Introduction to Artificial Intelligence

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Exercises, v2

5 Knowledge representation using formal logic

1 (T) When deciding what system of formal logic to use for knowledge representation, one has to make a trade-off between expresivity and decidability. This trade-off already exists when choosing between propositional logic (PL) and first order logic (FOL). **How is this trade-off manifest when choosing between PL and FOL?**

- A FOL is more expressive than PL because it has variables and quantifiers, but, unlike PL, FOL is not decidable, which means that there is no proof procedure in FOL which is both sound and complete
- B PL is less expressive than FOL, but PL is decidable, hence there is a procedure for proving deductive consequences which is both sound and complete, which means we can prove all logical consequences, while in FOL this is possible only with a complete proof strategy
- C PL is more expressive than FOL as every variable in PL can be expressed as an unary predicate in FOL, but PL is not decidable because the number of interpretations is exponential in the number of variables, while in FOL the number of interpretations is finite
- D FOL is more expressive than PL because it can model objects and relations, but, unlike PL, FOL is not decidable, which means that we cannot always determine that some FOL formula is not a logical consequence of the premises

2 (P) An interpretation is defined with domain $D = \{a, b, c\}$, mapping $f(a) = c, f(b) = a, f(c) = a$, and predicate extensions $\text{ext}(P) = \{(a, a), (a, c), (b, b), (c, a)\}$ and $\text{ext}(Q) = \{b, c\}$. This interpretation is the *model* of some formulas. **Which formula is this interpretation a model of?**

A $\exists x\big(Q(x) \to \forall y P(y, y)\big)$ B $\forall x Q(f(x))$ C $\forall x\big(P(a, x) \to Q(x)\big)$ D $\exists x \forall y P(x, y)$

3 (P) Let $F(x)$ denote *"x is a Frenchman"*, $C(x)$ denote *"x is cheese"*, and $L(x, y)$ denote *"x likes y"*. **Which of the following FOL formulas is representing the correct formalization of the (stereotypical) statement "All French like cheese"?**

- A $\forall x \forall y\Big(L(x, y) \to (F(x) \land C(y))\Big)$
- B $\exists x \exists y\Big((F(x) \land C(y)) \to L(x, y)\Big)$
- C $\forall x \forall y\Big((F(x) \land C(y)) \land L(x, y)\Big)$
- D $\forall x\Big(F(x) \to \forall y(C(y) \to L(x, y))\Big)$

6 Automated reasoning

4 (T) The semantic consequence is a fundamental concept in logic, but in pratice proving semantic consequences can be thorny. **What exactly is the problem with proving semantic consequences?**

A Intractability B Nondeterminicity C Non-soundness D Incompletness

5 (T) An inference method is ideally both sound and complete. **Which of the following is a necessary and sufficient condition for the completeness of an inference method**?

- A A complete set of inference rules and a complete search strategy
- B A complete set of inference rules and an optimal search strategy
- C An optimal search strategy
- D A complete and sound set of rules

6 (T) Some rules of inference are not sound. Abduction is one such rule. **Which of the following would have to hold for abduction to be a sound rule**?

A $A \to B,\ \neg B \models \neg A$ B $A \to B,\ A \models B$ C $A \to B,\ B \models A$ D $A \to B,\ B \to C \models A \to C$

7 (P) Soundness is an important property of an inference rule. **Which of the following rules of inference is sound?**

- A $A \to (B \to C) \vdash (A \lor B) \to C$
- B $B, A \to B \vdash A$
- C $A \to B, B \to C \vdash C \to A$
- D $A \to (B \lor C), \neg C \vdash A \to B$

8 (C) Consider the following premises: *"Ivy is happy (H) if she has money (M) or is on a vacation (V) or is in Zagreb (Z). If she's not on a vacation, Ivy answers phone calls (P). If Andrew is calling her on the phone (A), then Ivy is not answering the phone call. Ivy is not in Zagreb or she has money."* **Which statement logically follows from these premises?**

- A *If Ivy is happy, then Andrew is calling her on the phone.*
- B *If Ivy is happy, then she is not answering phone calls.*
- C *Ivy is happy if Andrew is calling her on the phone.*
- D *Ivy is not answering phone calls.*

9 (P) We're using refutation resolution with the set-of-support (SOS) strategy for doing automated inference in PL. We can look at such an inference procedure as a state space search problem, where states correspond to a set of all clauses (those that were given and those that were derived), while the transitions between states correspond to the application of the resolution rule to a pair of clauses. Such a search problem also has its branching factor, which depends on the depth of the tree, i.e., the inference step. Let the set of premises contain 10 clauses, and the negated goal 5 clauses. **What is the upper bound on the branching factor in the second inference step?** (Note: Once resolved, a pair of clauses will not be resolved again.)

A 74 B 60 C 104 D 119