3rd lecture overview

2.1.3 Nondeterministic finite automaton (NFA)

2.1.4 Nondeterministic finite automaton with ε-moves (ε-NFA)

Lecture overview

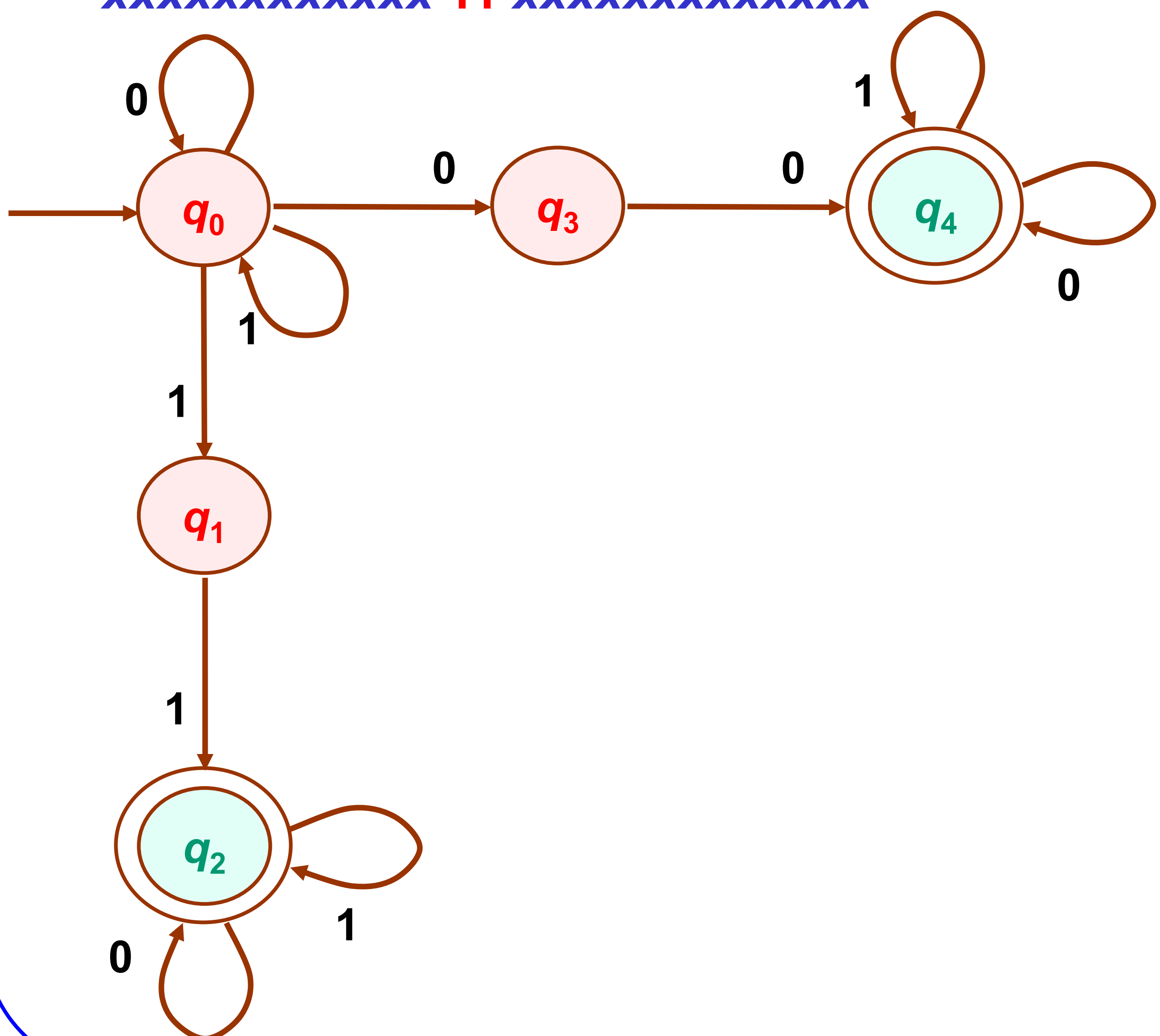
2.1.3 Nondeterministic finite automaton (NFA)

2.1.4 Nondeterministic finite automaton with ε-moves (ε-NFA)

Nondeterministic finite automaton (NFA)

xxxxxxxxxxxx **00** *xxxxxxxxxxxxx*

xxxxxxxxxxxx **11** *xxxxxxxxxxxxx*

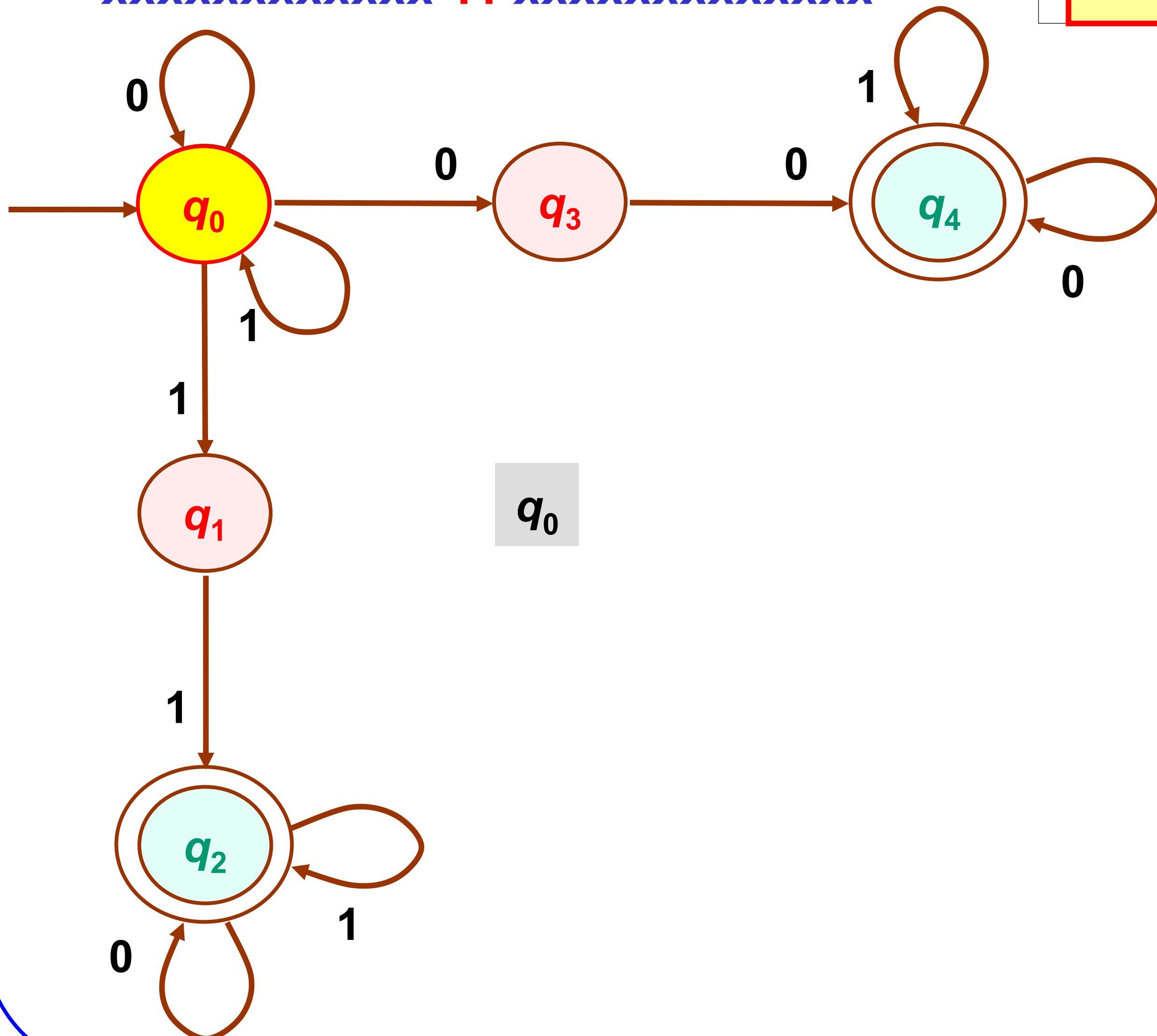


Nondeterministic finite automaton (NFA)

xxxxxxxxxxxxx **00** *xxxxxxxxxxxxxx*

xxxxxxxxxxxxx **11** *xxxxxxxxxxxxxx*

0	1	0	0	1



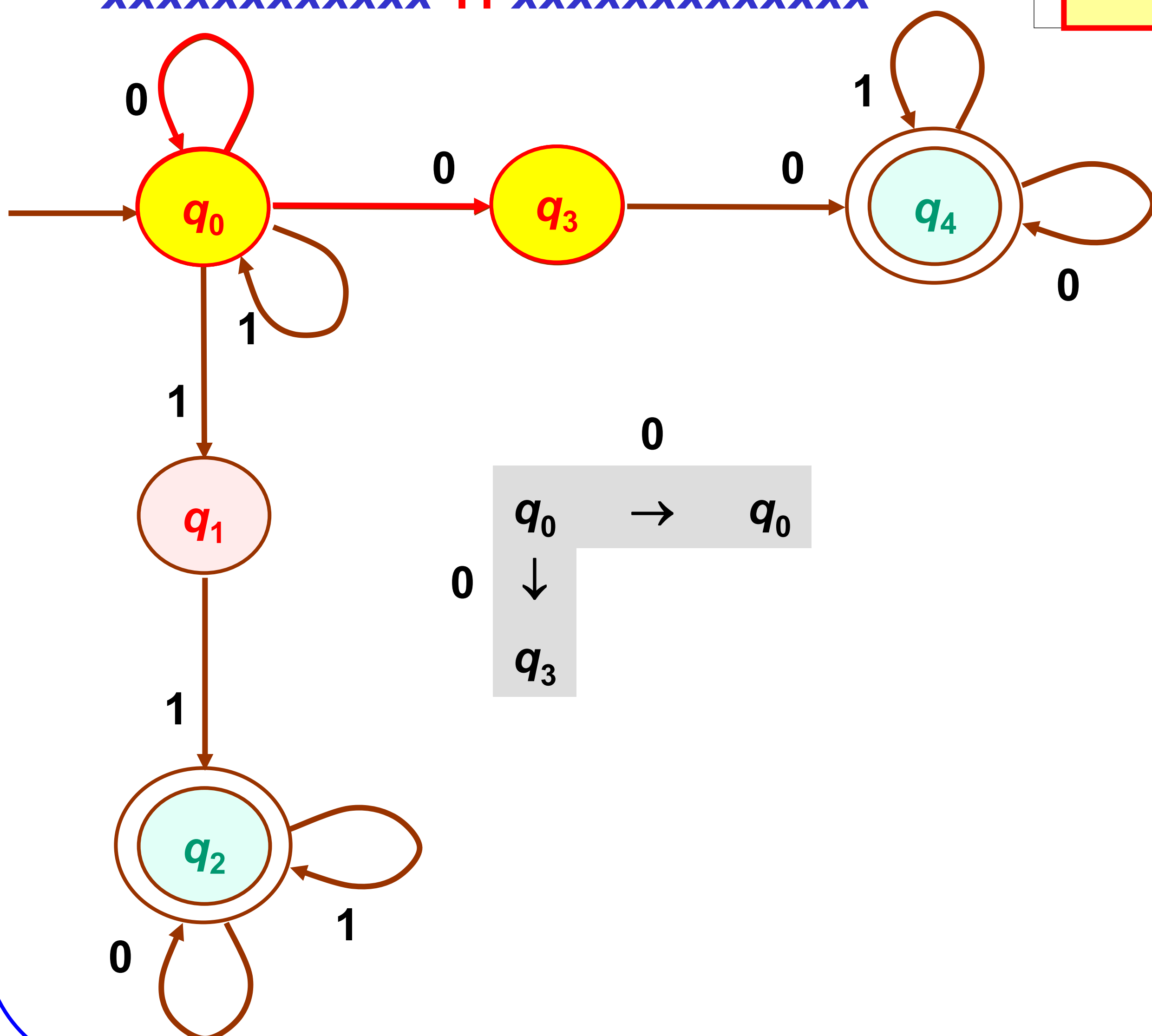
q_0

Nondeterministic finite automaton (NFA)

xxxxxxxxxxxx **00** *xxxxxxxxxxxxx*

xxxxxxxxxxxx **11** *xxxxxxxxxxxxx*

0	1	0	0	1

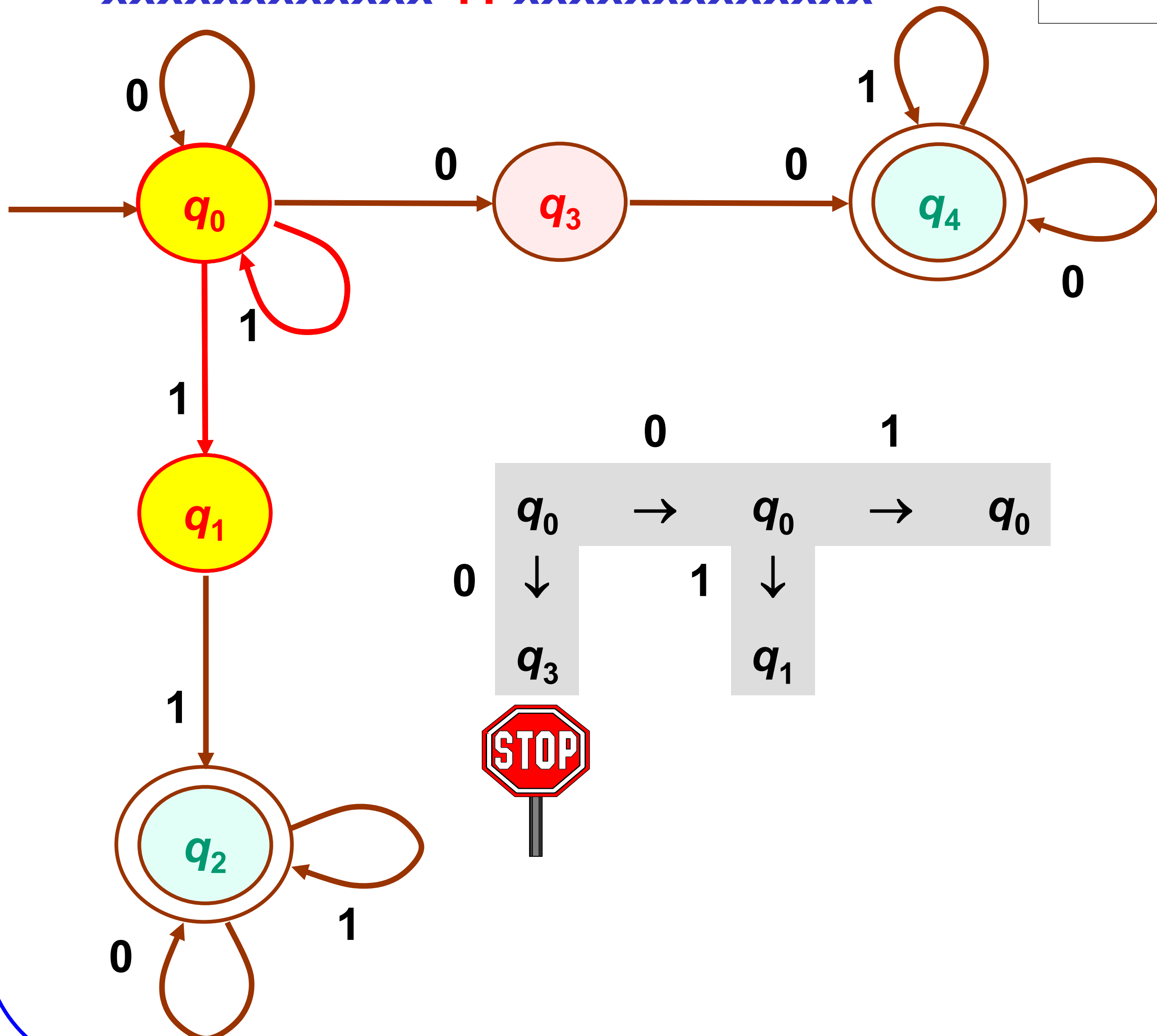


Nondeterministic finite automaton (NFA)

xxxxxxxxxxxxx 00 xxxxxxxxxxxxxx

xxxxxxxxxxxxx 11 xxxxxxxxxxxxxx

0	1	0	0	1

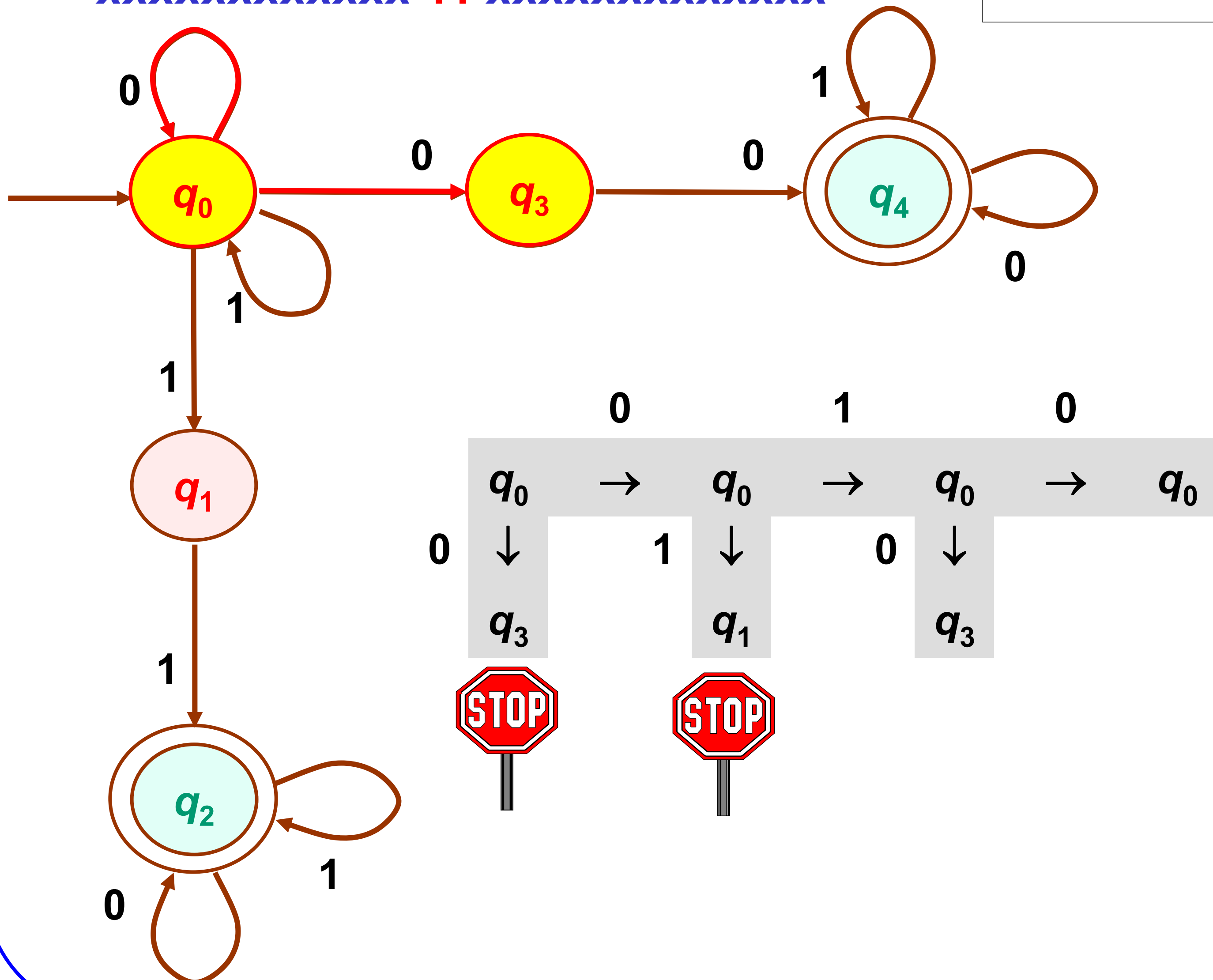


Nondeterministic finite automaton (NFA)

xxxxxxxxxxxx 00 *xxxxxxxxxxxxx*

xxxxxxxxxxxx 11 *xxxxxxxxxxxxx*

0	1	**0**	0	1

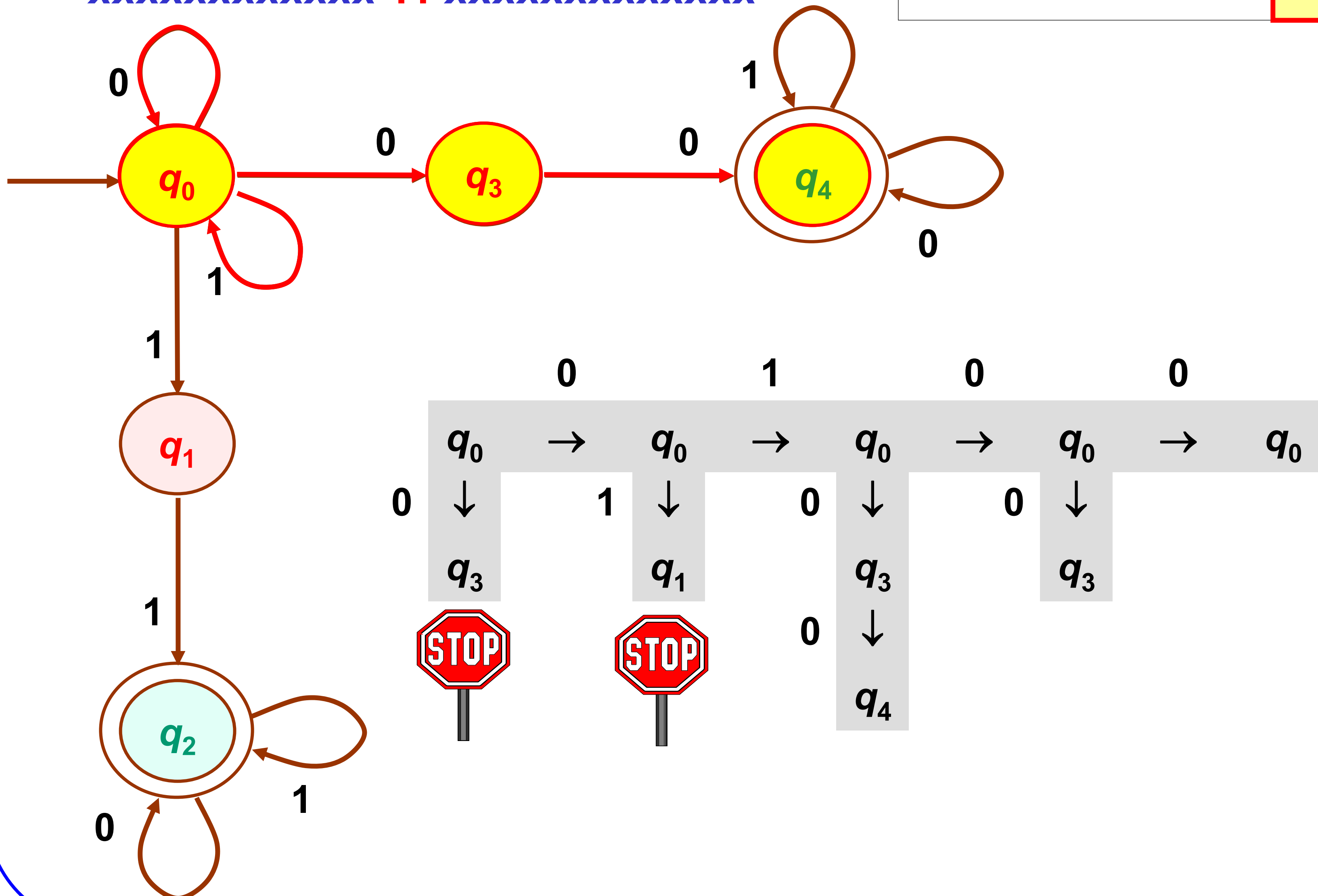


Nondeterministic finite automaton (NFA)

xxxxxxxxxxxx **00** *xxxxxxxxxxxxx*

xxxxxxxxxxxx **11** *xxxxxxxxxxxxx*

0	1	0	0	1

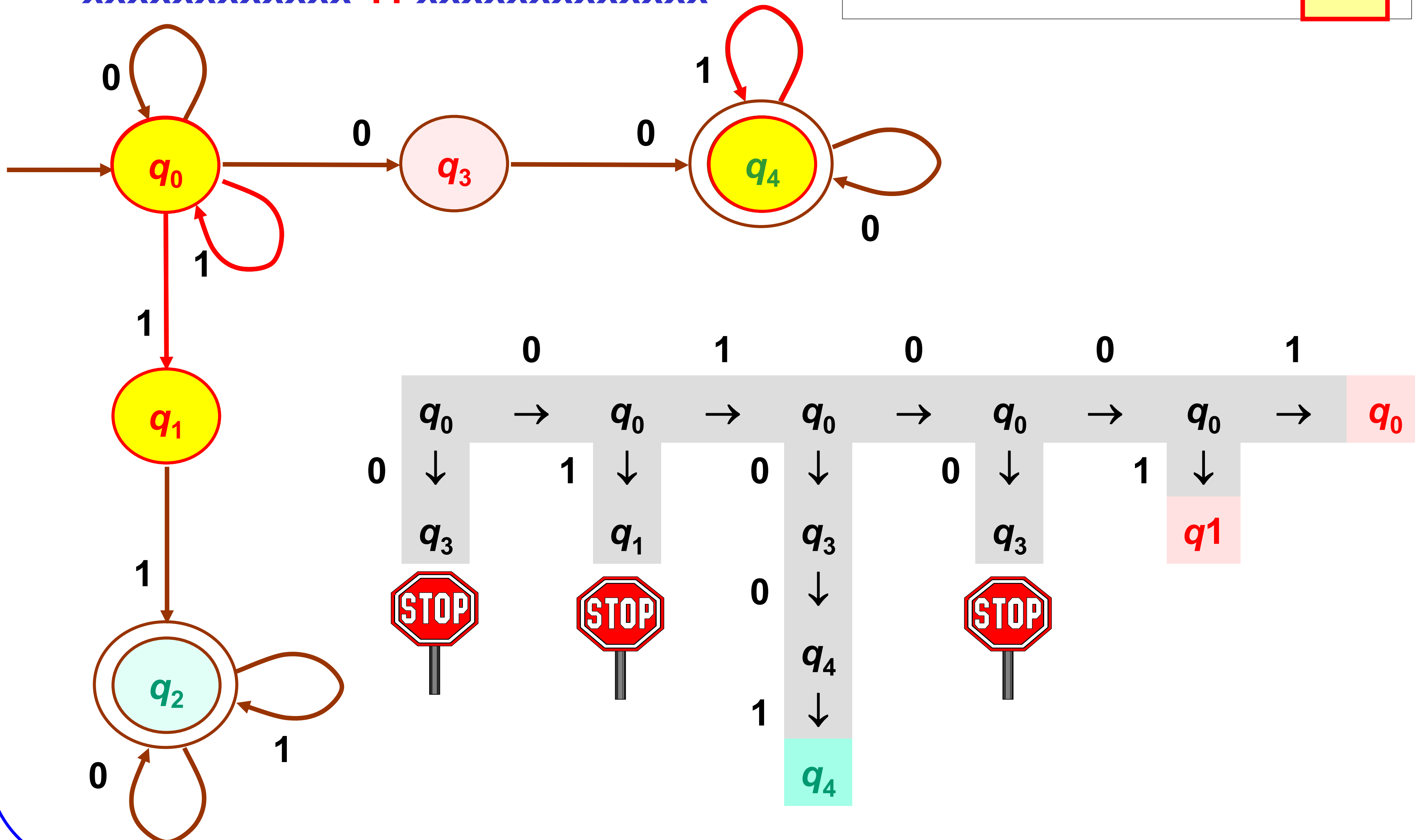


Nondeterministic finite automaton (NFA)

xxxxxxxxxxxx **00** *xxxxxxxxxxxxx*

xxxxxxxxxxxx **11** *xxxxxxxxxxxxx*

0 1 0 0 1



Nondeterministic finite automaton (NFA)

$$nfa = (Q, \Sigma, \delta, q_0, F)$$

Q	- finite set of states
Σ	- finite set of input symbols
δ	- transition function $Q \times \Sigma \rightarrow 2^Q$
$q_0 \in Q$	- start state
$F \subseteq Q$	- set of accept states

$$\delta\,(\mathit{State}, \mathit{InputSymbol}) = P \subseteq Q$$

Nondeterministic finite automaton (NFA)

$$\hat{\delta} : Q \times \Sigma^* \to 2^Q$$

(1) $\hat{\delta}(q, \varepsilon) = q$

(2) $\hat{\delta}(q, wa) = P =$

$\{p \mid$ for some state r from $\hat{\delta}(q, w)$, p is in $\delta(r, a)\}$,

$w \in \Sigma^*$, $a \in \Sigma$ i $P \subseteq Q$

$\hat{\delta}(q, a) = P = \{p \mid$ where p is from $\delta(q, a)\} = \delta(q, a)$

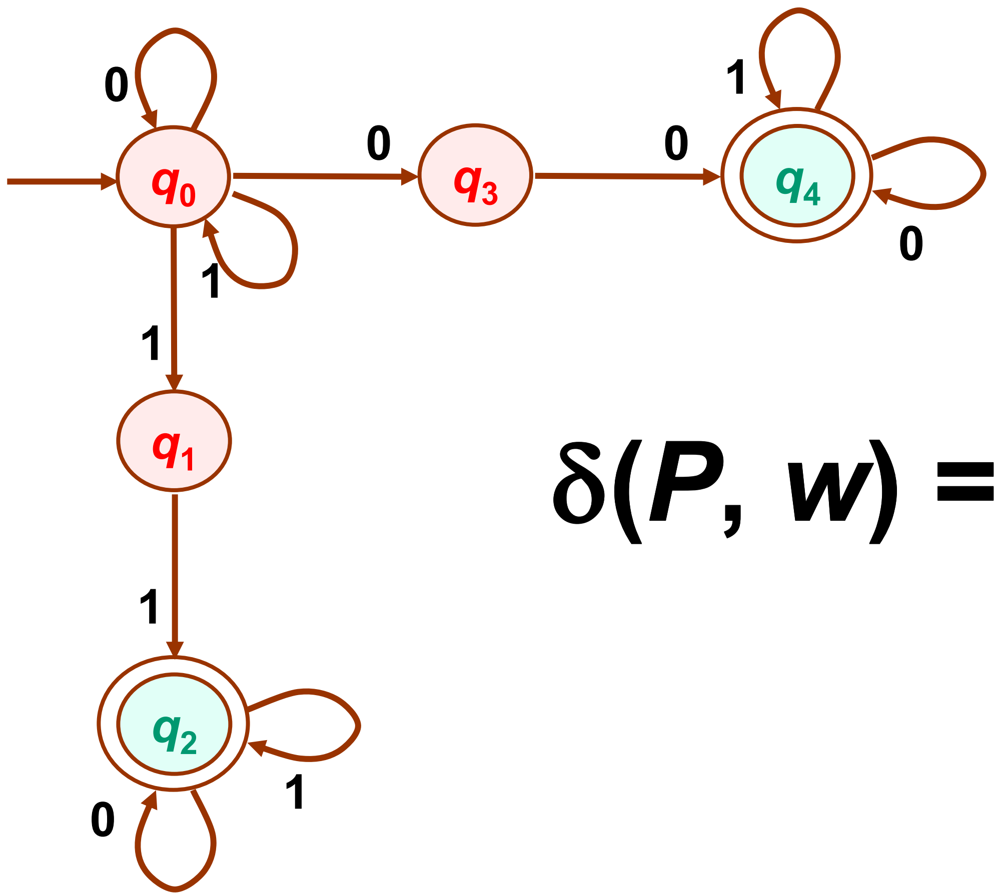
Nondeterministic finite automaton (NFA)

$$NFA = (Q, \Sigma, \delta, q_0, F)$$

NFA accepts string *w* if $\delta(q_0, w)$ contains at least one state from set *F*

NFA accepts language $L(NFA)=\{w \mid \delta(q_0, w)$ contains at least one state from set $F\}$

Nondeterministic finite automaton (NFA)



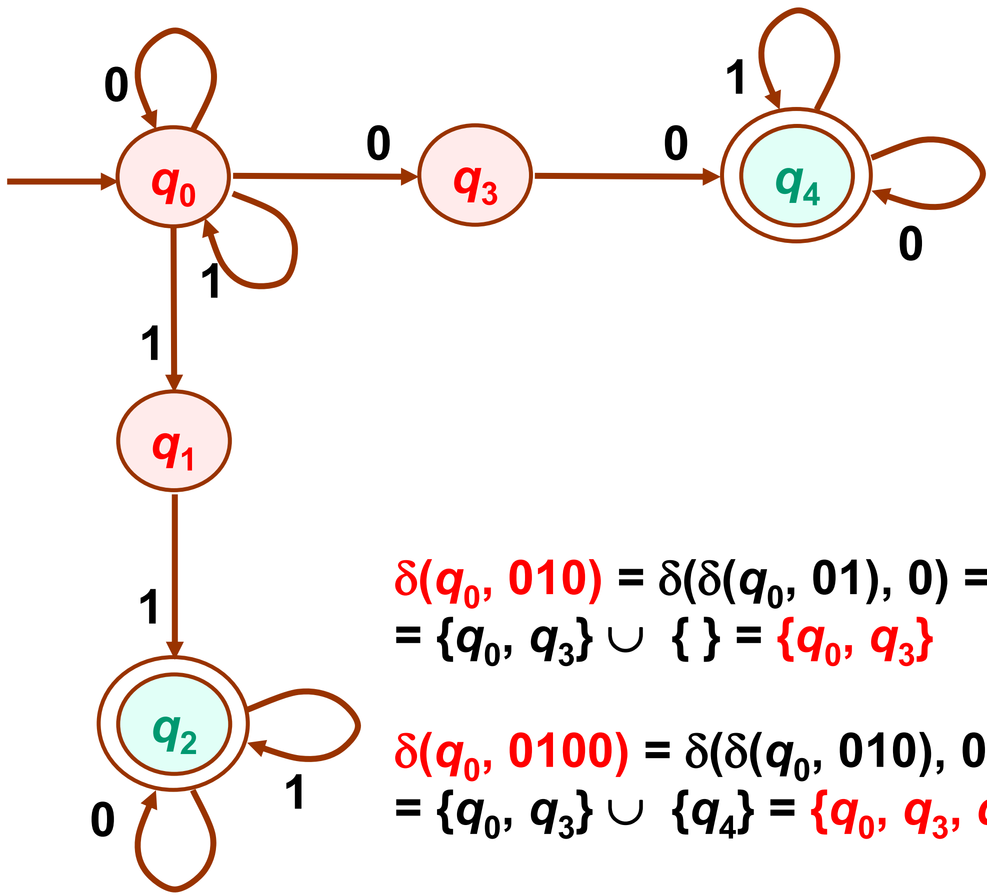
0 1 0 0 1

$$\delta(P, w) = \bigcup_{q \in P} \delta(q, w)$$

$\delta(q_0, 0) = \{q_0, q_3\}$

$\delta(q_0, 01) = \delta(\delta(q_0, 0), 1) = \delta(\{q_0, q_3\}, 1) = \delta(q_0, 1) \cup \delta(q_3, 1) = \{q_0, q_1\} \cup \{\ \} = \{q_0, q_1\}$

Nondeterministic finite automaton (NFA)



0 1 0 0 1

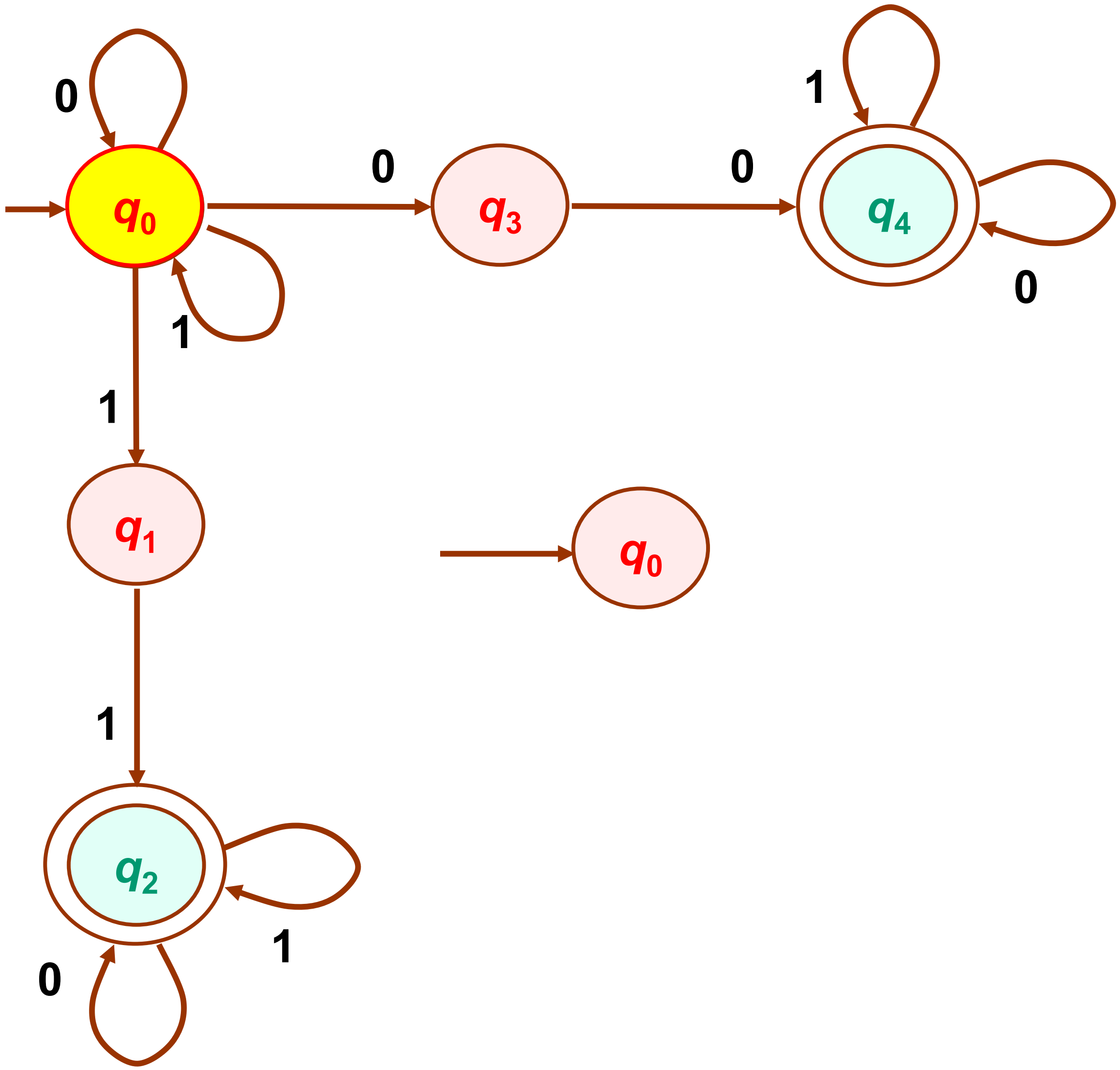
$\delta(q_0, 010) = \delta(\delta(q_0, 01), 0) = \delta(\{q_0, q_1\}, 0) = \delta(q_0, 0) \cup \delta(q_1, 0)$
$= \{q_0, q_3\} \cup \{\ \} = \{q_0, q_3\}$

$\delta(q_0, 0100) = \delta(\delta(q_0, 010), 0) = \delta(\{q_0, q_3\}, 0) = \delta(q_0, 0) \cup \delta(q_3, 0)$
$= \{q_0, q_3\} \cup \{q_4\} = \{q_0, q_3, q_4\}$

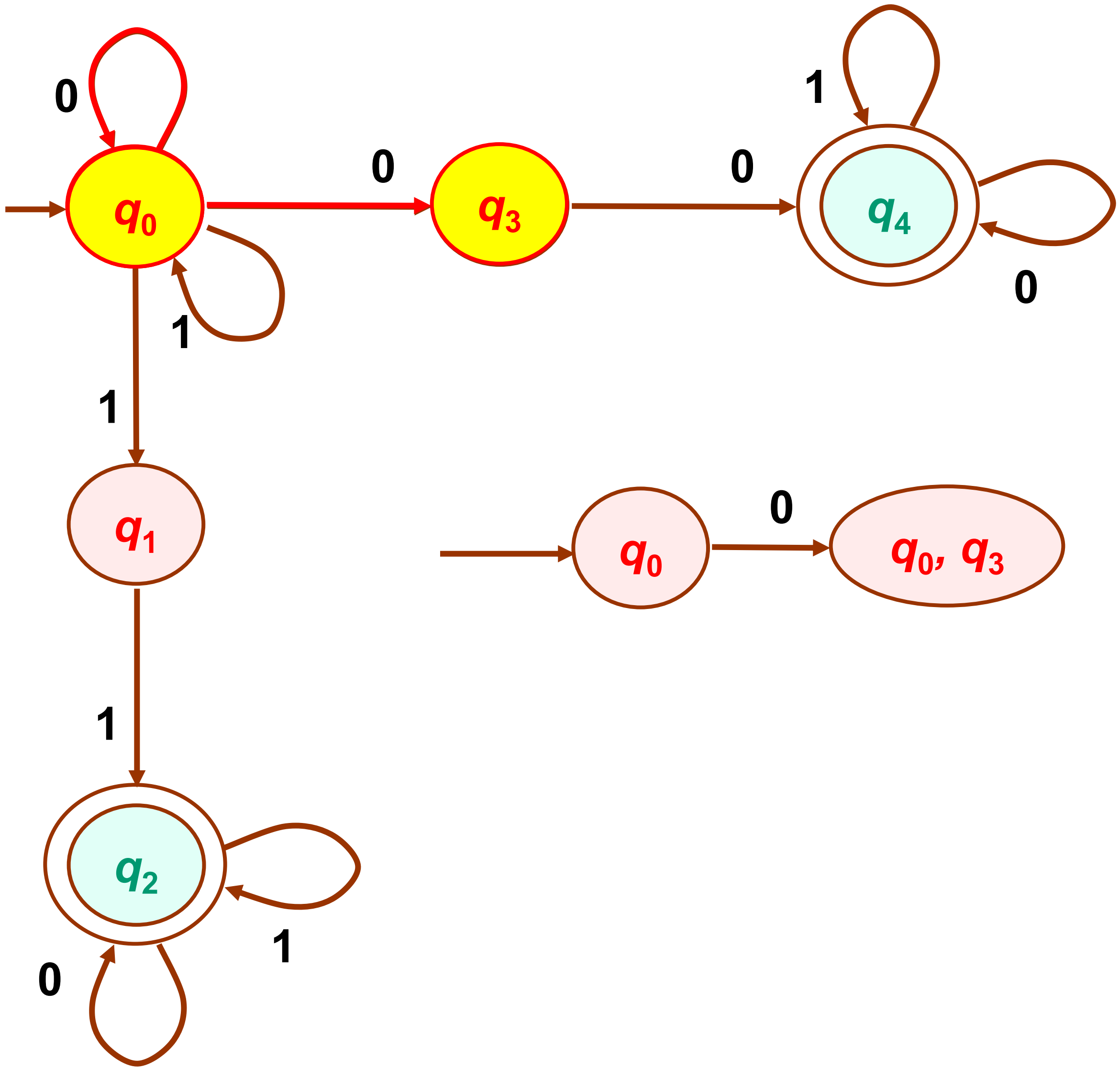
$\delta(q_0, 01001) = \delta(\delta(q_0, 0100), 1) = \delta(\{q_0, q_3, q_4\}, 1)$
$= \delta(q_0, 1) \cup \delta(q_3, 1) \cup \delta(q_4, 1) = \{q_0, q_1\} \cup \{\ \} \cup \{q_4\} = \{q_0, q_1, q_4\}$

$(q_4 \in \delta(q_0, 01001) \wedge q_4 \in F) \Rightarrow$ string 01001 is accepted

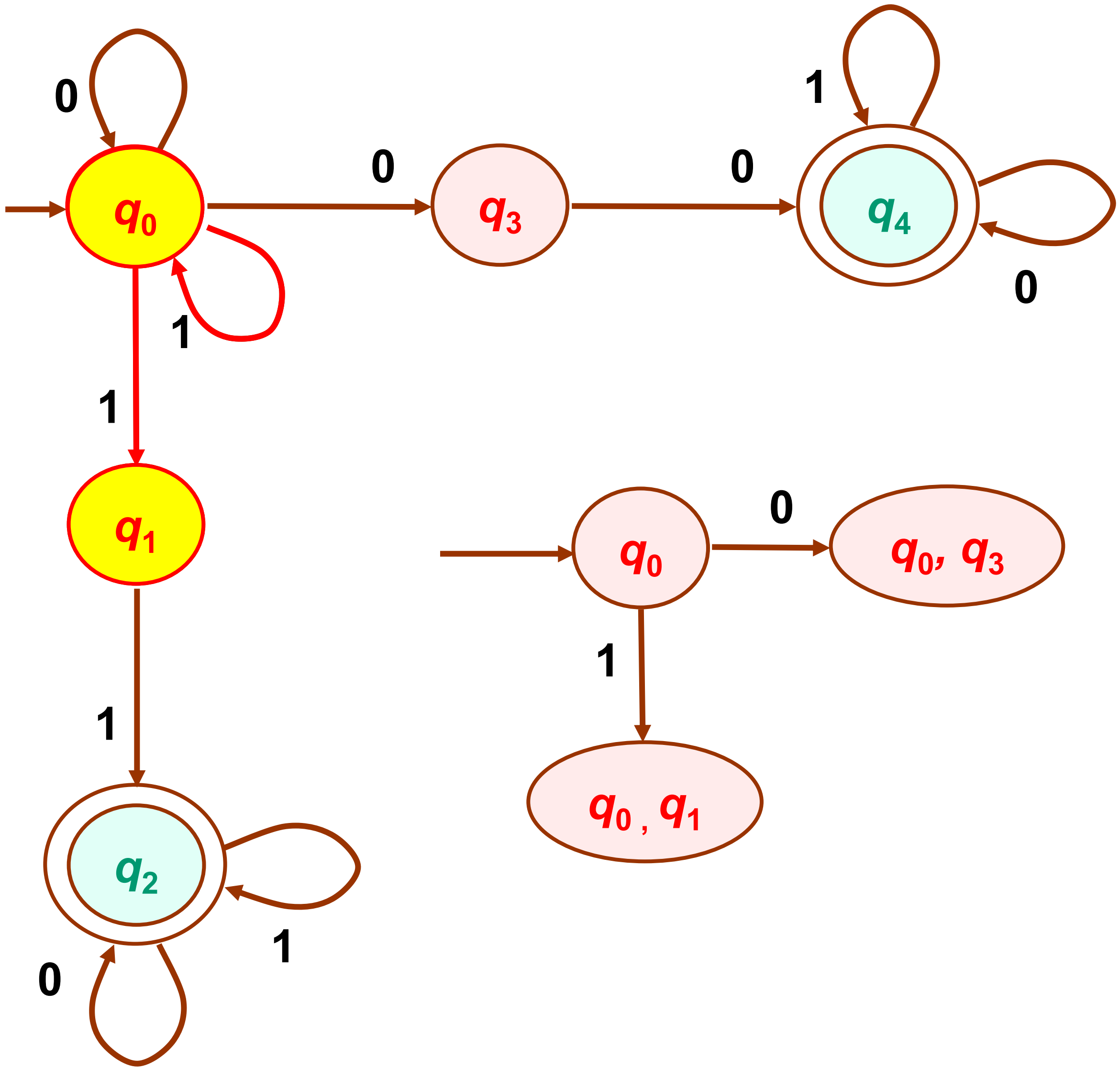
Constructing DFA equivalent to the given NFA



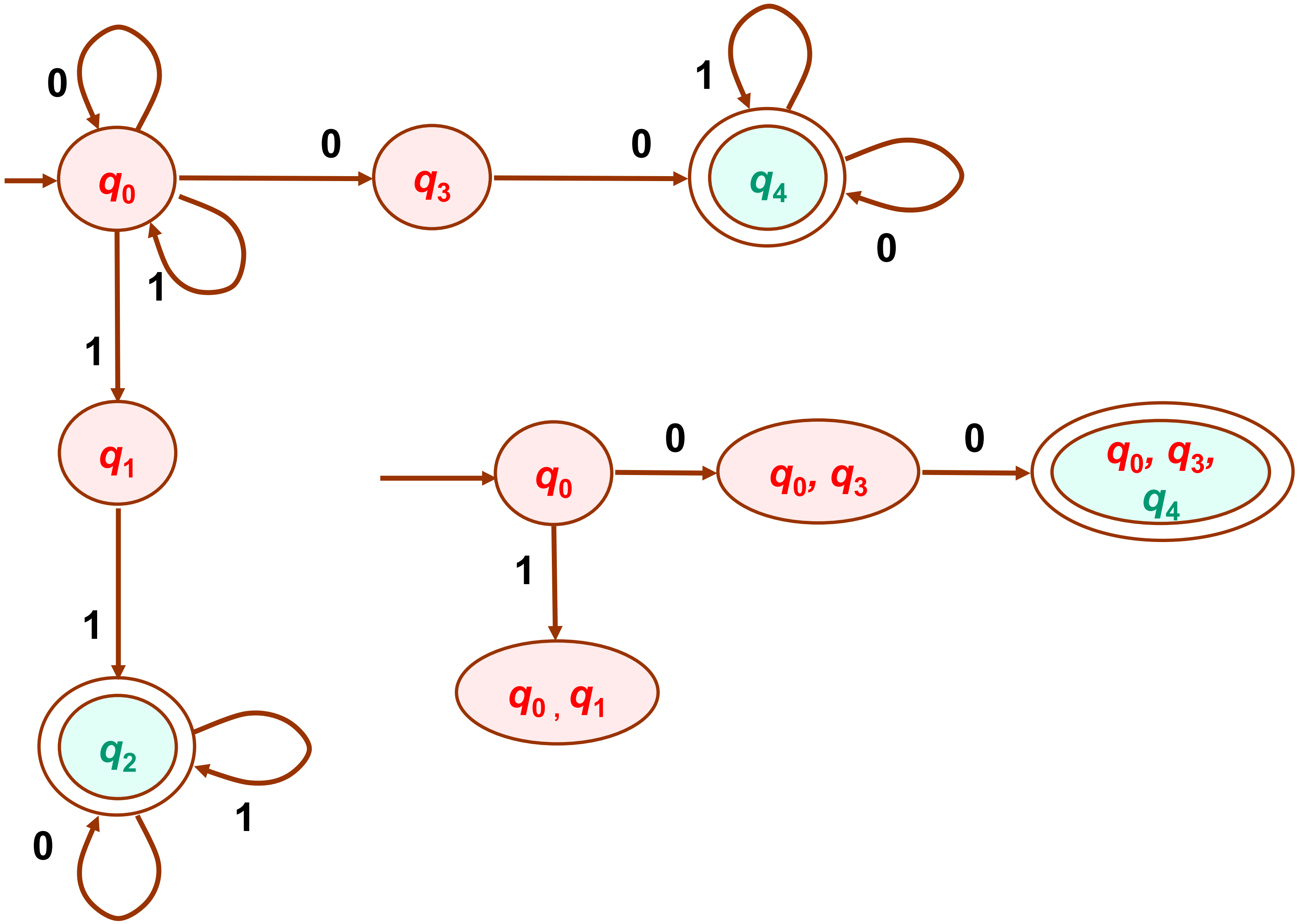
Constructing DFA equivalent to the given NFA



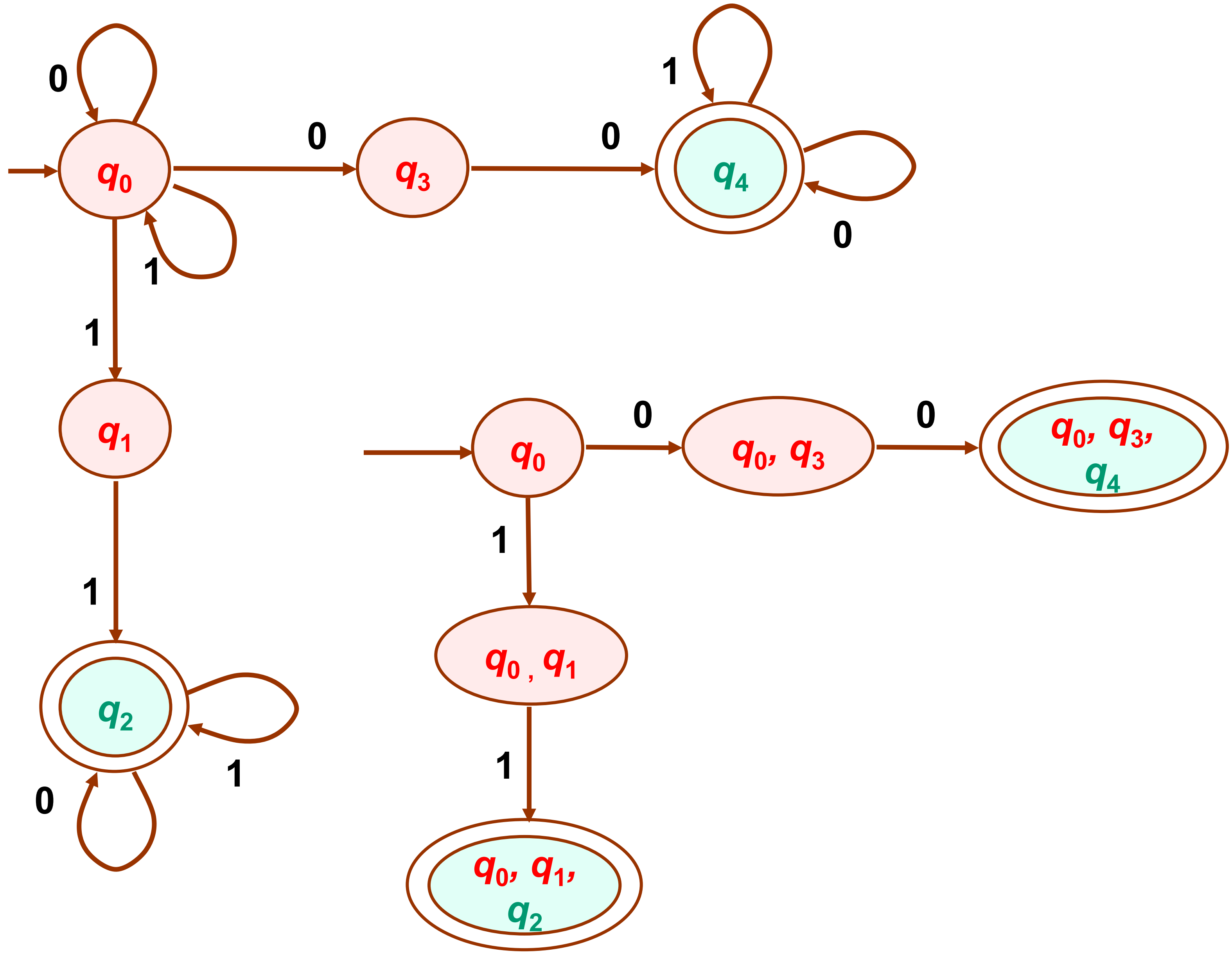
Constructing DFA equivalent to the given NFA



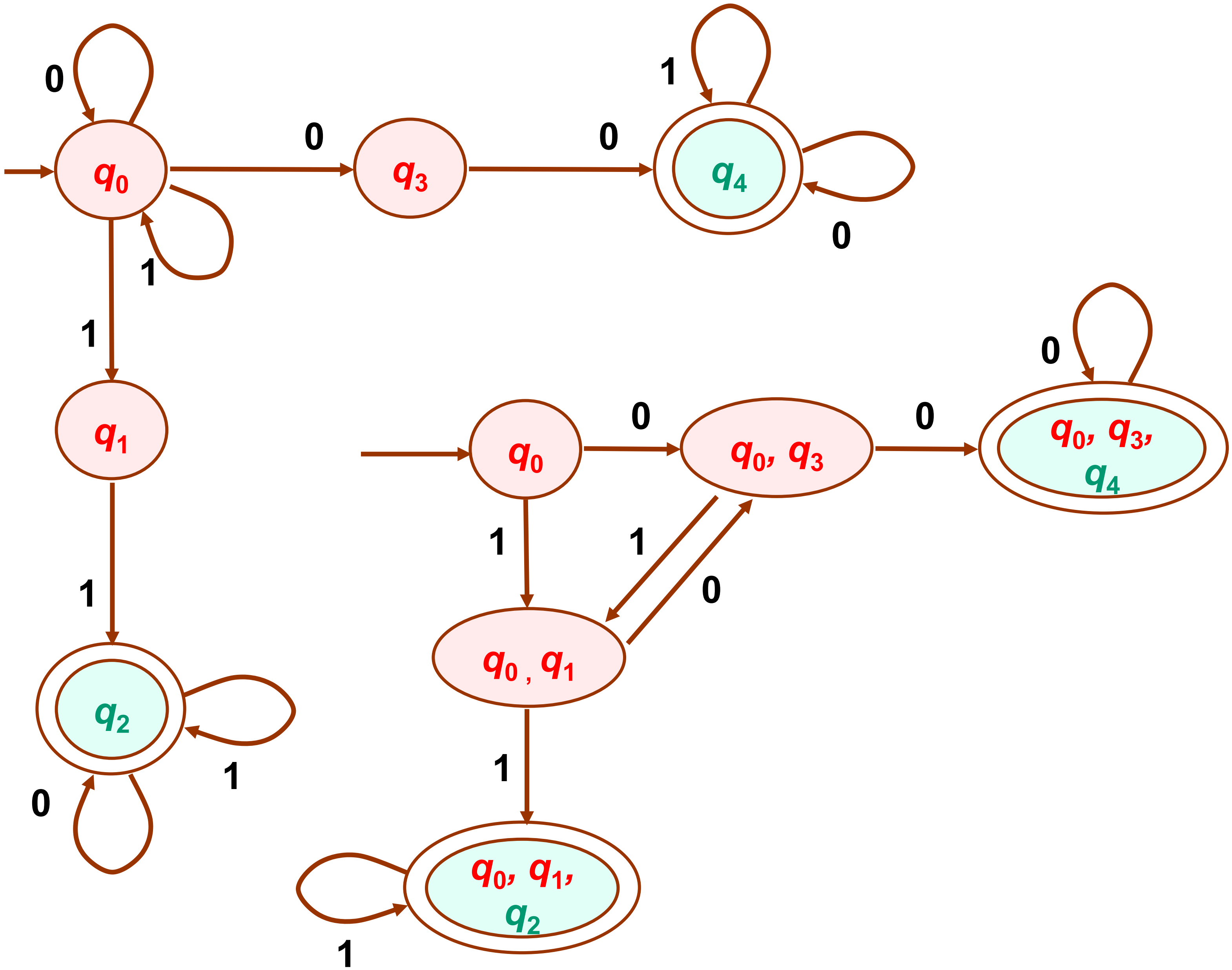
Constructing DFA equivalent to the given NFA



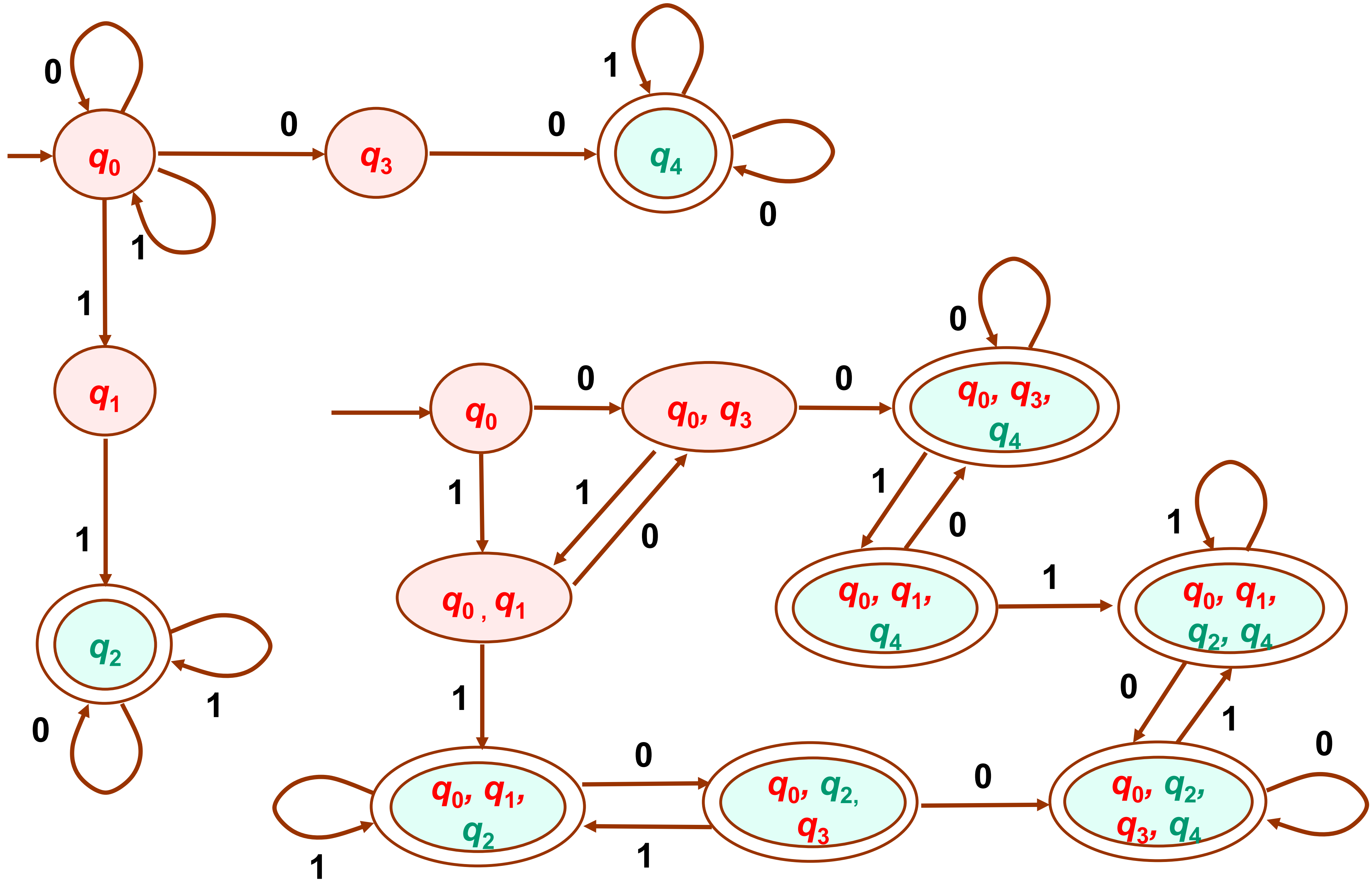
Constructing DFA equivalent to the given NFA



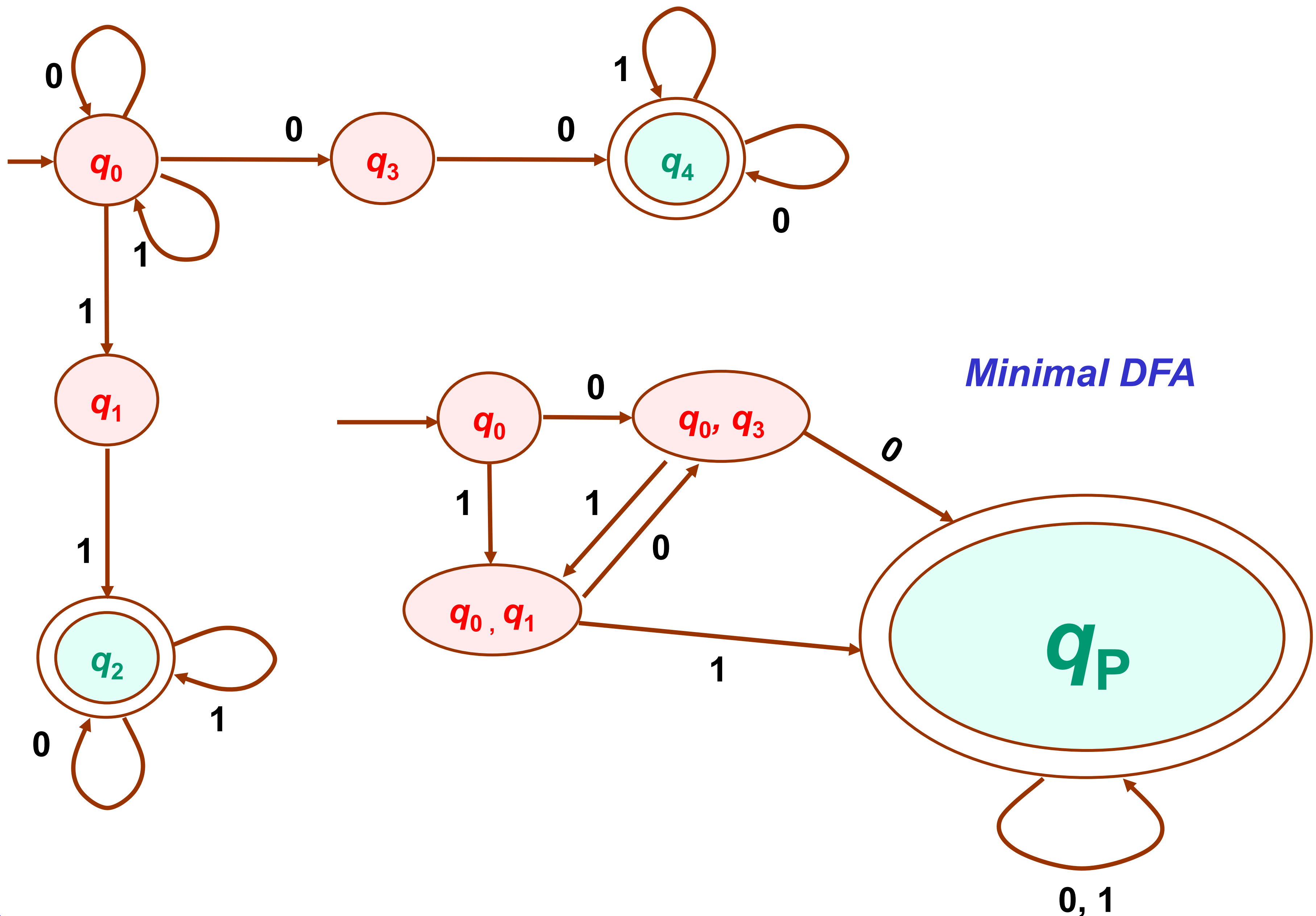
Constructing DFA equivalent to the given NFA



Constructing DFA equivalent to the given NFA



Constructing DFA equivalent to the given NFA



Constructing DFA equivalent to the given NFA

NKA $M=(Q, \Sigma, \delta, q_0, F)$

DKA $M'=(Q', \Sigma, \delta', q_0', F')$

$Q=\{q_0, q_1, q_2, \dots, q_i\}$

$Q'=\{[\varnothing], [q_0], [q_1], \dots, [q_i], [q_0, q_1], \dots, [q_{i-1}, q_i], [q_0, q_1, q_2], \dots, [q_0, q_1, q_2, \dots, q_i]\}$

$\delta(q_0, w)=\{p_0, p_1, \dots, p_j\}$

$\delta'([q_0], w)=[p_0, p_1, \dots, p_j]$

---

1) $Q' = 2^Q$

2) F' is the set of all states $[p_0, p_1, \dots, p_j]$ where at least one $p_k \in F$

3) $q_0' = [q_0]$

4) $\delta'([p_0, p_1, \dots, p_l], a) = [r_0, r_1, \dots, r_j]$ if and only if
$\delta(\{p_0, p_1, \dots, p_l\}, a) = \{r_0, r_1, \dots, r_j\}$

Constructing DFA equivalent to the NFA

	0	1	
$q0$	$\{q0, q1\}$	$\{q1\}$	0
$q1$	{ }	$\{q0, q1\}$	1

	0	1	
$[q0]$			0
$[q1]$			1
$[q0, q1]$			1
$[\varnothing]$			0

1) $Q' = \{ [\varnothing], [q_0], [q_1], [q_0, q_1] \}$

2) $F' = \{ [q_1], [q_0, q_1] \}$

3) $q_0' = [q_0]$

Constructing DFA equivalent to the NFA

	0	1	
$q0$	$\{q0, q1\}$	$\{q1\}$	0
$q1$	$\{ \}$	$\{q0, q1\}$	1

	0	1	
$[q0]$	$[q0, q1]$	$[q1]$	0
$[q1]$			1
$[q0, q1]$			1
$[\varnothing]$			0

4) $\delta'([q_0], 0) = [q_0, q_1]$ because $\delta(q_0, 0) = \{q_0, q_1\}$
$\delta'([q_0], 1) = [q_1]$ because $\delta(q_0, 1) = \{q_1\}$

Constructing DFA equivalent to the NFA

	0	1	
*q*0	{*q*0, *q*1}	{*q*1}	0
*q*1	{ }	{*q*0, *q*1}	1

	0	1	
[*q*0]	[*q*0, *q*1]	[*q*1]	0
[*q*1]	[∅]	[*q*0, *q*1]	1
[*q*0, *q*1]	[*q*0, *q*1]	[*q*0, *q*1]	1
[∅]	[∅]	[∅]	0

4)

$\delta'([q_0], 0) = [q_0, q_1]$ because $\delta(q_0, 0) = \{q_0, q_1\}$

$\delta'([q_0], 1) = [q_1]$ because $\delta(q_0, 1) = \{q_1\}$

$\delta'([q_1], 0) = [\varnothing]$ because $\delta(q_1, 0) = \{\ \}$

$\delta'([q_1], 1) = [q_0, q_1]$ because $\delta(q_1, 1) = \{q_0, q_1\}$

$\delta'([q_0, q_1], 0) = [q_0, q_1]$ because $\delta(\{q_0, q_1\}, 0) = \delta(q_0, 0) \cup \delta(q_1, 0)$
$= \{q_0, q_1\} \cup \{\ \} = \{q_0, q_1\}$

$\delta'([q_0, q_1], 1) = [q_0, q_1]$ because $\delta(\{q_0, q_1\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1)$
$= \{q_1\} \cup \{q_0, q_1\} = \{q_0, q_1\}$

$\delta'([\varnothing], 0) = [\varnothing]$ because $\delta(\{\}, 0) = \{\}$

$\delta'([\varnothing], 1) = [\varnothing]$ because $\delta(\{\}, 1) = \{\}$

Equivalence of DFA and NFA

NFA $M=(Q, \Sigma, \delta, q_0, F)$ DFA $M'=(Q', \Sigma, \delta', q_0', F')$

(*i*) $\delta'([q_0], w)=[r_0, r_1, \dots , r_j]$ if and only if $\delta(q_0, w)=\{r_0, r_1, \dots , r_j\}$

a) $|w|=0$, i.e. $w = \varepsilon$

$$\delta'([q_0], \varepsilon)=[q_0] \qquad \delta(q_0, \varepsilon)=\{q_0\}$$

b) We assume that (*i*) is valid for string $x \in \Sigma^*$,
and then we prove that (*i*) is valid for string $w=xa$, $a \in \Sigma$

According to the assumption that:

$\delta'([q_0], x)=[p_0, p_1, \dots , p_l]$ if and only if $\delta(q_0, x)=\{p_0, p_1, \dots , p_l\}$,

and based on definition (4) for the construction of the function δ' :

$\delta'([p_0, p_1, \dots , p_l], a) = [r_0, r_1, \dots , r_j]$ if and only if
$\delta(\{p_0, p_1, \dots , p_l\}, a) = \{r_0, r_1, \dots, r_j\}$,

we conclude that (*i*) is valid

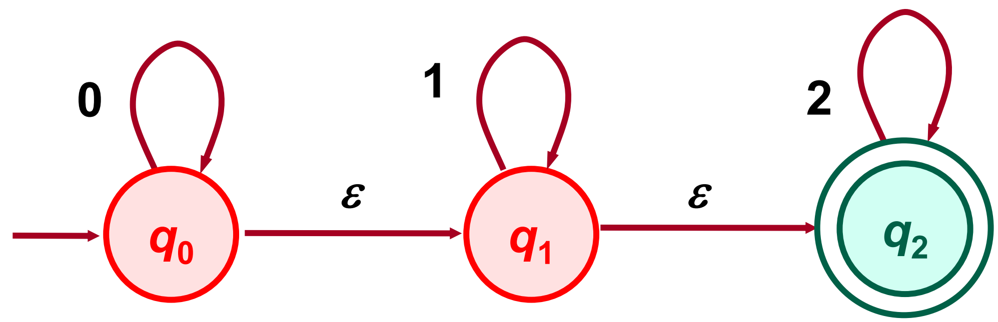
Lecture overview

2.1.3 Nondeterministic finite automaton (NFA)

2.1.4 Nondeterministic finite automaton with ε-moves (ε-NFA)

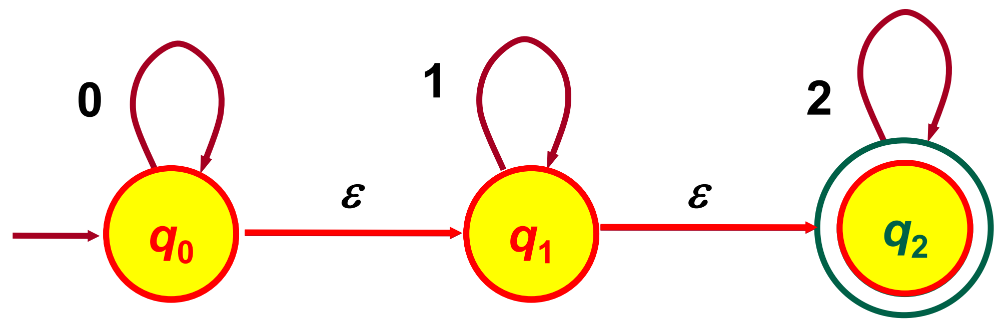
Nondeterministic finite automaton with ε moves (ε–NFA)

$L = \{0^n 1^m 2^l \mid n,m,l \geq 0\}$



Nondeterministic finite automaton with ε moves (ε–NFA)

$L = \{0^n 1^m 2^l \mid n,m,l \geq 0\}$

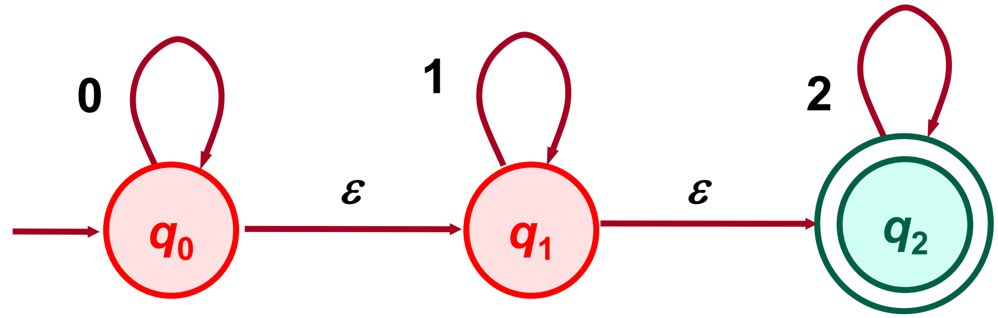


Empty string ε

$$q_0 \xrightarrow{\varepsilon} q_1 \xrightarrow{\varepsilon} q_2 \qquad q_2 \in F$$

Nondeterministic finite automaton with ε moves (ε–NFA)

$$L = \{0^n 1^m 2^l \mid n,m,l \geq 0\}$$



Empty string ε

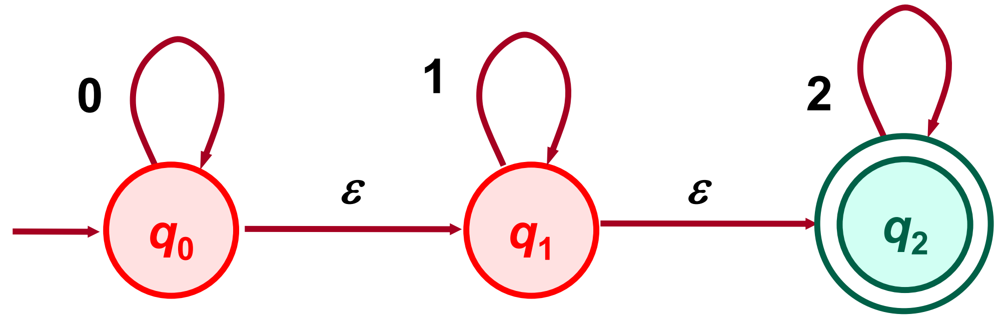
$$q_0 \xrightarrow{\varepsilon} q_1 \xrightarrow{\varepsilon} q_2 \qquad q_2 \in F$$

String 002

$$q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{\varepsilon} q_1 \xrightarrow{\varepsilon} q_2 \xrightarrow{2} q_2 \qquad q_2 \in F$$

Nondeterministic finite automaton with ε moves (ε–NFA)

$L = \{0^n1^m2^l \mid n,m,l \geq 0\}$



Empty string ε

$$q_0 \xrightarrow{\varepsilon} q_1 \xrightarrow{\varepsilon} q_2 \qquad q_2 \in F$$

String 002

$$q_0 \xrightarrow{0} q_0 \xrightarrow{0} q_0 \xrightarrow{\varepsilon} q_1 \xrightarrow{\varepsilon} q_2 \xrightarrow{2} q_2 \qquad q_2 \in F$$

String 01210 is not accepted

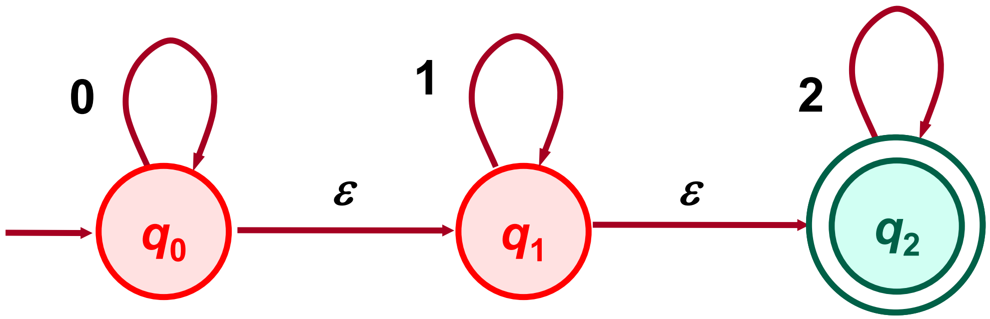
$$q_0 \xrightarrow{0} q_0 \xrightarrow{\varepsilon} q_1 \xrightarrow{1} q_1 \xrightarrow{\varepsilon} q_2 \xrightarrow{2} q_2 \xrightarrow{1}$$

Nondeterministic finite automaton with ε moves (ε–NFA)

$$\varepsilon\text{-}nfa = (Q, \Sigma, \delta, q_0, F)$$

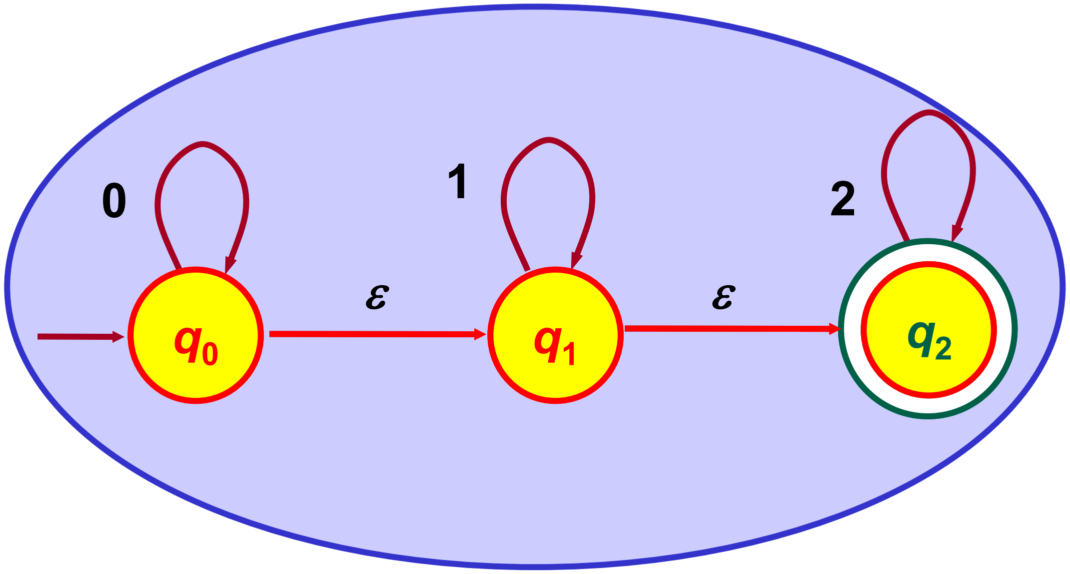
Q	- finite set of states
Σ	- finite set of input symbols
δ	- transition function $Q \times (\Sigma \cup \{\varepsilon\}) \rightarrow 2^Q$
$q_0 \in Q$	- start state
$F \subseteq Q$	- set of accept states

Nondeterministic finite automaton with ε moves (ε–NFA)



	0	1	2	ε	
$q0$	$\{q0\}$	$\emptyset$	$\emptyset$	$\{q1\}$	0
$q1$	$\emptyset$	$\{q1\}$	$\emptyset$	$\{q2\}$	0
$q2$	$\emptyset$	$\emptyset$	$\{q2\}$	$\emptyset$	1

Nondeterministic finite automaton with ε moves (ε–NFA)

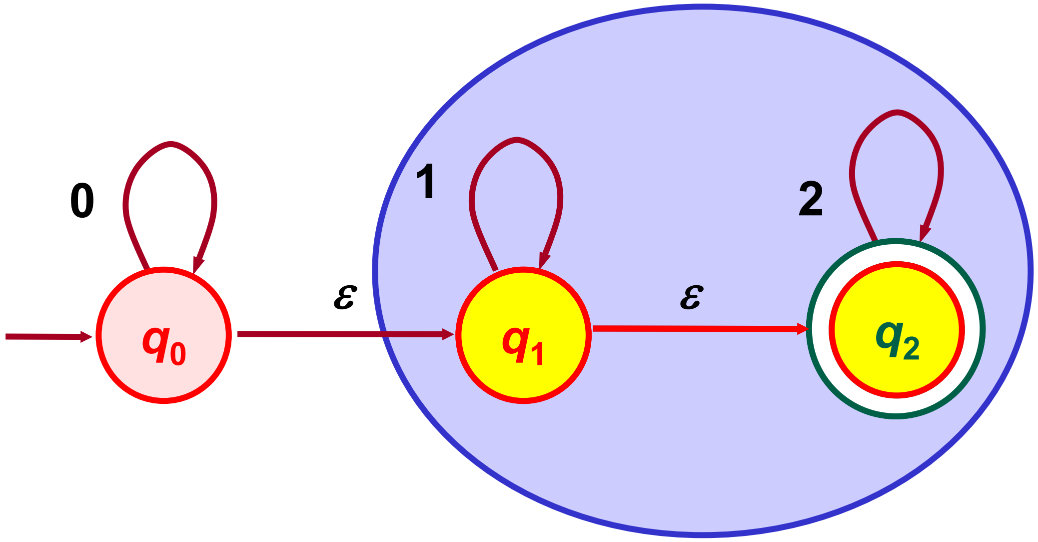


	0	1	2	ε	
$q0$	{$q0$}	∅	∅	{$q1$}	0
$q1$	∅	{$q1$}	∅	{$q2$}	0
$q2$	∅	∅	{$q2$}	∅	1

ε-CLOSURE(q) = {p | state p is either q or ε-NFA makes transition from state q to state p using exclusively ε-transitions}

ε-CLOSURE(q_0) = {q_0, q_1, q_2}

Nondeterministic finite automaton with ε moves (ε–NFA)



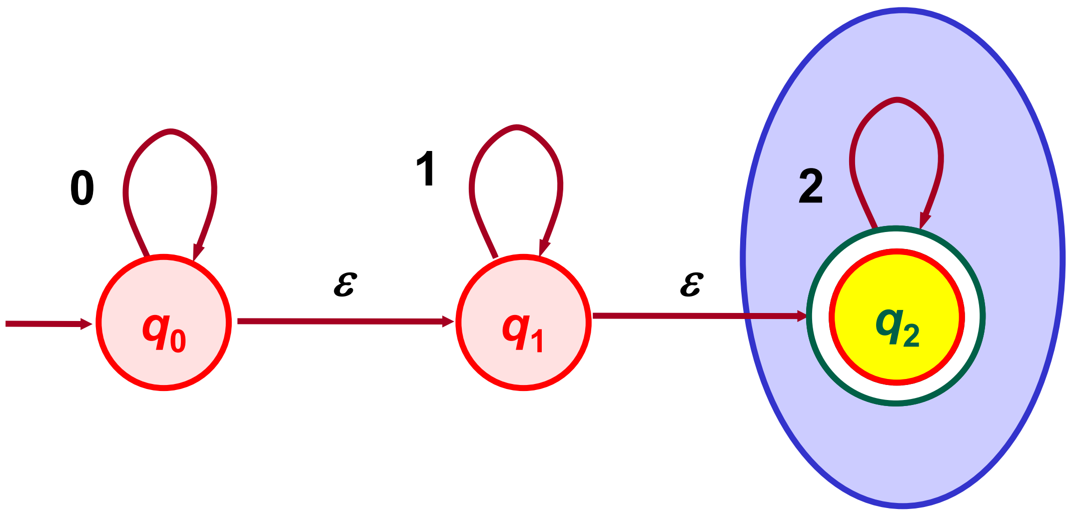
	0	1	2	ε	
$q0$	$\{q0\}$	$\varnothing$	$\varnothing$	$\{q1\}$	0
$q1$	$\varnothing$	$\{q1\}$	$\varnothing$	$\{q2\}$	0
$q2$	$\varnothing$	$\varnothing$	$\{q2\}$	$\varnothing$	1

ε-CLOSURE(q) = {p | state p is either q or ε-NFA makes transition from state q to state p using exclusively ε-transitions}

ε-CLOSURE$(q_0) = \{q_0, q_1, q_2\}$

ε-CLOSURE$(q_1) = \{q_1, q_2\}$

Nondeterministic finite automaton with ε moves (ε–NFA)



	0	1	2	ε	
$q0$	$\{q0\}$	$\varnothing$	$\varnothing$	$\{q1\}$	0
$q1$	$\varnothing$	$\{q1\}$	$\varnothing$	$\{q2\}$	0
$q2$	$\varnothing$	$\varnothing$	$\{q2\}$	$\varnothing$	1

ε-CLOSURE(q) = {p | state p is either q or ε-NFA makes transition from state q to state p using exclusively ε-transitions}

$$\varepsilon\text{-CLOSURE}(q_0) = \{q_0, q_1, q_2\}$$

$$\varepsilon\text{-CLOSURE}(q_1) = \{q_1, q_2\}$$

$$\varepsilon\text{-CLOSURE}(q_2) = \{q_2\}$$

Nondeterministic finite automaton with ε moves (ε–NFA)

$$\varepsilon\text{-CLOSURE}(P) = \bigcup_{q \in P} \varepsilon\text{-CLOSURE}(q)$$

(1) $\hat{\delta}(q, \varepsilon) = \varepsilon\text{-CLOSURE}\{q\}$

(2) $\hat{\delta}(q, wa) = \varepsilon\text{-CLOSURE}(P)$

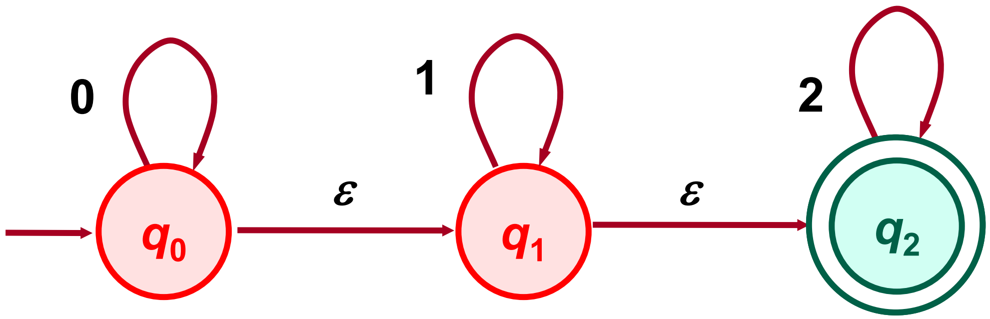
$P = \{ p \mid \text{for some state } r \text{ from } \hat{\delta}(q, w),\ p \text{ is in } \delta(r, a) \}$

$w \in \Sigma^*$, $a \in \Sigma$ and $P \subseteq Q$

$$\delta(R, a) = \bigcup_{q \in R} \delta(q, a)$$

$$\hat{\delta}(R, w) = \bigcup_{q \in R} \hat{\delta}(q, w),\ R \subseteq Q \text{ and } w \in \Sigma^*$$

Nondeterministic finite automaton with ε moves (ε–NFA)

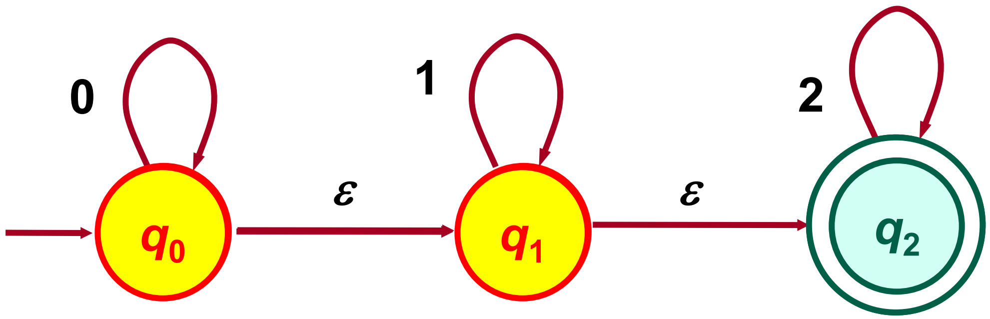


	0	1	2	ε	
q_0	$\{q_0\}$	$\varnothing$	$\varnothing$	$\{q_1\}$	0
q_1	$\varnothing$	$\{q_1\}$	$\varnothing$	$\{q_2\}$	0
q_2	$\varnothing$	$\varnothing$	$\{q_2\}$	$\varnothing$	1

$\delta(q_0, \varepsilon)$

$\hat{\delta}(q_0, \varepsilon)$

Nondeterministic finite automaton with ε moves (ε–NFA)

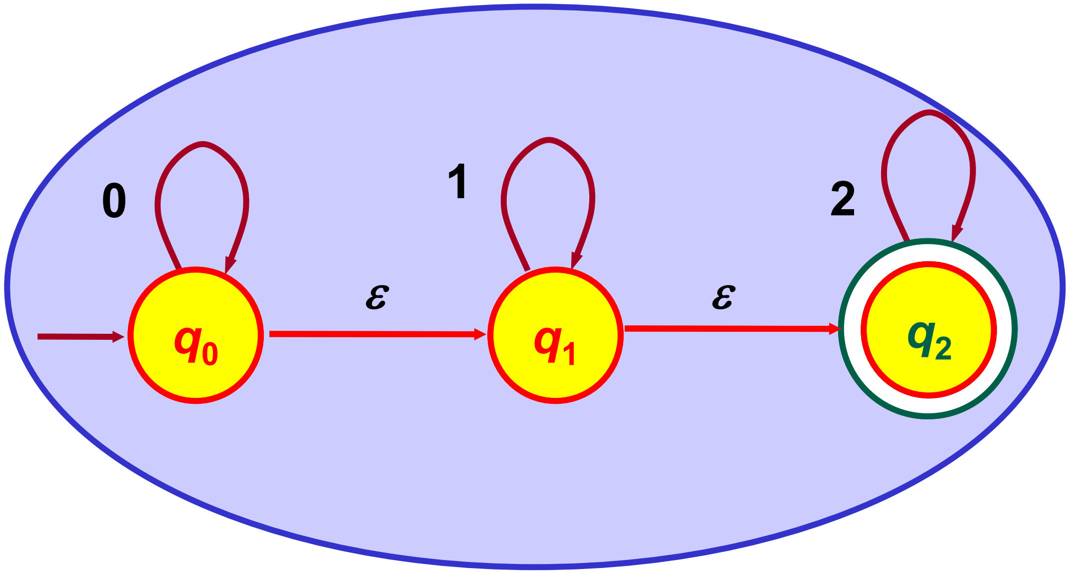


	0	1	2	ε	
q_0	$\{q_0\}$	$\emptyset$	$\emptyset$	$\{q_1\}$	0
q_1	$\emptyset$	$\{q_1\}$	$\emptyset$	$\{q_2\}$	0
q_2	$\emptyset$	$\emptyset$	$\{q_2\}$	$\emptyset$	1

$$\delta(q_0, \varepsilon) = \{q_1\}$$

$$\hat{\delta}(q_0, \varepsilon)$$

Nondeterministic finite automaton with ε moves (ε–NFA)



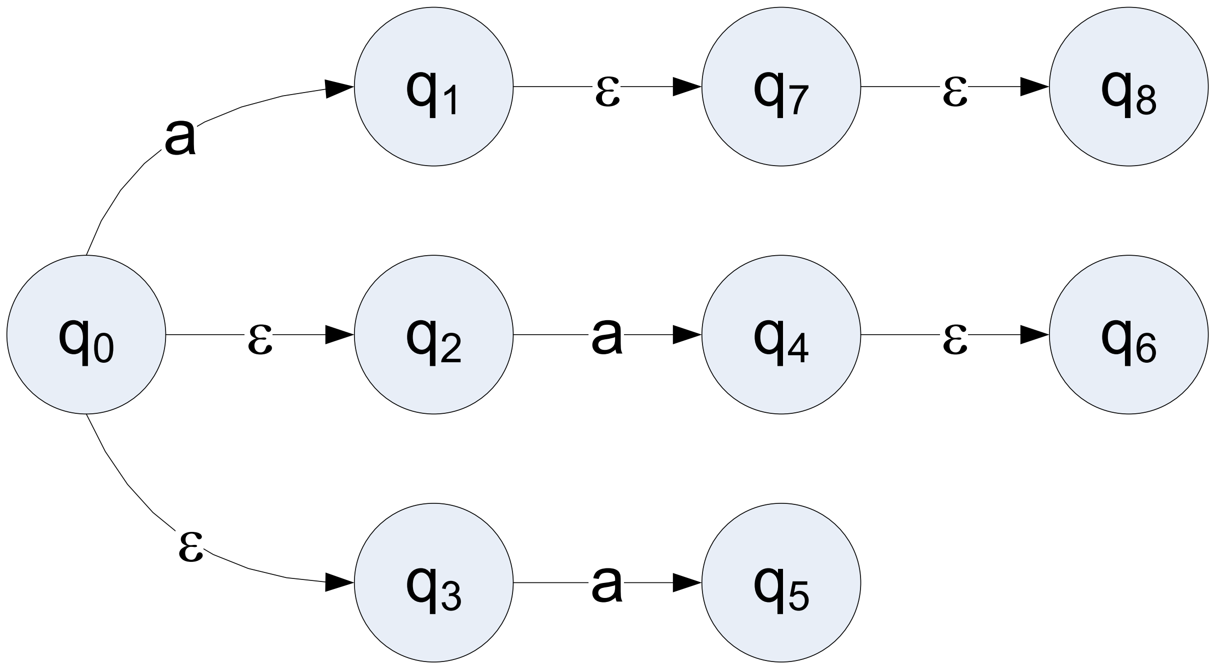
	0	1	2	ε	
q_0	$\{q_0\}$	$\varnothing$	$\varnothing$	$\{q_1\}$	0
q_1	$\varnothing$	$\{q_1\}$	$\varnothing$	$\{q_2\}$	0
q_2	$\varnothing$	$\varnothing$	$\{q_2\}$	$\varnothing$	1

$$\delta(q_0, \varepsilon) = \{q_1\}$$

$$\hat{\delta}(q_0, \varepsilon) = \varepsilon\text{-CLOSURE}(q_0) = \{q_0, q_1, q_2\}$$

Nondeterministic finite automaton with ε moves (ε–NFA)

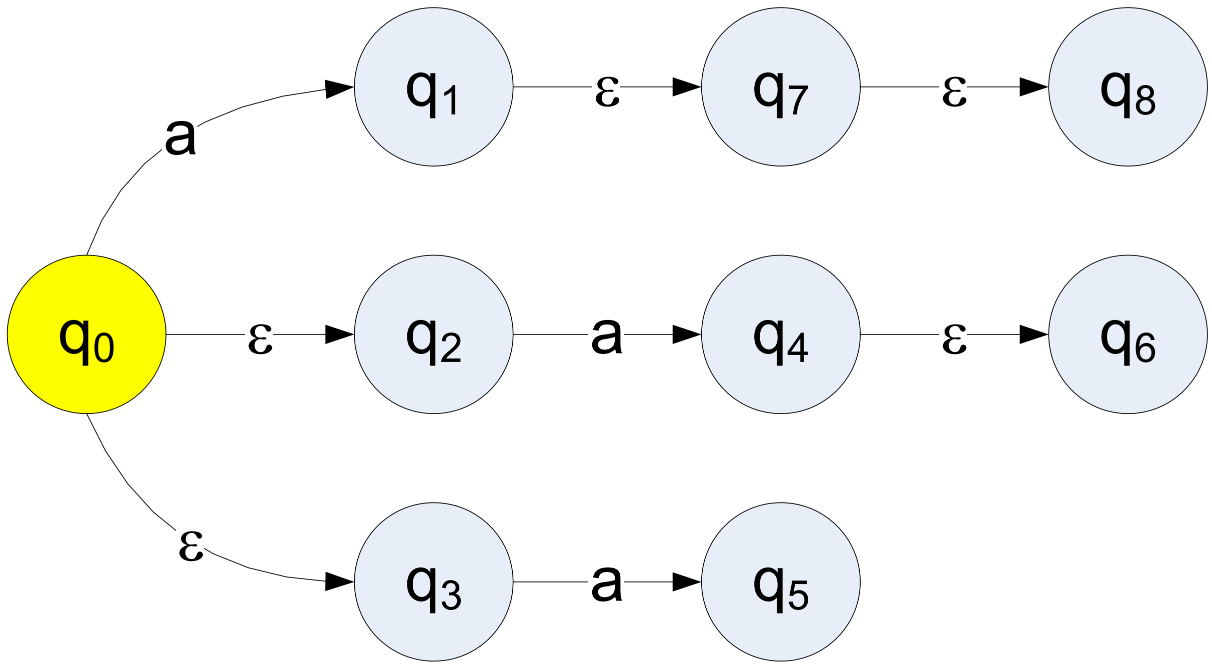
Determinig transition function $\hat{\delta}$ using transition function δ



$\delta(q_0, a) = q_1$

Nondeterministic finite automaton with ε moves (ε–NFA)

Determinig transition function $\hat{\delta}$ using transition function δ

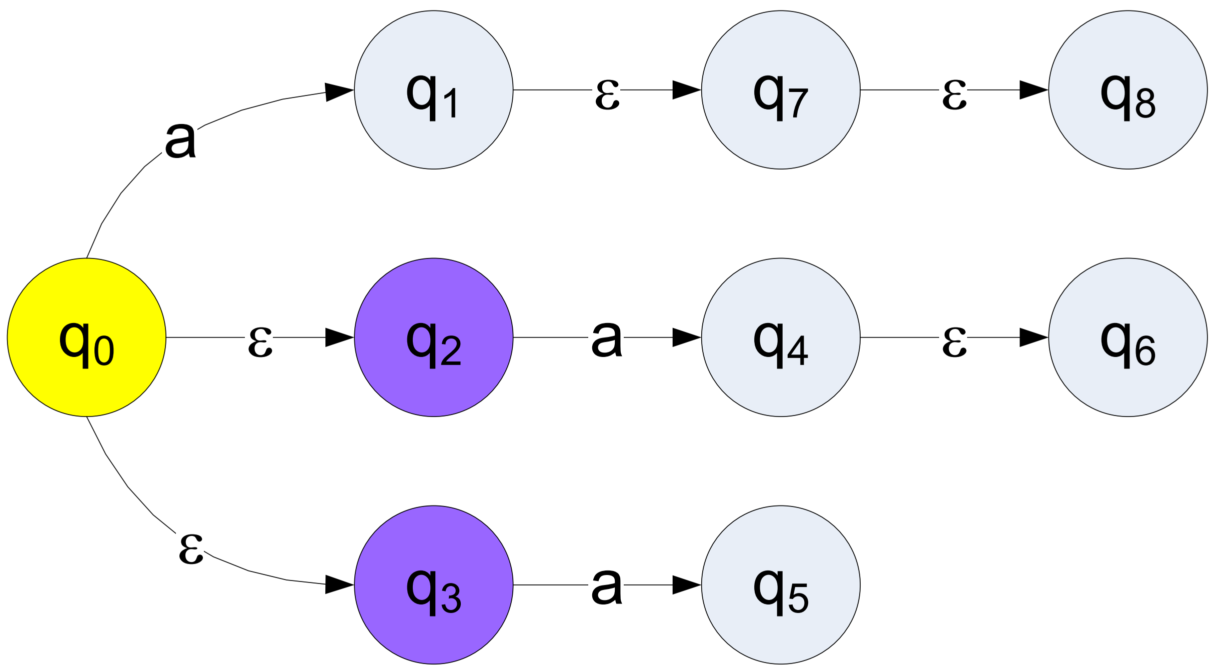


$\delta(q_0, a) = q_1$

$\hat{\delta}$ **(q_0, a)=** q_0

Nondeterministic finite automaton with ε moves (ε–NFA)

Determinig transition function $\hat{\delta}$ using transition function δ

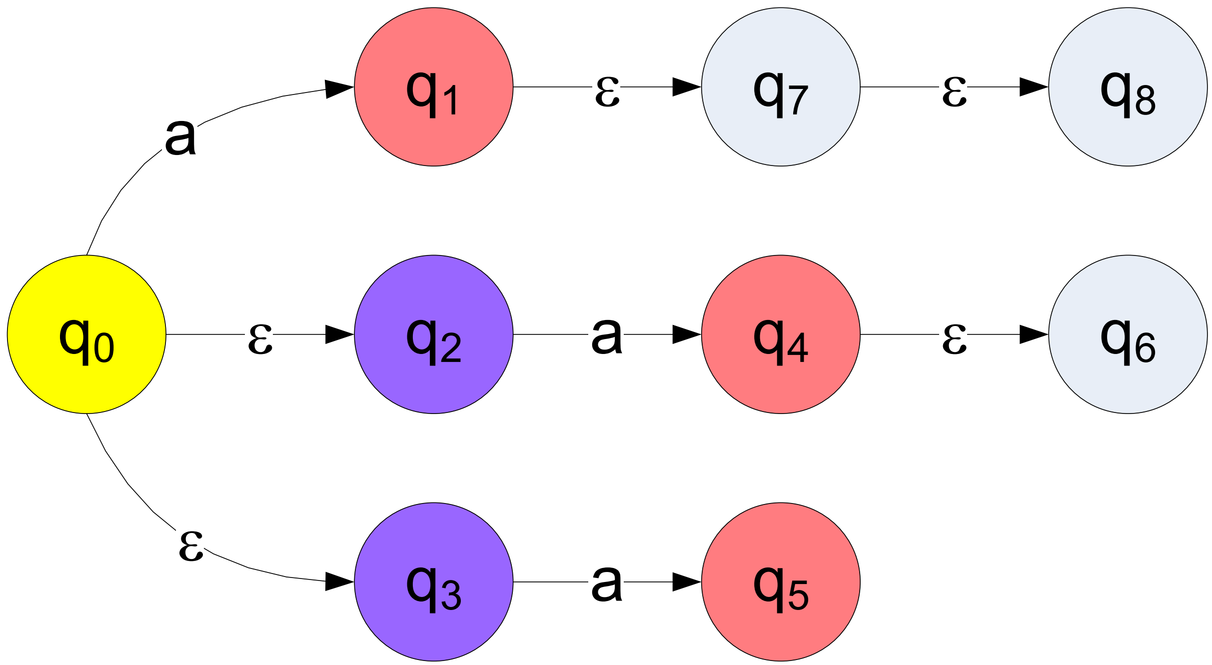


$\delta(q_0, a) = q_1$

$\hat{\delta}(q_0, a) =$ ε-CLOSURE(q_0)

Nondeterministic finite automaton with ε moves (ε–NFA)

Determinig transition function $\hat{\delta}$ using transition function δ

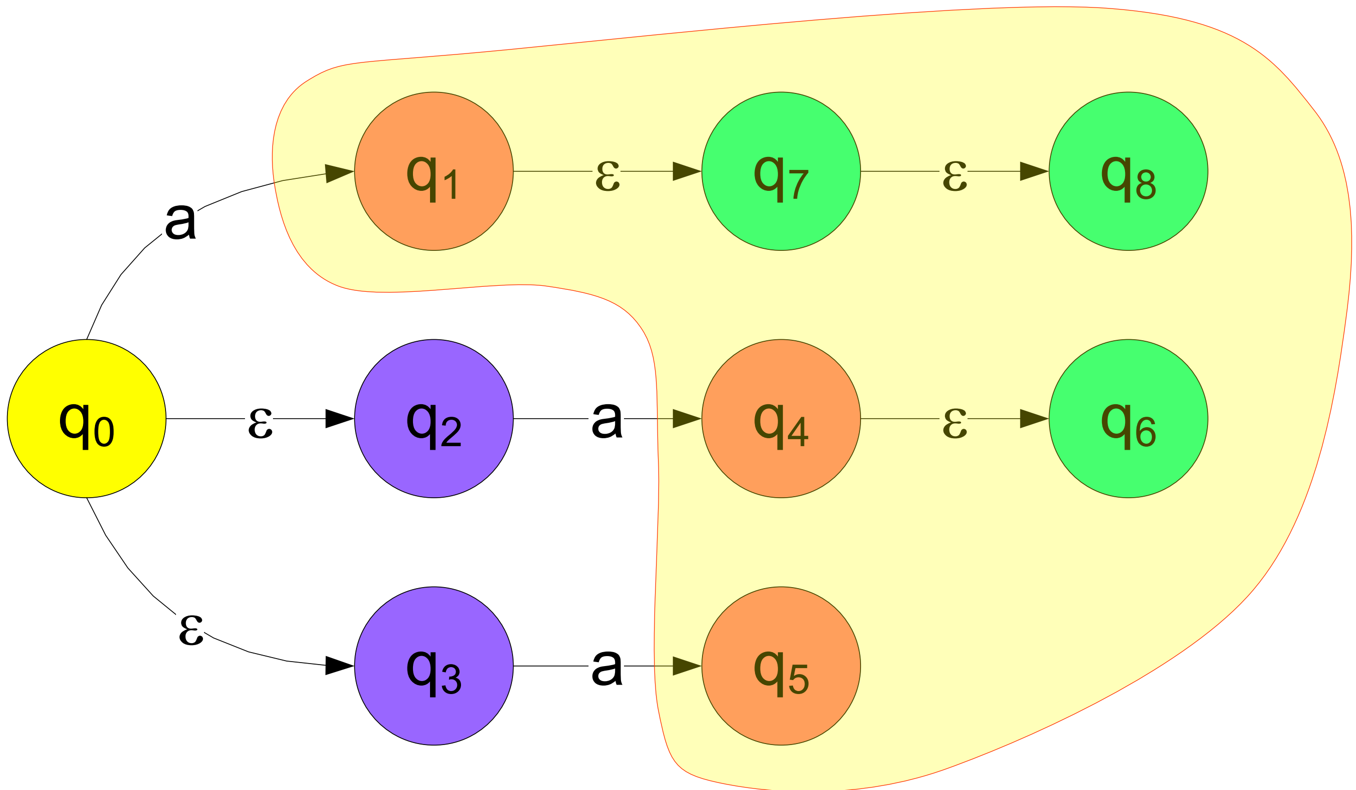


$\delta(q_0, a) = q_1$

$\hat{\delta}$ **$(q_0$, a$)$=** $\delta($ ε-CLOSURE(q_0) , a)

Nondeterministic finite automaton with ε moves (ε–NFA)

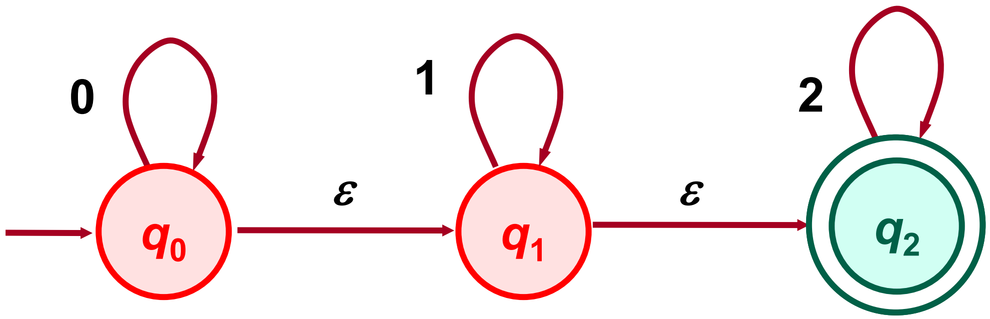
Determinig transition function $\hat{\delta}$ using transition function δ



$\delta(q_0, a) = q_1$

$\hat{\delta}$ **(q_0, a)=** ε-CLOSURE (δ (ε-CLOSURE(q_0) , a))

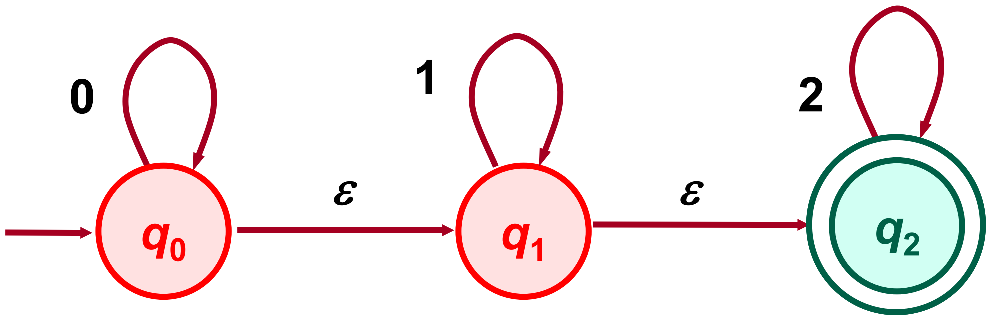
Nondeterministic finite automaton with ε moves (ε–NFA)



	0	1	2	ε	
q_0	$\{q_0\}$	$\varnothing$	$\varnothing$	$\{q_1\}$	0
q_1	$\varnothing$	$\{q_1\}$	$\varnothing$	$\{q_2\}$	0
q_2	$\varnothing$	$\varnothing$	$\{q_2\}$	$\varnothing$	1

$\delta(q_0, 1) = \varnothing = \{\ \}$

Nondeterministic finite automaton with ε moves (ε–NFA)



	0	1	2	ε	
q_0	$\{q_0\}$	$\varnothing$	$\varnothing$	$\{q_1\}$	0
q_1	$\varnothing$	$\{q_1\}$	$\varnothing$	$\{q_2\}$	0
q_2	$\varnothing$	$\varnothing$	$\{q_2\}$	$\varnothing$	1

$\delta(q_0, 1) = \varnothing = \{\ \}$

$\hat{\delta}(q_0, 1) = \hat{\delta}(q_0, \varepsilon 1) = \varepsilon\text{-CLOSURE}(\delta(\hat{\delta}(q_0, \varepsilon), 1))$

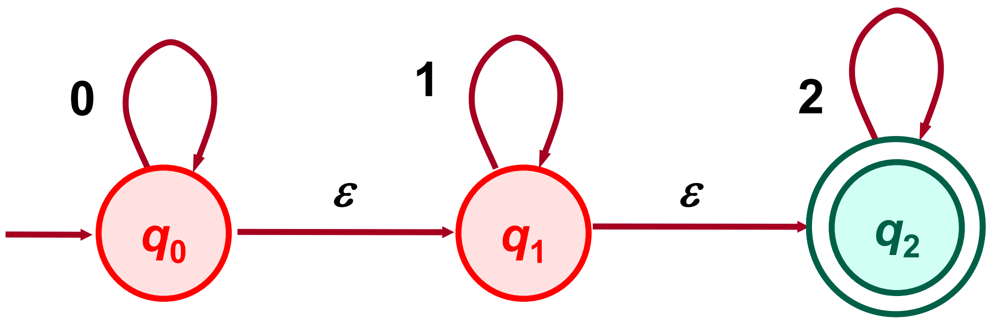
$= \varepsilon\text{-CLOSURE}(\delta(\{q_0, q_1, q_2\}, 1))$

$= \varepsilon\text{- CLOSURE}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1))$

$= \varepsilon\text{- CLOSURE}(\varnothing \cup \{q_1\} \cup \varnothing)$

$= \varepsilon\text{- CLOSURE}(\{q_1\})$

$= \varepsilon\text{- CLOSURE}(q_1) = \{q_1, q_2\}$



	0	1	2	ε	
q_0	$\{q_0\}$	$\varnothing$	$\varnothing$	$\{q_1\}$	0
q_1	$\varnothing$	$\{q_1\}$	$\varnothing$	$\{q_2\}$	0
q_2	$\varnothing$	$\varnothing$	$\{q_2\}$	$\varnothing$	1

String 01

$$\hat{\delta}(q_0, \varepsilon) = \varepsilon\text{-CLOSURE}(q_0) = \{q_0, q_1, q_2\}$$

$$\hat{\delta}(q_0, 0) = \hat{\delta}(q_0, \varepsilon 0) = \varepsilon\text{-CLOSURE}(\delta(\hat{\delta}(q_0, \varepsilon), 0))$$

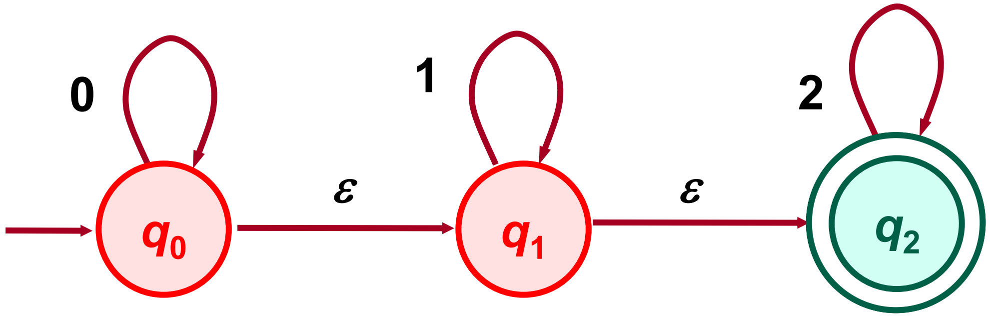
$$= \varepsilon\text{-CLOSURE}(\delta(\{q_0, q_1, q_2\}, 0))$$

$$= \varepsilon\text{-CLOSURE}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$$

$$= \varepsilon\text{-CLOSURE}(\{q_0\} \cup \varnothing \cup \varnothing)$$

$$= \varepsilon\text{-CLOSURE}(\{q_0\})$$

$$= \{q_0, q_1, q_2\}$$



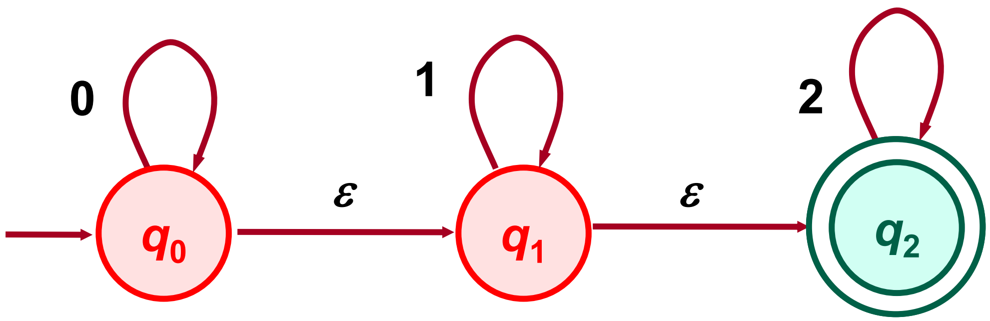
	0	1	2	ε	
q_0	$\{q_0\}$	$\emptyset$	$\emptyset$	$\{q_1\}$	0
q_1	$\emptyset$	$\{q_1\}$	$\emptyset$	$\{q_2\}$	0
q_2	$\emptyset$	$\emptyset$	$\{q_2\}$	$\emptyset$	1

String 01

$$\hat{\delta}(q_0, \varepsilon) = \varepsilon\text{-CLOSURE}(q_0) = \{q_0, q_1, q_2\}$$

$$\hat{\delta}(q_0, 0) = \{q_0, q_1, q_2\}$$

$$\begin{aligned}
\hat{\delta}(q_0, 01) &= \varepsilon\text{-CLOSURE}(\delta(\hat{\delta}(q_0,0),1)) \\
&= \varepsilon\text{-CLOSURE}(\delta(\{q_0, q_1, q_2\}, 1)) \\
&= \varepsilon\text{-CLOSURE}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)) \\
&= \varepsilon\text{-CLOSURE}(\emptyset \cup \{q_1\} \cup \emptyset) \\
&= \varepsilon\text{-CLOSURE}(\{q_1\}) \\
&= \{q_1, q_2\}, \quad q_2 \in F \quad \Rightarrow \quad \underline{\text{string 01 is accepted}}
\end{aligned}$$



	0	1	2	ε	
q_0	$\{q_0\}$	$\emptyset$	$\emptyset$	$\{q_1\}$	0
q_1	$\emptyset$	$\{q_1\}$	$\emptyset$	$\{q_2\}$	0
q_2	$\emptyset$	$\emptyset$	$\{q_2\}$	$\emptyset$	1

String 10

$$\hat{\delta}(q_0, \varepsilon) = \varepsilon\text{-CLOSURE}(q_0) = \{q_0, q_1, q_2\}$$

$$\begin{aligned}
\hat{\delta}(q_0, 1) = \hat{\delta}(q_0, \varepsilon 1) &= \varepsilon\text{-CLOSURE}(\delta(\hat{\delta}(q_0, \varepsilon), 1)) \\
&= \varepsilon\text{-CLOSURE}(\delta(\{q_0, q_1, q_2\}, 1)) \\
&= \varepsilon\text{-CLOSURE}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)) \\
&= \varepsilon\text{-CLOSURE}(\emptyset \cup \{q_1\} \cup \emptyset) \\
&= \varepsilon\text{-CLOSURE}(\{q_1\}) \\
&= \{q_1, q_2\}
\end{aligned}$$

$$\begin{aligned}
\hat{\delta}(q_0, 10) &= \varepsilon\text{-CLOSURE}(\delta(\hat{\delta}(q_0, 1), 0)) \\
&= \varepsilon\text{-CLOSURE}(\delta(\{q_1, q_2\}, 0)) \\
&= \varepsilon\text{-CLOSURE}(\delta(q_1, 0) \cup \delta(q_2, 0)) \\
&= \varepsilon\text{-CLOSURE}(\emptyset \cup \emptyset) \\
&= \varepsilon\text{-CLOSURE}(\emptyset) = \emptyset, \Rightarrow
\end{aligned}$$

string 10 is NOT accepted

Constructing NFA for the given ε-NFA

ε-NFA $M=(Q, \Sigma, \delta, q_0, F)$

NFA $M'=(Q', \Sigma, \delta', q_0', F')$

1) $Q' = Q$

2) $q_0' = q_0$

3) $\delta'(q, a) = \hat{\delta}(q, a), \forall a \in \Sigma$ and $\forall q \in Q$

4) If ε-CLOSURE(q_0) contains at least one state from F

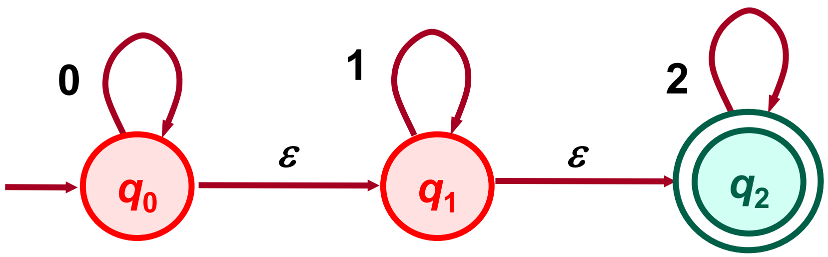
then

$F' = F \cup \{q_0\}$

else

$F' = F$

Constructing NFA for the given ε-NFA



	0	1	2	
$q0$				1
$q1$				0
$q2$				1

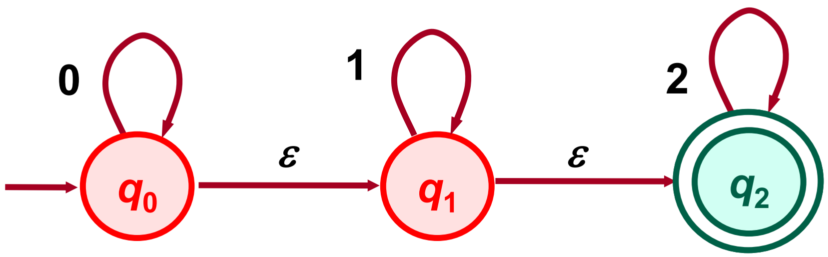
1) $Q' = Q = \{q_0, q_1, q_2\}$

2) $q_0' = q_0$

3) $F' = F \cup \{q_0\} = \{q_2\} \cup \{q_0\} = \{q_0, q_2\}$

because ε-CLOSURE$(q_0) \cap F = \{q_0, q_1, q_2\} \cap \{q_2\} = \{q_2\}$

Constructing NFA for the given ε-NFA

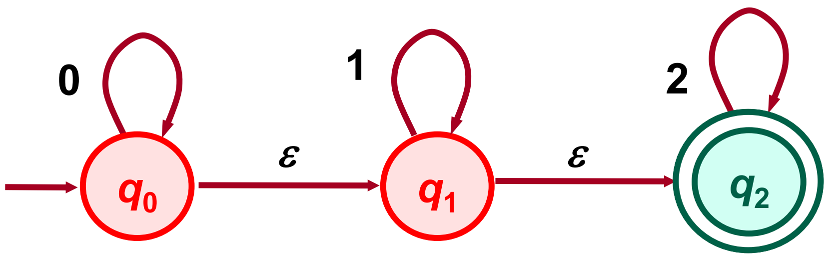


	0	1	2	
$q0$	$\{q0, q1, q2\}$			1
$q1$				0
$q2$				1

$\delta'(q_0, 0)=\{q_0, q_1, q_2\}$, **because**

$$\hat{\delta}(q_0, 0)= \varepsilon\text{-CLOSURE}(\delta(\hat{\delta}(q_0, \varepsilon), 0))$$
$$= \varepsilon\text{-CLOSURE}(\delta(\varepsilon\text{-CLOSURE}(q_0), 0))$$
$$=\varepsilon\text{-CLOSURE}(\delta(\{q_0, q_1, q_2\}, 0))$$
$$=\varepsilon\text{-CLOSURE}(\{q_0\} \cup \emptyset \cup \emptyset)$$
$$=\{q_0, q_1, q_2\}$$

Constructing NFA for the given ε-NFA



	0	1	2	
$q0$	$\{q0, q1, q2\}$	$\{q1, q2\}$		1
$q1$				0
$q2$				1

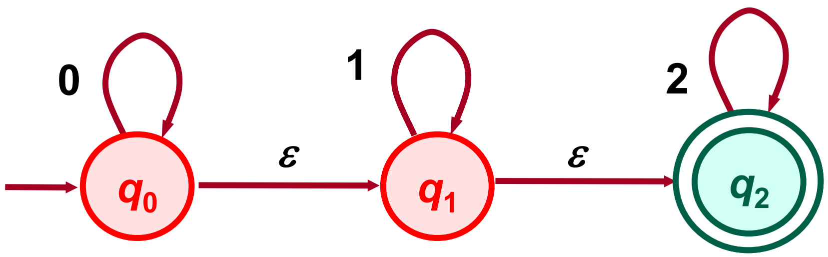
$\delta'(q_0, 0)=\{q_0, q_1, q_2\}$, because

$$\hat{\delta}(q_0, 0)= \varepsilon\text{-CLOSURE}(\delta(\hat{\delta}(q_0, \varepsilon), 0))$$
$$= \varepsilon\text{-CLOSURE}(\delta(\varepsilon\text{-CLOSURE}(q_0), 0))$$
$$=\varepsilon\text{-CLOSURE}(\delta(\{q_0, q_1, q_2\}, 0))$$
$$=\varepsilon\text{-CLOSURE}(\{q_0\} \cup \varnothing \cup \varnothing)$$
$$=\{q_0, q_1, q_2\}$$

$\delta'(q_0, 1)=\{q_1, q_2\}$, because

$$\hat{\delta}(q_0, 1)= \varepsilon\text{-CLOSURE}(\delta(\hat{\delta}(q_0, \varepsilon), 1))$$
$$= \varepsilon\text{-CLOSURE}(\delta(\varepsilon\text{-CLOSURE}(q_0), 1))$$
$$=\varepsilon\text{-CLOSURE}(\delta(\{q_0, q_1, q_2\}, 1))$$
$$=\varepsilon\text{-CLOSURE}(\varnothing \cup \{q_1\} \cup \varnothing)$$
$$=\{q_1, q_2\}$$

Constructing NFA for the given ε-NFA



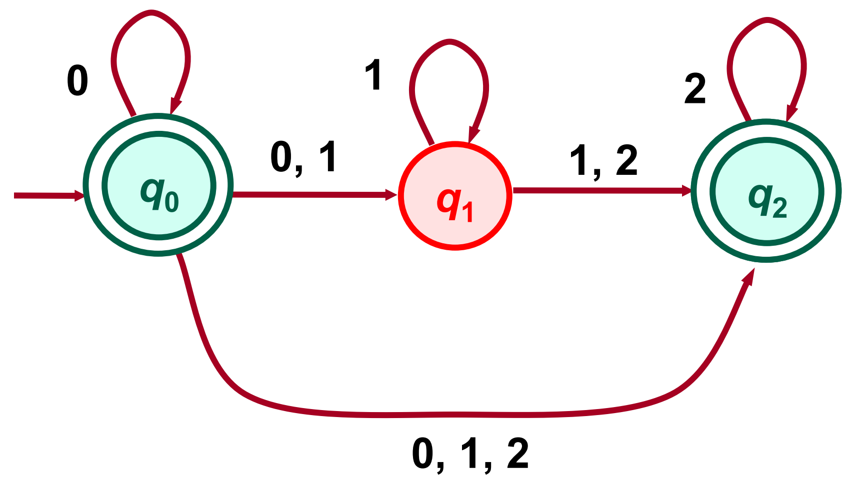
	0	1	2	
$q0$	$\{q0, q1, q2\}$	$\{q1, q2\}$	$\{q2\}$	1
$q1$	$\varnothing$	$\{q1, q2\}$	$\{q2\}$	0
$q2$	$\varnothing$	$\varnothing$	$\{q2\}$	1

$\delta'(q_0, 0)=\{q_0, q_1, q_2\}$, because

$$\hat{\delta}(q_0, 0)= \varepsilon\text{-CLOSURE}(\delta(\hat{\delta}(q_0, \varepsilon), 0))$$
$$= \varepsilon\text{-CLOSURE}(\delta(\varepsilon\text{-CLOSURE}(q_0), 0))$$
$$=\varepsilon\text{-CLOSURE}(\delta(\{q_0, q_1, q_2\}, 0))$$
$$=\varepsilon\text{-CLOSURE}(\{q_0\} \cup \varnothing \cup \varnothing)$$
$$=\{q_0, q_1, q_2\}$$

$\delta'(q_0, 1)=\{q_1, q_2\}$, because

$$\hat{\delta}(q_0, 1)= \varepsilon\text{-CLOSURE}(\delta(\hat{\delta}(q_0, \varepsilon), 1))$$
$$= \varepsilon\text{-CLOSURE}(\delta(\varepsilon\text{-CLOSURE}(q_0), 1))$$
$$=\varepsilon\text{-CLOSURE}(\delta(\{q_0, q_1, q_2\}, 1))$$
$$=\varepsilon\text{-CLOSURE}(\varnothing \cup \{q_1\} \cup \varnothing)$$
$$=\{q_1, q_2\}$$



Equivalence of NFA with ε–NFA

$$\delta'(q_0, \varepsilon) \neq \hat{\delta}(q_0, \varepsilon)$$

a) $|x|=1,\ \delta'(q_0, a) = \hat{\delta}(q_0, a)$ - follows from the NFA construction

b) We assume the induction hypothesis:

$$P = \delta'(q_0, w) = \hat{\delta}(q_0, w)$$

According to the the definition of the function δ' of the NKA:

$$\delta'(q_0, wa) = \delta'(\delta'(q_0, w), a) = \delta'(P, a) = \bigcup_{q \in P} \delta'(q, a) = \bigcup_{q \in P} \hat{\delta}(q, a)$$

$$= \hat{\delta}(P, a) = \hat{\delta}(\hat{\delta}(q_0, w), a) = \hat{\delta}(q_0, wa)$$

Equivalence of NFA with ε–NFA

We prove:

$$q_0 \in \delta'(q_0, x) \quad \Rightarrow \quad \varepsilon\text{-CLOSURE}(q_0) \subseteq \hat{\delta}(q_0, x)$$

Empty string $x = \varepsilon$

$$\delta'(q_0, \varepsilon) = q_0$$

$$\hat{\delta}(q_0, \varepsilon) = \varepsilon\text{-CLOSURE}(q_0)$$

String $x=wa$

$$\hat{\delta}(q_0, x) = \varepsilon\text{-CLOSURE}(\delta(\hat{\delta}(q_0, w), a))$$