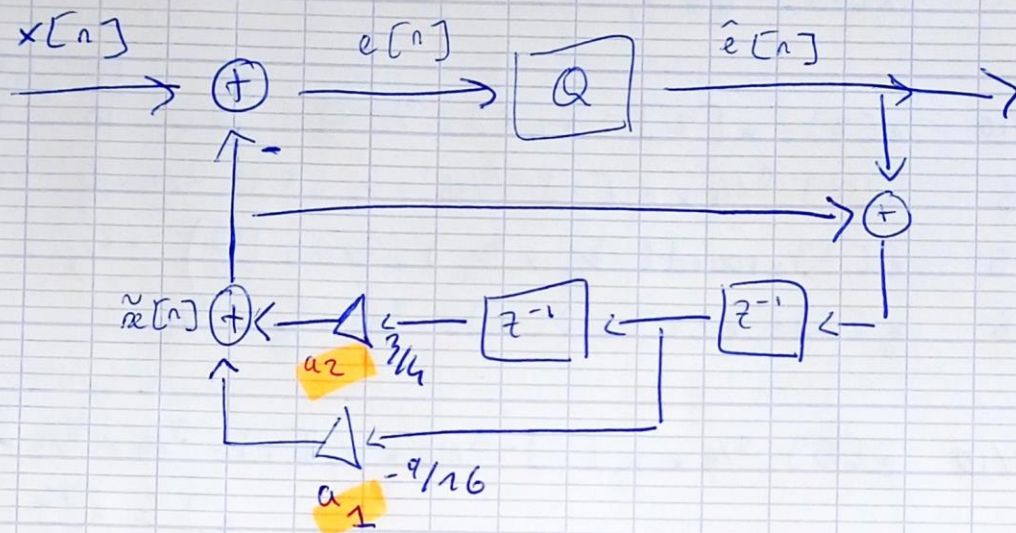


Task 5:

we want first 6 samples $e[n]$ and $\hat{e}[n]$ closed loop
 where $x[n] = \sqrt{3} \times \sin(\frac{\pi}{3} \times n)$ with $a_1 = 3/4$ $a_2 = -9/16$
 quantization step $\Delta = 0.4$



$$e[n] = x[n] - \hat{x}[n]$$

$$\hat{x}[n] = a_1 x_r[n-1] + a_2 x_r[n-2]$$

$$x_r[n] = \hat{e}[n] + \hat{x}[n]$$

$$\hat{e}[n] = \text{round}\left(e[n] \times \frac{2}{5}\right) \times \frac{5}{2} \quad \leftarrow \quad \text{round}\left(e[n] \times \frac{1}{\Delta}\right) \times \Delta$$

$$x[0] = \sqrt{3} \times \sin\left(\frac{\pi}{3} \times 0\right) = \sqrt{3} \times 0 = 0$$

$$x[1] = \sqrt{3} \times \sin\left(\frac{\pi}{3} \times 1\right) = \sqrt{3} \times \frac{\sqrt{3}}{2} = 3/2$$

$$x[2] = \sqrt{3} \times \sin\left(\frac{\pi}{3} \times 2\right) = 3/2$$

$$x[3] = \sqrt{3} \times \sin\left(\frac{3\pi}{3}\right) = \sqrt{3} \times 0 = 0$$

$$x[4] = \sqrt{3} \times \sin\left(\frac{4\pi}{3}\right) = \sqrt{3} \times -\frac{\sqrt{3}}{2} = -3/2$$

$$x[5] = \sqrt{3} \times \sin\left(\frac{5\pi}{3}\right) = \sqrt{3} \times -\frac{\sqrt{3}}{2} = -3/2$$

$$x[6] = \sqrt{3} \times \sin\left(\frac{6\pi}{3}\right) = \sqrt{3} \times 0 = 0$$

0

$$\begin{aligned}
 e[0] &= 0 & e[0] &= x[0] - \hat{x}[0] = 0 - 0 = 0 \\
 \hat{e}[0] &= 0 & \hat{e}[0] &= \text{round}(0 \times 5/2) \times 2/5 = 0 \\
 x_r[0] &= 0 & x_r[0] &= \hat{e}[0] + \hat{x}[0] = 0 + 0 = 0 \\
 \hat{x}[0] &= 0 & \hat{x}[0] &= 0
 \end{aligned}$$

$$\begin{aligned}
 e[1] &= 3/2 & e[1] &= x[1] - \hat{x}[1] = x[1] - 3/4 x_r[0] - 9/16 x_r[-1] \\
 & & &= 3/2 - 0 = 3/2 \\
 \hat{e}[1] &= 8/5 & \hat{e}[1] &= \text{round}(3/2 \times 5/2) \times 2/5 = \text{round}(15/4) \times 2/5 = 4 \times 2/5 = 8/5 \\
 x_r[1] &= 8/5 & x_r[1] &= \hat{e}[1] + \hat{x}[1] = 8/5 + 0 = 8/5 \\
 \hat{x}[1] &= 0 & \hat{x}[1] &= 3/4 x_r[0] - 9/16 x_r[-1] = 0
 \end{aligned}$$

$$\begin{aligned}
 e[2] &= 3/10 & e[2] &= x[2] - \hat{x}[2] = 3/2 - 6/5 = \frac{15}{10} - \frac{12}{10} = \frac{3}{10} \\
 \hat{e}[2] &= 6/5 & \hat{e}[2] &= 3/4 x_r[1] - 9/16 x_r[0] = 3/4 \times \frac{8}{5} - 9/16 \times 0 = \frac{3}{4} \times \frac{8}{5} = \frac{24}{20} = \frac{6}{5} \\
 \hat{x}[2] &= 2/5 & \hat{x}[2] &= \text{round}(\frac{3}{10} \times \frac{5}{2}) \times 2/5 = (\frac{15}{20}) \times 2/5 = 1 \times 2/5 = 2/5 \\
 x_r[2] &= 8/5 & x_r[2] &= \hat{e}[2] + \hat{x}[2] = \frac{6}{5} + \frac{2}{5} = \frac{8}{5}
 \end{aligned}$$

$$\begin{aligned}
 \hat{x}[3] &= 3/10 & \hat{x}[3] &= 3/4 x_r[2] - 9/16 x_r[1] = 3/4 \times \frac{8}{5} - 9/16 \times \frac{8}{5} = \frac{24}{20} - \frac{72}{80} \\
 & & &= \frac{96}{80} - \frac{72}{80} = \frac{24}{80} = \frac{3}{10}
 \end{aligned}$$

$$\begin{aligned}
 e[3] &= -3/10 & e[3] &= x[3] - \hat{x}[3] = -3/10 \\
 \hat{e}[3] &= -2/5 & \hat{e}[3] &= \text{round}(-3/10 \times 5/2) \times 2/5 = (\frac{-15}{20}) \times 2/5 = -1 \times 2/5 = -2/5 \\
 x_r[3] &= -1/10 & x_r[3] &= \hat{e}[3] + \hat{x}[3] = \frac{-2}{5} + \frac{3}{10} = \frac{-1}{10}
 \end{aligned}$$

$$\begin{aligned}
 \hat{x}[4] &= -39/40 & \hat{x}[4] &= 3/4 x_r[3] - 9/16 x_r[2] = 3/4 \times \frac{-1}{10} - 9/16 \times \frac{8}{5} = \frac{-3}{40} - \frac{72}{80} \\
 & & &= \frac{-6}{80} - \frac{72}{80} = \frac{-78}{80} = \frac{-39}{40}
 \end{aligned}$$

$$\begin{aligned}
 e[4] &= -21/40 & e[4] &= -3/2 + \frac{39}{40} = \frac{-60}{40} + \frac{39}{40} = \frac{-21}{40} \\
 \hat{e}[4] &= -2/5 & \hat{e}[4] &= \text{round}(\frac{-21}{40} \times \frac{5}{2}) \times 2/5 = \text{round}(\frac{-105}{80}) \times 2/5 = -2/5 \\
 x_r[4] &= -11/8 & x_r[4] &= \hat{e}[4] + \hat{x}[4] = \frac{-2}{5} - \frac{39}{40} = \frac{-16-39}{40} = \frac{-55}{40} = \frac{-11}{8}
 \end{aligned}$$

$$\begin{aligned}
 \hat{x}[5] &= -39/40 & \hat{x}[5] &= 3/4 x_r[4] - 9/16 x_r[3] = 3/4 \times \frac{-11}{8} - 9/16 \times \frac{-1}{10} = \frac{-33}{32} + \frac{9}{160} \\
 & & &= \frac{-165 + 9}{160} = \frac{-156}{160} = \frac{-39}{40} \\
 e[5] &= -21/40 & e[5] &= -3/2 + \frac{39}{40} = \frac{-60}{40} + \frac{39}{40} = \frac{-21}{40} \\
 \hat{e}[5] &= -2/5 & \hat{e}[5] &= \text{round}(\frac{-21}{40} \times \frac{5}{2}) \times 2/5 = \text{round}(\frac{-105}{80}) \times 2/5 = -2/5
 \end{aligned}$$