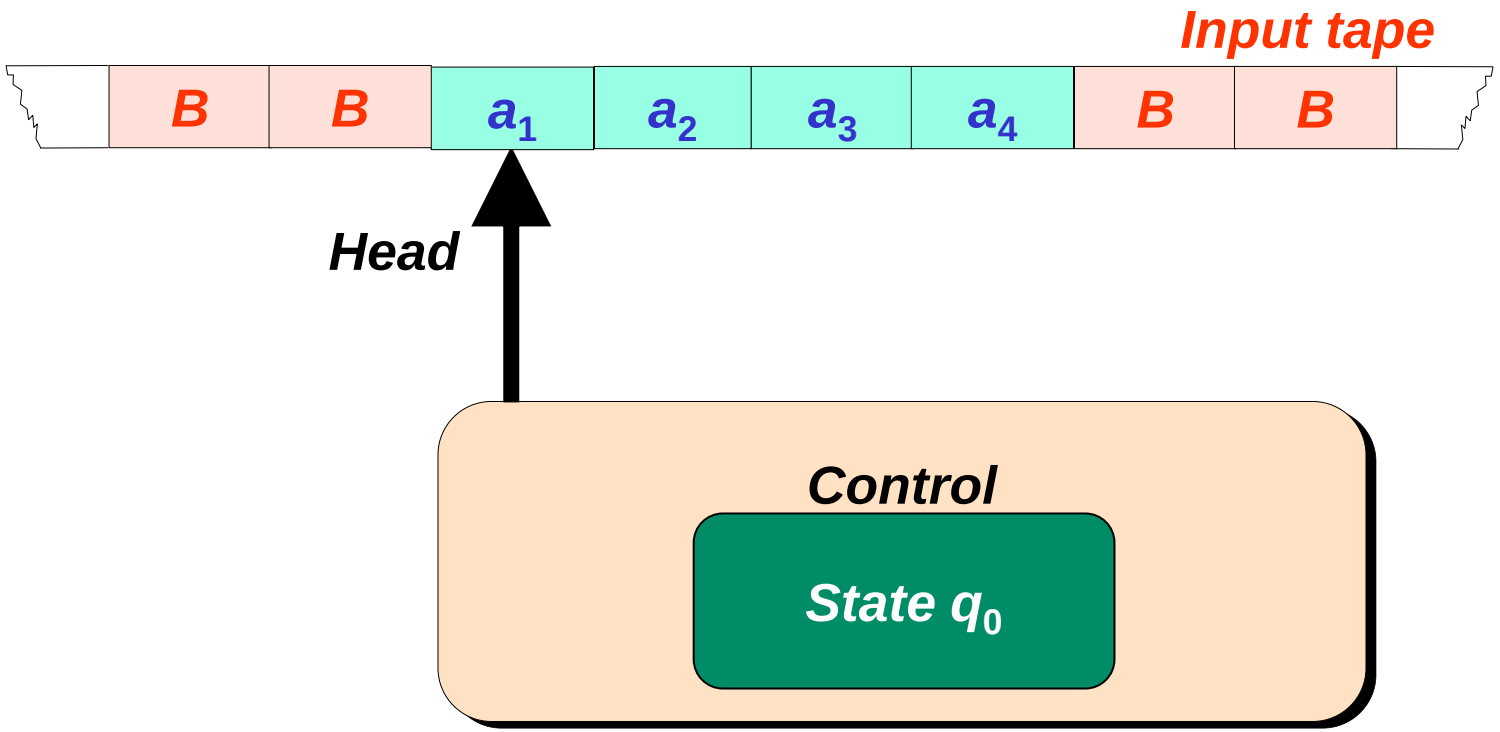
# Multitape TM





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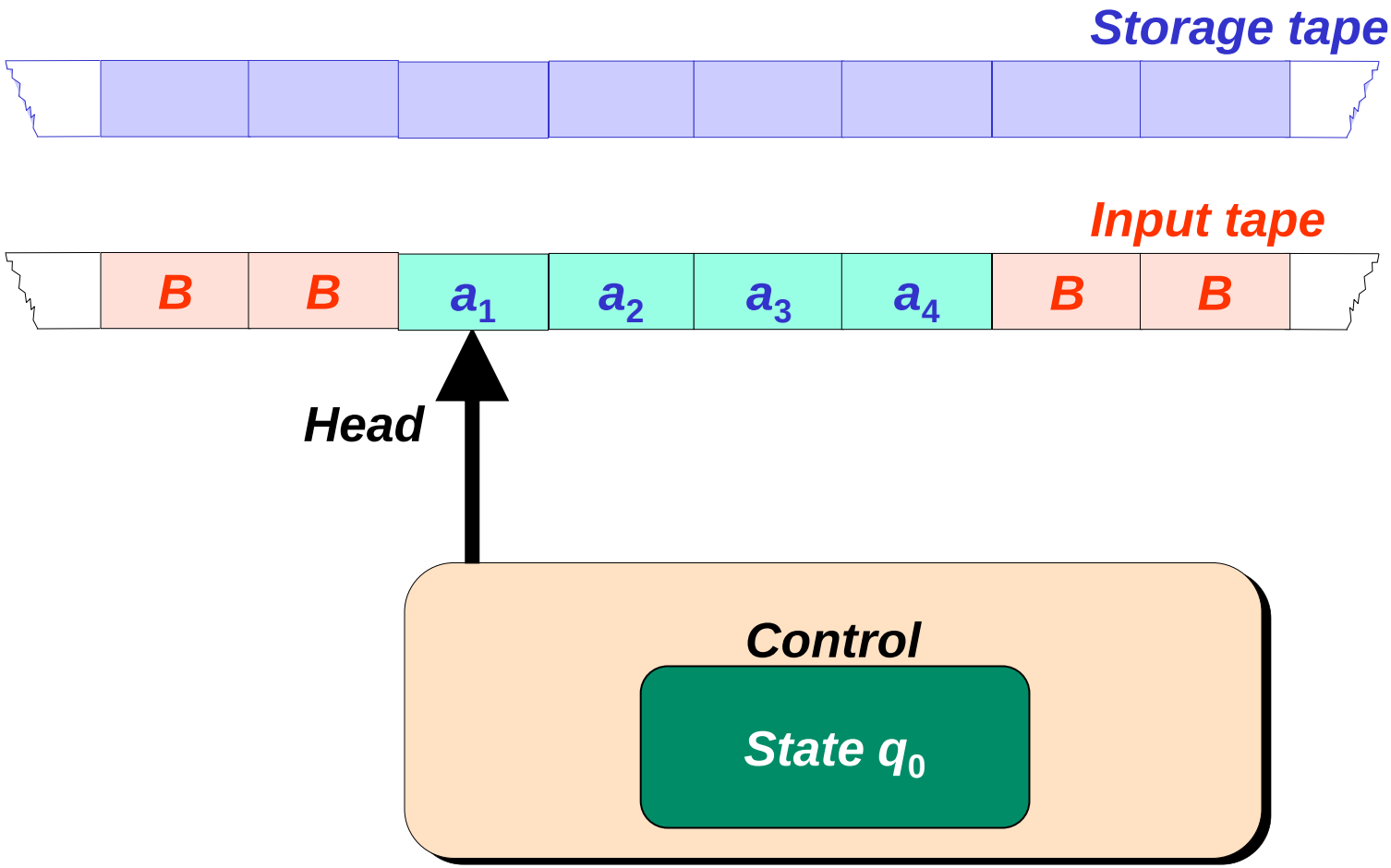
# Multitape TM





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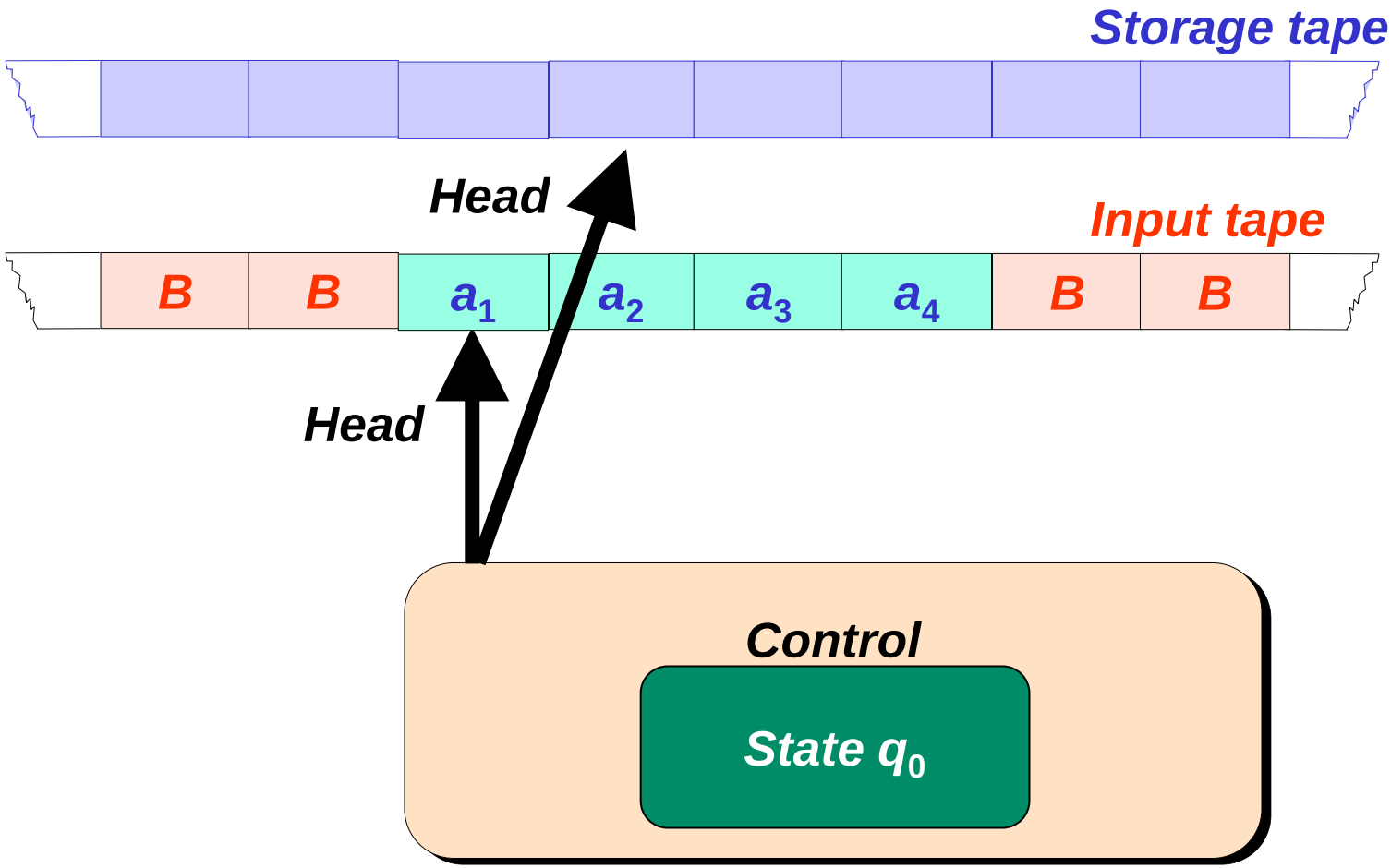



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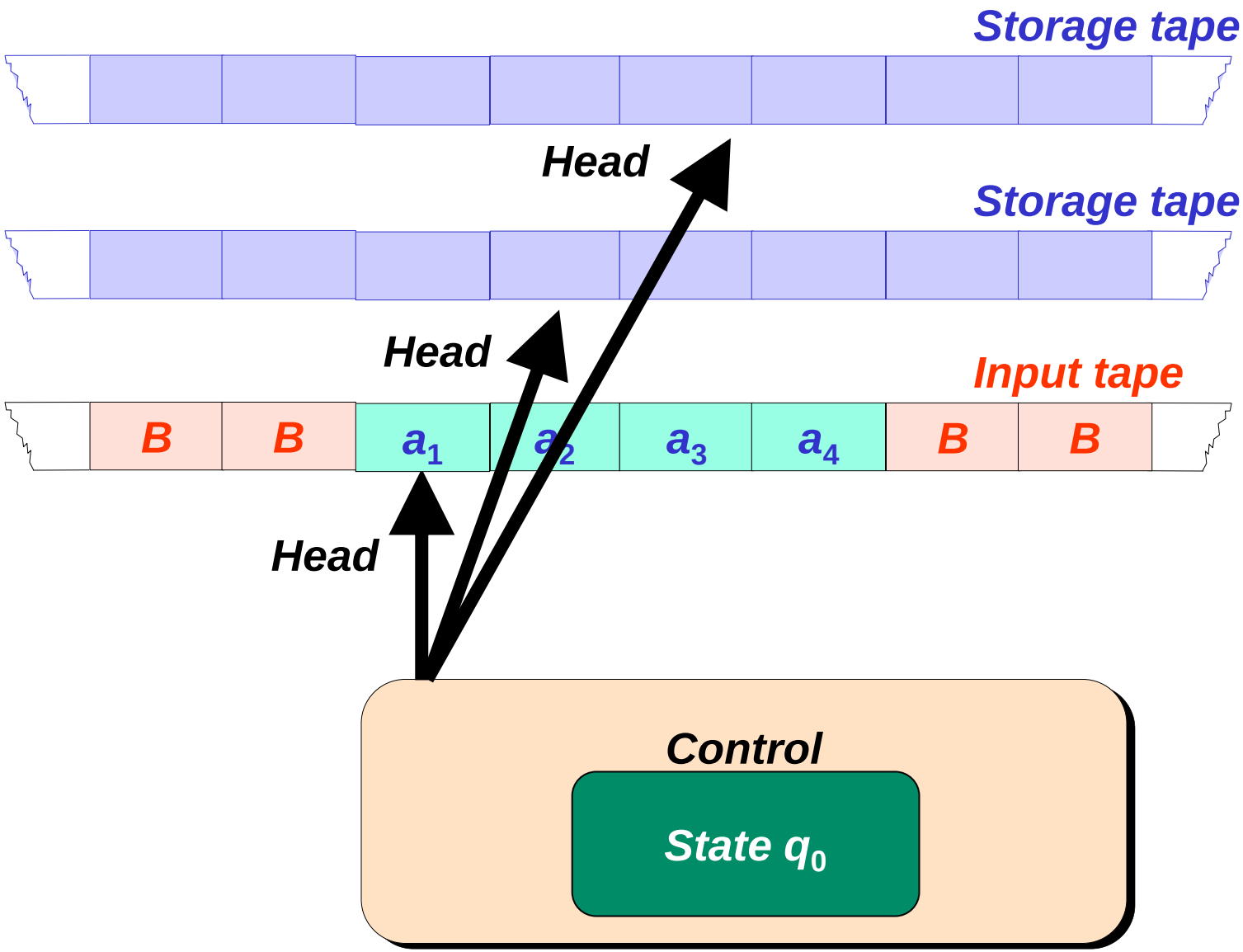
# Multitape TM





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# Multitape TM


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# Multitape TM

- **Decision**


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# Multitape TM

- **Decision**
  - Input


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# Multitape TM

- **Decision**
  - Input
    1. **State**


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# Multitape TM

- **Decision**
  - Input
    1. **State**
    2. **Reading *k* symbols from *k* tapes**


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# Multitape TM

- **Decision**
  - Input
    1. **State**
    2. **Reading *k* symbols from *k* tapes**
  - Output


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# Multitape TM

- **Decision**
  - Input
    1. **State**
    2. **Reading *k* symbols from *k* tapes**
  - Output
    1. **New state**


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# Multitape TM

- **Decision**
  - Input
    1. **State**
    2. **Reading *k* symbols from *k* tapes**
  - Output
    1. **New state**
    2. **Writing *k* symbols on *k* tapes**


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# Multitape TM

- **Decision**
  - Input
    1. **State**
    2. **Reading *k* symbols from *k* tapes**
  - Output
    1. **New state**
    2. **Writing *k* symbols on *k* tapes**
    3. **Independent movement of *k* heads**


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# Multitape TM


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# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**


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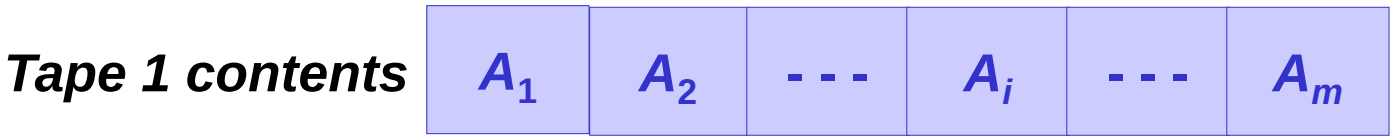
# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**





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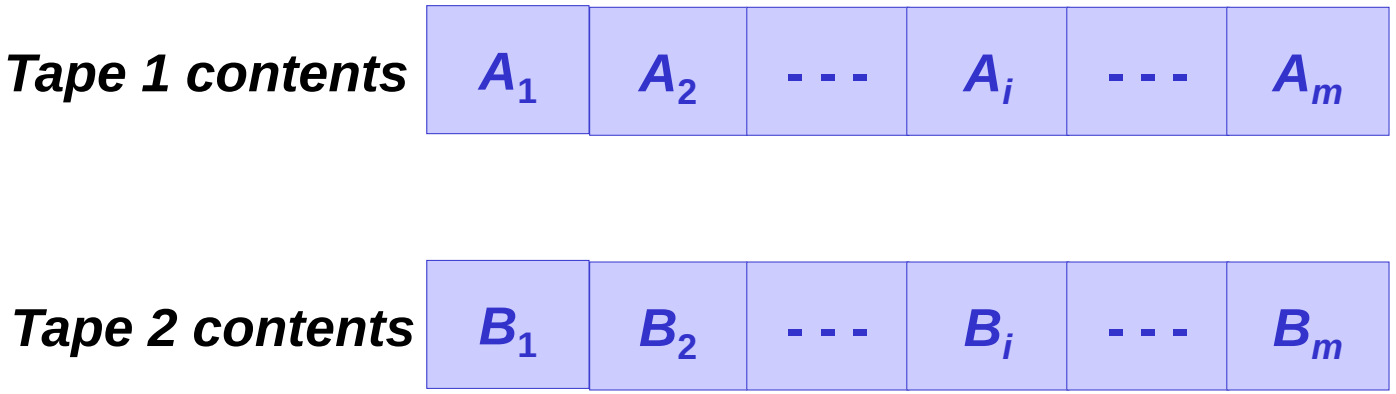
**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**





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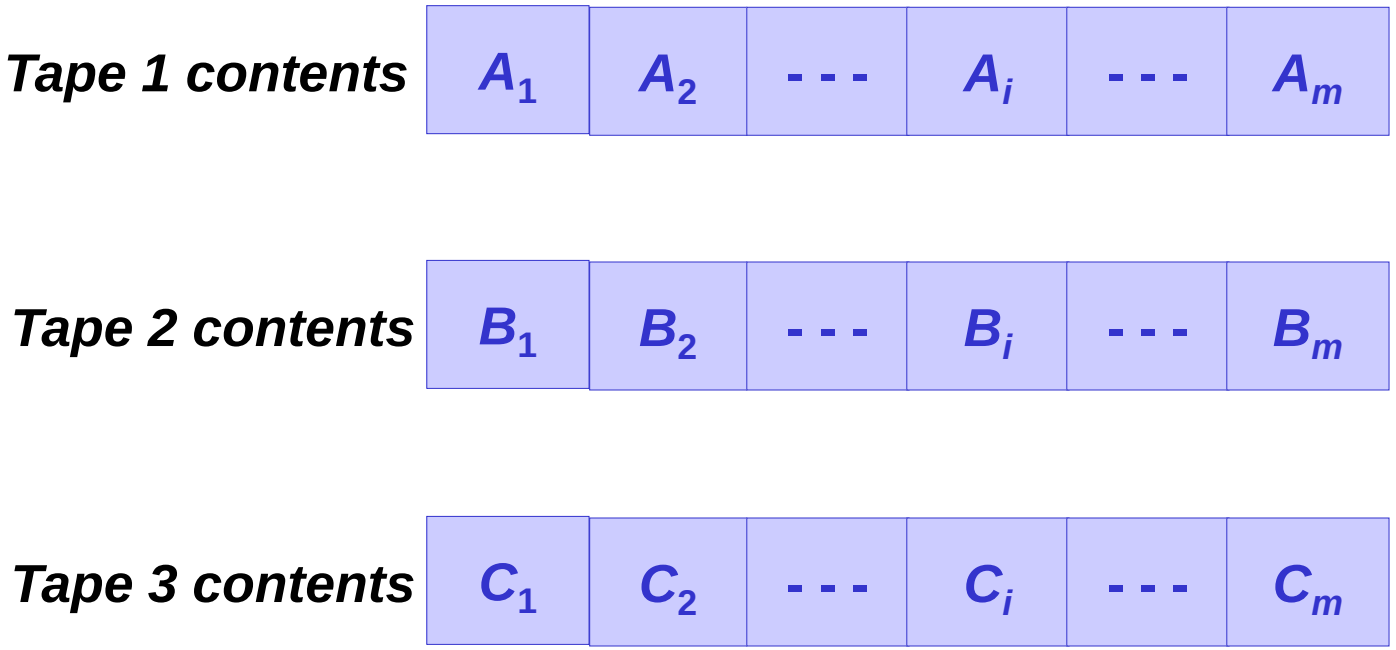
**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**





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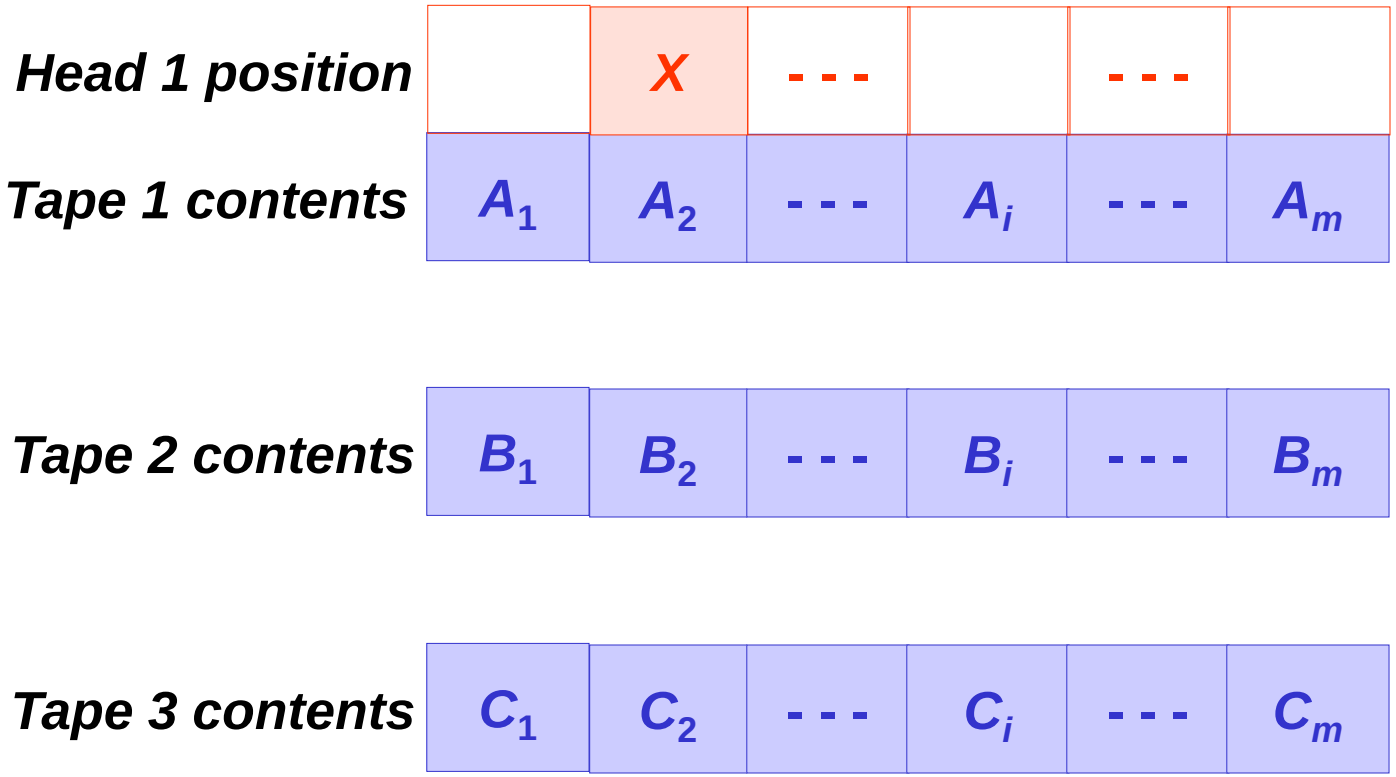
**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**





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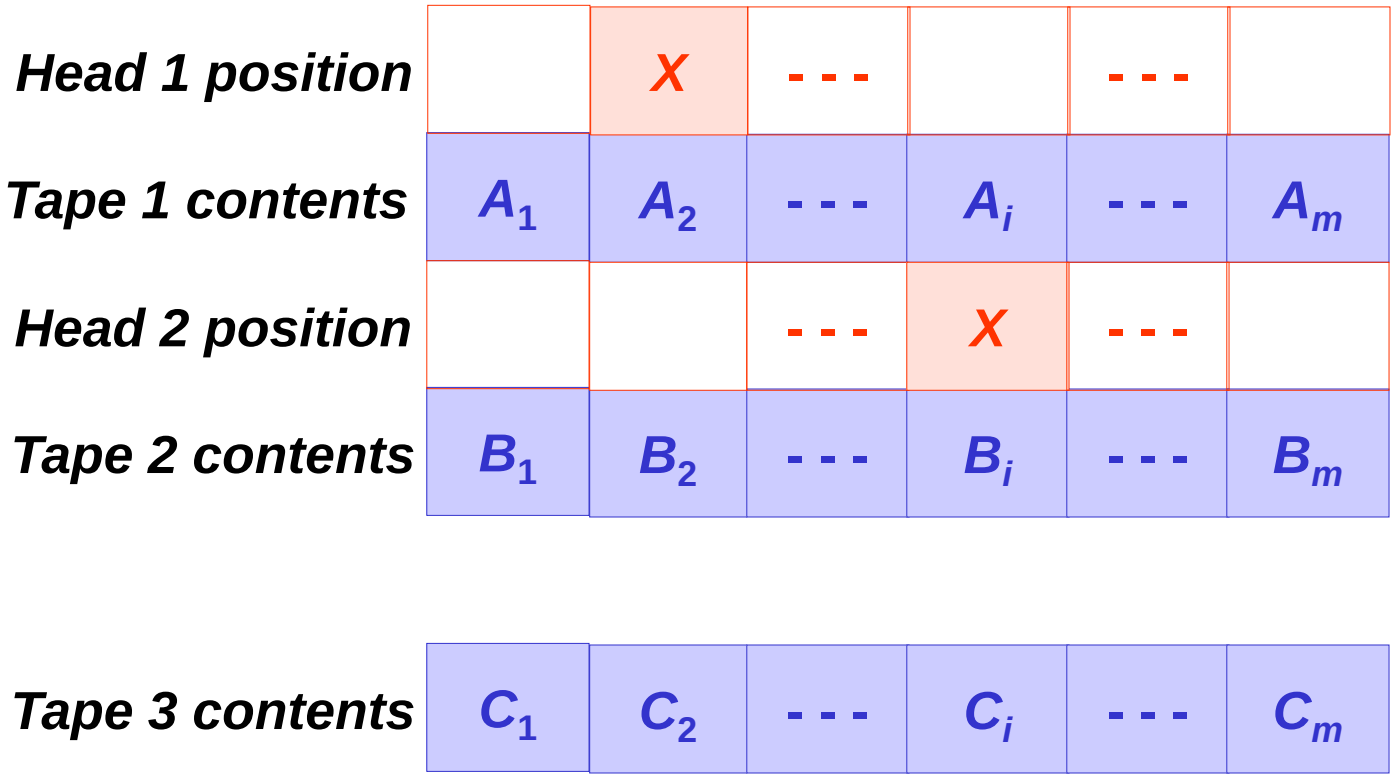
# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**





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# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

| | | | | | | |
|---|---|---|---|---|---|---|
| ***Head 1 position*** | | $X$ | - - - | | - - - | |
| ***Tape 1 contents*** | $A_1$ | $A_2$ | - - - | $A_i$ | - - - | $A_m$ |
| ***Head 2 position*** | | | - - - | $X$ | - - - | |
| ***Tape 2 contents*** | $B_1$ | $B_2$ | - - - | $B_i$ | - - - | $B_m$ |
| ***Head 3 position*** | $X$ | | - - - | | - - - | |
| ***Tape 3 contents*** | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |


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# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

| | | | | | | |
|---|---|---|---|---|---|---|
| *Head 1 position* | | $X$ | - - - | | - - - | |
| *Tape 1 contents* | $A_1$ | $A_2$ | - - - | $A_i$ | - - - | $A_m$ |
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| *Tape 2 contents* | $B_1$ | $B_2$ | - - - | $B_i$ | - - - | $B_m$ |
| *Head 3 position* | $X$ | | - - - | | - - - | |
| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

**[ State TS $M_1$,**


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# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

| | | | | | | |
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| ***Head 1 position*** | | $X$ | - - - | | - - - | |
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| ***Tape 3 contents*** | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

**[ State TS $M_1$, Counter,**


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# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

| | | | | | | |
|---|---|---|---|---|---|---|
| *Head 1 position* | | $X$ | - - - | | - - - | |
| *Tape 1 contents* | $A_1$ | $A_2$ | - - - | $A_i$ | - - - | $A_m$ |
| *Head 2 position* | | | - - - | $X$ | - - - | |
| *Tape 2 contents* | $B_1$ | $B_2$ | - - - | $B_i$ | - - - | $B_m$ |
| *Head 3 position* | $X$ | | - - - | | - - - | |
| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

**[ State TS $M_1$, Counter, Tape 1 contents, ..., Tape k contents]**


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# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

| | | | | | | |
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| | | | | | | |
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[ $q_0$ ,


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| | | | | | | |
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| *Tape 2 contents* | $B_1$ | $B_2$ | - - - | $B_i$ | - - - | $B_m$ |
| *Head 3 position* | $X$ | | - - - | | - - - | |
| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

$[ \quad q_0 \quad , \quad 3 \quad ,$


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**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

| | | | | | | |
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| *Head 3 position* | $X$ | | - - - | | - - - | |
| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

$$[\ q_0\ ,\ 3\ ,\ B\ ,\ B\ ,\ B\ ]$$


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| | | | | | | |
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| *Tape 2 contents* | $B_1$ | $B_2$ | - - - | $B_i$ | - - - | $B_m$ |
| *Head 3 position* | $X$ | | - - - | | - - - | |
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$$[\ q_0\ ,\ 3\ ,\ B\ ,\ B\ ,\ B\ ]$$


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$$[\ q_0\ ,\ 3\ ,\ B\ ,\ B\ ,\ B\ ]$$


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| | | | | | | |
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| *Head 2 position* | | | - - - | $X$ | - - - | |
| *Tape 2 contents* | $B_1$ | $B_2$ | - - - | $B_i$ | - - - | $B_m$ |
| *Head 3 position* | $X$ | | - - - | | - - - | |
| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

$$[\ q_0\ ,\ 2\ ,\ B\ ,\ B\ ,\ B\ ]$$


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# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

| | | | | | | |
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$$[\ q_0\ ,\ 2\ ,\ B\ ,\ B\ ,\ B\ ]$$


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| *Head 3 position* | $X$ | | - - - | | - - - | |
| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

$$[\ q_0\ ,\ 2\ ,\ B\ ,\ B\ ,\ C_1\ ]$$


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| *Head 2 position* | | | - - - | $X$ | - - - | |
| *Tape 2 contents* | $B_1$ | $B_2$ | - - - | $B_i$ | - - - | $B_m$ |
| *Head 3 position* | $X$ | | - - - | | - - - | |
| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

$$[\ q_0\ ,\ 2\ ,\ B\ ,\ B\ ,\ C_1\ ]$$


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# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

| | | | | | | |
|---|---|---|---|---|---|---|
| *Head 1 position* | | $X$ | - - - | | - - - | |
| *Tape 1 contents* | $A_1$ | $A_2$ | - - - | $A_i$ | - - - | $A_m$ |
| *Head 2 position* | | | - - - | $X$ | - - - | |
| *Tape 2 contents* | $B_1$ | $B_2$ | - - - | $B_i$ | - - - | $B_m$ |
| *Head 3 position* | $X$ | | - - - | | - - - | |
| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

$$[\ q_0\ ,\ 2\ ,\ B\ ,\ B\ ,\ C_1\ ]$$


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**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

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| *Head 1 position* | | $X$ | - - - | | - - - | |
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| *Head 3 position* | $X$ | | - - - | | - - - | |
| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

$$[\ q_0\ ,\ 1\ ,\ B\ ,\ B\ ,\ C_1\ ]$$


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| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

$$[\ q_0\ ,\ 1\ ,\ A_2\ ,\ B\ ,\ C_1\ ]$$


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**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

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| *Head 1 position* | | $X$ | - - - | | - - - | |
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$$[\ q_0\ ,\ 1\ ,\ A_2\ ,\ B\ ,\ C_1\ ]$$


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$$[\ q_0\ ,\ 1\ ,\ A_2\ ,\ B\ ,\ C_1\ ]$$


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$$[\ q_0\ ,\ 1\ ,\ A_2\ ,\ B\ ,\ C_1\ ]$$


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**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

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| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

$$[\ q_0\ ,\ 0\ ,\ A_2\ ,\ B\ ,\ C_1\ ]$$


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$$[\ q_0\ ,\ 0\ ,\ A_2\ ,\ B\ ,\ C_1\ ]$$


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$$[\ q_0\ ,\ 0\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$


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| *Head 3 position* | $X$ | | - - - | | - - - | |
| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

$$[\ q\ ,\ 3\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$


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**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

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| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

$$[\ q\ ,\ 2\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$


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$$[\ q\ ,\ 2\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$


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**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

| | | | | | | |
|---|---|---|---|---|---|---|
| *Head 1 position* | | $X$ | - - - | | - - - | |
| *Tape 1 contents* | $A_1$ | $A_2$ | - - - | $A_i$ | - - - | $A_m$ |
| *Head 2 position* | | | - - - | | $X$ | |
| *Tape 2 contents* | $B_1$ | $B_2$ | - - - | $Z_i$ | - - - | $B_m$ |
| *Head 3 position* | $X$ | | - - - | | - - - | |
| *Tape 3 contents* | $C_1$ | $C_2$ | - - - | $C_i$ | - - - | $C_m$ |

$$[\ q\ ,\ 2\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$


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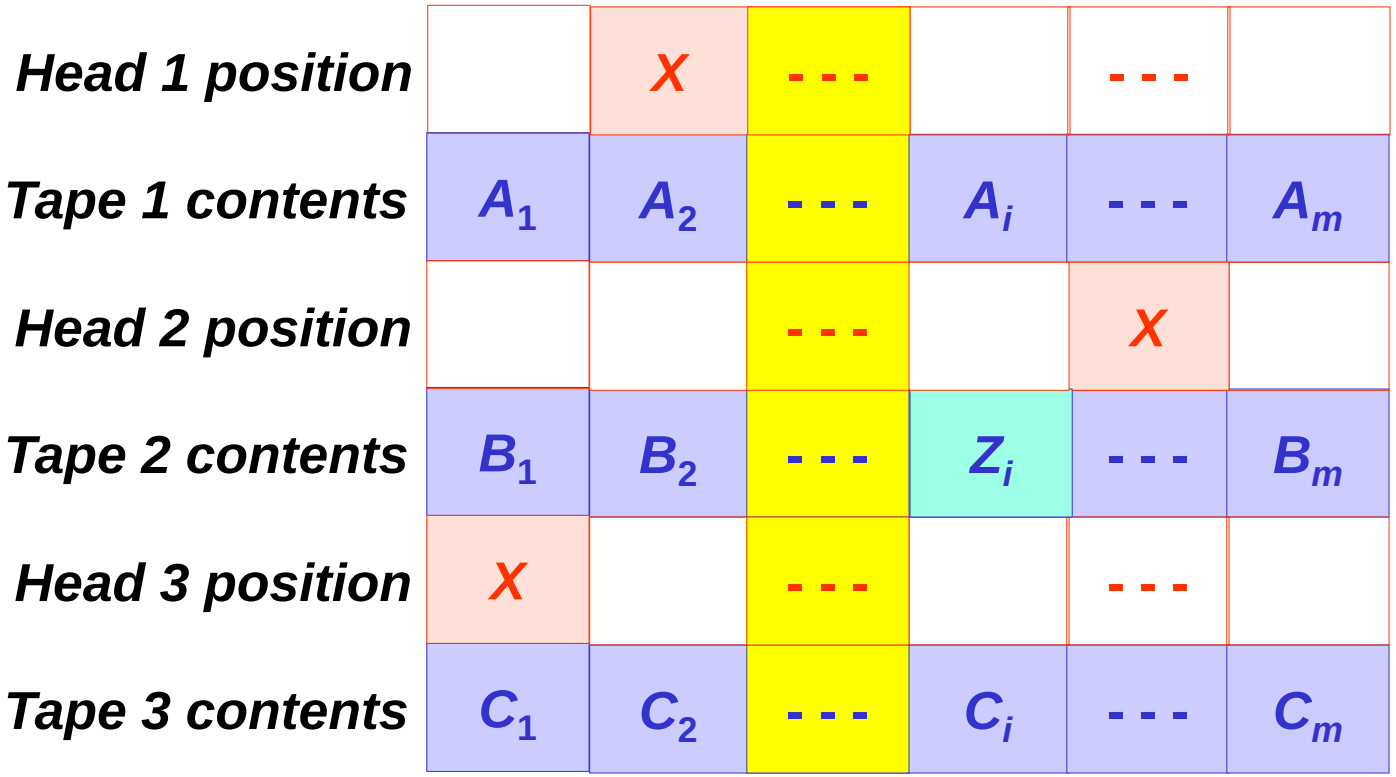
# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**




$$[\ q\ ,\ 2\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$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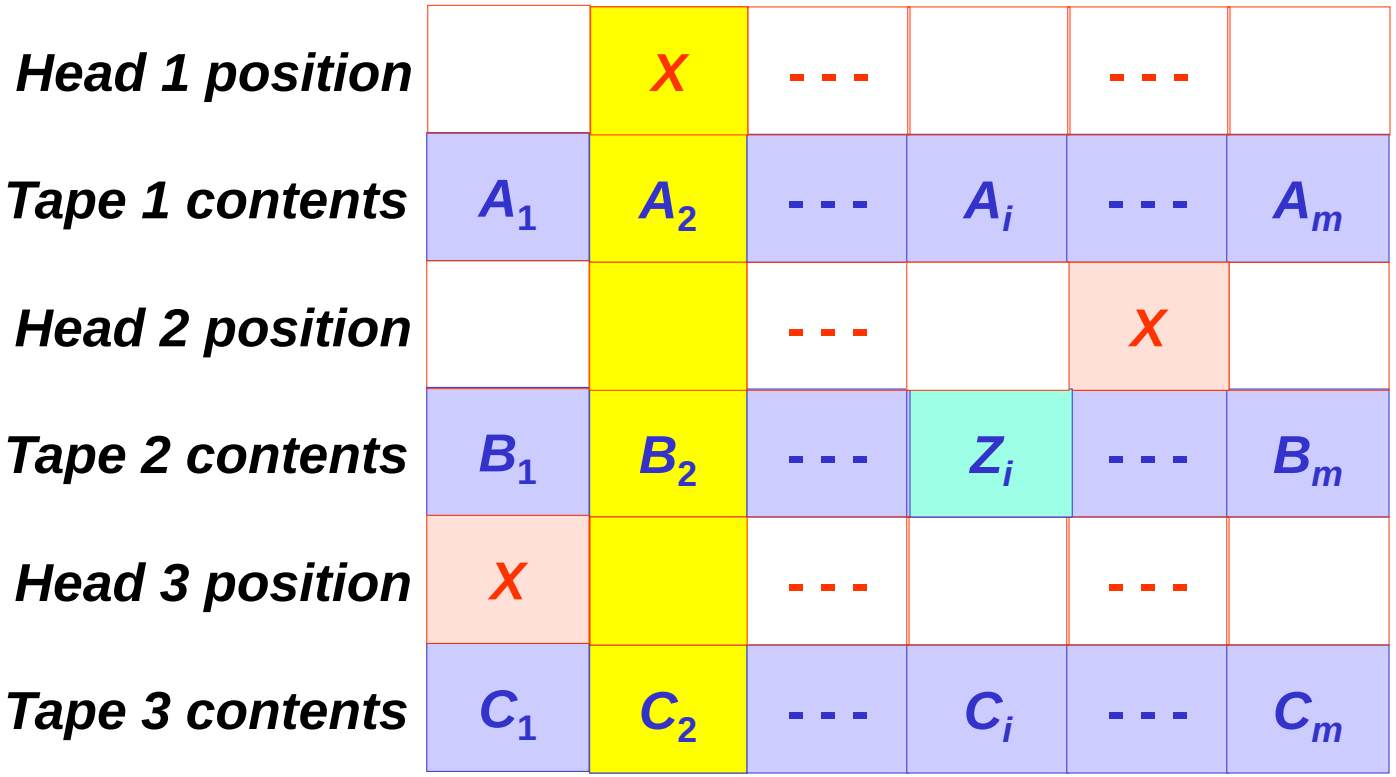
# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**




$$[\ q\ ,\ 2\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$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# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**

$$[\ q\ ,\ 1\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$


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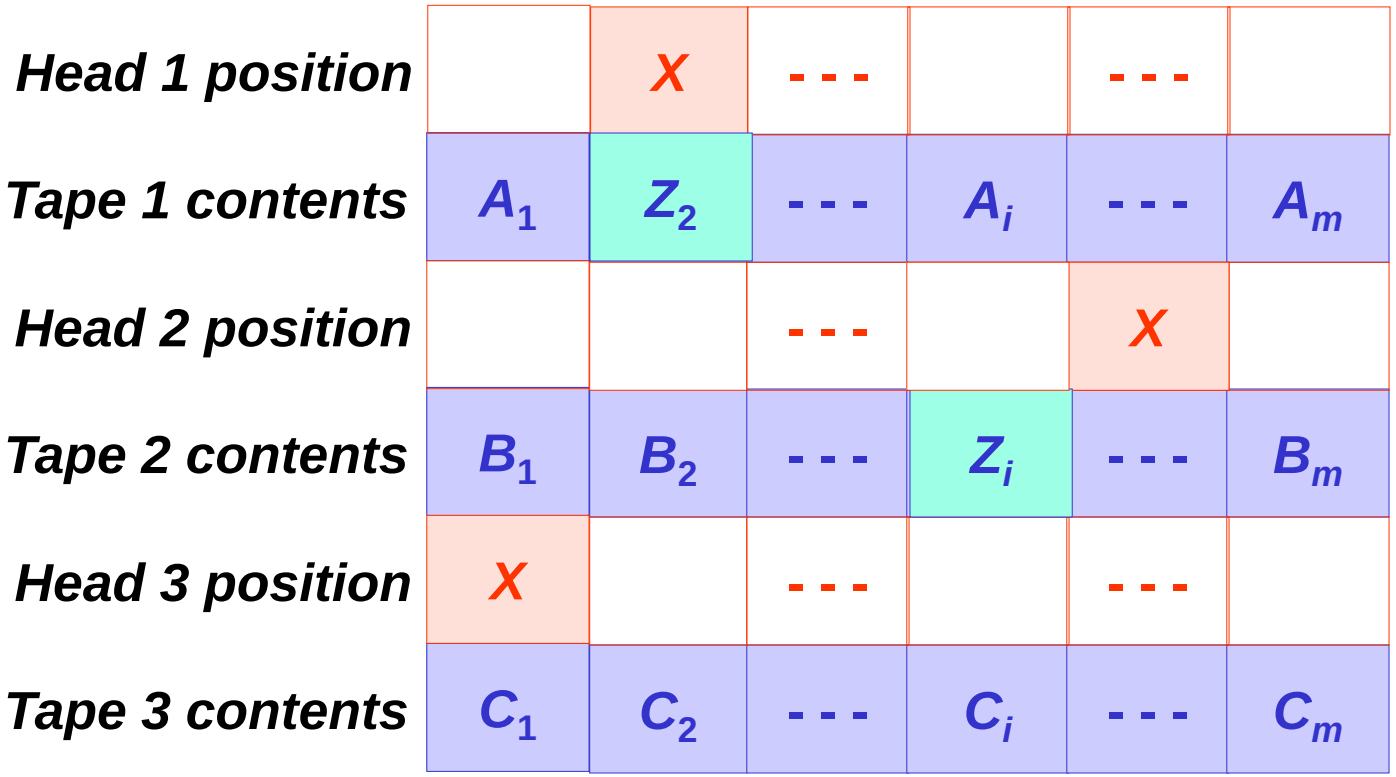
# Multitape TM

**Simulating 3-tape TM $M_1$ using 6 tracks of the single-tape TM $M_2$**




$$[\ q\ ,\ 1\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$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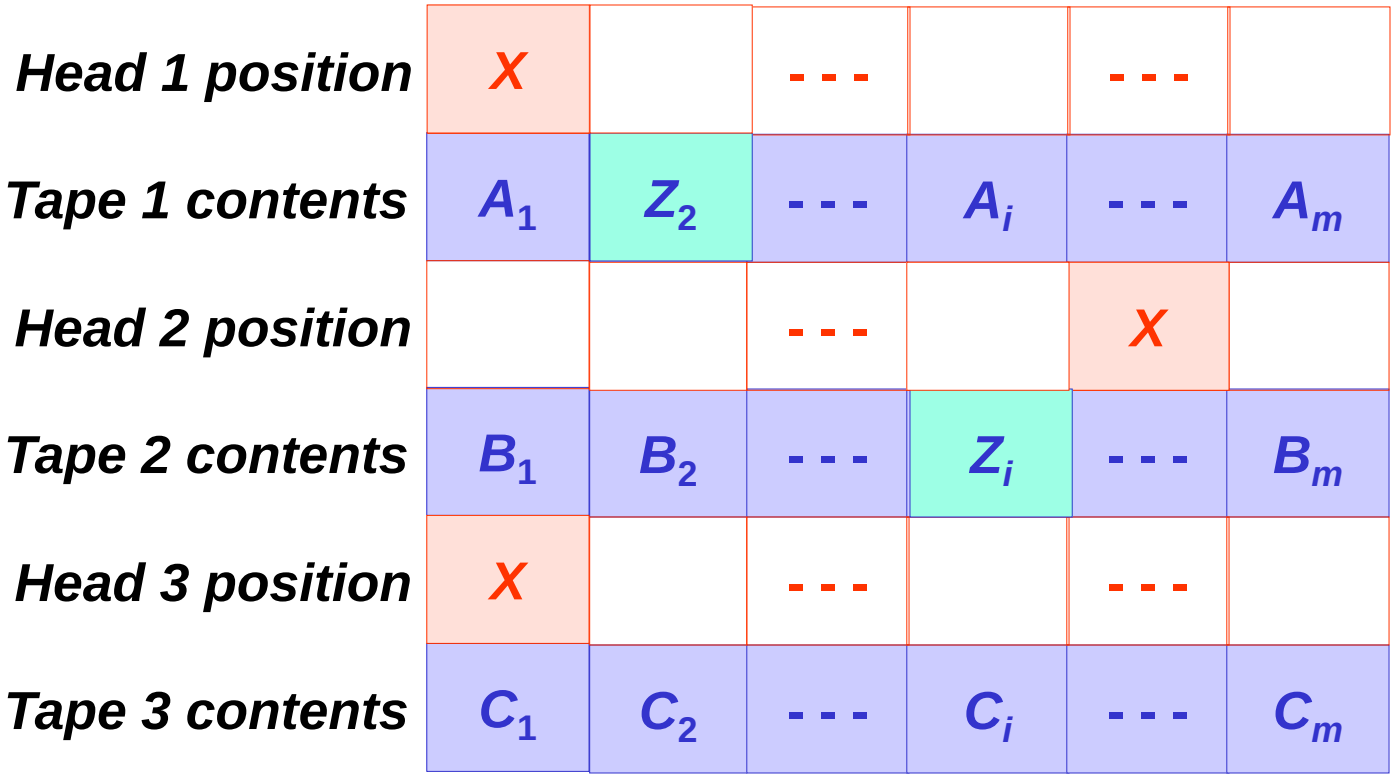


$$[\ q\ ,\ 1\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$


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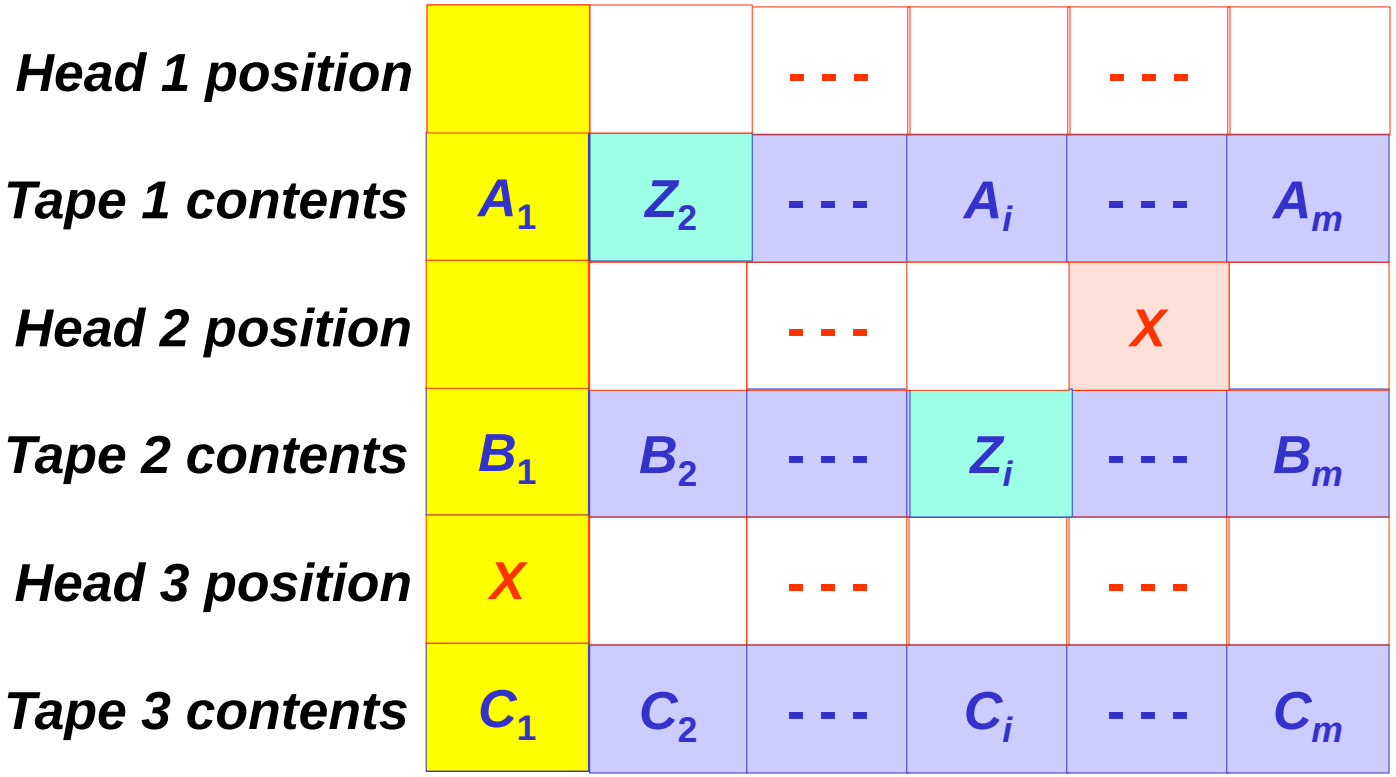


$$[\ q\ ,\ 1\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$


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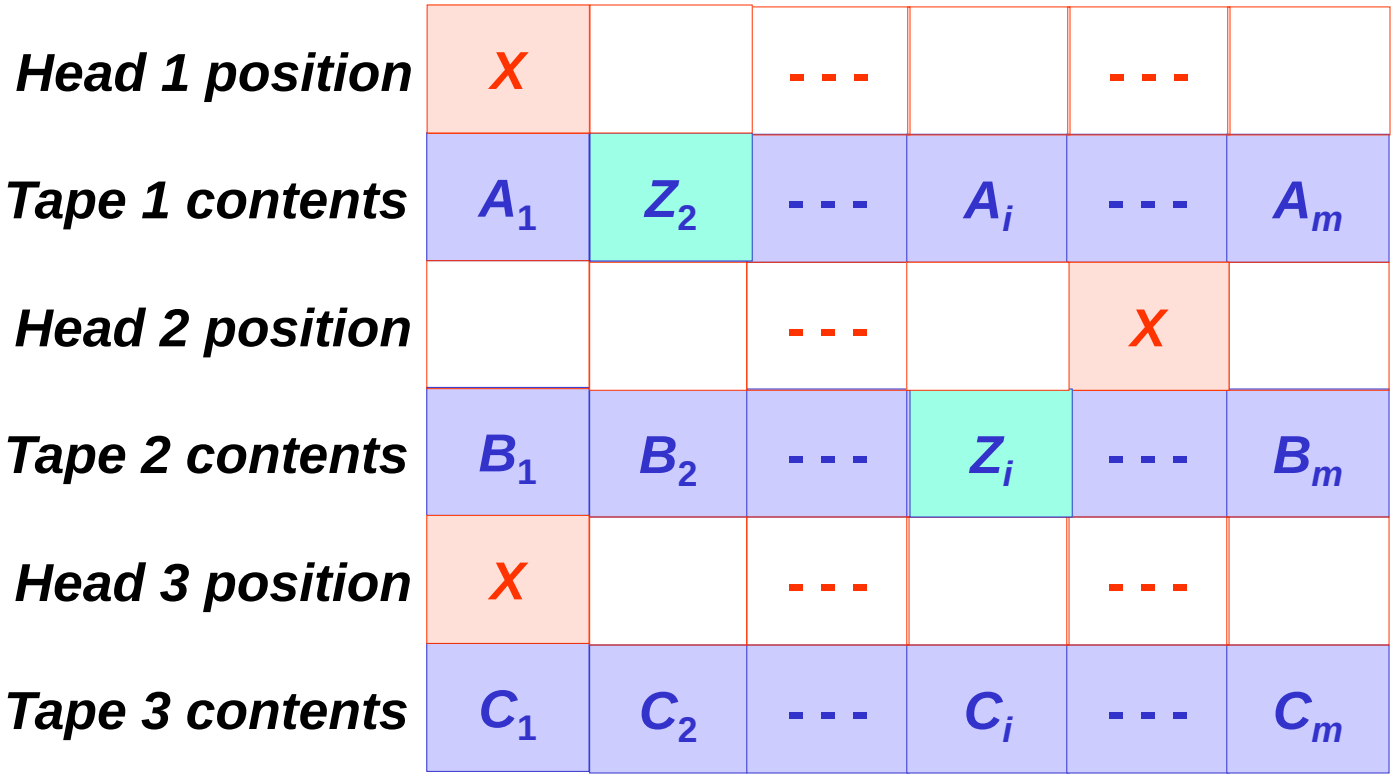


$$[\ q\ ,\ 0\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$


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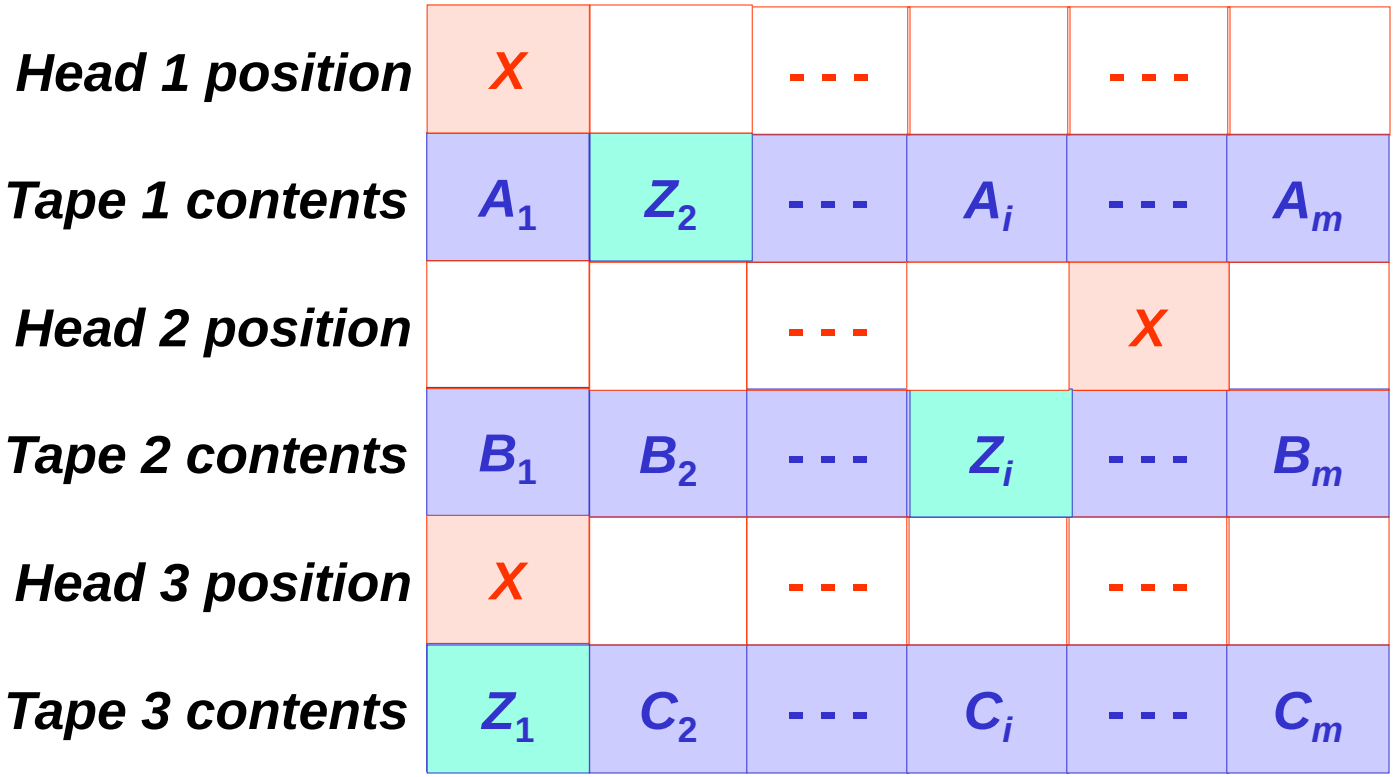


$$[\ q\ ,\ 0\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$


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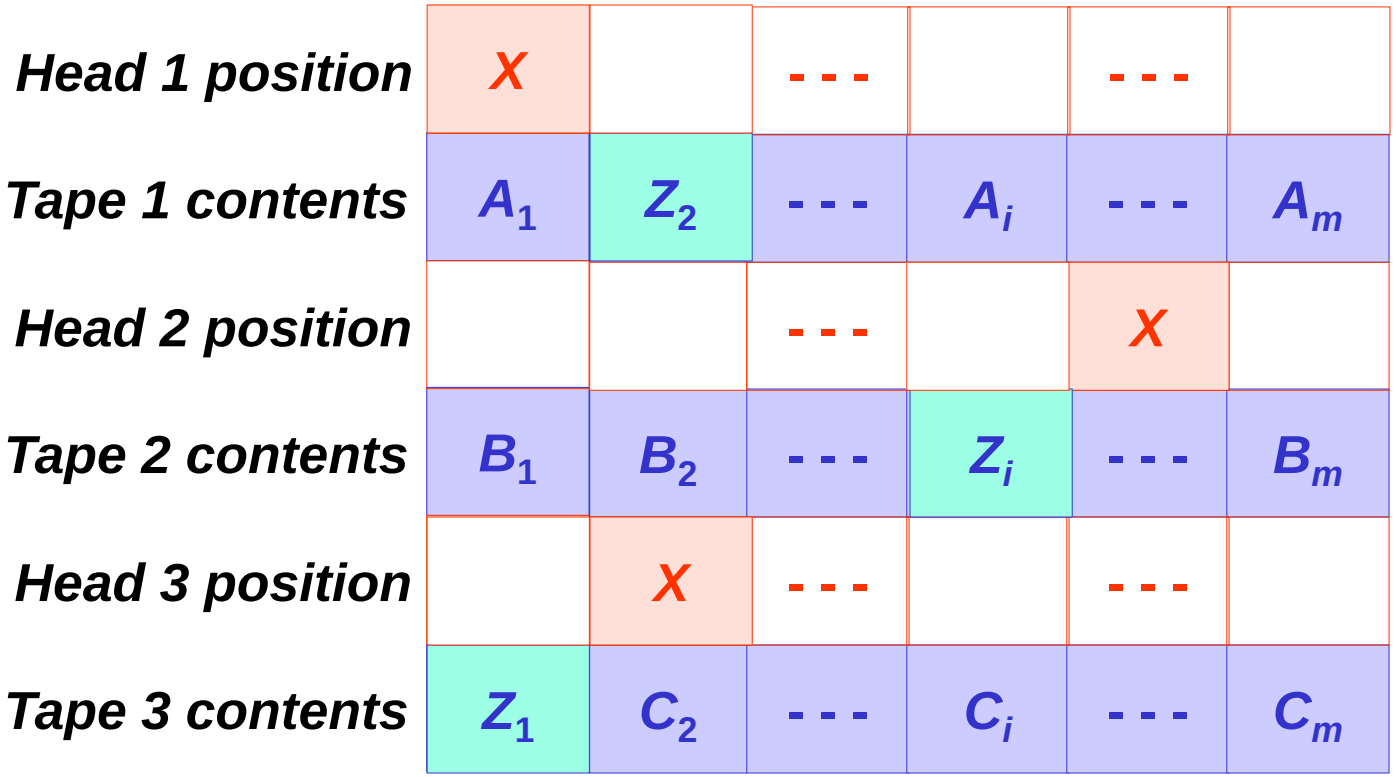


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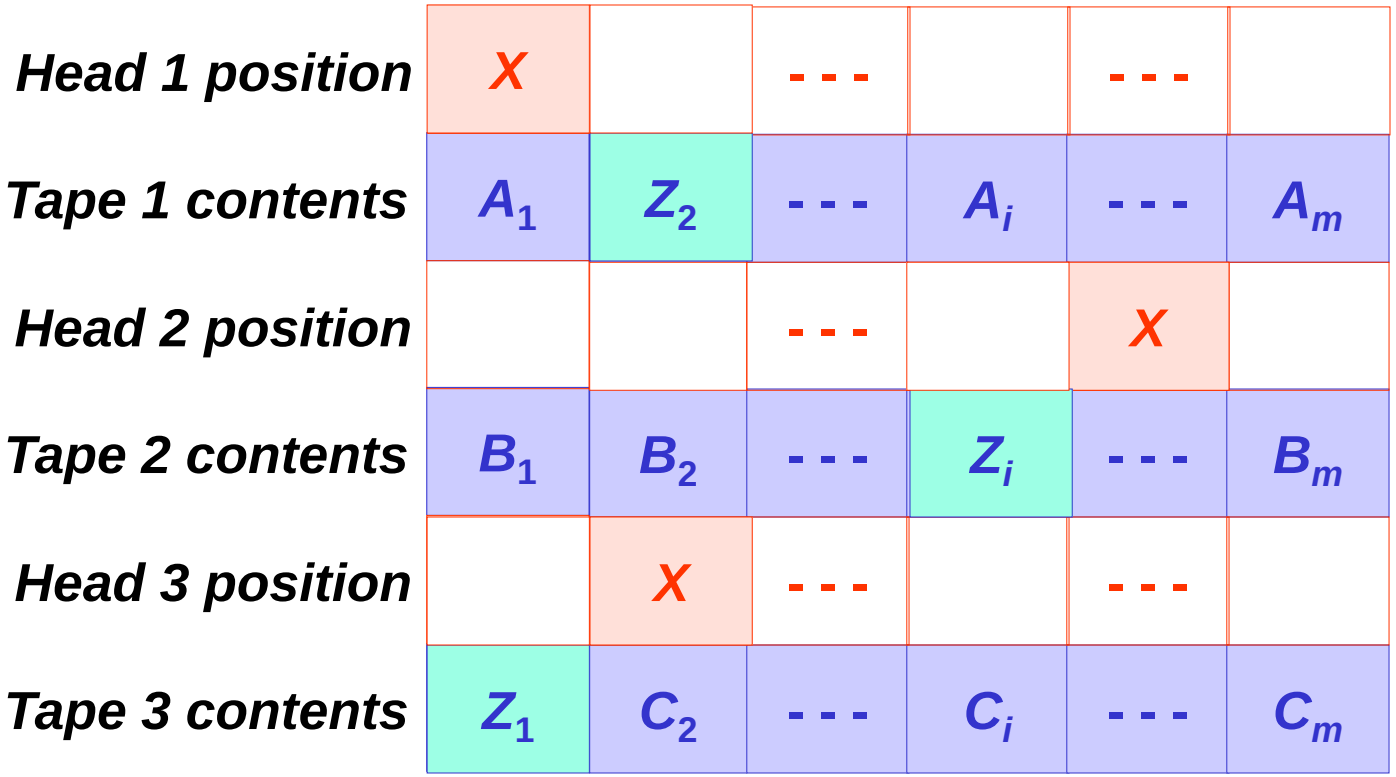


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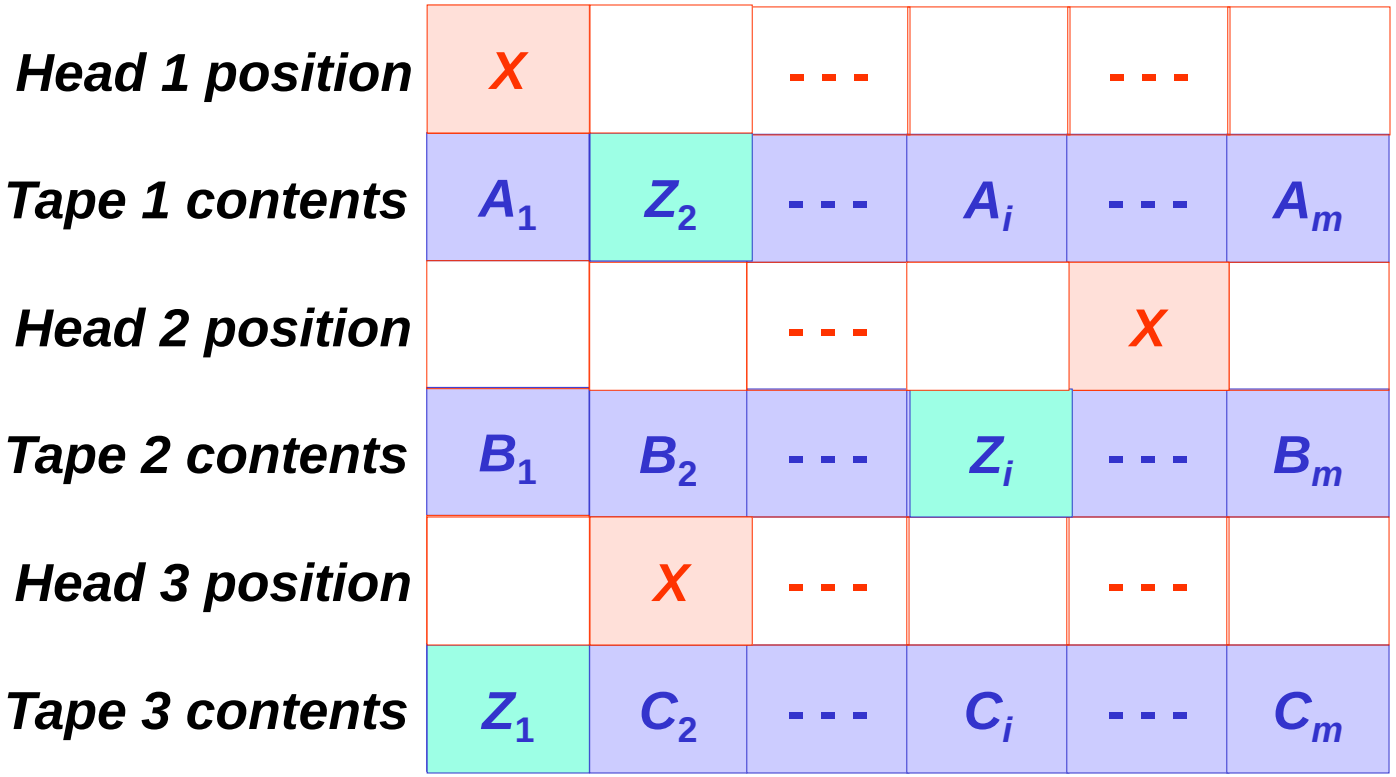


$$[\ q\ ,\ 3\ ,\ A_2\ ,\ B_i\ ,\ C_1\ ]$$


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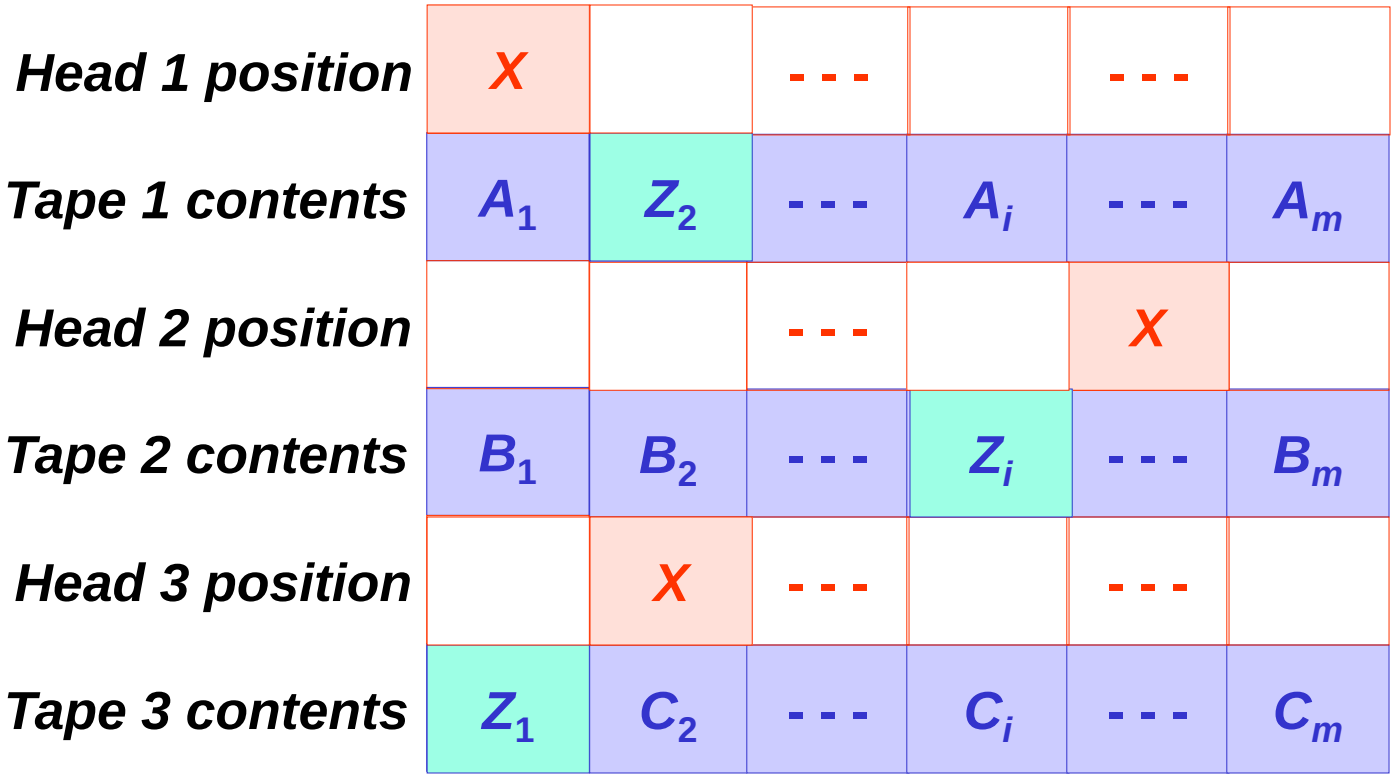


$$[\ q\ ,\ 3\ ,\ B\ ,\ B\ ,\ B\ ]$$


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# Multitape TM


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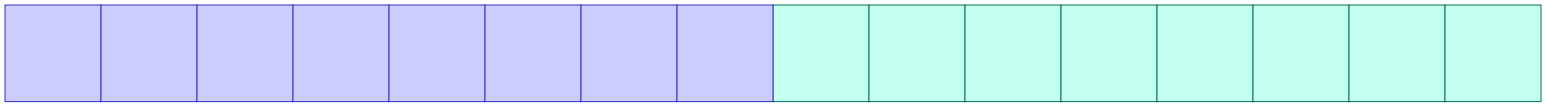


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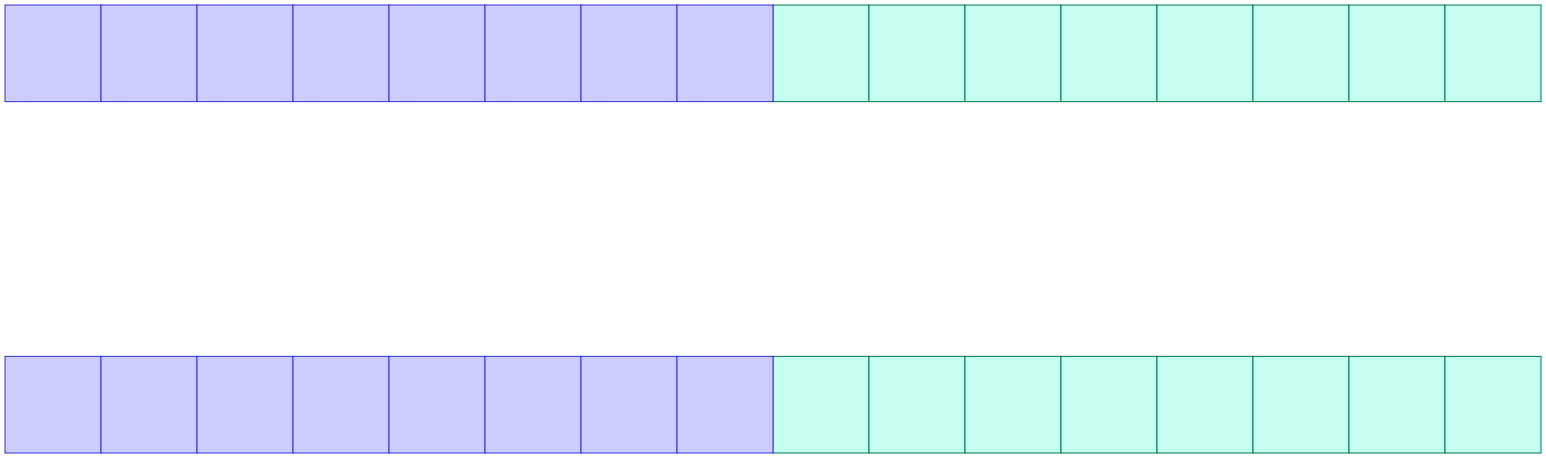


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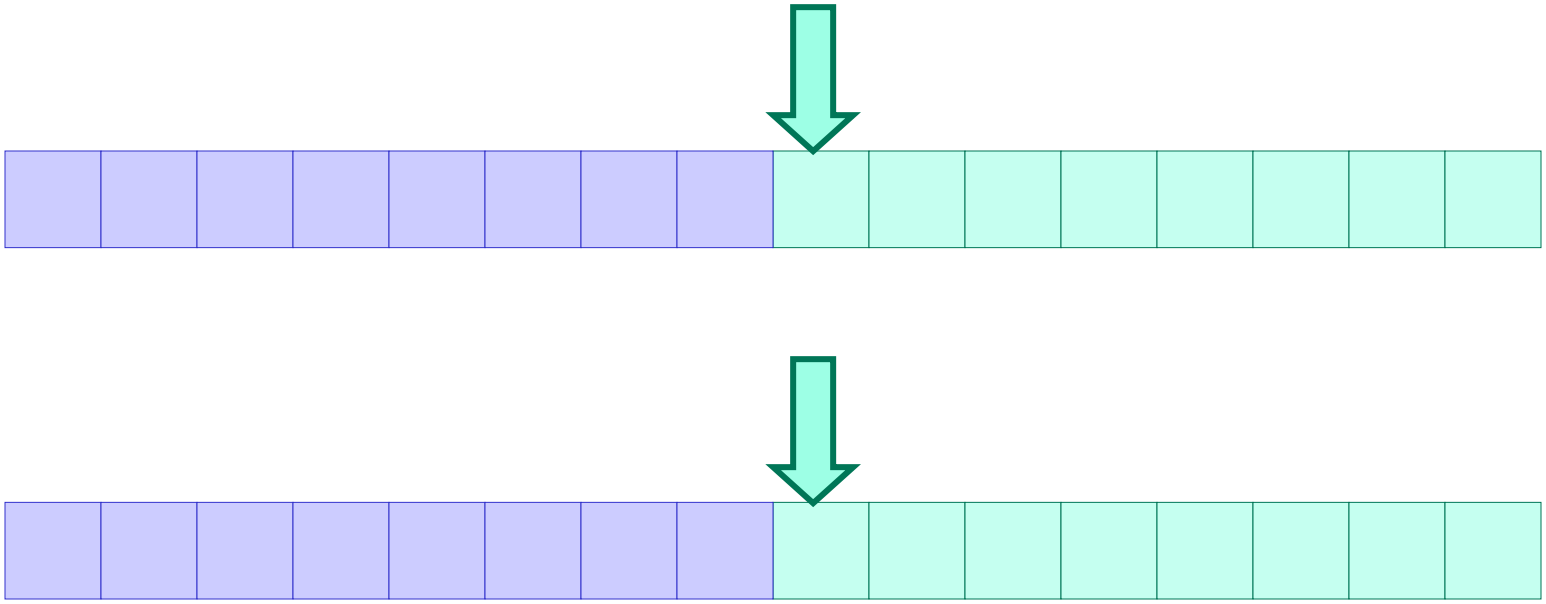

**Initial step**

**Distance: 0**


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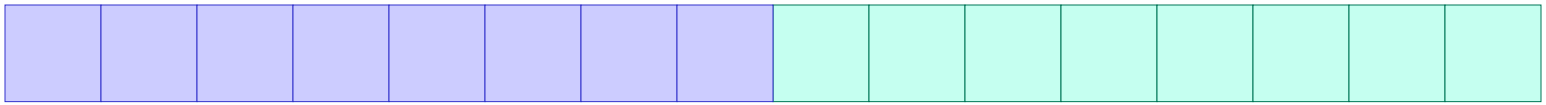
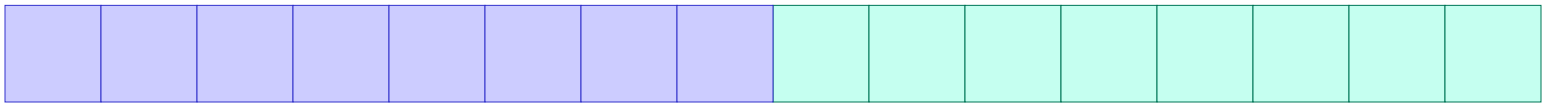
# Multitape TM





**Initial step**

**Distance: 0**

**Step *i***



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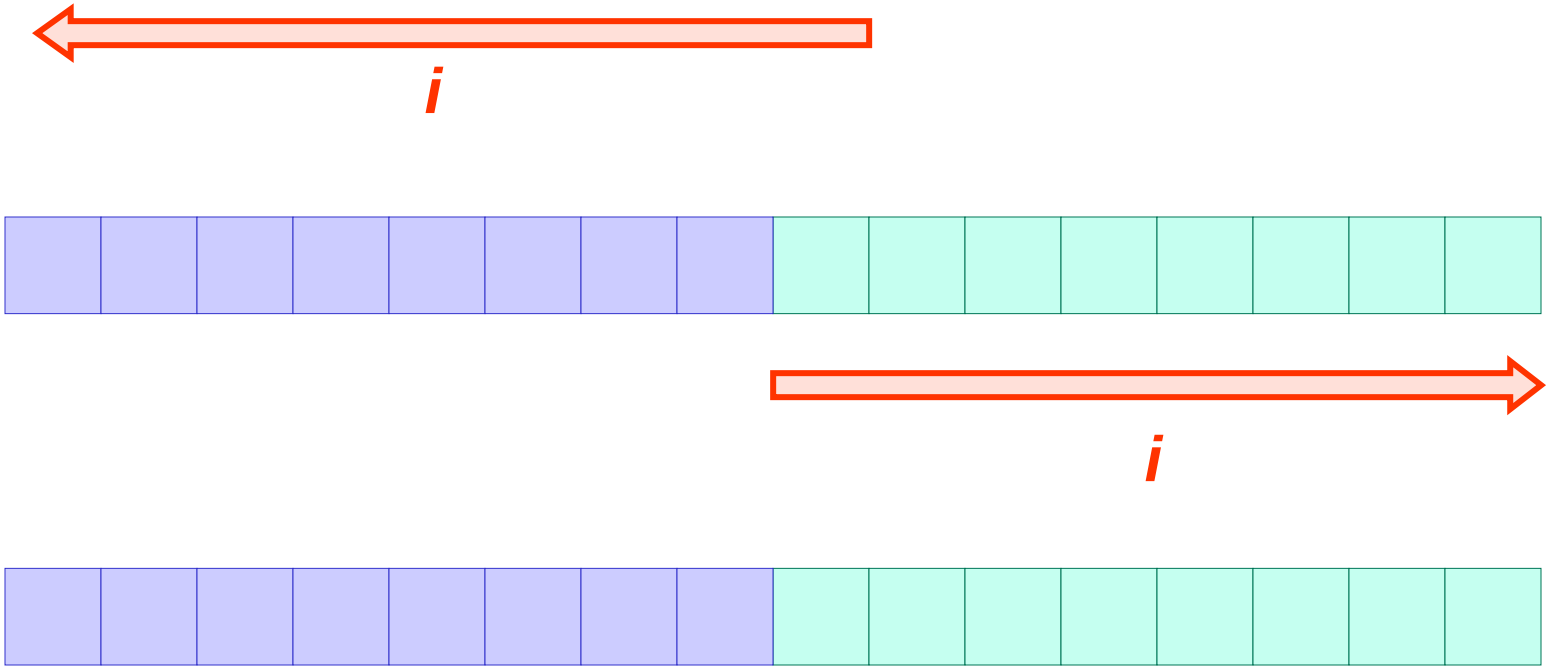
# Multitape TM




**Initial step**

**Distance: 0**

**Step *i***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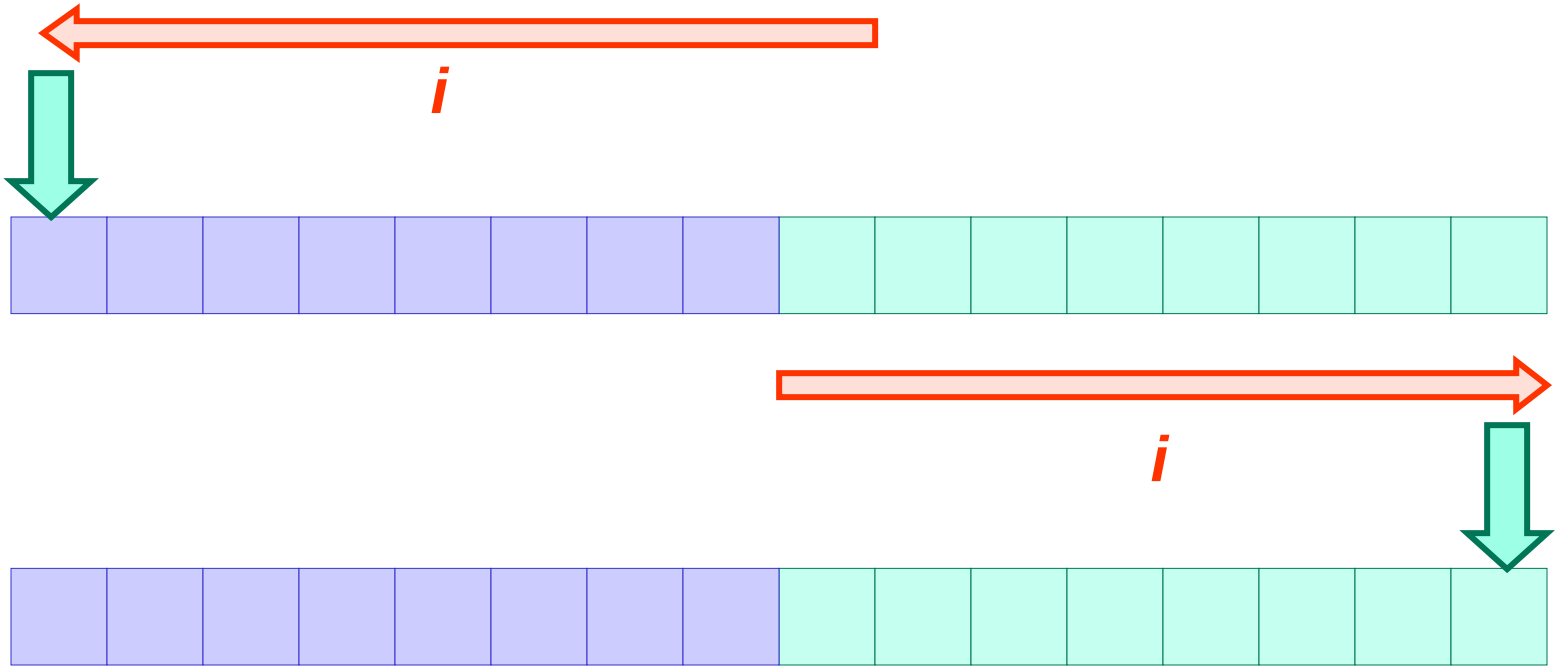
# Multitape TM




**Initial step**

**Distance: 0**

**Step *i***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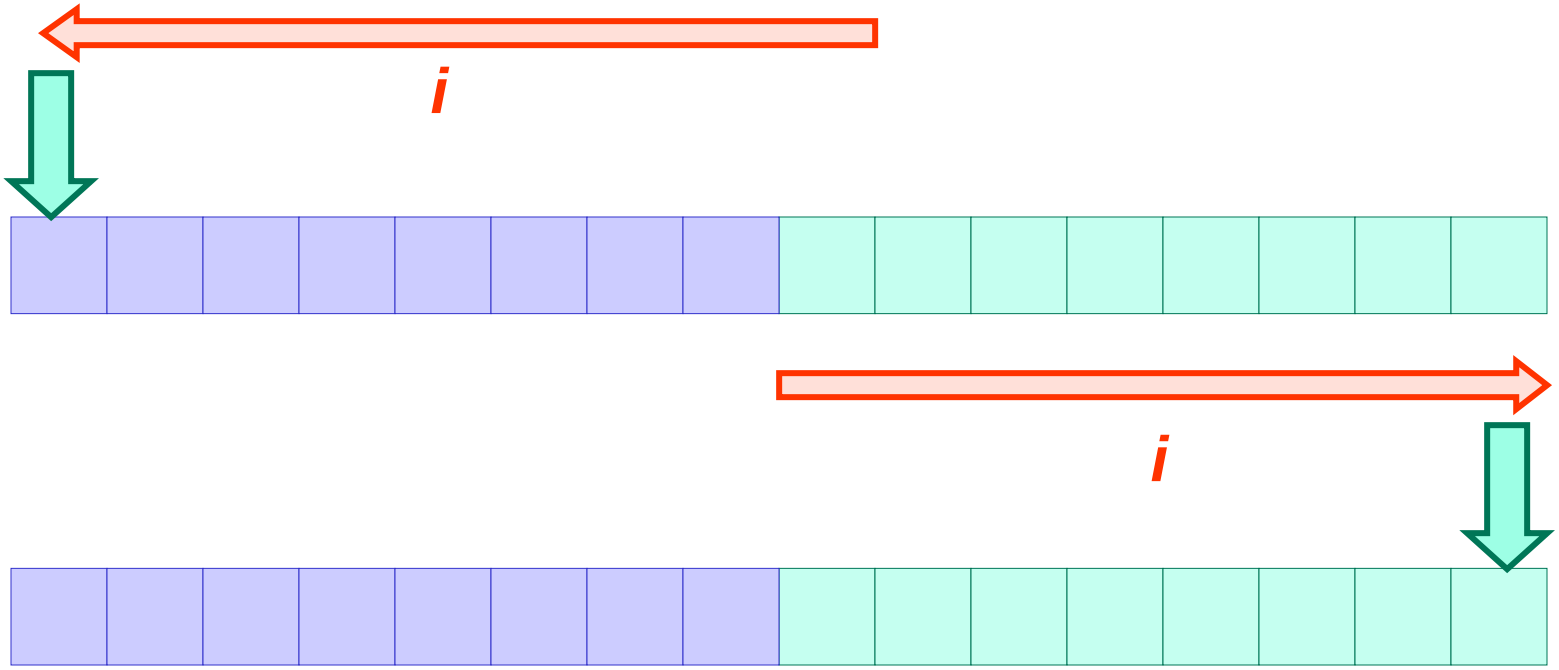
# Multitape TM




**Initial step**
**Distance: 0**

**Step *i***
**Distance: 2*i***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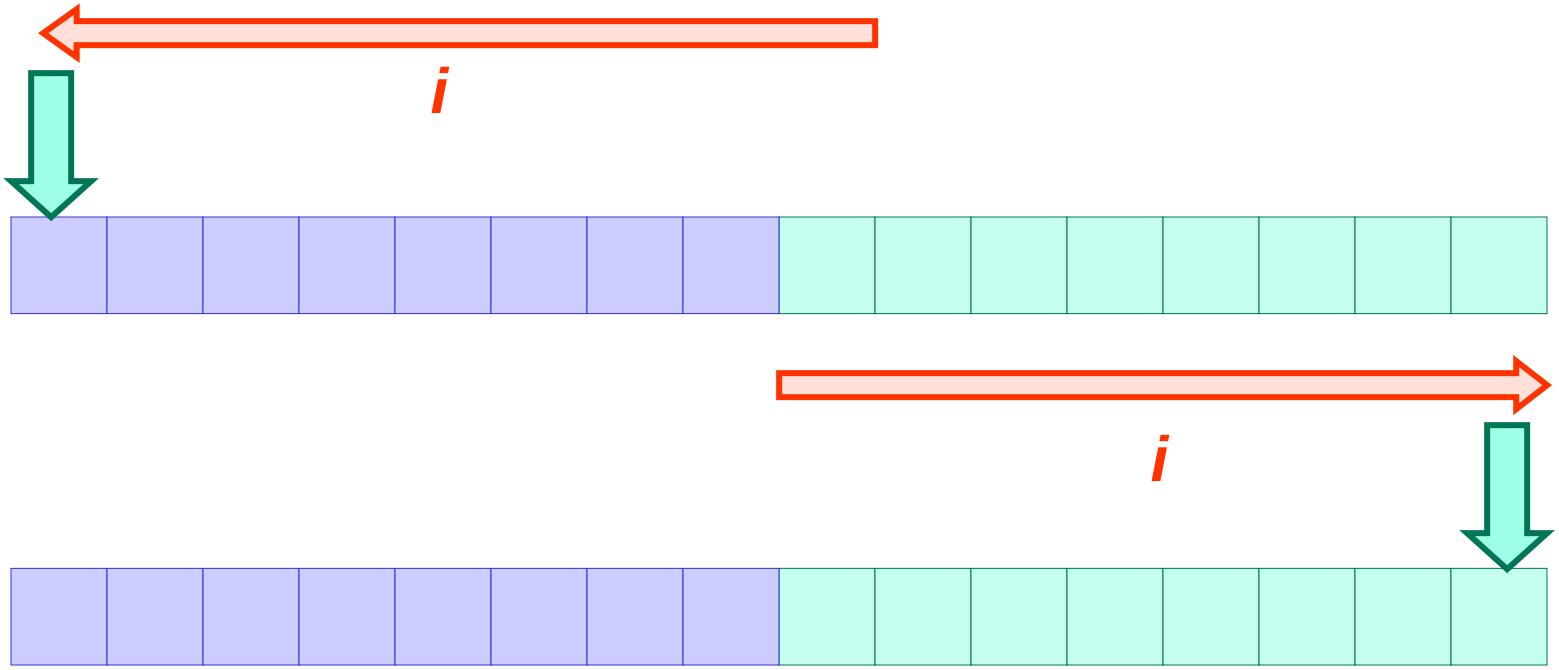
# Multitape TM




**Initial step**
**Distance: 0**

**Step *i***
**Distance: 2*i***

**Simulating single transition: min. 4*i* head movements**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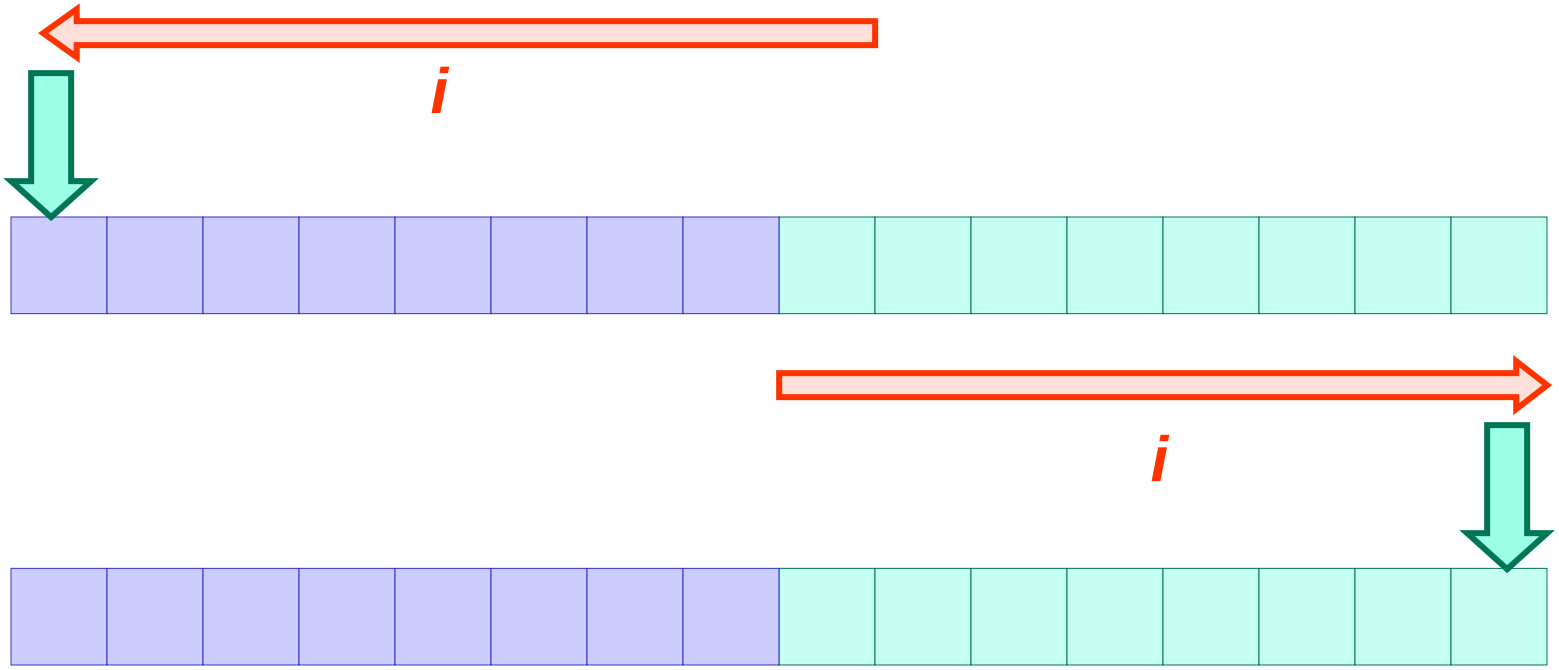
# Multitape TM




**Initial step**
**Distance: 0**

**Step *i***
**Distance: 2*i***

**Simulating single transition: min. 4*i* head movements**

***m* head movements**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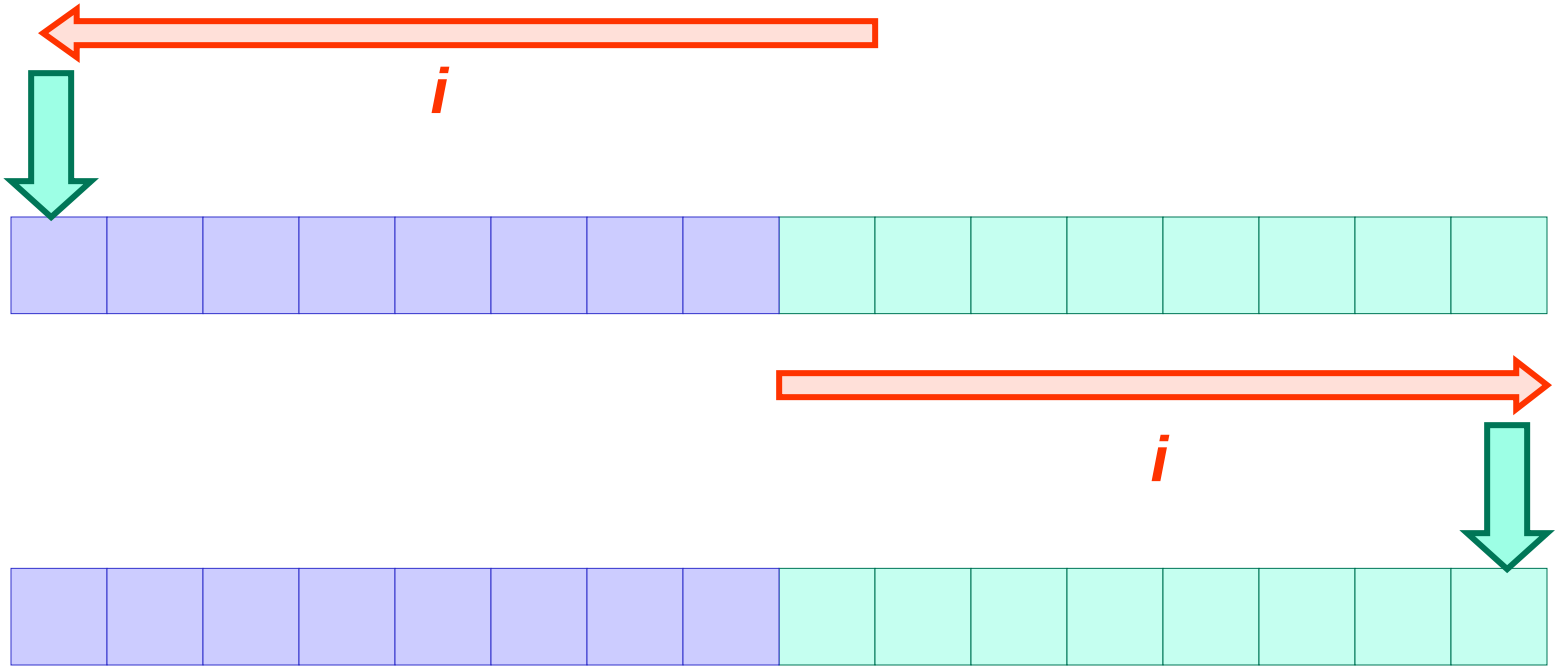
# Multitape TM




**Initial step**
**Distance: 0**

**Step *i***
**Distance: 2*i***

**Simulating single transition: min. 4*i* head movements**

***m* head movements** $\sum_{i=1}^{m} 4i \approx 2m^2$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# Multitrack TM


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# Multitrack TM

$$\{ ww^R \mid w \in ( a+b )^* \}$$


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# Multitrack TM

$$\{ ww^R \mid w \in ( a+b )^* \}$$

*Input tape*

| a | a | b | a | a | a | b | b |
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# Multitrack TM

$$\{ ww^R \mid w \in ( a+b )^* \}$$

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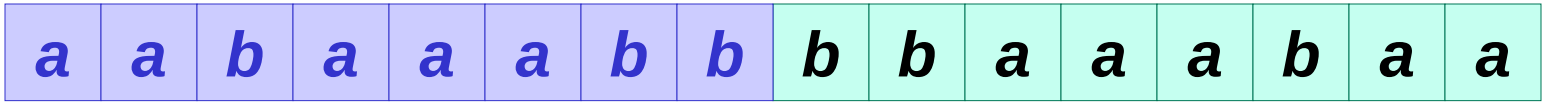



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# Multitrack TM

$$\{ ww^R \mid w \in ( a+b )^* \}$$

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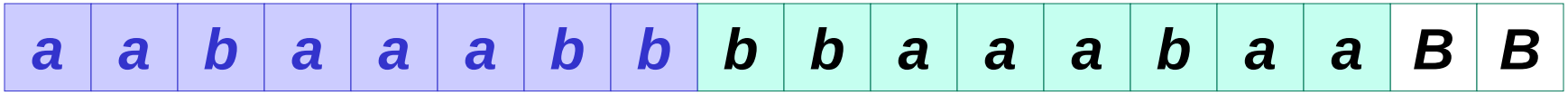



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# Multitrack TM

$$\{ ww^R \mid w \in ( a+b )^* \}$$

*Input tape*

| a | a | b | a | a | a | b | b | b | b | a | a | a | b | a | a | B | B |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B |


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# Multitrack TM

$$\{ ww^R \mid w \in ( a+b )^* \}$$

*Input tape*

| a | a | b | a | a | a | b | b | b | b | a | a | a | b | a | a | B | B |
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# Multitrack TM

$$\{ ww^R \mid w \in ( a+b )^* \}$$

*Input tape*

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# Multitrack TM

$$\{ ww^R \mid w \in ( a+b )^* \}$$

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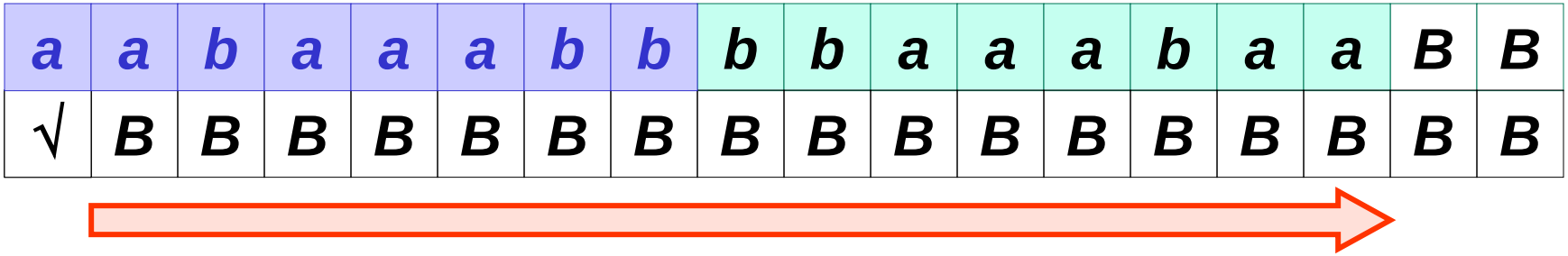



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$$\{ ww^R \mid w \in ( a+b )^* \}$$

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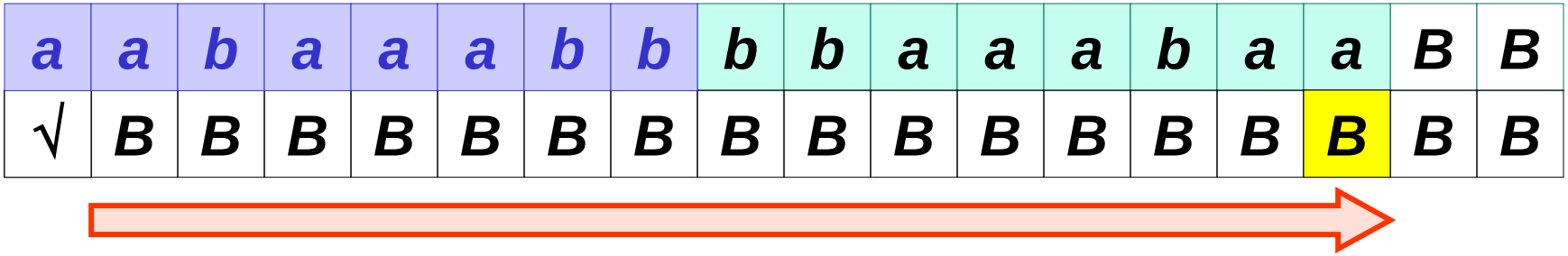



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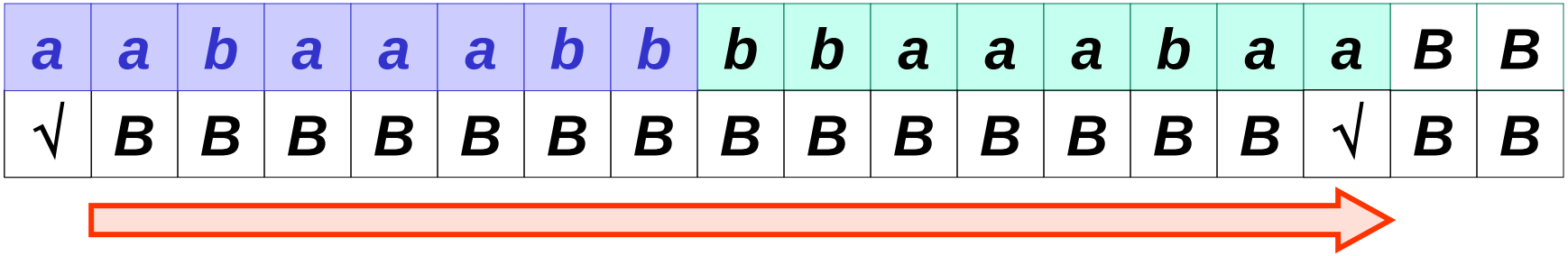



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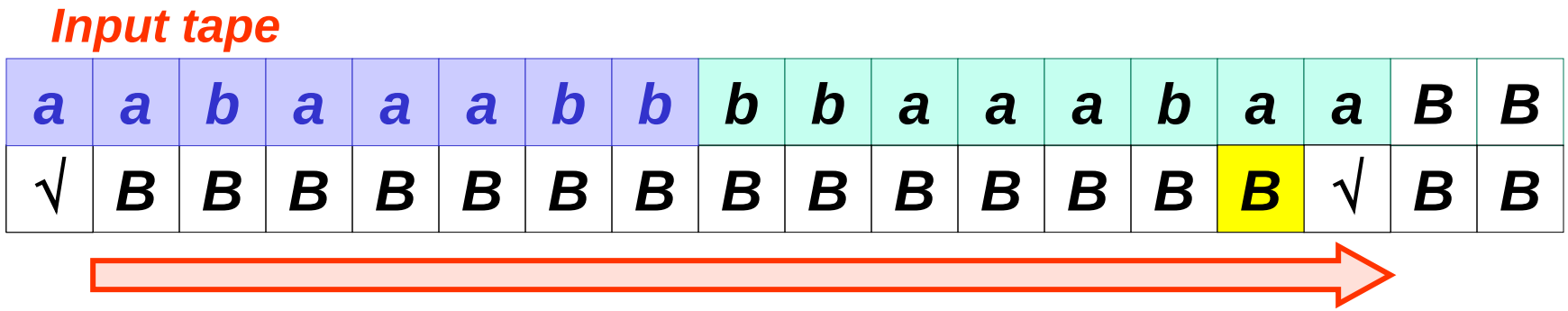



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# Multitrack TM

$$\{ ww^R \mid w \in ( a+b )^* \}$$

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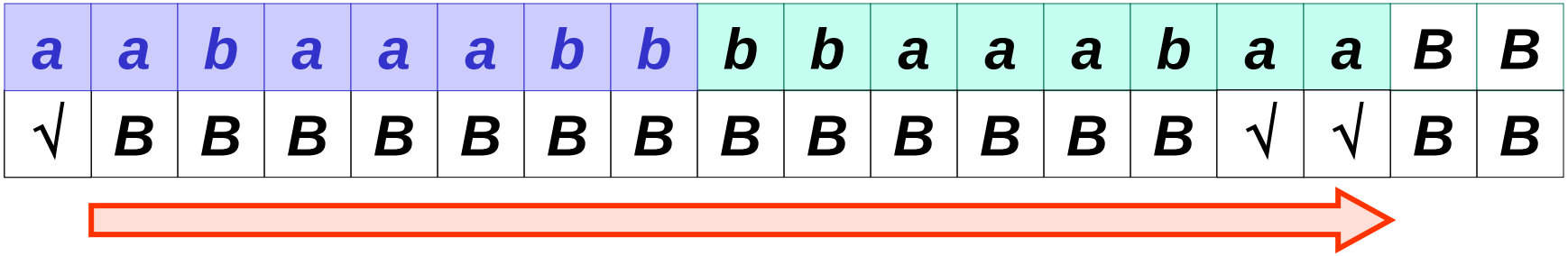



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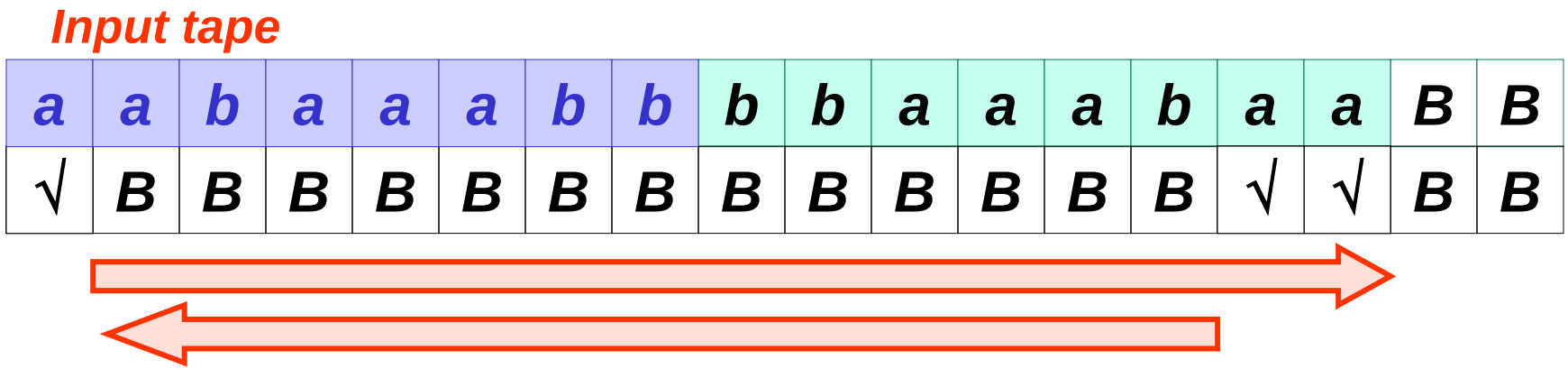



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$$\{ ww^R \mid w \in ( a+b )^* \}$$

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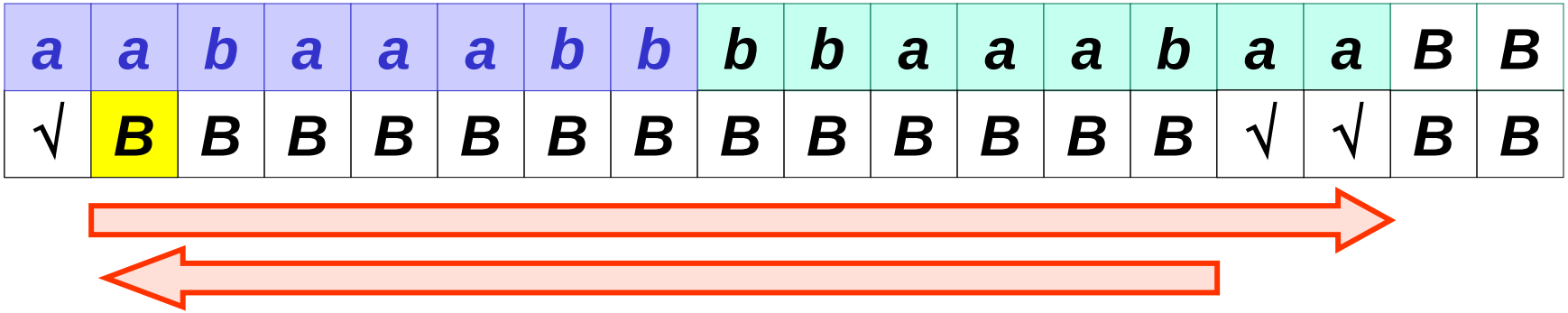



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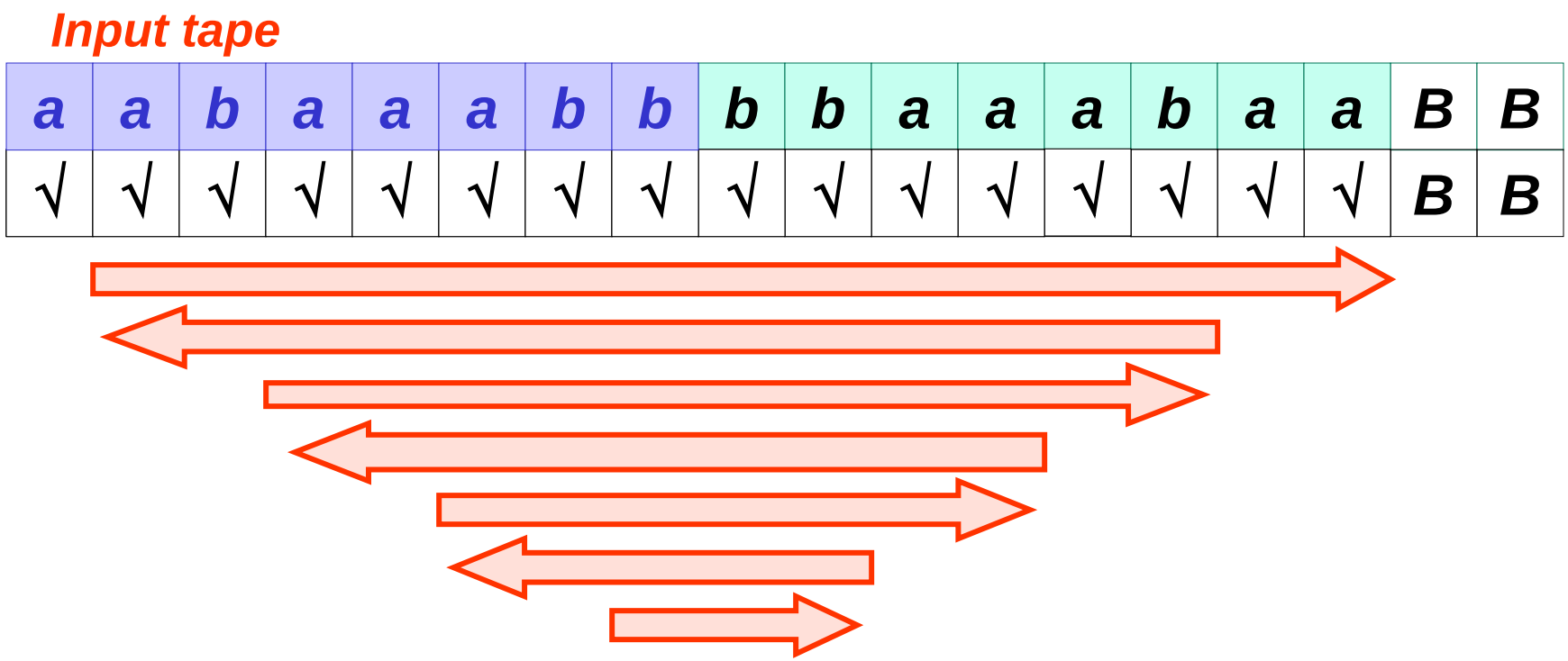



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# Multitape TM

$$\{ ww^R \mid w \in ( a+b )^* \}$$


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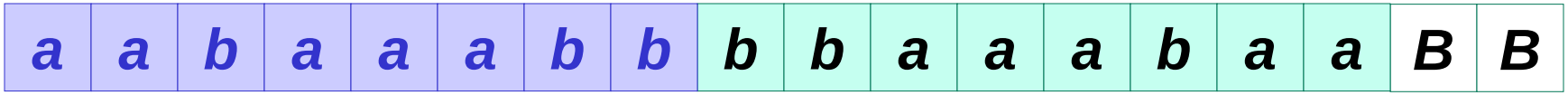


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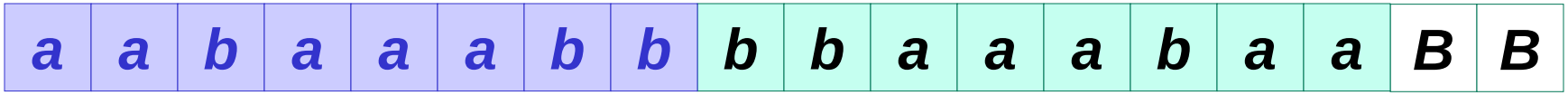


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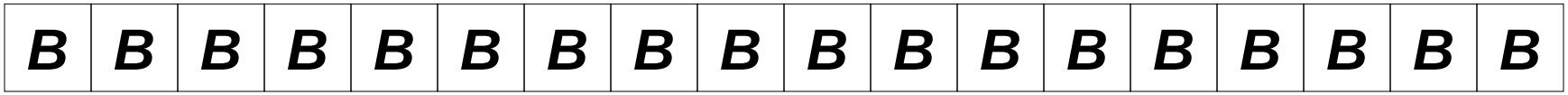


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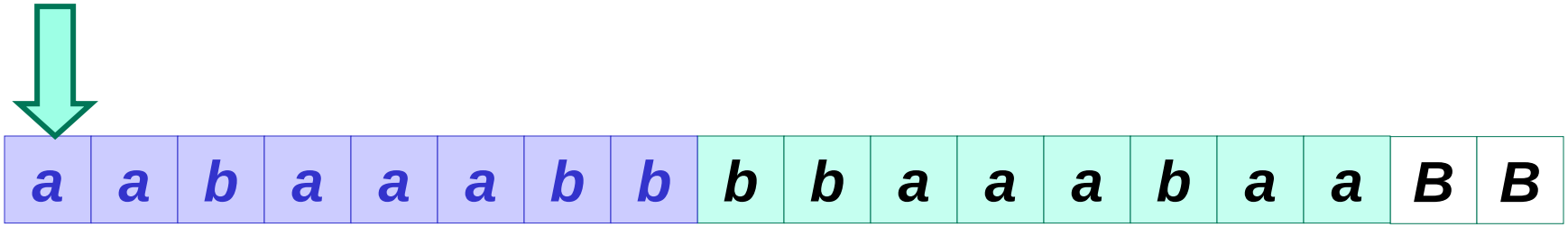


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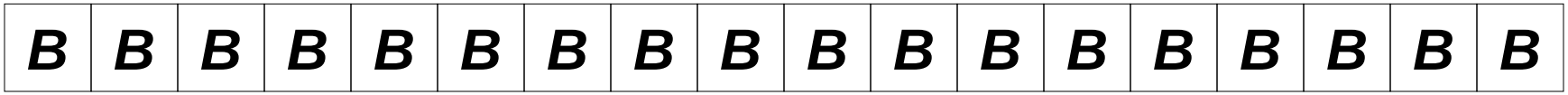


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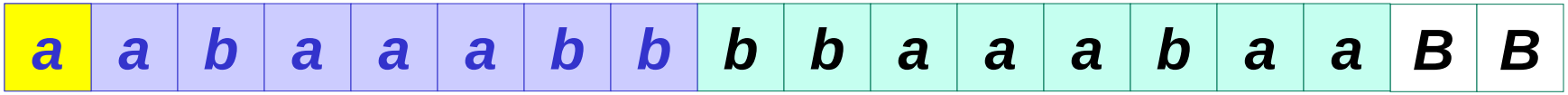


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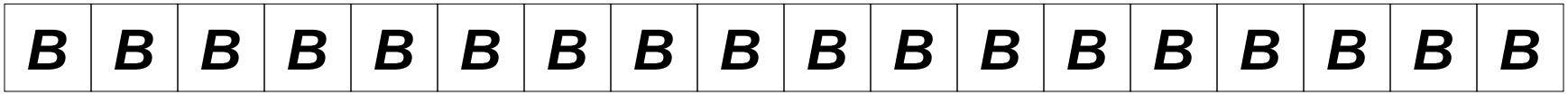


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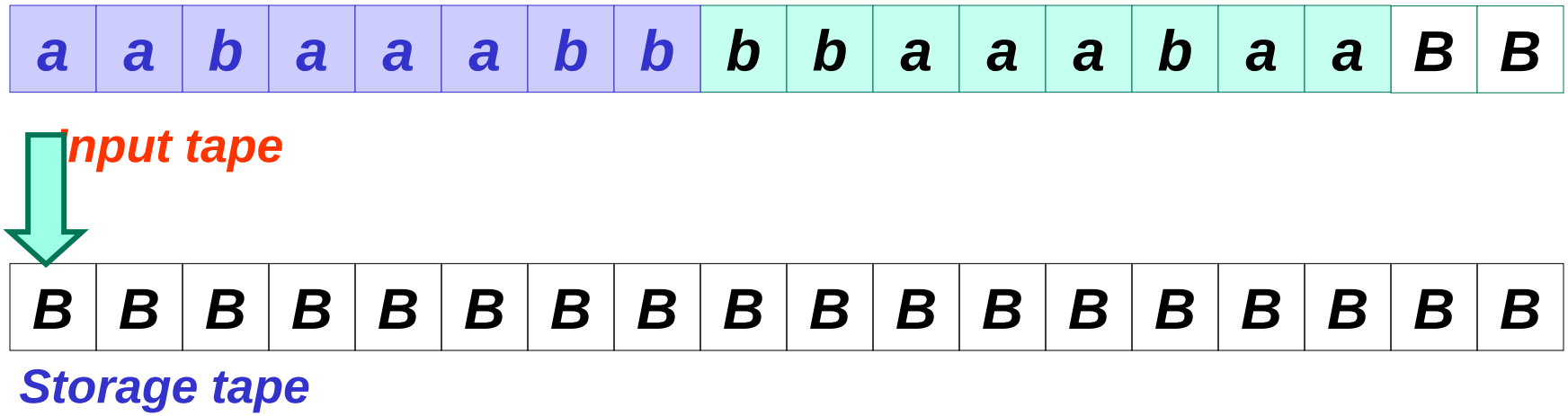



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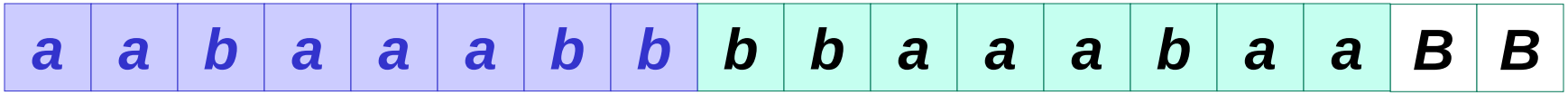


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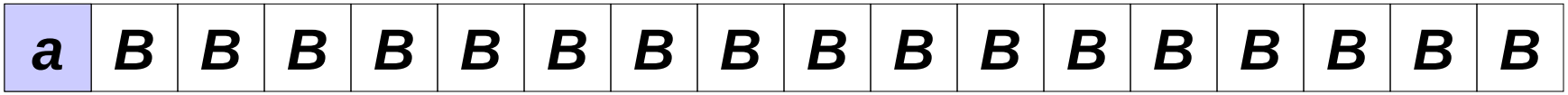


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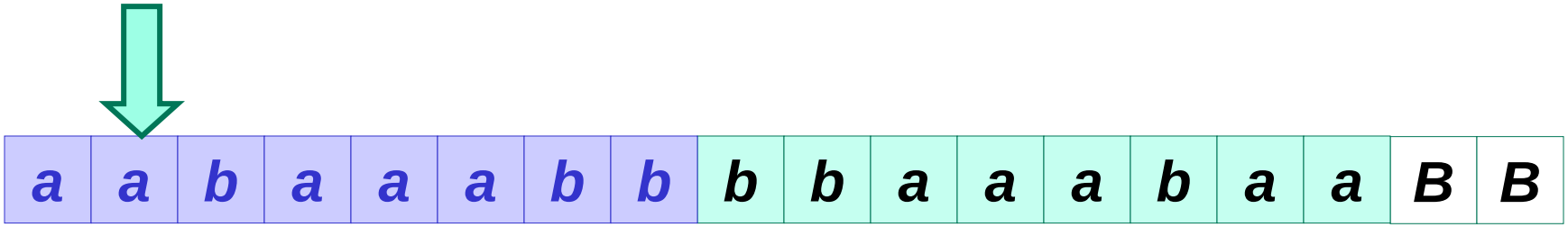


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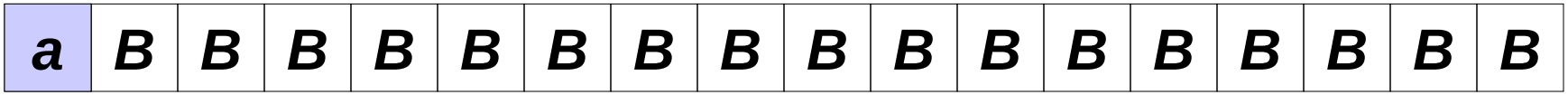


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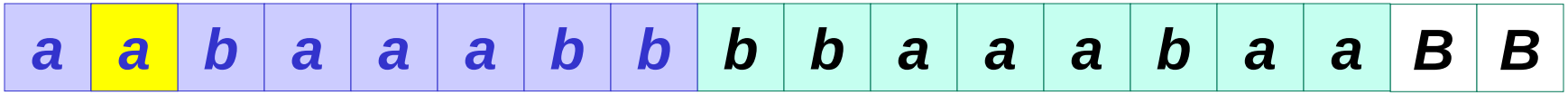


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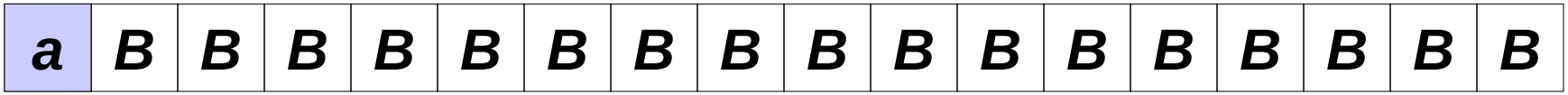


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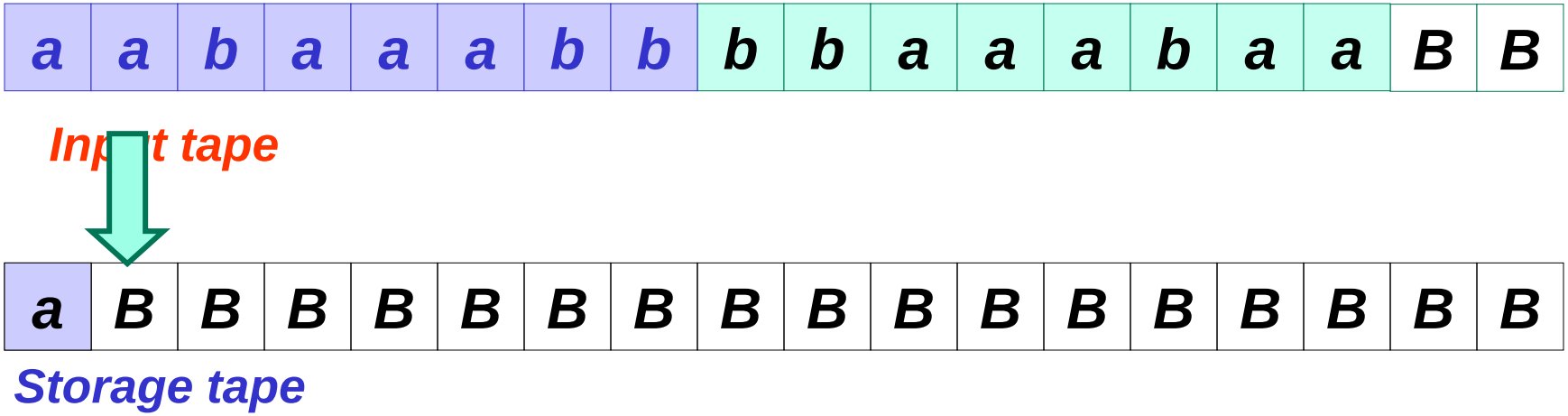
# Multitape TM

$\{ ww^R \mid w \in ( a+b )^* \}$





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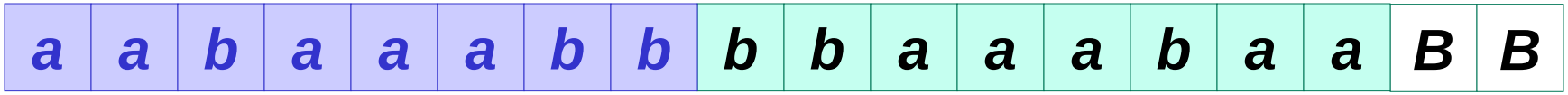
# Multitape TM

$$\{ ww^R \mid w \in ( a+b )^* \}$$

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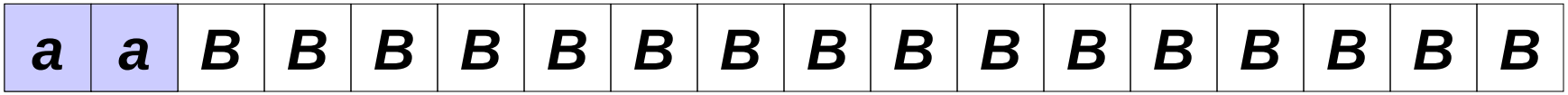


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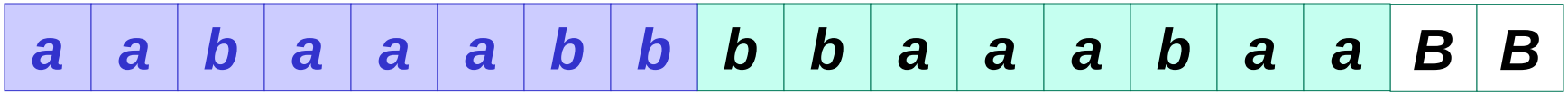
# Multitape TM

$\{ ww^R \mid w \in ( a+b )^* \}$

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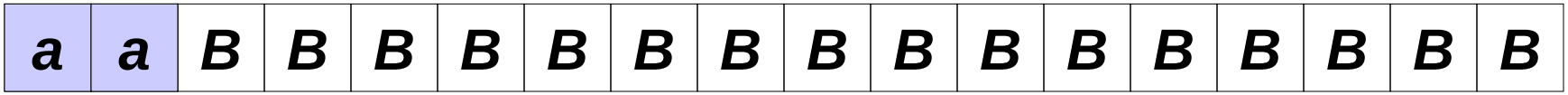


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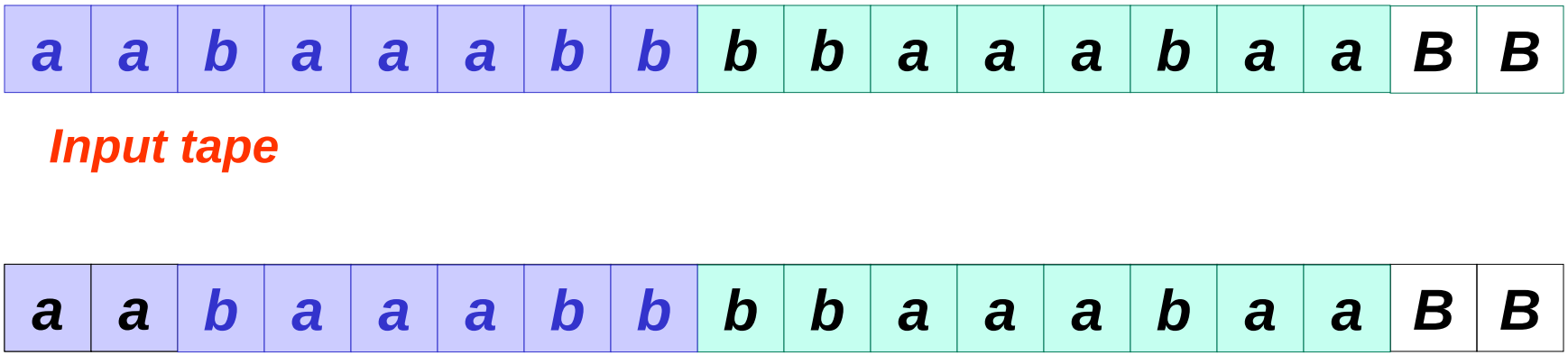
$\{ ww^R \mid w \in ( a+b )^* \}$





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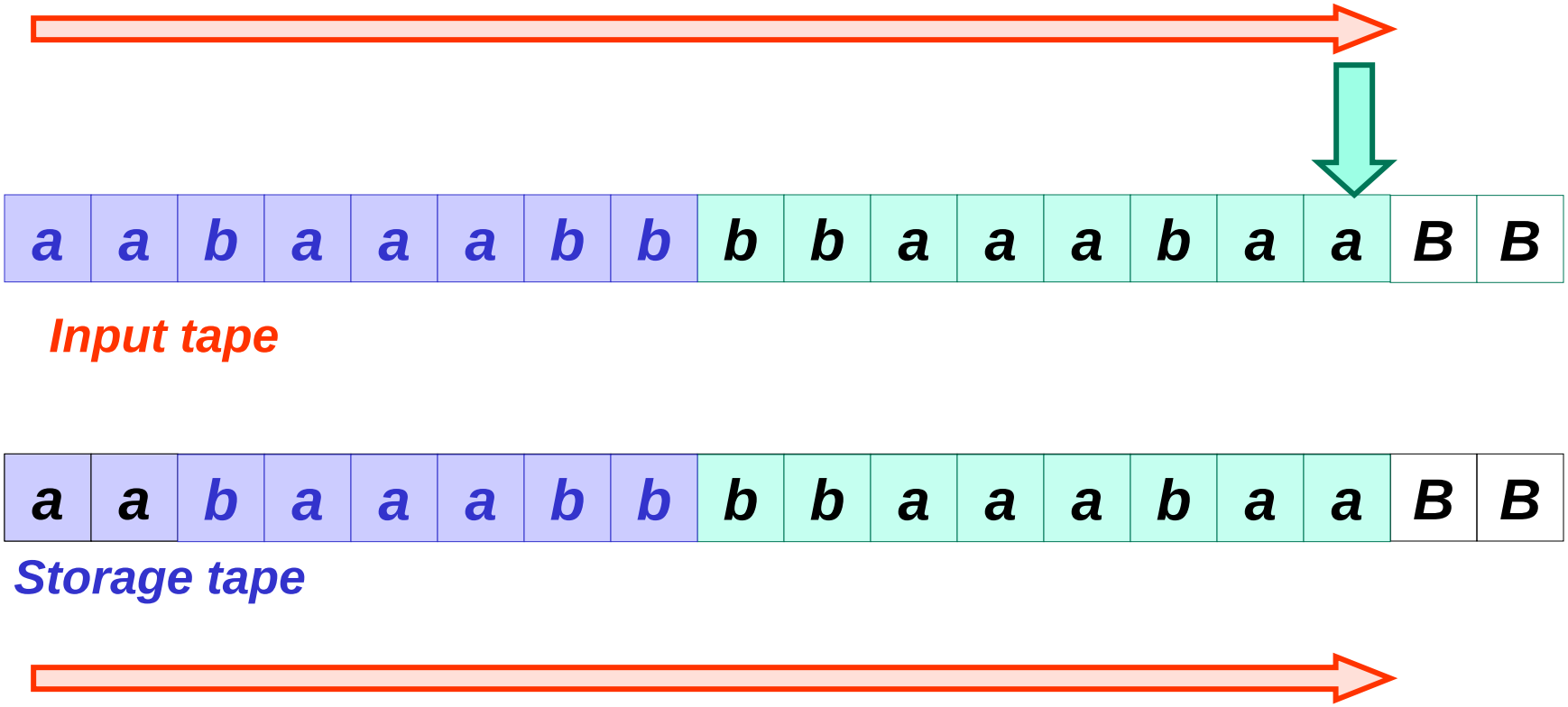



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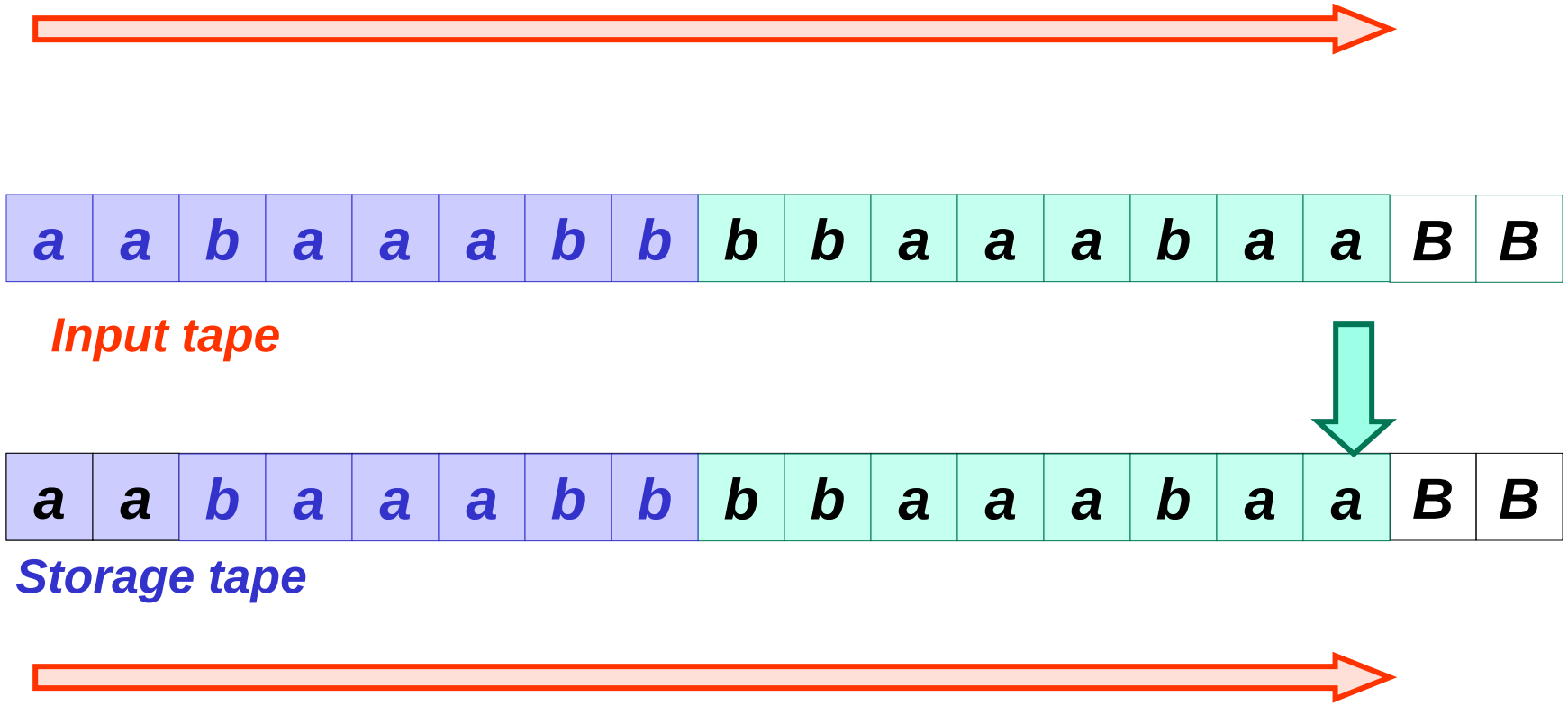



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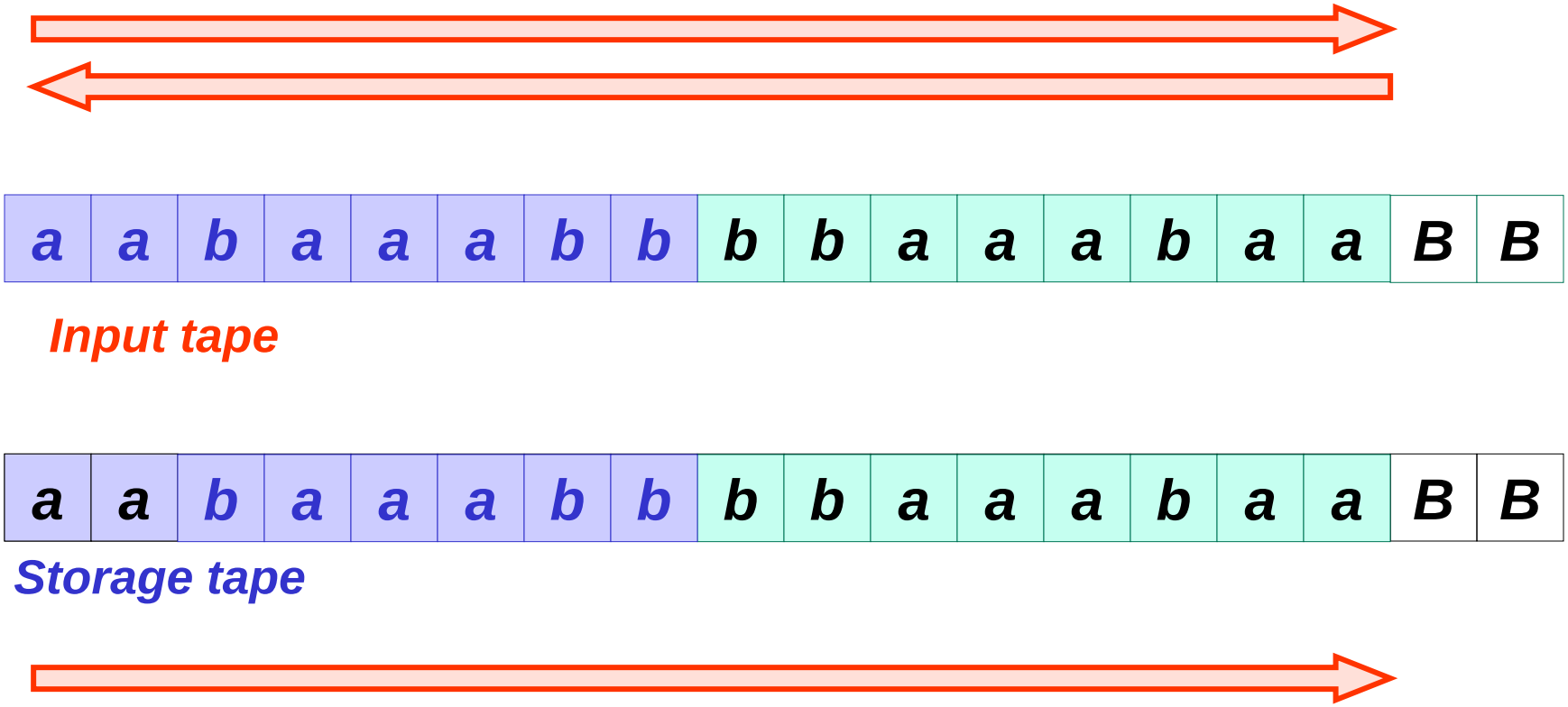



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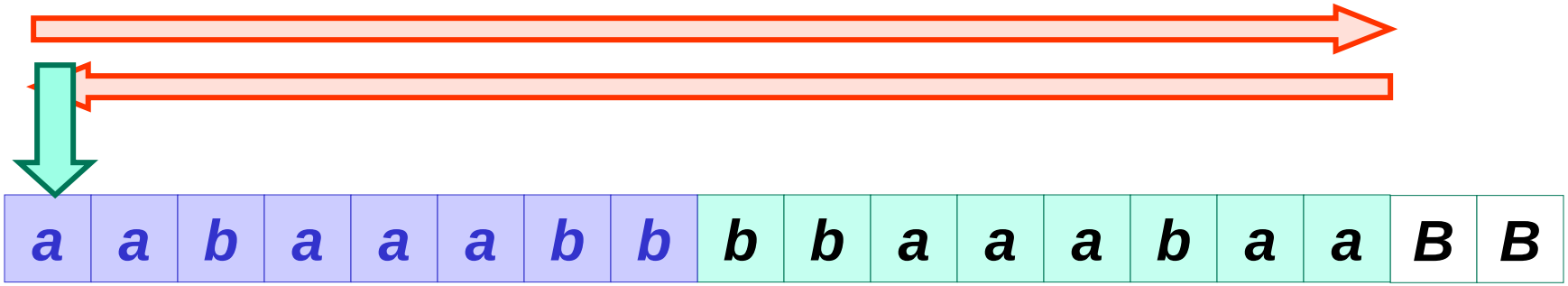


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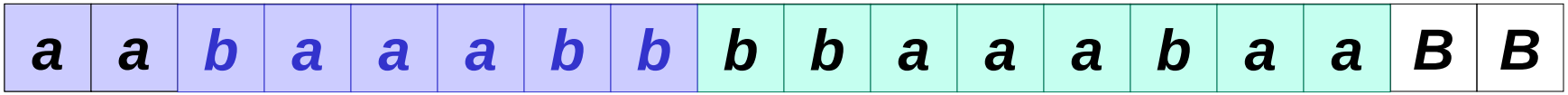


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# Multitape TM

$\{ ww^R \mid w \in ( a+b )^* \}$

| a | a | b | a | a | a | b | b | b | b | a | a | a | b | a | a | B | B |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|

Input tape

| a | a | b | a | a | a | b | b | b | b | a | a | a | b | a | a | B | B |
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Storage tape

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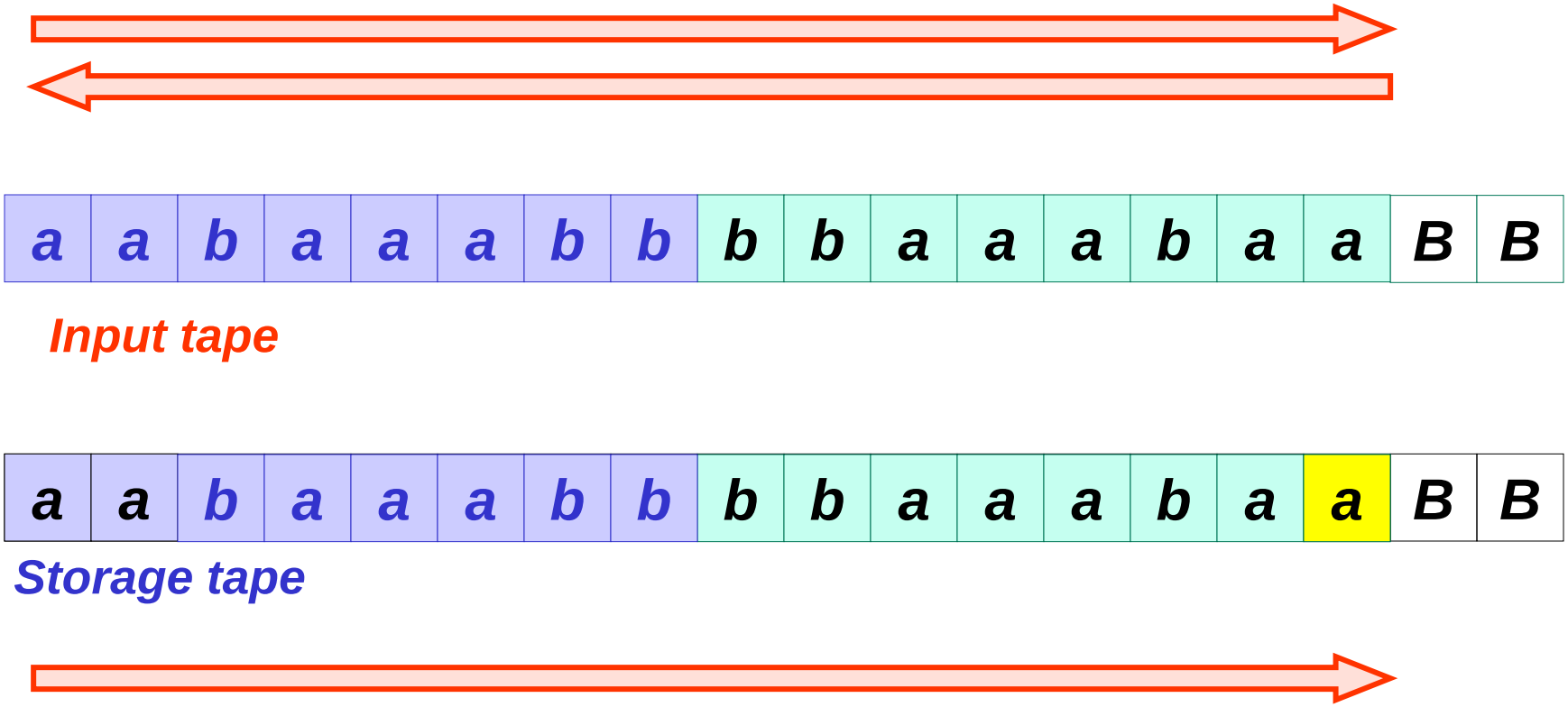
# Multitape TM

$\{ ww^R \mid w \in ( a+b )^* \}$





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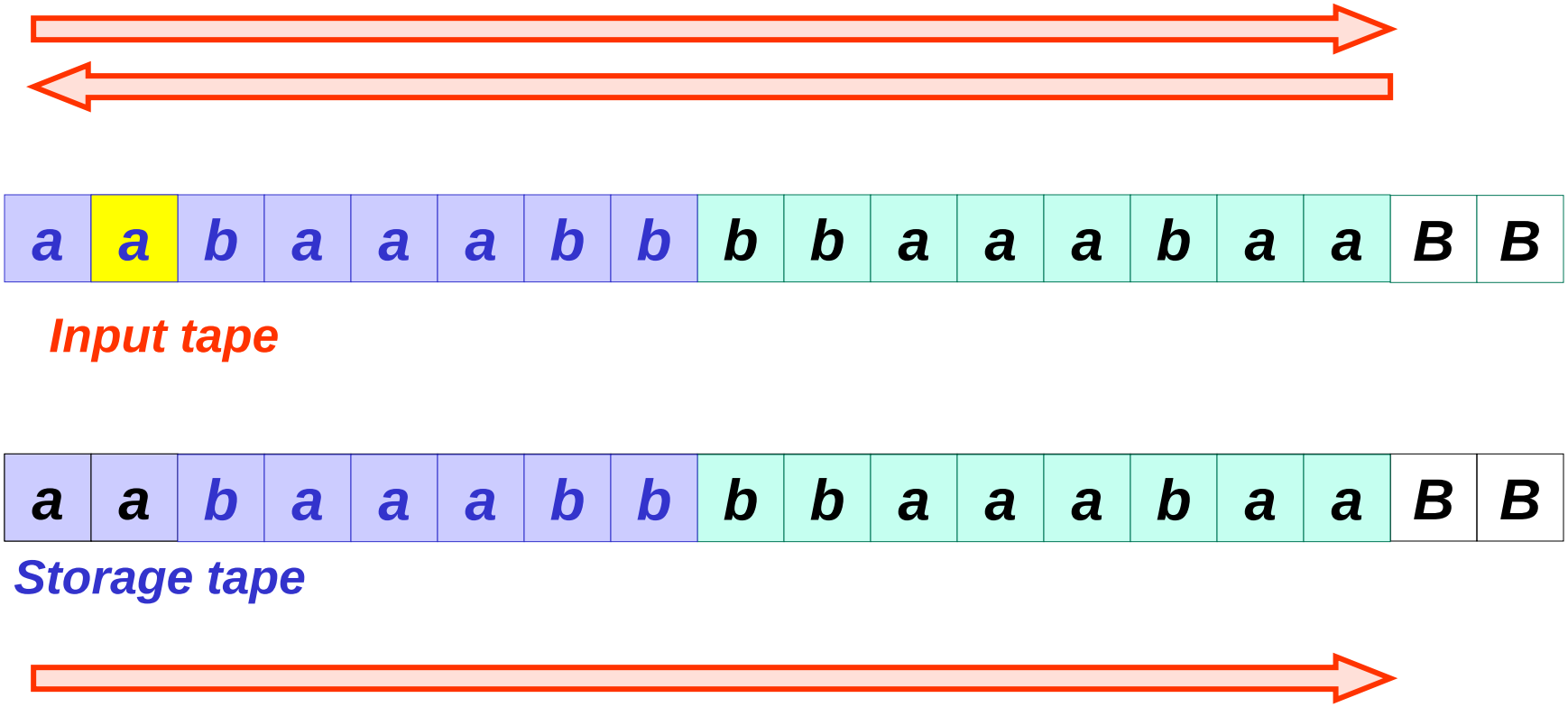
# Multitape TM

$\{ ww^R \mid w \in ( a+b )^* \}$





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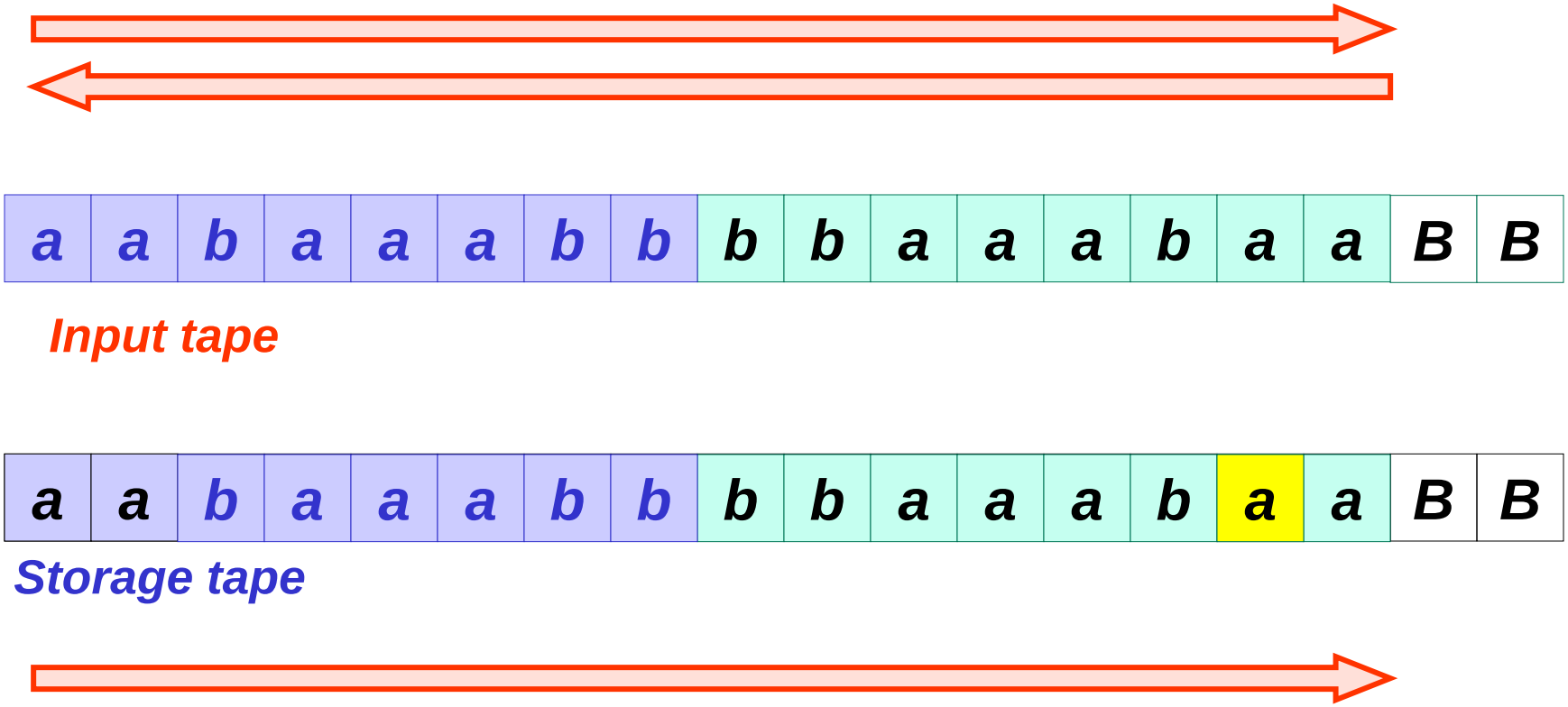
# Multitape TM

$\{ ww^R \mid w \in ( a+b )^* \}$





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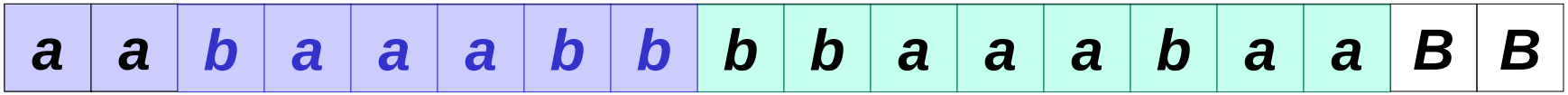
# Multitape TM

$\{ ww^R \mid w \in ( a+b )^* \}$

*Input tape*




*Storage tape*

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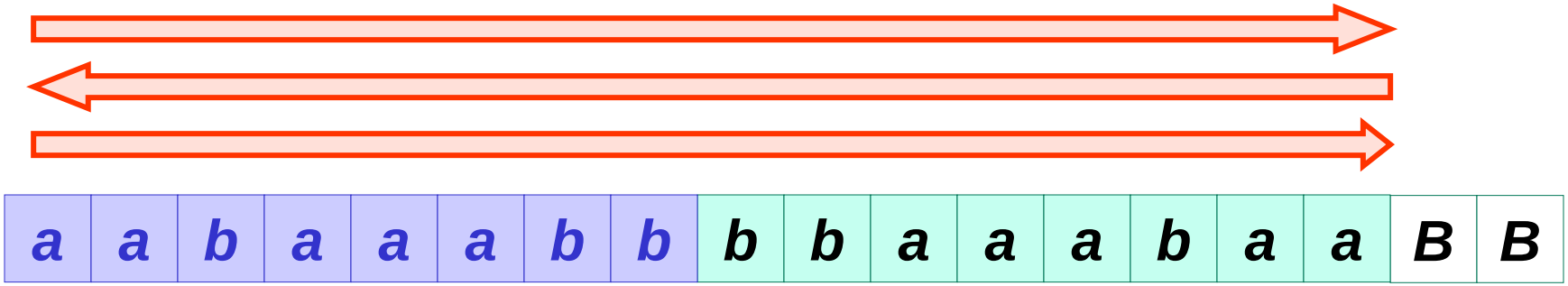
# Multitape TM

$\{ ww^R \mid w \in ( a+b )^* \}$

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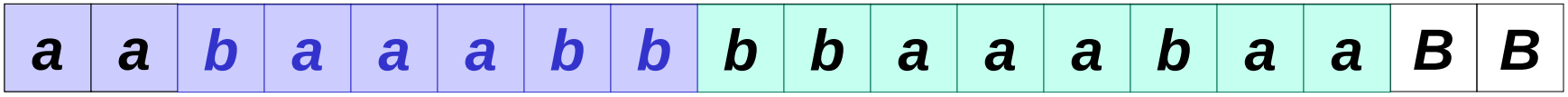


*Storage tape*

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# Nondeterministic TM


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# Nondeterministic TM

$\delta(q, X) =$


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# Nondeterministic TM

$\delta(q, X) = \{$ $\}$


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# Nondeterministic TM

$\delta(q, X) = \{ \ (p_1, Z_1, D_1) \ , \qquad \}$


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# Nondeterministic TM

$\delta(q, X) = \{ \ (p_1, Z_1, D_1) \ , (p_2, Z_2, D_2) \ , \ ..., \ \}$


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# Nondeterministic TM

$$\delta(q, X) = \{ \ (p_1, Z_1, D_1) \ , (p_2, Z_2, D_2) \ , \ ..., \ (p_k, Z_k, D_k) \ \}$$


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# Nondeterministic TM

$$\delta(q, X) = \{ \ (p_1, Z_1, D_1) \ , (p_2, Z_2, D_2) \ , \ ..., \ (p_k, Z_k, D_k) \ \}$$

**Equivalent to deterministic TM**


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# Nondeterministic TM


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# Nondeterministic TM

$\delta(q, X) = \{ (p_1, Z_1, D_1) , (p_2, Z_2, D_2) , ..., (p_k, Z_k, D_k) \}$


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# Nondeterministic TM

$$\delta(q, X) = \{ (p_1, Z_1, D_1) , (p_2, Z_2, D_2) , ..., (p_k, Z_k, D_k) \}$$

$$(q, X)$$


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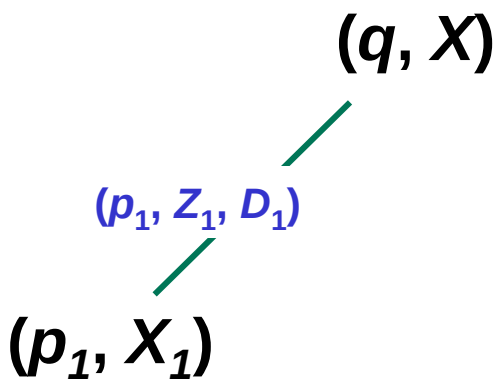
# Nondeterministic TM

$\delta(q, X) = \{ (p_1, Z_1, D_1) , (p_2, Z_2, D_2) , ..., (p_k, Z_k, D_k) \}$





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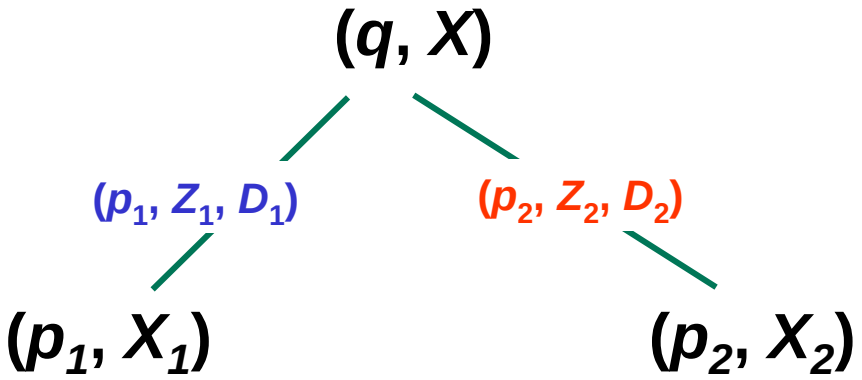
# Nondeterministic TM

$\delta(q, X) = \{ (p_1, Z_1, D_1) , (p_2, Z_2, D_2) , ..., (p_k, Z_k, D_k) \}$





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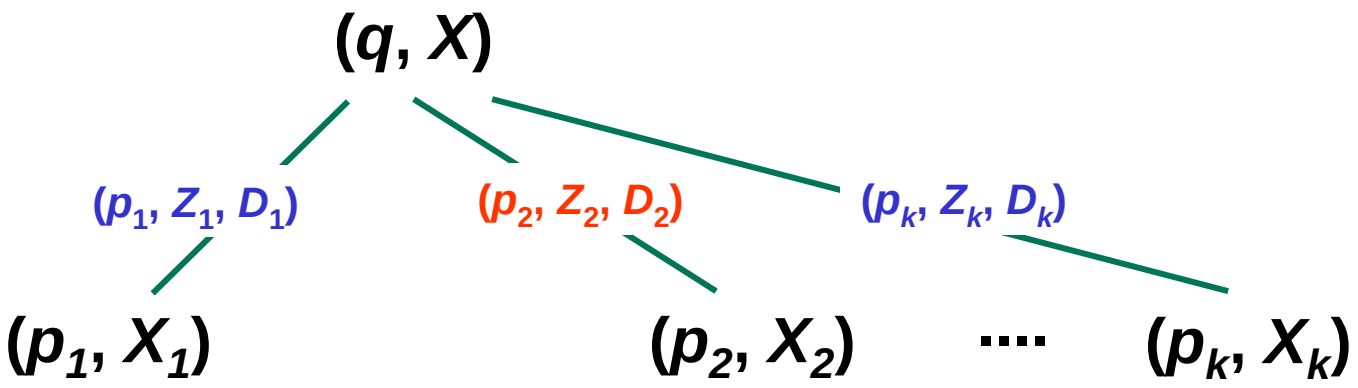
# Nondeterministic TM

$\delta(q, X) = \{ (p_1, Z_1, D_1) , (p_2, Z_2, D_2) , ..., (p_k, Z_k, D_k) \}$





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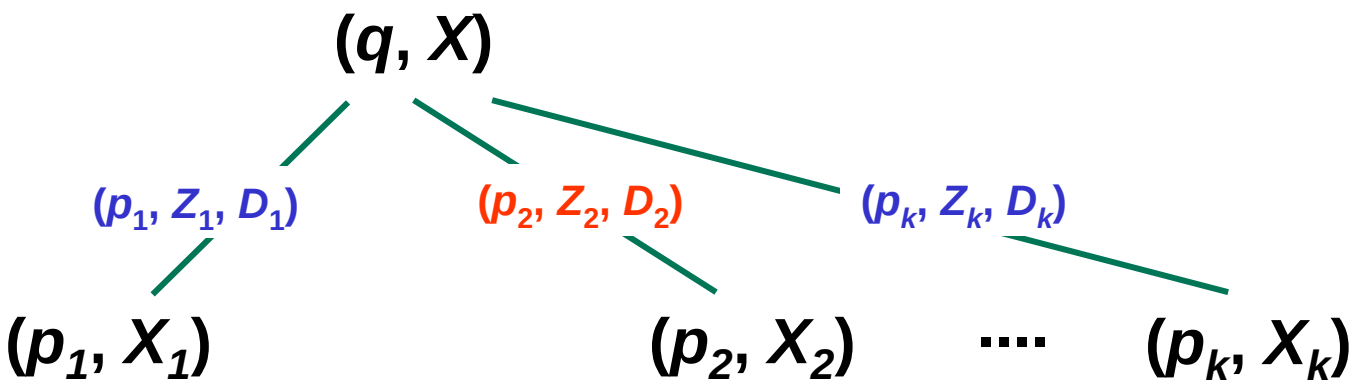
# Nondeterministic TM

$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1) , (r_2, T_2, Y_2) , ..., (r_m, T_m, Y_m) \}$





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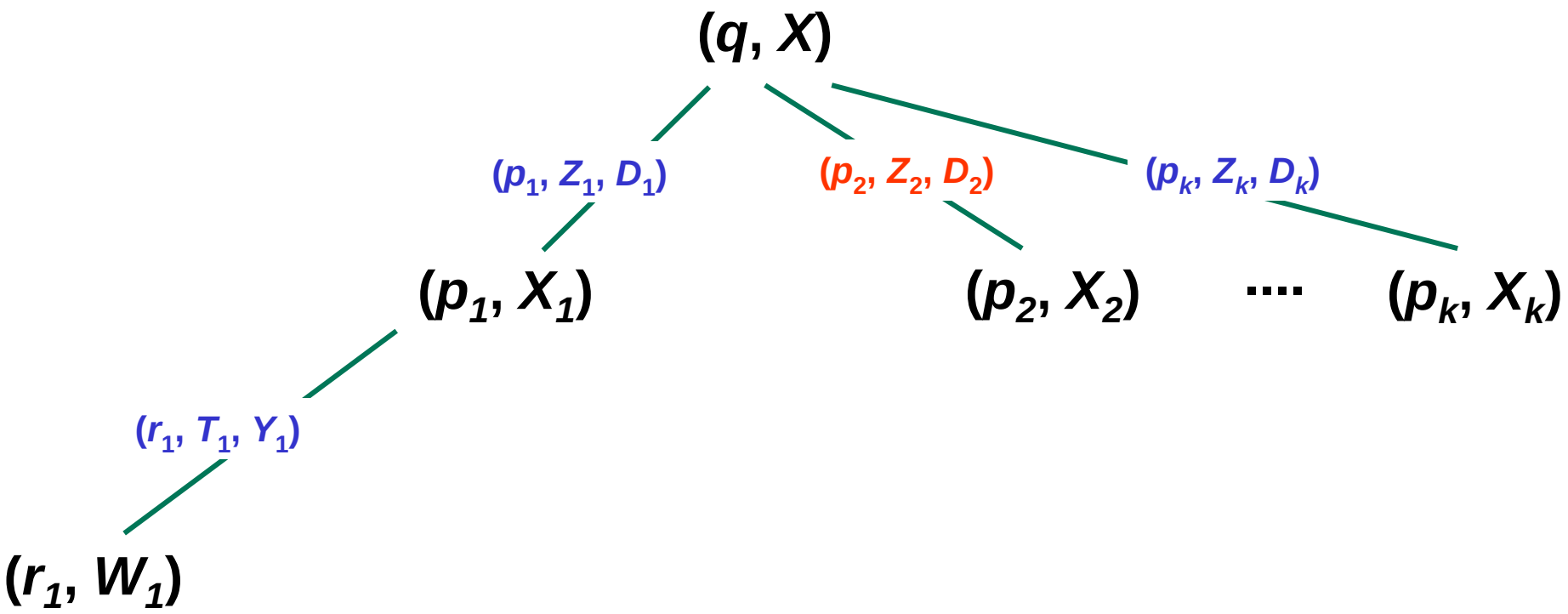



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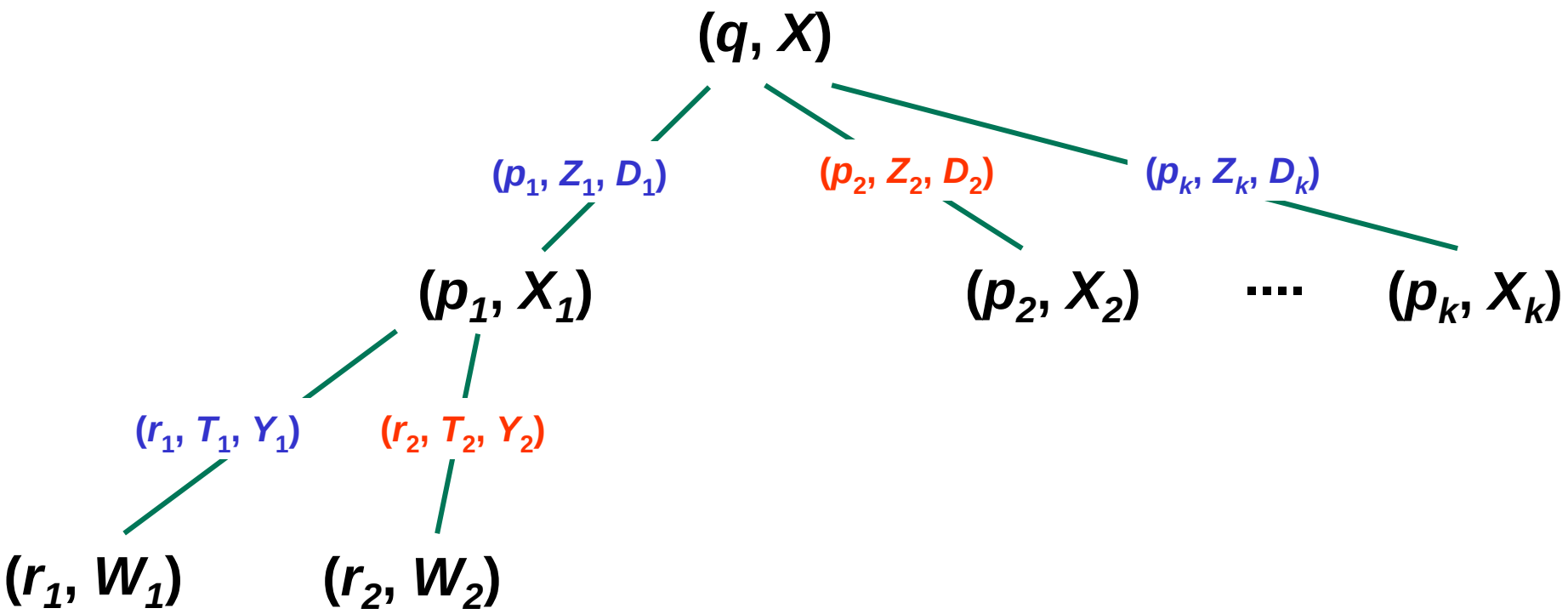



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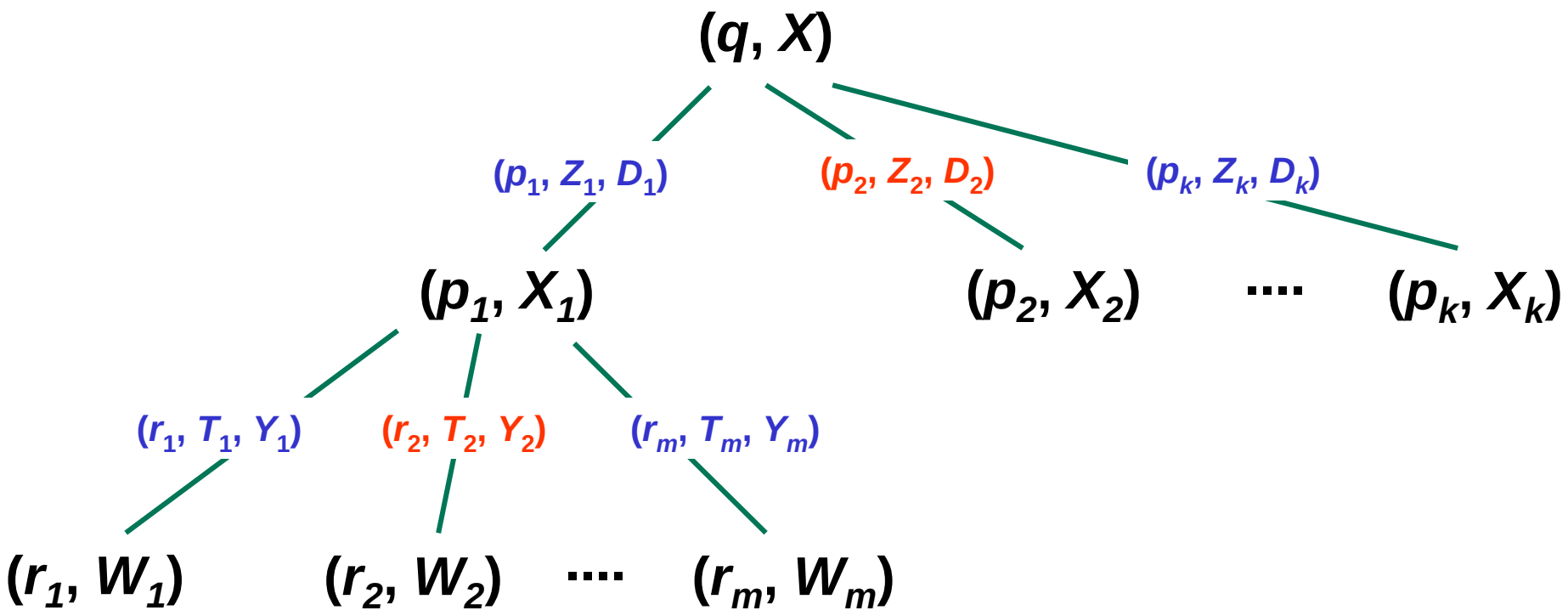
$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1) , (r_2, T_2, Y_2) , ..., (r_m, T_m, Y_m) \}$





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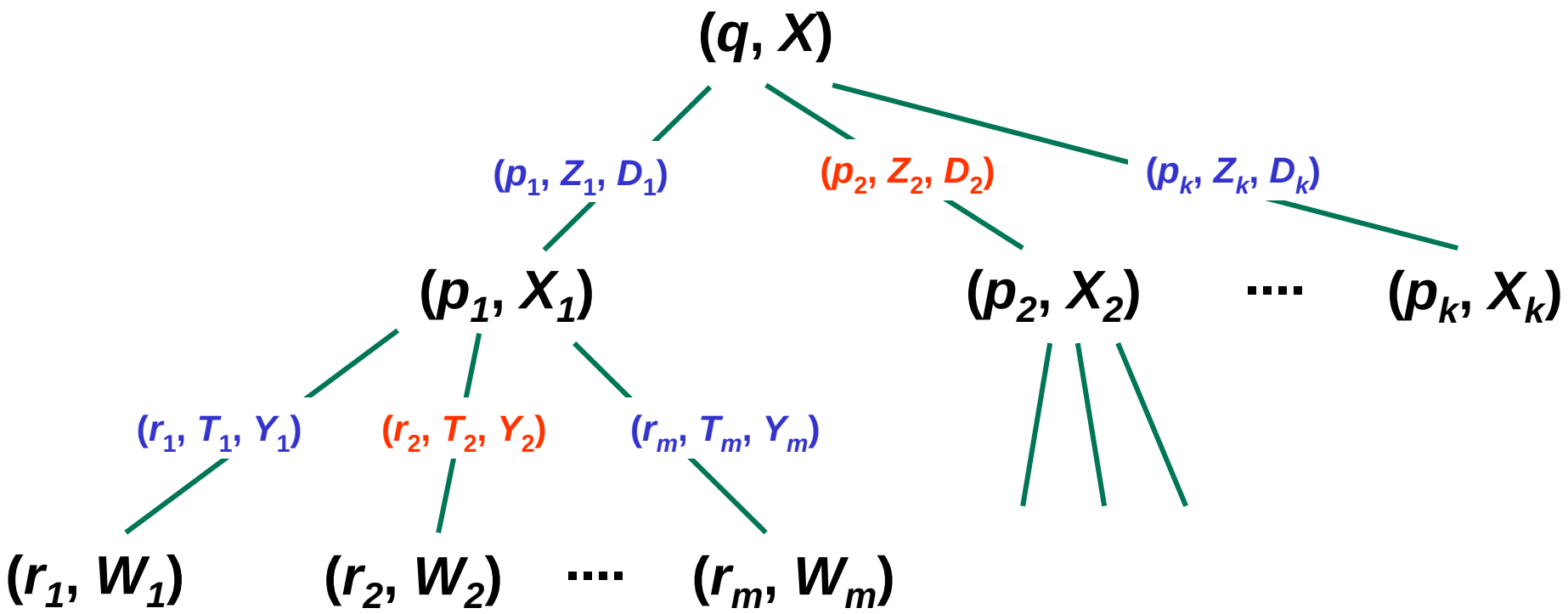
$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1) , (r_2, T_2, Y_2) , ..., (r_m, T_m, Y_m) \}$





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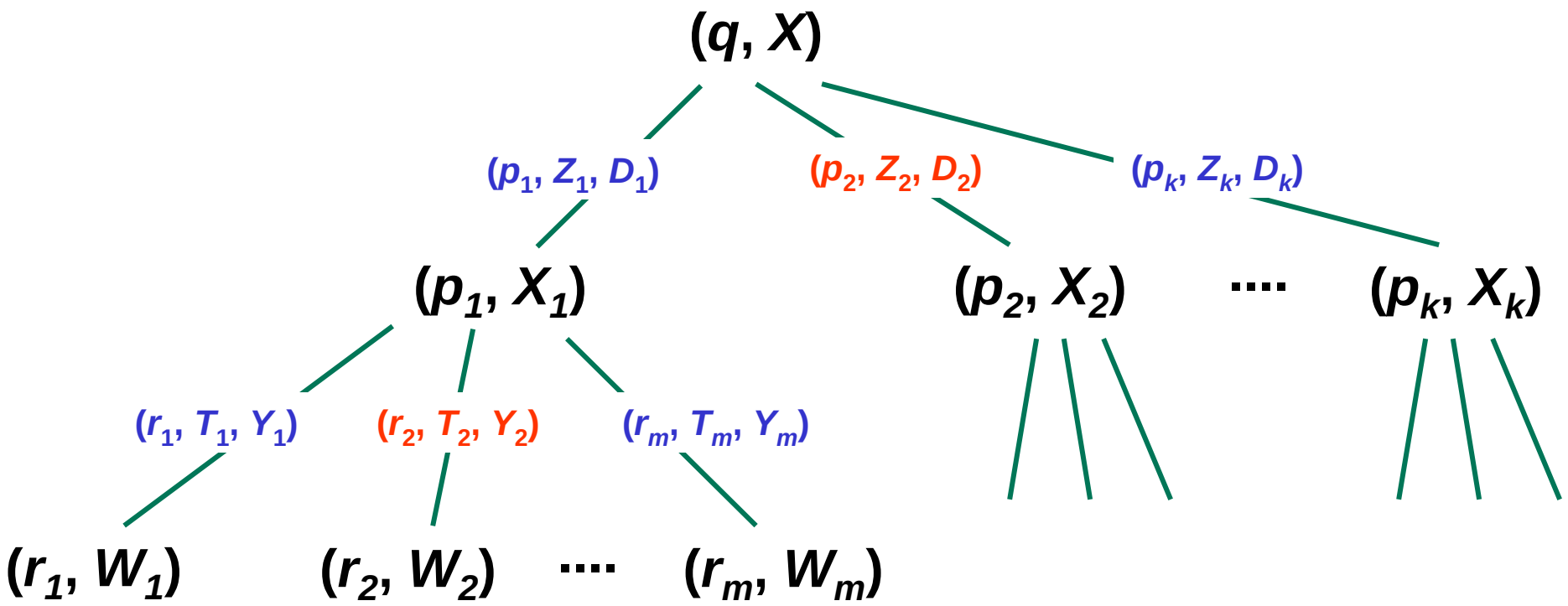



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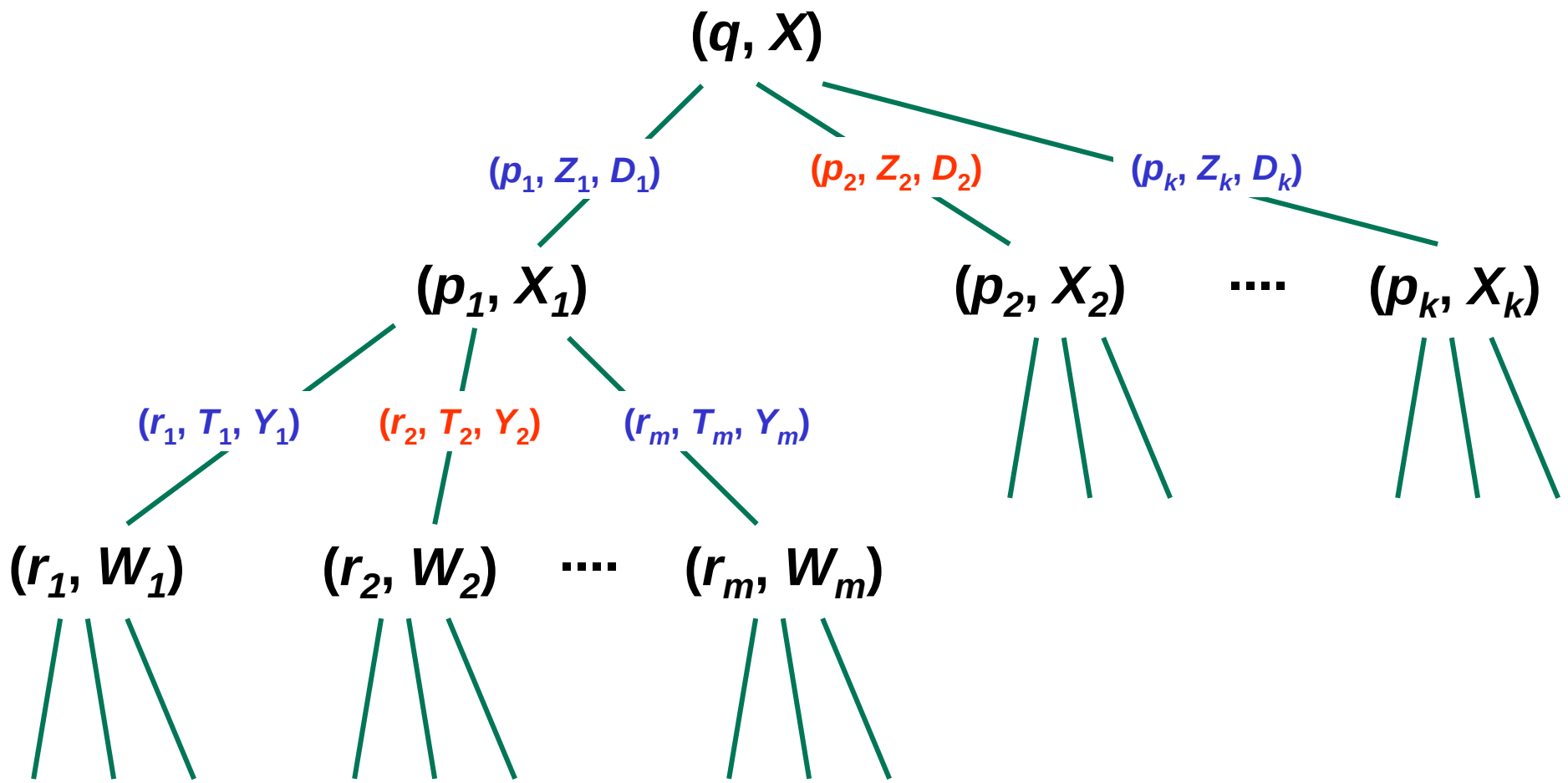
# Nondeterministic TM

$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1) , (r_2, T_2, Y_2) , ..., (r_m, T_m, Y_m) \}$





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# Nondeterministic TM

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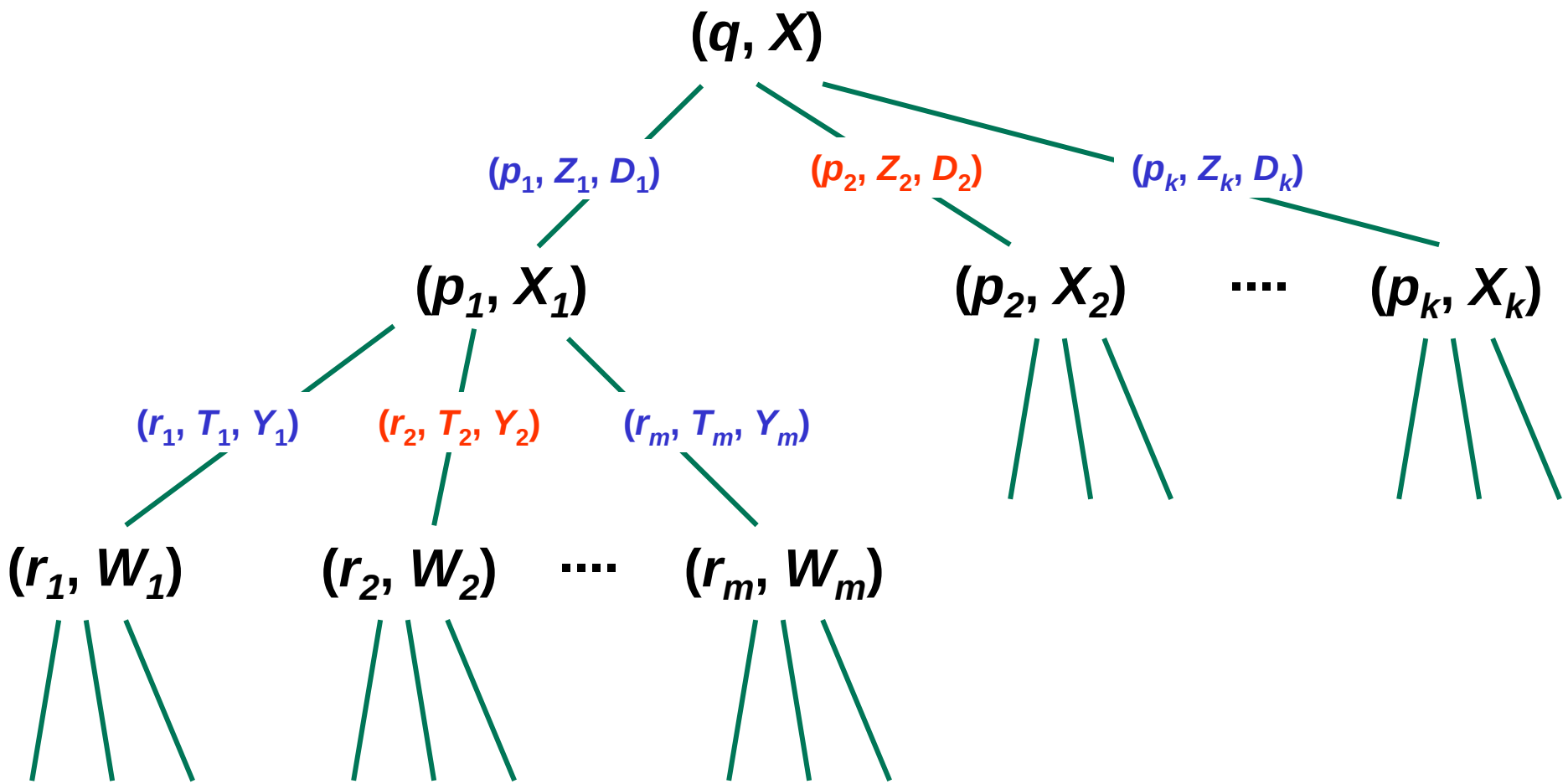


**TS that never halts**


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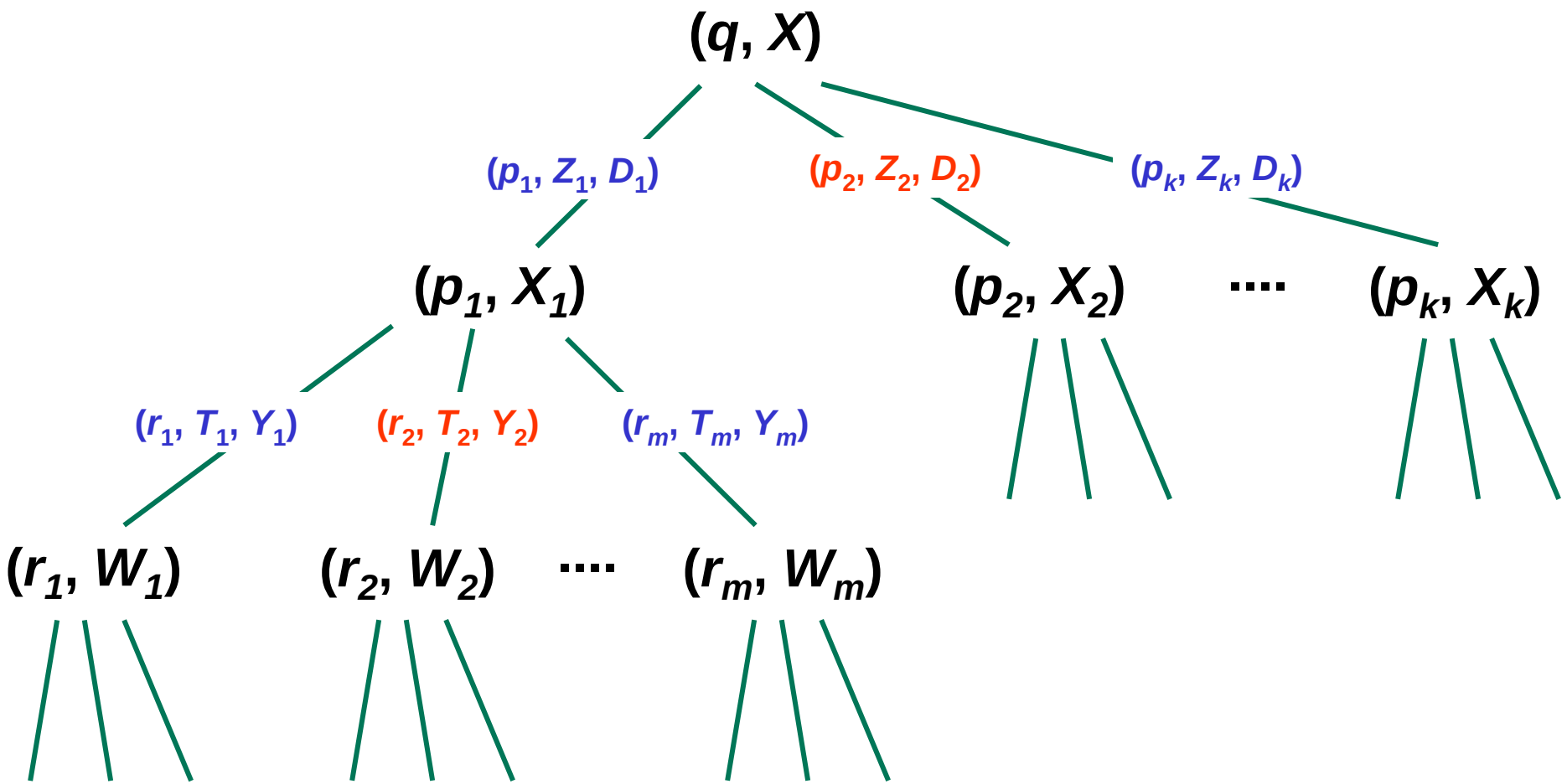
# Nondeterministic TM

$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1) , (r_2, T_2, Y_2) , ..., (r_m, T_m, Y_m) \}$




**TS that never halts** ⇒ **Infinite depth tree**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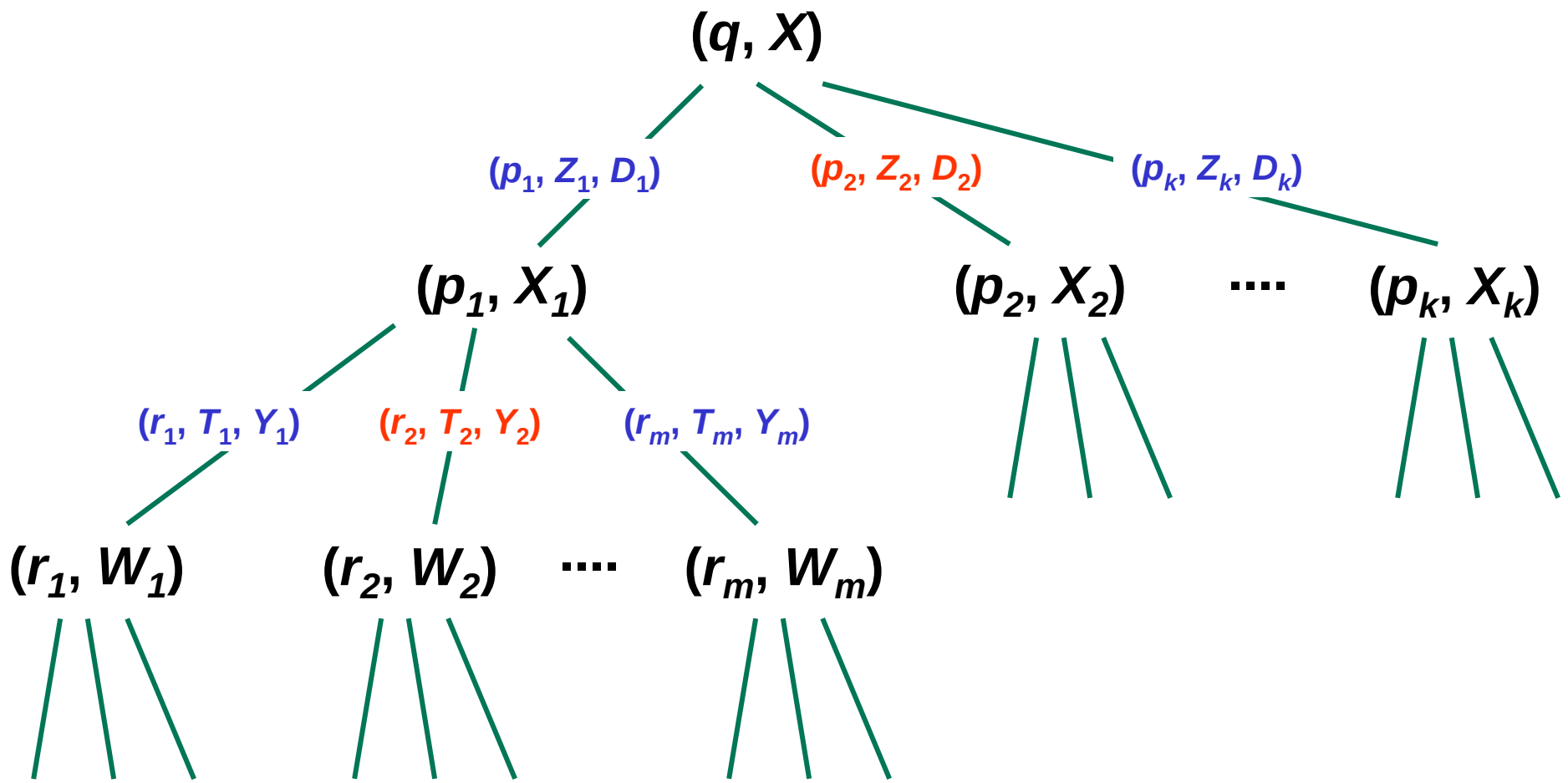
# Nondeterministic TM

$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1) , (r_2, T_2, Y_2) , ..., (r_m, T_m, Y_m) \}$




**TS that never halts** ⇒ **Infinite depth tree** ⇒ **Breadth first search**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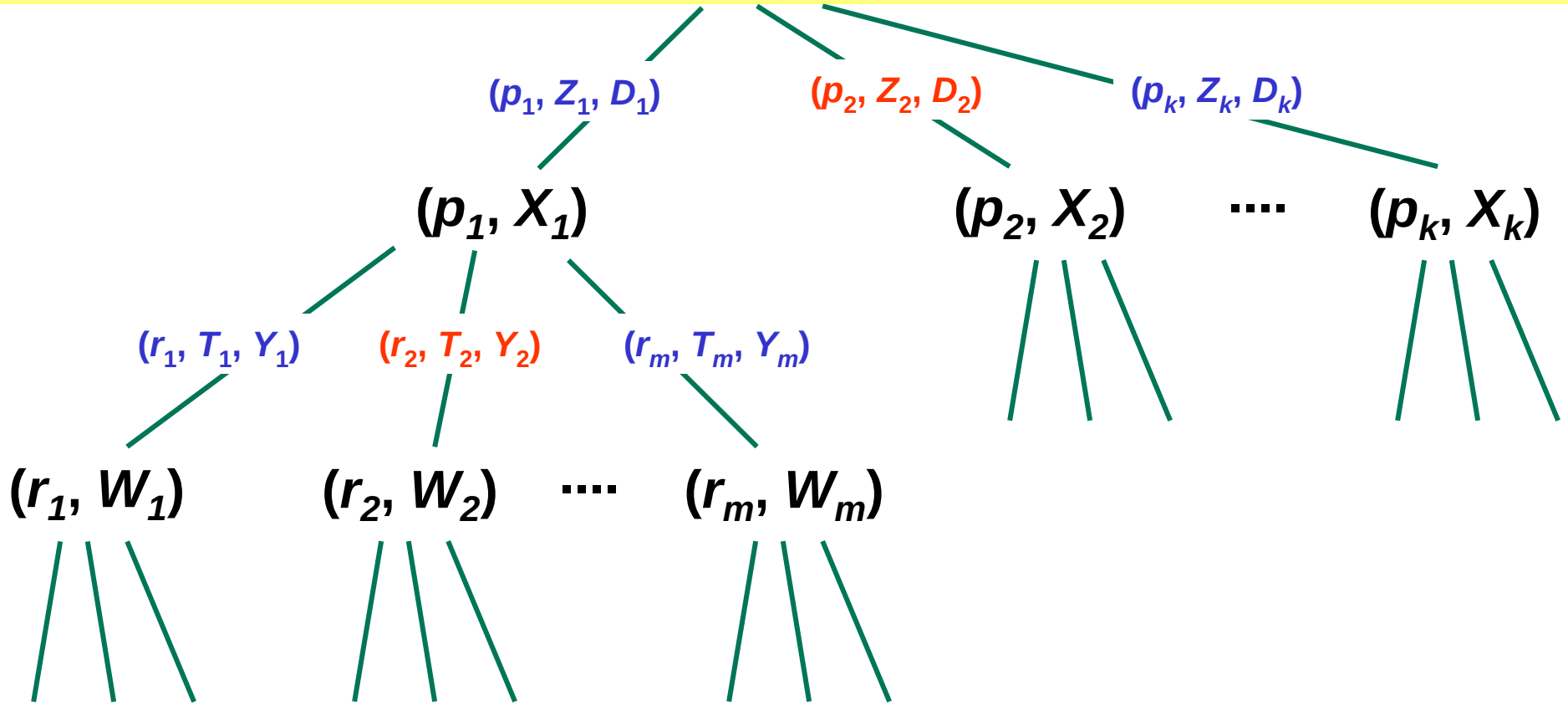
# Nondeterministic TM

$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1) , (r_2, T_2, Y_2) , ..., (r_m, T_m, Y_m) \}$




**TS that never halts** ⇒ **Infinite depth tree** ⇒ **Breadth first search**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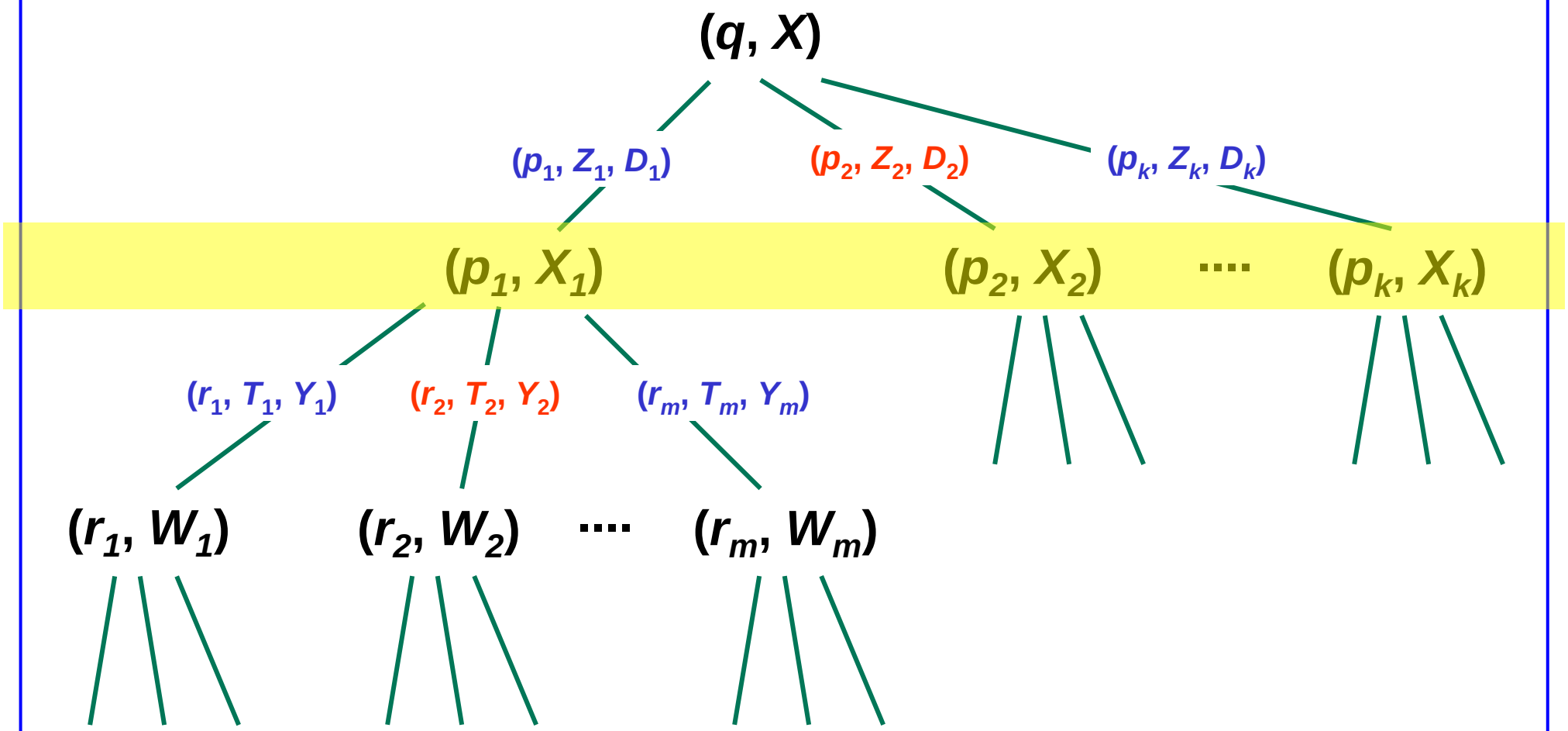
# Nondeterministic TM

$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1) , (r_2, T_2, Y_2) , ..., (r_m, T_m, Y_m) \}$




**TS that never halts** ⇒ **Infinite depth tree** ⇒ **Breadth first search**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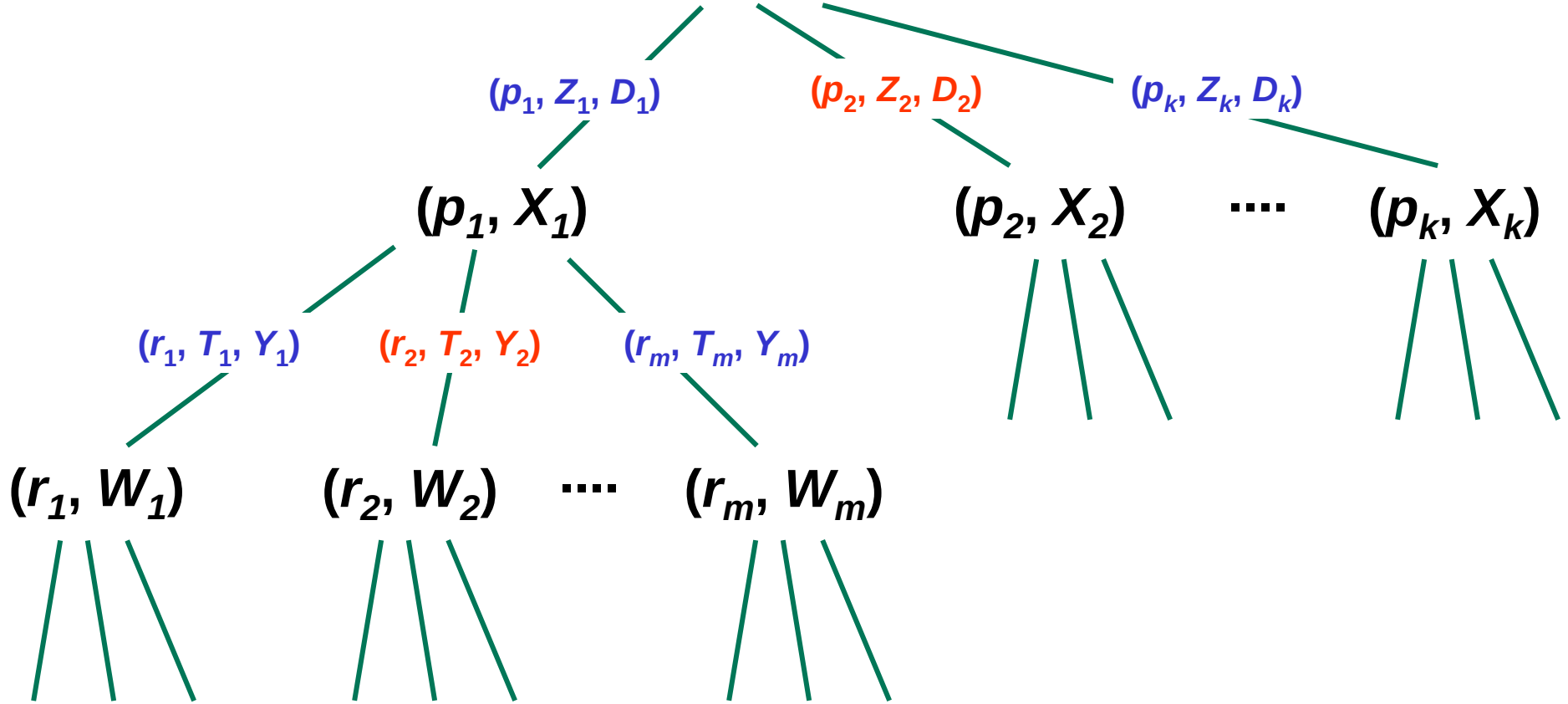
# Nondeterministic TM

$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1) , (r_2, T_2, Y_2) , ..., (r_m, T_m, Y_m) \}$




**TS that never halts** ⇒ **Infinite depth tree** ⇒ **Breadth first search**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# Nondeterministic TM

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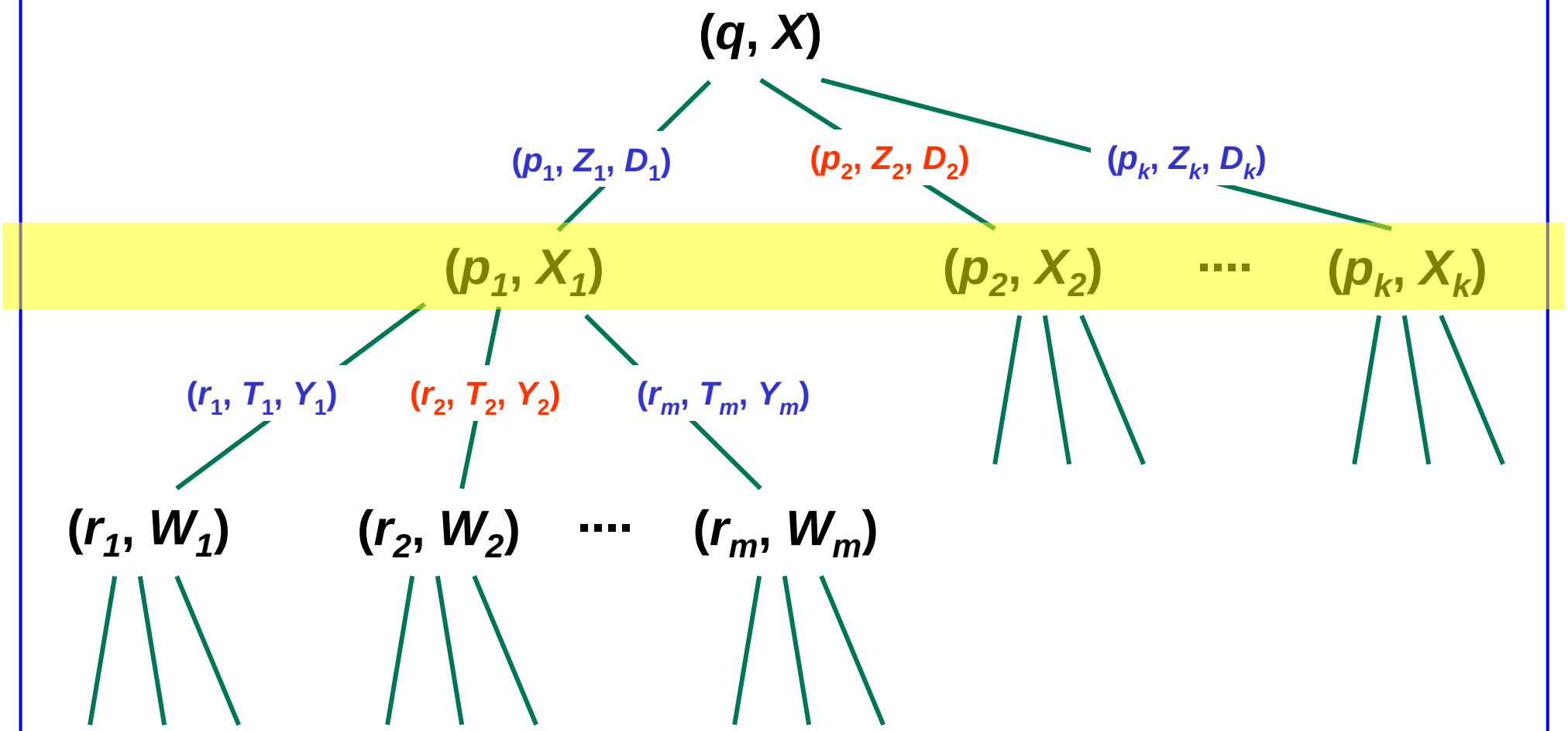


**TS that never halts** ⇒ **Infinite depth tree** ⇒ **Breadth first search**


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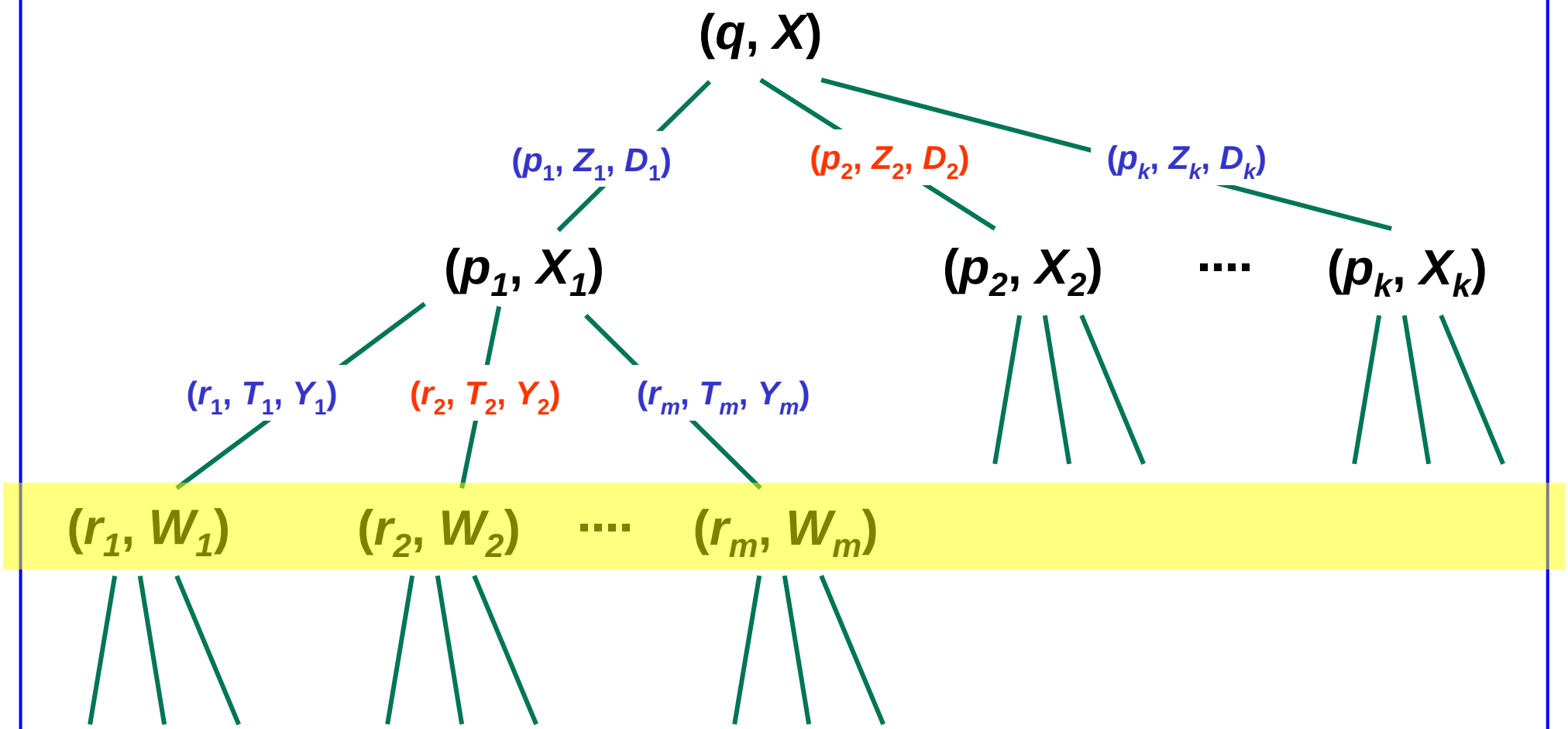


**TS that never halts** ⇒ **Infinite depth tree** ⇒ **Breadth first search**


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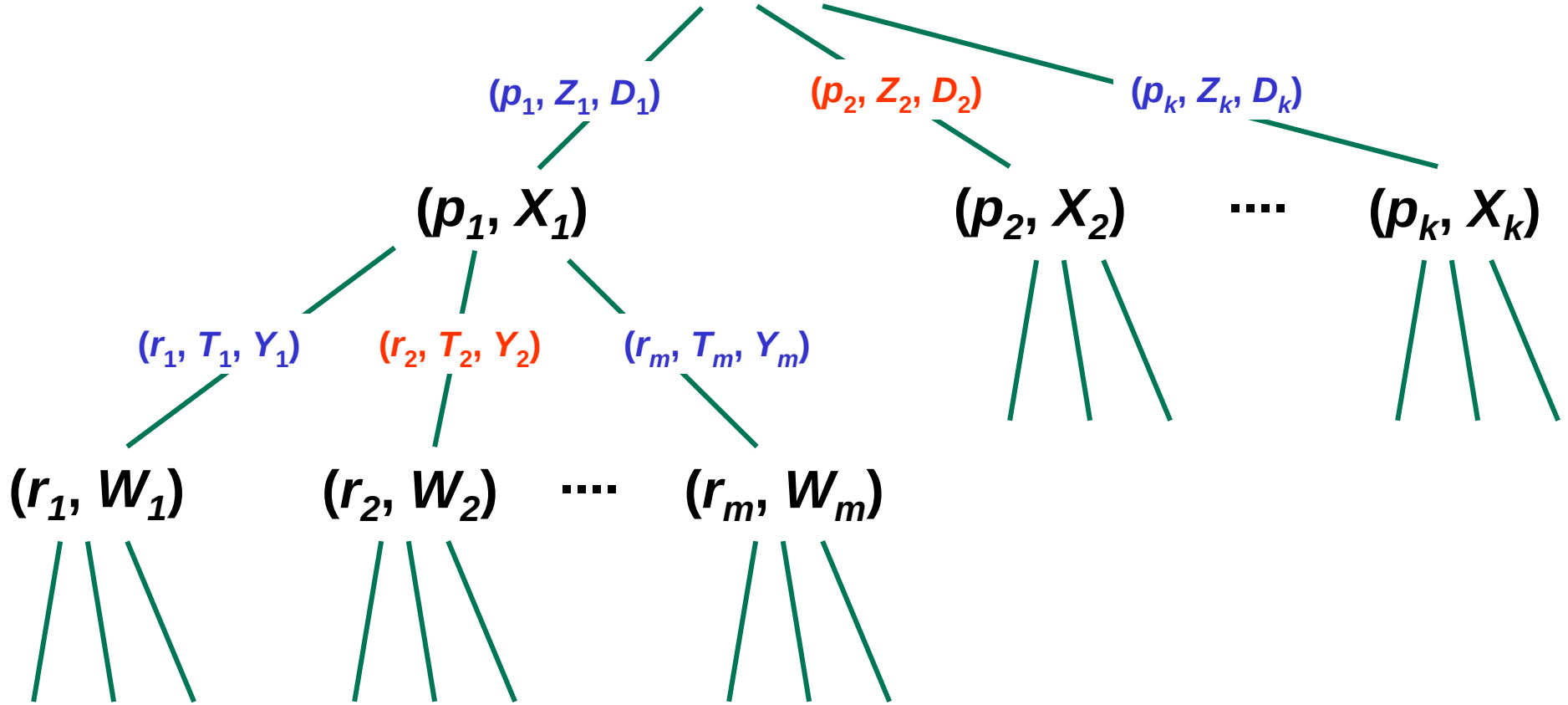


**TS that never halts** ⇒ **Infinite depth tree** ⇒ **Breadth first search**


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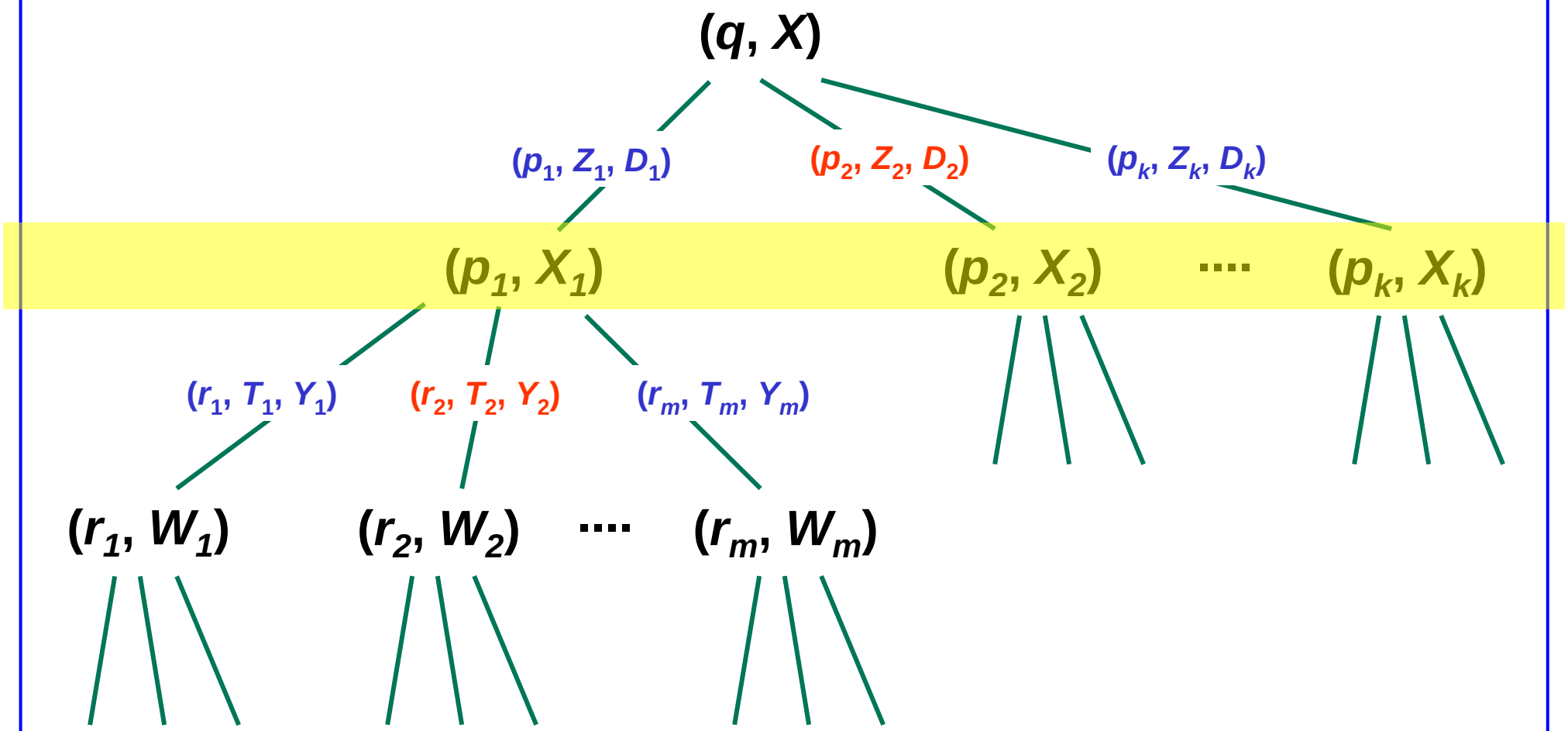


**TS that never halts** ⇒ **Infinite depth tree** ⇒ **Breadth first search**


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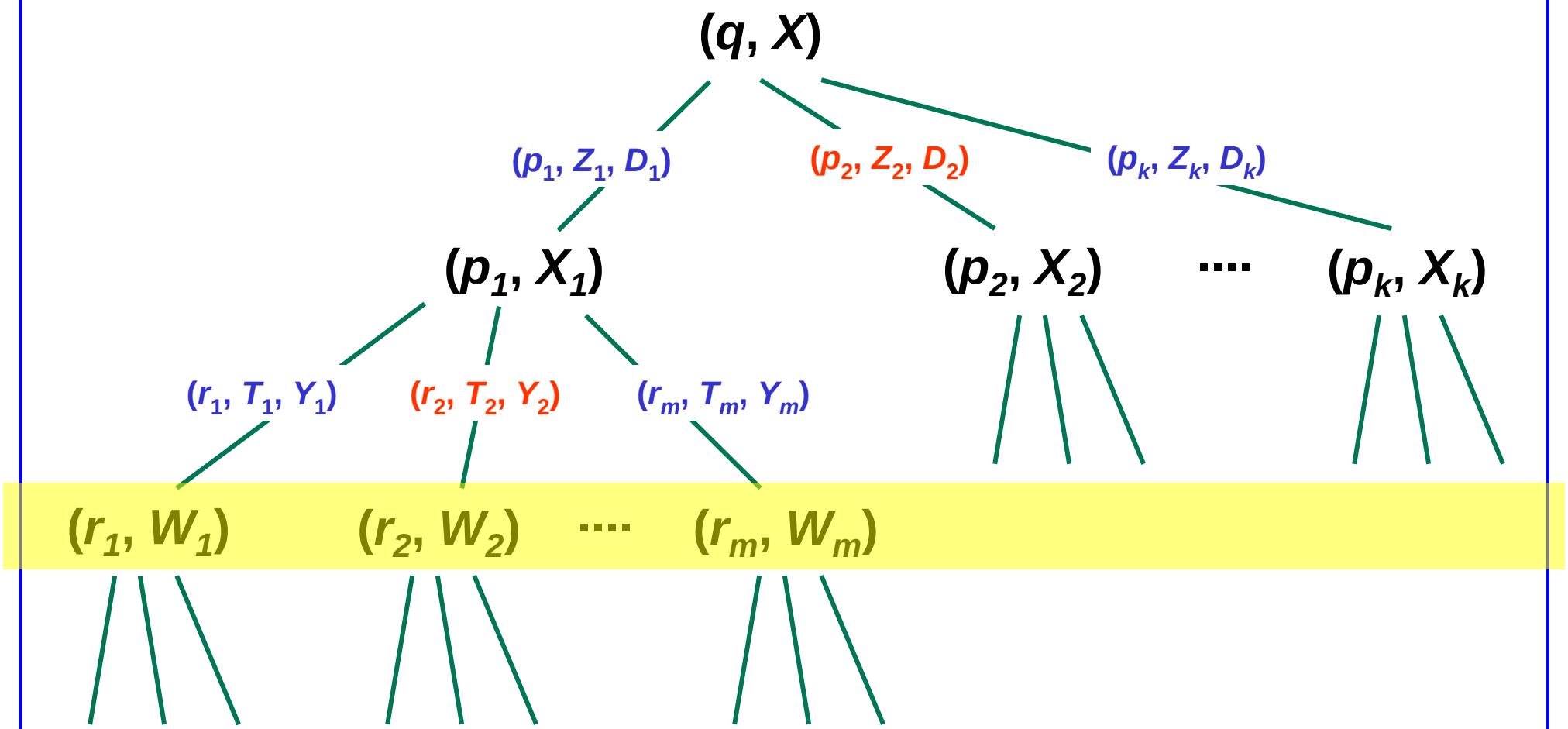


**TS that never halts** ⇒ **Infinite depth tree** ⇒ **Breadth first search**


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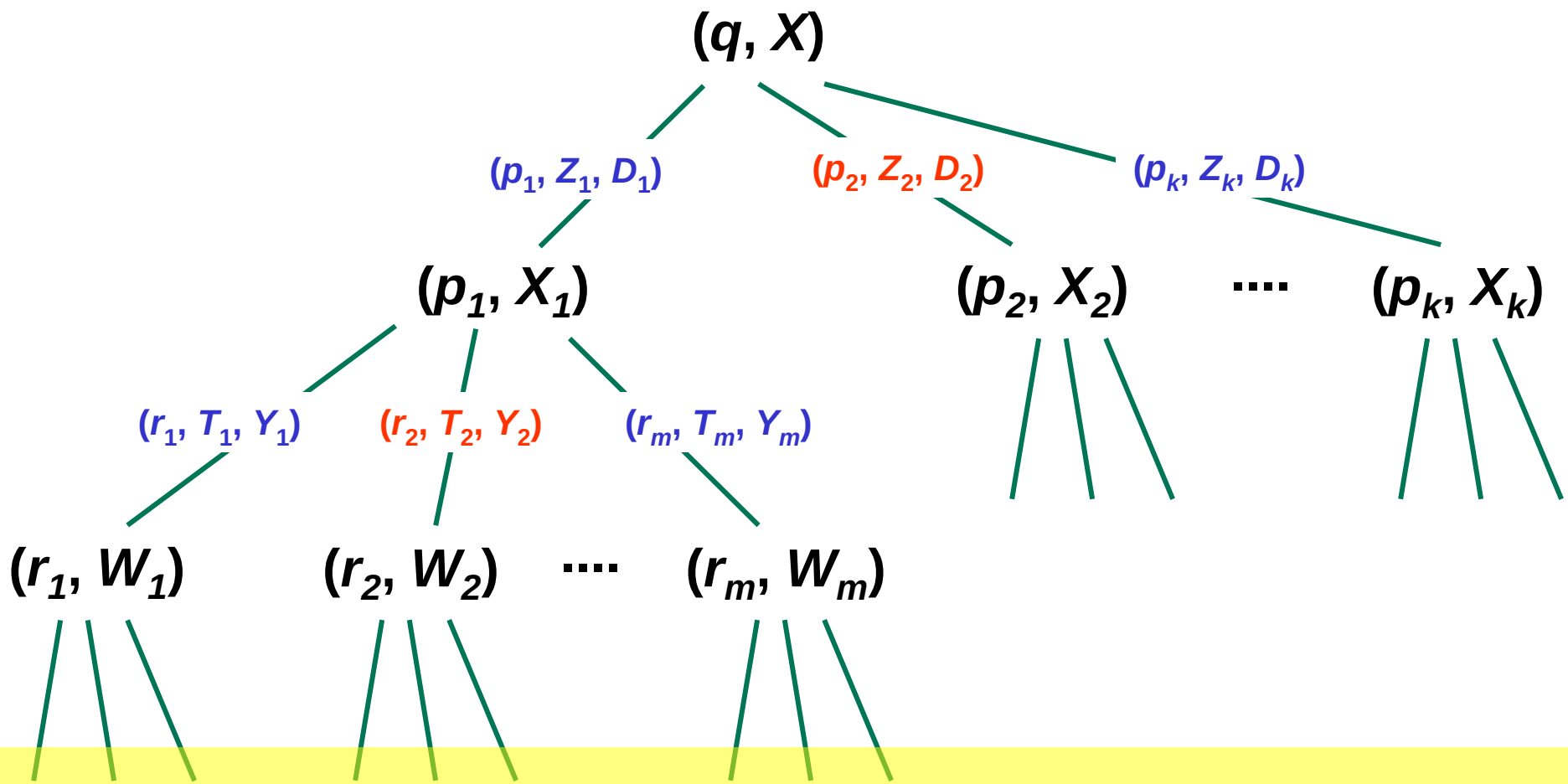
$\delta(p_1, X_1) = \{ (r_1, T_1, Y_1) , (r_2, T_2, Y_2) , ..., (r_m, T_m, Y_m) \}$




**TS that never halts** ⇒ **Infinite depth tree** ⇒ **Breadth first search**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# Nondeterministic TM

$$\delta(q, X) = \{ \ (p_1, Z_1, D_1) \ , (p_2, Z_2, D_2) \ , \ ..., \ (p_k, Z_k, D_k) \ \}$$


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# Nondeterministic TM

$$\delta(q, X) = \{ \ (p_1, Z_1, D_1) \ , (p_2, Z_2, D_2) \ , \ ..., \ (p_k, Z_k, D_k) \ \}$$

$$r = max( \ card \ ( \ \delta(q_i, X_j) \ ) \ )$$


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# Nondeterministic TM

$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), \ldots, (p_k, Z_k, D_k) \}$$

$$r = max( card ( \delta(q_i, X_j) ) )$$

$$n_1, n_2, \ldots, n_m$$


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# Nondeterministic TM

$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), \ldots, (p_k, Z_k, D_k) \}$$

$$r = max( card ( \delta(q_i, X_j) ) )$$

$$n_1, n_2, \ldots, n_m$$

$$1 \leq n_i \leq k$$


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# Nondeterministic TM

$$\delta(q, X) = \{ (p_1, Z_1, D_1), (p_2, Z_2, D_2), \ldots, (p_k, Z_k, D_k) \}$$

$$r = max( card ( \delta(q_i, X_j) ) )$$

$n_1, n_2, \ldots, n_m$

$$1 \leq n_i \leq k$$

$$1 \leq k \leq r$$


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# Nondeterministic TM


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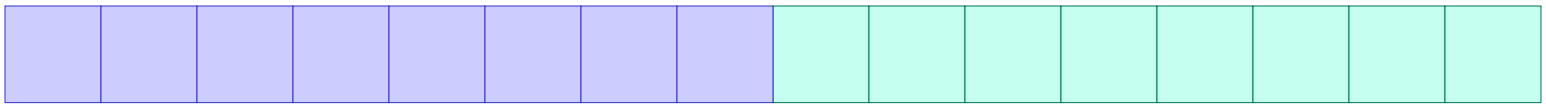
# Nondeterministic TM

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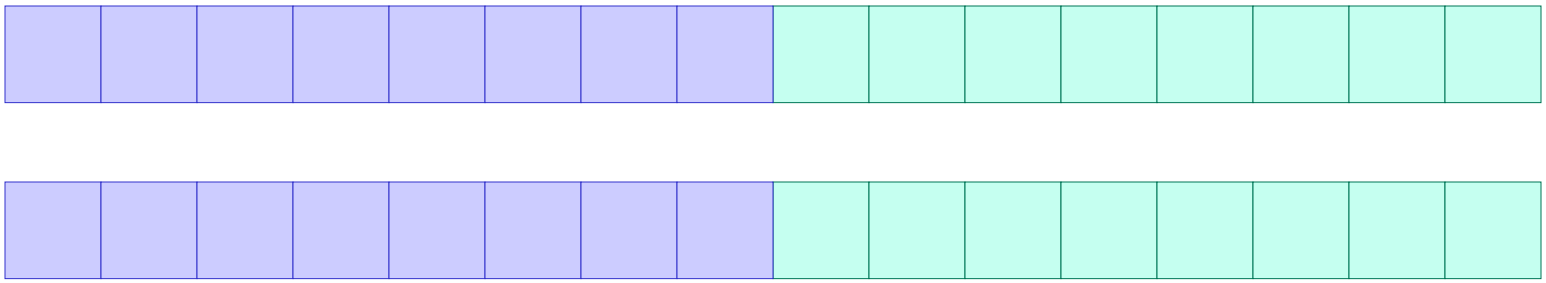


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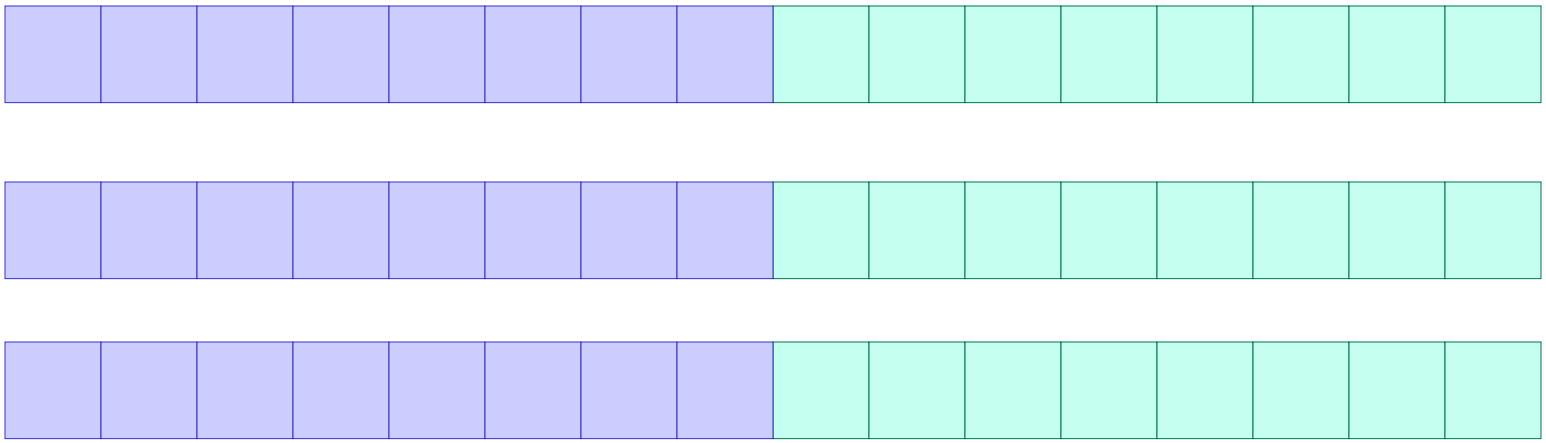
# Nondeterministic TM

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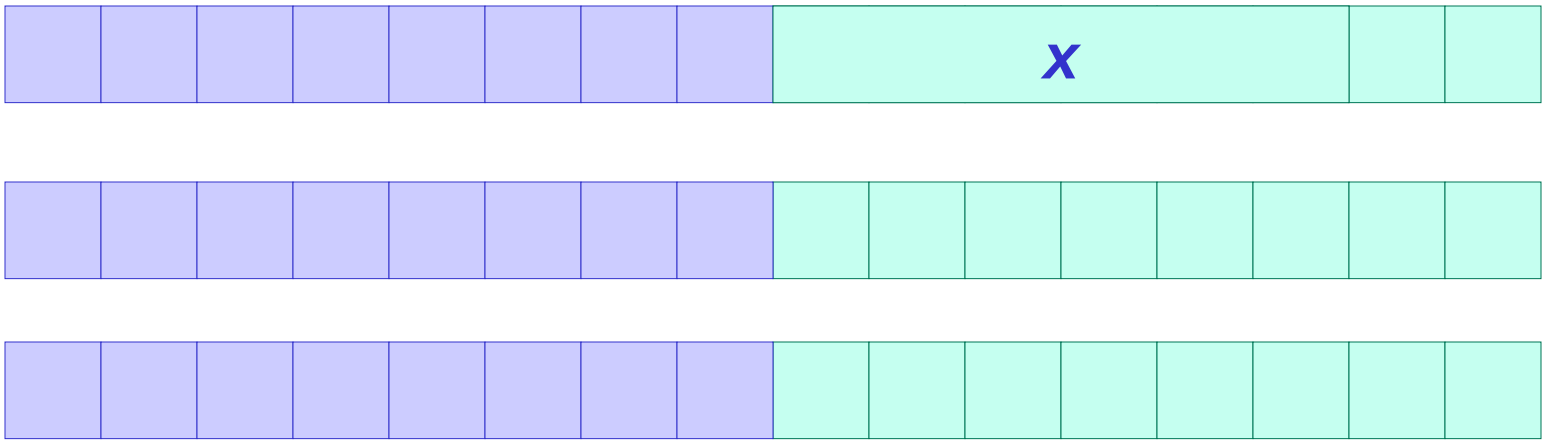



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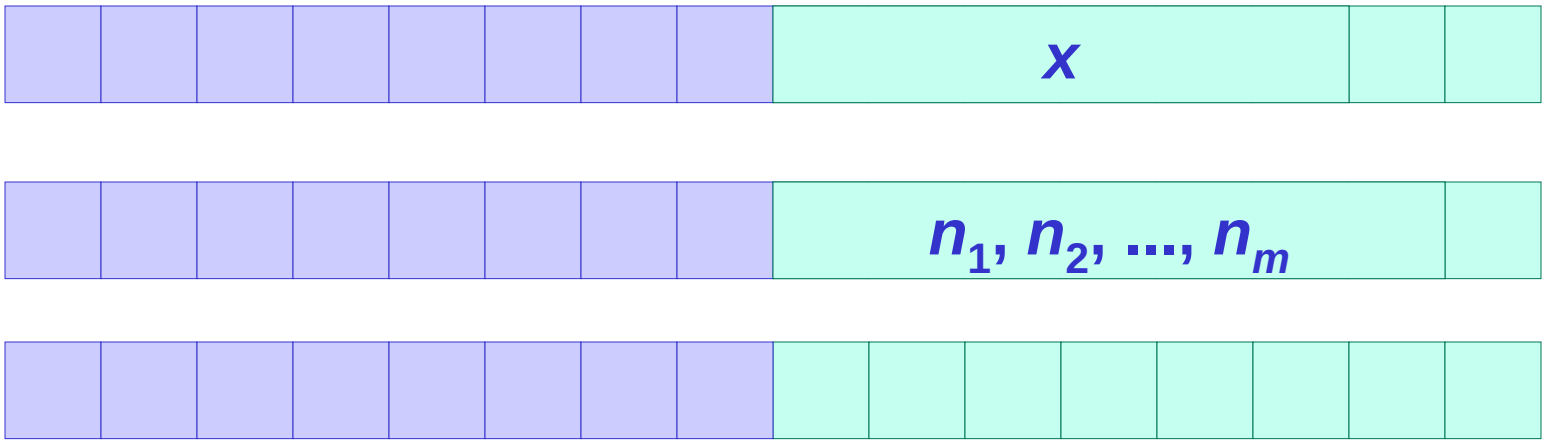



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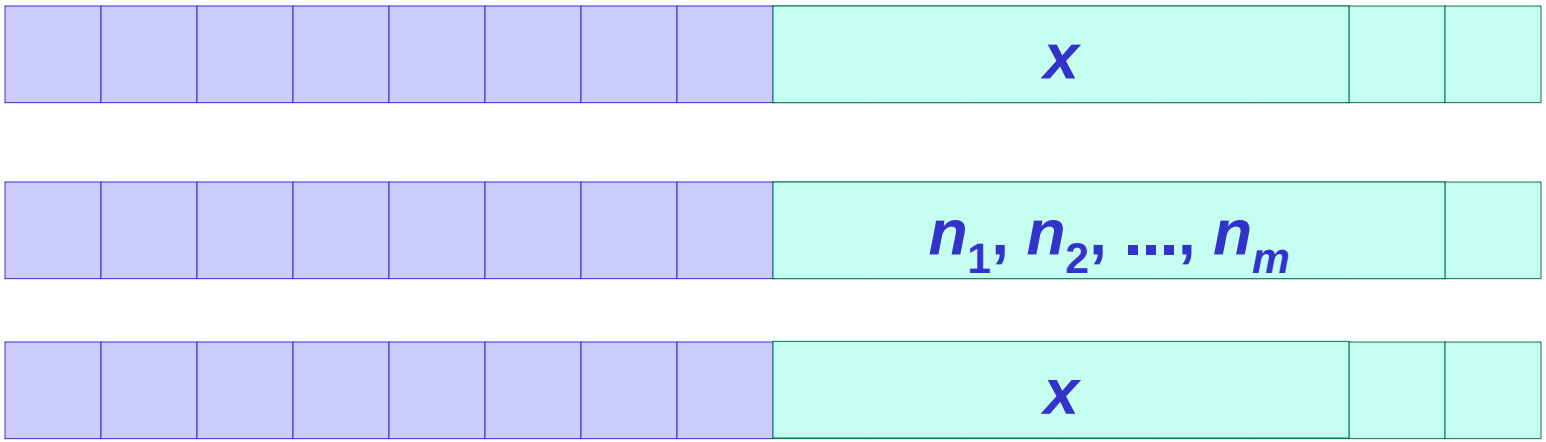



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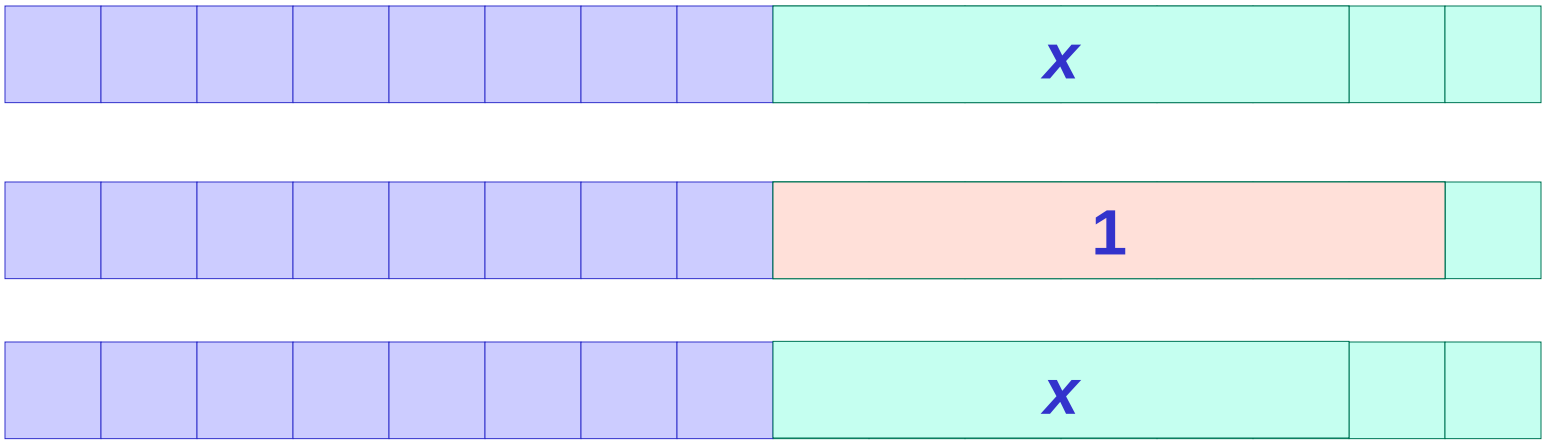



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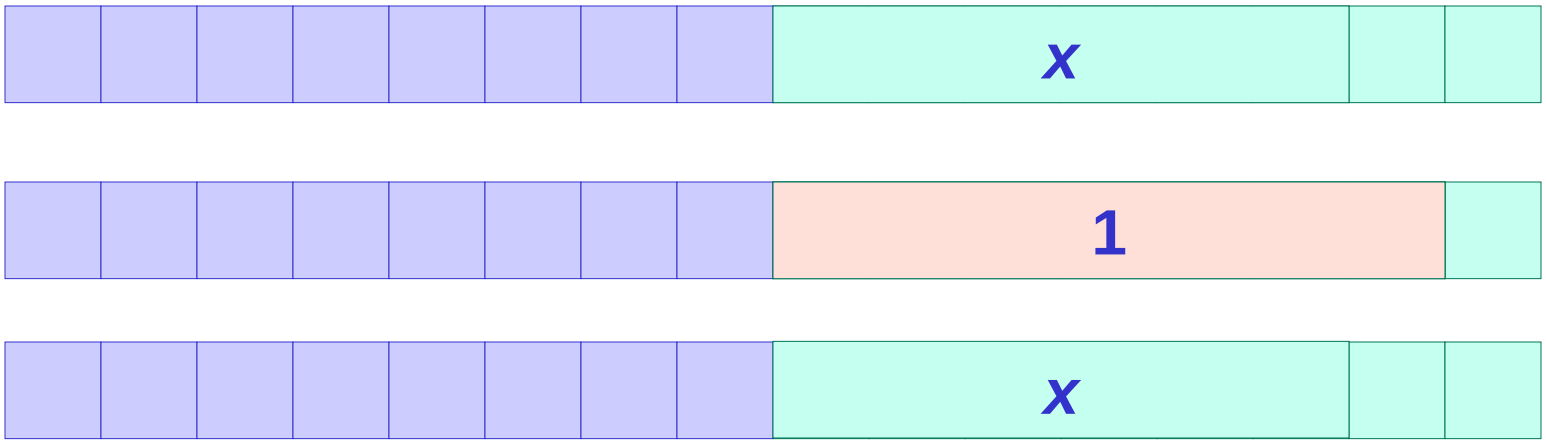


$$\delta(q_1, X_1) =$$

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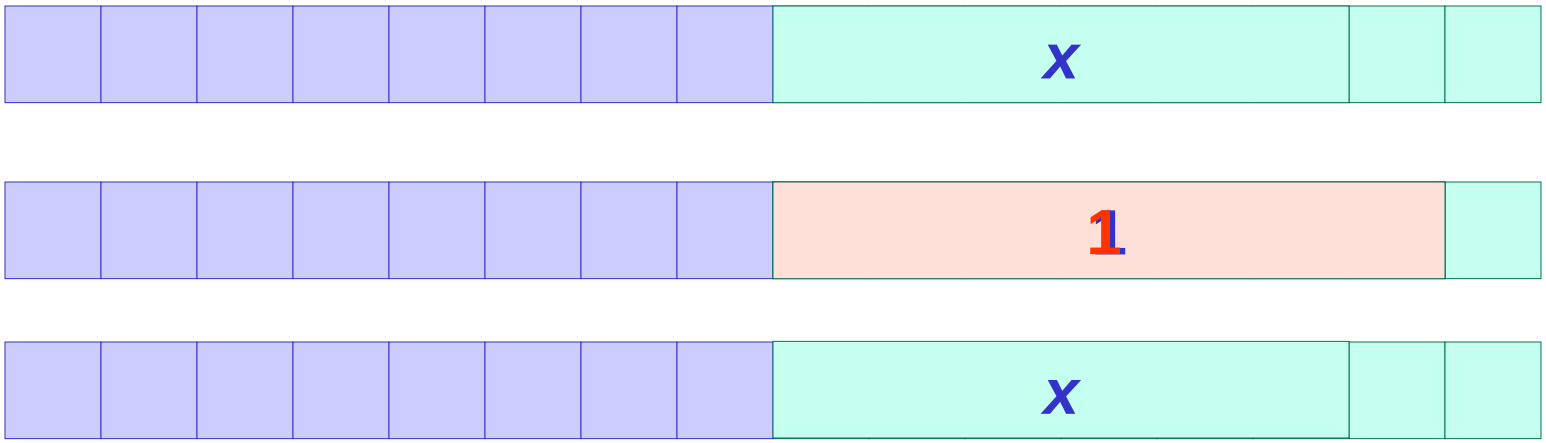


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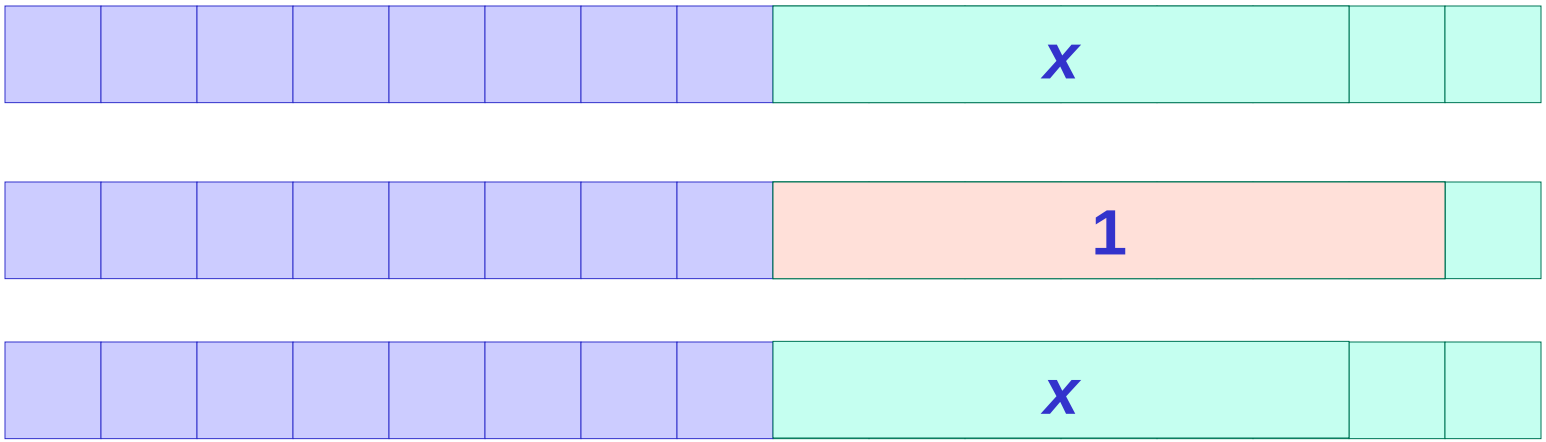


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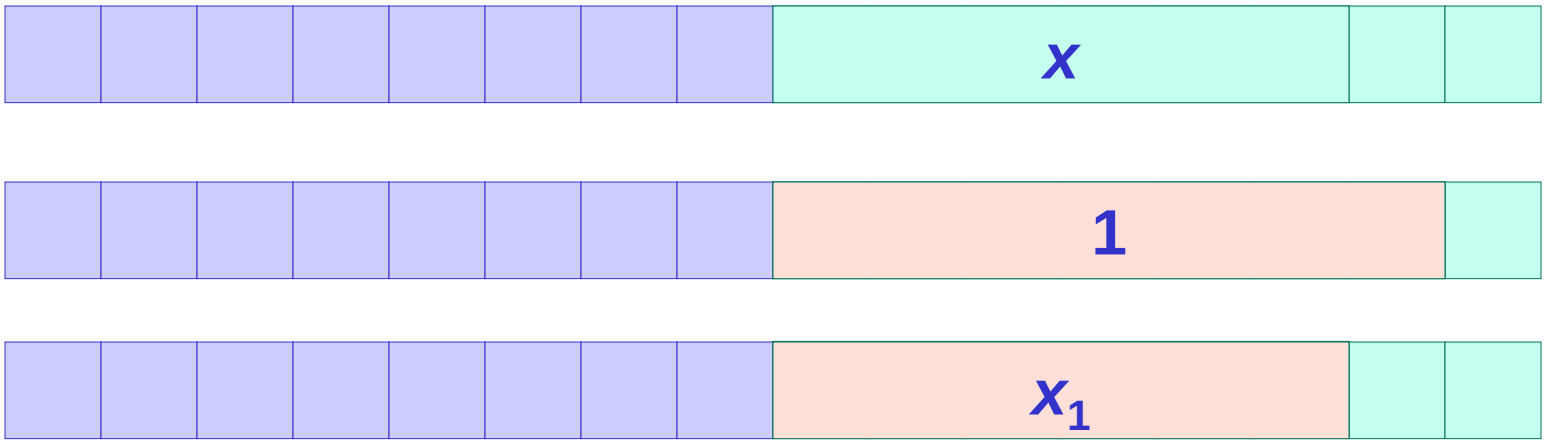


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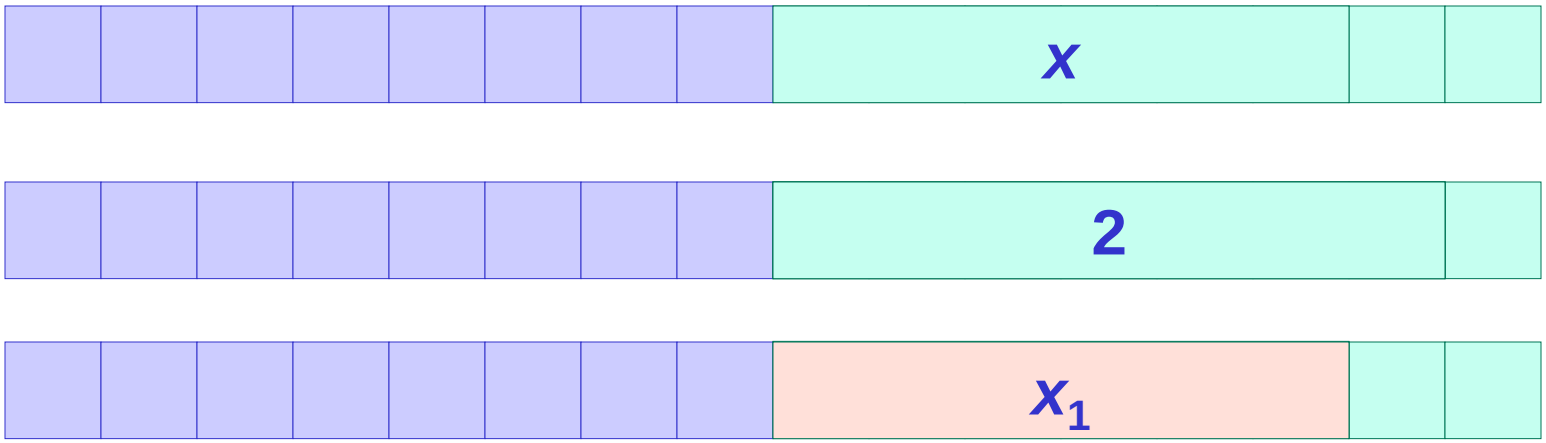


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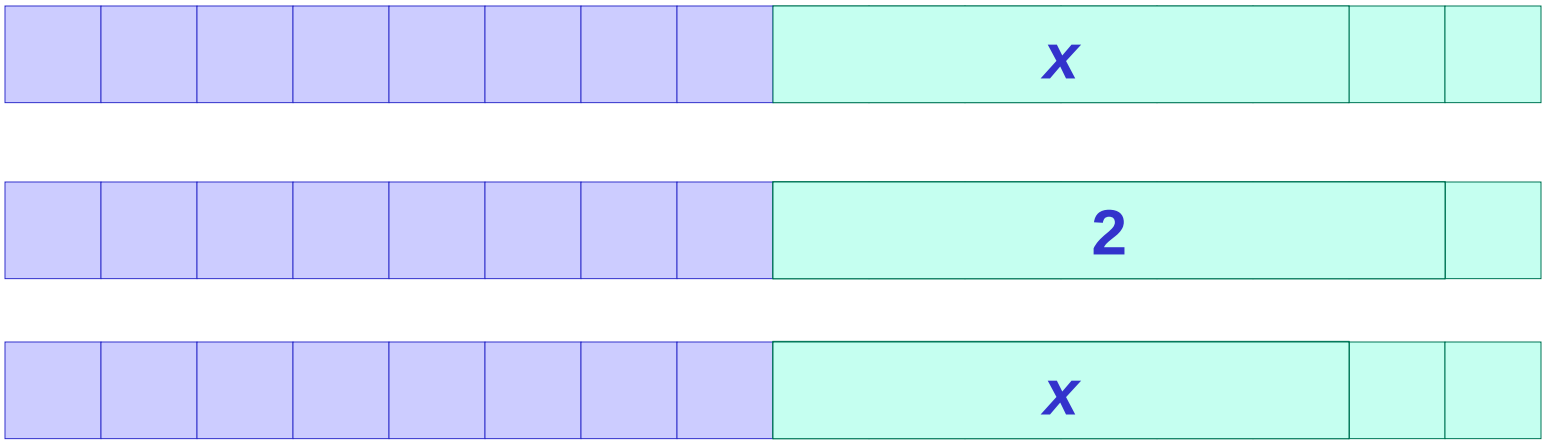


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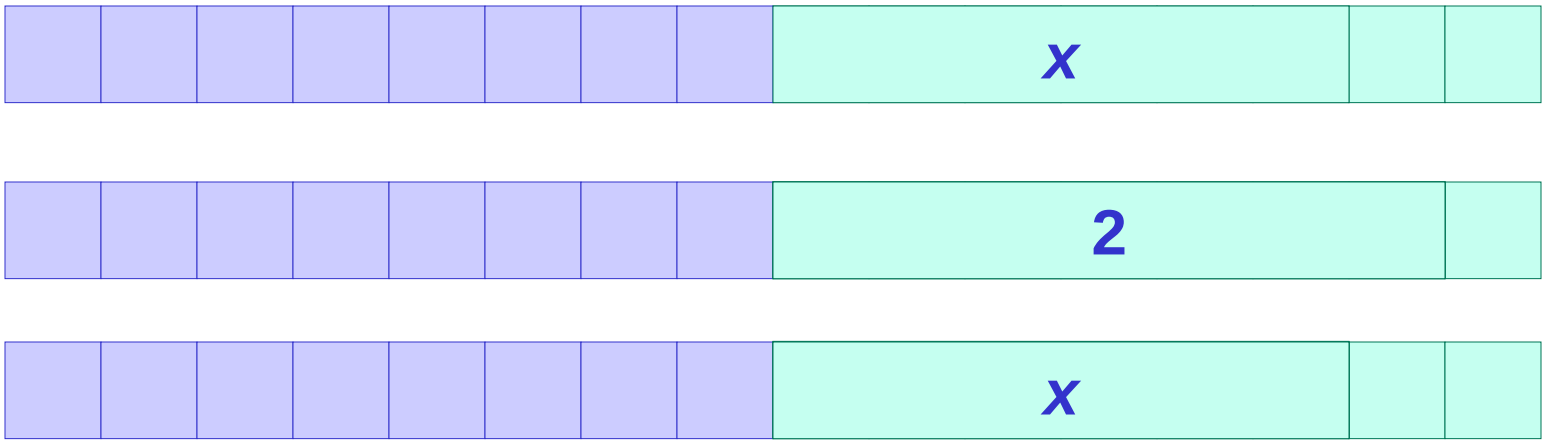


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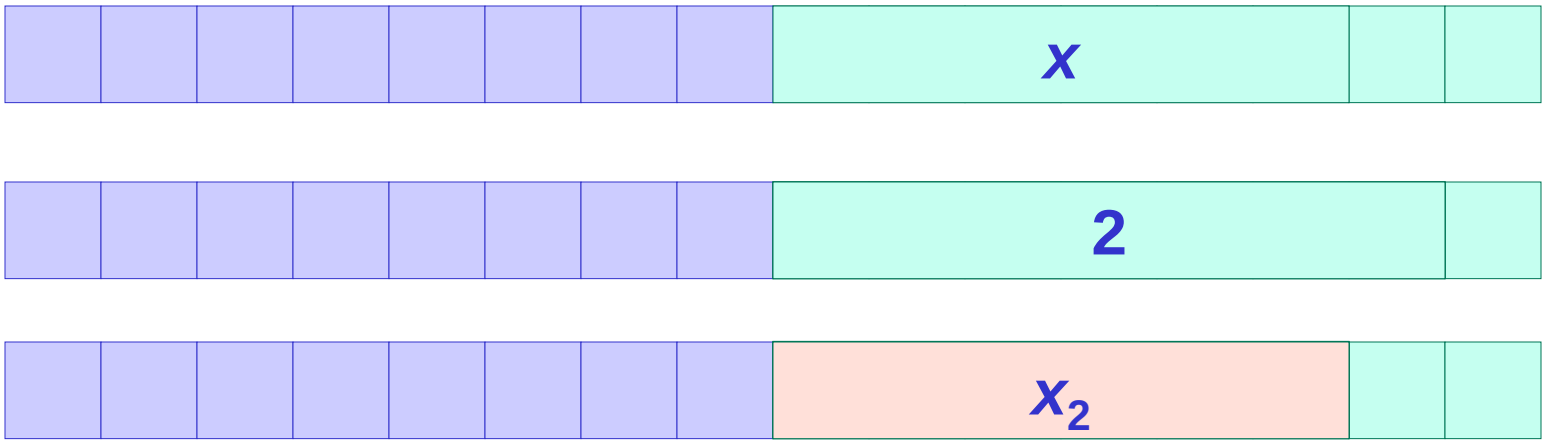


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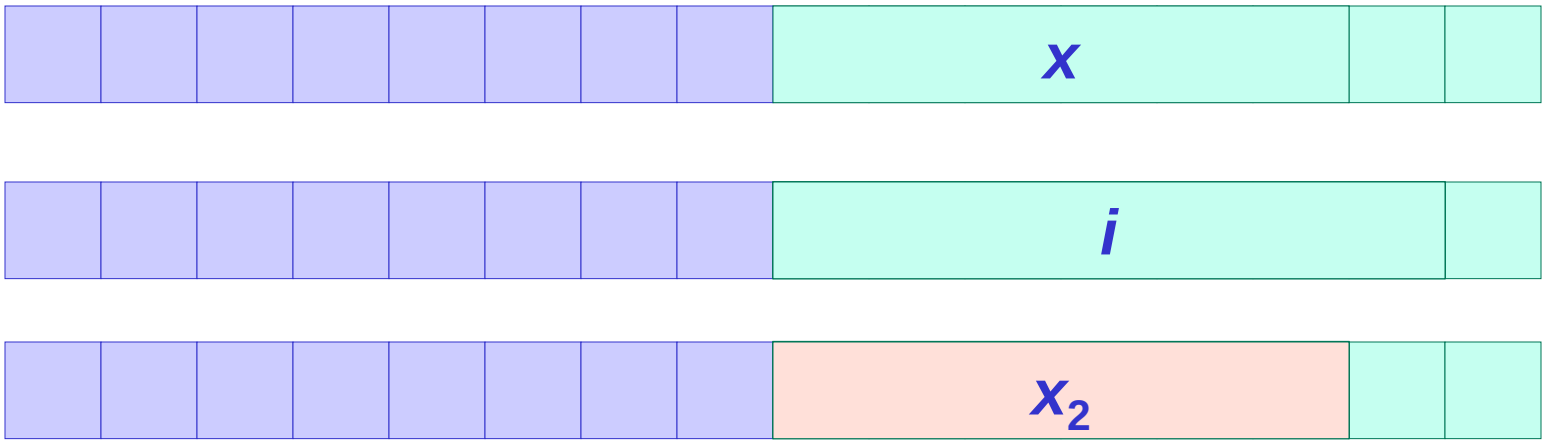


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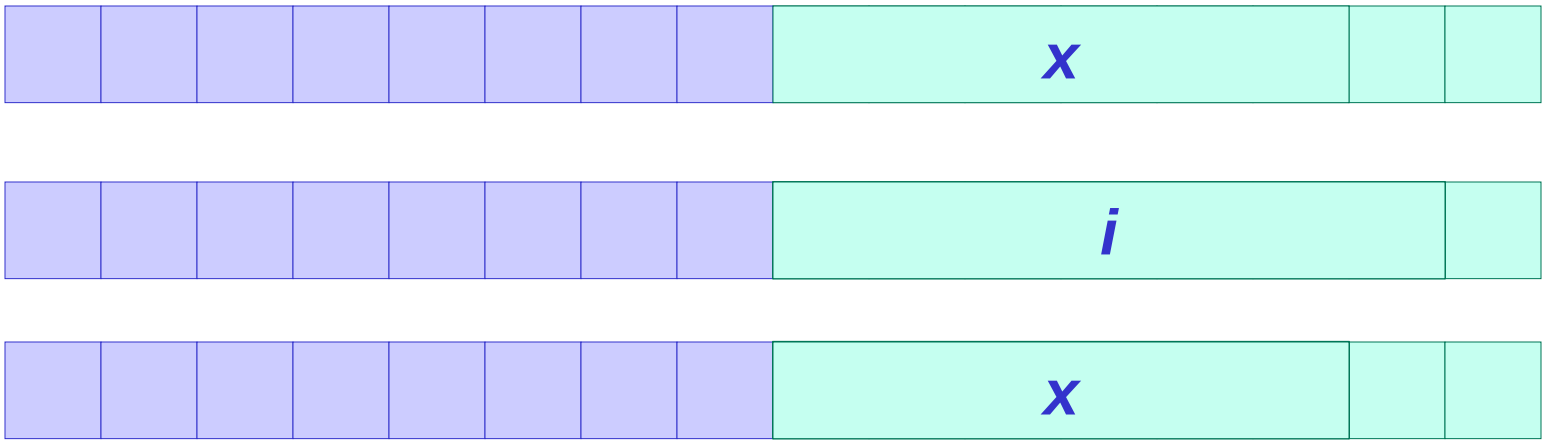


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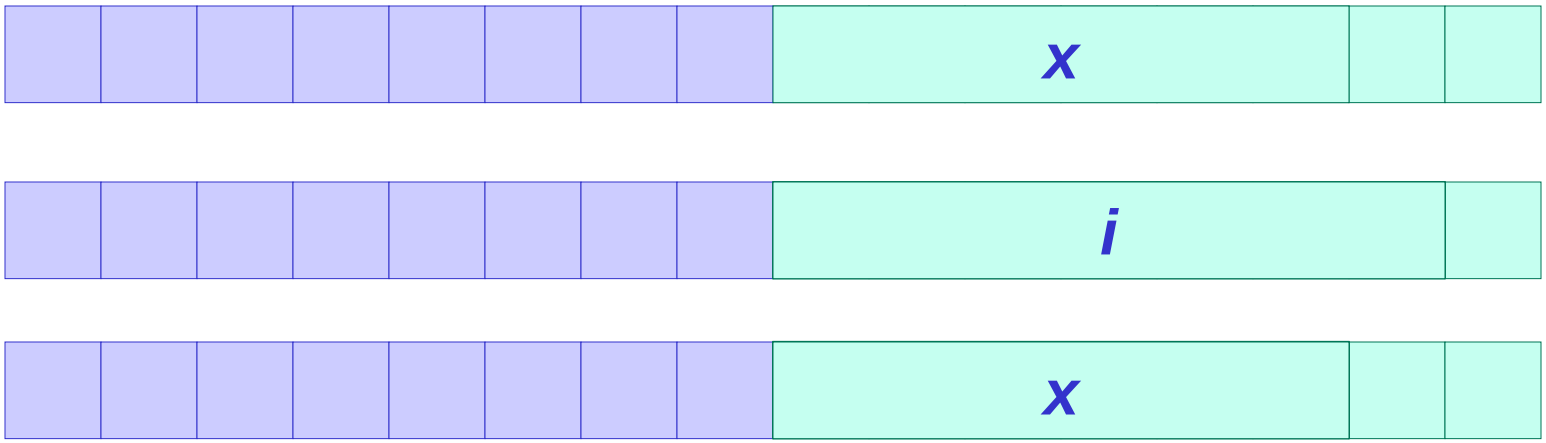


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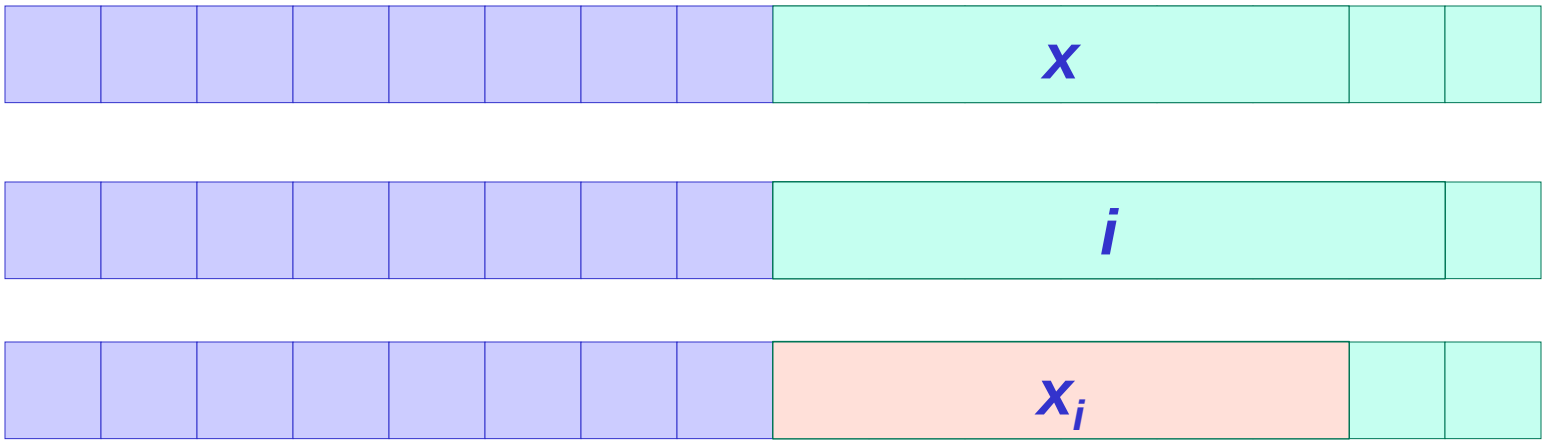


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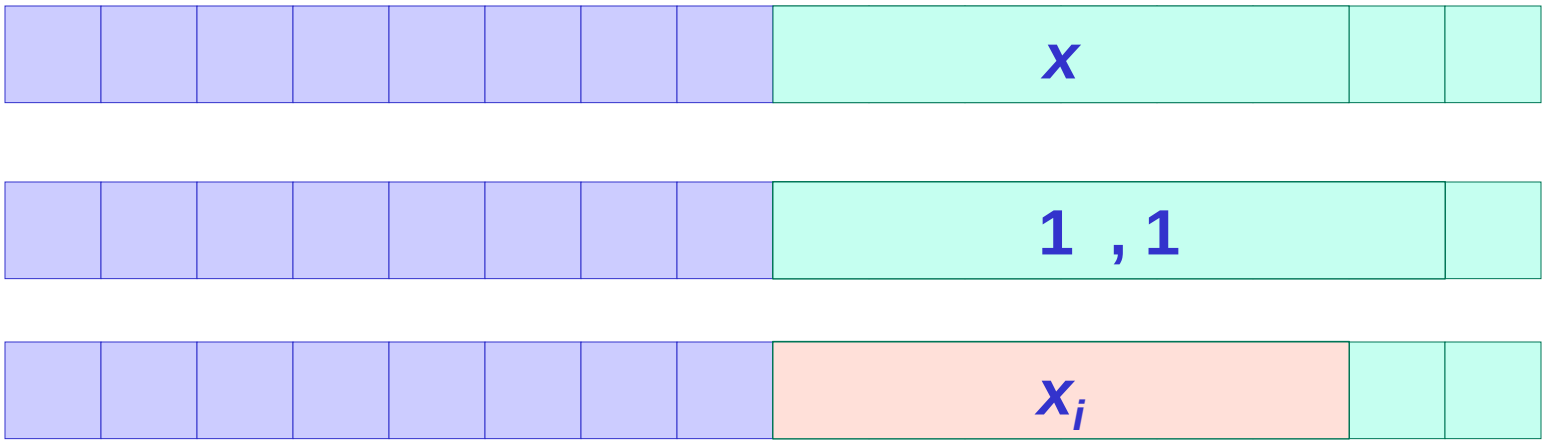


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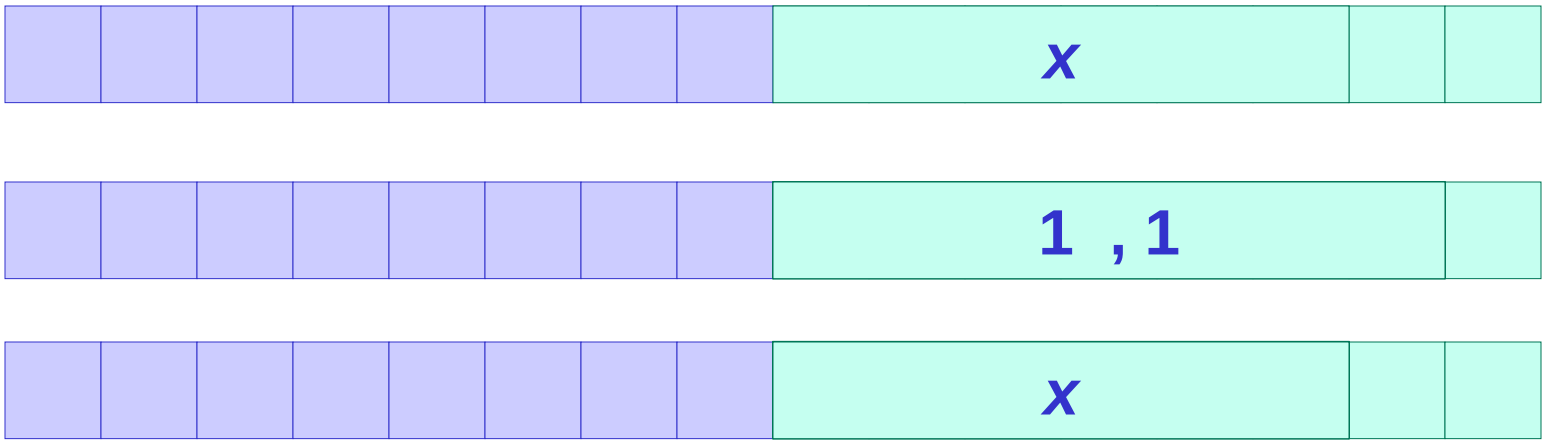


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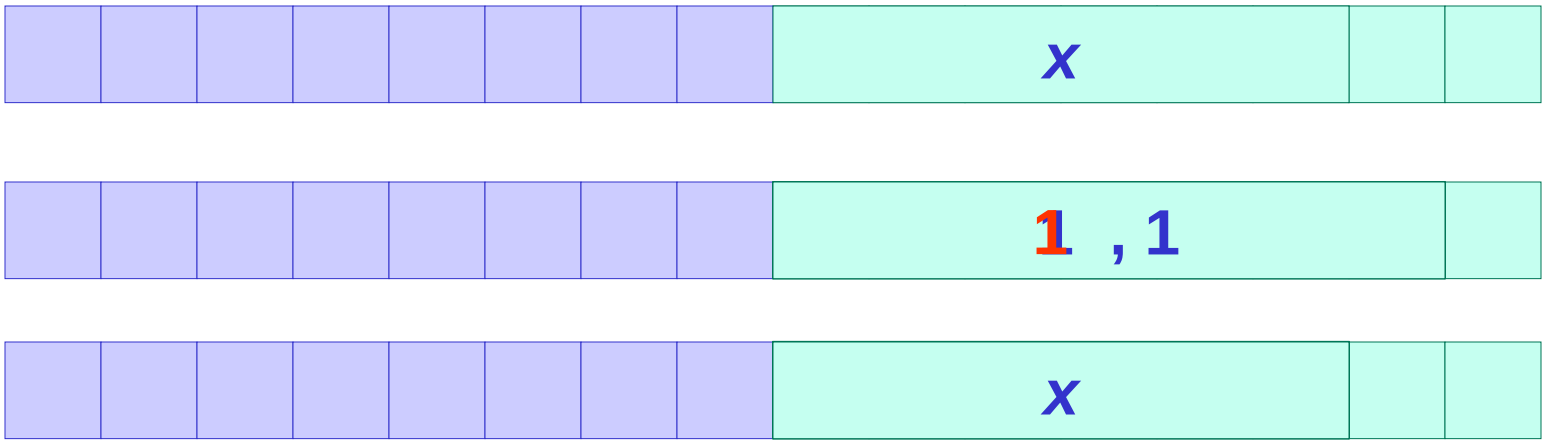


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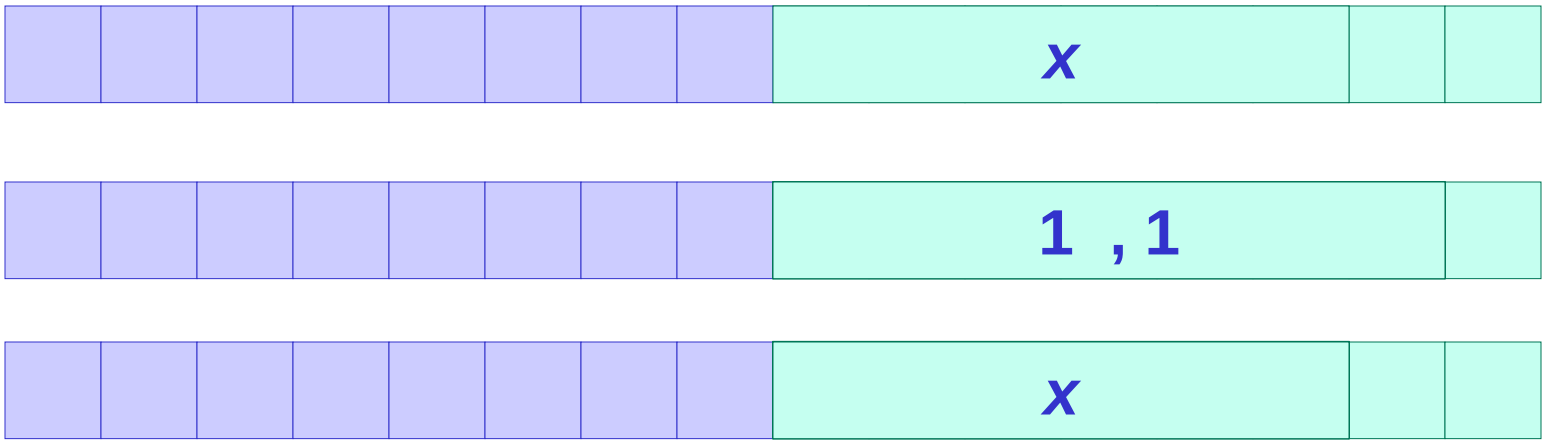


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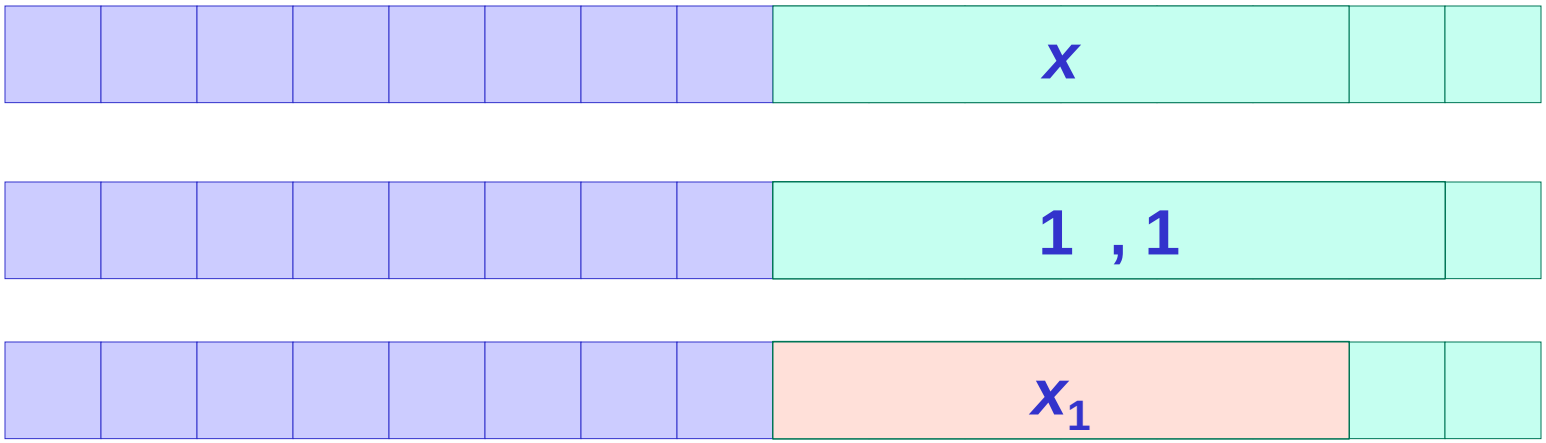


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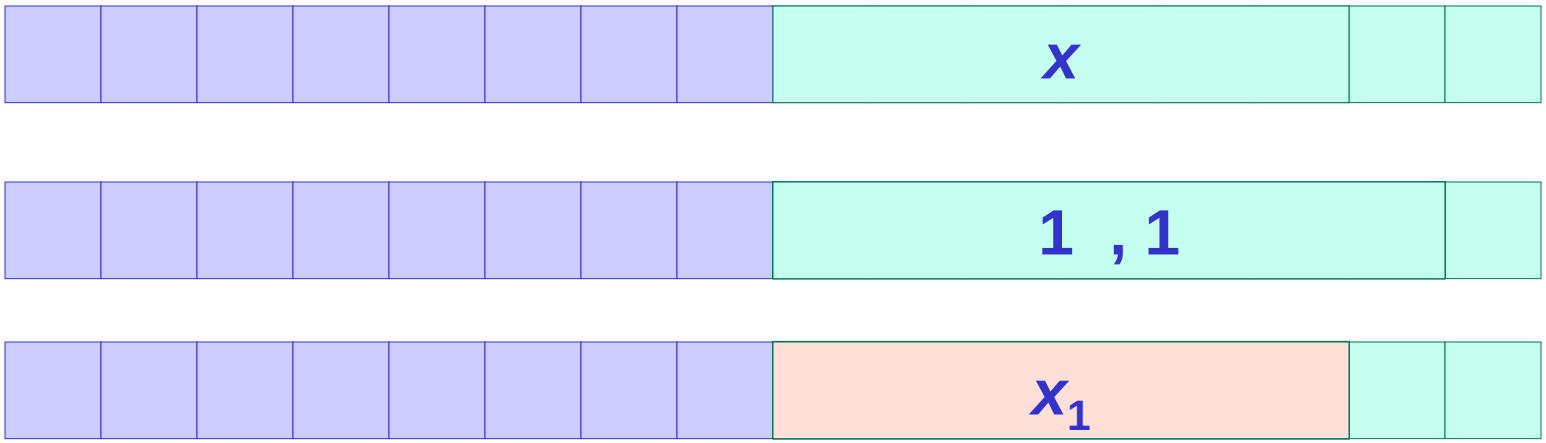


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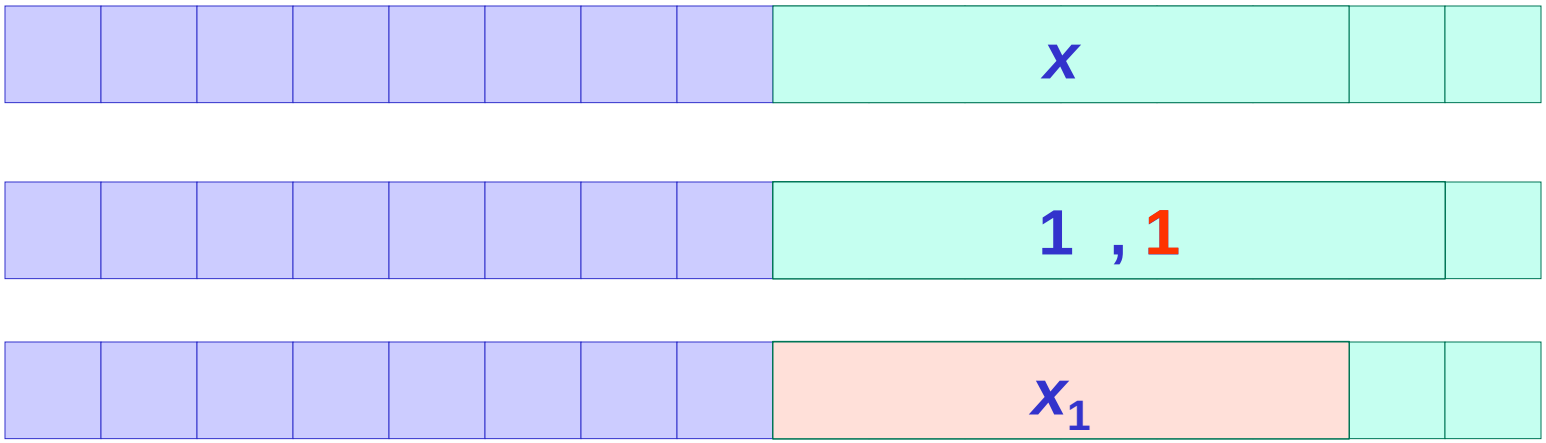


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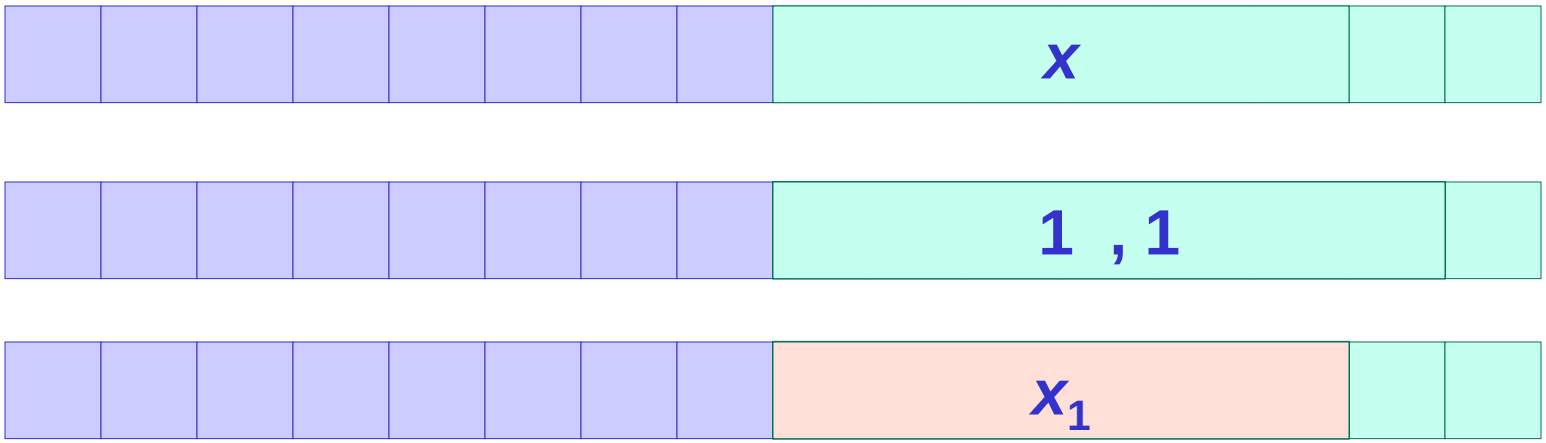


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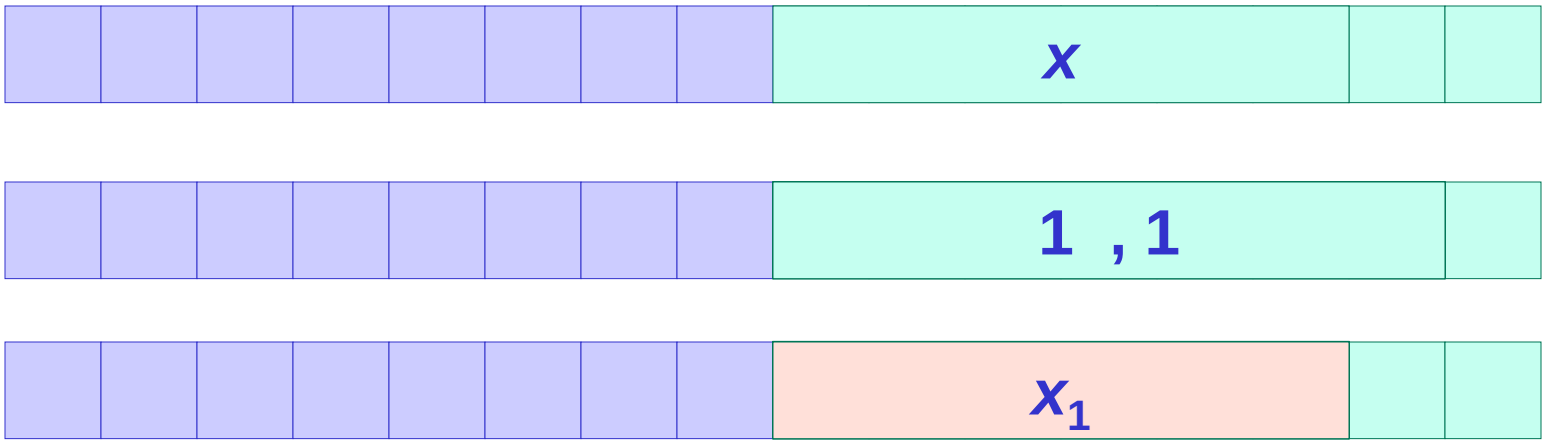


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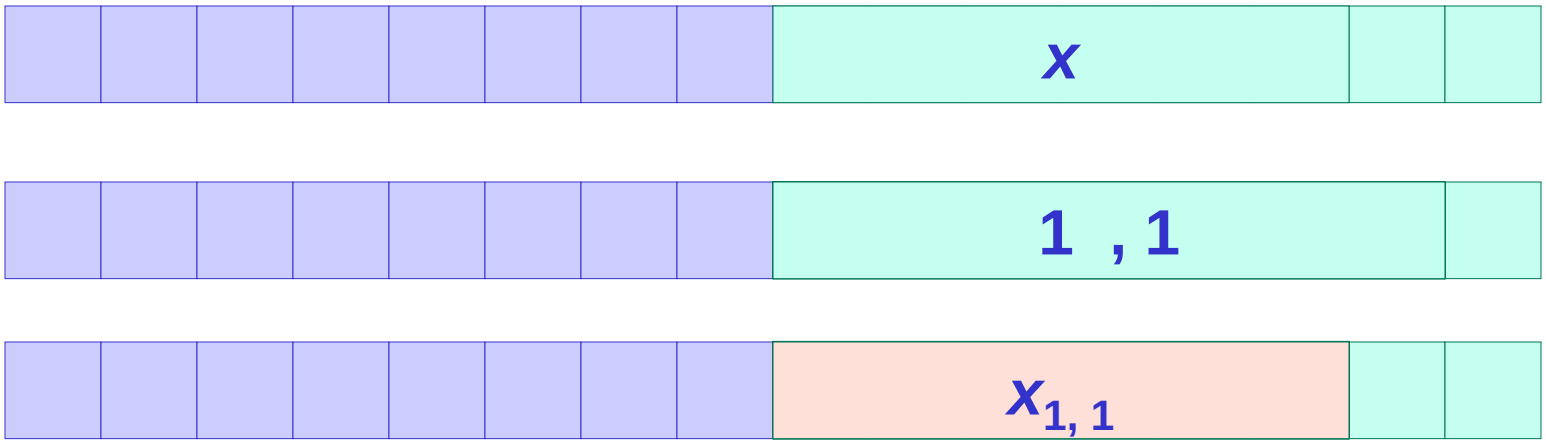


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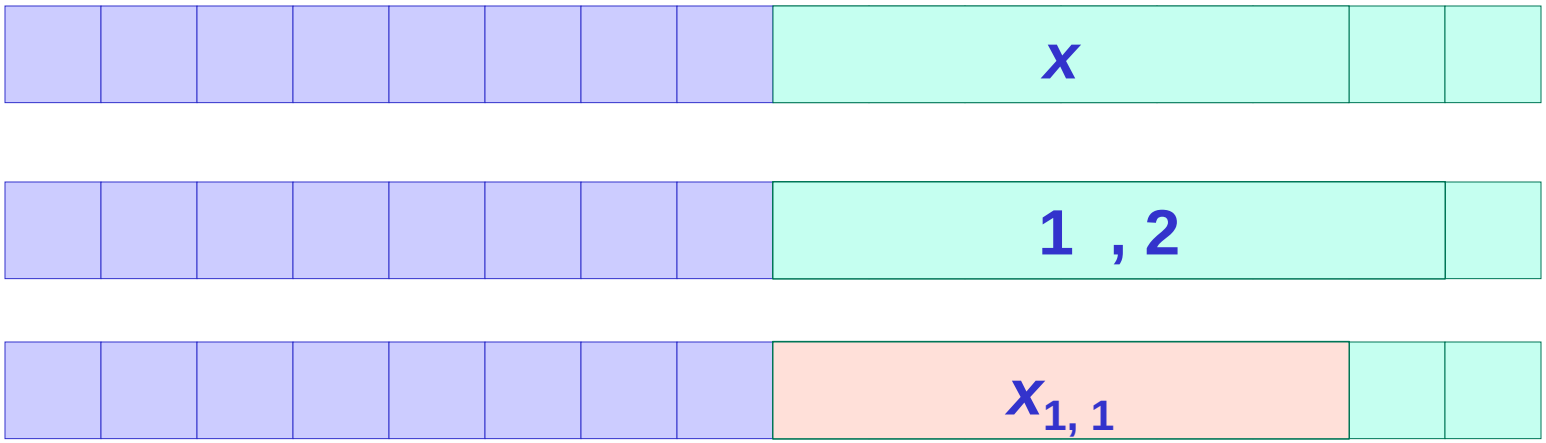


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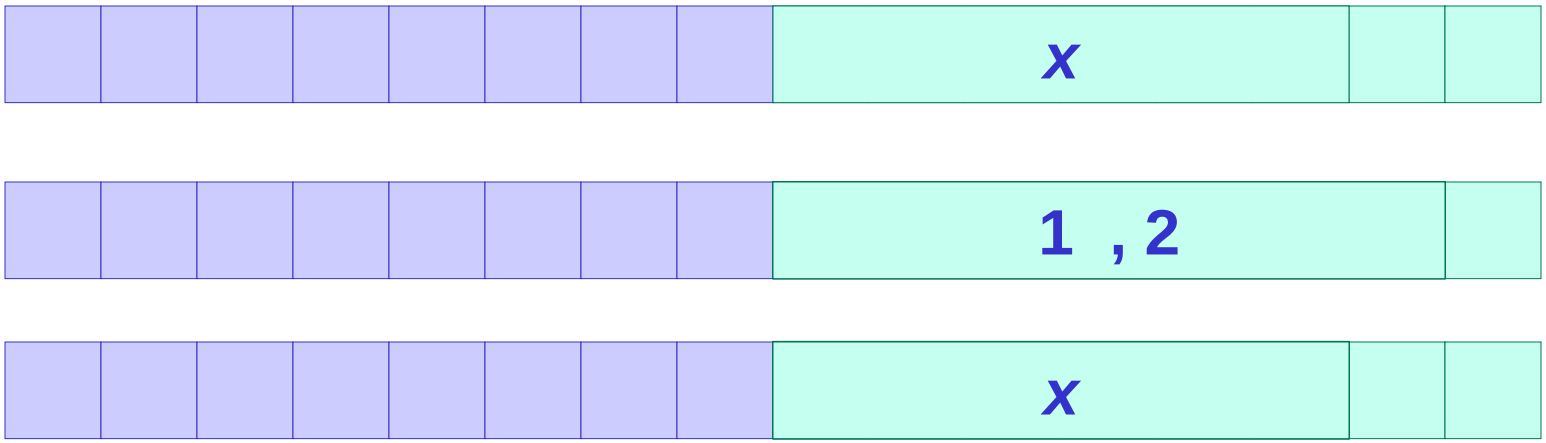


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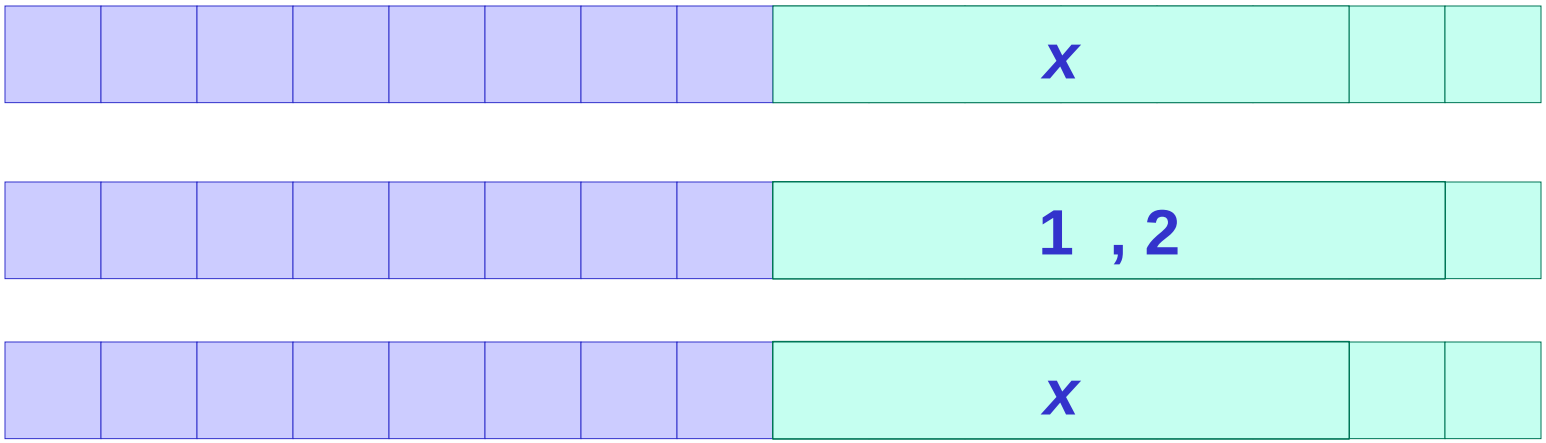


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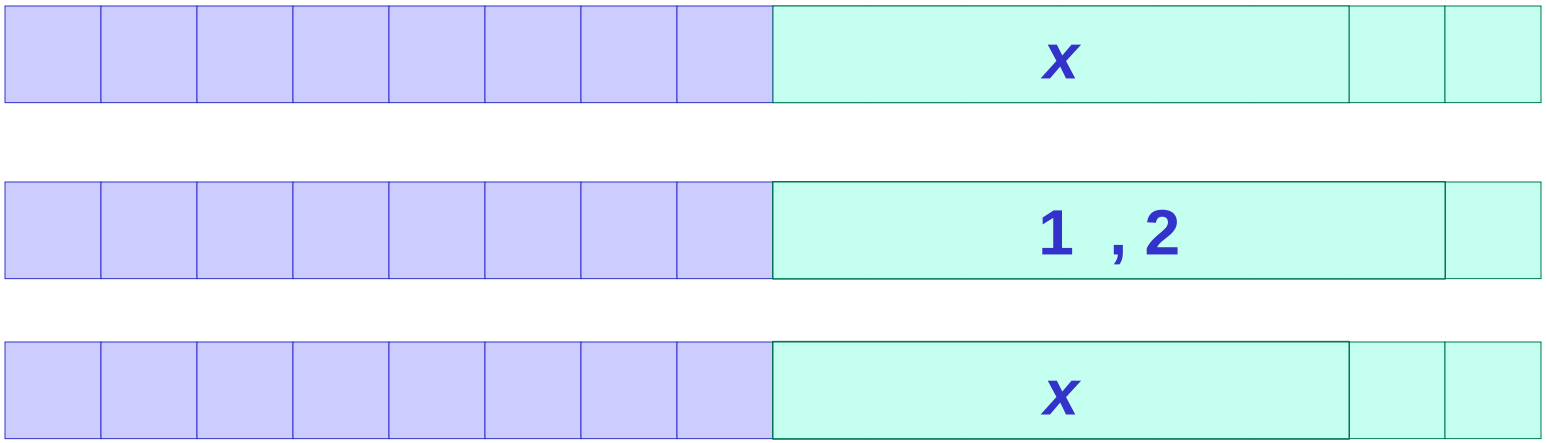


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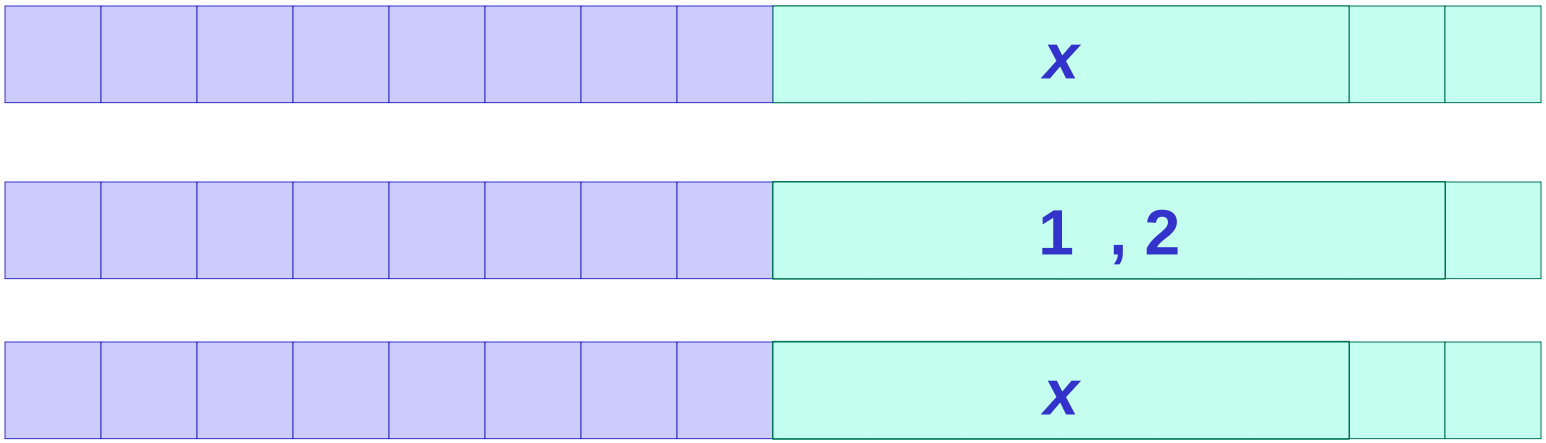


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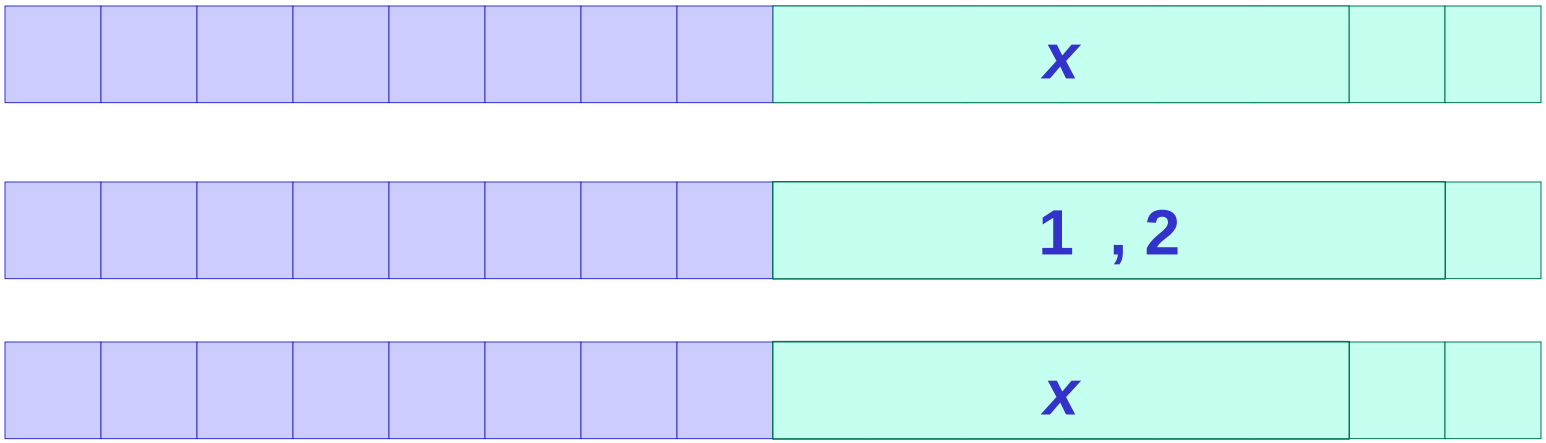


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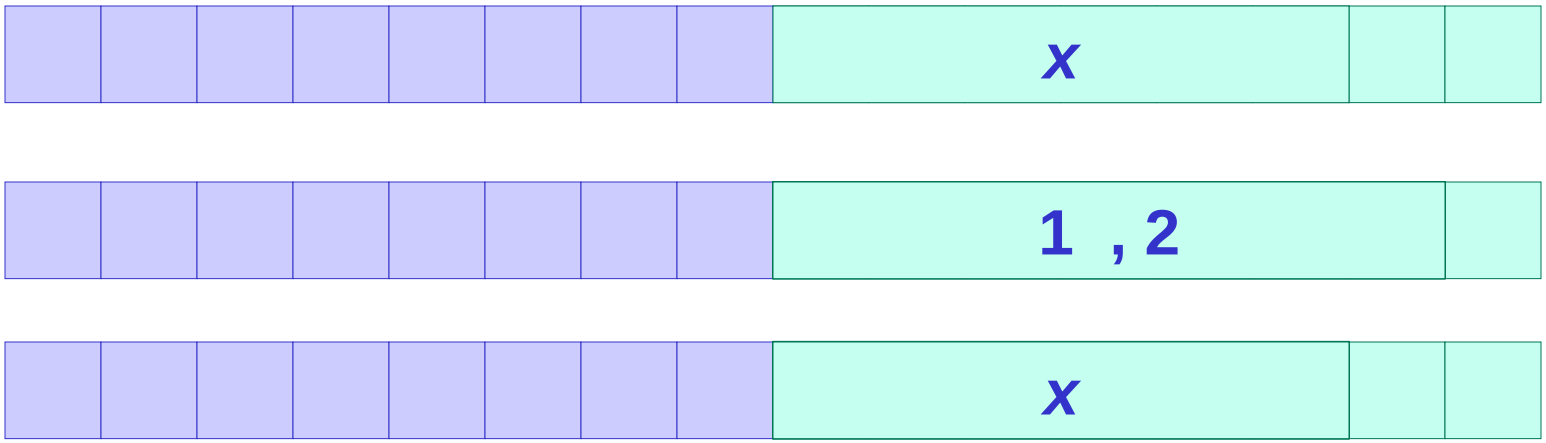
# Nondeterministic TM




$$\delta(p_1, X_2) =$$

$$\{ (r_1, T_1, Y_1) \quad , \quad (r_2, T_2, Y_2) \quad , \ldots, \quad (r_{ni}, T_{ni}, Y_{ni}) \quad , \ldots, (r_k, T_k, Y_k) \quad \}$$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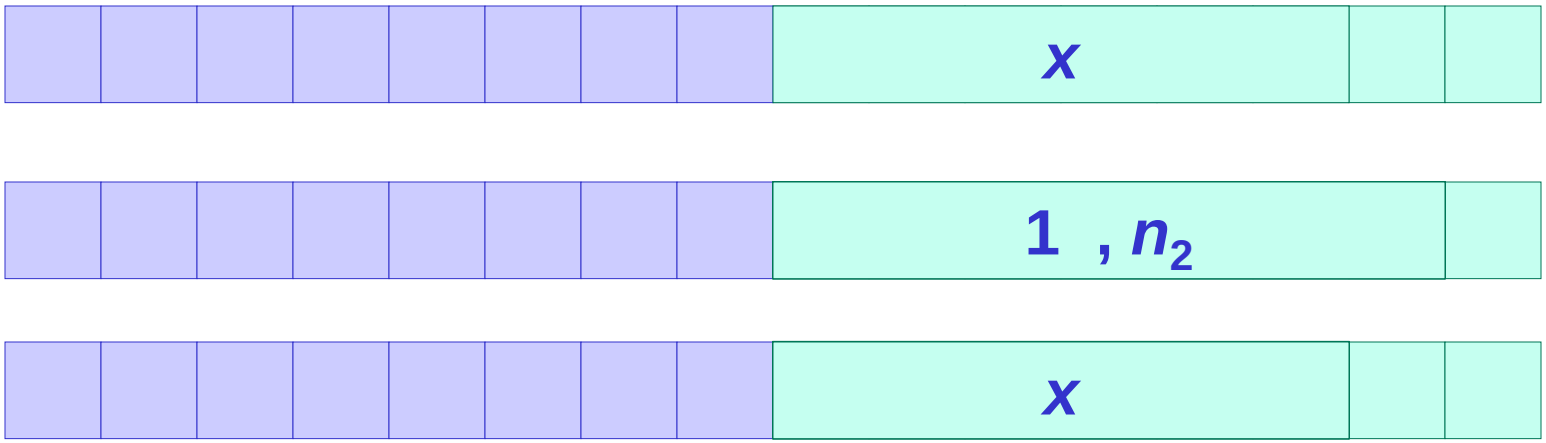


$$\delta(p_1, X_2) =$$

$$\{ (r_1, T_1, Y_1) \quad , \quad (r_2, T_2, Y_2) \quad , \ldots, \quad (r_{ni}, T_{ni}, Y_{ni}) \quad , \ldots, (r_k, T_k, Y_k) \quad \}$$


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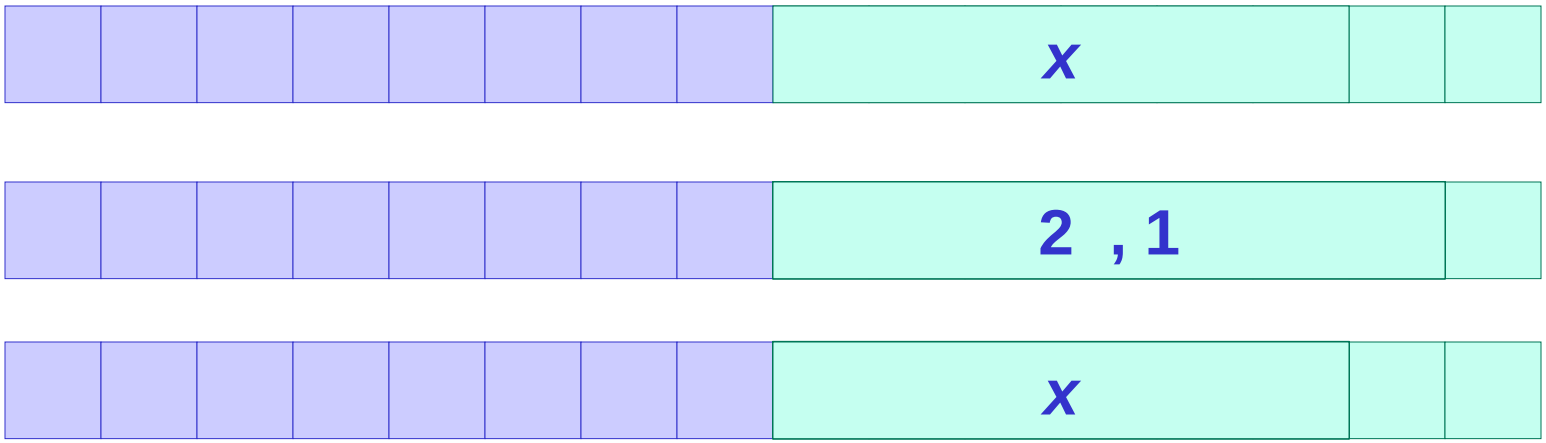


$$\delta(p_1, X_2) =$$

$$\{ (r_1, T_1, Y_1) \quad , \quad (r_2, T_2, Y_2) \quad , \ldots, \quad (r_{ni}, T_{ni}, Y_{ni}) \quad , \ldots, (r_k, T_k, Y_k) \quad \}$$


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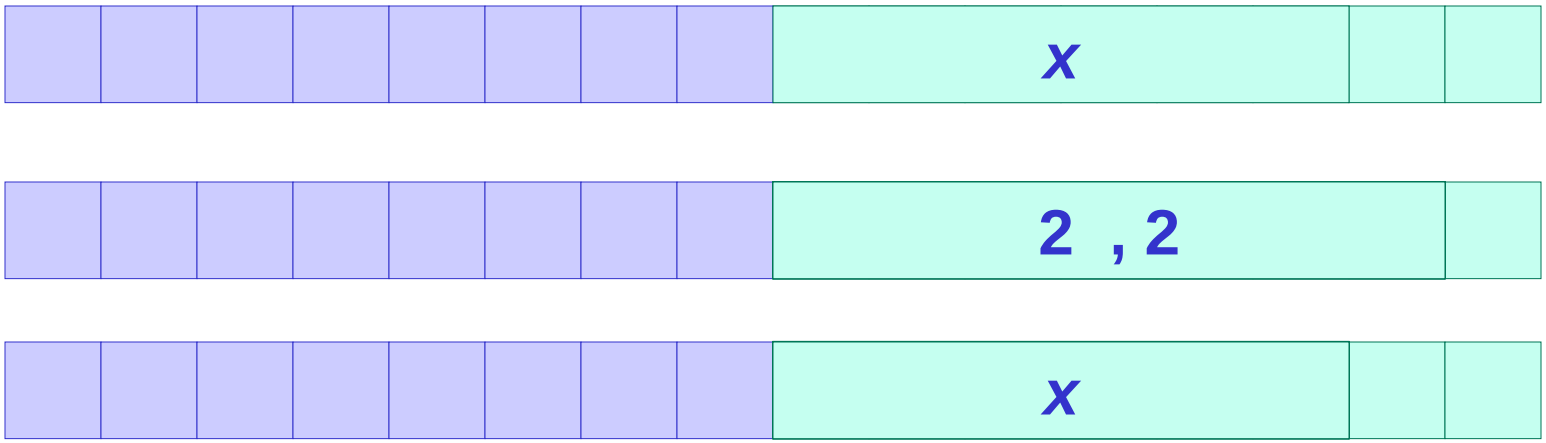


$$\delta(p_1, X_2) =$$

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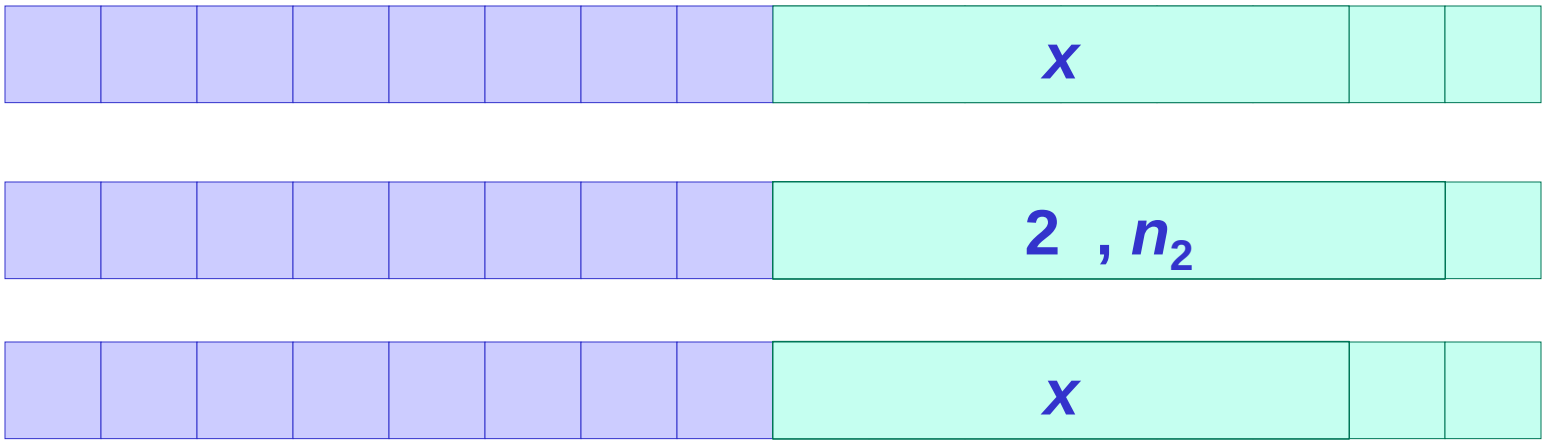


$$\delta(p_1, X_2) =$$

$$\{ (r_1, T_1, Y_1) \quad , \quad (r_2, T_2, Y_2) \quad , \ldots, \quad (r_{ni}, T_{ni}, Y_{ni}) \quad , \ldots, (r_k, T_k, Y_k) \quad \}$$


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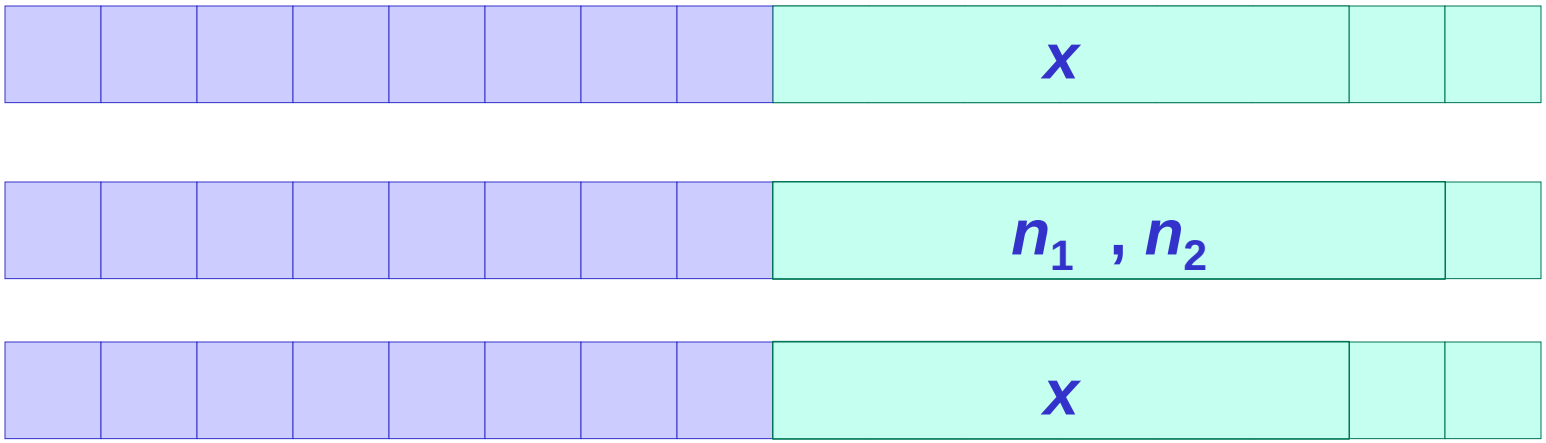


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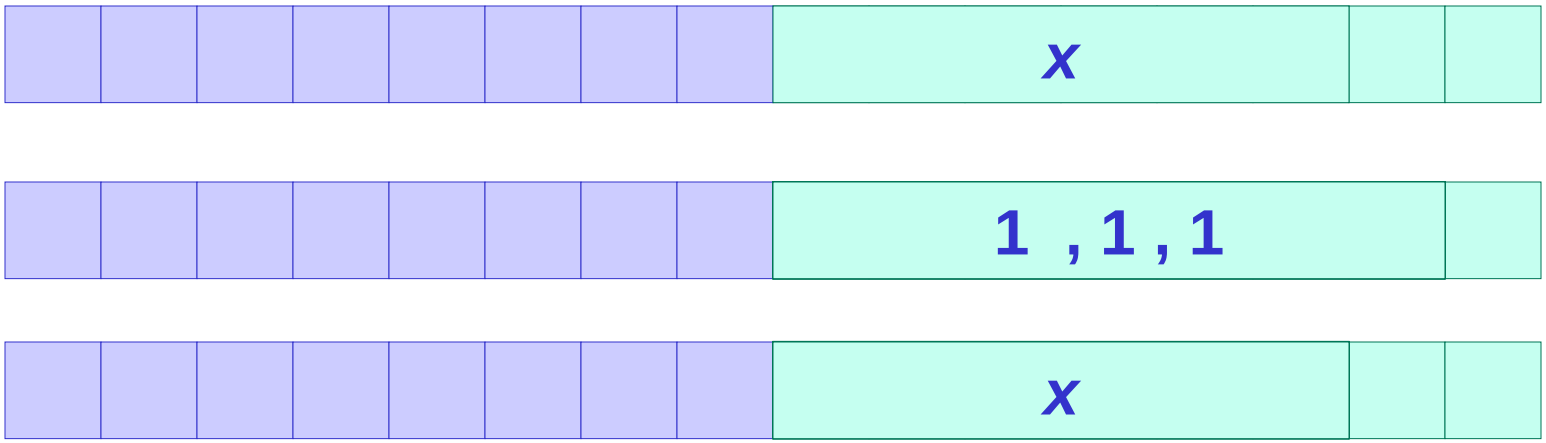


$$\delta(p_1, X_2) =$$

$$\{ (r_1, T_1, Y_1) \quad , \quad (r_2, T_2, Y_2) \quad , \ldots, \quad (r_{ni}, T_{ni}, Y_{ni}) \quad , \ldots, (r_k, T_k, Y_k) \quad \}$$


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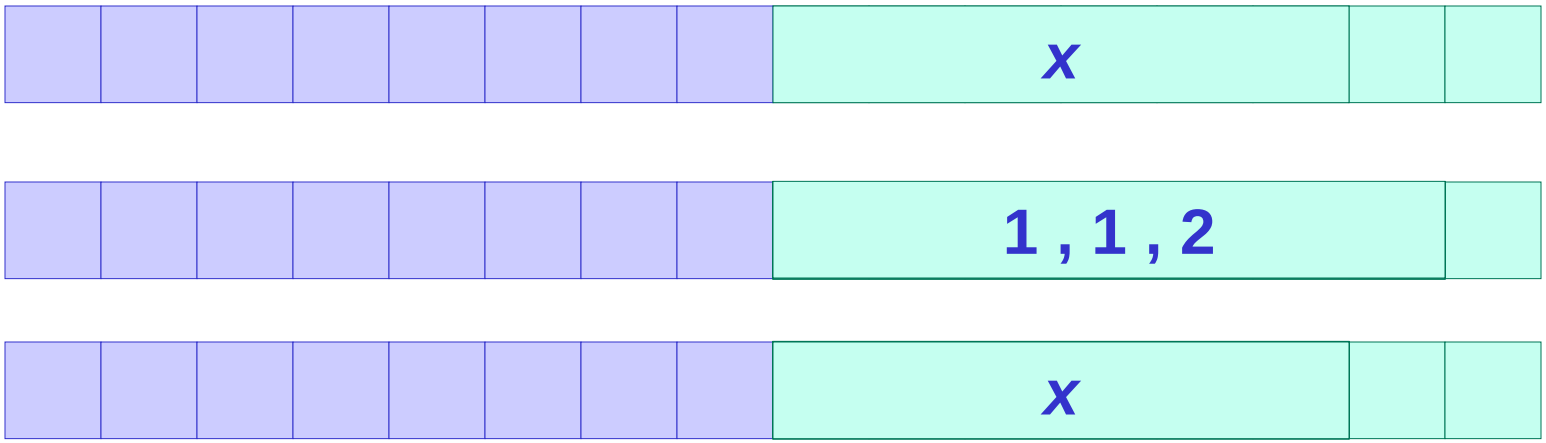


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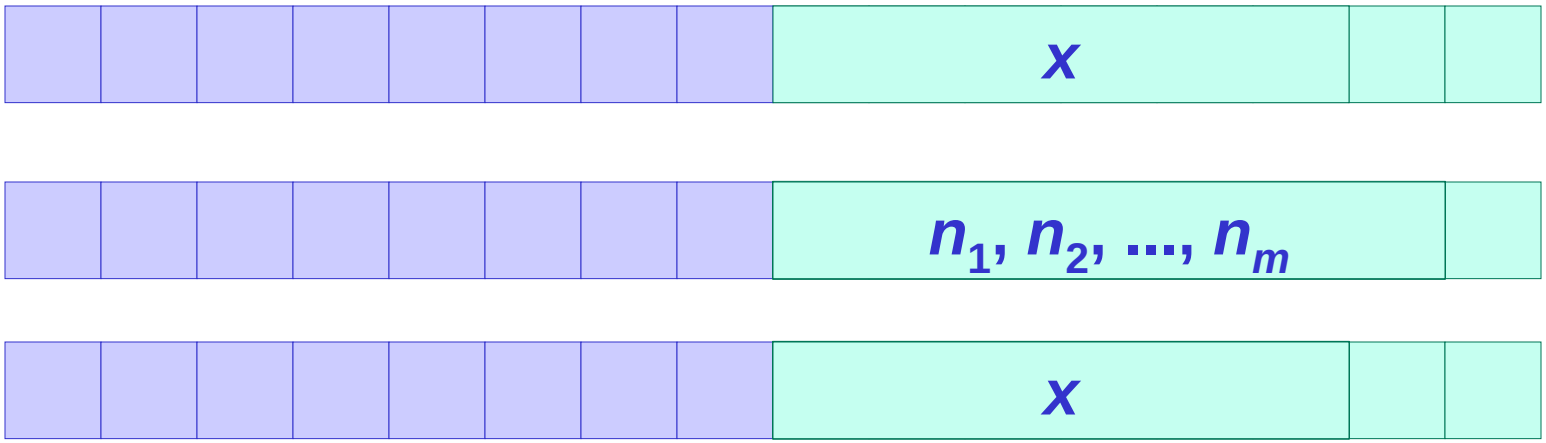


$$\delta(p_1, X_2) =$$

$$\{ (r_1, T_1, Y_1) \quad , \quad (r_2, T_2, Y_2) \quad , ..., \quad (r_{ni}, T_{ni}, Y_{ni}) \quad , ..., (r_k, T_k, Y_k) \quad \}$$


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# TM with a multidimensional input


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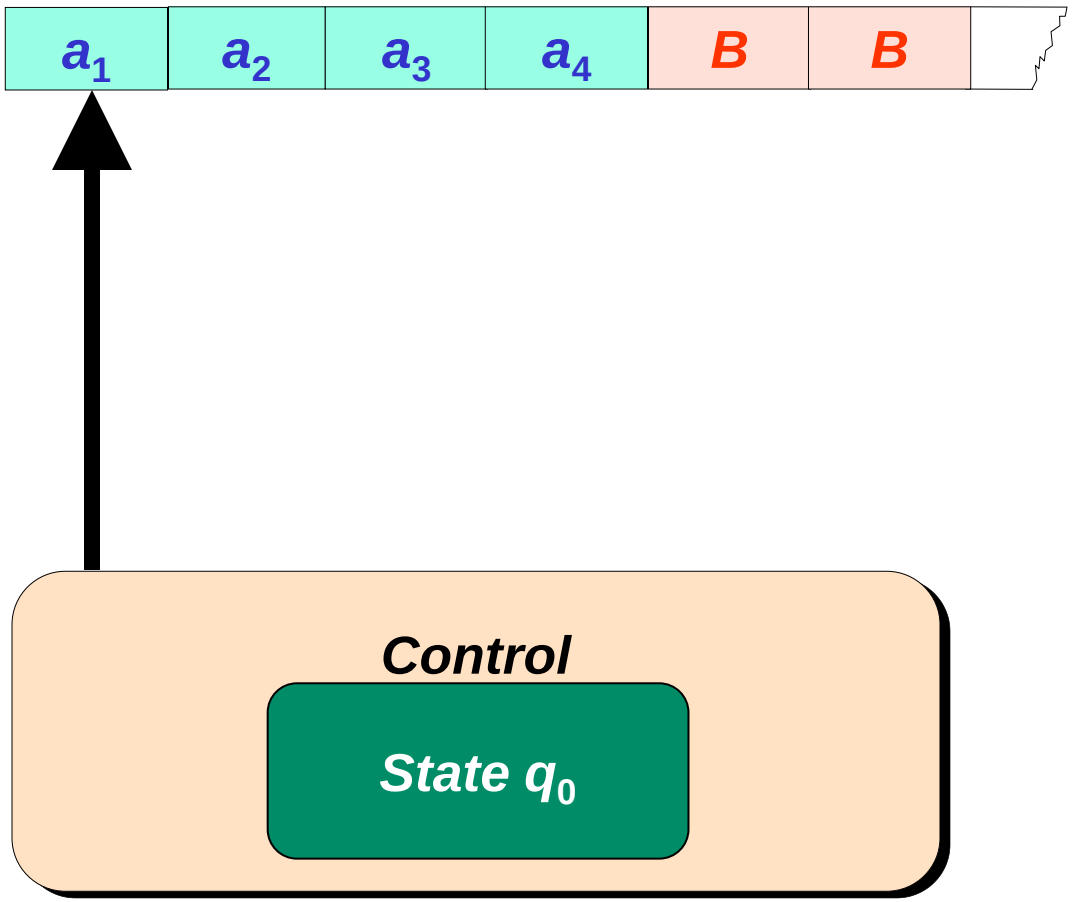
# TM with a multidimensional input





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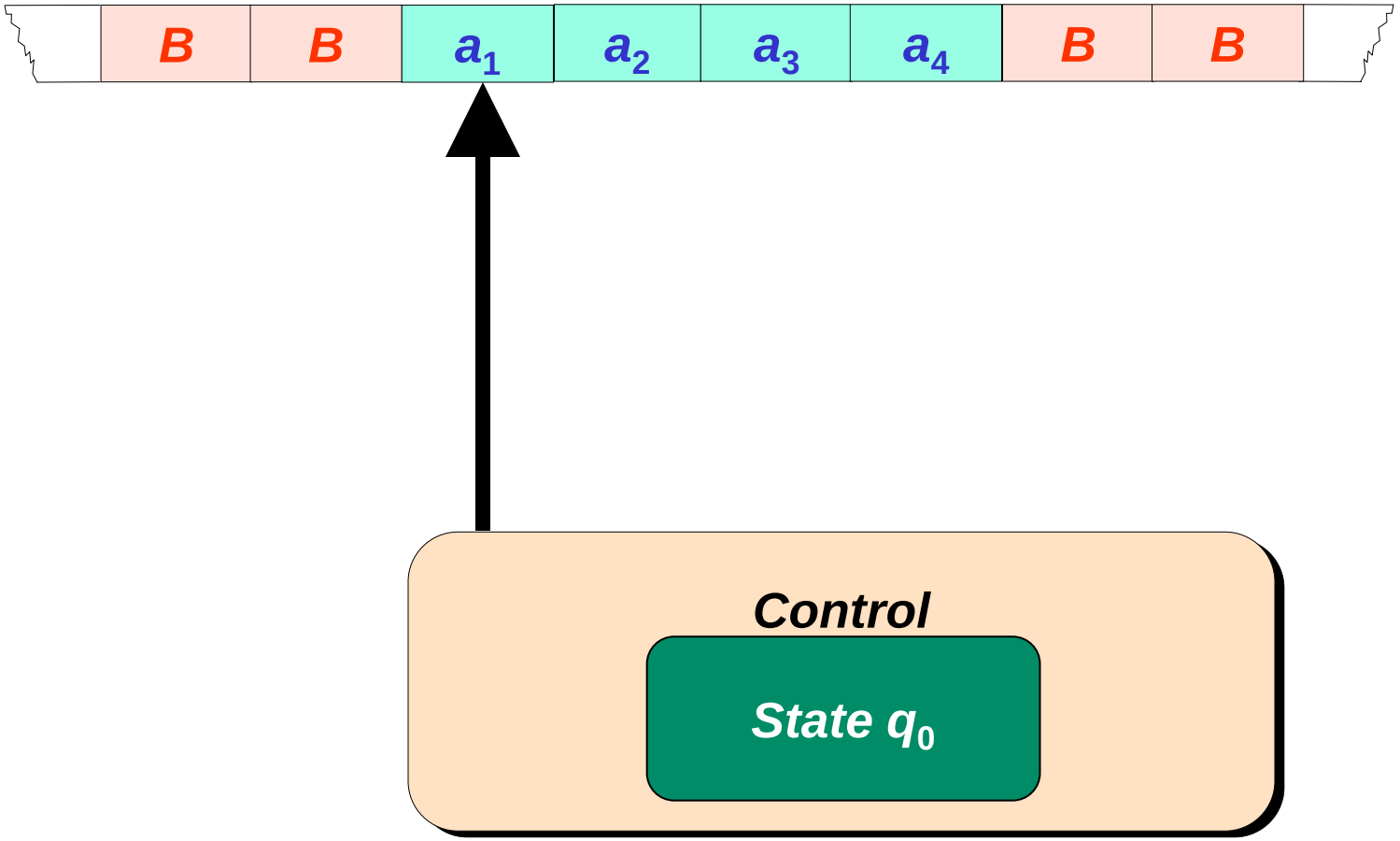



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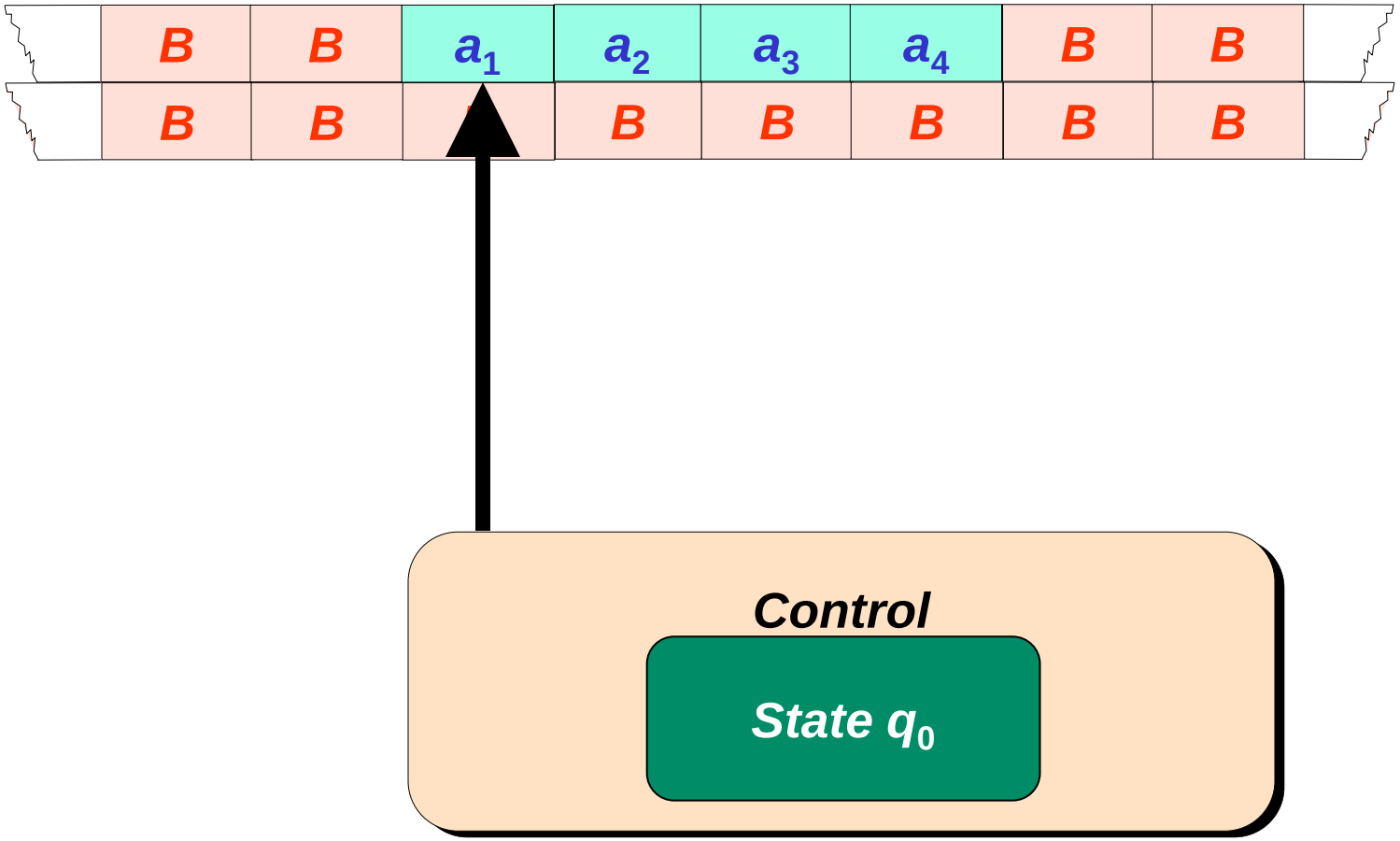



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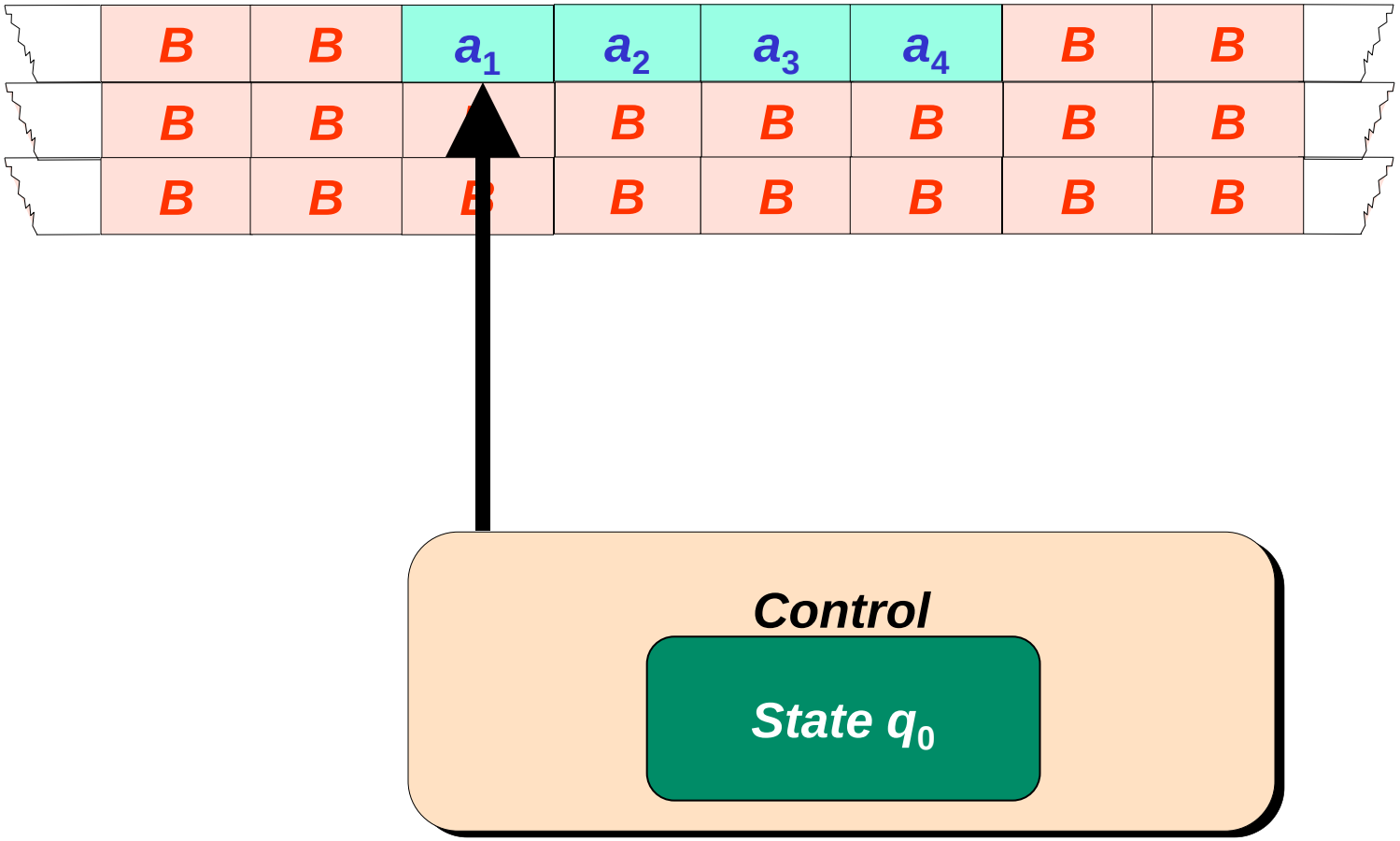



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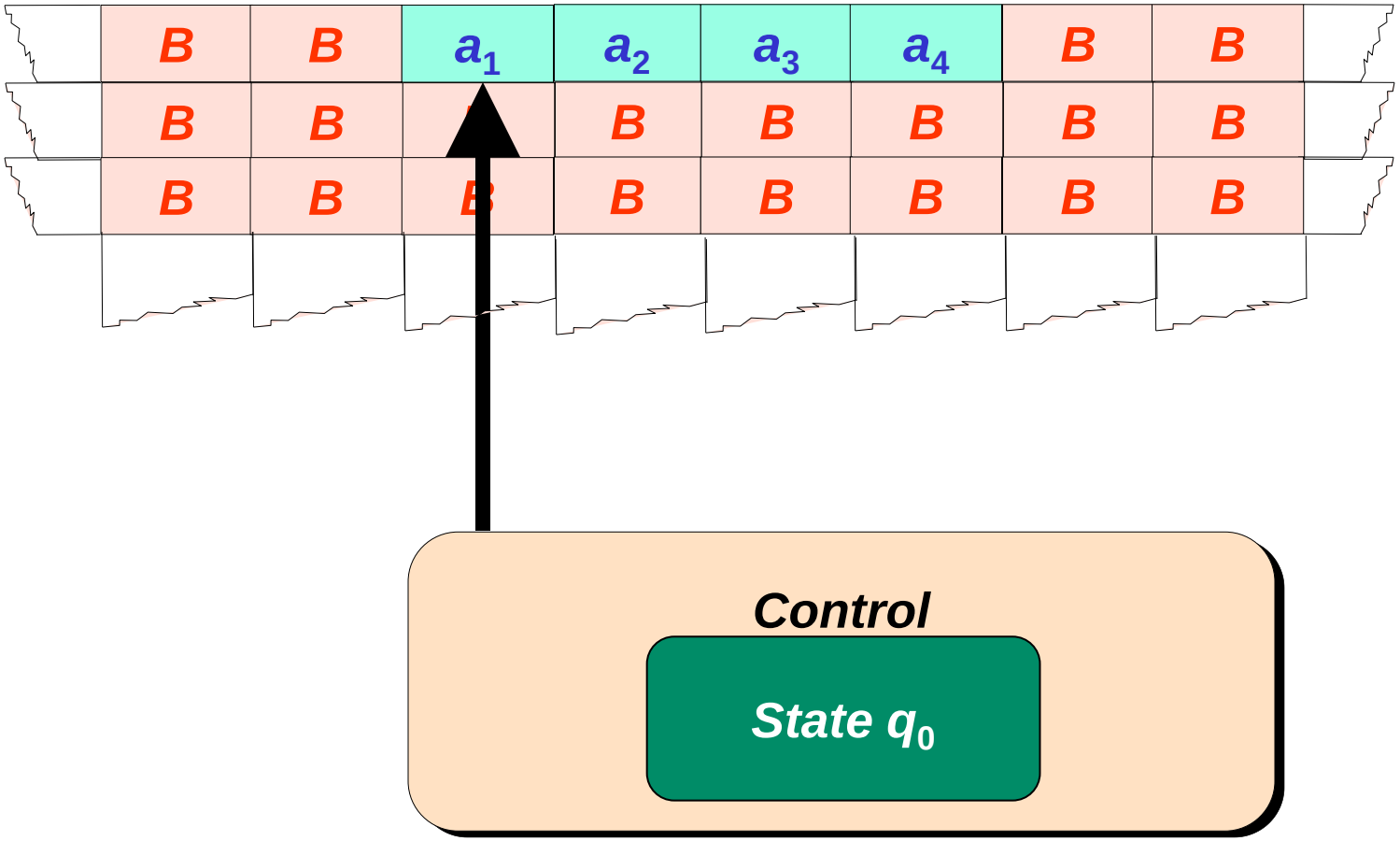



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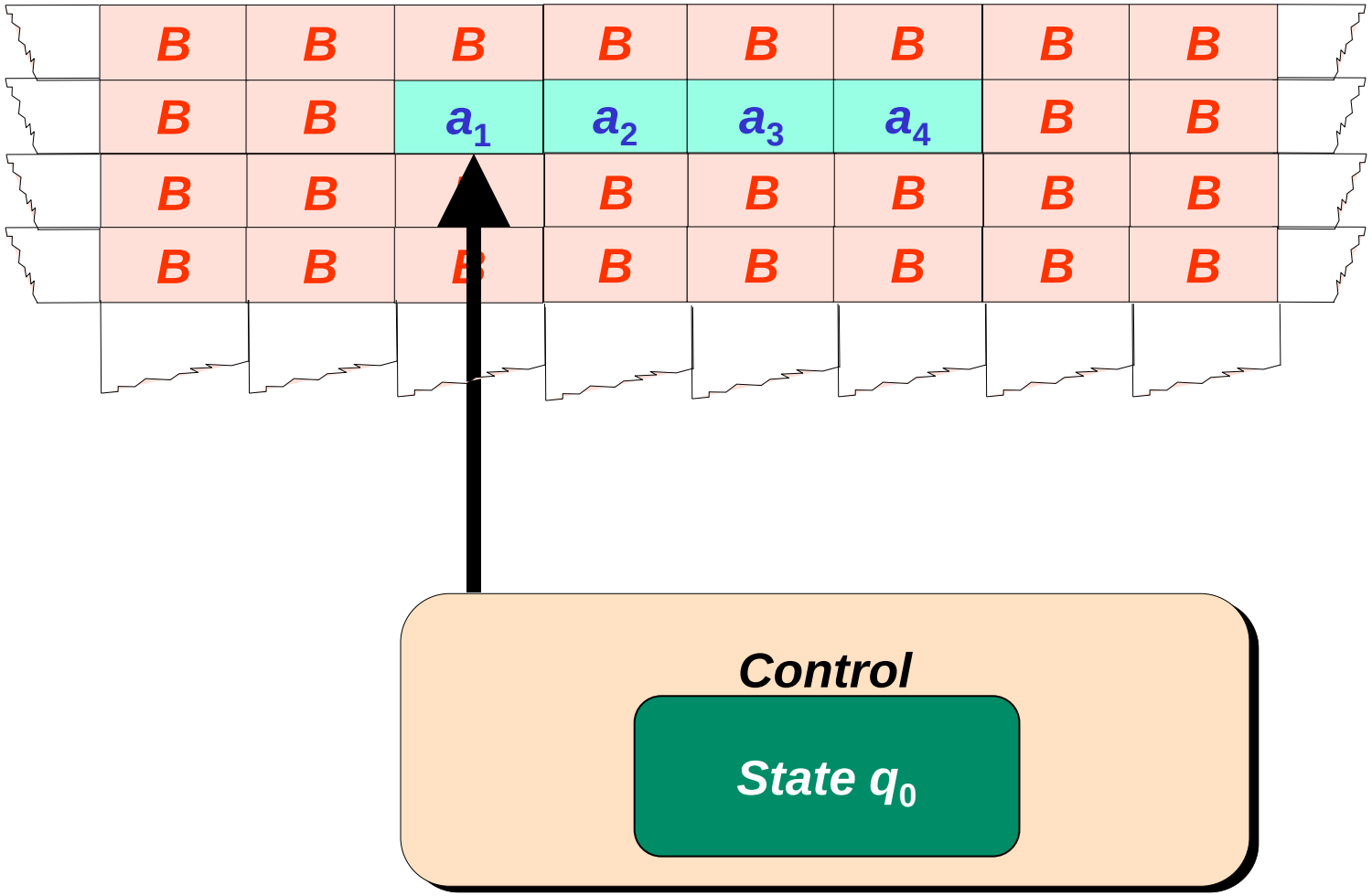



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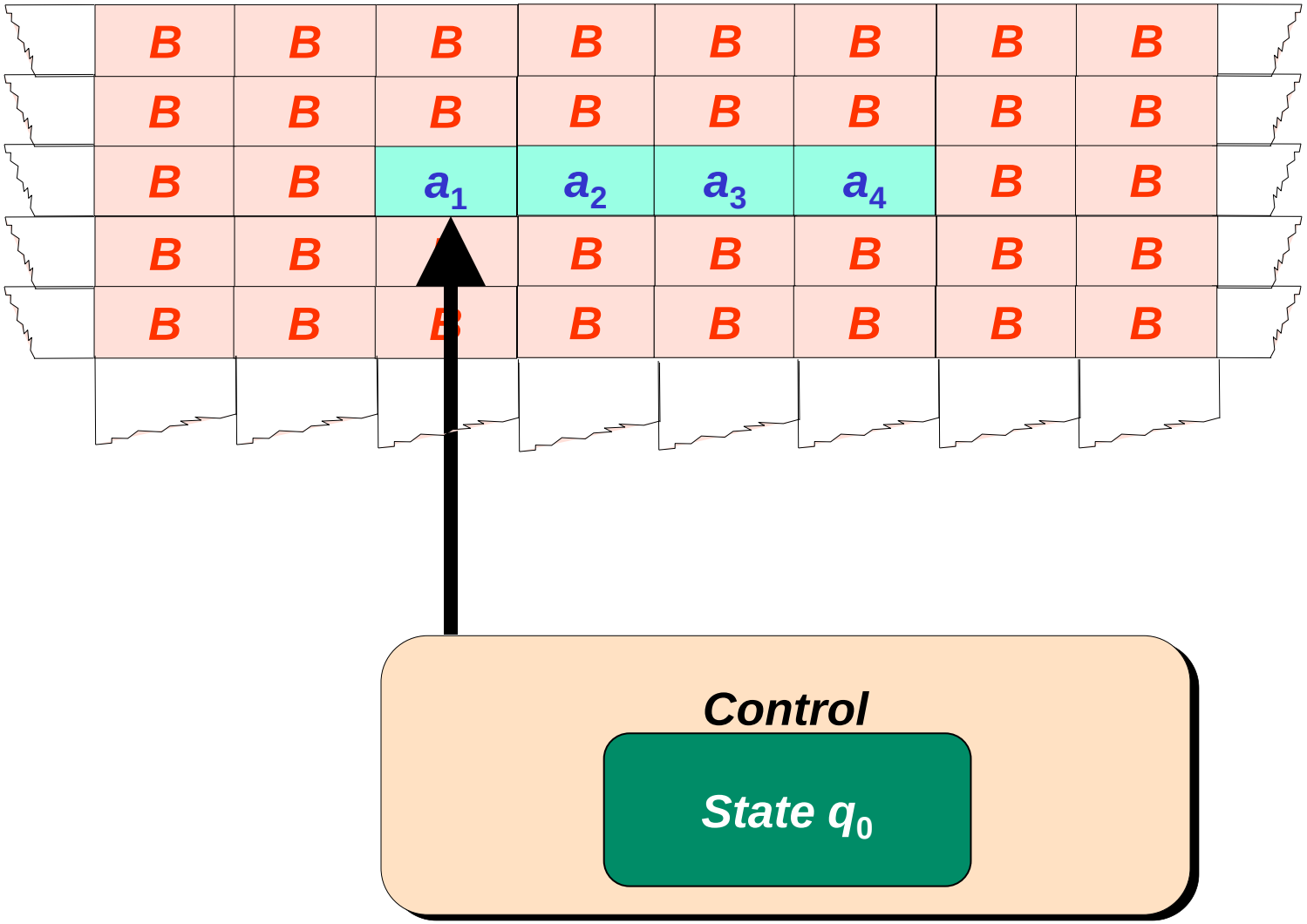



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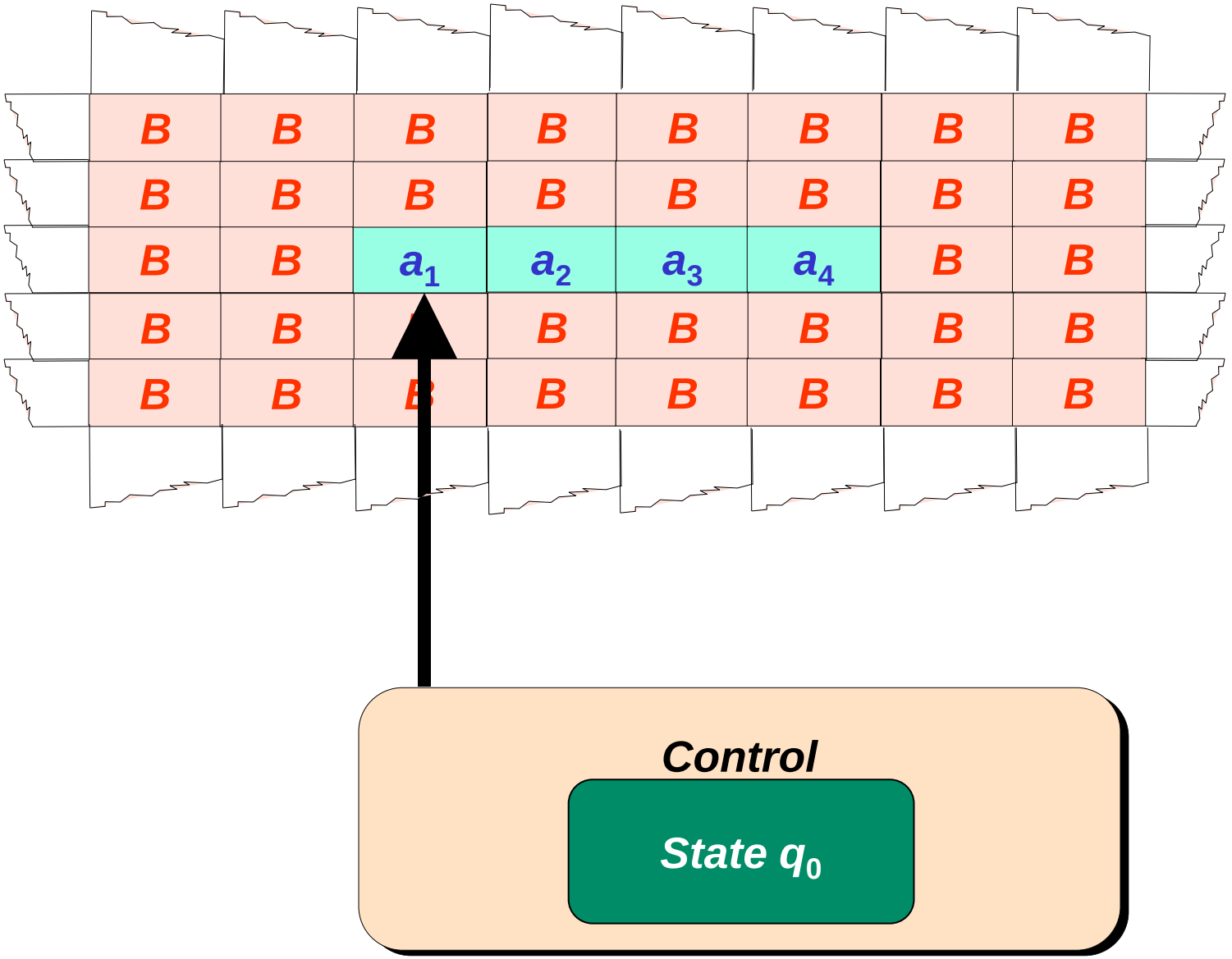



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# TM with a multidimensional input


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |


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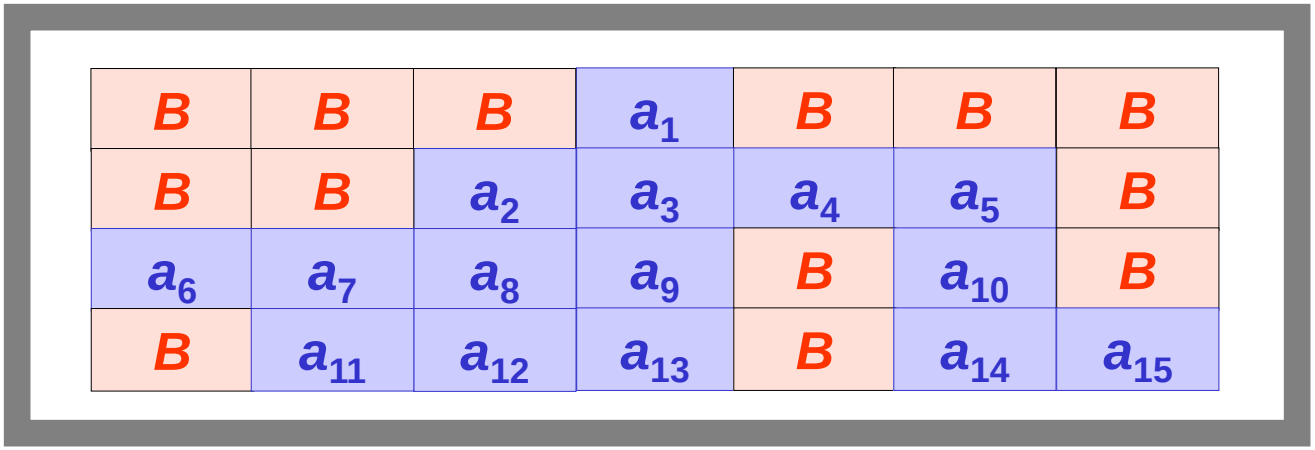



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|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |

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|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
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** $BBBa_1BBB$ *


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| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
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| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
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|---|---|---|---|---|---|---|
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** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ *


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|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
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** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ *


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***Horizontal movement within the rectangle***


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|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
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|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
| $B$ | $B$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $B$ |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | $B$ | $a_{10}$ | $B$ |
| $B$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $B$ | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
| $B$ | $B$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $B$ |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | $B$ | $a_{10}$ | $B$ |
| $B$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $B$ | $a_{14}$ | $a_{15}$ |

$$** BBBa_1BBB * BBa_2a_3a_4a_5B * a_6a_7a_8a_9Ba_{10}B * Ba_{11}a_{12}a_{13}Ba_{14}a_{15} **$$

***Horizontal movement within the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
| $B$ | $B$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $B$ |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | $B$ | $a_{10}$ | $B$ |
| $B$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $B$ | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
| $B$ | $B$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $B$ |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | $B$ | $a_{10}$ | $B$ |
| $B$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $B$ | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
| $B$ | $B$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $B$ |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | $B$ | $a_{10}$ | $B$ |
| $B$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $B$ | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
| $B$ | $B$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $B$ |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | $B$ | $a_{10}$ | $B$ |
| $B$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $B$ | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
| $B$ | $B$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $B$ |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | $B$ | $a_{10}$ | $B$ |
| $B$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $B$ | $a_{14}$ | $a_{15}$ |

$$** BBBa_1BBB * BBa_2a_3a_4a_5B * a_6a_7a_8a_9Ba_{10}B * Ba_{11}a_{12}a_{13}Ba_{14}a_{15} **$$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
| $B$ | $B$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $B$ |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | $B$ | $a_{10}$ | $B$ |
| $B$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $B$ | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***


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# TM with a multidimensional input

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|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
| $B$ | $B$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $B$ |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | $B$ | $a_{10}$ | $B$ |
| $B$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $B$ | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***


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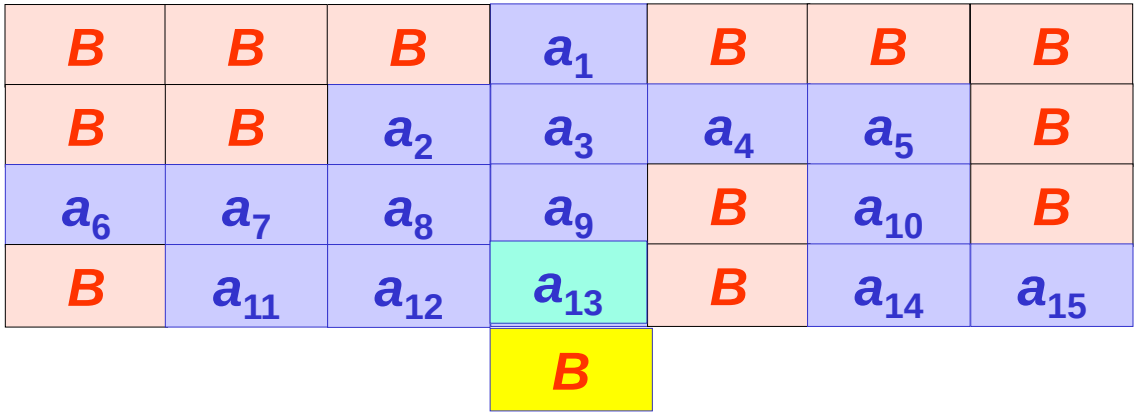
# TM with a multidimensional input




** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |
| B | B | B | B | B | B | B |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***


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# TM with a multidimensional input

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|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |
| B | B | B | B | B | B | B |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |
| B | B | B | B | B | B | B |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |
| B | B | B | B | B | B | B |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ * BBBBBBB **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |
| B | B | B | B | B | B | B |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ * BBBBBBB **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ |
| B | B | B | B | B | B | B |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$ * BBBBBBB **

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***


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# TM with a multidimensional input

| | | | | | | |
|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
| $B$ | $B$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $B$ |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | $B$ | $a_{10}$ | $B$ |
| $B$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $B$ | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***


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# TM with a multidimensional input

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|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
| $B$ | $B$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $B$ |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | $B$ | $a_{10}$ | $B$ |
| $B$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $B$ | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***

***Horizontal movement outside the rectangle***


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# TM with a multidimensional input

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|---|---|---|---|---|---|---|
| $B$ | $B$ | $B$ | $a_1$ | $B$ | $B$ | $B$ |
| $B$ | $B$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $B$ |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | $B$ | $a_{10}$ | $B$ |
| $B$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $B$ | $a_{14}$ | $a_{15}$ |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***

***Horizontal movement outside the rectangle***


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# TM with a multidimensional input

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B | |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B | |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B | |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ | B |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$

*Horizontal movement within the rectangle*

*Vertical movement within the rectangle*

*Vertical movement outside the rectangle*

*Horizontal movement outside the rectangle*


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# TM with a multidimensional input

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|---|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ | B |

** $BBBa_1BBB$ * $BBa_2a_3a_4a_5B$ * $a_6a_7a_8a_9Ba_{10}B$ * $Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$

*Horizontal movement within the rectangle*

*Vertical movement within the rectangle*

*Vertical movement outside the rectangle*

*Horizontal movement outside the rectangle*


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# TM with a multidimensional input

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|---|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ | B |

** $BBBa_1BBBB * BBa_2a_3a_4a_5B * a_6a_7a_8a_9Ba_{10}B * Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***

***Horizontal movement outside the rectangle***


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# TM with a multidimensional input

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|---|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ | B |

** $BBBa_1BBBB * BBa_2a_3a_4a_5BB * a_6a_7a_8a_9Ba_{10}B * Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***

***Horizontal movement outside the rectangle***


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# TM with a multidimensional input

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|---|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ | B |

** $BBBa_1BBBB * BBa_2a_3a_4a_5BB * a_6a_7a_8a_9Ba_{10}BB * Ba_{11}a_{12}a_{13}Ba_{14}a_{15}$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***

***Horizontal movement outside the rectangle***


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# TM with a multidimensional input

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ | B |

** $BBBa_1BBBB * BBa_2a_3a_4a_5BB * a_6a_7a_8a_9Ba_{10}BB * Ba_{11}a_{12}a_{13}Ba_{14}a_{15}B$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***

***Horizontal movement outside the rectangle***


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# TM with a multidimensional input

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ | B |

** $BBBa_1BBBB * BBa_2a_3a_4a_5BB * a_6a_7a_8a_9Ba_{10}BB * Ba_{11}a_{12}a_{13}Ba_{14}a_{15}B$

*Horizontal movement within the rectangle*

*Vertical movement within the rectangle*

*Vertical movement outside the rectangle*

*Horizontal movement outside the rectangle*


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# TM with a multidimensional input

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ | B |

** $BBBa_1BBBB * BBa_2a_3a_4a_5BB * a_6a_7a_8a_9Ba_{10}BB * Ba_{11}a_{12}a_{13}Ba_{14}a_{15}B$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***

***Horizontal movement outside the rectangle***


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# TM with a multidimensional input

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ | B |

** $BBBa_1BBBB * BBa_2a_3a_4a_5BB * a_6a_7a_8a_9Ba_{10}BB * Ba_{11}a_{12}a_{13}Ba_{14}a_{15}B$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***

***Horizontal movement outside the rectangle***


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# TM with a multidimensional input

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| B | B | B | $a_1$ | B | B | B | B |
| B | B | $a_2$ | $a_3$ | $a_4$ | $a_5$ | B | B |
| $a_6$ | $a_7$ | $a_8$ | $a_9$ | B | $a_{10}$ | B | B |
| B | $a_{11}$ | $a_{12}$ | $a_{13}$ | B | $a_{14}$ | $a_{15}$ | B |

** $BBBa_1BBBB * BBa_2a_3a_4a_5BB * a_6a_7a_8a_9Ba_{10}BB * Ba_{11}a_{12}a_{13}Ba_{14}a_{15}B$

***Horizontal movement within the rectangle***

***Vertical movement within the rectangle***

***Vertical movement outside the rectangle***

***Horizontal movement outside the rectangle***


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