Lecture 18

6.2.3 Language classes with respect to time and space complexity 187

Language hierarchy based on the complexity of accepting the language

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i. ***Infinity***

- language hierarchy is infinite

Language hierarchy based on the complexity of accepting the language

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ii. ***Continuity*** **for fully space-constructible functions**

- language hierarchy is continuous for fully space-constructible functions

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iii. ***Continuity* for fully time-constructible functions**

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Language hierarchy based on the complexity of accepting the language

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- language hierarchy is continuous for fully space-constructible functions

iii. ***Continuity*** **for fully time-constructible functions**

- language hierarchy is continuous for fully time-constructible functions

iv. ***Gaps*** **in hierarchy**

- language hierarchy *is not* continuous for the general case of functions which are not space and time constructible

Language hierarchy based on the complexity of accepting the language

i. ***Infinity***

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- language hierarchy *is not* continuous for the general case of functions which are not space and time constructible

v. ***Optimal* TM**

- there is a language for which there is no optimal TM which accepts it in minimal time or minimal space

Language hierarchy based on the complexity of accepting the language

i. ***Infinity***

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- language hierarchy *is not* continuous for the general case of functions which are not space and time constructible

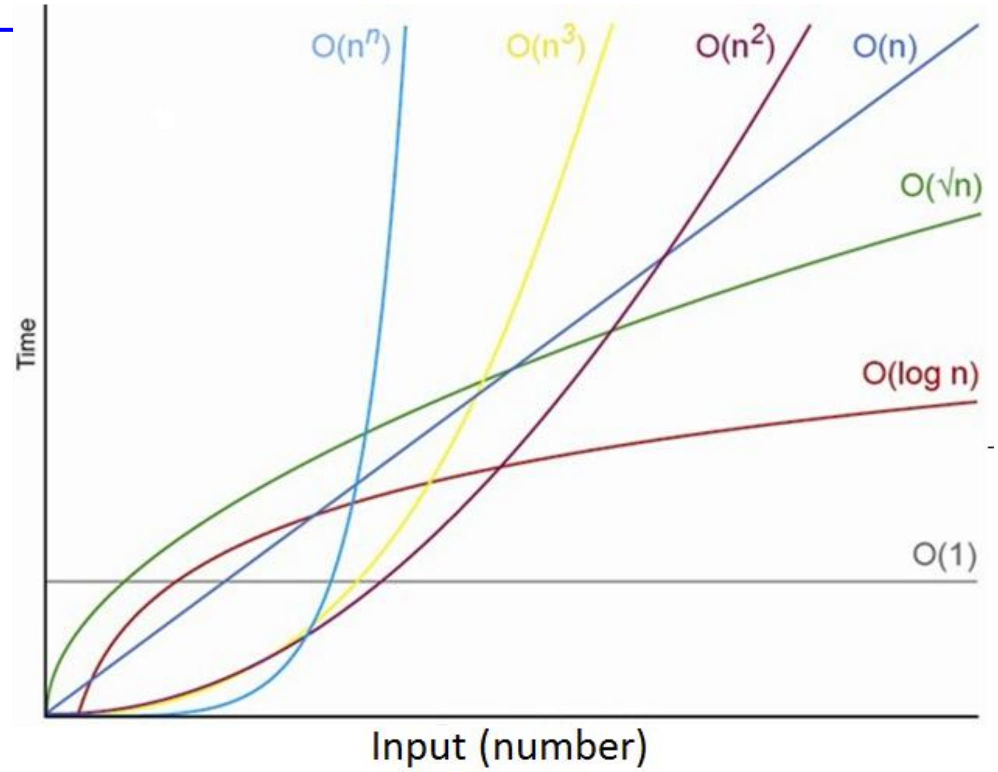
v. ***Optimal*** **TM**

- there is a language for which there is no optimal TM which accepts it in minimal time or minimal space

vi. ***Union*** **of language classes**

- there is a complexity function which covers all languages from the union

Infinity



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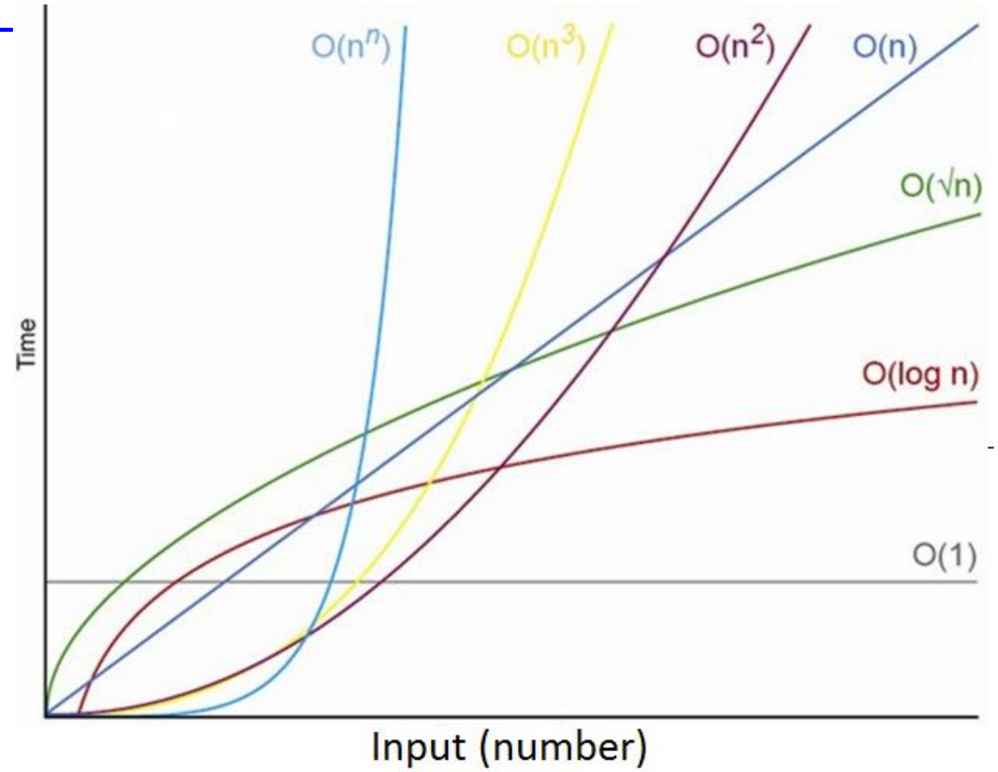
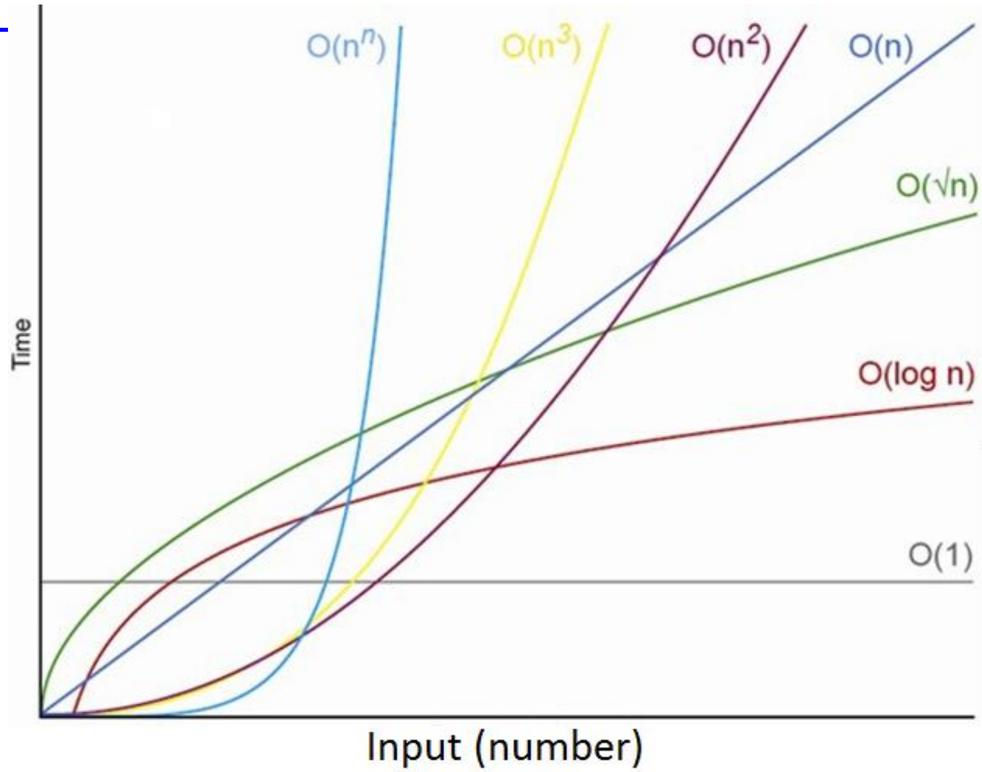


DTIME ($f_1(n)$)

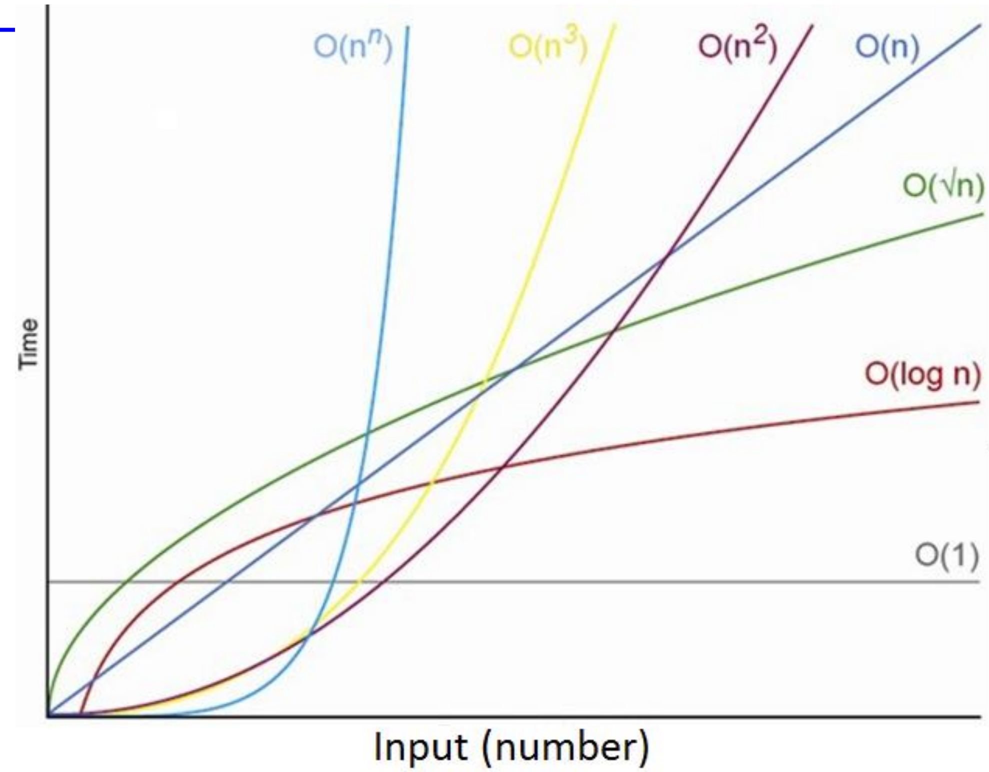
Infinity



$L_1 \notin \mathrm{DTIME}(f_1(n))$

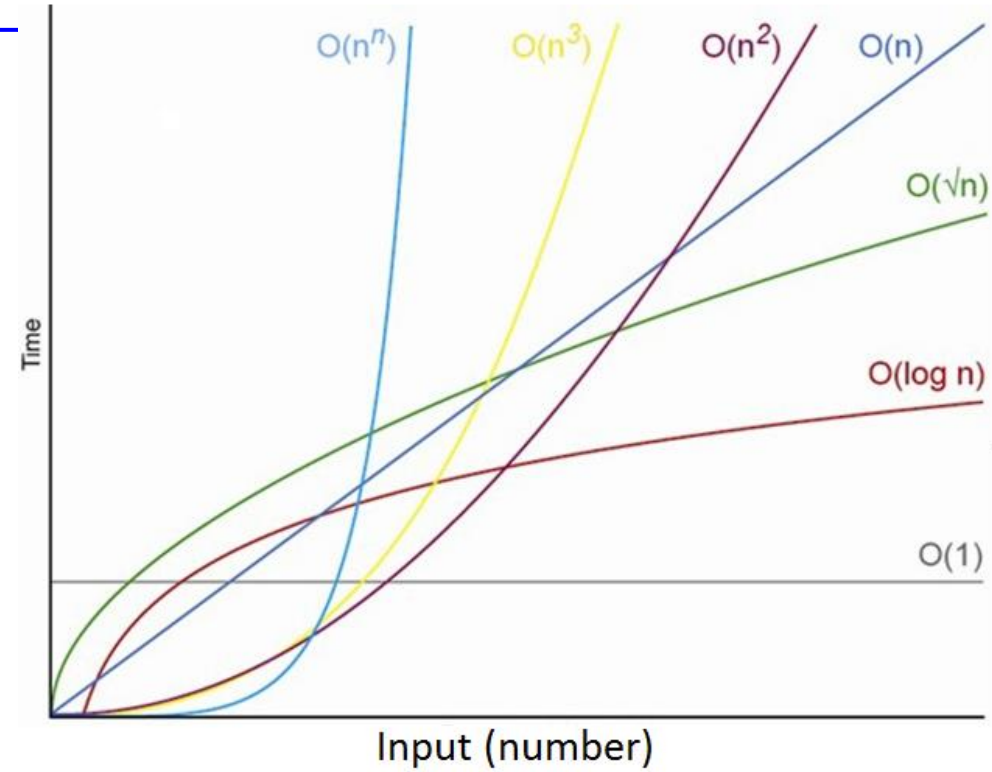
DTIME ($f_1(n)$)

Infinity

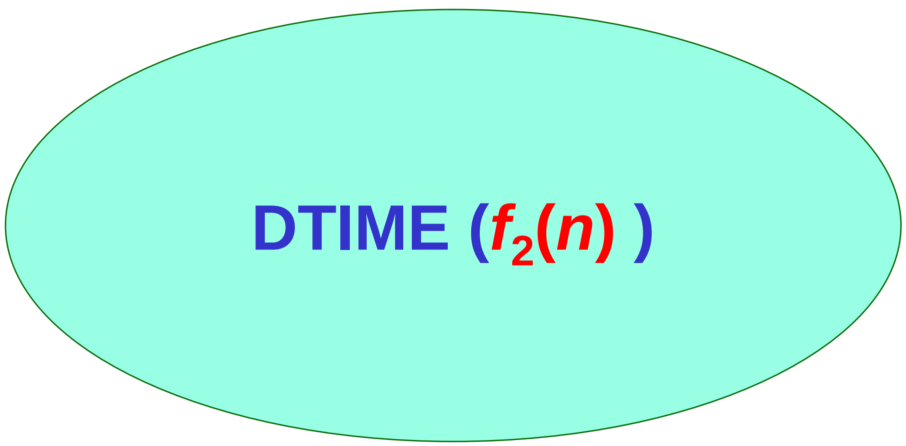


DTIME ($f_2(n)$)

Infinity



$L_2 \notin \mathrm{DTIME}(f_2(n))$



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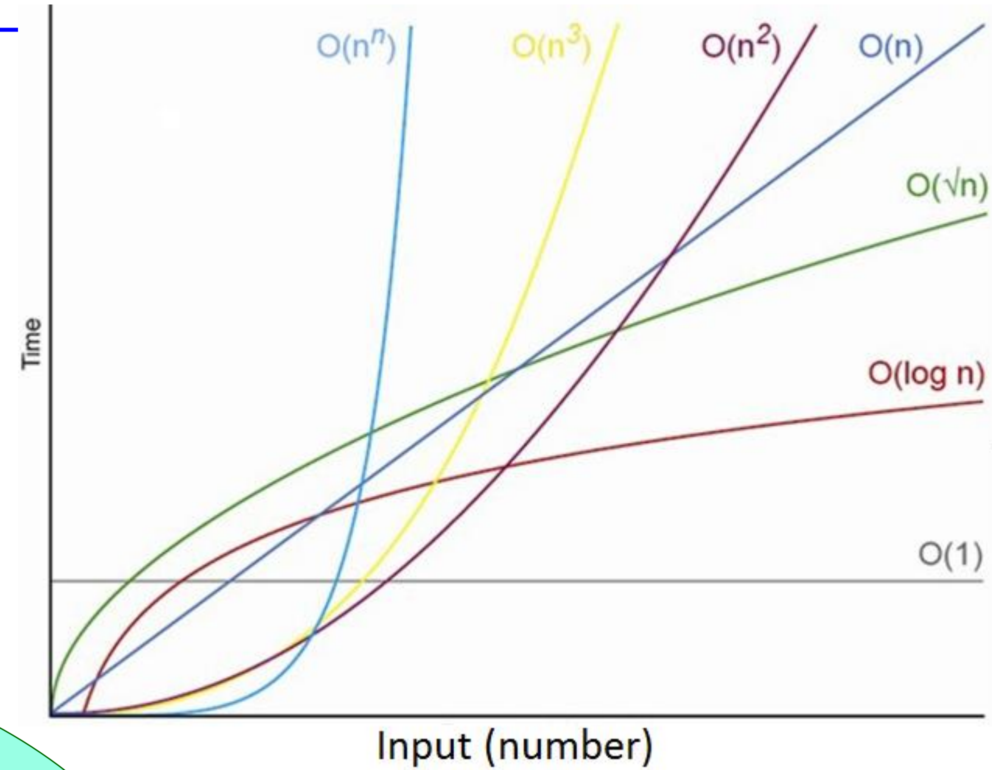
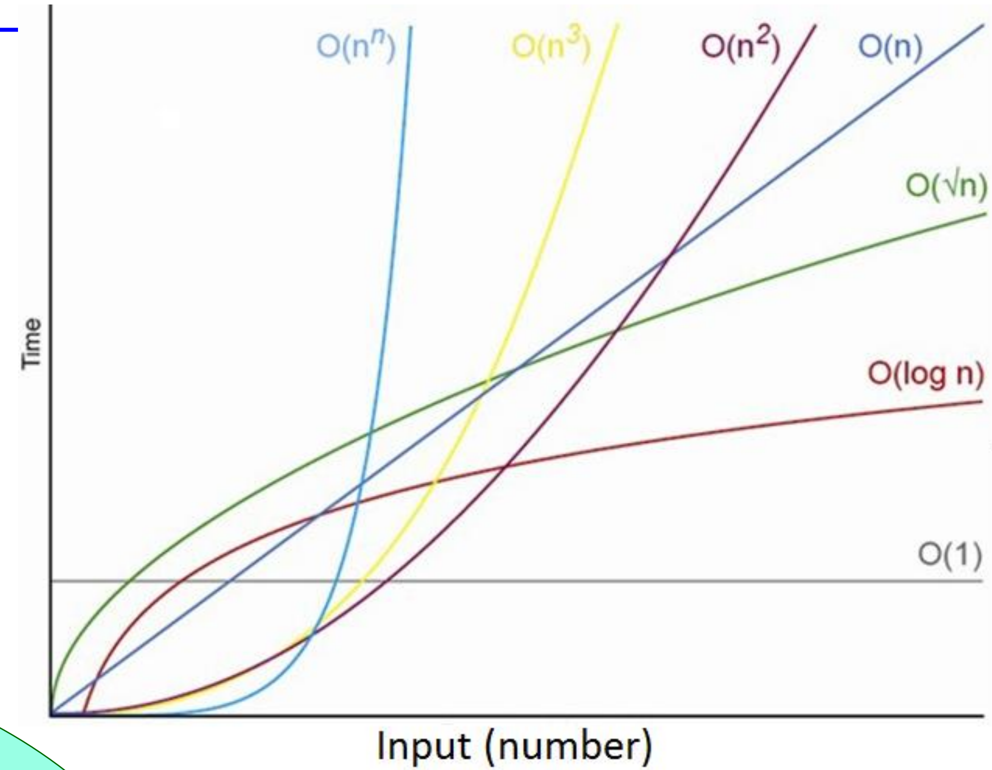
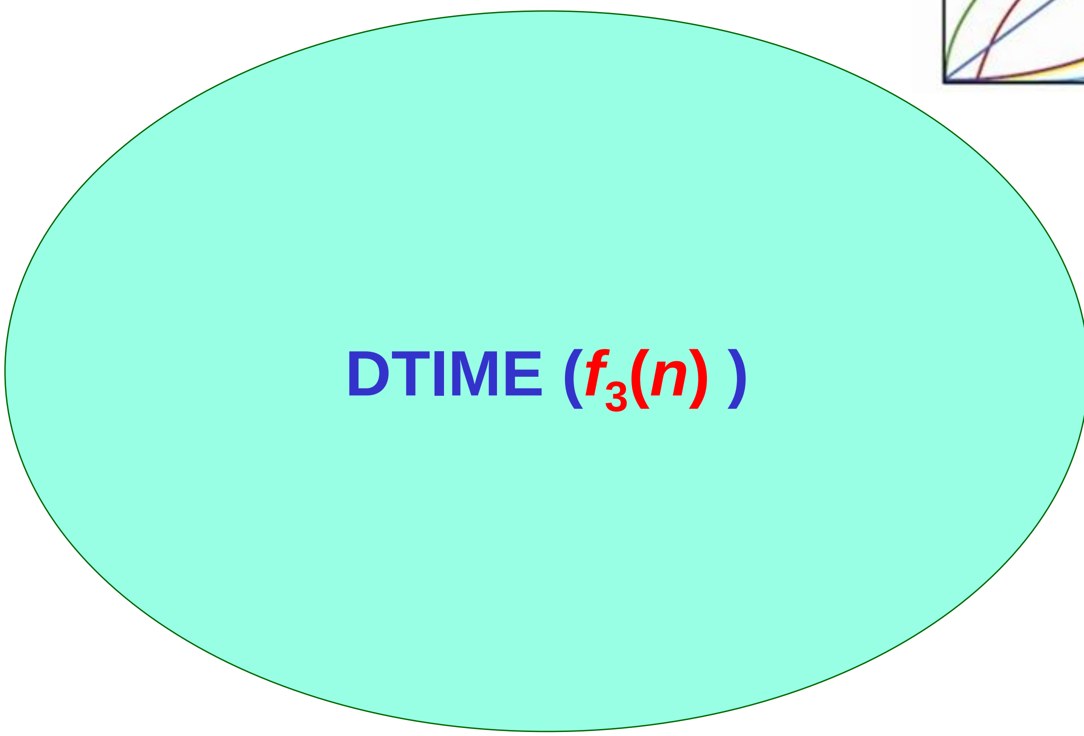


DTIME ($f_3(n)$)

Infinity



$L_3 \notin \mathrm{DTIME}(f_3(n))$



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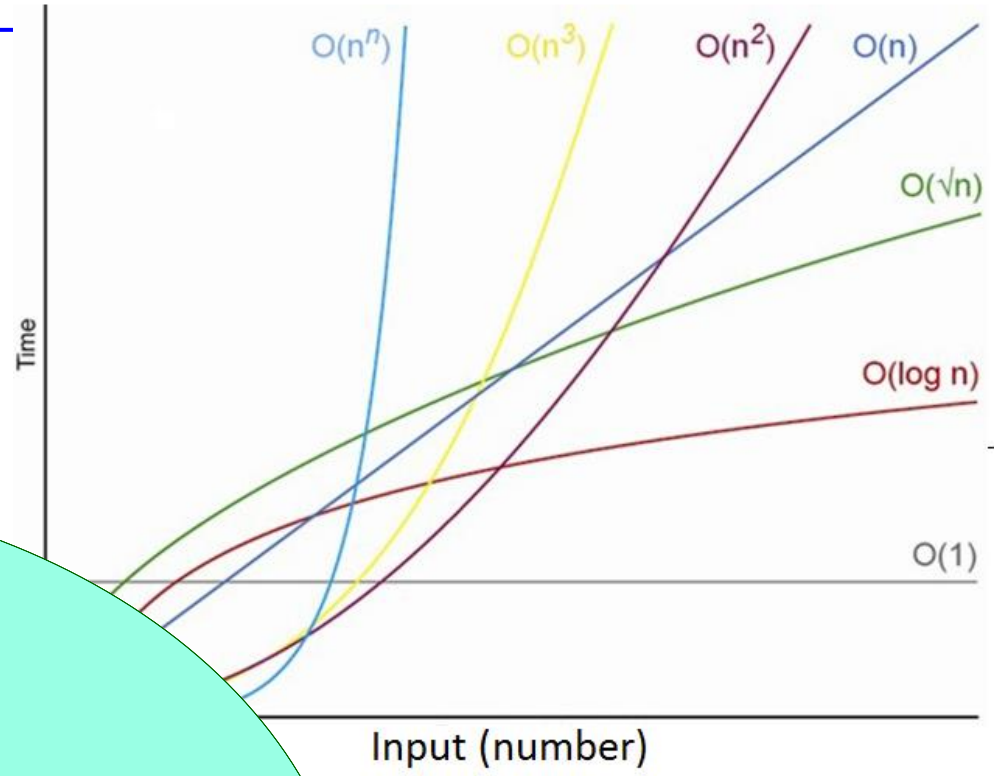
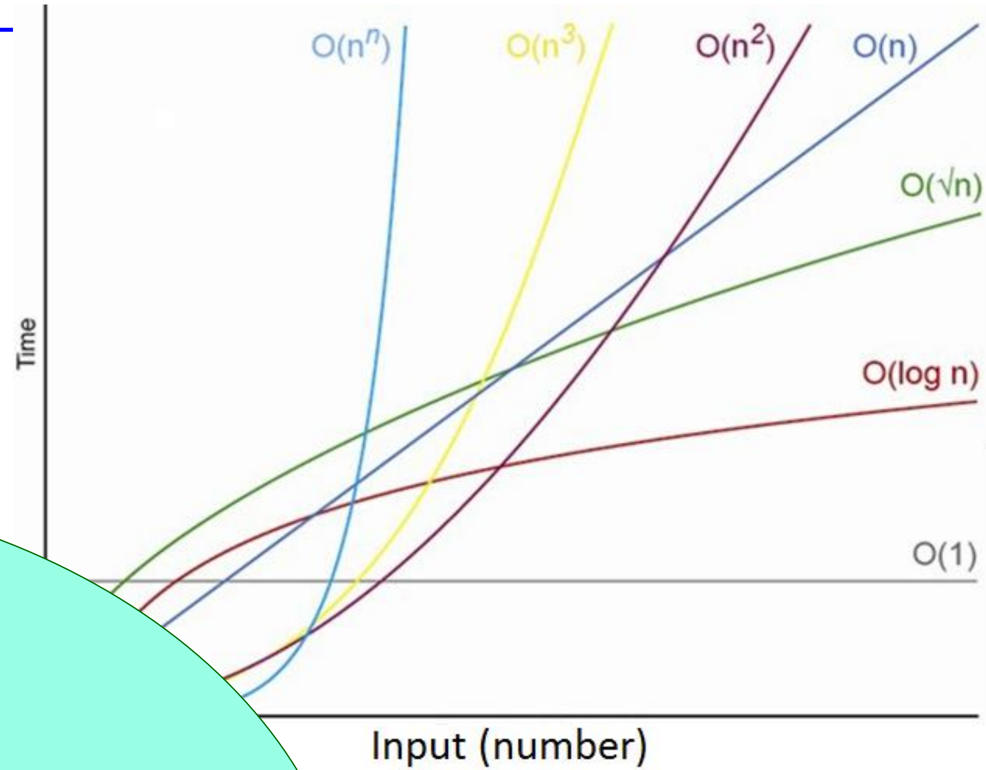


DTIME ($f_4(n)$)

Infinity



$L_4 \notin \text{DTIME}(f_4(n))$

DTIME ($f_4(n)$)

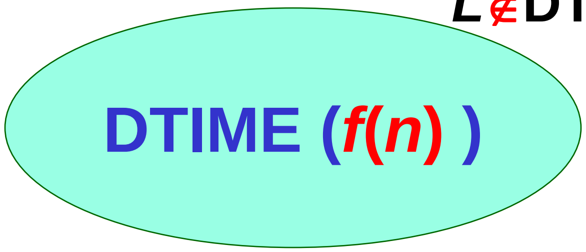
Infinity

$L \notin \text{DTIME}(\ f(n)\ )$

DTIME ($f(n)$)

Infinity

$L \notin$ DTIME($f(n)$)



- If

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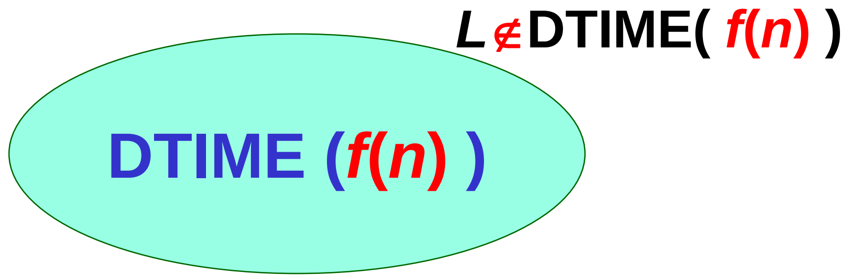
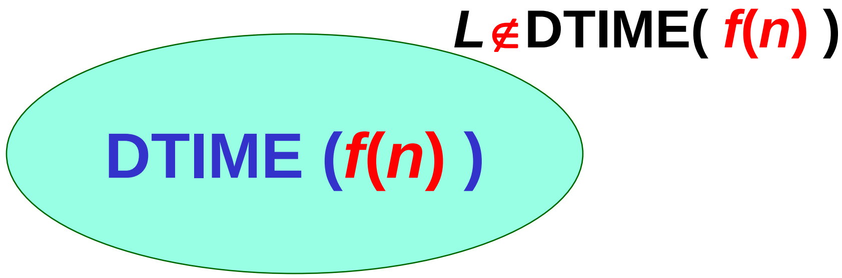


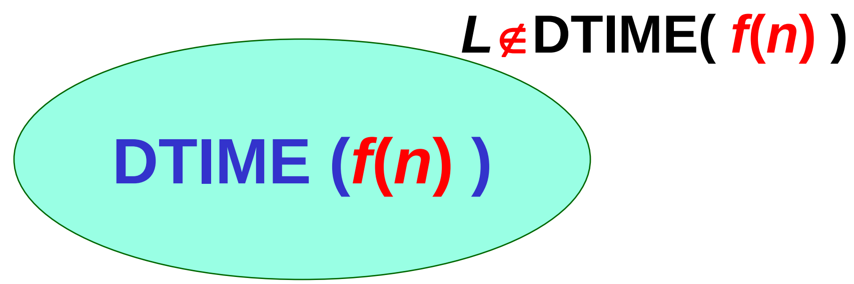
- **If**
  - **$f(n)$ is a total recursive function**

Infinity



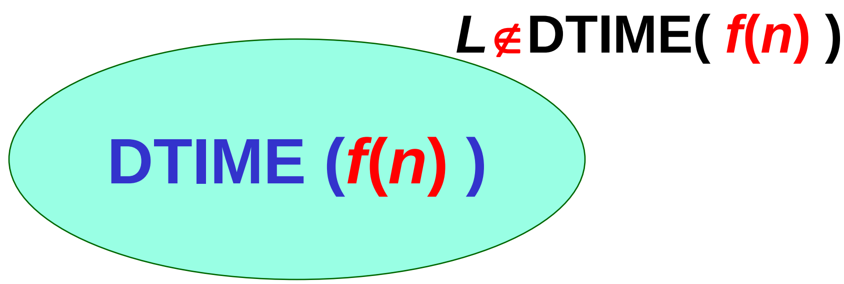
- **If**
  - **$f(n)$ is a total recursive function**
- **Then there is a language L**

Infinity



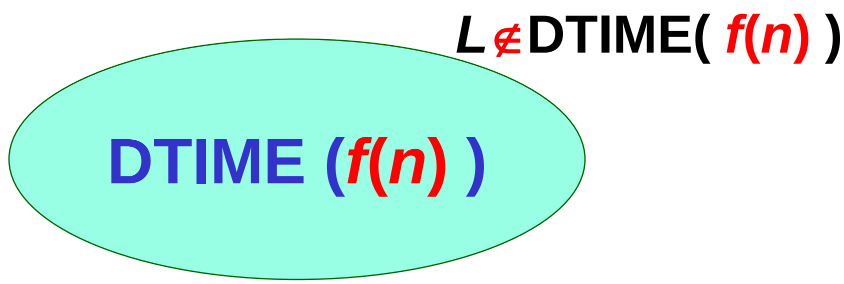
- **If**
  - **$f(n)$ is a total recursive function**
- **Then there is a language L**
  - **$L \notin \text{DTIME}(f(n))$**

Infinity



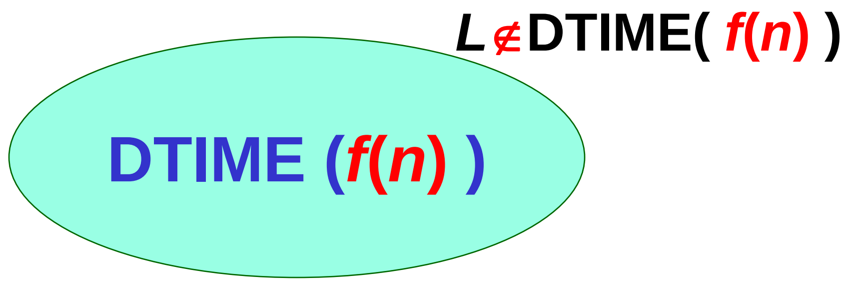
- **If**
  - **$f(n)$ is a total recursive function**
- **Then there is a language L**
  - **$L \notin \mathrm{DTIME}(f(n))$**
  - **$L \notin \mathrm{NTIME}(f(n))$**

Infinity



- If
  - $f(n)$ is a total recursive function
- Then there is a language L
  - $L \notin \text{DTIME}(f(n))$
  - $L \notin \text{NTIME}(f(n))$
  - $L \notin \text{DSPACE}(f(n))$

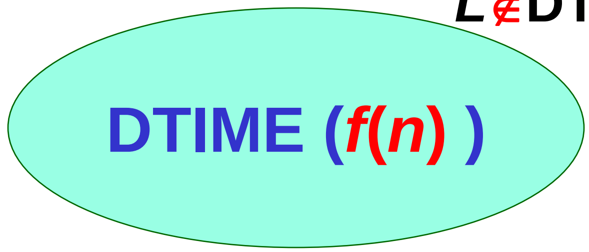
Infinity



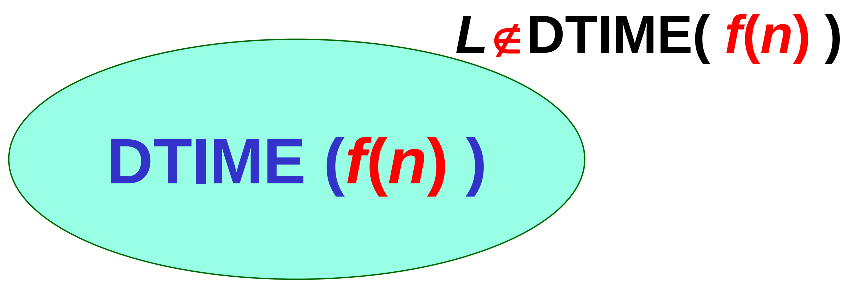
- If
  - $f(n)$ is a total recursive function
- Then there is a language L
  - $L \notin \text{DTIME}(f(n))$
  - $L \notin \text{NTIME}(f(n))$
  - $L \notin \text{DSPACE}(f(n))$
  - $L \notin \text{NSPACE}(f(n))$

Infinity

$L \notin \mathrm{DTIME}(\ f(n)\ )$

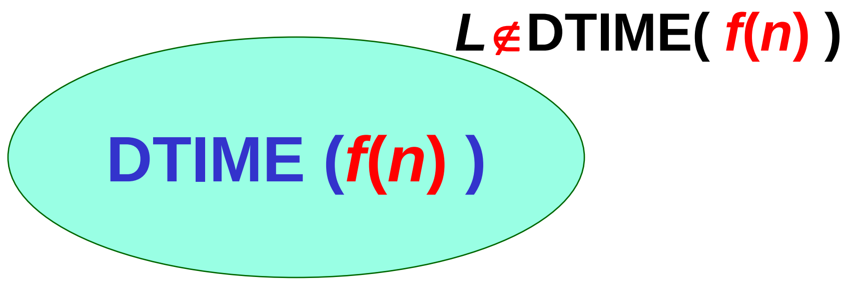


Infinity



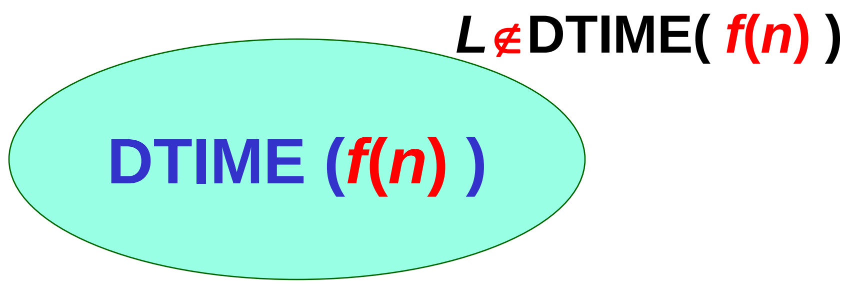
- **Encoding of TM M**
  - **tape symbols: $\{0, 1, B, X_4, X_5, \ldots X_m\}$**
  - **symbol X_k is encoded by the string of zeros 0^k**

Infinity



- Encoding of TM M
  - tape symbols: $\{0, 1, B, X_4, X_5, \ldots X_m\}$
  - symbol X_k is encoded by the string of zeros 0^k
- String w_i
  - string w_i is the i-th string in a canonical sequence of all strings

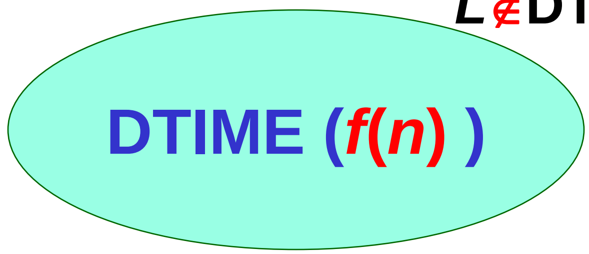
Infinity



- **Encoding of TM *M***
  - **tape symbols: $\{0, 1, B, X_4, X_5, \ldots X_m\}$**
  - **symbol X_k is encoded by the string of zeros 0^k**
- **String w_i**
  - **string w_i is the i-th string in a canonical sequence of all strings**
- **TM M_i**
  - **index value i in TM M_i equals to the integer value of its binary encoding**

Infinity

$L \notin \mathrm{DTIME}(\, f(n)\,)$



Infinity

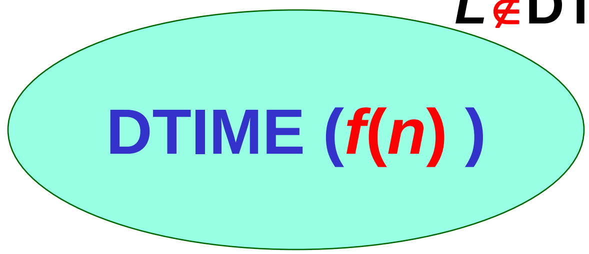
$L \notin \text{DTIME}(f(n))$

$\text{DTIME}(f(n))$

- **Language definition is based on diagonalization**

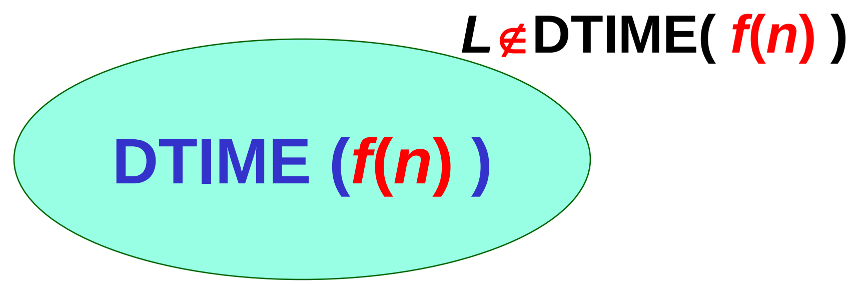
Infinity

$L \notin \mathrm{DTIME}(\,f(n)\,)$



- **Language definition is based on diagonalization**
  - $L = \{\; w_i \;|$

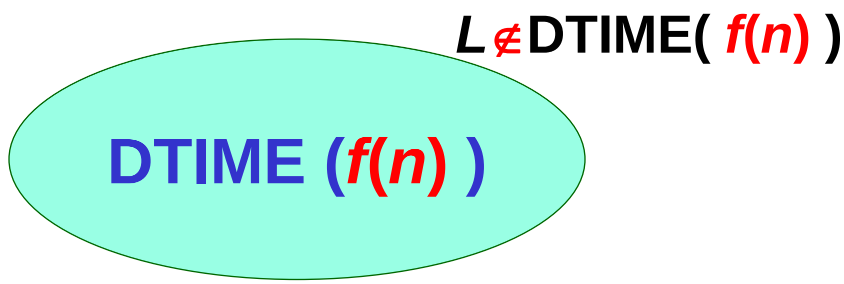
Infinity



- Language definition is based on diagonalization

  - $L = \{\ w_i \mid$
    TM M_i does not accept w_i

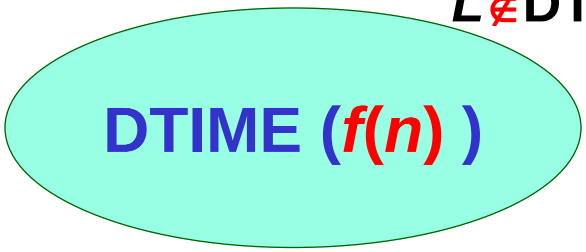
Infinity



- **Language definition is based on diagonalization**
  - **$L = \{\ w_i\ |$ TM M_i does not accept w_i in less than $f(\ |w_i|\ )$ head moves$\}$**

Infinity

$L \notin \mathrm{DTIME}(f(n))$



Infinity

$L \notin \mathrm{DTIME}(f(n))$

$\mathrm{DTIME}(f(n))$

- **Language *L* is recursive**

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

TM M accepts L and always halts

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

TM M accepts L and always halts

B	*B*	*B*	*B*	*B*	*B*

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- **Language L is recursive**

TM M accepts L and always halts

TM M computes the input length $n = \|w\|$	B	B

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- **Language L is recursive**

TM M accepts L and always halts

TM M computes the input length $n = \|w\|$	*B*	*B*

B	*B*	*B*	*B*	*B*	*B*

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

We calculate $f(n)$

$f(n)$ is a total recursive function

TM M accepts L and always halts

TM M computes the input length $n = \|w\|$				*B*	*B*	
B	*B*	*B*	*B*	*B*	*B*	

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

TM M accepts L and always halts

TM M computes the input length $n = \|w\|$	B	B	
TM N computes the value $f(n)$ and always halts		B	

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

TM M accepts L and always halts

TM M computes the input length $n = \|w\|$	B	B	

TM N computes the value $f(n)$ and always halts	B	

B	B	B	B	B	B	

Infinity

$L \notin \mathrm{DTIME}(\, f(n)\,)$

DTIME $(f(n))$

- Language L is recursive

TM M accepts L and always halts

TM M computes the input length $n = \lvert w \rvert$	B	B	

TM N computes the value $f(n)$ and always halts	B	

Head moves counter $f(n)$		B	B	B	

Infinity

$L \notin \text{DTIME}(\ f(n)\ )$

DTIME ($f(n)$)

- **Language *L* is recursive**

TM *M* accepts *L* and always halts

TM *M* computes the input length $n = \|w\|$	*B*	*B*	
TM *N* computes the value $f(n)$ and always halts	*B*		

B	*B*	*B*	*B*	*B*	*B*	

Head moves counter $f(n)$		*B*	*B*	*B*	

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

TM M accepts L and always halts

TM M computes the input length $n = \|w\|$	B	B	
TM N computes the value $f(n)$ and always halts	B		
TM M finds the index i of string w_i in the canonical sequence TM M finds the binary encoding of i which is also the encoding of TM M_i			
Head moves counter $f(n)$	B	B	B

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

TM M accepts L and always halts

TM M computes the input length $n = |w|$ | B | B

TM N computes the value $f(n)$ and always halts | B

TM M finds the index i of string w_i in the canonical sequence
TM M finds the binary encoding of i which is also the encoding of TM M_i

B | B | B | B | B | B

Head moves counter $f(n)$ | B | B | B

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

TM M accepts L and always halts

TM M computes the input length $n = \|w\|$	B	B
TM N computes the value $f(n)$ and always halts		B
TM M finds the index i of string w_i in the canonical sequence TM M finds the binary encoding of i which is also the encoding of TM M_i		
Invalid code - TM M_i has no defined transitions		
Head moves counter $f(n)$	B	B B

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

TM M accepts L and always halts

TM M computes the input length $n = \|w\|$	B	B	
TM N computes the value $f(n)$ and always halts	B		
TM M finds the index i of string w_i in the canonical sequence TM M finds the binary encoding of i which is also the encoding of TM M_i			
Invalid code - TM M_i has no defined transitions			
Head moves counter $f(n)$	B	B	B

YES

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

TM M accepts L and always halts

TM M computes the input length $n = |w|$ *B* *B*

TM N computes the value $f(n)$ and always halts *B*

TM M finds the index i of string w_i in the canonical sequence
TM M finds the binary encoding of i which is also the encoding of TM M_i

TM M_i halts and does not accept the string

Head moves counter $f(n)$ *B* *B* *B*

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- **Language L is recursive**

TM M accepts L and always halts

TM M computes the input length $n = \|w\|$	B	B
TM N computes the value $f(n)$ and always halts		B

TM M finds the index i of string w_i in the canonical sequence
TM M finds the binary encoding of i which is also the encoding of TM M_i

TM M_i halts and does not accept the string

YES

Head moves counter $f(n)$ B B B

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- **Language L is recursive**

TM M accepts L and always halts

TM M computes the input length $n = |w|$ *B* *B*

TM N computes the value $f(n)$ and always halts *B*

TM M finds the index i of string w_i in the canonical sequence

TM M finds the binary encoding of i which is also the encoding of TM M_i

TM M_i halts and accepts the string

Head moves counter $f(n)$ *B* *B* *B*

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

TM M accepts L and always halts

TM M computes the input length $n = \|w\|$	B	B	
TM N computes the value $f(n)$ and always halts	B		
TM M finds the index i of string w_i in the canonical sequence TM M finds the binary encoding of i which is also the encoding of TM M_i			
TM M_i halts and accepts the string			
Head moves counter $f(n)$	B	B	B

NO

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

TM M accepts L and always halts

TM M computes the input length $n = |w|$ | B | B

TM N computes the value $f(n)$ and always halts | B

TM M finds the index i of string w_i in the canonical sequence

TM M finds the binary encoding of i which is also the encoding of TM M_i

TM M_i does not accept the string, head moves > $f(n)$ times

Head moves counter $f(n)$ | B | B | B

Infinity

$L \notin$ DTIME($f(n)$)

DTIME ($f(n)$)

- Language L is recursive

TM M accepts L and always halts

TM M computes the input length $n = |w|$ | B | B

TM N computes the value $f(n)$ and always halts | B

TM M finds the index i of string w_i in the canonical sequence
TM M finds the binary encoding of i which is also the encoding of TM M_i

TM M_i does not accept the string, head moves > $f(n)$ times → YES

Head moves counter $f(n)$ | B | B | B

Infinity

$L \notin$ DTIME($f(n)$)

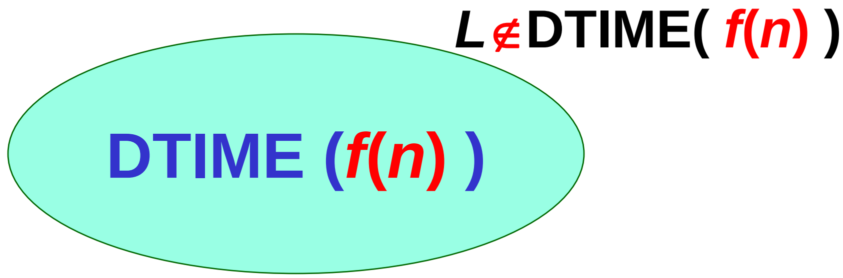
DTIME ($f(n)$)

- **Language L is recursive**

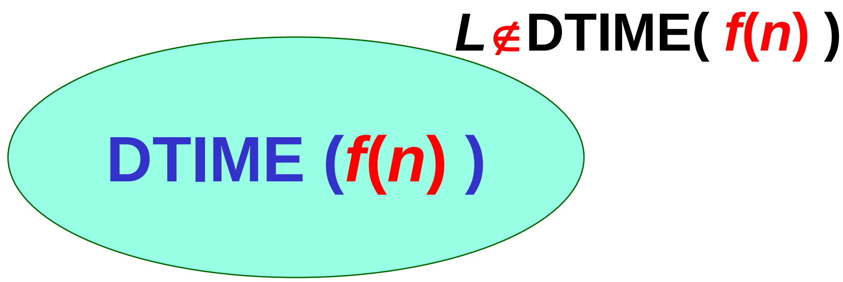
TM M accepts L and always halts

TM M computes the input length $n = \|w\|$	B	B
TM N computes the value $f(n)$ and always halts		B
TM M finds the index i of string w_i in the canonical sequence TM M finds the binary encoding of i which is also the encoding of TM M_i		
TM M_i does not accept the string, head moves > $f(n)$ times		
Head moves counter $f(n)$	B	B B

Infinity

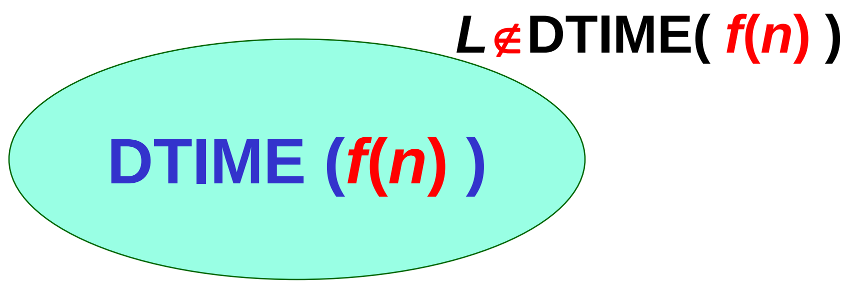


Infinity



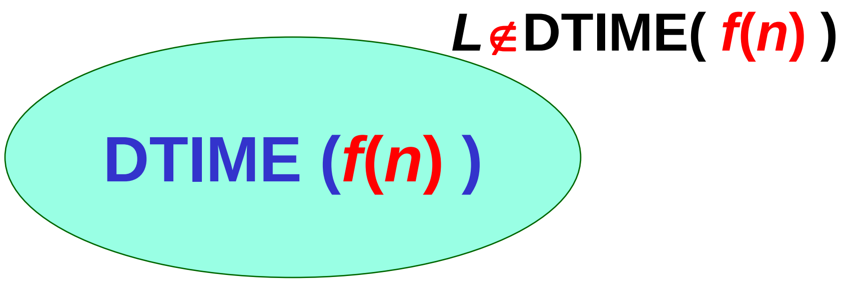
- A proof that $L \notin \text{DTIME}(f(n))$

Infinity



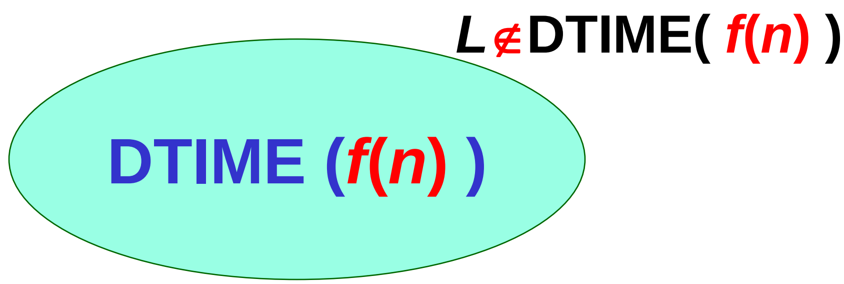
- A proof that $L \notin \mathrm{DTIME}(f(n))$
  - Assumption: $L=L(M_i)$

Infinity



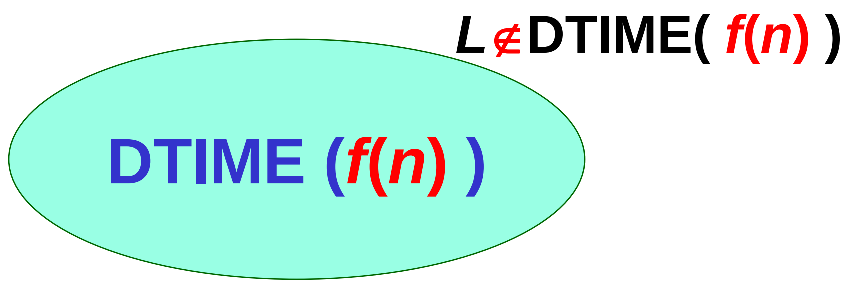
- A proof that $L \notin$ DTIME($f(n)$)
  - Assumption: $L = L(M_i)$
    - —language L is accepted by TM M_i of time complexity $f(n)$

Infinity



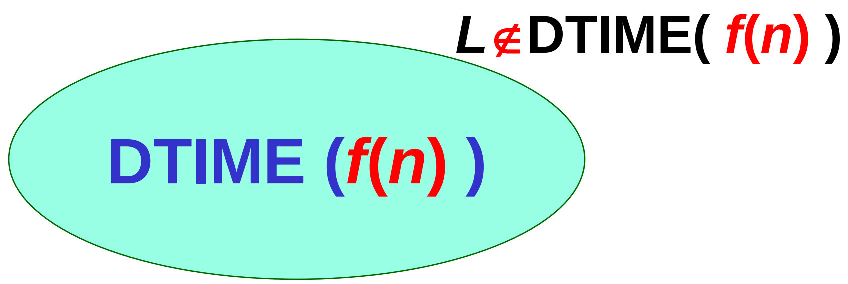
- A proof that $L \notin \mathrm{DTIME}(f(n))$
  - Assumption: $L = L(M_i)$
    - —language L is accepted by TM M_i of time complexity $f(n)$
  - $|w_i| = n$

Infinity



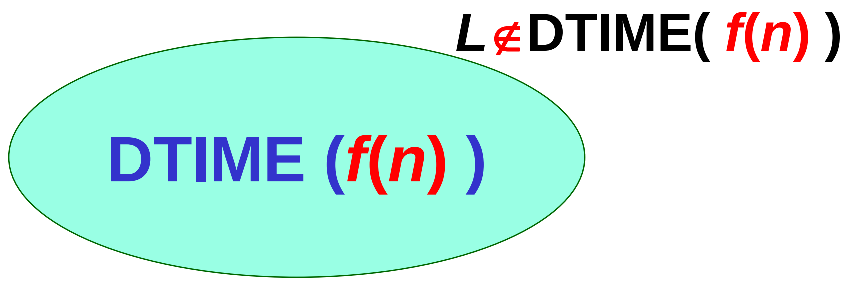
- A proof that $L \notin \text{DTIME}(f(n))$
  - Assumption: $L = L(M_i)$
    - —language L is accepted by TM M_i of time complexity $f(n)$
  - $|w_i| = n$
  - Assumption : $w_i \in L(M_i)$

Infinity



- A proof that $L \notin \mathrm{DTIME}(f(n))$
  - Assumption: $L=L(M_i)$
    - —language L is accepted by TM M_i of time complexity $f(n)$
  - $|w_i| = n$
  - Assumption : $w_i \in L(M_i)$
    - $\Rightarrow$ TM M_i accepts the string w_i in less than $f(n)$ head moves

Infinity



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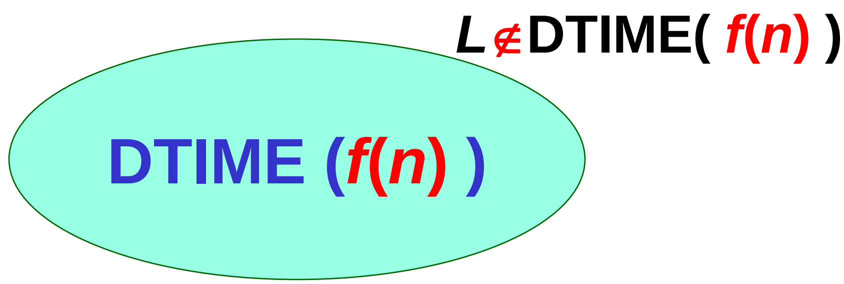
  - $|w_i| = n$
  - Assumption : $w_i \in L(M_i)$
    - $\Rightarrow$ TM M_i accepts the string w_i in less than $f(n)$ head moves
    - $\Rightarrow$ $w_i \notin L$, because $L=\{w_i \mid M_i$ does not accept w_i in less than $f(|w_i|)$ head moves$\}$

Infinity



- A proof that $L \notin \mathrm{DTIME}(f(n))$
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    - —language L is accepted by TM M_i of time complexity $f(n)$
  - $|w_i| = n$
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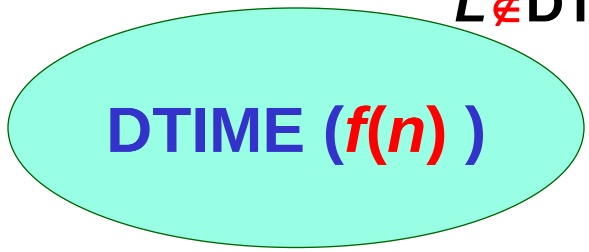
    - $\Rightarrow$ TM M_i accepts the string w_i in less than $f(n)$ head moves
    - $\Rightarrow$ $w_i \notin L$, because $L=\{w_i \mid M_i$ does not accept w_i in less than $f(|w_i|)$ head moves$\}$
    - $\Rightarrow$ $w_i \notin L(M_i)$, because we assume that $L=L(M_i)$

Infinity

$L \notin \mathrm{DTIME}(\ f(n)\ )$



- A proof that $L \notin \mathrm{DTIME}(f(n))$
  - Assumption: $L=L(M_i)$
    - —language L is accepted by TM M_i of time complexity $f(n)$
  - $|w_i| = n$
  - Assumption : $w_i \in L(M_i)$
    - $\Rightarrow$ TM M_i accepts the string w_i in less than $f(n)$ head moves
    - $\Rightarrow$ $w_i \notin L$, because $L=\{w_i \mid M_i$ does not accept w_i in less than $f(|w_i|)$ head moves$\}$
    - $\Rightarrow$ $w_i \notin L(M_i)$, because we assume that $L=L(M_i)$
    - $\Rightarrow$ contradiction

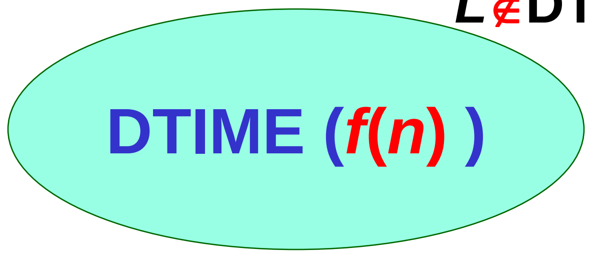
Infinity

$L \notin \mathrm{DTIME}(\, f(n) \,)$

$\mathrm{DTIME}\ (f(n))$

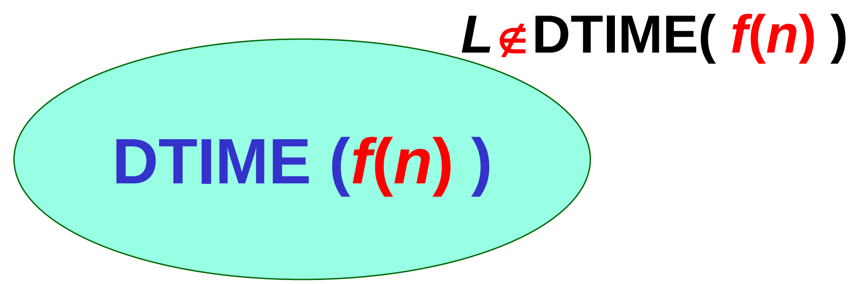
Infinity

$L \notin \mathrm{DTIME}(\, f(n) \,)$



- **Assumption:** $w_i \notin L(M_i)$

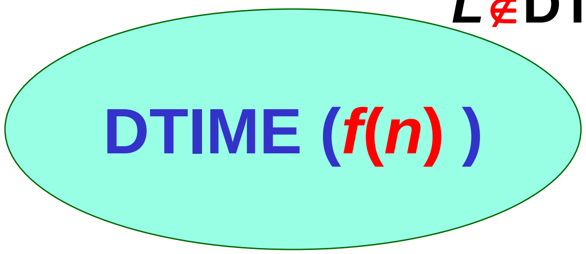
Infinity



- **Assumption:** $w_i \notin L(M_i)$
  - $\Rightarrow$ **TM M_i does not accept the string w_i in less than $f(n)$ head moves**

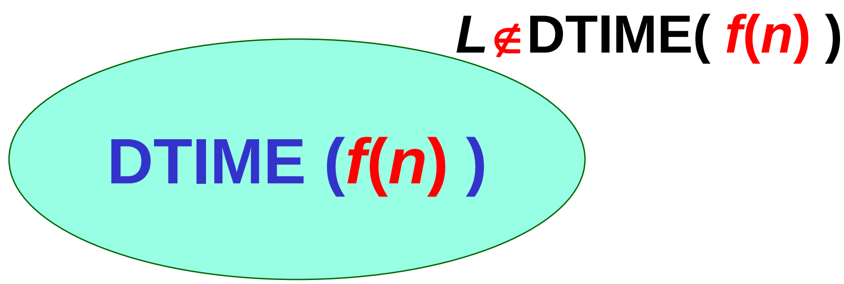
Infinity

$L \notin \mathrm{DTIME}(\, f(n)\,)$



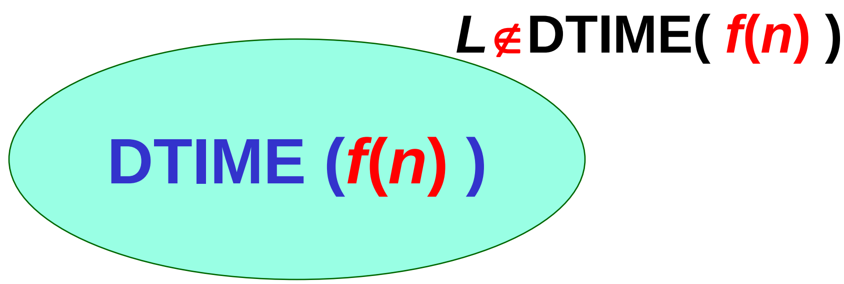
- **Assumption:** $w_i \notin L(M_i)$
  - $\Rightarrow$ **TM M_i does not accept the string w_i in less than $f(n)$ head moves**
  - $\Rightarrow$ $w_i \in L$, **because $L=\{w_i \mid M_i$ does not accept w_i in less than $f(|w_i|)$ head moves$\}$**

Infinity



- **Assumption:** $w_i \notin L(M_i)$
  - $\Rightarrow$ **TM M_i does not accept the string w_i in less than $f(n)$ head moves**
  - $\Rightarrow$ $w_i \in L$, **because $L=\{w_i \mid M_i$ does not accept w_i in less than $f(|w_i|)$ head moves$\}$**
  - $\Rightarrow$ $w_i \in L(M_i)$, **because we assume that $L=L(M_i)$**

Infinity



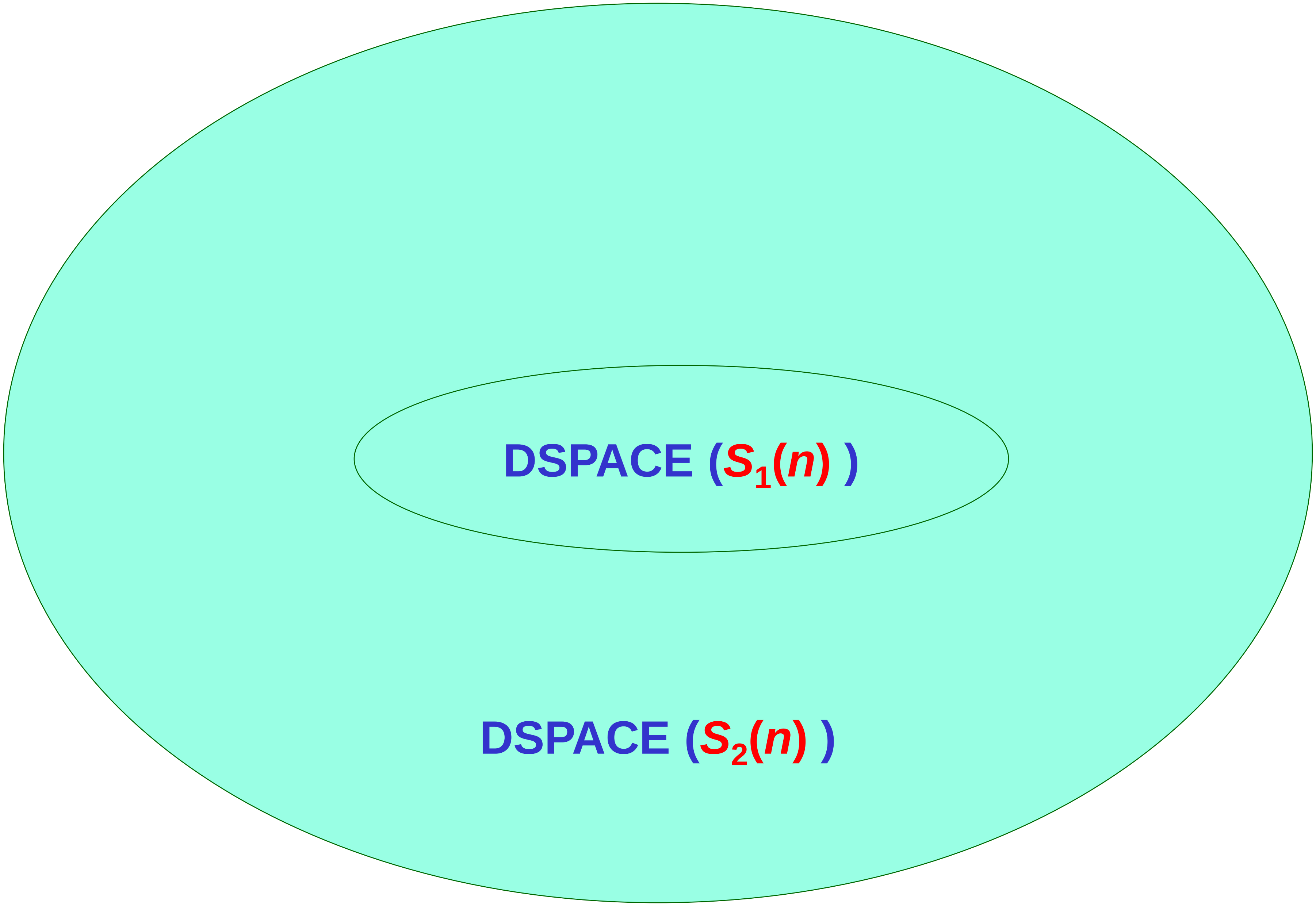
- **Assumption:** $w_i \notin L(M_i)$
  - $\Rightarrow$ **TM M_i does not accept the string w_i in less than $f(n)$ head moves**
  - $\Rightarrow$ $w_i \in L$, **because $L=\{w_i \mid M_i$ does not accept w_i in less than $f(|w_i|)$ head moves$\}$**
  - $\Rightarrow$ $w_i \in L(M_i)$, **because we assume that $L=L(M_i)$**
  - $\Rightarrow$ **contradiction**

Continuity of the hierarchy for fully space-constructible functions

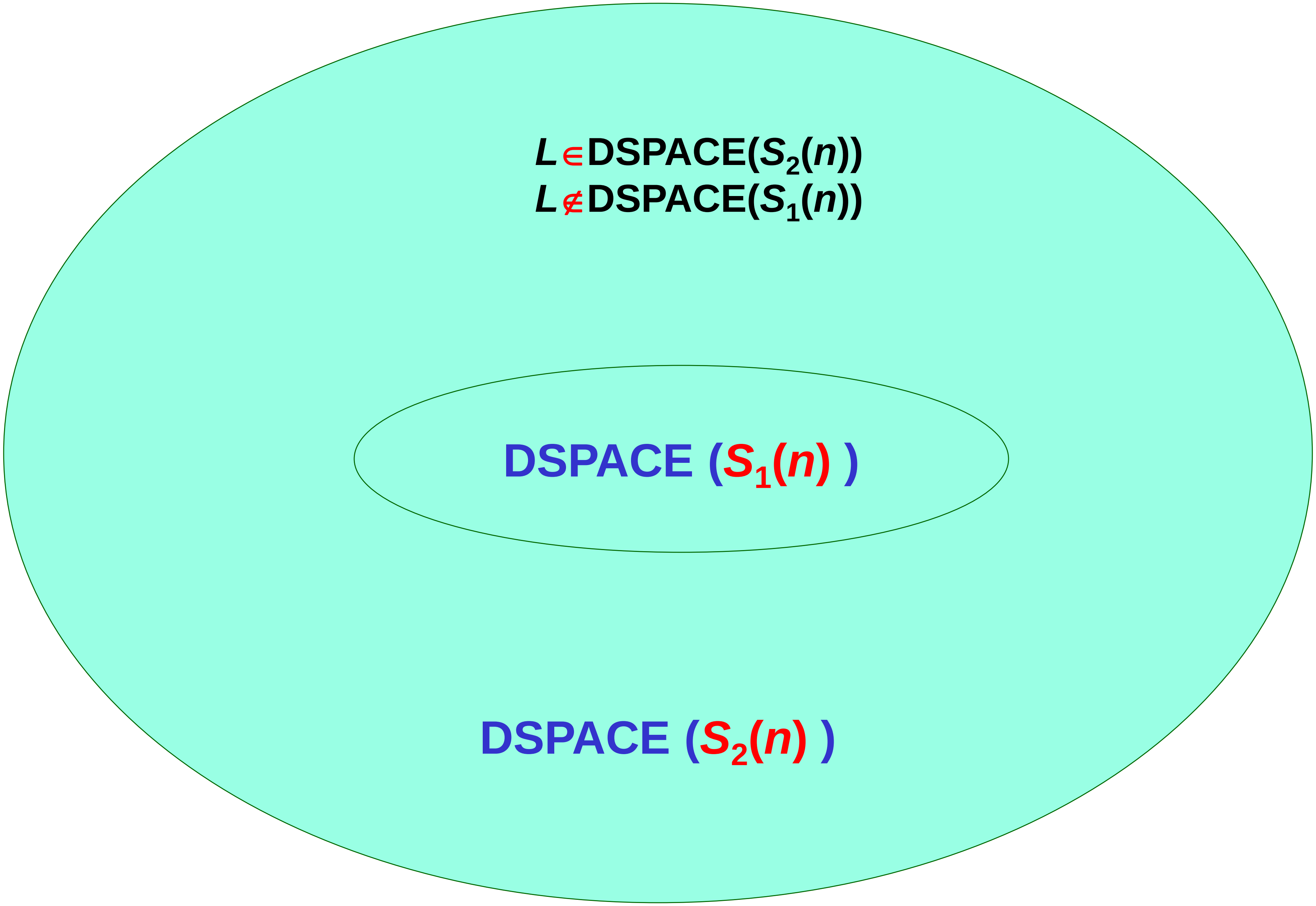
Continuity of the hierarchy for fully space-constructible functions

DSPACE ($S_1(n)$)

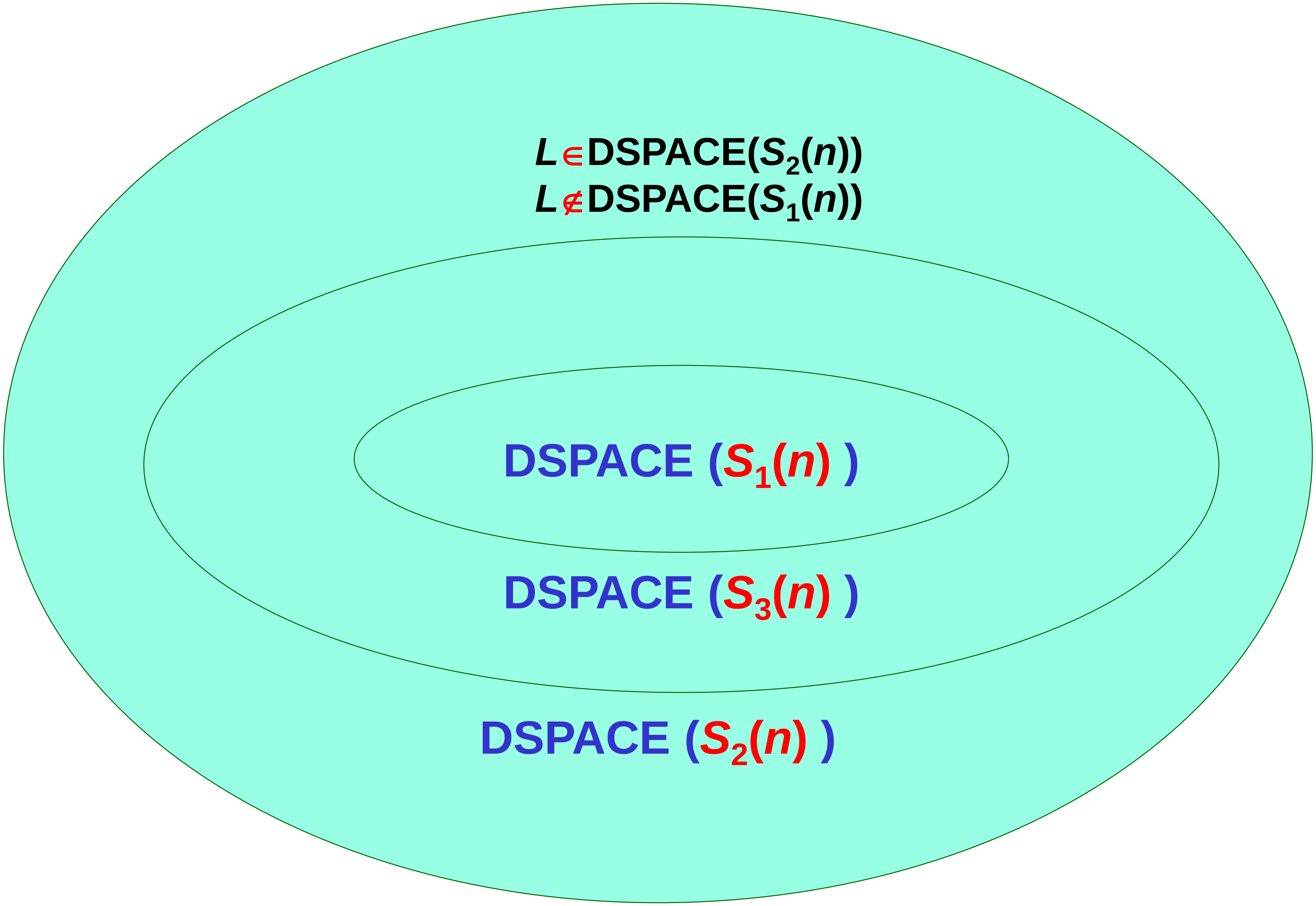
Continuity of the hierarchy for fully space-constructible functions



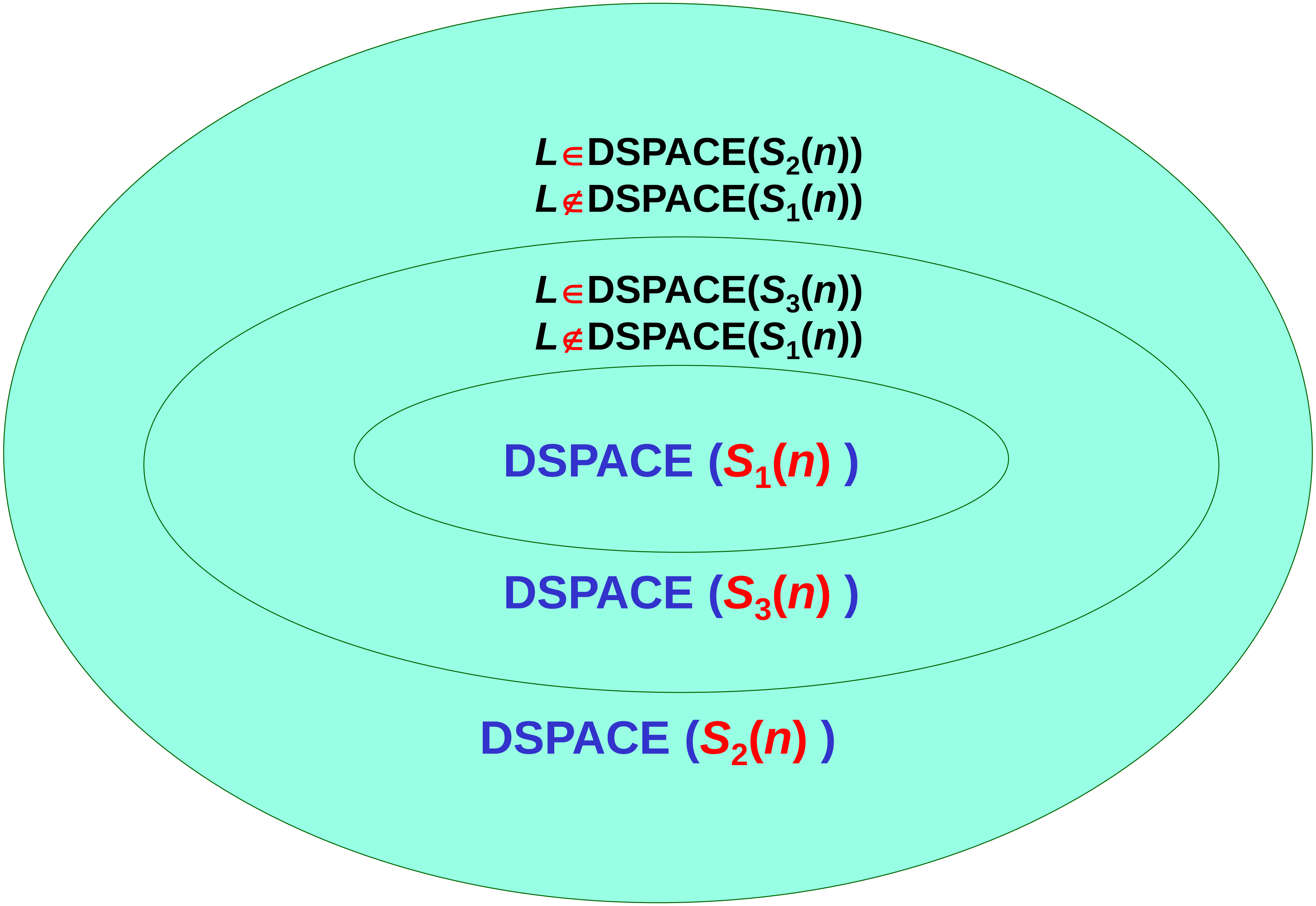
Continuity of the hierarchy for fully space-constructible functions



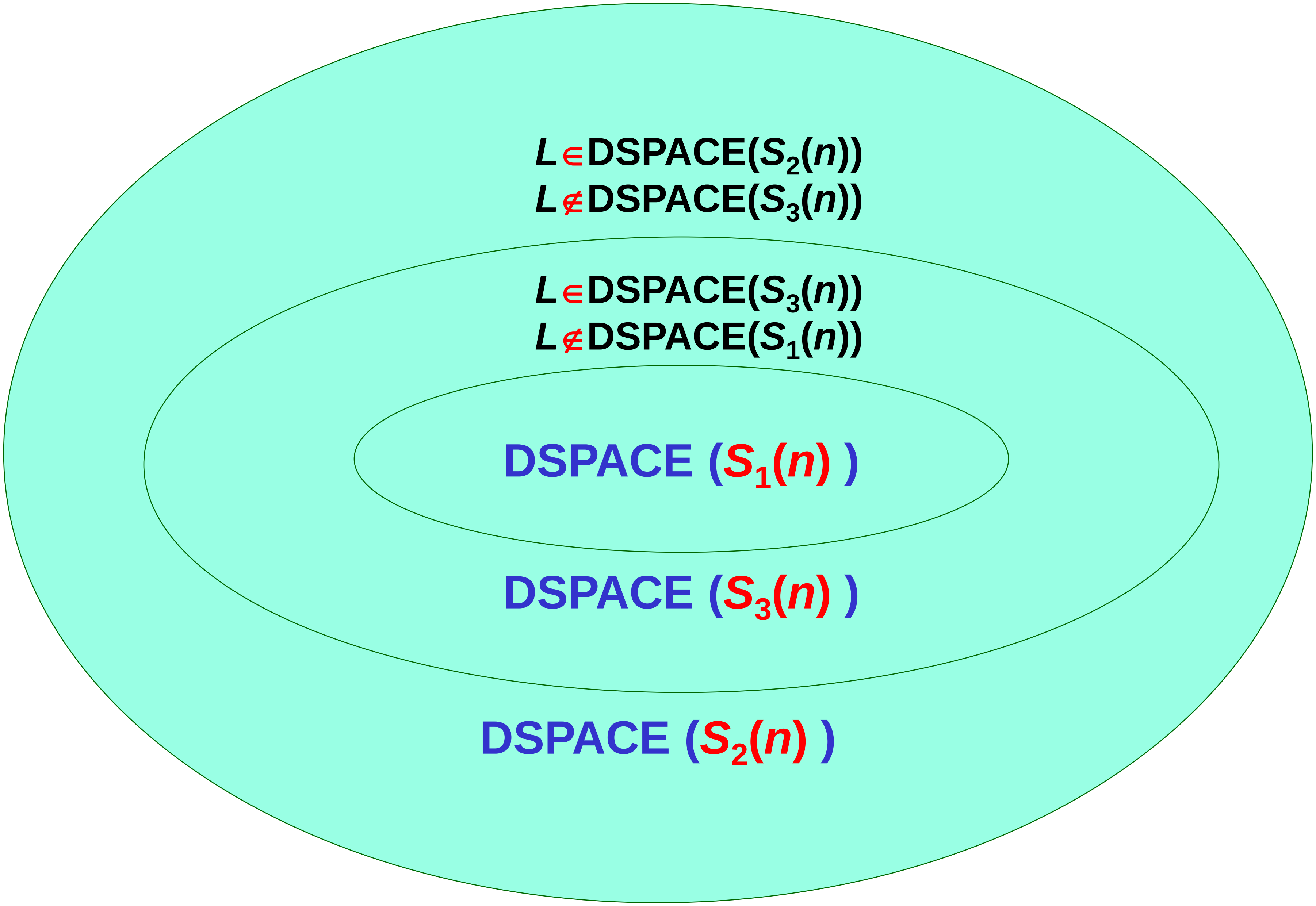
Continuity of the hierarchy for fully space-constructible functions



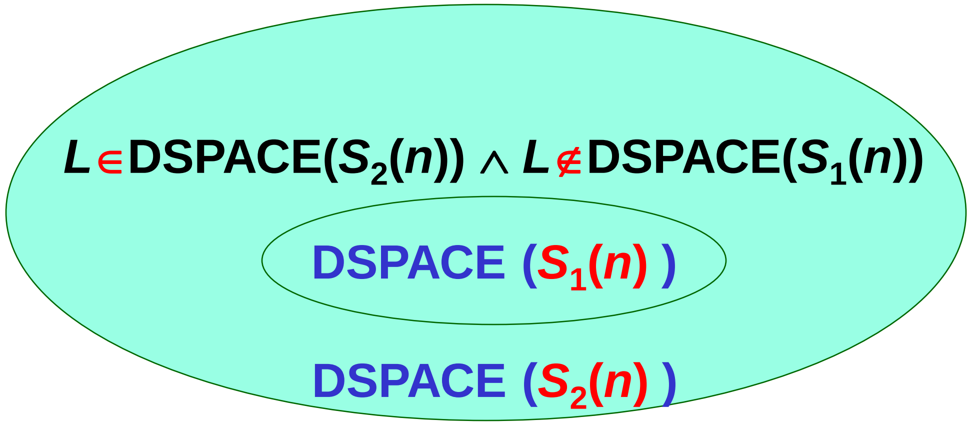
Continuity of the hierarchy for fully space-constructible functions



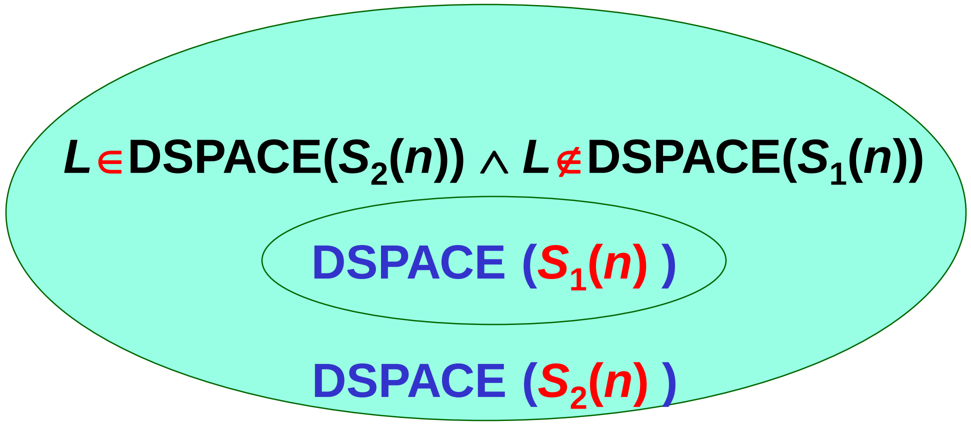
Continuity of the hierarchy for fully space-constructible functions



Continuity of the hierarchy for fully space-constructible functions

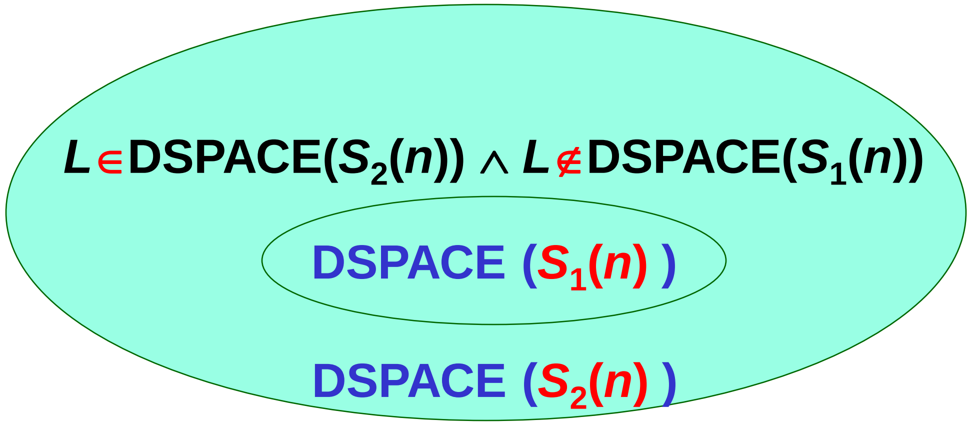


Continuity of the hierarchy for fully space-constructible functions



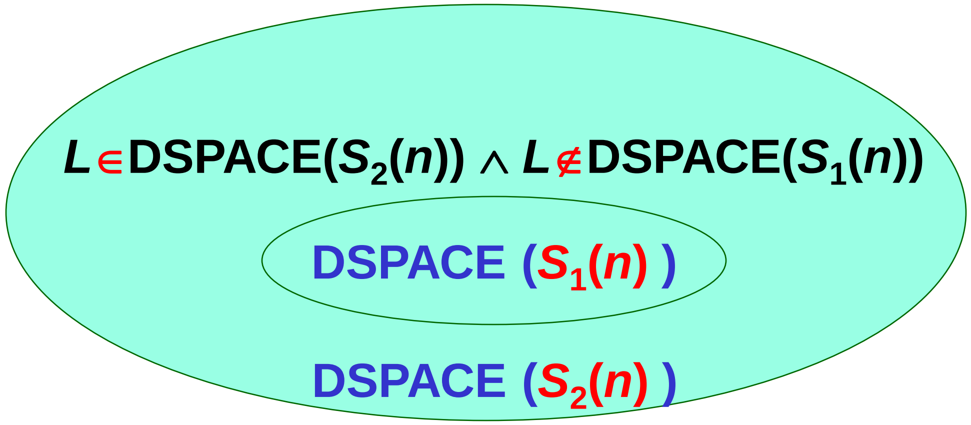
- **If**

Continuity of the hierarchy for fully space-constructible functions



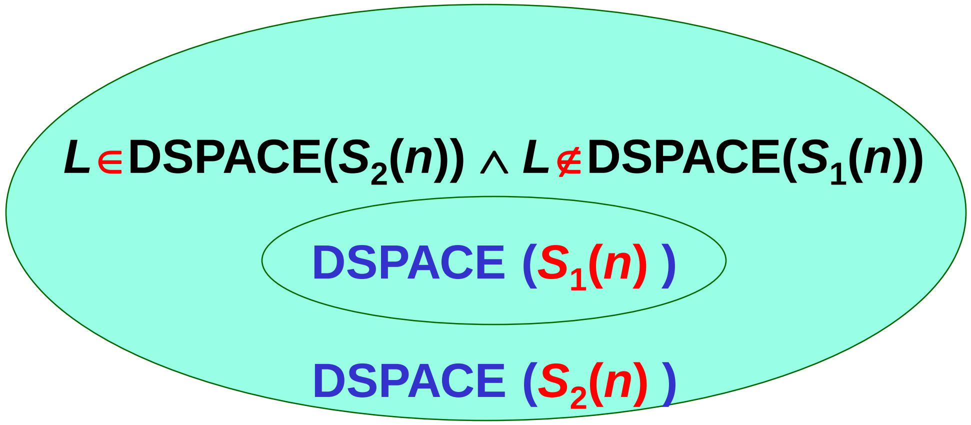
- **If**
  - **$S_2(n)$ is a fully space-constructible function**

Continuity of the hierarchy for fully space-constructible functions



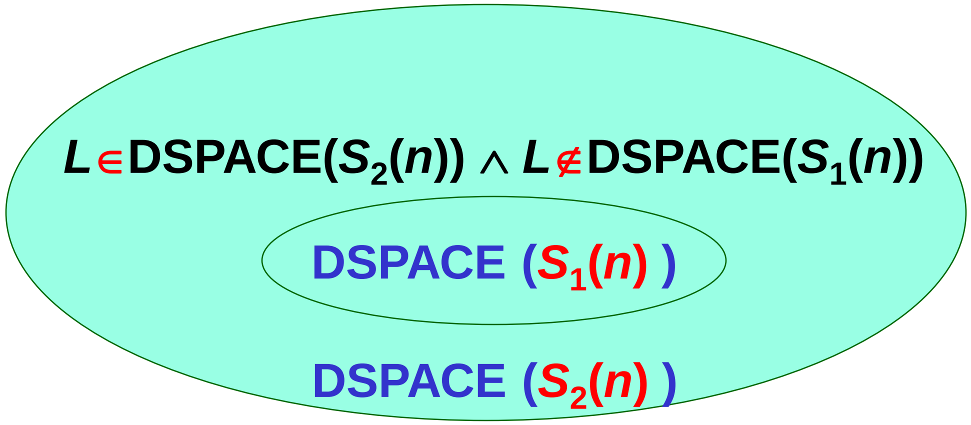
- **If**
    - **$S_2(n)$ is a fully space-constructible function**
    - **$\inf_{n\to\infty} S_1(n)/S_2(n) = 0$**

Continuity of the hierarchy for fully space-constructible functions



- **If**
  - $S_2(n)$ **is a fully space-constructible function**
  - $\inf_{n\to\infty} S_1(n)/S_2(n) = 0$
  - $S_1(n)$ **and** $S_2(n)$ **are at least** $\log_2 n$

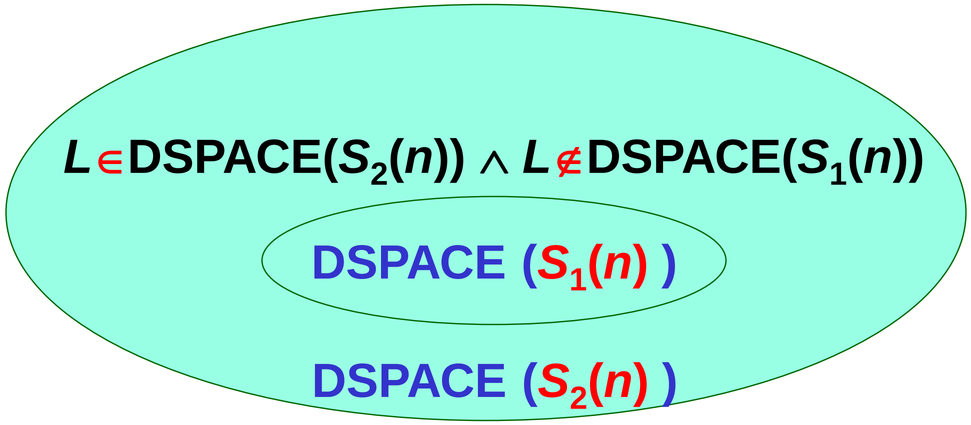
Continuity of the hierarchy for fully space-constructible functions



- **If**
  - **$S_2(n)$ is a fully space-constructible function**
  - **$\inf_{n\to\infty} S_1(n)/S_2(n) = 0$**
  - **$S_1(n)$ and $S_2(n)$ are at least $\log_2 n$**

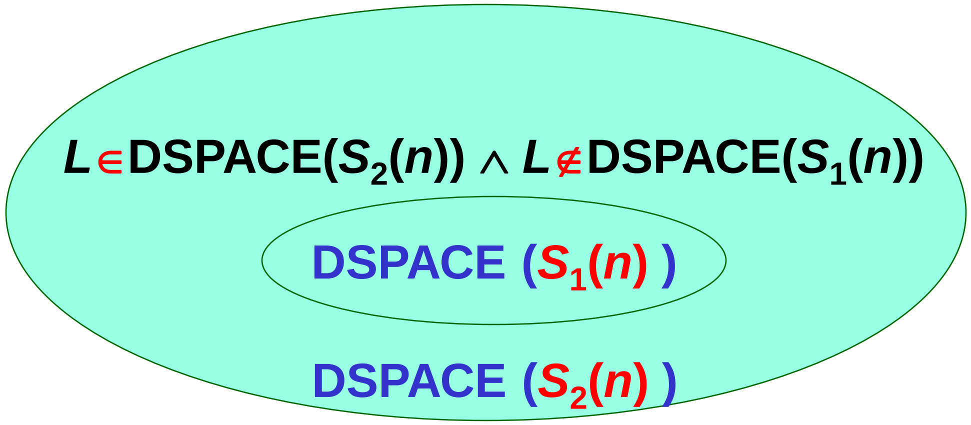
- **Then there is a language L**

Continuity of the hierarchy for fully space-constructible functions

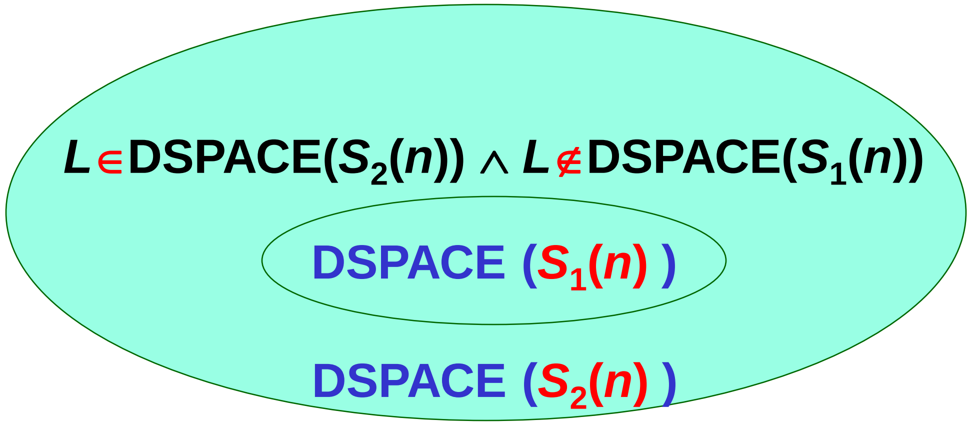


- **If**
  - **$S_2(n)$ is a fully space-constructible function**
  - **$\inf_{n\to\infty} S_1(n)/S_2(n) = 0$**
  - **$S_1(n)$ and $S_2(n)$ are at least $\log_2 n$**
- **Then there is a language L**
  - **$L \in \text{DSPACE}(S_2(n)) \wedge L \notin \text{DSPACE}(S_1(n))$**

Continuity of the hierarchy for fully space-constructible functions

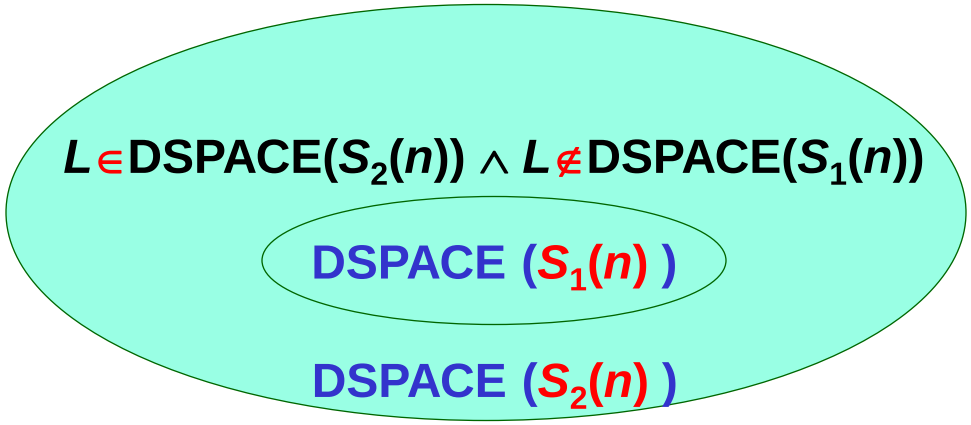


Continuity of the hierarchy for fully space-constructible functions



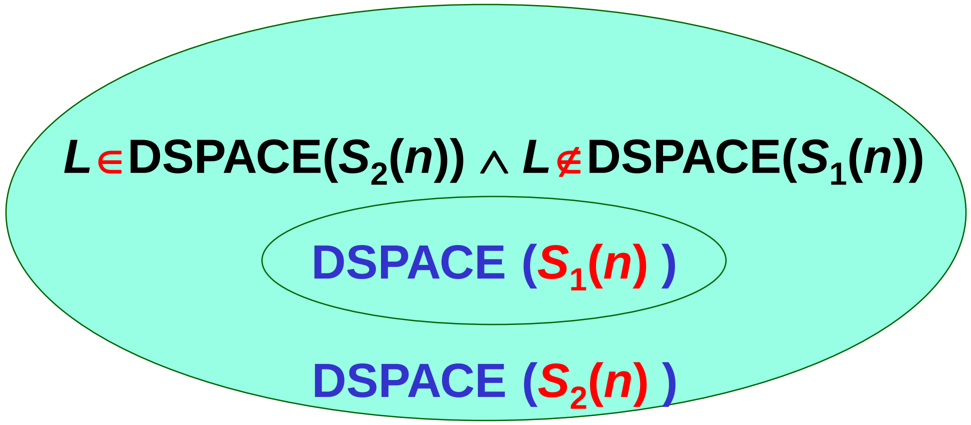
- **We construct TM *M* for which:**

Continuity of the hierarchy for fully space-constructible functions



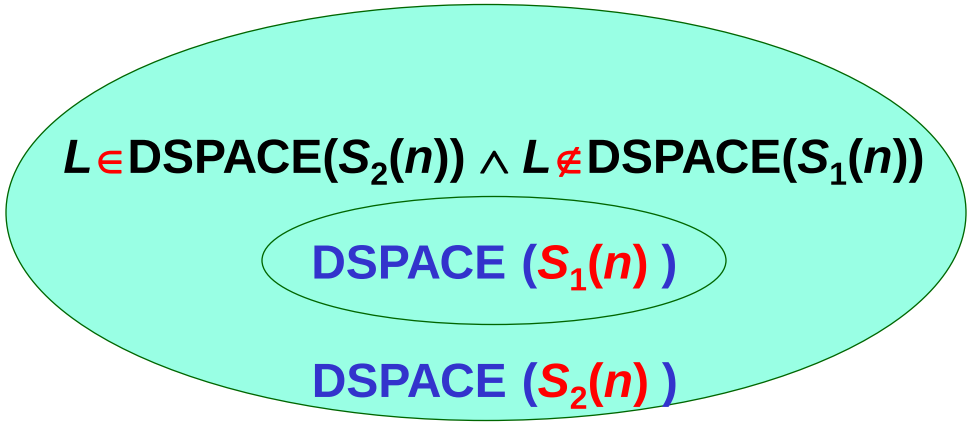
- **We construct TM *M* for which:**
  - **TM *M* has space complexity of $S_2(n)$**

Continuity of the hierarchy for fully space-constructible functions

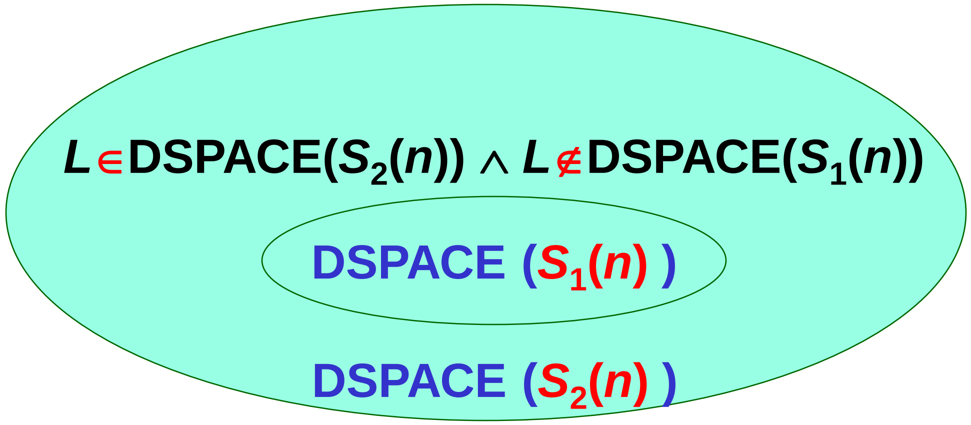


- **We construct TM *M* for which:**
  - **TM *M* has space complexity of $S_2(n)$**
  - **TM *M* gives the opposite decision than any TM with space complexity $S_1(n)$ for at least one input string**

Continuity of the hierarchy for fully space-constructible functions

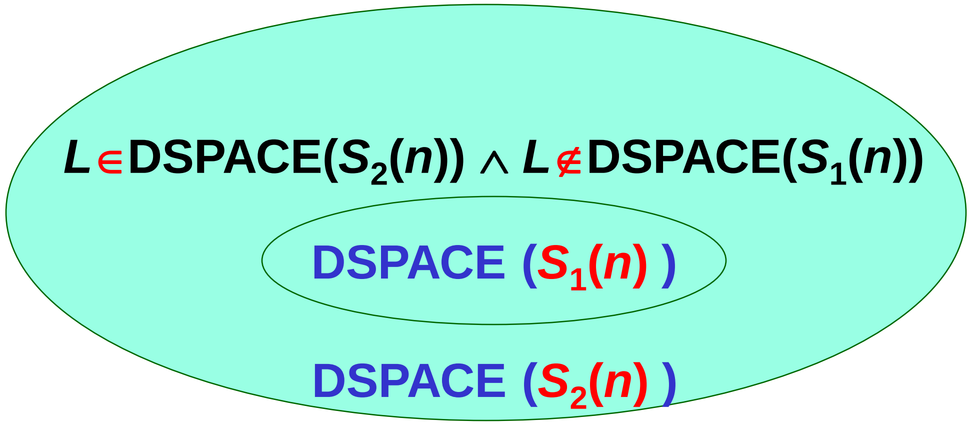


Continuity of the hierarchy for fully space-constructible functions



- **TM *M* – ensure a space complexity $S_2(n)$**

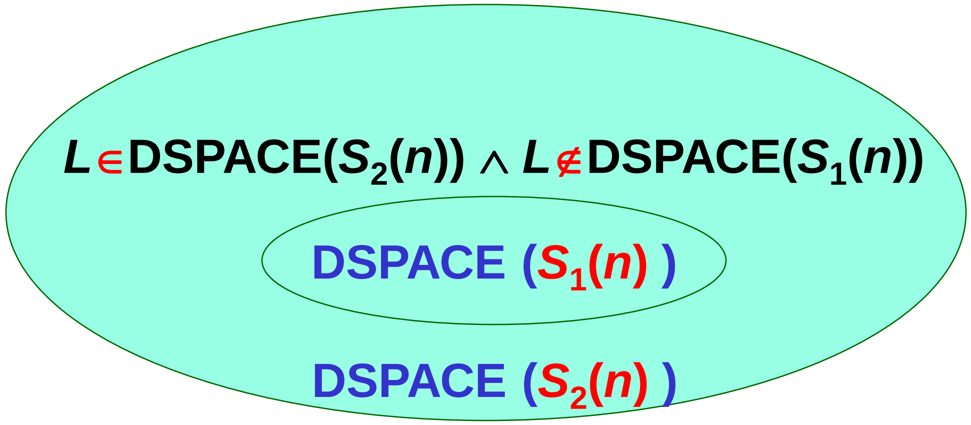
Continuity of the hierarchy for fully space-constructible functions



- **TM *M* – ensure a space complexity $S_2(n)$**

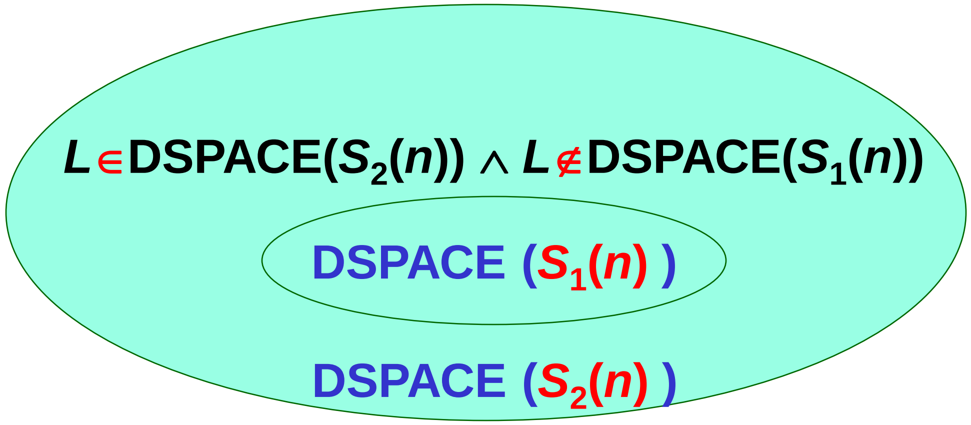
  - **We simulate any TM M_{S2} with space complexity $S_2(n)$**

Continuity of the hierarchy for fully space-constructible functions



- **TM *M* – ensure a space complexity $S_2(n)$**
  - **We simulate any TM M_{S2} with space complexity $S_2(n)$**
  - **$S_2(n)$ is fully space-constructible**

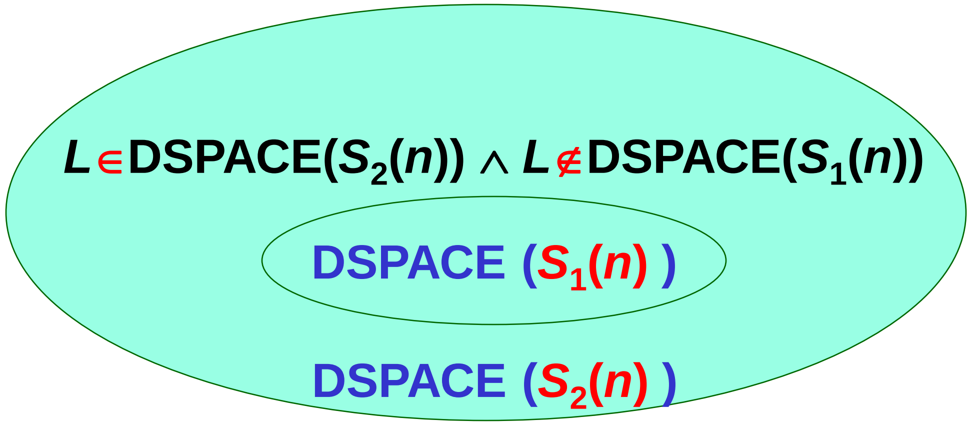
Continuity of the hierarchy for fully space-constructible functions



- **TM M – ensure a space complexity $S_2(n)$**
  - **We simulate any TM M_{S2} with space complexity $S_2(n)$**
  - **$S_2(n)$ is fully space-constructible**

    $\Rightarrow$ **TM M_{S2} for any string of length n uses all $S_2(n)$ cells**

Continuity of the hierarchy for fully space-constructible functions



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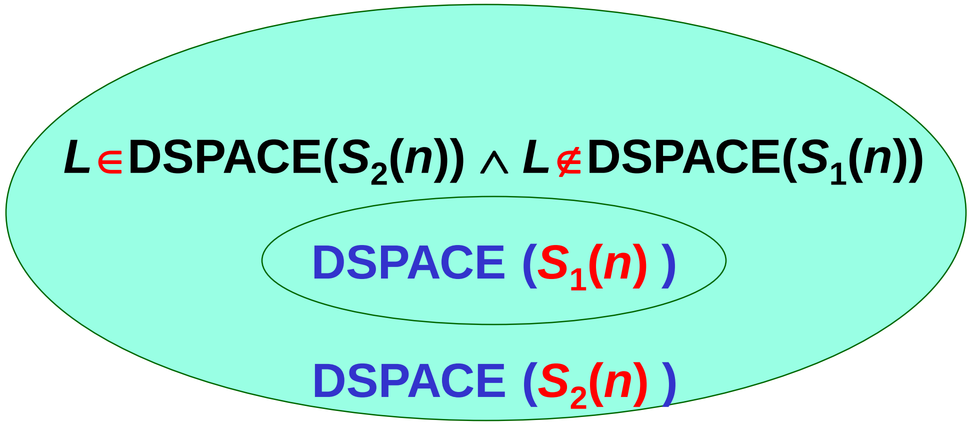
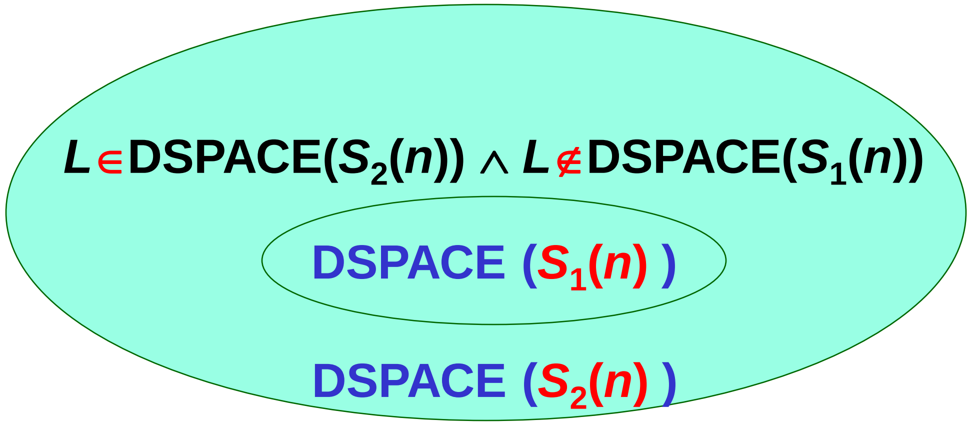
Continuity of the hierarchy for fully space-constructible functions



- **TM *M* - ensure a space complexity $S_2(n)$**

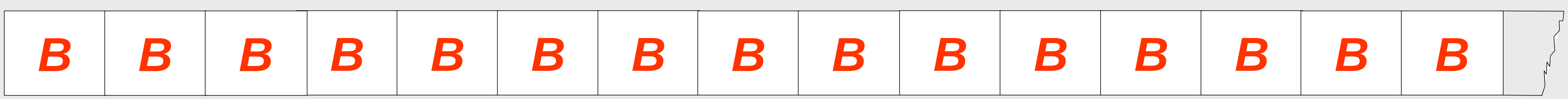
TM *M* which accepts the language *L*

Continuity of the hierarchy for fully space-constructible functions

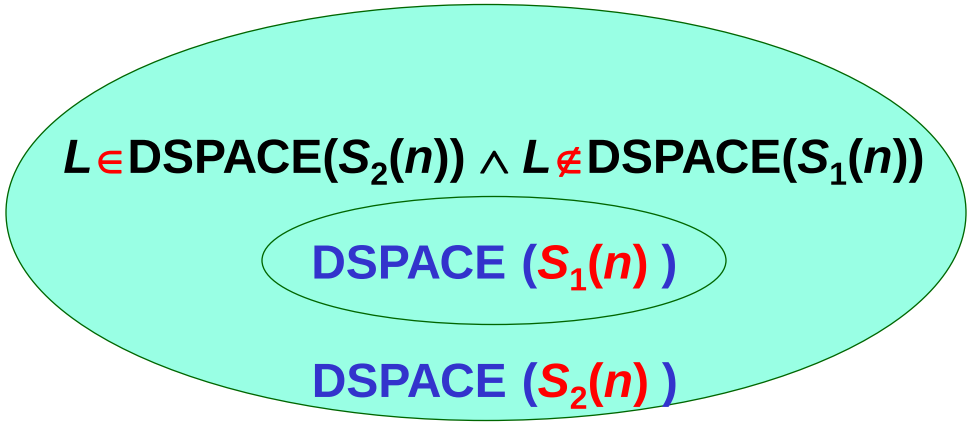


- TM M - ensure a space complexity $S_2(n)$

TM *M* which accepts the language *L*

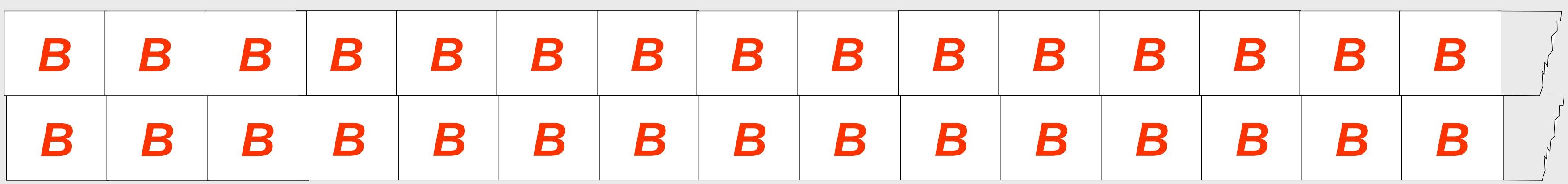


Continuity of the hierarchy for fully space-constructible functions

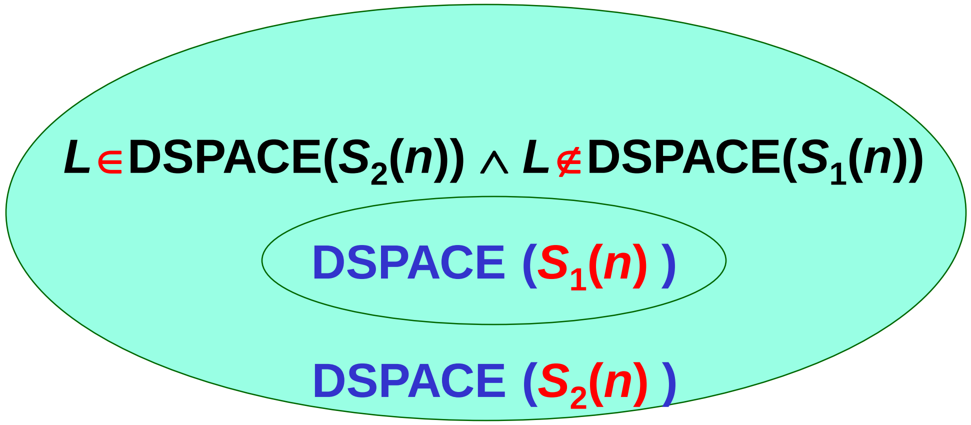


- **TM M - ensure a space complexity $S_2(n)$**

TM M which accepts the language L



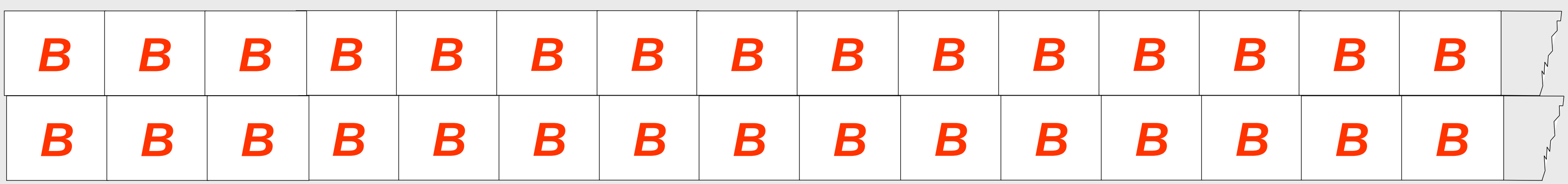
Continuity of the hierarchy for fully space-constructible functions

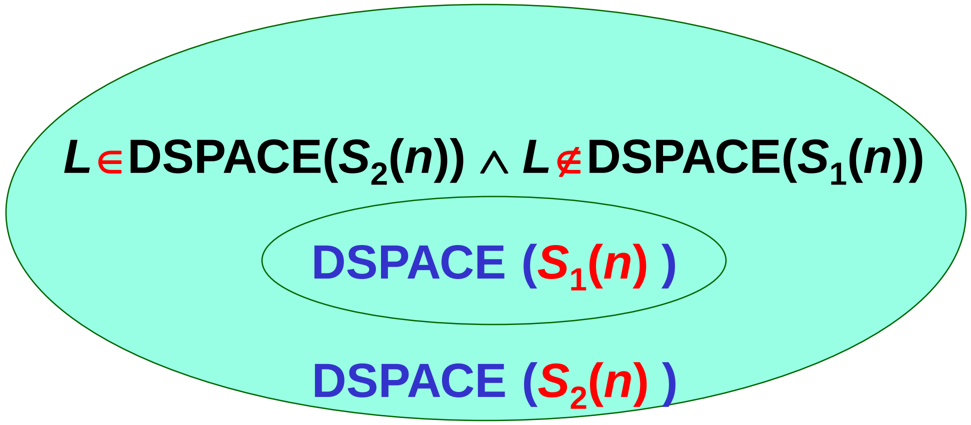


- **TM *M* - ensure a space complexity $S_2(n)$**

TM *M* which accepts the language *L*

Simulation of TM M_{S2}

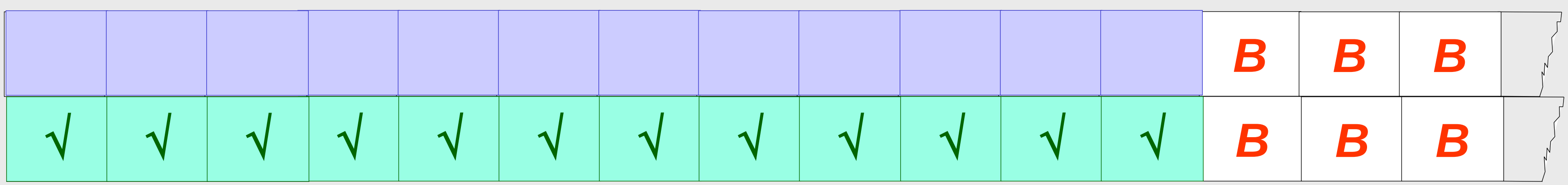


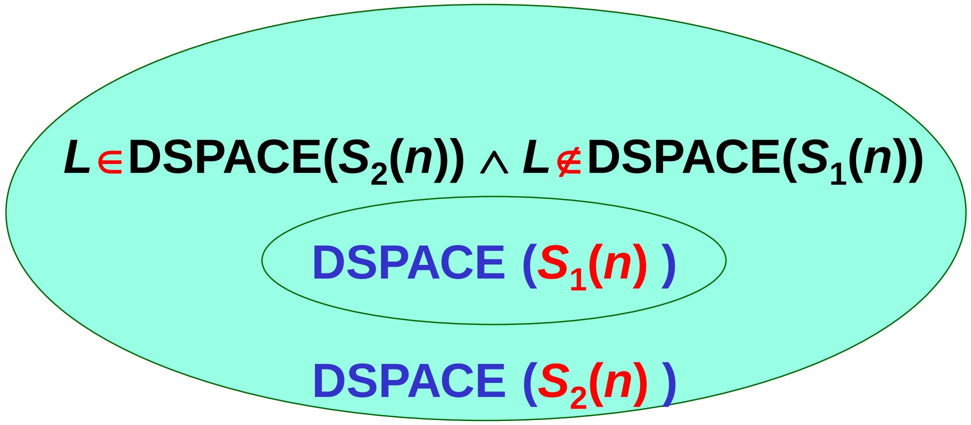


- TM M - ensure a space complexity $S_2(n)$

TM *M* which accepts the language *L*

Simulation of TM M_{S2}

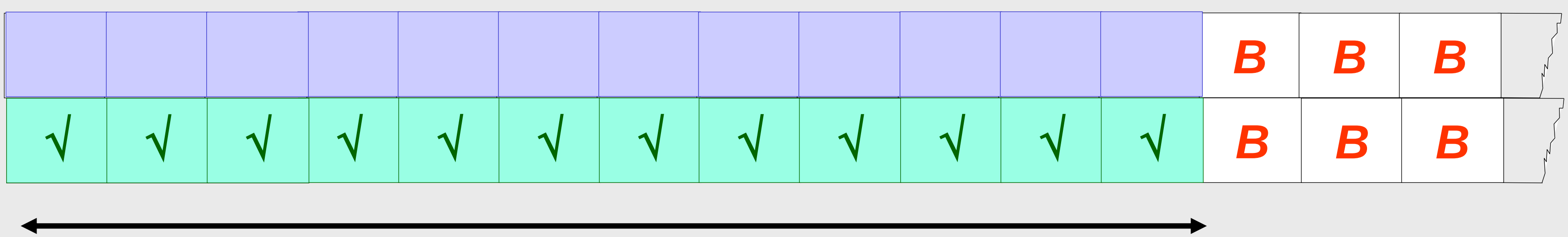




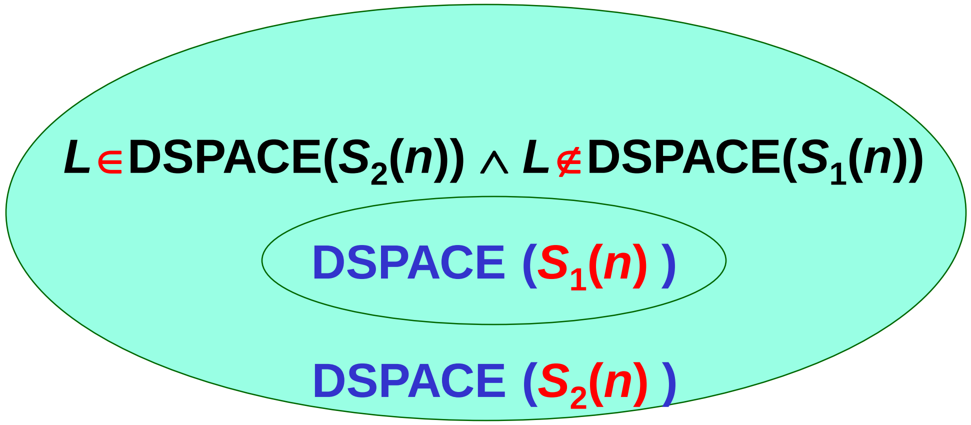
- TM *M* - ensure a space complexity $S_2(n)$

TM *M* which accepts the language *L*

Simulation of TM M_{S2}

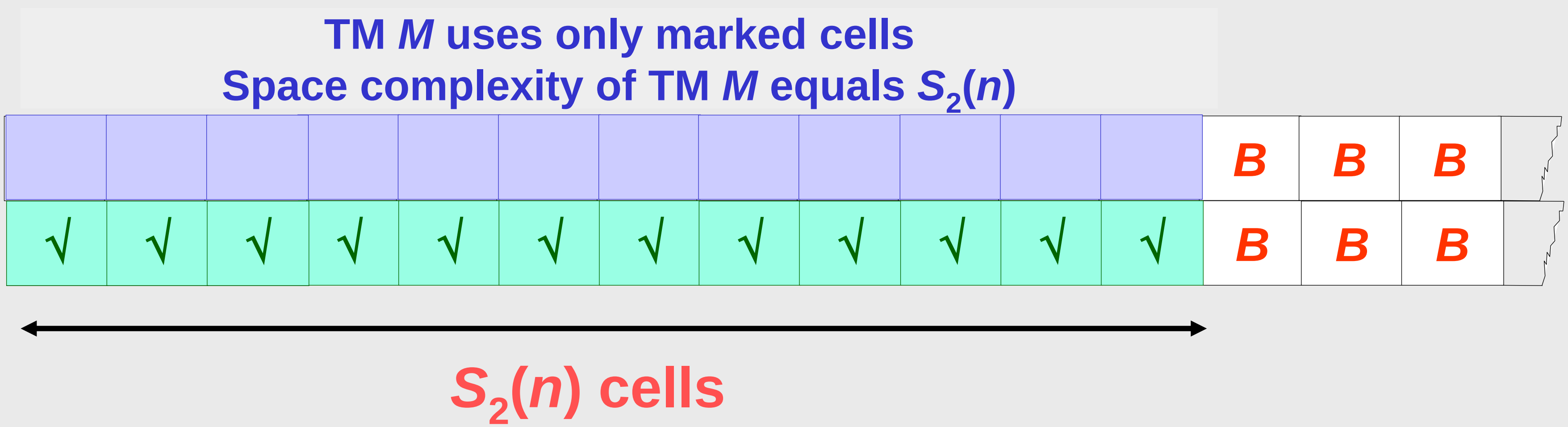


$S_2(n)$ cells

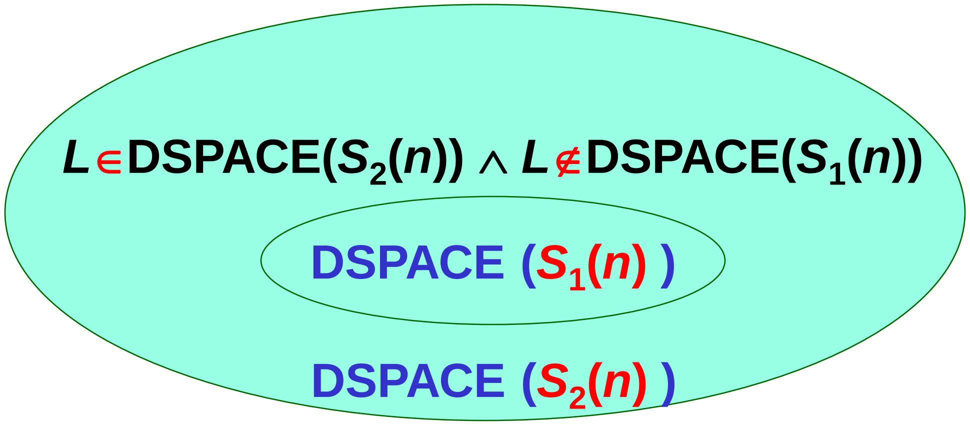


- **TM *M* - ensure a space complexity $S_2(n)$**

TM *M* which accepts the language *L*

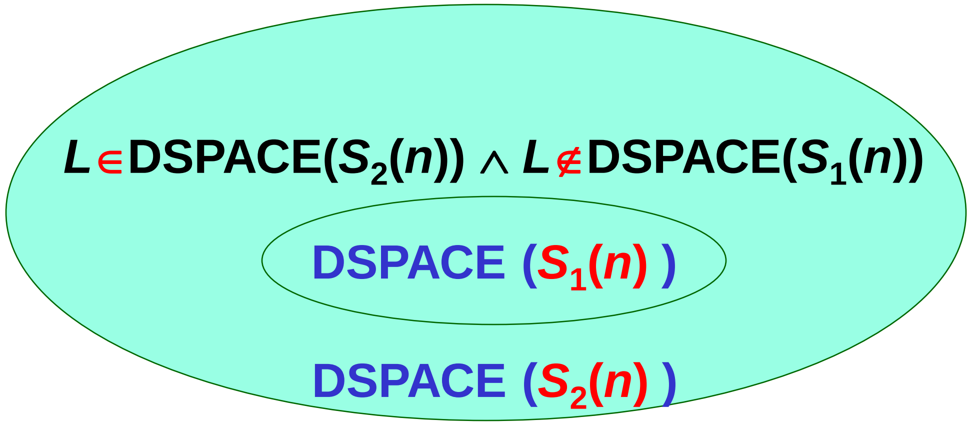


Continuity of the hierarchy for fully space-constructible functions



- **Opposite decision than any TM in class DSPACE($S_1(n)$)**

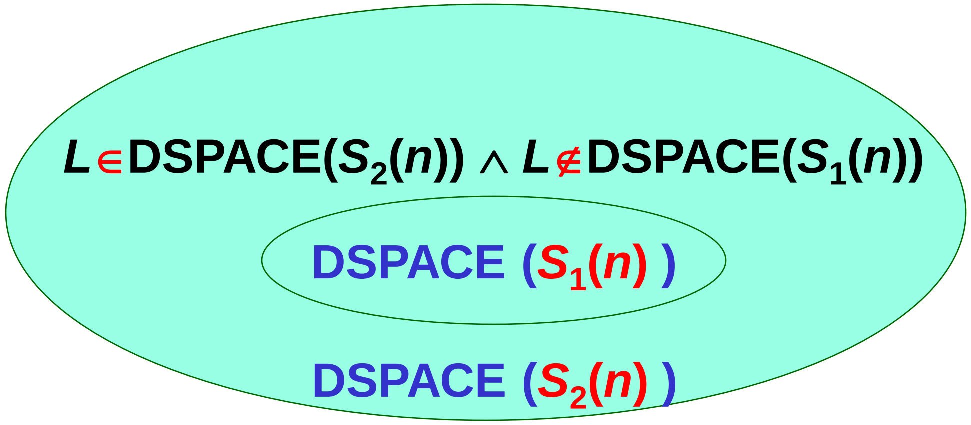
Continuity of the hierarchy for fully space-constructible functions



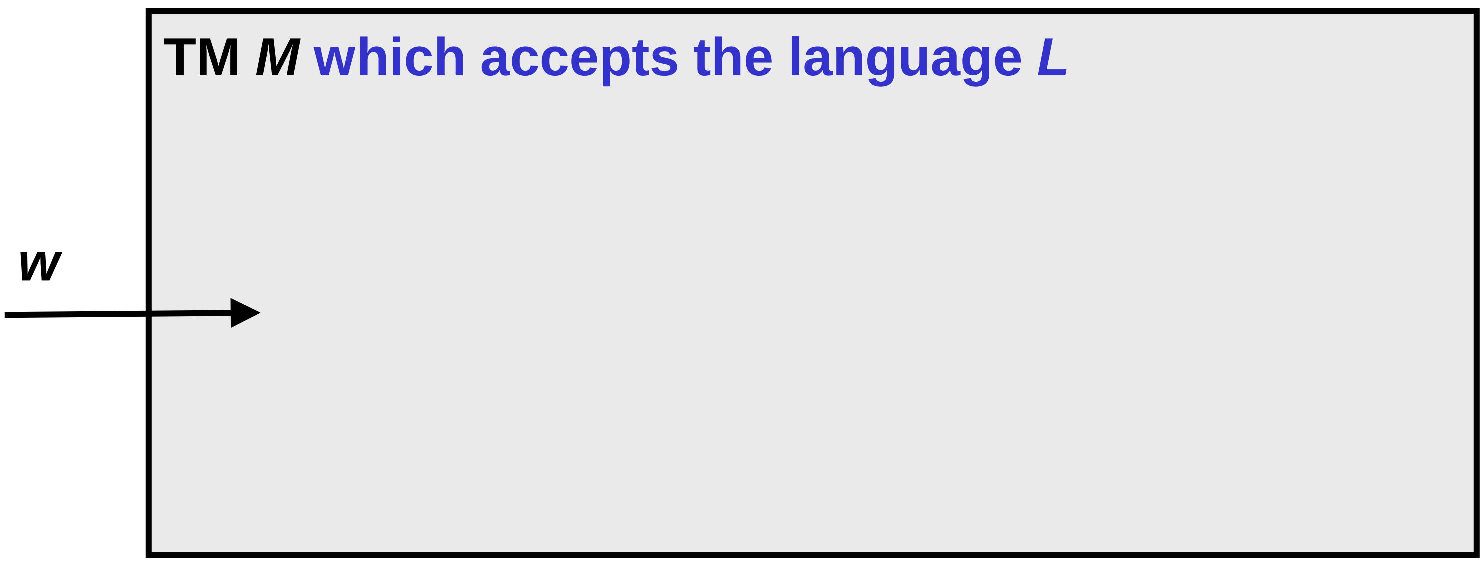
- **Opposite decision than any TM in class DSPACE($S_1(n)$)**

TM *M* which accepts the language *L*

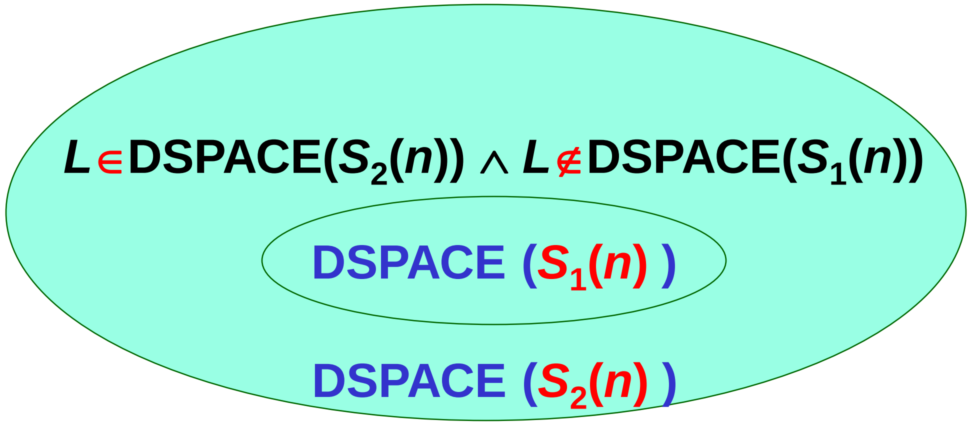
Continuity of the hierarchy for fully space-constructible functions



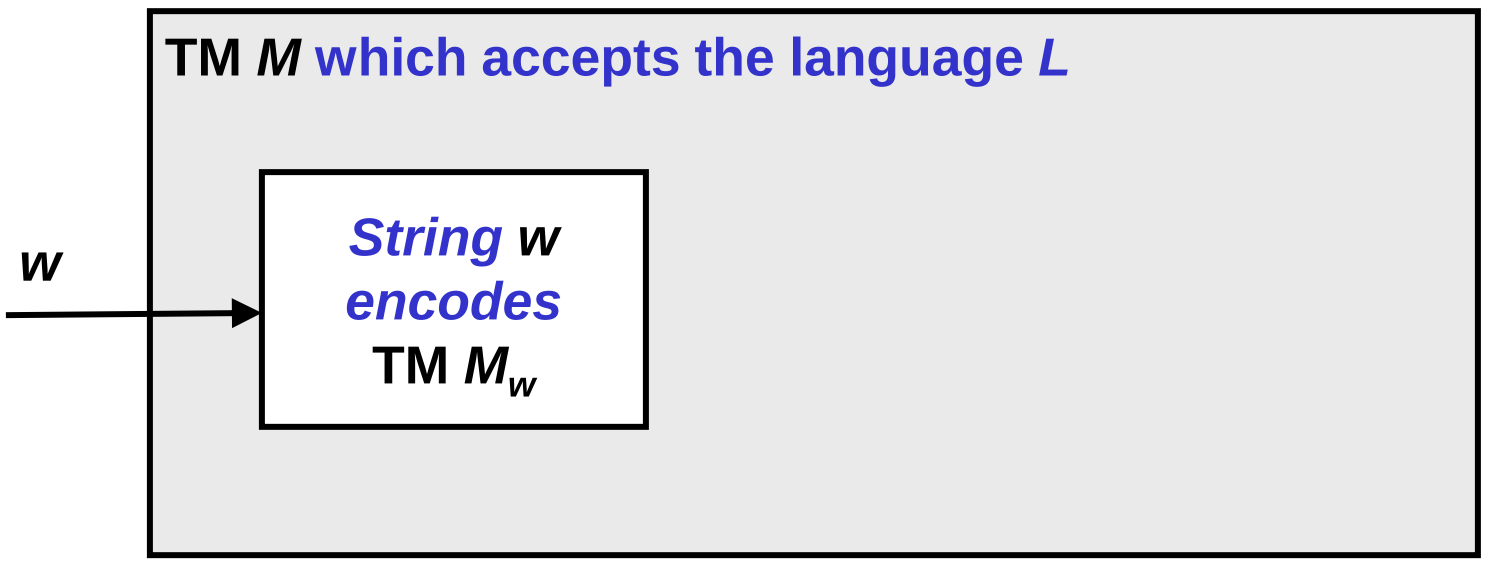
- **Opposite decision than any TM in class DSPACE($S_1(n)$)**



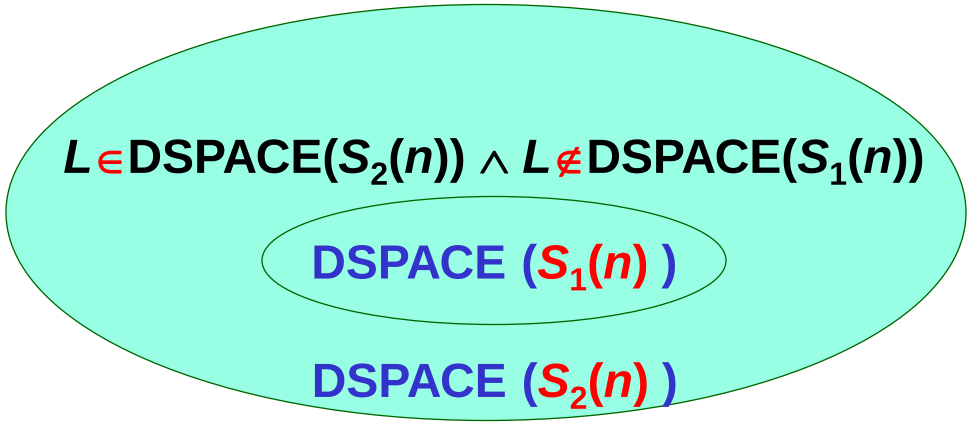
Continuity of the hierarchy for fully space-constructible functions



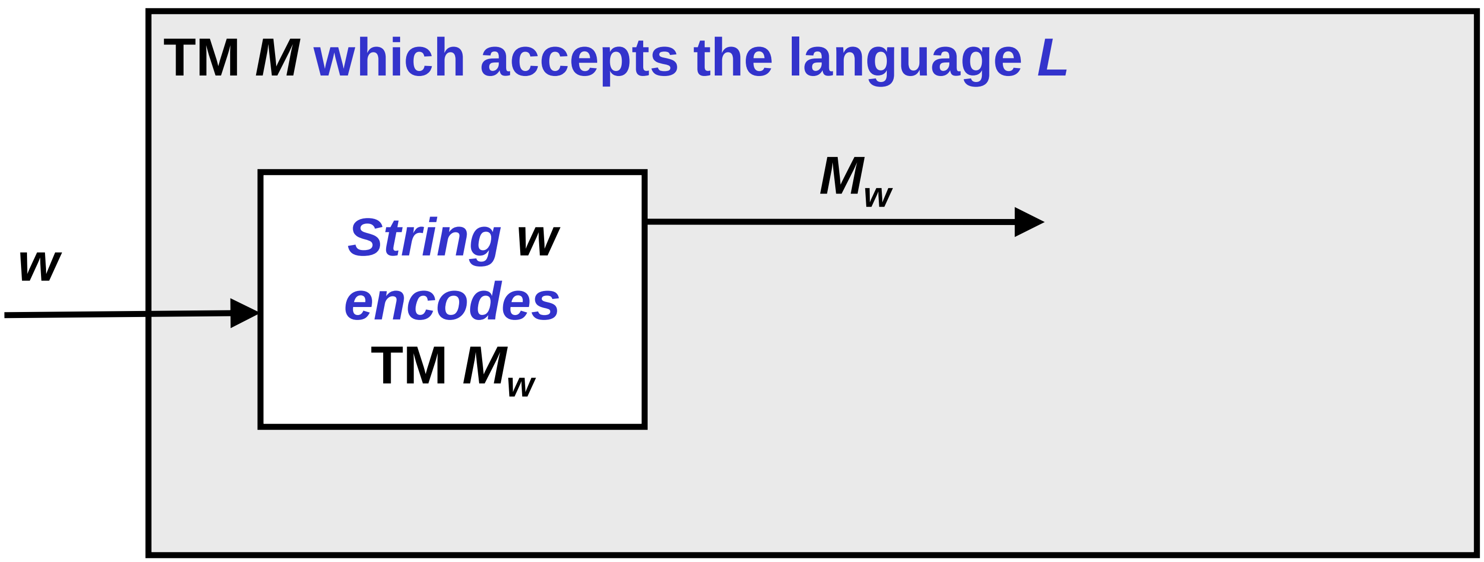
- Opposite decision than any TM in class DSPACE($S_1(n)$)



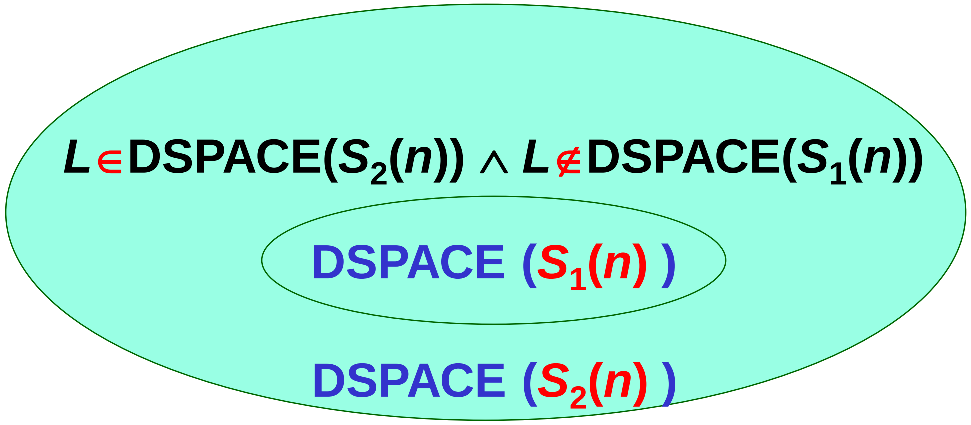
Continuity of the hierarchy for fully space-constructible functions



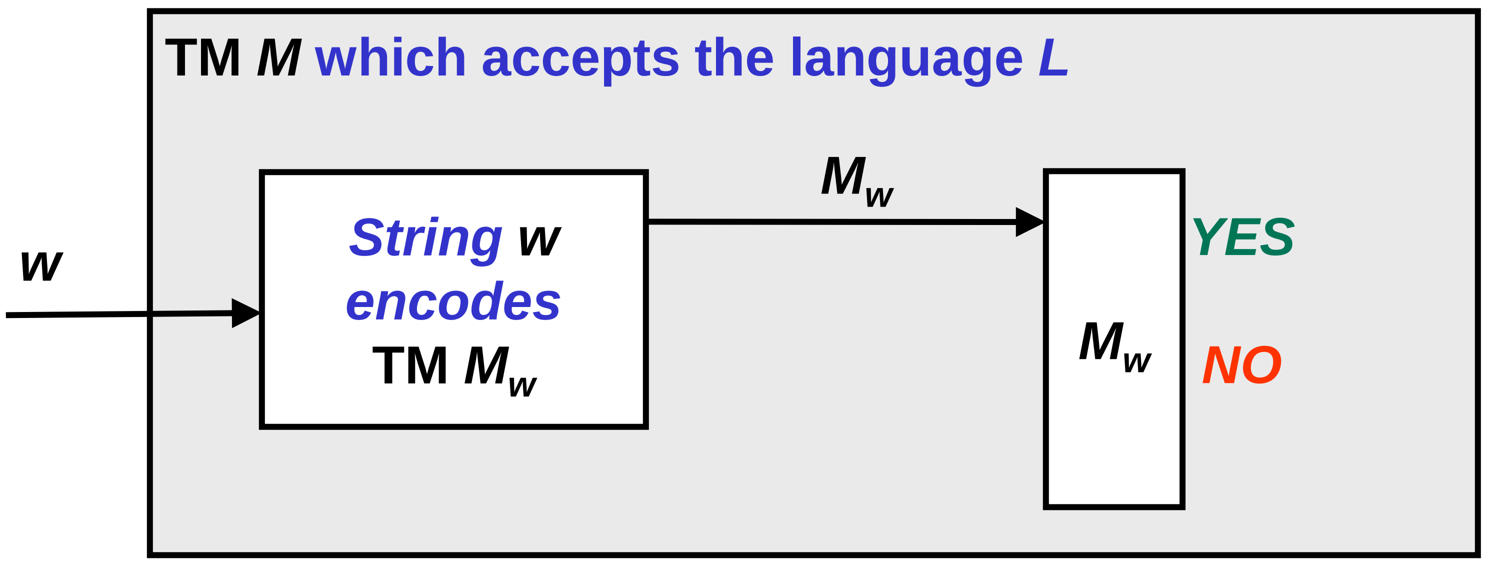
- Opposite decision than any TM in class DSPACE($S_1(n)$)



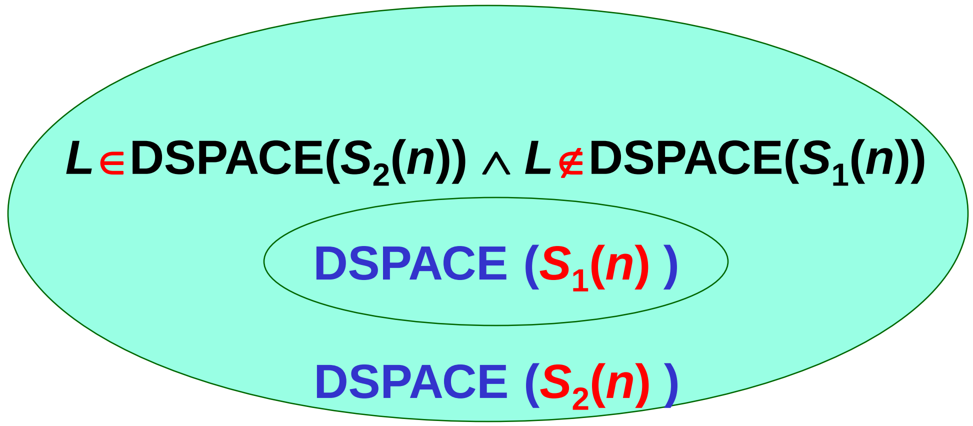
Continuity of the hierarchy for fully space-constructible functions



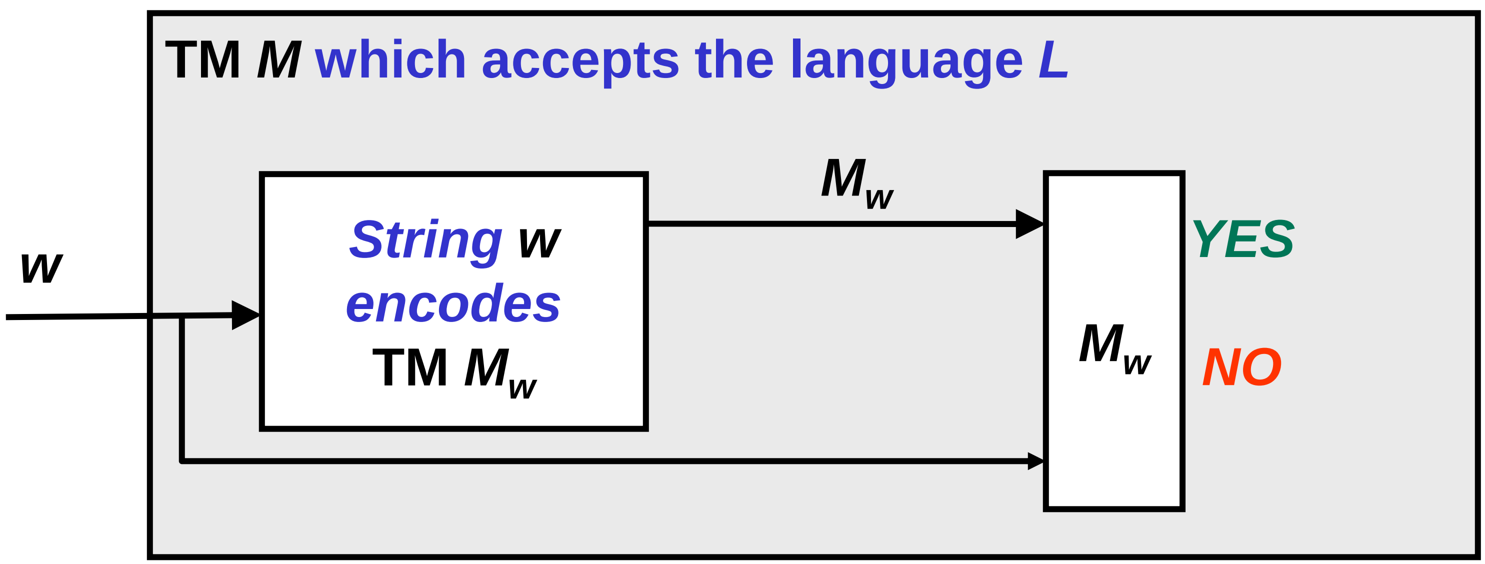
- Opposite decision than any TM in class DSPACE($S_1(n)$)



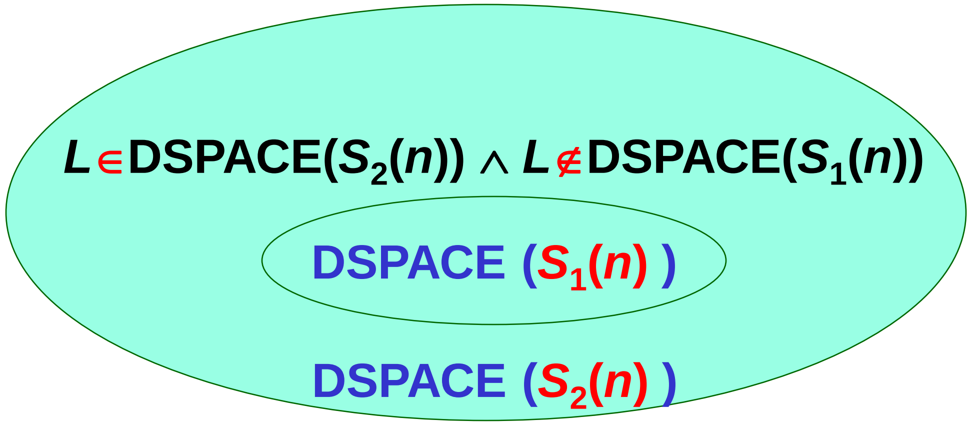
Continuity of the hierarchy for fully space-constructible functions



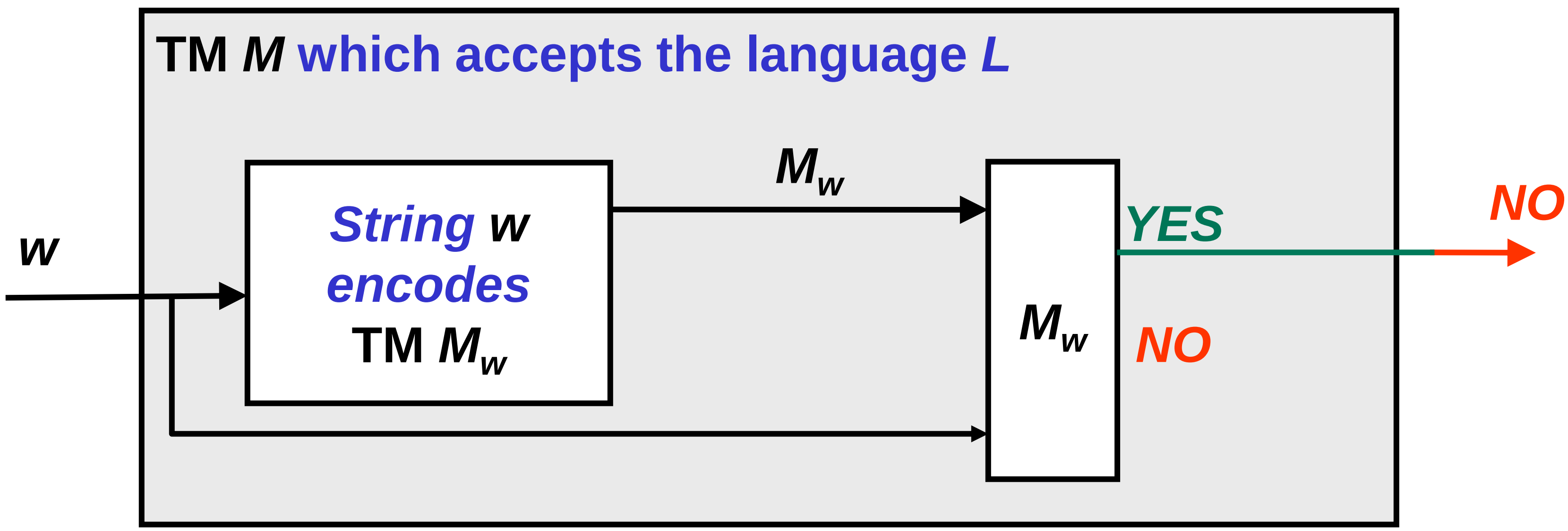
- Opposite decision than any TM in class DSPACE($S_1(n)$)



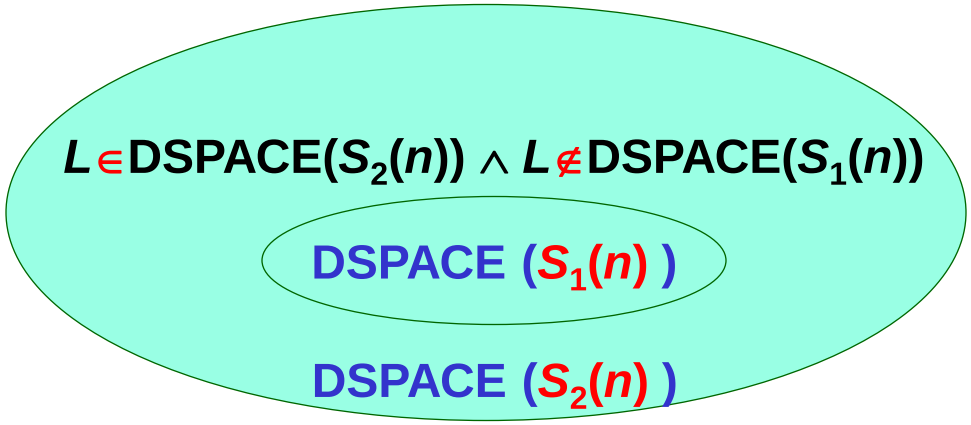
Continuity of the hierarchy for fully space-constructible functions



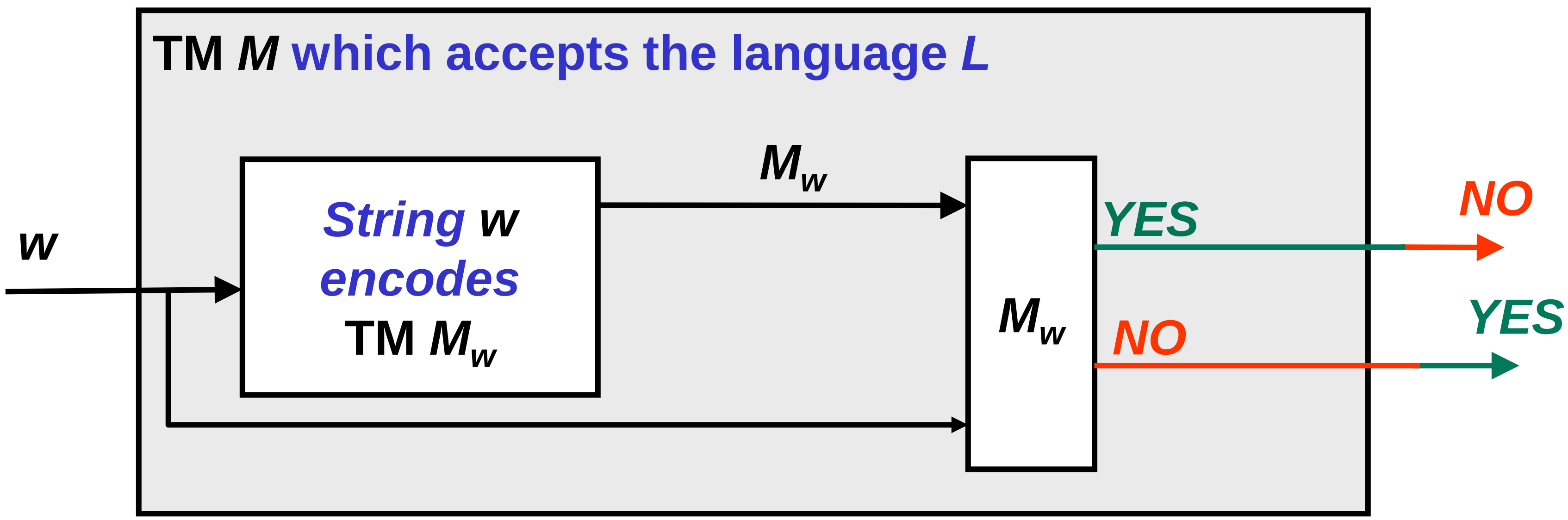
- Opposite decision than any TM in class DSPACE($S_1(n)$)



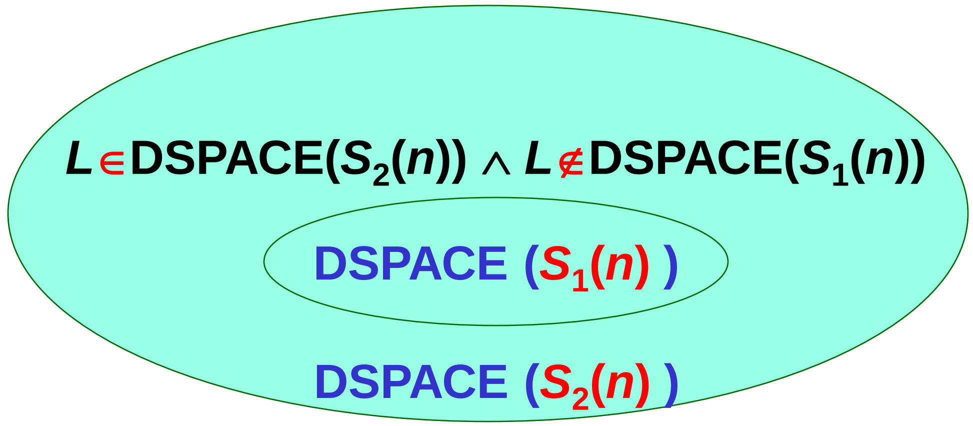
Continuity of the hierarchy for fully space-constructible functions



- Opposite decision than any TM in class DSPACE($S_1(n)$)

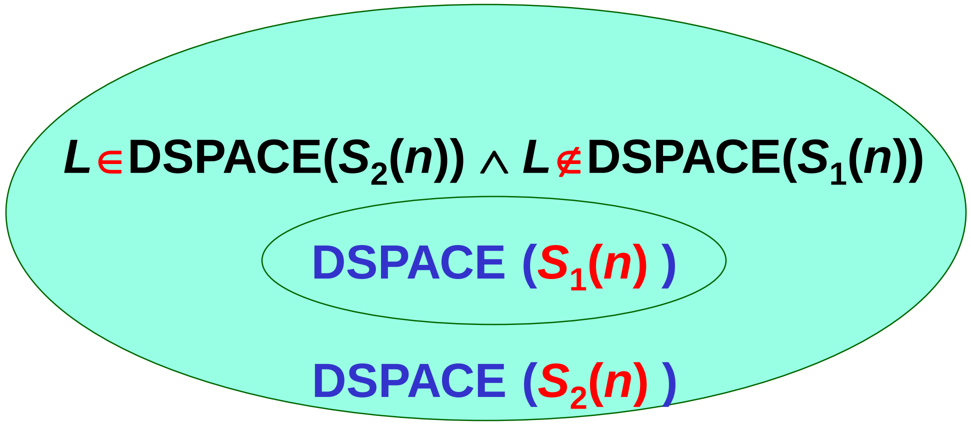


Continuity of the hierarchy for fully space-constructible functions



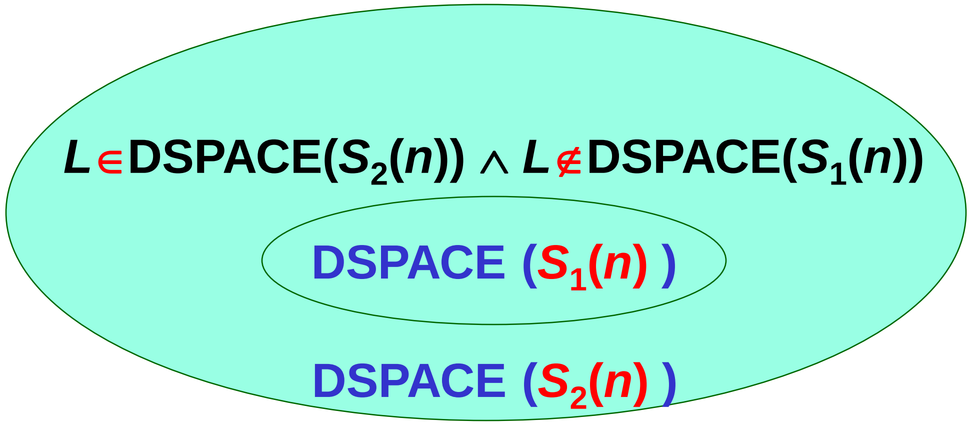
- **Simulating only those TM with space complexity $S_1(n)$**

Continuity of the hierarchy for fully space-constructible functions



- **Simulating only those TM with space complexity $S_1(n)$**
  - **$\mathrm{DSPACE}(\ S_1(n)\ ) \Rightarrow \mathrm{DTIME}(\ c^{S1(n)}\ )$**

Continuity of the hierarchy for fully space-constructible functions

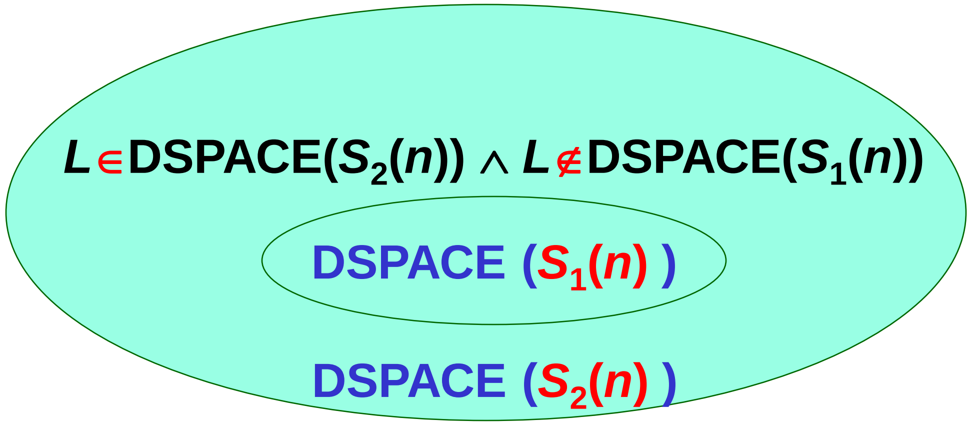


- Simulating only those TM with space complexity $S_1(n)$

- $\mathrm{DSPACE}(\ S_1(n)\ ) \Rightarrow \mathrm{DTIME}(\ c^{S1(n)}\ )$

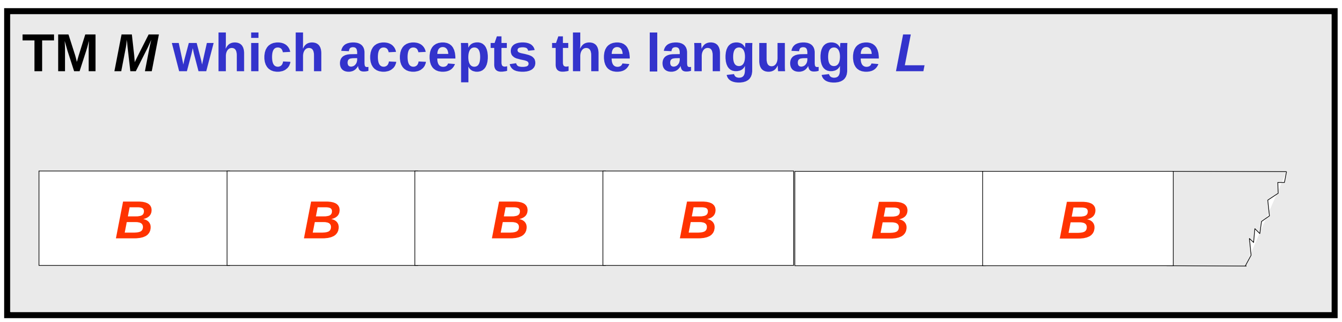
TM *M* which accepts the language *L*

Continuity of the hierarchy for fully space-constructible functions

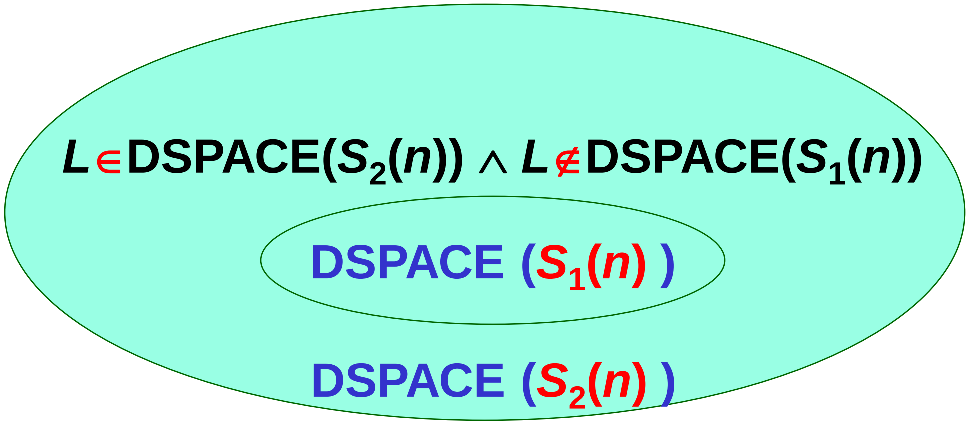


- Simulating only those TM with space complexity $S_1(n)$

- **DSPACE($S_1(n)$) $\Rightarrow$ DTIME($c^{S1(n)}$)**

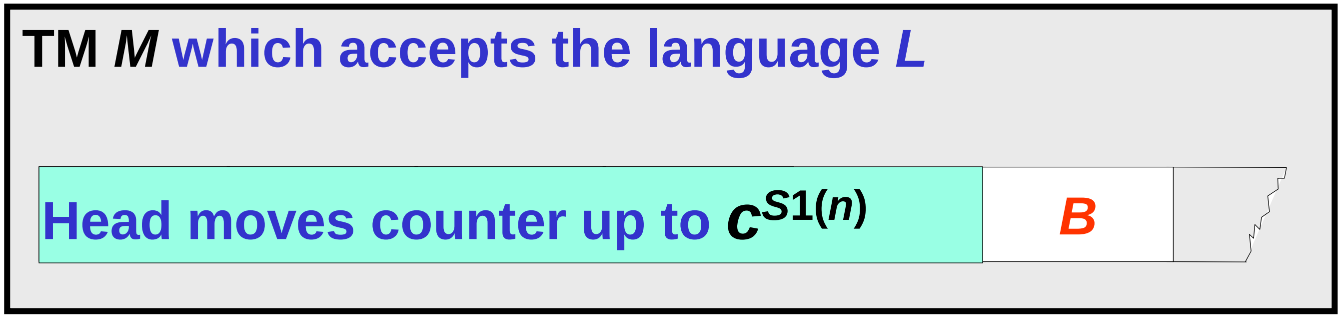


Continuity of the hierarchy for fully space-constructible functions

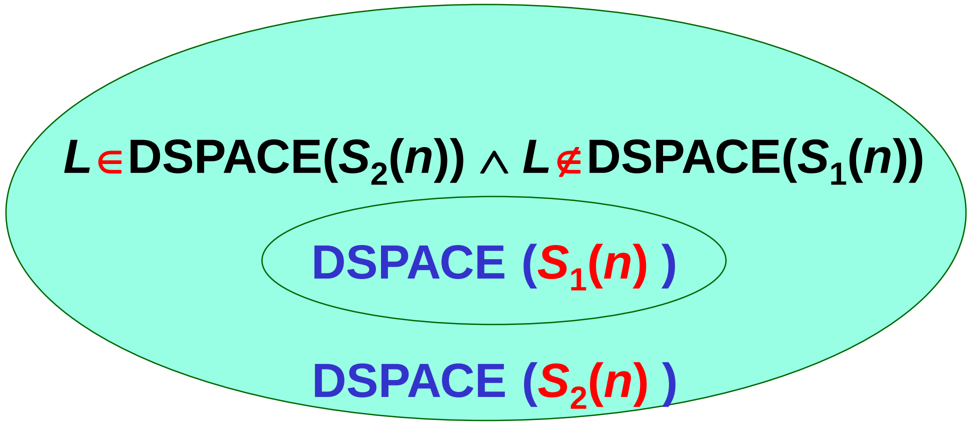


- Simulating only those TM with space complexity $S_1(n)$

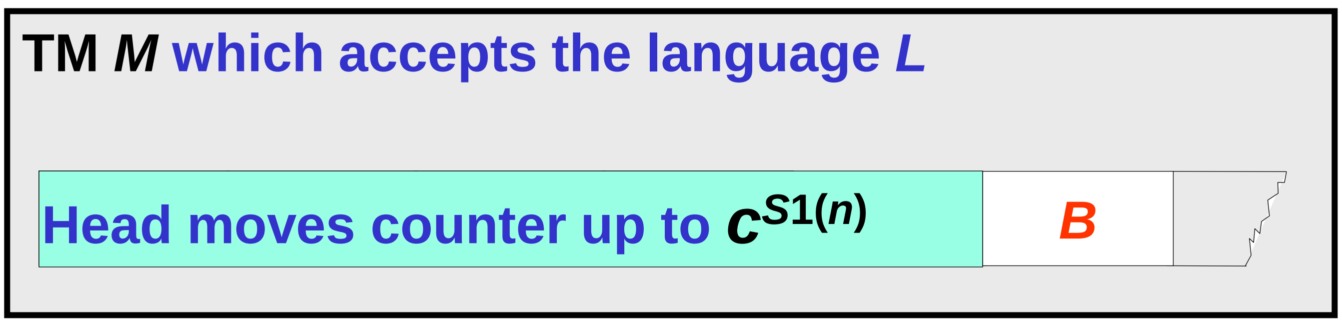
- **DSPACE($S_1(n)$) $\Rightarrow$ DTIME($c^{S1(n)}$)**



Continuity of the hierarchy for fully space-constructible functions

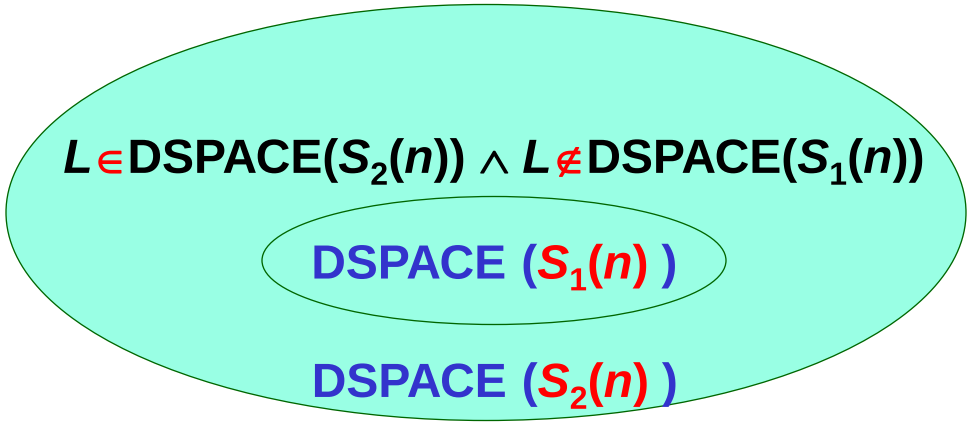


- Simulating only those TM with space complexity $S_1(n)$
- $\mathrm{DSPACE}(\ S_1(n)\ ) \Rightarrow \mathrm{DTIME}(\ c^{S1(n)}\ )$



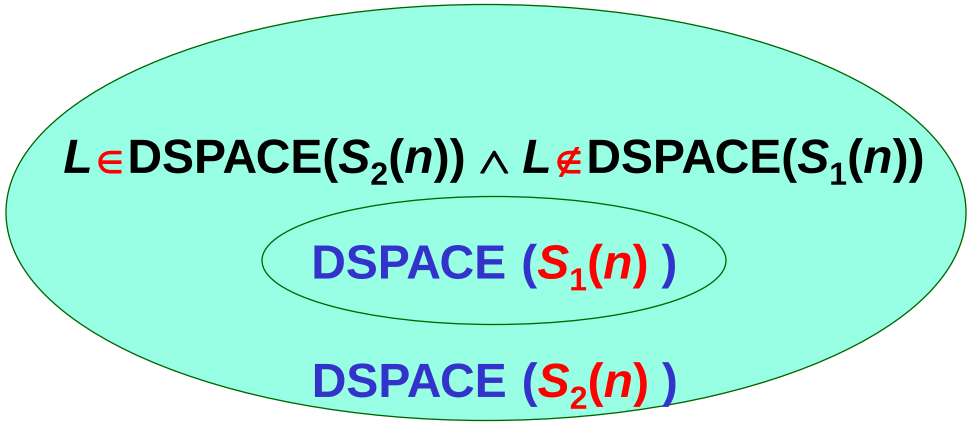
- Choosing the counter number base
  - Counter length < values of functions $S_1(n)$ and $S_2(n)$

Continuity of the hierarchy for fully space-constructible functions



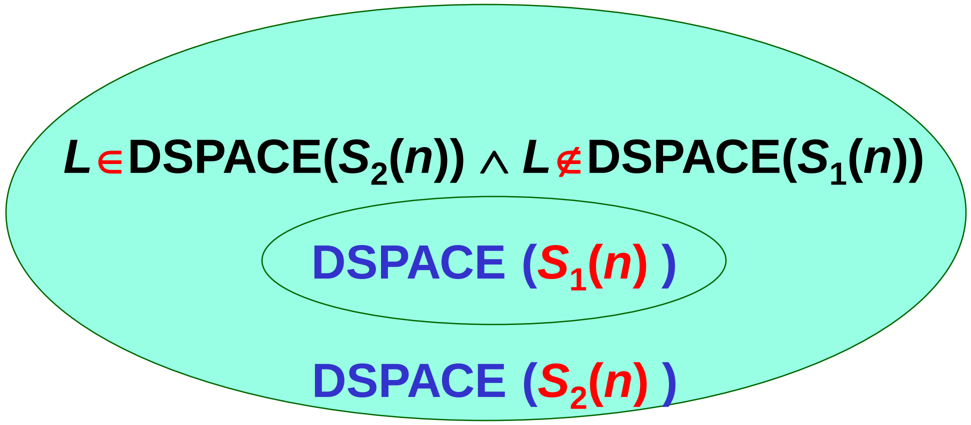
- **Ensure that *M* simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells**

Continuity of the hierarchy for fully space-constructible functions



- **Ensure that *M* simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells**
  - **Number of cells used by simulating a TM from class DSPACE($S_1(n)$) :**

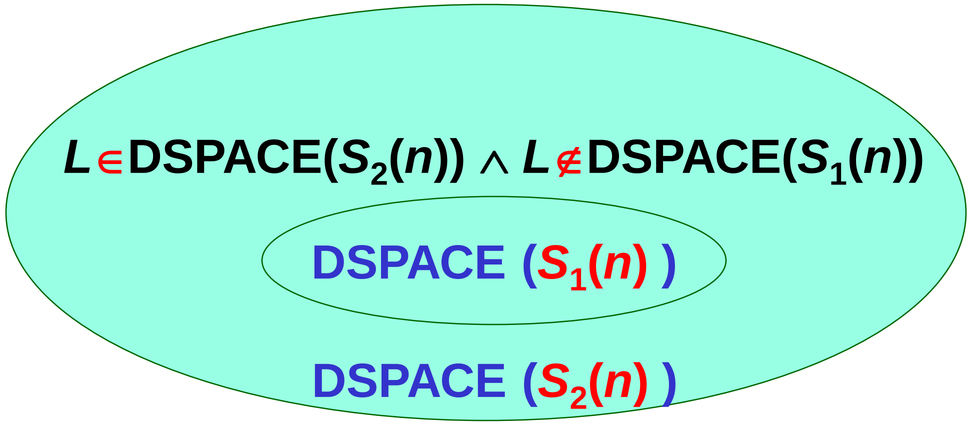
Continuity of the hierarchy for fully space-constructible functions



- **Ensure that *M* simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells**
  - **Number of cells used by simulating a TM from class DSPACE($S_1(n)$) :**

t **– number of tape symbols of a TM from class DSPACE($S_1(n)$)**

Continuity of the hierarchy for fully space-constructible functions

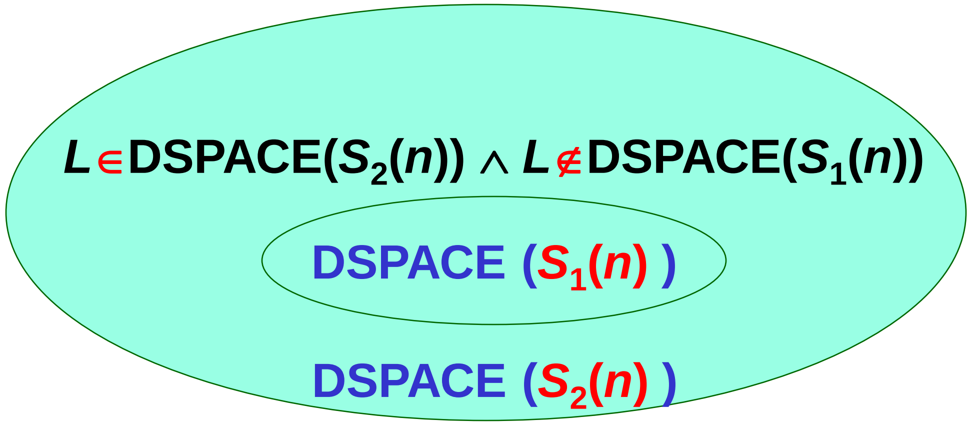


- **Ensure that M simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells**
  - **Number of cells used by simulating a TM from class DSPACE($S_1(n)$) :**

t **– number of tape symbols of a TM from class DSPACE($S_1(n)$)**

{0, 1, *B*} – tape symbols of TM *M*

Continuity of the hierarchy for fully space-constructible functions



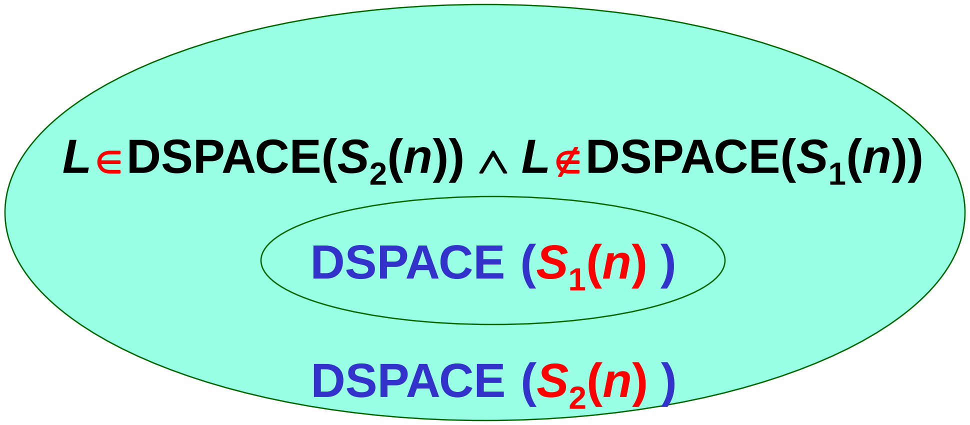
- **Ensure that *M* simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells**
  - **Number of cells used by simulating a TM from class DSPACE($S_1(n)$) :**

$$t$$

t **– number of tape symbols of a TM from class DSPACE($S_1(n)$)**

{0, 1, *B*} – tape symbols of TM *M*

Continuity of the hierarchy for fully space-constructible functions



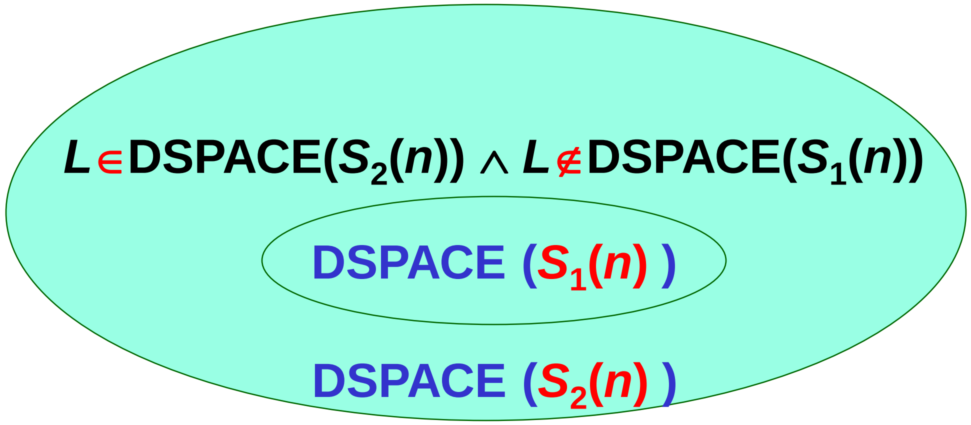
- **Ensure that M simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells**
  - **Number of cells used by simulating a TM from class DSPACE($S_1(n)$) :**

$$\log_2 t$$

t **– number of tape symbols of a TM from class DSPACE($S_1(n)$)**

{0, 1, *B*} – tape symbols of TM *M*

Continuity of the hierarchy for fully space-constructible functions



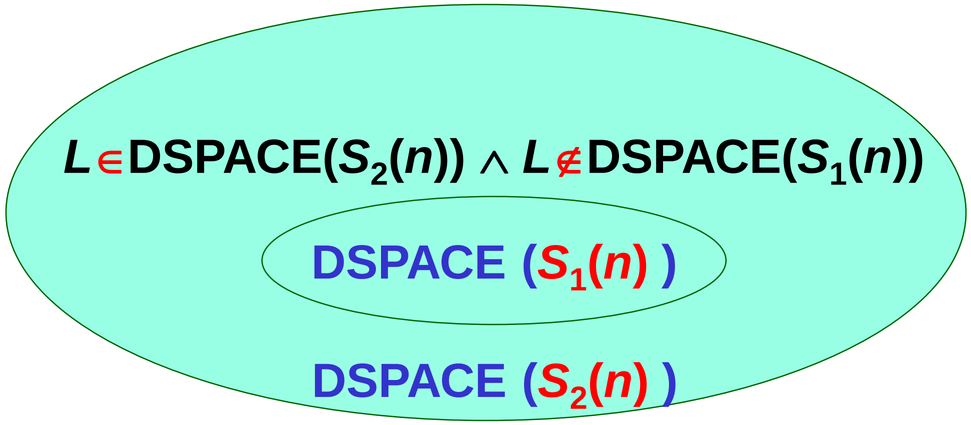
- Ensure that M simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells
  - Number of cells used by simulating a TM from class DSPACE($S_1(n)$) :

$$\lceil \log_2 t \rceil$$

t – number of tape symbols of a TM from class DSPACE($S_1(n)$)

$\{0, 1, B\}$ – tape symbols of TM M

Continuity of the hierarchy for fully space-constructible functions



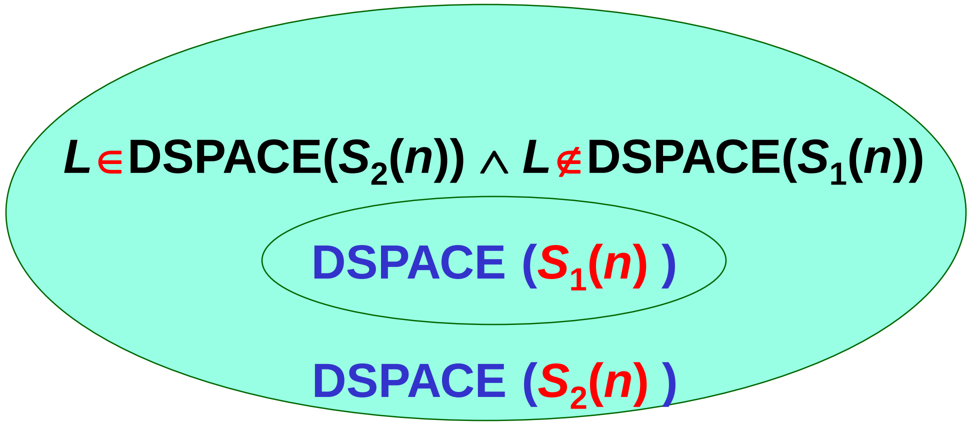
- **Ensure that *M* simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells**
  - **Number of cells used by simulating a TM from class DSPACE($S_1(n)$) :**

$$\lceil \log_2 t \rceil \, S_1(n)$$

t **– number of tape symbols of a TM from class DSPACE($S_1(n)$)**

{0, 1, *B*} – tape symbols of TM *M*

Continuity of the hierarchy for fully space-constructible functions



- **Ensure that *M* simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells**

Continuity of the hierarchy for fully space-constructible functions

$L \in \text{DSPACE}(S_2(n)) \wedge L \notin \text{DSPACE}(S_1(n))$

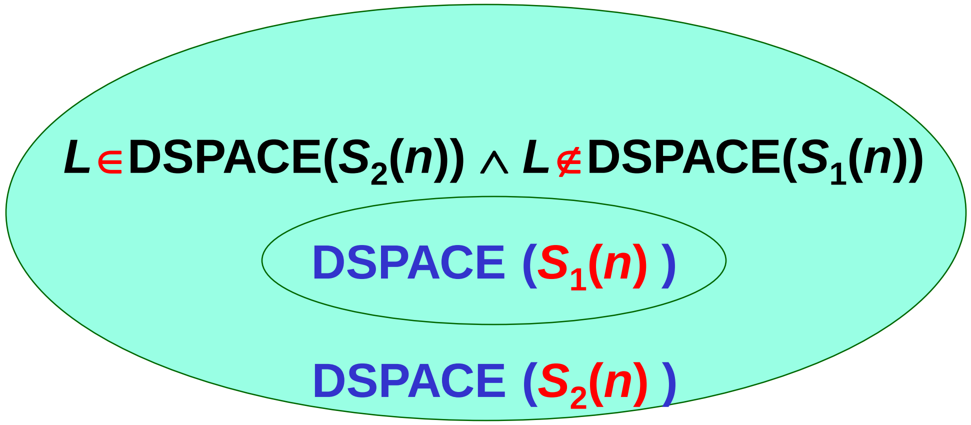
DSPACE ($S_1(n)$)

DSPACE ($S_2(n)$)

- Ensure that M simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells

- $\left(\inf_{n\to\infty} S_1(n)/S_2(n) = 0 \right) \Rightarrow \left(\lceil \log_2 t \rceil\, S_1(n) < S_2(n) \right)$

Continuity of the hierarchy for fully space-constructible functions

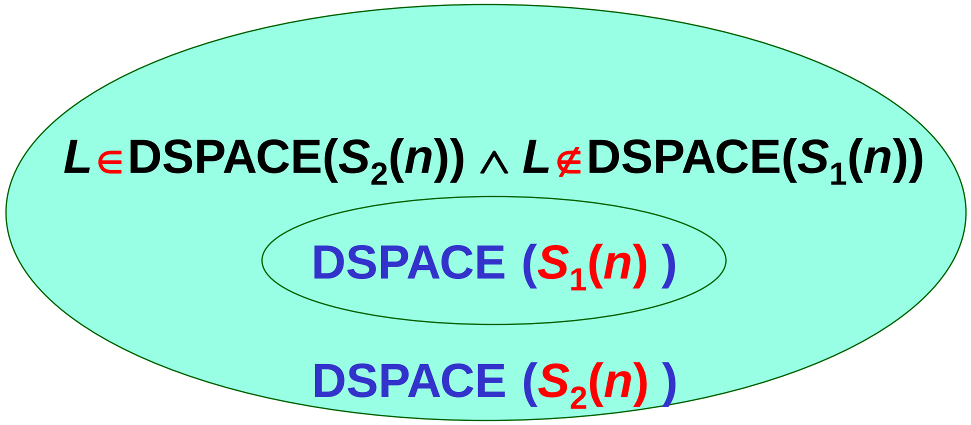


- **Ensure that *M* simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells**

- **($\inf_{n \to \infty} S_1(n)/S_2(n) = 0$) $\Rightarrow$ ($\lceil \log_2 t \rceil S_1(n) < S_2(n)$)**

- **We expand the encoding procedure of TM:**

Continuity of the hierarchy for fully space-constructible functions



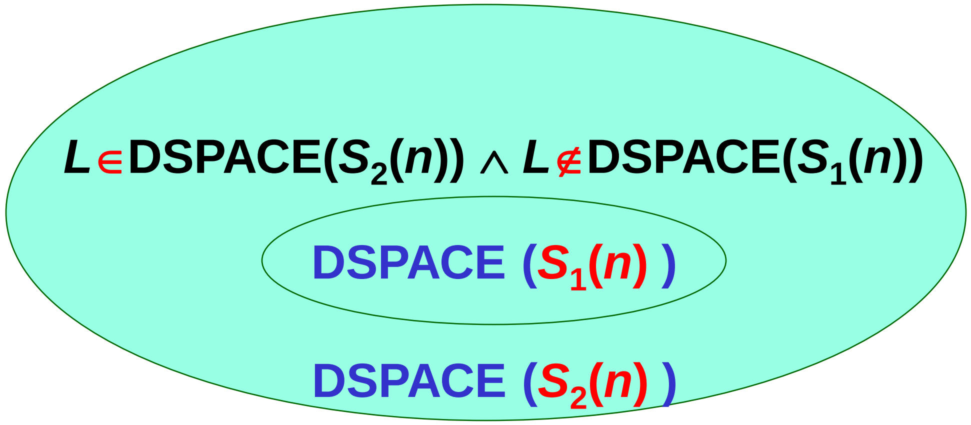
- Ensure that M simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells

  - $\left(\inf_{n\to\infty} S_1(n)/S_2(n) = 0 \right) \Rightarrow \left(\lceil \log_2 t \rceil\, S_1(n) < S_2(n) \right)$

  - We expand the encoding procedure of TM:

$$111 \;\; code_1 \;\; 11 \;\; code_2 \;\; 11 \; \text{- - -} \; 11 \;\; code_r \;\; 111$$

Continuity of the hierarchy for fully space-constructible functions



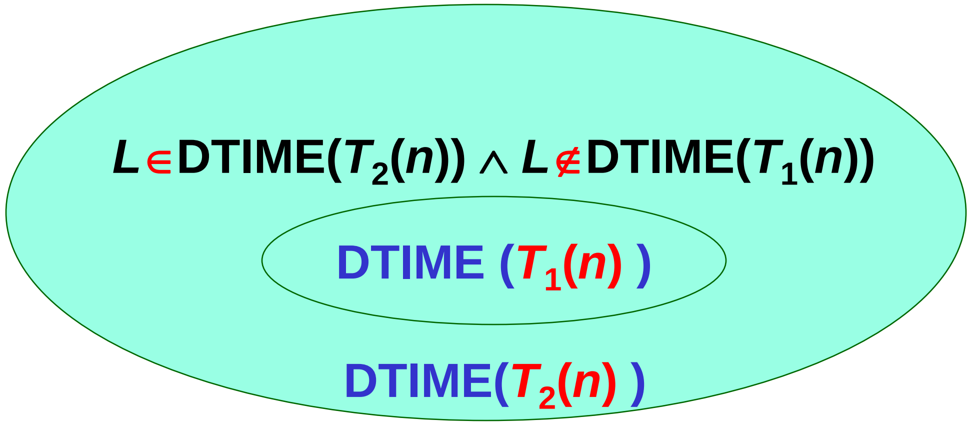
- Ensure that M simulates M_w from DSPACE($S_1(n)$) in $S_2(n)$ cells

  - $(\ \inf_{n\to\infty} S_1(n)/S_2(n) = 0\ ) \Rightarrow (\ \lceil \log_2 t \rceil\, S_1(n) < S_2(n)\ )$

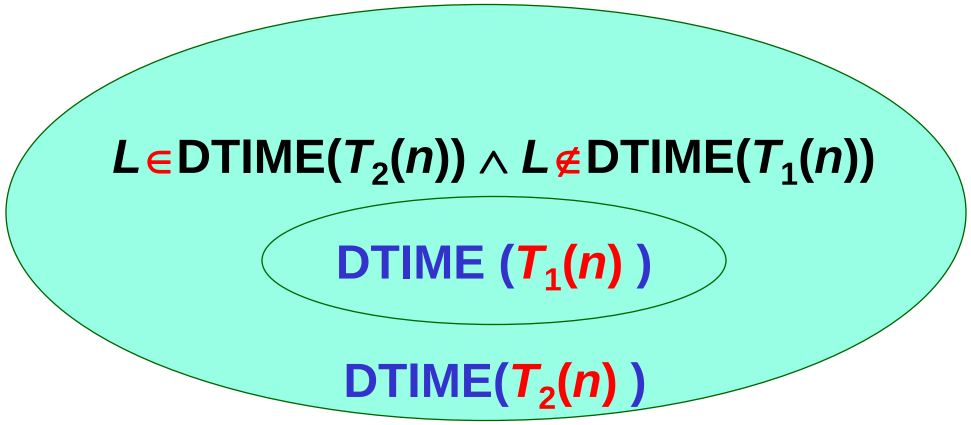
  - **We expand the encoding procedure of TM:**

111111111111111111111 *code*$_1$ **11** *code*$_2$ **11 - - - 11** *code*$_r$ **111**

Continuity of the hierarchy for fully time-constructible functions

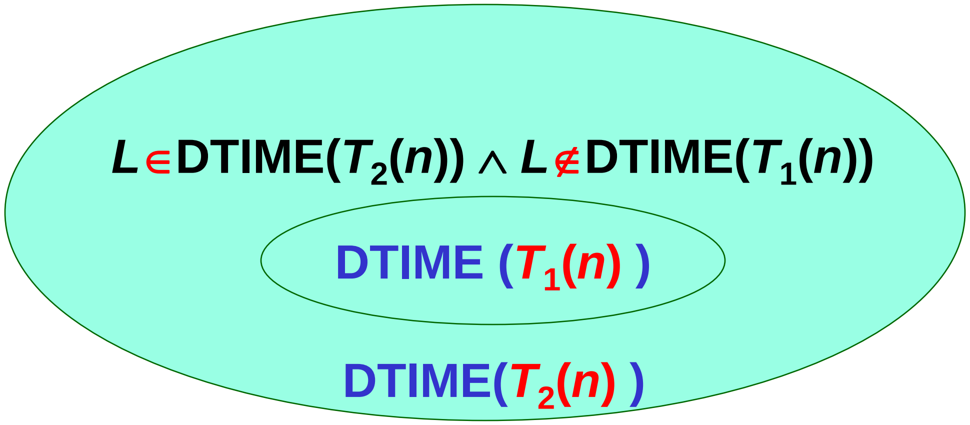


Continuity of the hierarchy for fully time-constructible functions



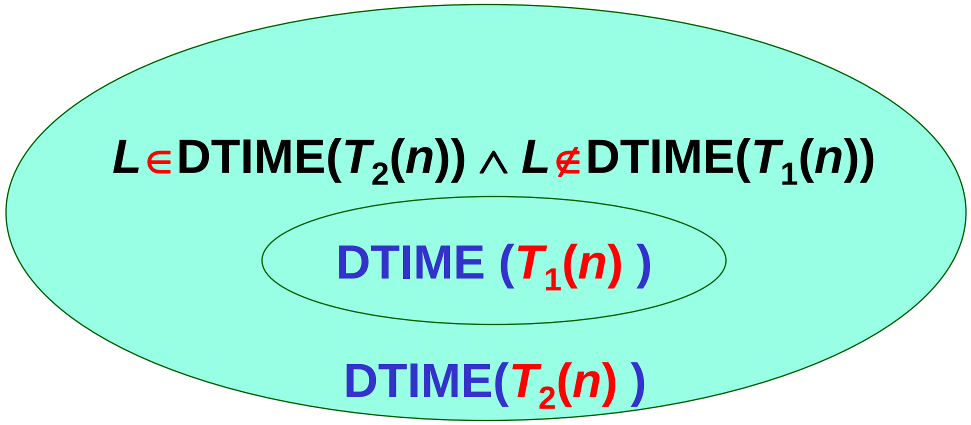
- If

Continuity of the hierarchy for fully time-constructible functions



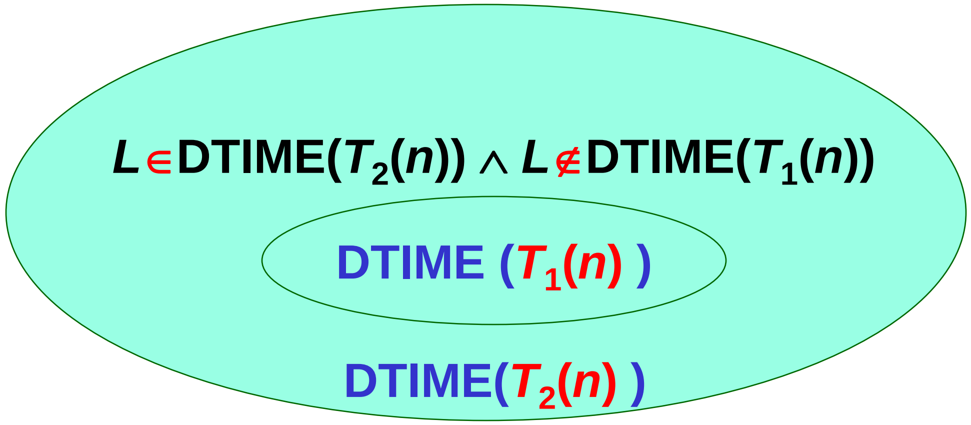
- **If**
  - **$T_2(n)$ is a fully time-constructible function**

Continuity of the hierarchy for fully time-constructible functions



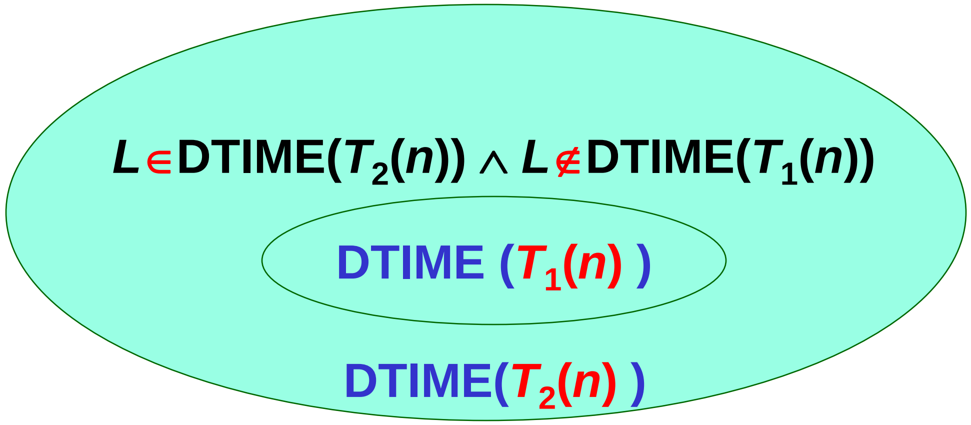
- If
  - $T_2(n)$ is a fully time-constructible function
  - $\inf_{n\to\infty} T_1(n) \log T_1(n) / T_2(n) = 0$

Continuity of the hierarchy for fully time-constructible functions



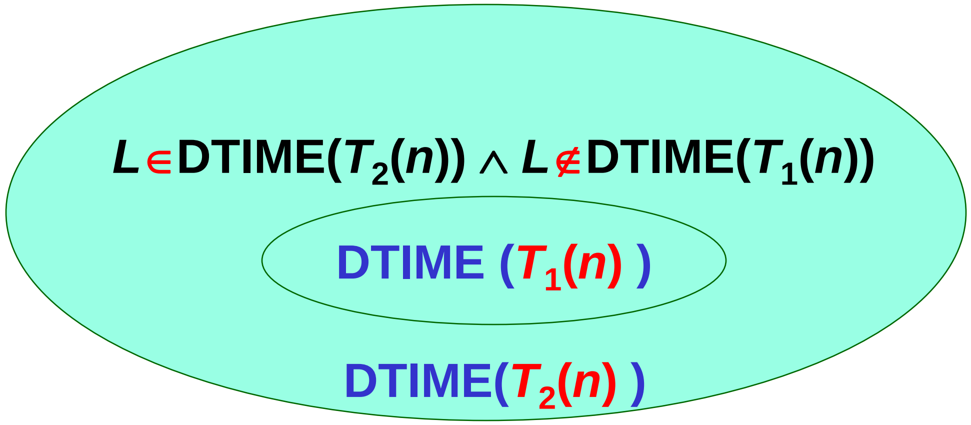
- **If**
  - **$T_2(n)$ is a fully time-constructible function**
  - **$\inf_{n\to\infty} T_1(n) \log T_1(n)/T_2(n) = 0$**
- **Then there is a language *L***

Continuity of the hierarchy for fully time-constructible functions

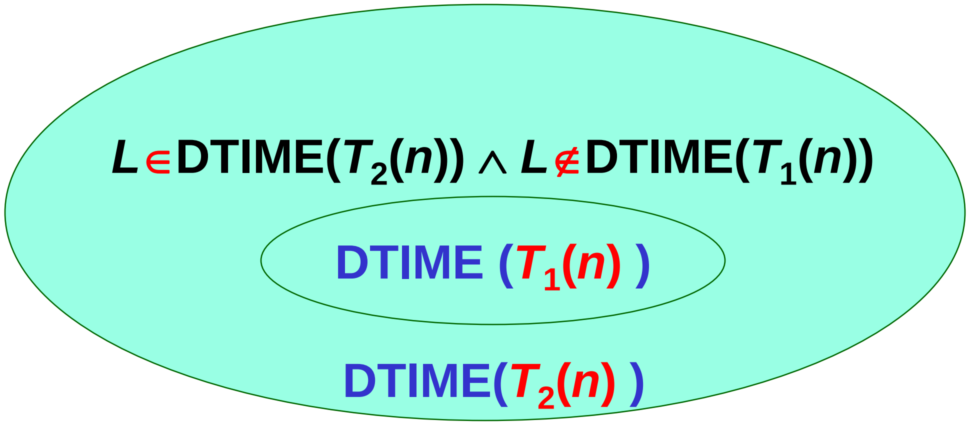


- **If**
  - $T_2(n)$ **is a fully time-constructible function**
  - $\inf_{n\to\infty} T_1(n) \log T_1(n) / T_2(n) = 0$
- **Then there is a language** L
  - $L \in \mathrm{DTIME}(T_2(n)) \wedge L \notin \mathrm{DTIME}(T_1(n))$

Continuity of the hierarchy for fully time-constructible functions

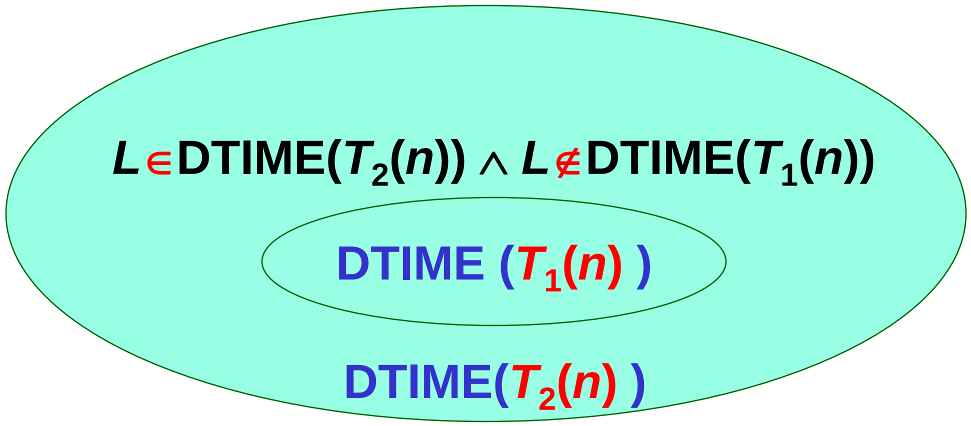


Continuity of the hierarchy for fully time-constructible functions



- **We construct a TM *M* for which:**

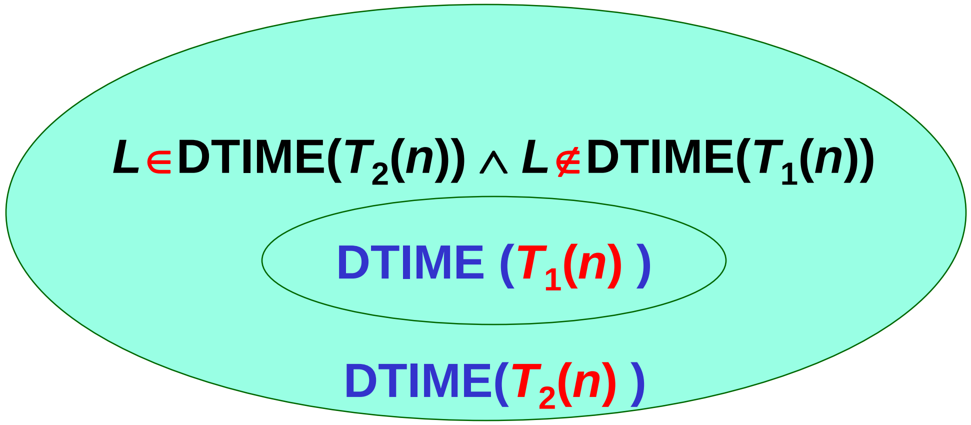
Continuity of the hierarchy for fully time-constructible functions



- **We construct a TM *M* for which:**

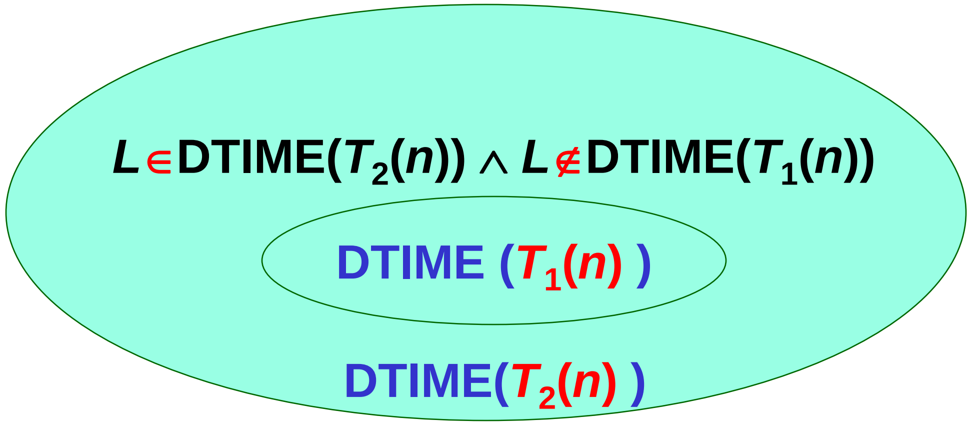
  - **TM *M* has time complexity $T_2(n)$**

Continuity of the hierarchy for fully time-constructible functions

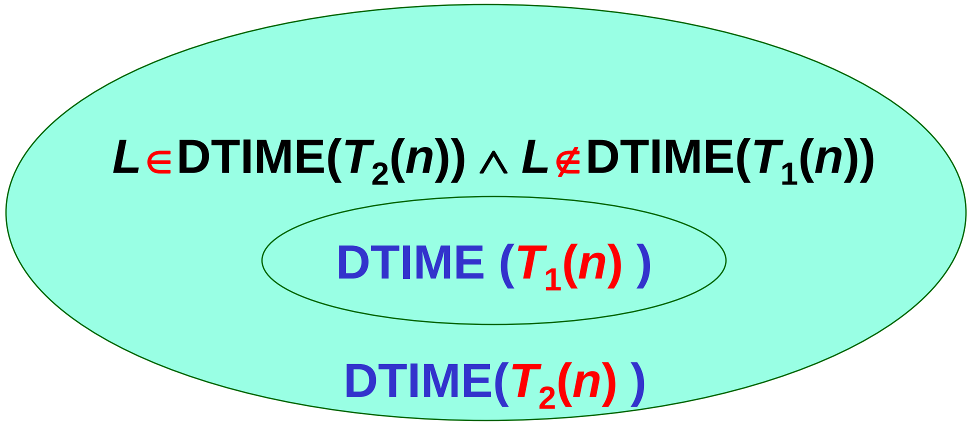


- **We construct a TM *M* for which:**
  - **TM *M* has time complexity $T_2(n)$**
  - **TM *M* gives the opposite decision than any TM with time complexity $T_1(n)$ for at least one input string**

Continuity of the hierarchy for fully time-constructible functions

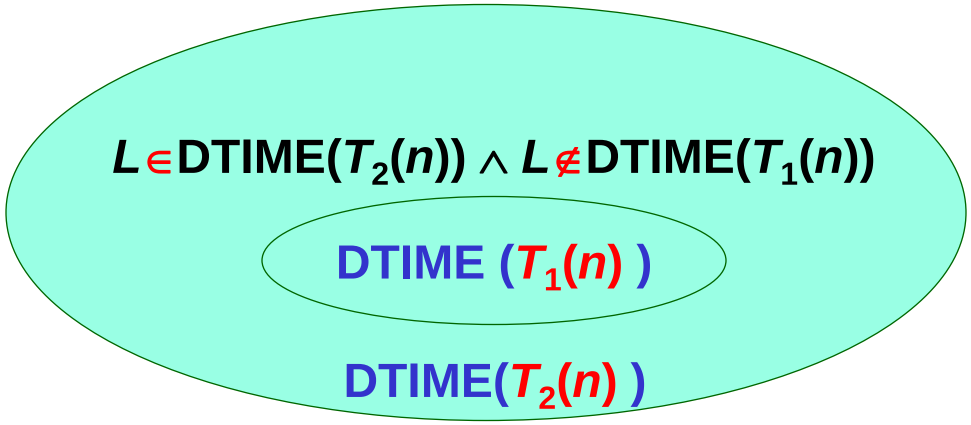


Continuity of the hierarchy for fully time-constructible functions



- **TM *M* – ensure a time complexity of $T_2(n)$**

Continuity of the hierarchy for fully time-constructible functions



- **TM *M* – ensure a time complexity of $T_2(n)$**
  - **Parallel simulation**

Continuity of the hierarchy for fully time-constructible functions

$L \in \mathrm{DTIME}(T_2(n)) \wedge L \notin \mathrm{DTIME}(T_1(n))$

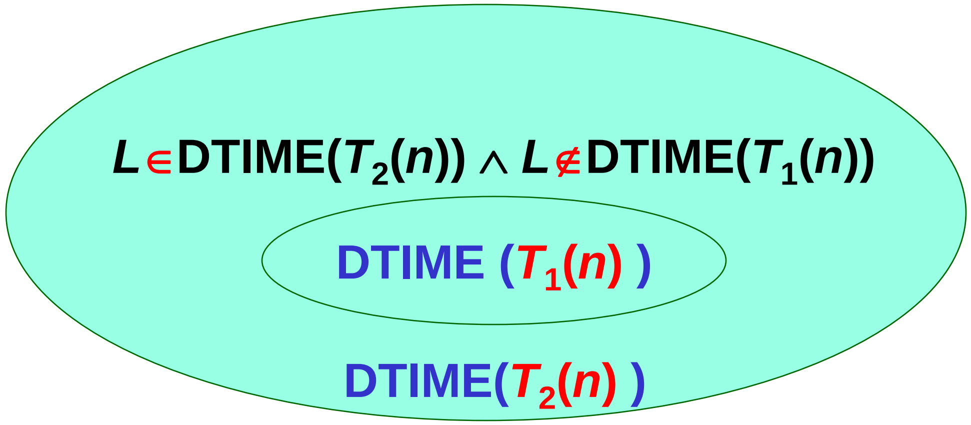
DTIME $(T_1(n))$

DTIME$(T_2(n))$

- TM M – ensure a time complexity of $T_2(n)$

  - **Parallel simulation**
    - — work of TM M_w for an input string w

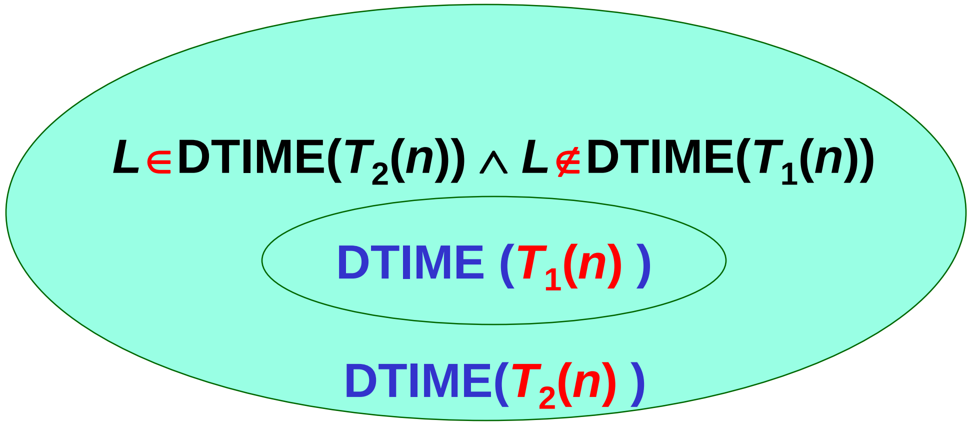
Continuity of the hierarchy for fully time-constructible functions



- TM M – ensure a time complexity of $T_2(n)$

  - **Parallel simulation**
    - — work of TM M_w for an input string w
    - — work of any TM M_{T2} with time complexity $T_2(n)$

Continuity of the hierarchy for fully time-constructible functions

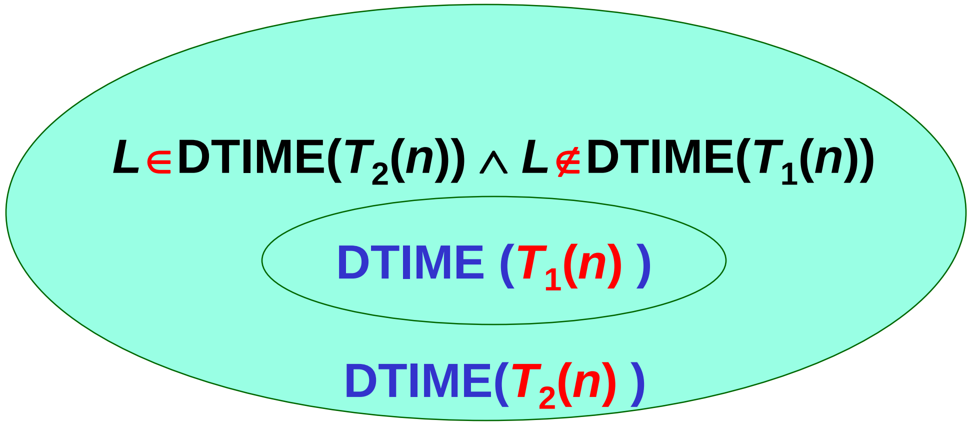


- TM M – ensure a time complexity of $T_2(n)$

  - **Parallel simulation**
    - work of TM M_w for an input string w
    - work of any TM M_{T2} with time complexity $T_2(n)$

  - **$T_2(n)$ is fully time-constructible**

Continuity of the hierarchy for fully time-constructible functions



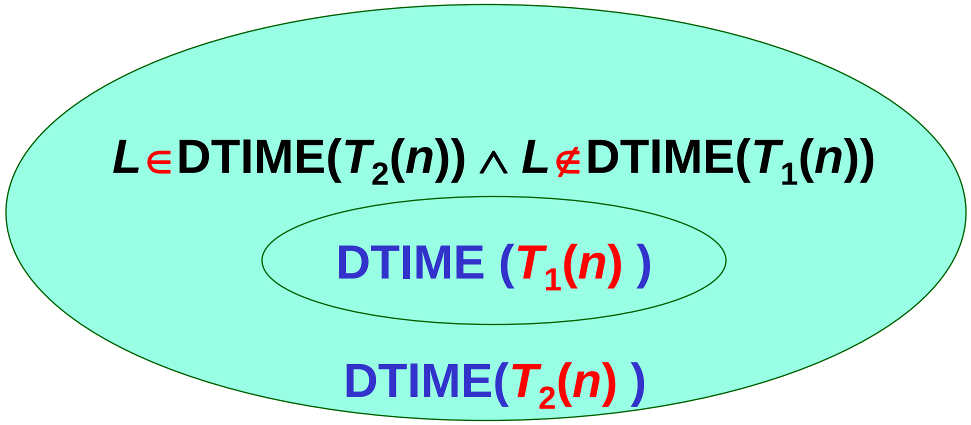
- **TM M – ensure a time complexity of $T_2(n)$**

  - **Parallel simulation**
    - — **work of TM M_w for an input string w**
    - — **work of any TM M_{T2} with time complexity $T_2(n)$**

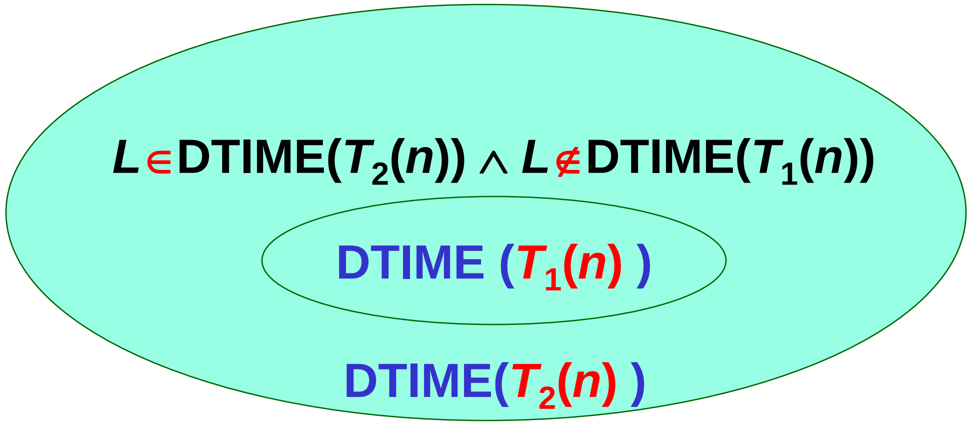
  - **$T_2(n)$ is fully time-constructible**

    $\Rightarrow$ **TM M_{T2} for any string of length n does $T_2(n)$ moves**

Continuity of the hierarchy for fully time-constructible functions

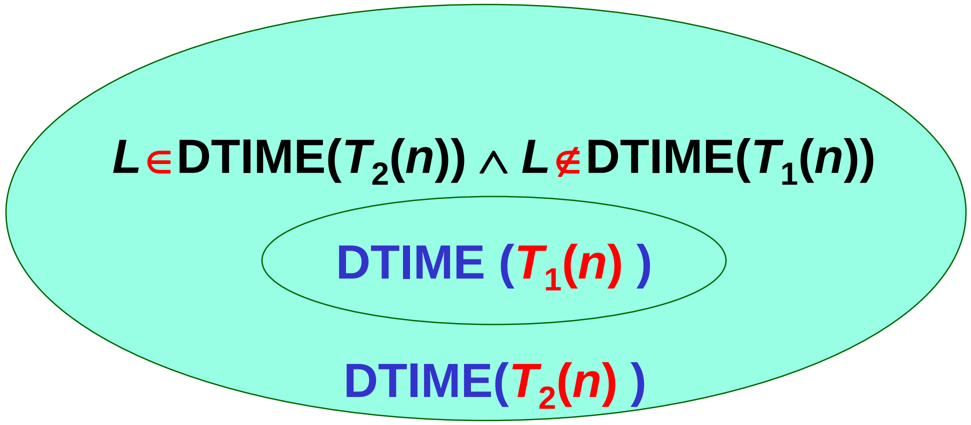


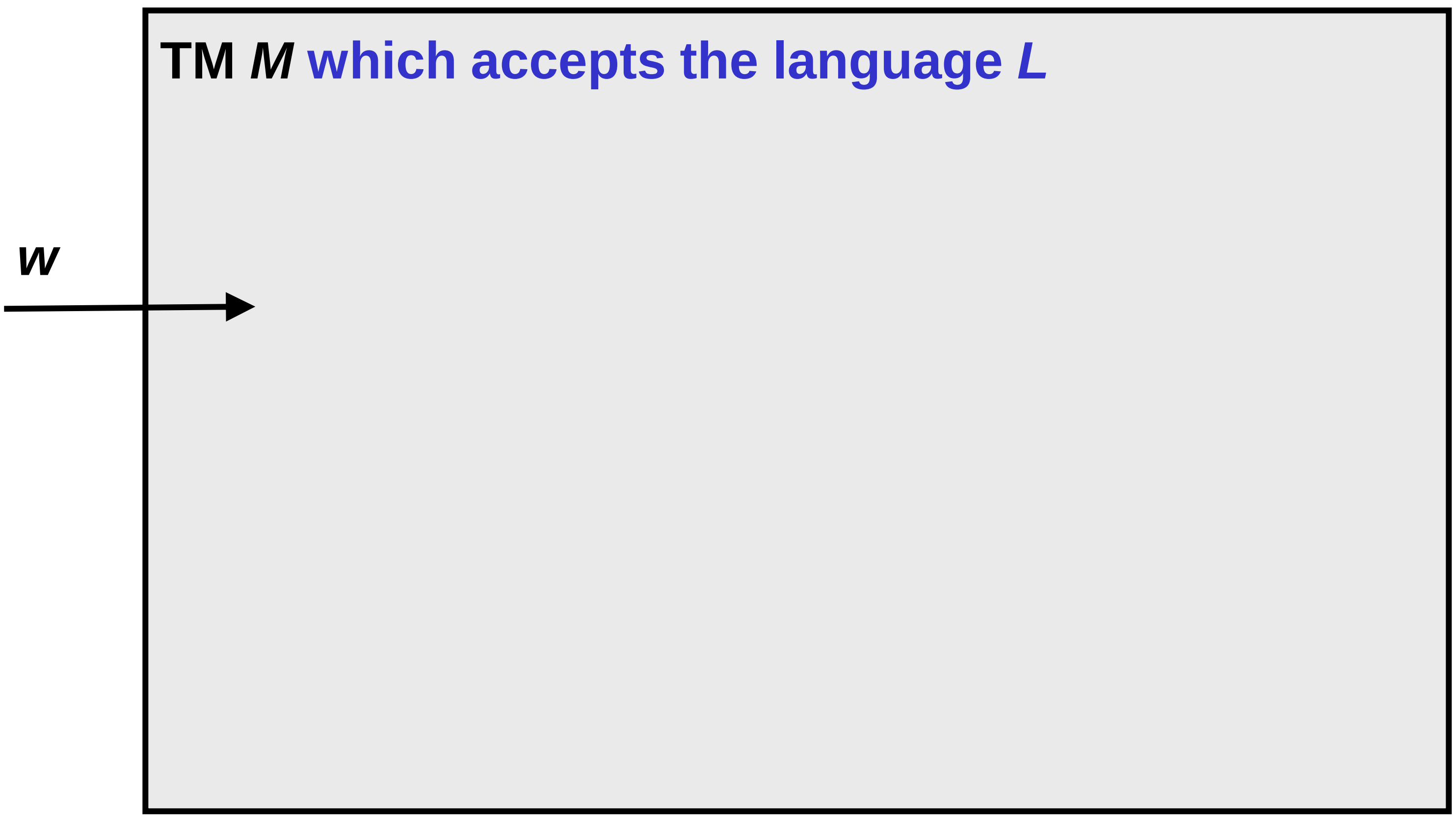
Continuity of the hierarchy for fully time-constructible functions



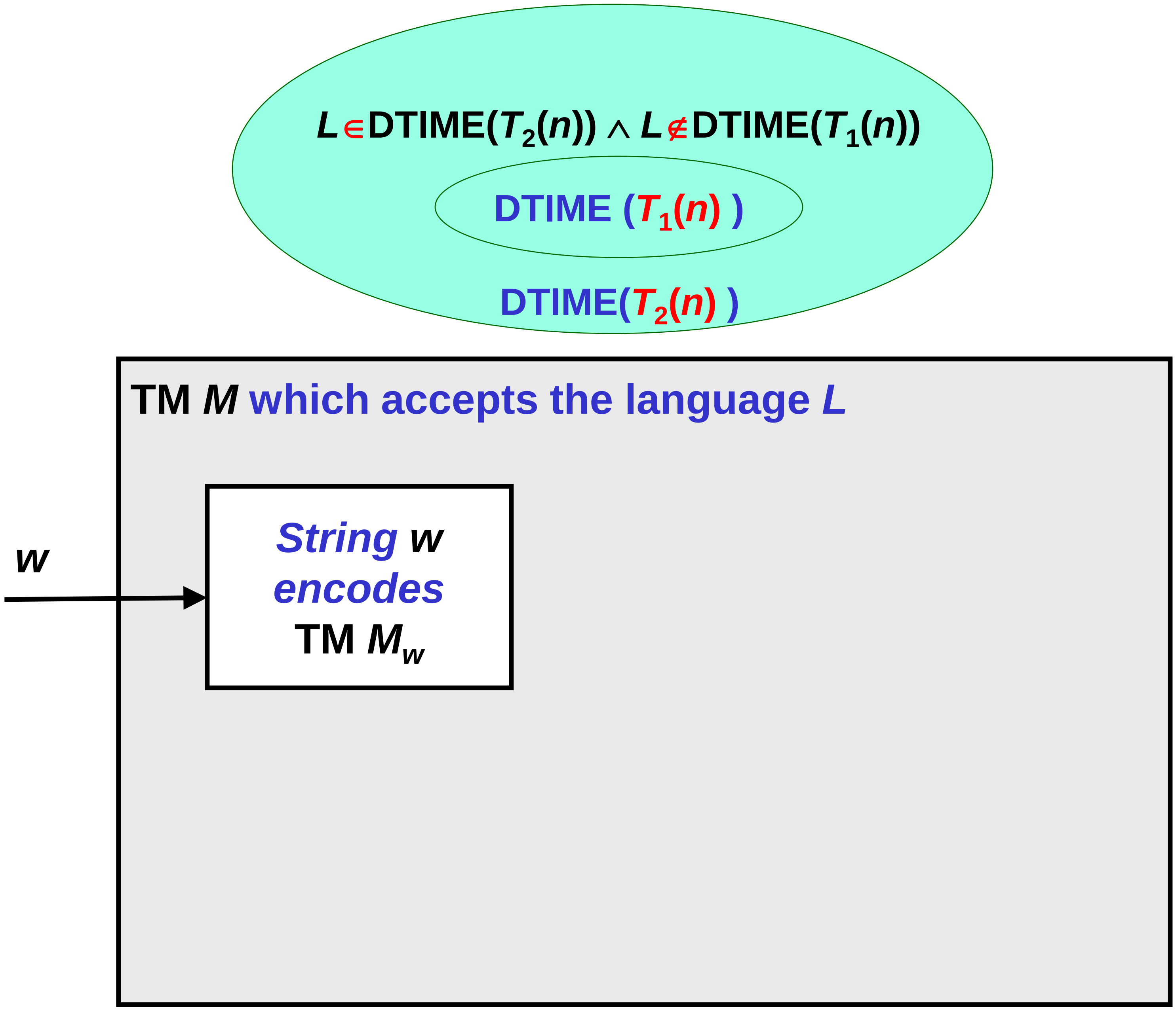
TM *M* which accepts the language *L*

Continuity of the hierarchy for fully time-constructible functions





Continuity of the hierarchy for fully time-constructible functions



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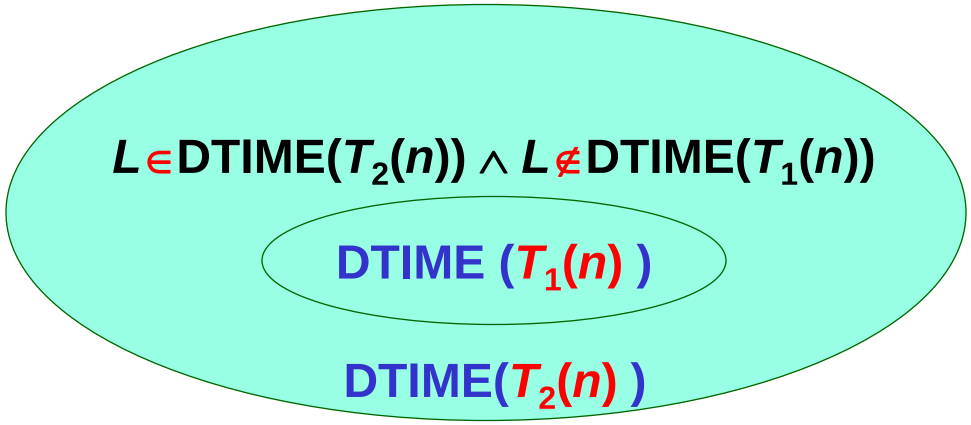

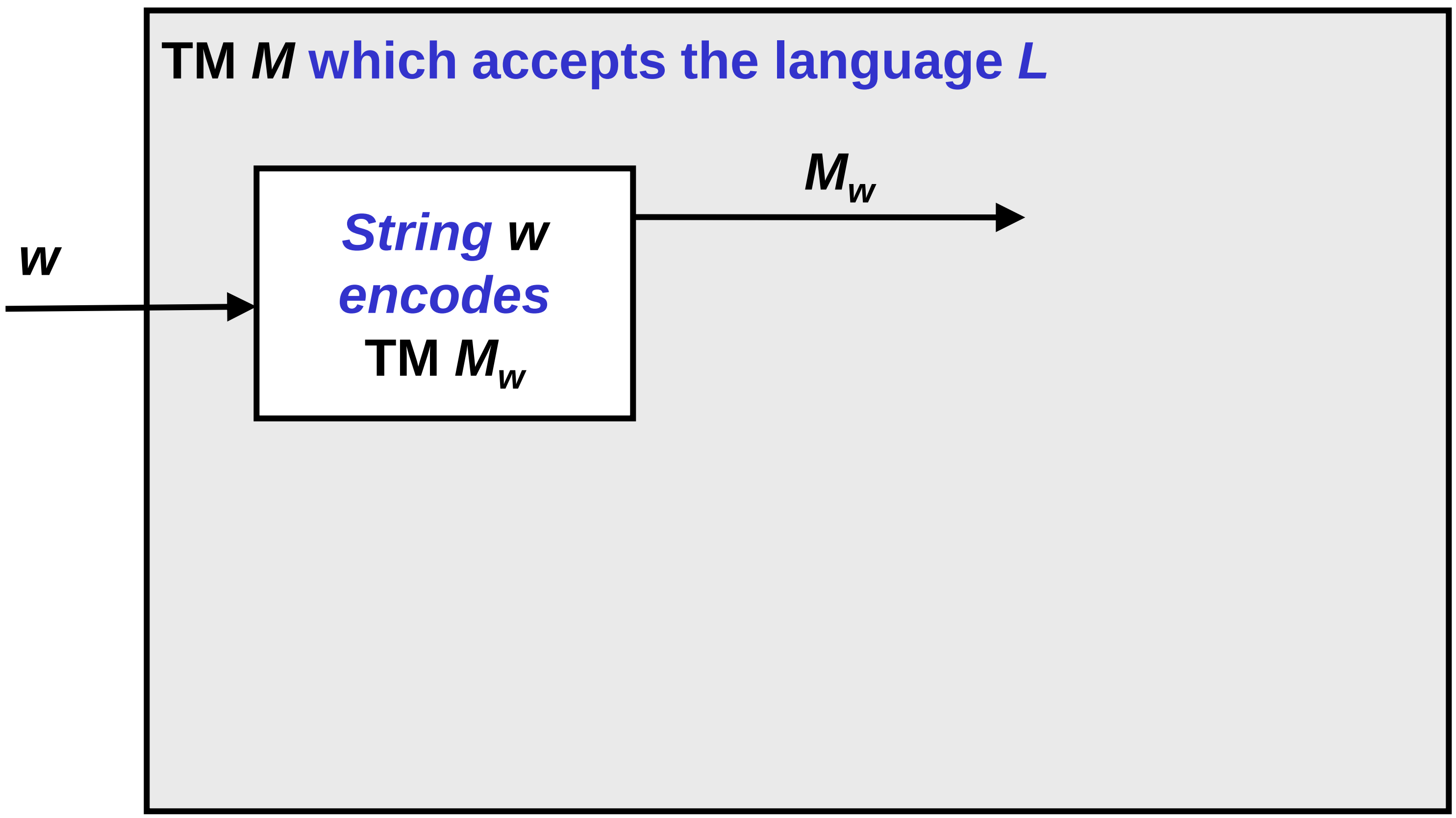

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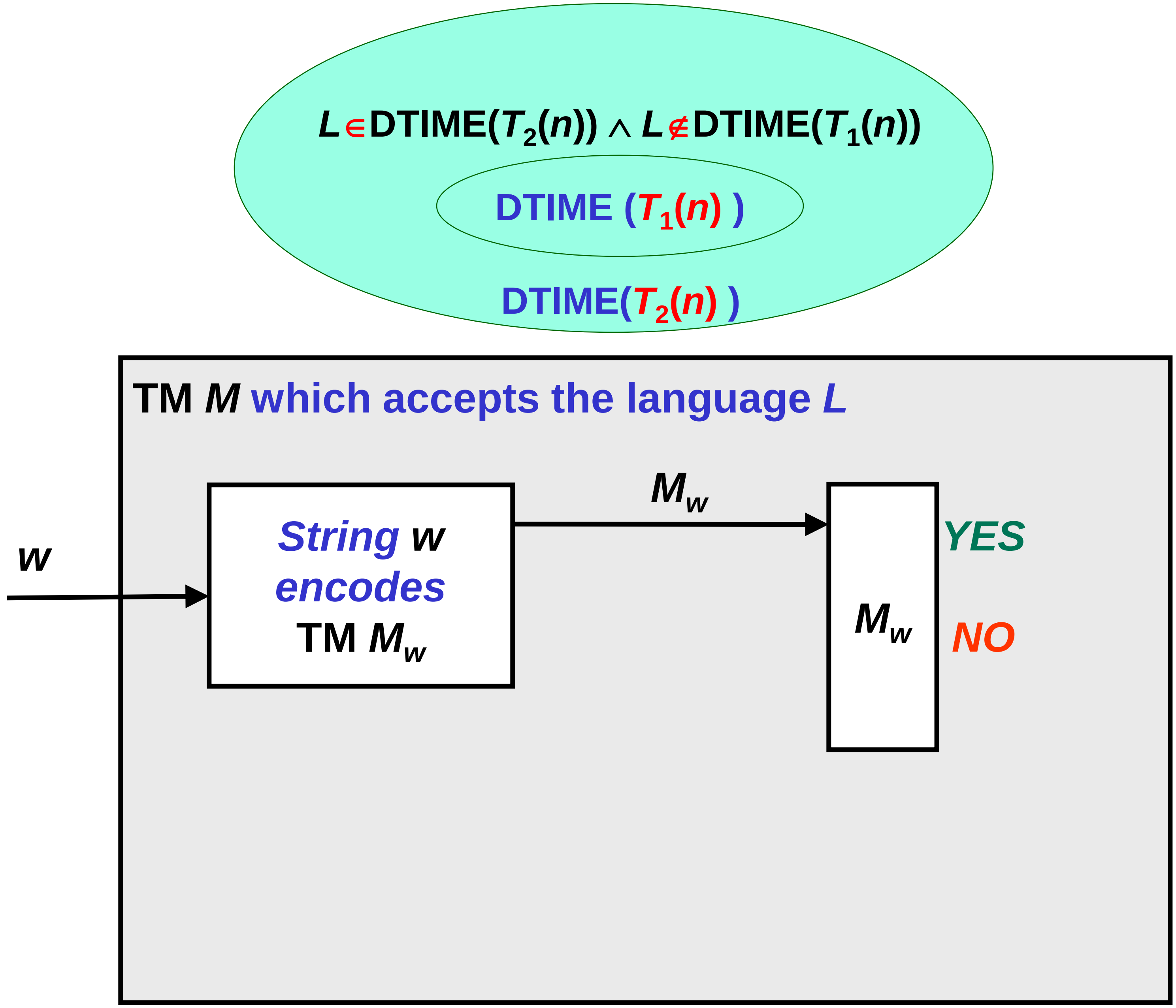

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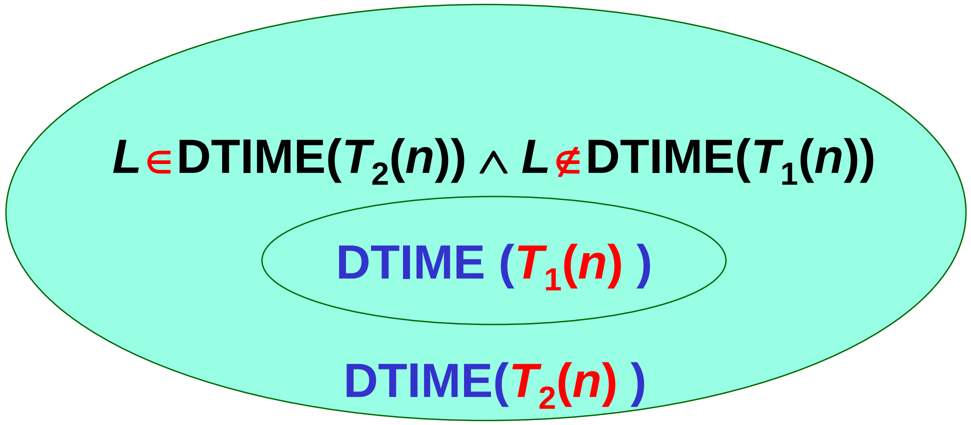

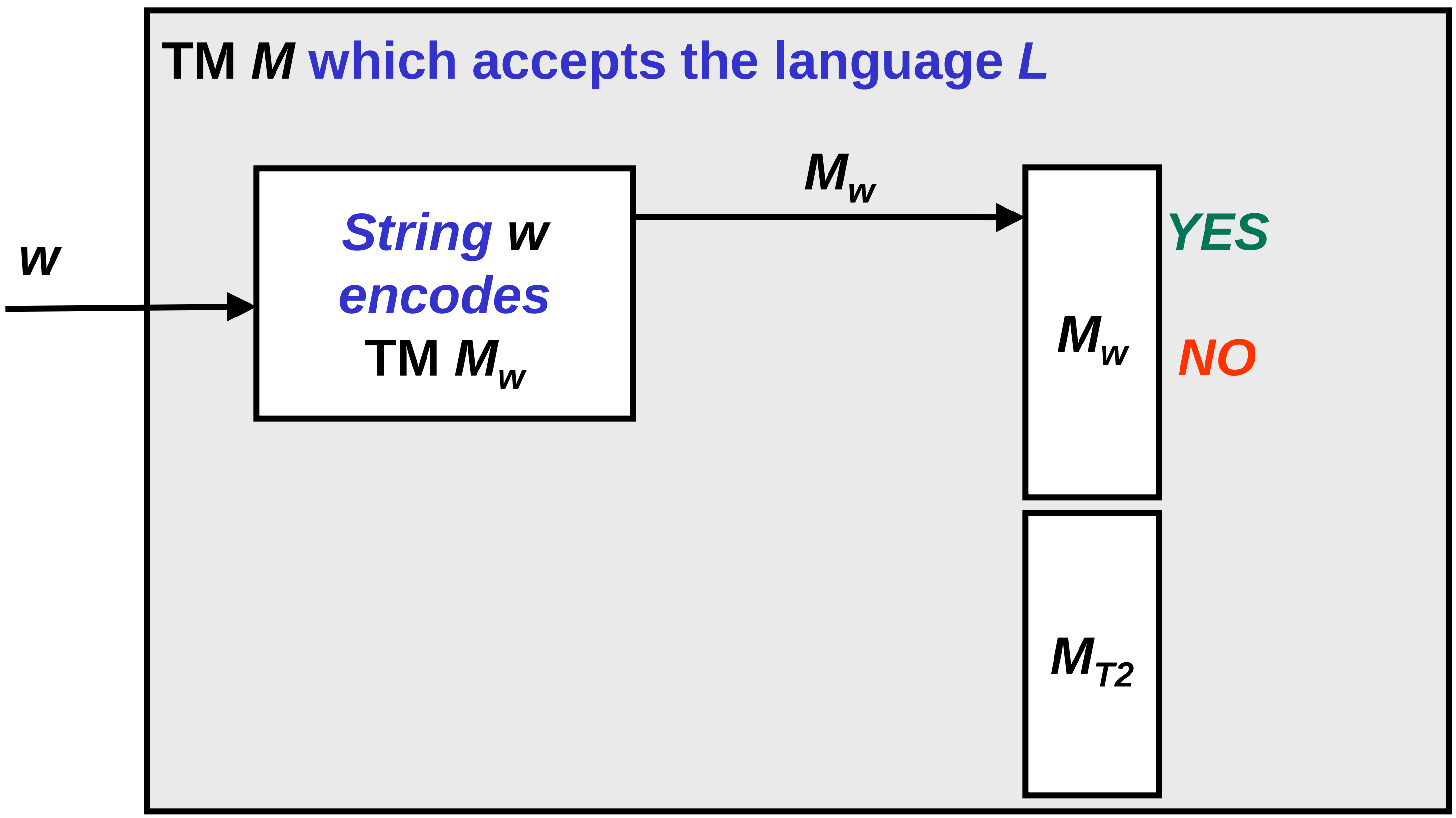

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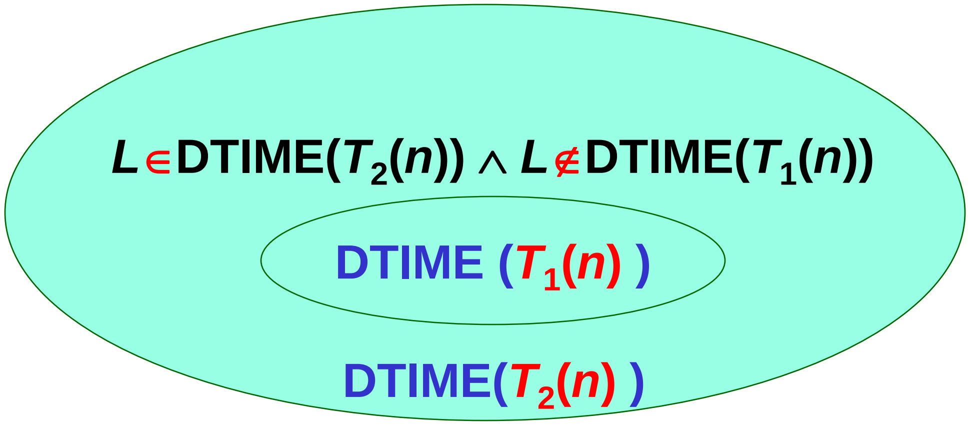

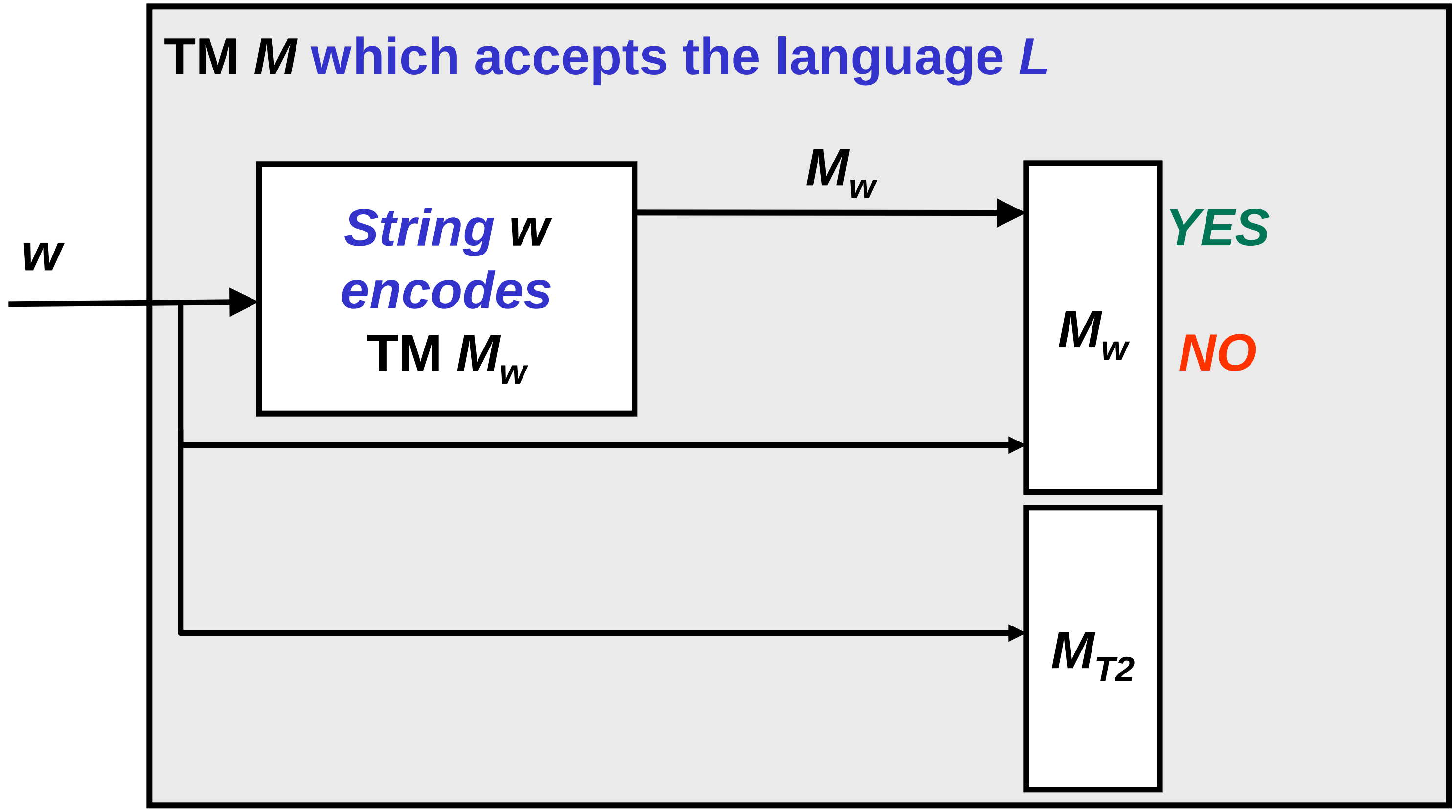

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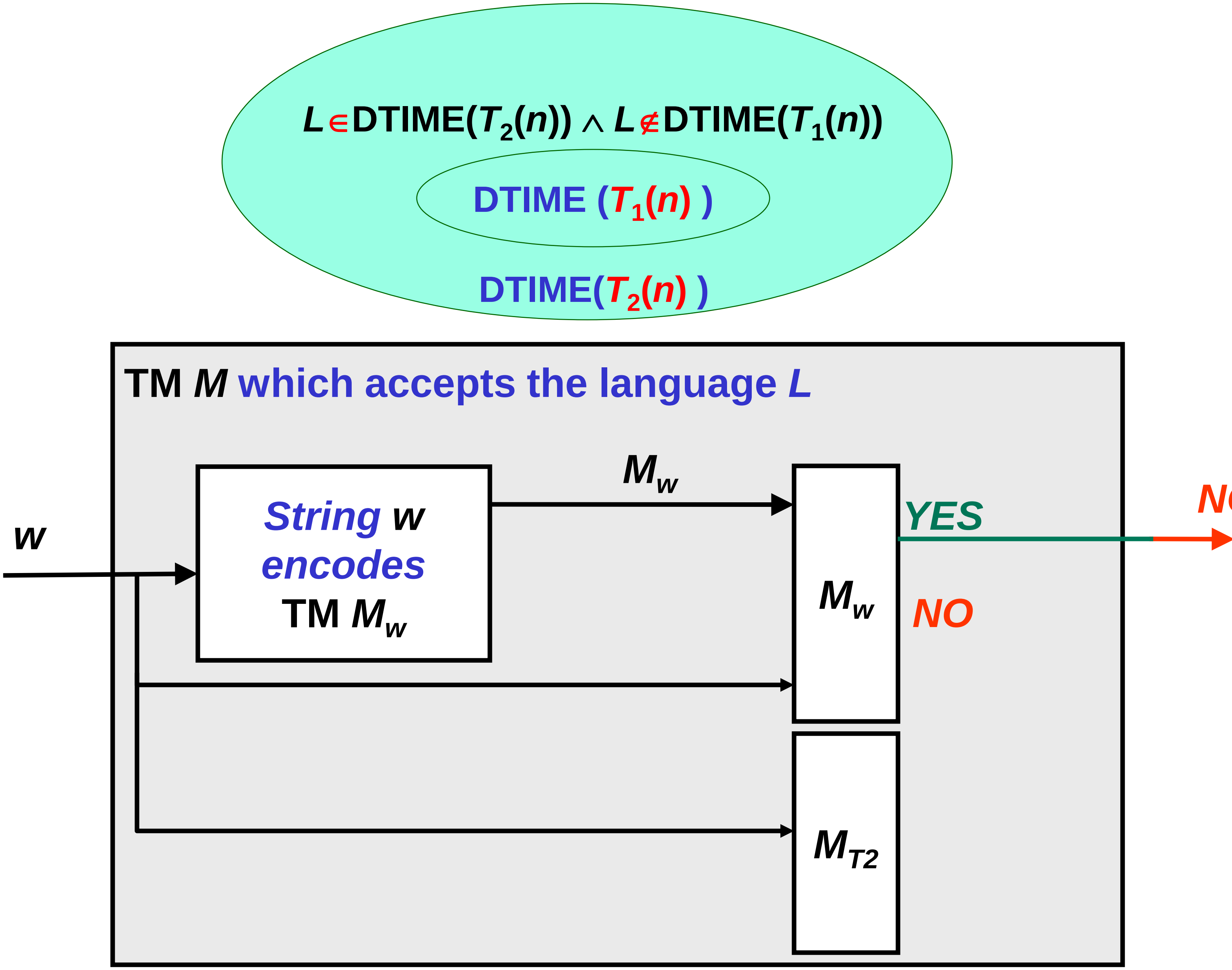

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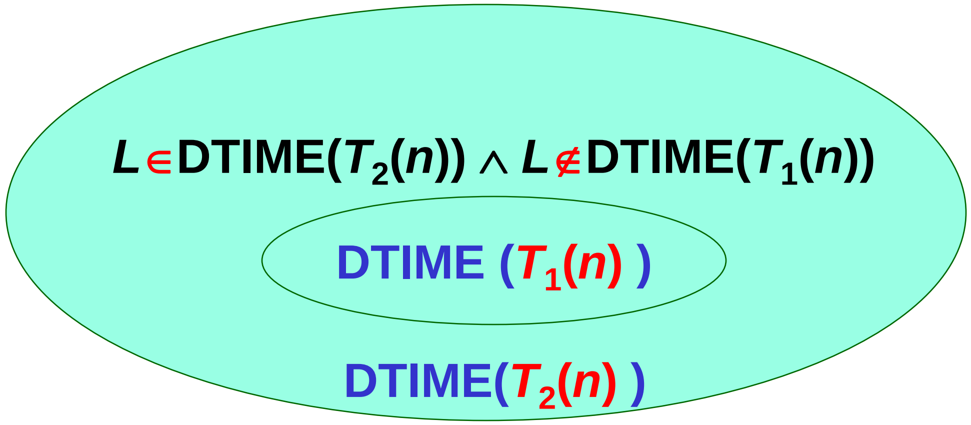

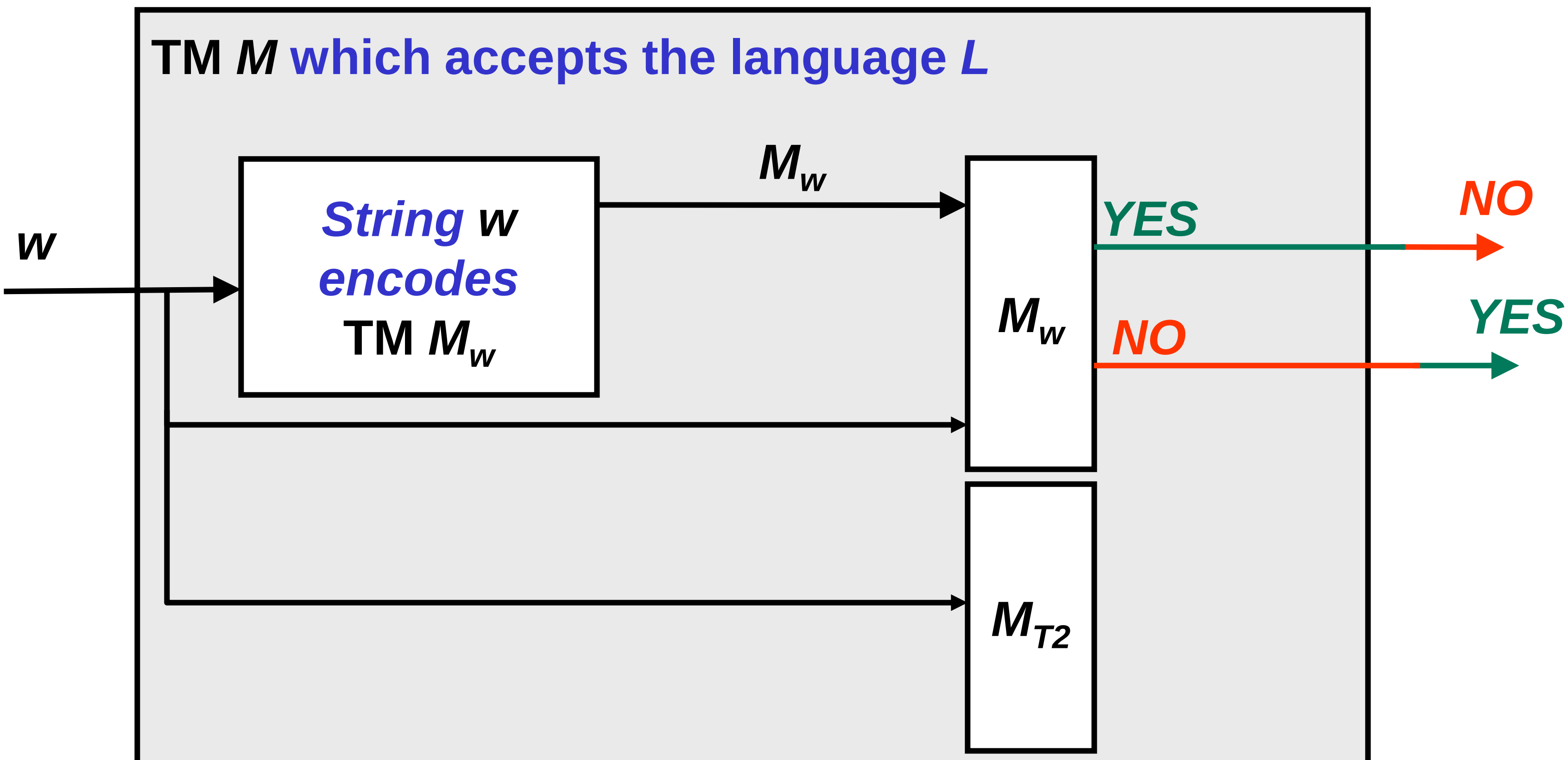

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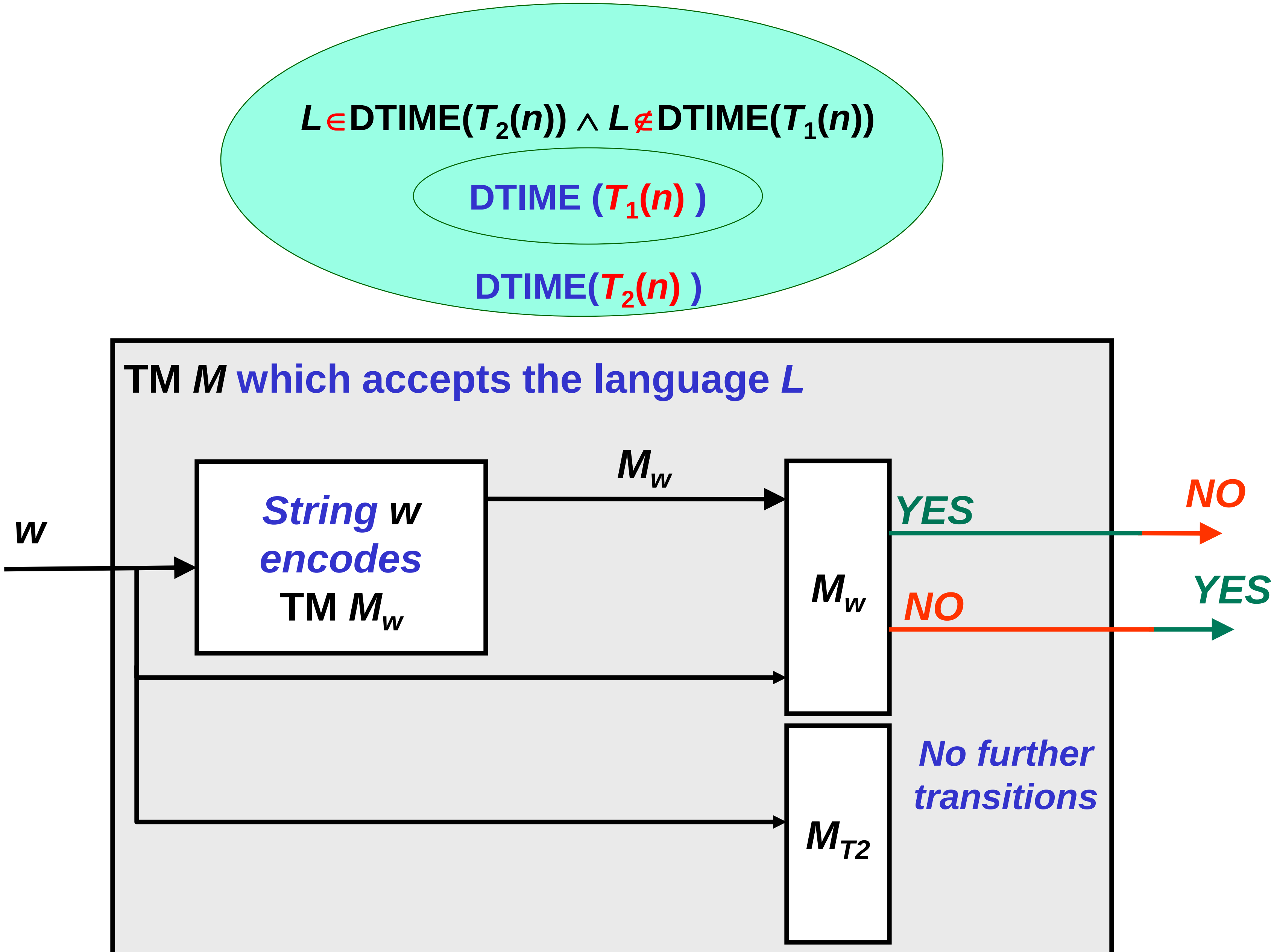
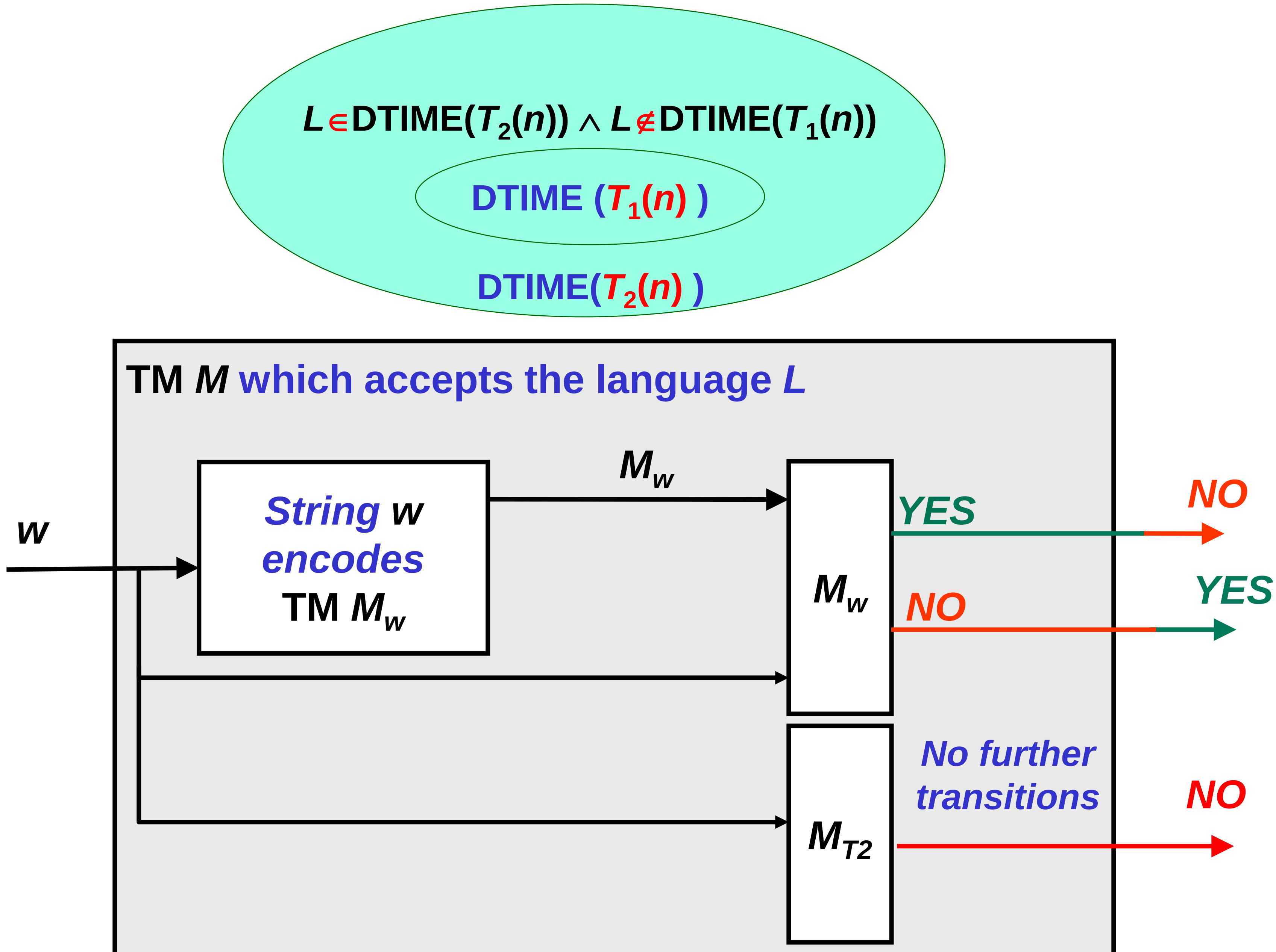


Continuity of the hierarchy for fully time-constructible functions



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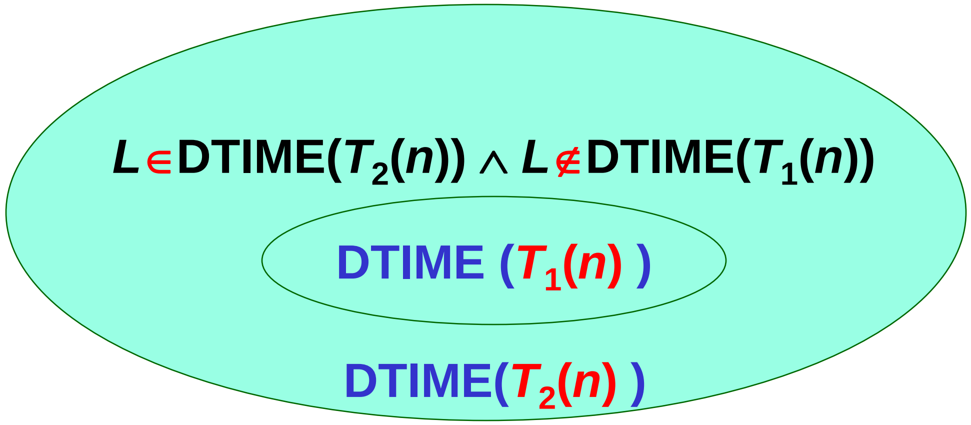

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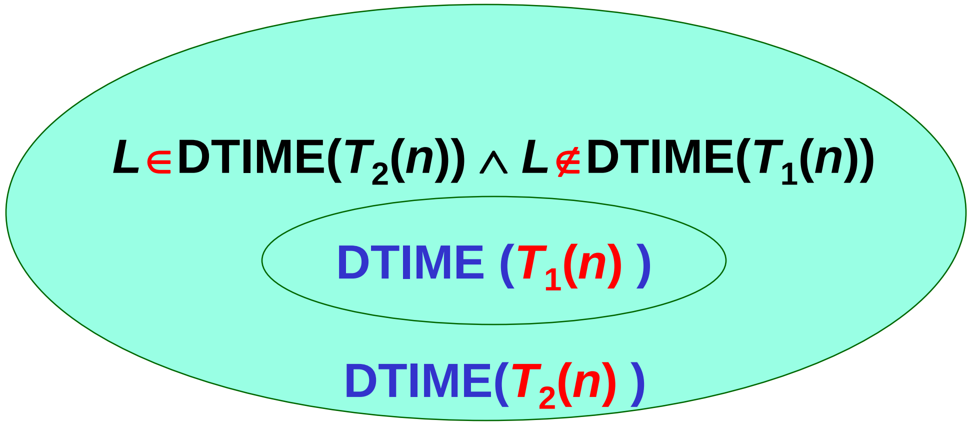
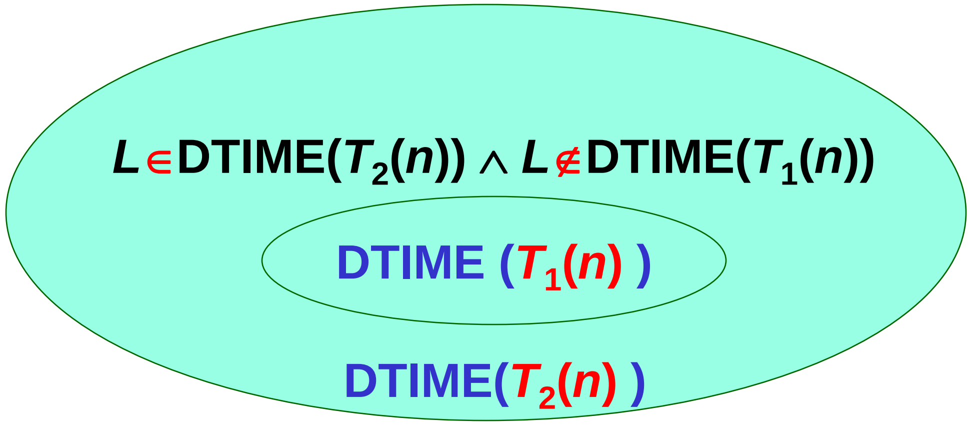


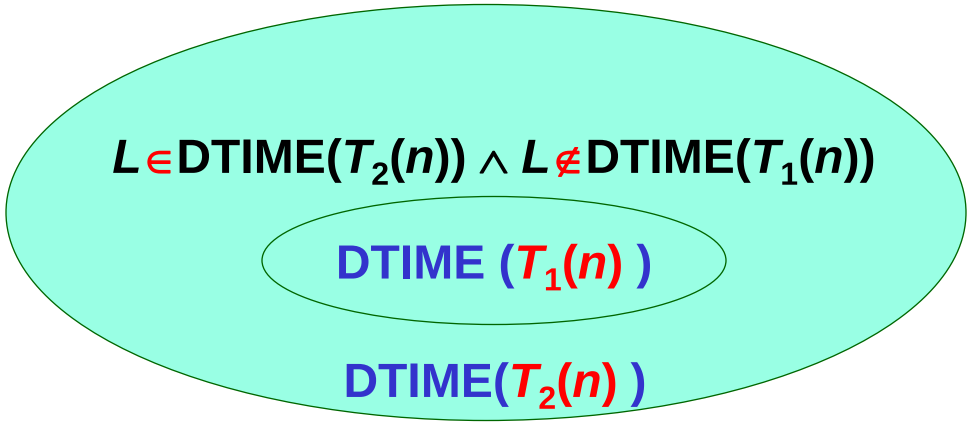
- **Ensure that M simulates M_w from class DTIME$(T_1(n))$ in $T_2(n)$ moves**

Continuity of the hierarchy for fully time-constructible functions



- **Ensure that *M* simulates M_w from class DTIME($T_1(n)$) in $T_2(n)$ moves**
  - **Multiple tapes of TM M_w are reduced to two tapes**
    - **To enable simulating TM M_w with an arbitrary number of tapes by the TM *M* with a limited number of tapes**
    - $T_1(n) \Rightarrow T_1(n)\log T_1(n)$

Continuity of the hierarchy for fully time-constructible functions



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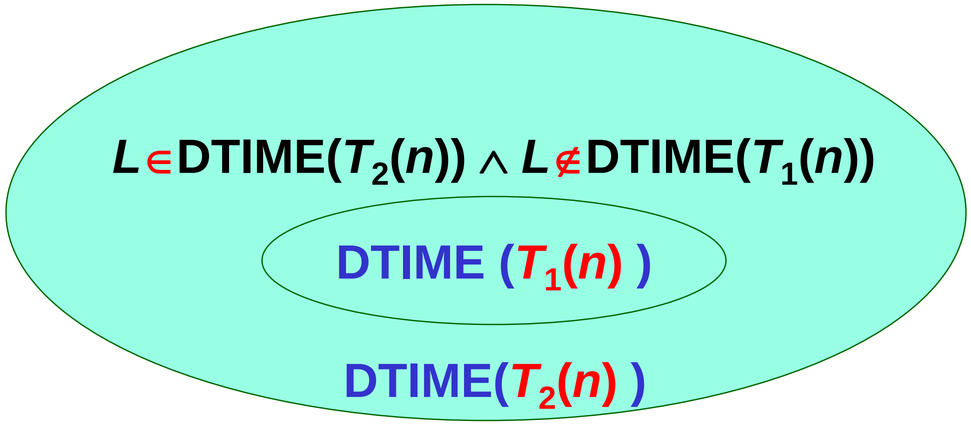
  - Multiple tapes of TM M_w are reduced to two tapes
    - To enable simulating TM M_w with an arbitrary number of tapes by the TM M with a limited number of tapes
    - $T_1(n) \Rightarrow T_1(n)\log T_1(n)$
  - $\left(\inf_{n\to\infty} T_1(n)\log T_1(n)/T_2(n) = 0 \right) \Rightarrow \left(T_1(n)\log T_1(n) < T_2(n) \right)$

Continuity of the hierarchy for fully time-constructible functions



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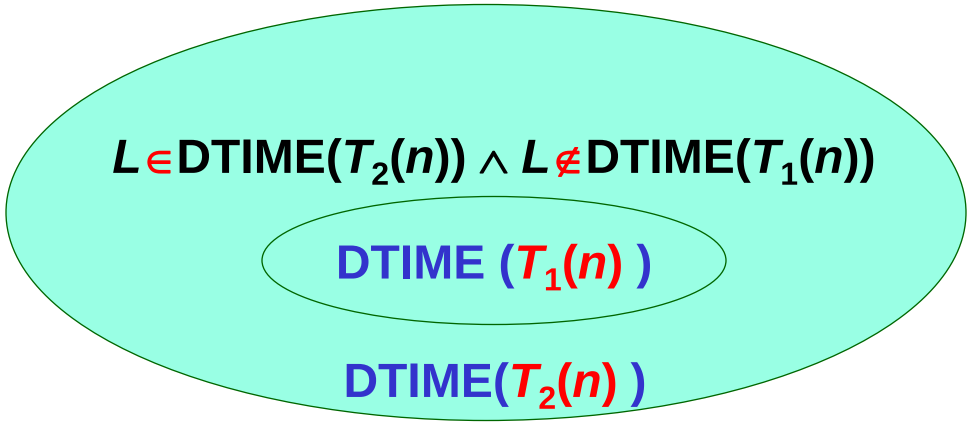
  - **Encoding procedure of TM:**

Continuity of the hierarchy for fully time-constructible functions



- **Ensure that M simulates M_w from class DTIME($T_1(n)$) in $T_2(n)$ moves**
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  - **Encoding procedure of TM:**

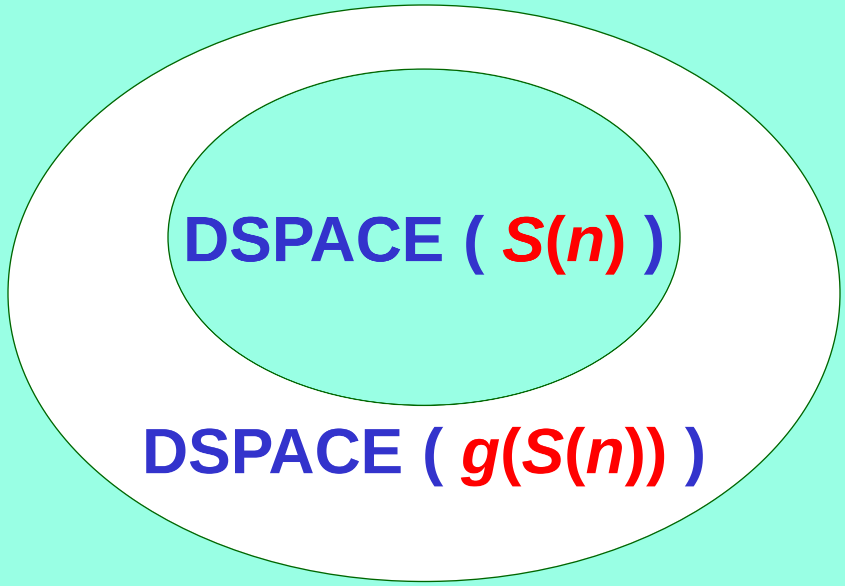
1111111111111111111111 $code_1$ **11** $code_2$ **11 - - - 11** $code_r$ **111**

Gaps in the hierarchy

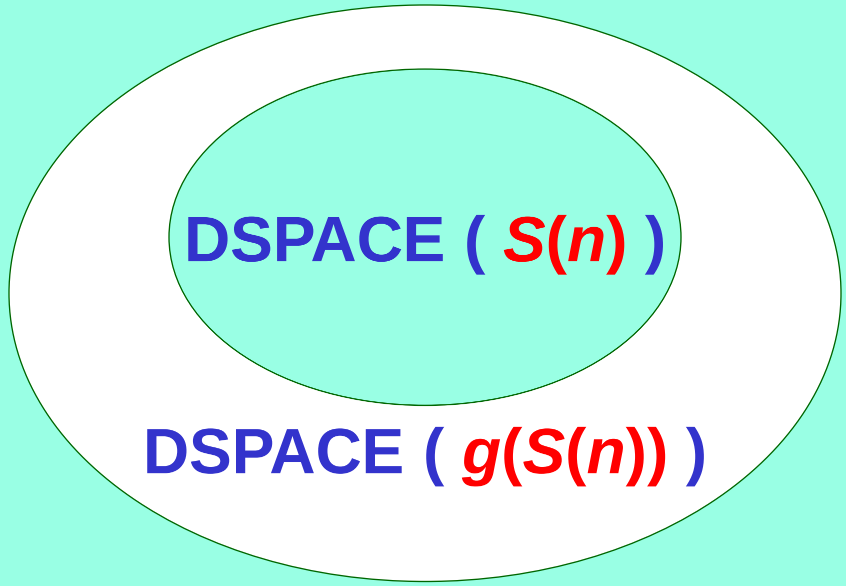
Gaps in the hierarchy

DSPACE (*S*(*n*))

Gaps in the hierarchy

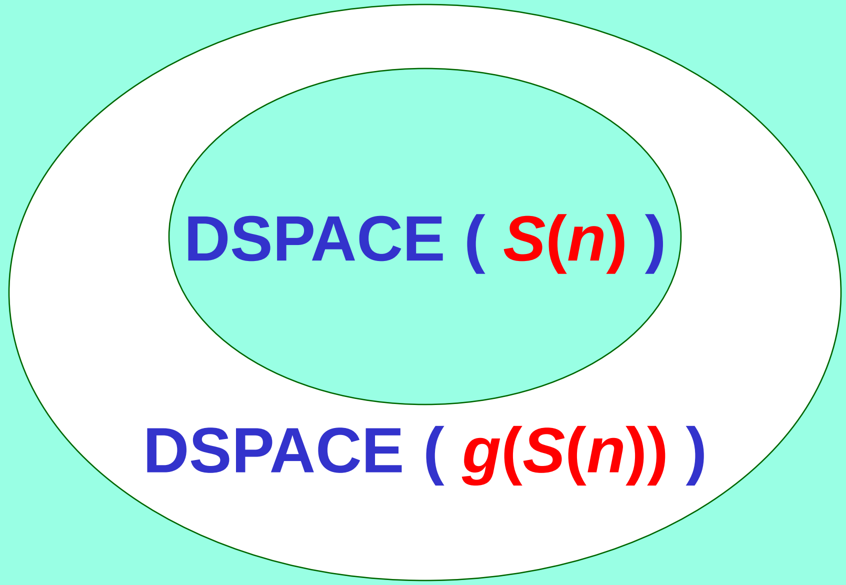


Gaps in the hierarchy



- $g(n)$ is an arbitrary total recursive function, $g(n) \geq n$

Gaps in the hierarchy



- ***g*(*n*) is an arbitrary total recursive function, $g(n) \geq n$**
  - ***g(n)*** is not fully time or space constructible

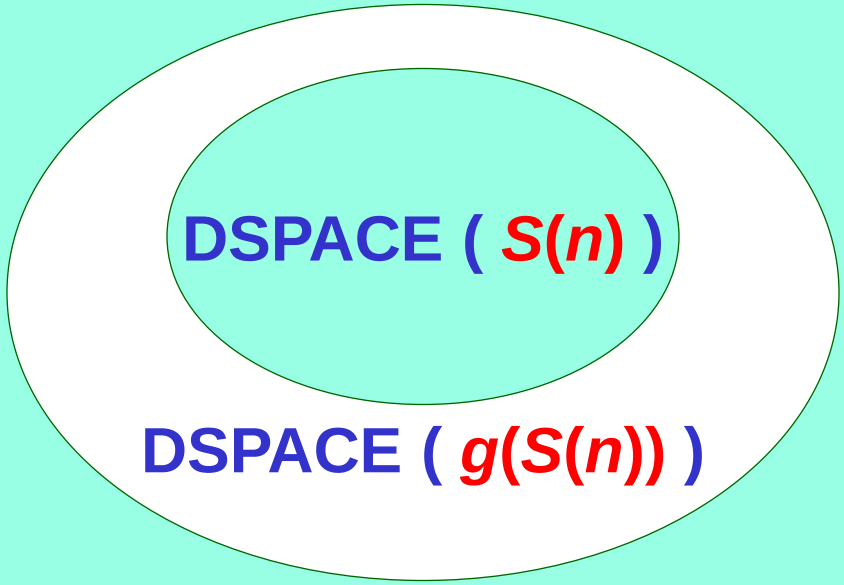
Gaps in the hierarchy

$$\mathrm{DSPACE}\,(\,S(n)\,) \subset \mathrm{DSPACE}\,(\,g(S(n))\,)$$

- $g(n)$ **is an arbitrary total recursive function,** $g(n) \geq n$
  - $g(n)$ is not fully time or space constructible

- ***It is possible to construct*** **a total recursive function** $S(n)$

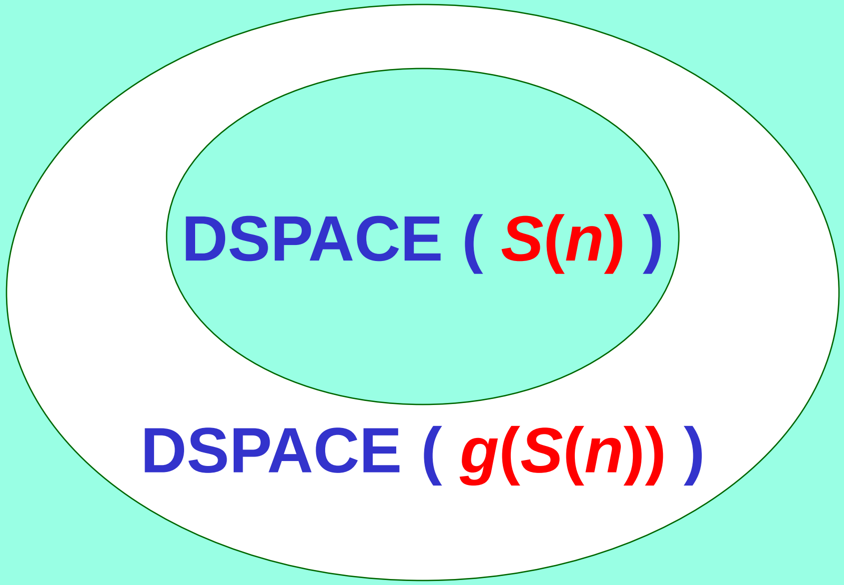
Gaps in the hierarchy



- ***g*(*n*)** **is an arbitrary total recursive function,** ***g*(*n*)≥*n***
    - ***g(n)*** is not fully time or space constructible

- ***It is possible to construct*** **a total recursive function** ***S*(*n*)**
    - DSPACE($S(n)$) = DSPACE($g(S(n))$)

Gaps in the hierarchy



- ***g*(*n*)** **is an arbitrary total recursive function, *g*(*n*)≥*n***
  - ***g(n)*** is not fully time or space constructible

- ***It is possible to construct* a total recursive function *S*(*n*)**
  - DSPACE($S(n)$) = DSPACE($g(S(n))$)
  - DTIME($f(n)$) = NTIME($f(n)$) = DSPACE($f(n)$) = NSPACE($f(n)$)

Optimal TM

Optimal TM

- $r(n)$ – any total recursive function

Optimal TM

- ***r*(*n*) – any total recursive function**
- ***It is possible to construct* a language *L***

Optimal TM

- ***r*(*n*) – any total recursive function**
- ***It is possible to construct* a language *L***
    - for any TM M_i with complexity $S_i(n)$ that accepts the language $L(M_i)=L$

Optimal TM

- ***r*(*n*) – any total recursive function**
- ***It is possible to construct* a language *L***
  - for any TM M_i with complexity $S_i(n)$ that accepts the language $L(M_i)=L$
  - there is a TM M_j with complexity $S_j(n)$ that accepts the language $L(M_j)=L(M_i)=L$

Optimal TM

- ***r*(*n*) – any total recursive function**
- ***It is possible to construct* a language *L***
  - for any TM M_i with complexity $S_i(n)$ that accepts the language $L(M_i)=L$
  - there is a TM M_j with complexity $S_j(n)$ that accepts the language $L(M_j)=L(M_i)=L$
  - $S_i(n) \geq r(S_j(n))$ for almost all values of n

Optimal TM

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Code TM M_1

Optimal TM

- **$r(n)$ – any total recursive function**
- ***It is possible to construct* a language *L***
  - for any TM M_i with complexity $S_i(n)$ that accepts the language $L(M_i)=L$
  - there is a TM M_j with complexity $S_j(n)$ that accepts the language $L(M_j)=L(M_i)=L$
  - $S_i(n) \geq r(S_j(n))$ for almost all values of n

Code TM M_1	**Code TM M_2**

Optimal TM

- $r(n)$ – **any total recursive function**
- ***It is possible to construct* a language L**
  - for any TM M_i with complexity $S_i(n)$ that accepts the language $L(M_i)=L$
  - there is a TM M_j with complexity $S_j(n)$ that accepts the language $L(M_j)=L(M_i)=L$
  - $S_i(n) \geq r(S_j(n))$ for almost all values of n

Code TM M_1	**Code TM M_2**	**Code TM M_3**

Optimal TM

- ***r*(*n*) – any total recursive function**
- ***It is possible to construct* a language *L***
  - for any TM M_i with complexity $S_i(n)$ that accepts the language $L(M_i)=L$
  - there is a TM M_j with complexity $S_j(n)$ that accepts the language $L(M_j)=L(M_i)=L$
  - $S_i(n) \geq r(S_j(n))$ for almost all values of n

Code TM M_1	**Code TM M_2**	**Code TM M_3**	**Code TM M_4**

Optimal TM

- **$r(n)$ – any total recursive function**
- ***It is possible to construct* a language L**
  - for any TM M_i with complexity $S_i(n)$ that accepts the language $L(M_i)=L$
  - there is a TM M_j with complexity $S_j(n)$ that accepts the language $L(M_j)=L(M_i)=L$
  - $S_i(n) \geq r(S_j(n))$ for almost all values of n

Code TM M_1	**Code TM M_2**	**Code TM M_3**	**Code TM M_4**	**Code TM M_5**

Optimal TM

- ***r*(*n*) – any total recursive function**
- ***It is possible to construct* a language *L***
  - for any TM M_i with complexity $S_i(n)$ that accepts the language $L(M_i)=L$
  - there is a TM M_j with complexity $S_j(n)$ that accepts the language $L(M_j)=L(M_i)=L$
  - $S_i(n) \geq r(S_j(n))$ for almost all values of n

Code TM M_1	**Code TM M_2**	**Code TM M_3**	**Code TM M_4**	**Code TM M_5**	**Code TM M_6**

- ***r*(*n*) – any total recursive function**
- ***It is possible to construct* a language *L***
  - for any TM M_i with complexity $S_i(n)$ that accepts the language $L(M_i)=L$
  - there is a TM M_j with complexity $S_j(n)$ that accepts the language $L(M_j)=L(M_i)=L$
  - $S_i(n) \geq r(S_j(n))$ for almost all values of n

Code TM M_1	**Code TM M_2**	**Code TM M_3**	**Code TM M_4**	**Code TM M_5**	**Code TM M_6**

$L(M_2) = L$

Optimal TM

- ***r*(*n*) – any total recursive function**
- ***It is possible to construct* a language *L***
  - for any TM M_i with complexity $S_i(n)$ that accepts the language $L(M_i)=L$
  - there is a TM M_j with complexity $S_j(n)$ that accepts the language $L(M_j)=L(M_i)=L$
  - $S_i(n) \geq r(S_j(n))$ for almost all values of n

Code TM M_1	**Code TM M_2**	**Code TM M_3**	**Code TM M_4**	**Code TM M_5**	**Code TM M_6**

$L(M_2) = L$ $L(M_4) = L$

Optimal TM

- ***r*(*n*)** **– any total recursive function**
- ***It is possible to construct* a language *L***
  - for any TM M_i with complexity $S_i(n)$ that accepts the language $L(M_i)=L$
  - there is a TM M_j with complexity $S_j(n)$ that accepts the language $L(M_j)=L(M_i)=L$
  - $S_i(n) \geq r(S_j(n))$ for almost all values of n

Code TM M_1	**Code TM M_2**	**Code TM M_3**	**Code TM M_4**	**Code TM M_5**	**Code TM M_6**

$L(M_2) = L$ $\qquad$ $L(M_4) = L$ $\qquad$ $L(M_6) = L$

Optimal TM

- ***r*(*n*)** **– any total recursive function**
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Code TM M_1	**Code TM M_2**	**Code TM M_3**	**Code TM M_4**	**Code TM M_5**	**Code TM M_6**

$L(M_2) = L$ $\quad$ $L(M_4) = L$ $\quad$ $L(M_6) = L$

$$S_2(n) \geq r(S_4(n))$$

Optimal TM

- ***r*(*n*) – any total recursive function**
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  - for any TM M_i with complexity $S_i(n)$ that accepts the language $L(M_i)=L$
  - there is a TM M_j with complexity $S_j(n)$ that accepts the language $L(M_j)=L(M_i)=L$
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Code TM M_1	**Code TM M_2**	**Code TM M_3**	**Code TM M_4**	**Code TM M_5**	**Code TM M_6**

$L(M_2) = L$ $\qquad$ $L(M_4) = L$ $\qquad$ $L(M_6) = L$

$S_2(n) \geq r(S_4(n))$ $\qquad$ $S_4(n) \geq r(S_6(n))$

Union of language classes

Union of language classes

- $\{f_i(n) \mid i=1, 2, \ldots\}$ – a set of recursive functions

Union of language classes

- $\{f_i(n) \mid i=1, 2, \ldots\}$ – a set of recursive functions
- If we can construct a TM *M* that outputs the list

Union of language classes

- **$\{f_i(n) \mid i=1, 2, ...\}$ – a set of recursive functions**
- **If we can construct a TM *M* that outputs the list**
    - TM M_1, M_2, ... compute $f_1(n)$, $f_2(n)$, ...

Union of language classes

- **$\{f_i(n) \mid i=1, 2, ...\}$ – a set of recursive functions**
- **If we can construct a TM M that outputs the list**
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Union of language classes

- $\{f_i(n) \mid i=1, 2, \ldots\}$ **– a set of recursive functions**
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Union of language classes

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Union of language classes

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$$\mathrm{DSPACE}(\, S(n) \,) = \bigcup_{i \geq 1} \mathrm{DSPACE}(\, f_i(n) \,)$$

Union of language classes

- $\{f_i(n) \mid i=1, 2, \ldots\}$ – a set of recursive functions
- If we can construct a TM *M* that outputs the list
  - TM $M_1, M_2, \ldots$ compute $f_1(n), f_2(n), \ldots$.
- and if for all n and i it holds that
  - $f_i(n) < f_{i+1}(n)$
- then there is a recursive function $S(n)$ for which

$$\mathrm{DSPACE}(\ S(n)\ ) = \bigcup_{i \geq 1} \mathrm{DSPACE}(\ f_i(n)\ )$$

DSPACE ($f_1(n)$)

Union of language classes

- $\{f_i(n) \mid i=1, 2, \ldots\}$ – a set of recursive functions
- If we can construct a TM *M* that outputs the list
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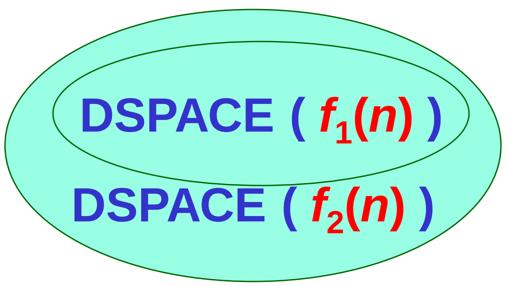

Union of language classes

- **$\{f_i(n) \mid i=1, 2, \ldots\}$ – a set of recursive functions**
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  - TM M_1, M_2, ... compute $f_1(n)$, $f_2(n)$, ...
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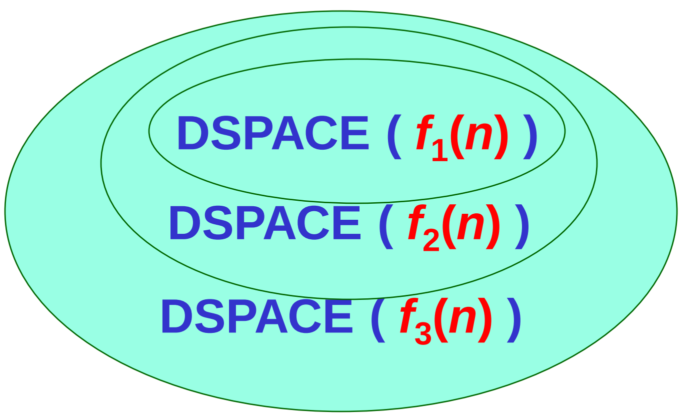
  - $f_i(n) < f_{i+1}(n)$
- **then there is a recursive function $S(n)$ for which**

$$\mathrm{DSPACE}(\, S(n)\,) = \bigcup_{i \geq 1} \mathrm{DSPACE}(\, f_i(n)\,)$$



Union of language classes

- $\{f_i(n) \mid i=1, 2, \ldots\}$ – a set of recursive functions
- If we can construct a TM M that outputs the list
  - TM M_1, M_2, ... compute $f_1(n)$, $f_2(n)$, ...
- and if for all n and i it holds that
  - $f_i(n) < f_{i+1}(n)$
- then there is a recursive function $S(n)$ for which

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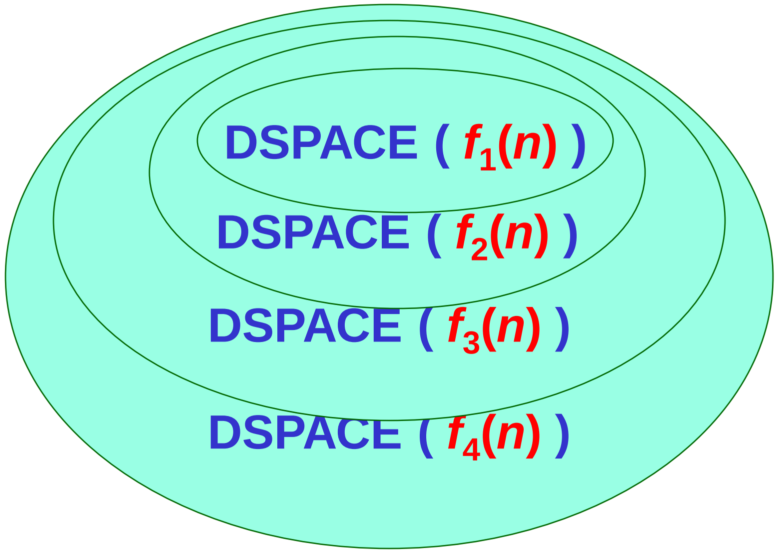

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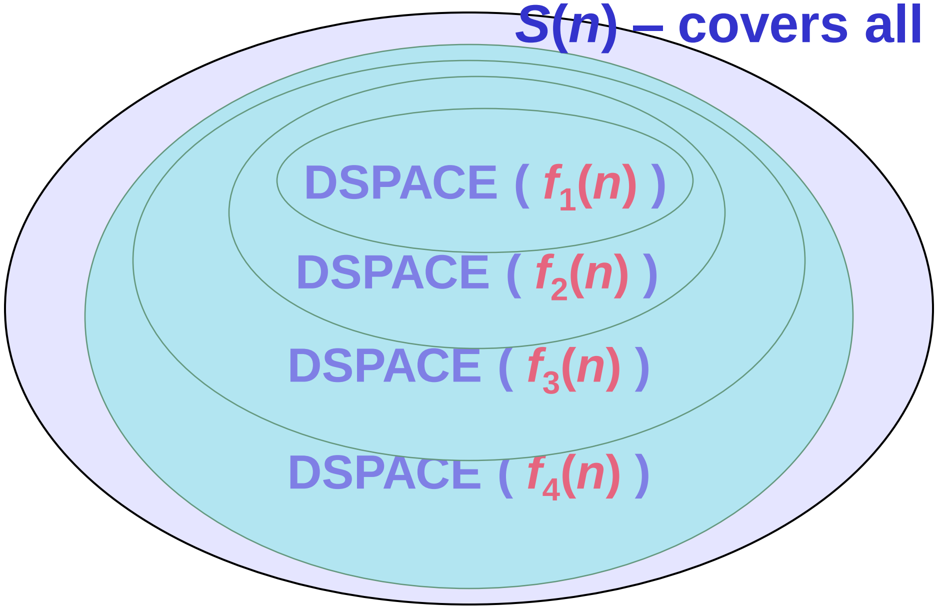
$S(n)$ – covers all languages for all classes $f_i(n)$



Union of language classes

- $\{f_i(n) \mid i=1, 2, ...\}$ – a set of recursive functions
- If we can construct a TM M that outputs the list
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- and if for all n and i it holds that
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- then there is a recursive function $S(n)$ for which

$$\text{DSPACE}(\, S(n)\,) = \bigcup_{i \geq 1} \text{DSPACE}(\, f_i(n)\,)$$

$S(n)$ – does not cover any other langauge