Lecture 2

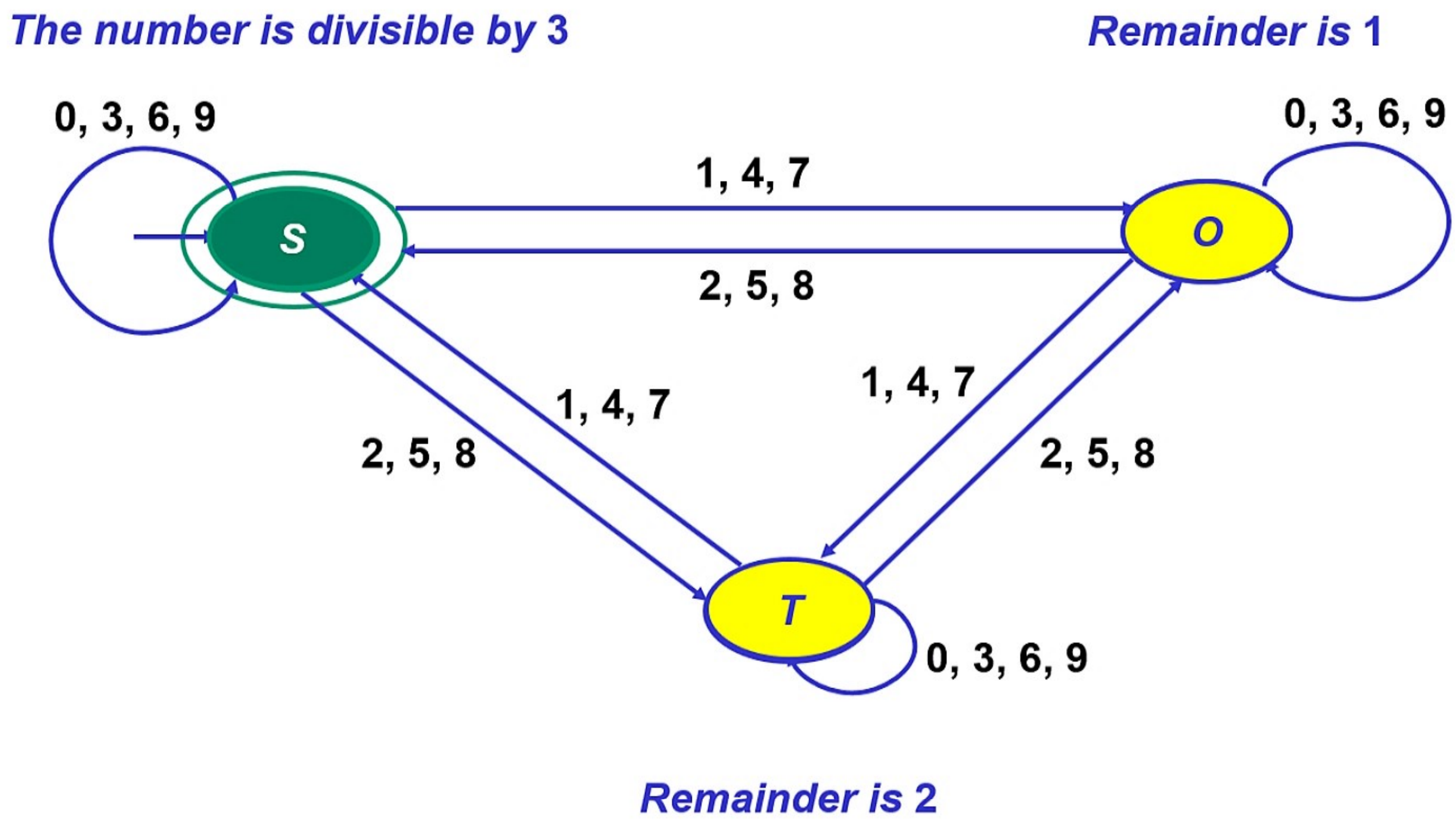
2 REGULAR LANGUAGES

2.1 FINITE AUTOMATA

2.1.1 Deterministic finite automaton (DFA)

2.1.2 Minimization of deterministic finite automaton

Deterministic Finite Automaton (DFA)



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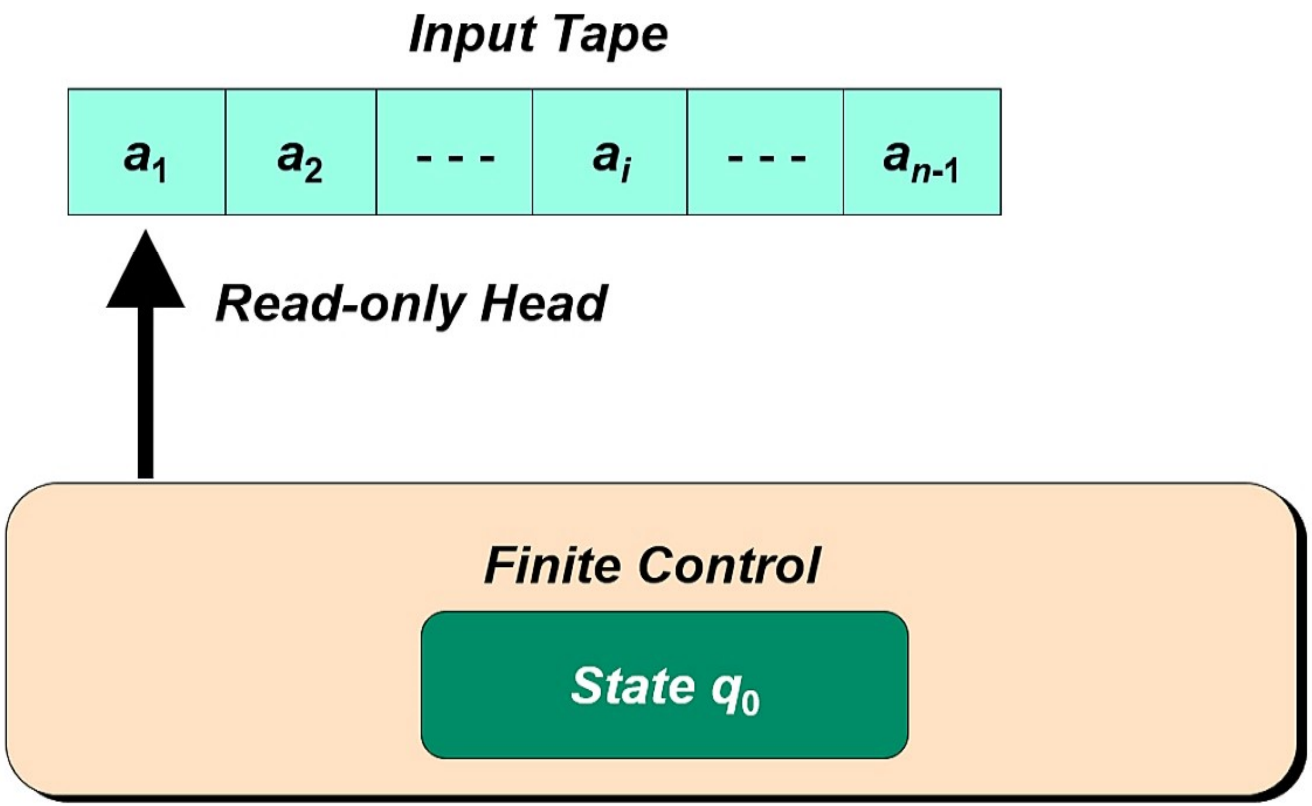
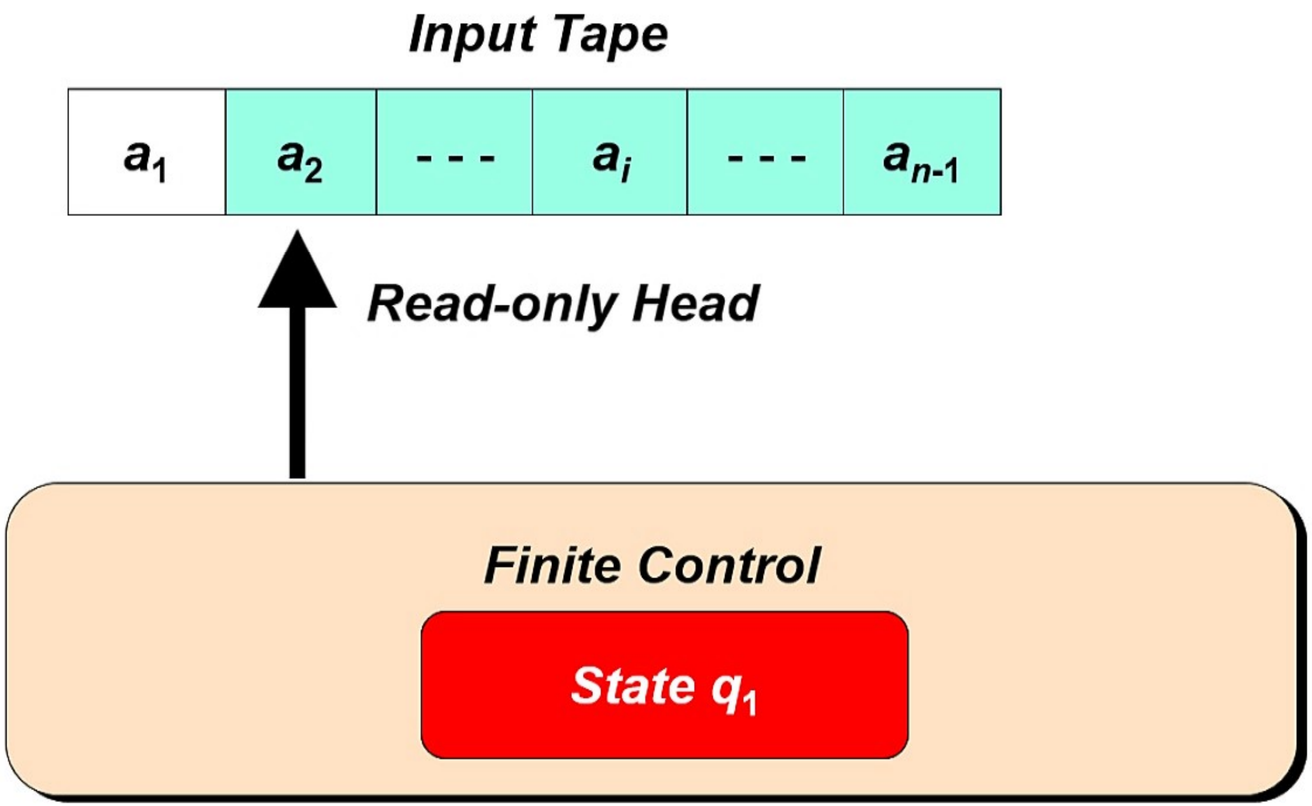
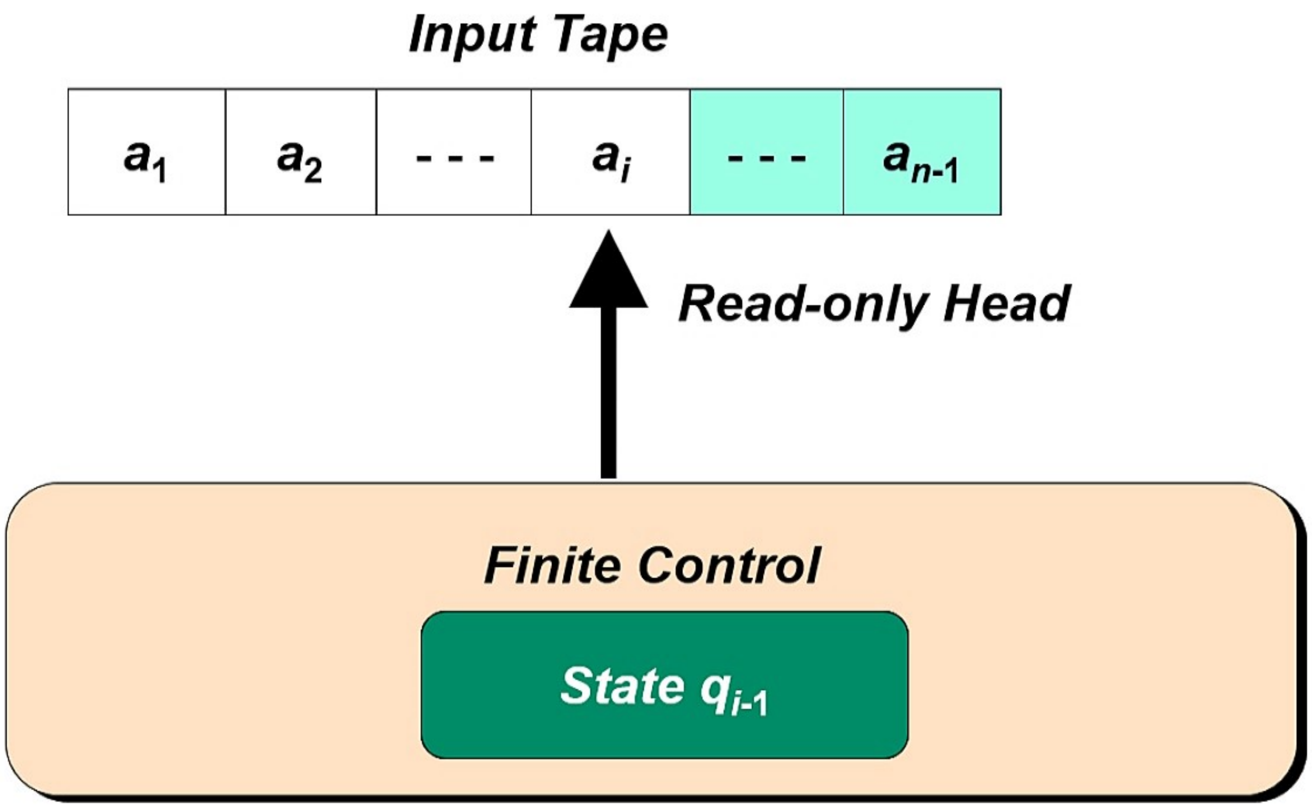
Deterministic Finite Automaton (DFA)



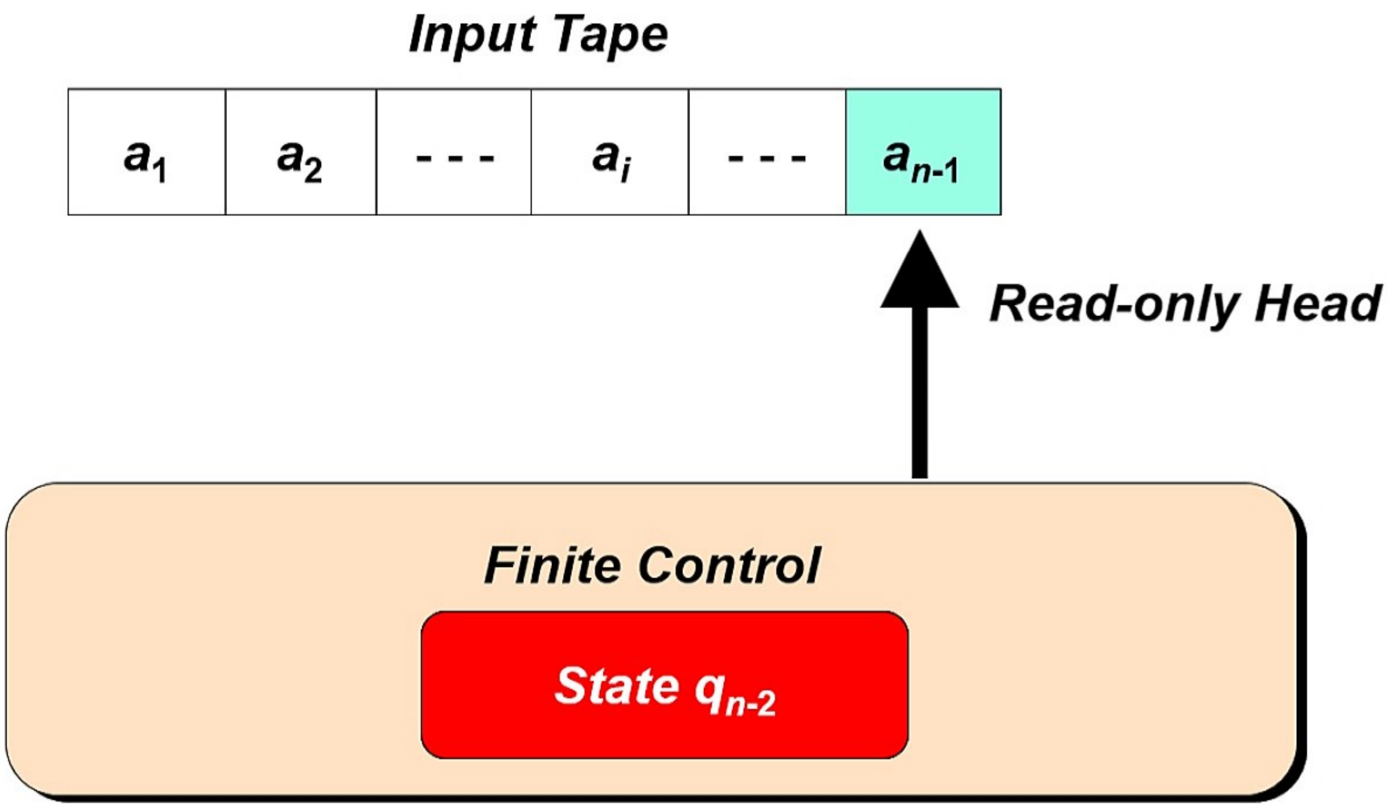
Deterministic Finite Automaton (DFA)



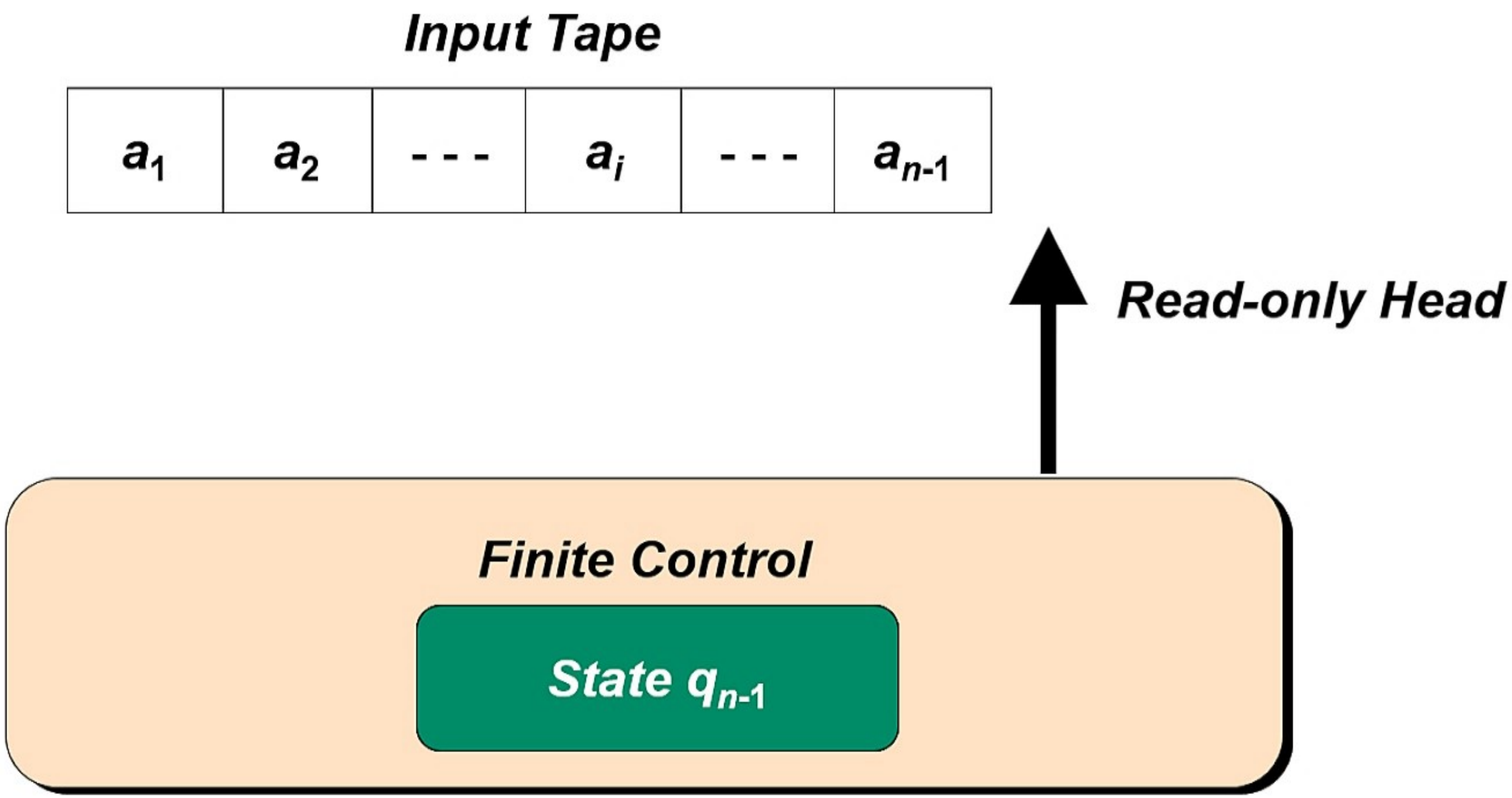
Deterministic Finite Automaton (DFA)



Deterministic Finite Automaton (DFA)



Deterministic Finite Automaton (DFA)



Deterministic Finite Automaton (DFA)

$$dfa = (Q, \Sigma, \delta, q_0, F)$$

Q	- a finite set of states
Σ	- a finite set of input symbols
δ	- a transition function $Q \times \Sigma \rightarrow Q$
$q_0 \in Q$	- an initial (start) state
$F \subseteq Q$	- a set of accepting states

$$NextState = \delta\,(CurrentState, InputSymbol)$$

Deterministic Finite Automaton (DFA)

$$\hat{\delta} : Q \times \Sigma^* \rightarrow Q$$

$$(1) \quad \hat{\delta}(q, \varepsilon) = q$$

$$(2) \quad \hat{\delta}(q, wa) = \delta(\hat{\delta}(q, w), a)$$

$$\hat{\delta}(q, a) = \delta(\hat{\delta}(q, \varepsilon), a) = \delta(q, a)$$

String and Language Acceptance (DFA)

$$DFA = (Q, \Sigma, \delta, q_0, F)$$

DFA **accepts the string** w **if** $\delta(q_0, w)=p$ **and** $p \in F$

DFA **accepts the language** $L(DFA)=\{w \mid \delta(q_0, w) \in F\} \subseteq \Sigma^*$

Example of a DFA

$\delta(ee, 1) = eo$ $\quad \delta(eo, 1) = ee$ $\quad \delta(oe, 1) = oo$ $\quad \delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\quad \delta(eo, 0) = oo$ $\quad \delta(oe, 0) = ee$ $\quad \delta(oo, 0) = eo$

$$\delta(ee, \varepsilon) = ee$$

Example of a DFA

$\delta(ee, 1) = eo$ $\delta(eo, 1) = ee$ $\delta(oe, 1) = oo$ $\delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\delta(eo, 0) = oo$ $\delta(oe, 0) = ee$ $\delta(oo, 0) = eo$

$$\delta(ee, \varepsilon) = ee$$

$$\delta(ee, 1) = \delta(\ \delta(ee, \varepsilon)\ , 1) =$$

Example of a DFA

$\delta(ee, 1) = eo$ $\delta(eo, 1) = ee$ $\delta(oe, 1) = oo$ $\delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\delta(eo, 0) = oo$ $\delta(oe, 0) = ee$ $\delta(oo, 0) = eo$

$$\boxed{\delta(ee, \varepsilon) = ee}$$

$$\delta(ee, 1) = \delta(\ \delta(ee, \varepsilon)\ , 1) =$$

Example of a DFA

$\delta(ee, 1) = eo$ $\quad$ $\delta(eo, 1) = ee$ $\quad$ $\delta(oe, 1) = oo$ $\quad$ $\delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\quad$ $\delta(eo, 0) = oo$ $\quad$ $\delta(oe, 0) = ee$ $\quad$ $\delta(oo, 0) = eo$

$$\delta(ee, \varepsilon) = ee$$

$$\delta(ee, 1) = \delta(\ \delta(ee, \varepsilon)\ , 1) = \delta(\ ee\ , 1) =$$

Example of a DFA

$\delta(ee, 1) = eo$ $\quad$ $\delta(eo, 1) = ee$ $\quad$ $\delta(oe, 1) = oo$ $\quad$ $\delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\quad$ $\delta(eo, 0) = oo$ $\quad$ $\delta(oe, 0) = ee$ $\quad$ $\delta(oo, 0) = eo$

$$\delta(ee, \varepsilon) = ee$$

$$\delta(ee, 1) = \delta(\ \delta(ee, \varepsilon)\ , 1) = \delta(\ ee\ , 1) = eo$$

Example of a DFA

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$$\delta(ee, \varepsilon) = ee$$

$$\delta(ee, 1) = \delta(\ \delta(ee, \varepsilon)\ , 1) = \delta(\ ee\ , 1) = eo$$

$$\delta(ee, 1101) =$$

$$\delta(ee, 1101)$$

Example of a DFA

$\delta(ee, 1) = eo$ $\quad \delta(eo, 1) = ee$ $\quad \delta(oe, 1) = oo$ $\quad \delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\quad \delta(eo, 0) = oo$ $\quad \delta(oe, 0) = ee$ $\quad \delta(oo, 0) = eo$

$$\delta(ee, \varepsilon) = ee$$

$$\delta(ee, 1) = \delta(\ \delta(ee, \varepsilon)\ , 1) = \delta(\ ee\ , 1) = eo$$

$$\delta(ee, 1101) =$$

$$\delta(\ \ \delta(ee, 110)\ \ , 1)$$

Example of a DFA

$\delta(ee, 1) = eo$ $\delta(eo, 1) = ee$ $\delta(oe, 1) = oo$ $\delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\delta(eo, 0) = oo$ $\delta(oe, 0) = ee$ $\delta(oo, 0) = eo$

$$\delta(ee, \varepsilon) = ee$$

$$\delta(ee, 1) = \delta(\ \delta(ee, \varepsilon)\ , 1) = \delta(\ ee\ , 1) = eo$$

$$\delta(ee, 1101) =$$

$$\delta(\delta(\delta(ee, 11), 0), 1)$$

Example of a DFA

$\delta(ee, 1) = eo$ $\quad$ $\delta(eo, 1) = ee$ $\quad$ $\delta(oe, 1) = oo$ $\quad$ $\delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\quad$ $\delta(eo, 0) = oo$ $\quad$ $\delta(oe, 0) = ee$ $\quad$ $\delta(oo, 0) = eo$

$$\delta(ee, \varepsilon) = ee$$

$$\delta(ee, 1) = \delta(\ \delta(ee, \varepsilon)\ , 1) = \delta(\ ee\ , 1) = eo$$

$$\delta(ee, 1101) =$$

$$\delta(\delta(\delta(\delta(ee, 1), 1), 0), 1)$$

Example of a DFA

$\delta(ee, 1) = eo$ $\quad$ $\delta(eo, 1) = ee$ $\quad$ $\delta(oe, 1) = oo$ $\quad$ $\delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\quad$ $\delta(eo, 0) = oo$ $\quad$ $\delta(oe, 0) = ee$ $\quad$ $\delta(oo, 0) = eo$

$$\delta(ee, \varepsilon) = ee$$

$$\delta(ee, 1) = \delta(\ \delta(ee, \varepsilon)\ , 1) = \delta(\ ee\ , 1) = eo$$

$$\delta(ee, 1101) =$$

$$\delta(\delta(\delta(eo, 1), 0), 1)$$

Example of a DFA

$\delta(ee, 1) = eo$ $\delta(eo, 1) = ee$ $\delta(oe, 1) = oo$ $\delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\delta(eo, 0) = oo$ $\delta(oe, 0) = ee$ $\delta(oo, 0) = eo$

$$\delta(ee, \varepsilon) = ee$$

$$\delta(ee, 1) = \delta(\ \delta(ee, \varepsilon)\ , 1) = \delta(\ ee\ , 1) = eo$$

$$\delta(ee, 1101) =$$

$$\delta(\delta(ee, 0), 1)$$

Example of a DFA

$\delta(ee, 1) = eo$ $\delta(eo, 1) = ee$ $\delta(oe, 1) = oo$ $\delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\delta(eo, 0) = oo$ $\delta(oe, 0) = ee$ $\delta(oo, 0) = eo$

$$\delta(ee, \varepsilon) = ee$$

$$\delta(ee, 1) = \delta(\ \delta(ee, \varepsilon)\ , 1) = \delta(\ ee\ , 1) = eo$$

$$\delta(ee, 1101) =$$

$$\delta(oe, 1)$$

Example of a DFA

$\delta(ee, 1) = eo$ $\delta(eo, 1) = ee$ $\delta(oe, 1) = oo$ $\delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\delta(eo, 0) = oo$ $\delta(oe, 0) = ee$ $\delta(oo, 0) = eo$

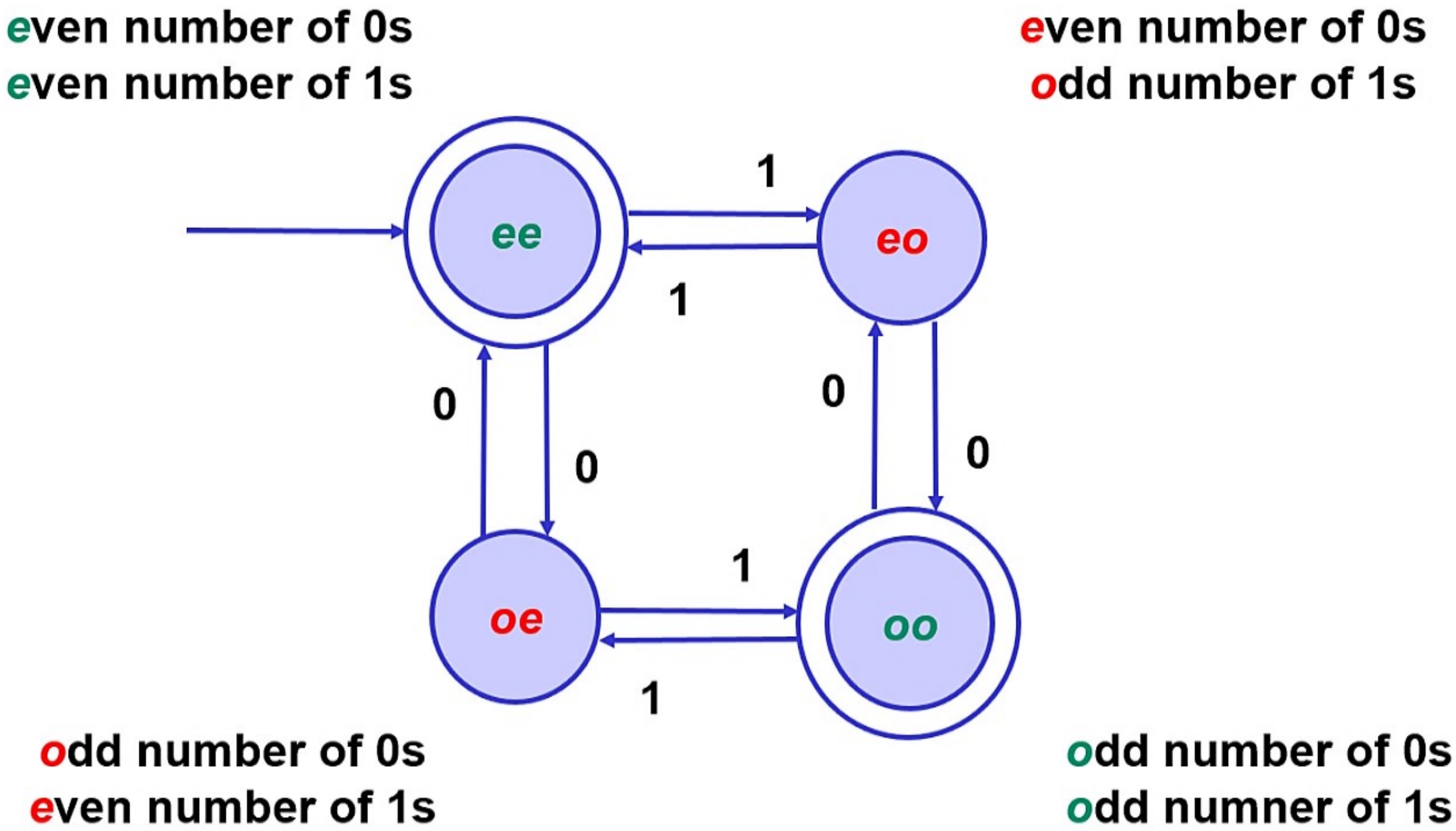
$$\delta(ee, \varepsilon) = ee$$

$$\delta(ee, 1) = \delta(\delta(ee, \varepsilon) , 1) = \delta(ee , 1) = eo$$

$$\delta(ee, 1101) =$$

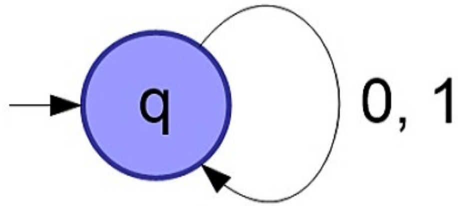
$$oo$$

Example of a DFA



Example of a DFA

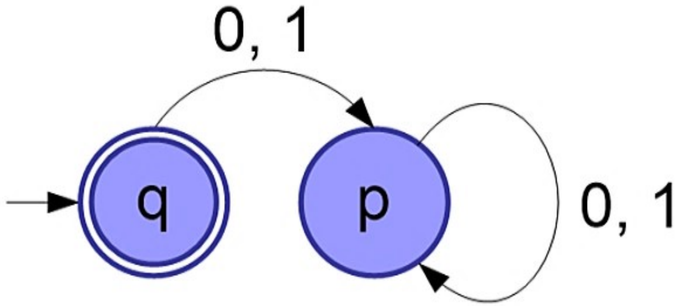
$M_1 = (\{q\}, \{0, 1\}, \{\delta(q, 0) = q, \delta(q, 1) = q\}, q, \{\ \})$



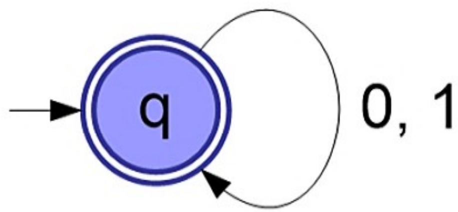
$L(M_1) = \{\ \}$

$M_2 = (\{q, p\}, \{0, 1\}, \{\delta(q, 0) = p, \delta(q, 1) = p, \delta(p, 0) = p, \delta(p, 1) = p\}, q, \{ q \})$

$L(M_2) = \{ \varepsilon \}$

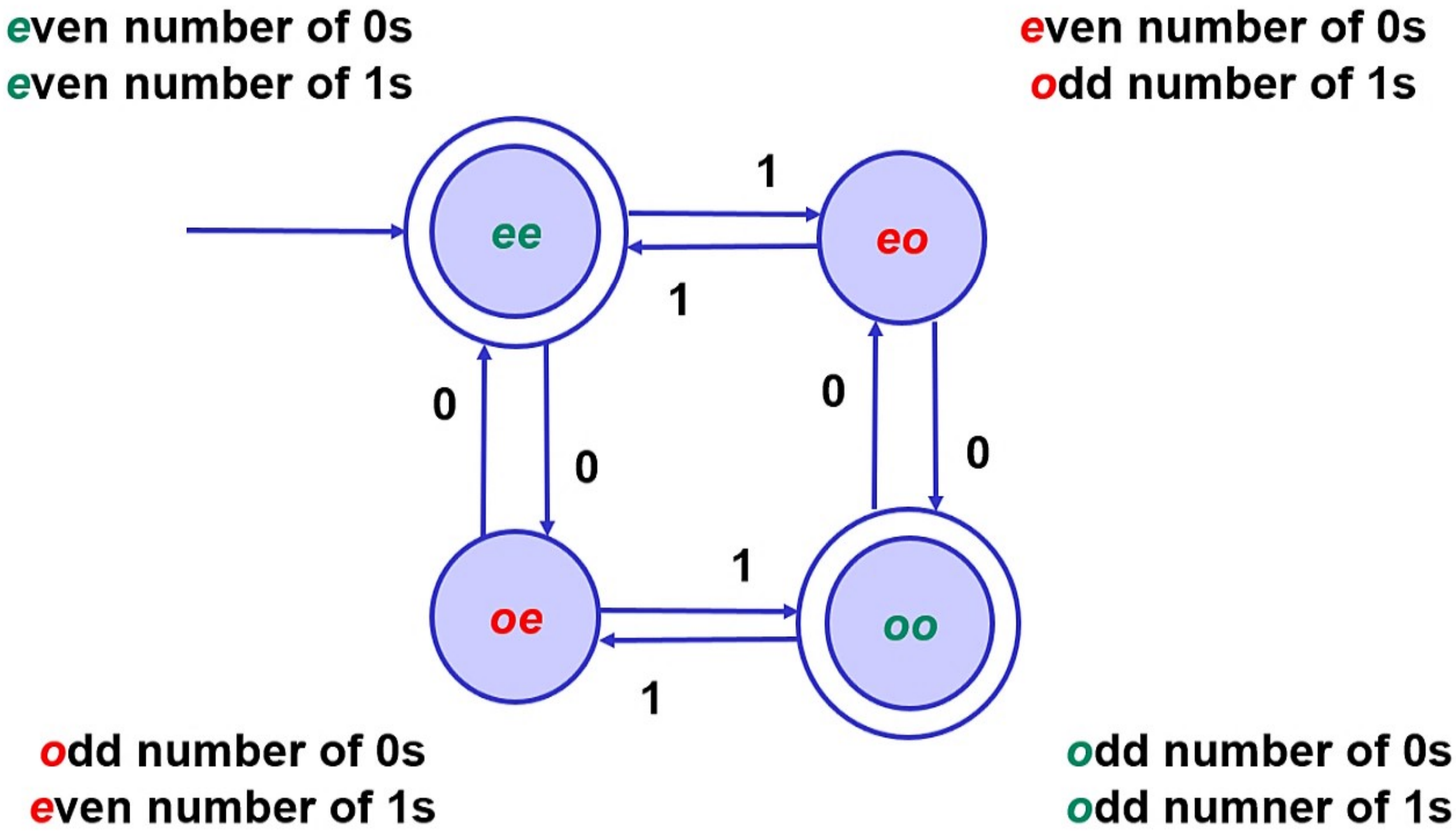


$M_3 = (\{q\}, \{0, 1\}, \{\delta(q, 0) = q, \delta(q, 1) = q\}, q, \{ q \})$



$L(M_3) = \Sigma^*$

Software Implementation of DFA



State Representation Method – EXPLICIT

$\delta(ee, 0) = oe$
$\delta(ee, 1) = eo$
$\delta(eo, 0) = oo$
$\delta(eo, 1) = ee$
$\delta(oe, 0) = ee$
$\delta(oe, 1) = oo$
$\delta(oo, 0) = eo$
$\delta(oo, 1) = oe$

```
Table [EE, 0] = OE;
Table [EE, 1] = EO;
Table [EE, ⊥] = 1;
Table [OE, 0] = EE;
Table [OE, 1] = OO;
Table [OE, ⊥] = 0;
Table [EO, 0] = OO;
Table [EO, 1] = EE;
Table [EO, ⊥] = 0;
Table [OO, 0] = EO;
Table [OO, 1] = OE;
Table [OO, ⊥] = 1;

State = EE;
Read(Symbol);

while (Symbol != ⊥)
{
    State = Table [State, Symbol];
    Read(Symbol);
}

Print(Table [State, ⊥], State);
```

State Representation Method – IMPLICIT

$\delta(ee, 1) = eo$ $\delta(eo, 1) = ee$ $\delta(oe, 1) = oo$ $\delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\delta(eo, 0) = oo$ $\delta(oe, 0) = ee$ $\delta(oo, 0) = eo$

```
EE: Read(Symbol);

    if (Symbol == ⊥)
        Print("STRING IS ACCEPTABLE, EVEN:0, EVEN:1");
    if (Symbol == 0)
        goto OE;
    else
        goto EO;

OE: Read(Symbol);

    if (Symbol == ⊥)
        Print("STRING IS NOT ACCEPTABLE, ODD:0, EVEN:1");
    if (Symbol == 0)
        goto EE;
    else
        goto OO;
```

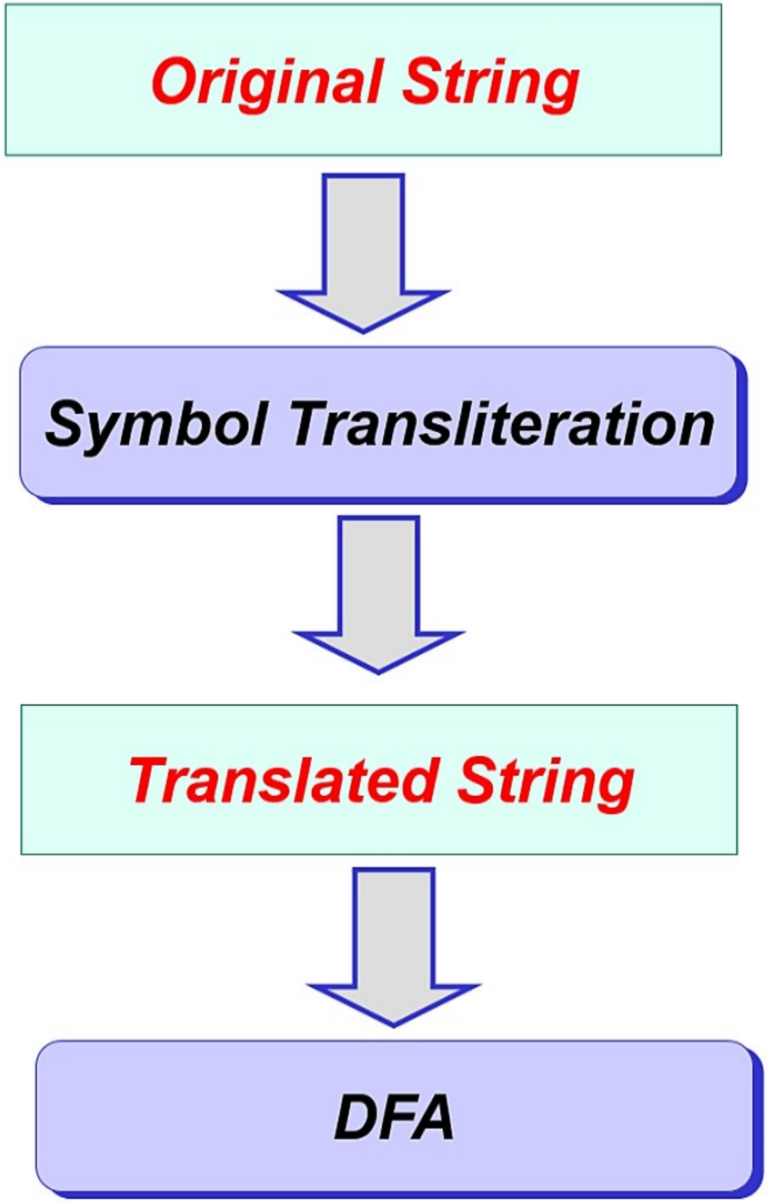
State Representation Method – IMPLICIT

$\delta(ee, 1) = eo$ $\delta(eo, 1) = ee$ $\delta(oe, 1) = oo$ $\delta(oo, 1) = oe$

$\delta(ee, 0) = oe$ $\delta(eo, 0) = oo$ $\delta(oe, 0) = ee$ $\delta(oo, 0) = eo$

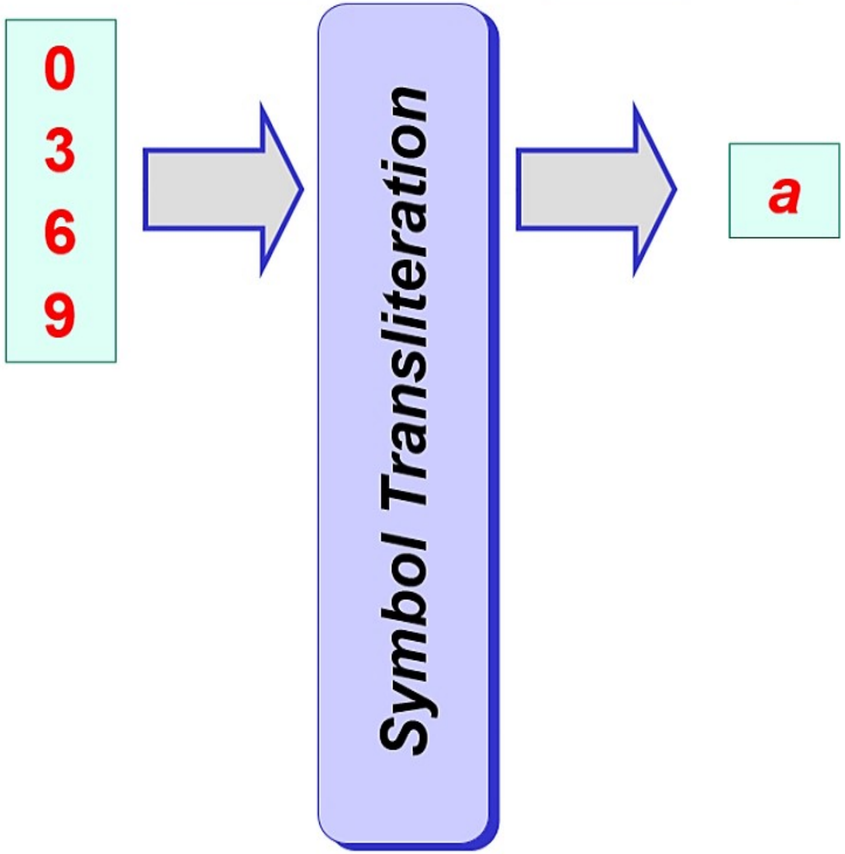
```
EO: Read(Symbol);

    if (Symbol == ⊥)
        Print("STRING IS NOT ACCEPTABLE, EVEN:0, ODD:1");
    if (Symbol == 0)
        goto OO;
    else
        goto EE;

OO: Read(Symbol);

    if (Symbol == ⊥)
        Print("STRING IS ACCEPTABLE, ODD:0, ODD:1");
    if (Symbol == 0)
        goto EO;
    else
        goto OE;
```

Symbol Transliteration



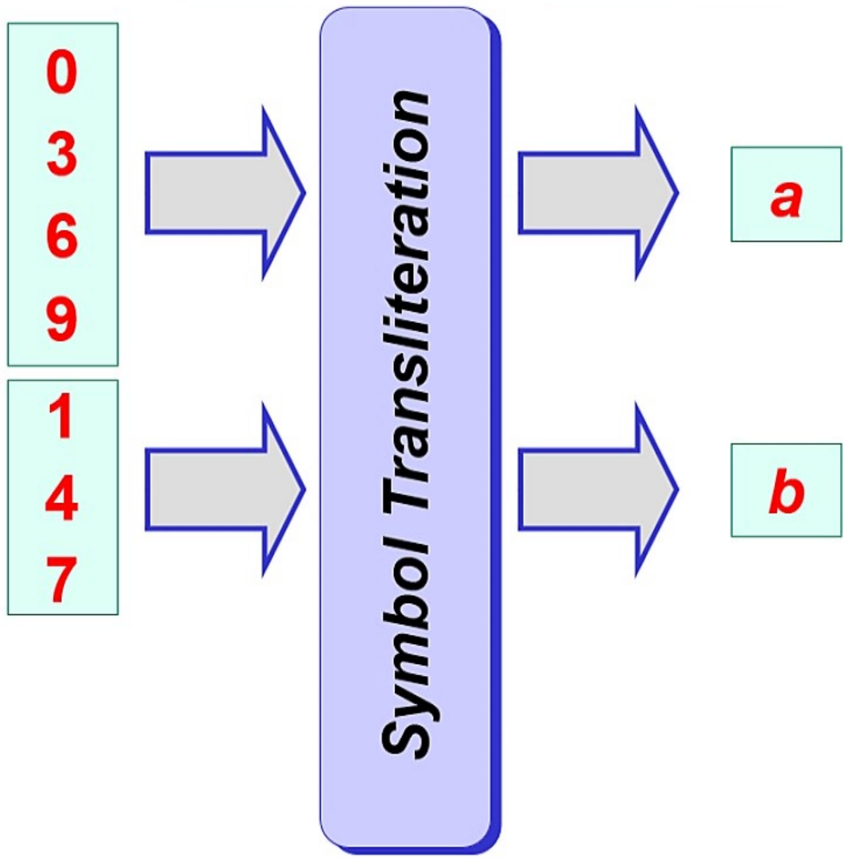
Example of Symbol Transliteration

	0	1	2	3	4	5	6	7	8	9	
S	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	1
O	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	0
T	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	0



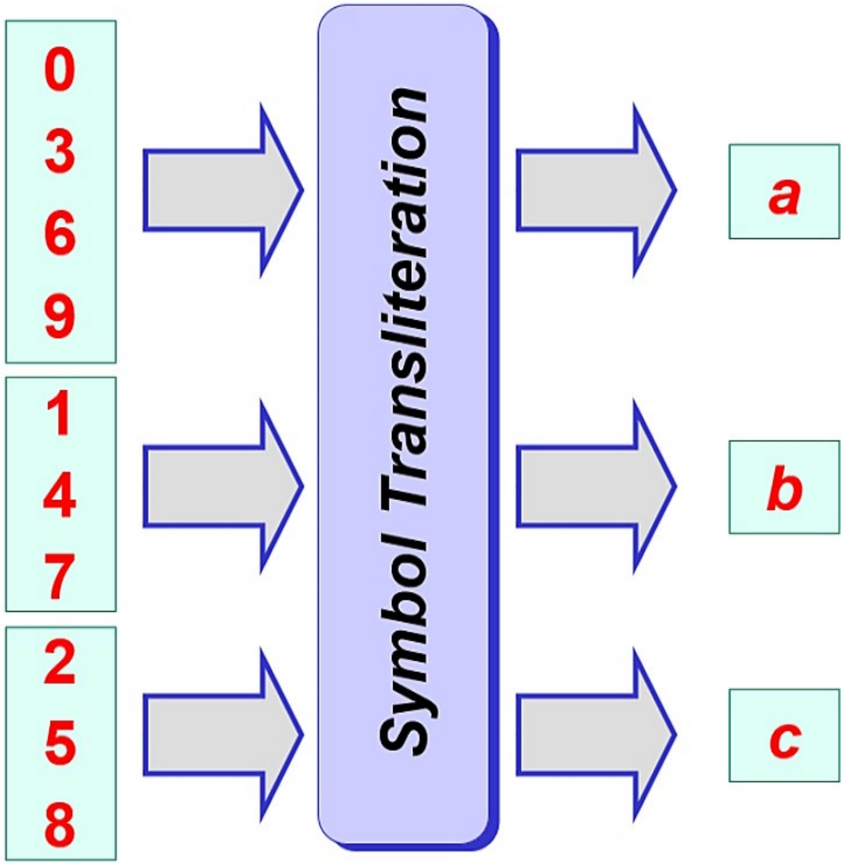
Example of Symbol Transliteration

	a	1	2	*a*	4	5	*a*	7	8	*a*	
S	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	1
O	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	0
T	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	0



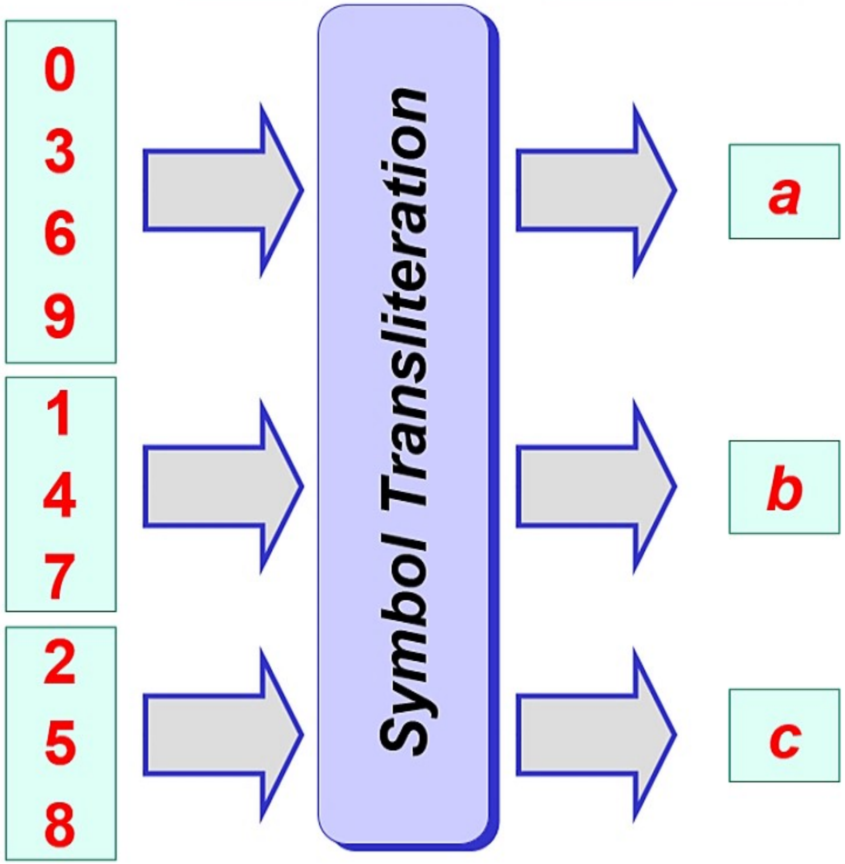
Example of Symbol Transliteration

	a	b	2	a	b	5	a	b	8	a	
S	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	1
O	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	0
T	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	0



Example of Symbol Transliteration

	a	b	c	a	b	c	a	b	c	a	
S	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	1
O	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	0
T	*T*	*S*	*O*	*T*	*S*	*O*	*T*	*S*	*O*	*T*	0



	a	b	c	
S	*S*	*O*	*T*	1
O	*O*	*T*	*S*	0
T	*T*	*S*	*O*	0

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2.1.2 Minimization of deterministic finite automaton

Minimization of DFA

- **Efficient software implementation**

  - **Requires DFA with a minimum possible number of states**

    For a regular language ***L*** there exists a ***DFA M*** with less of equal number of states than any other ***DFA M'*** that accepts that same language ***L***.

- **Equivalence of states**

  State ***p*** of a ***DFA*** $M=(Q, \Sigma, \delta, q_0, F)$ is equivalent to the state ***p'*** of a ***DFA*** $M'=(Q', \Sigma, \delta', q_0', F')$ if and only if ***DFA M*** from the state ***p*** accepts the same set of strings as ***DFA M'*** accepts from the state ***p'***.

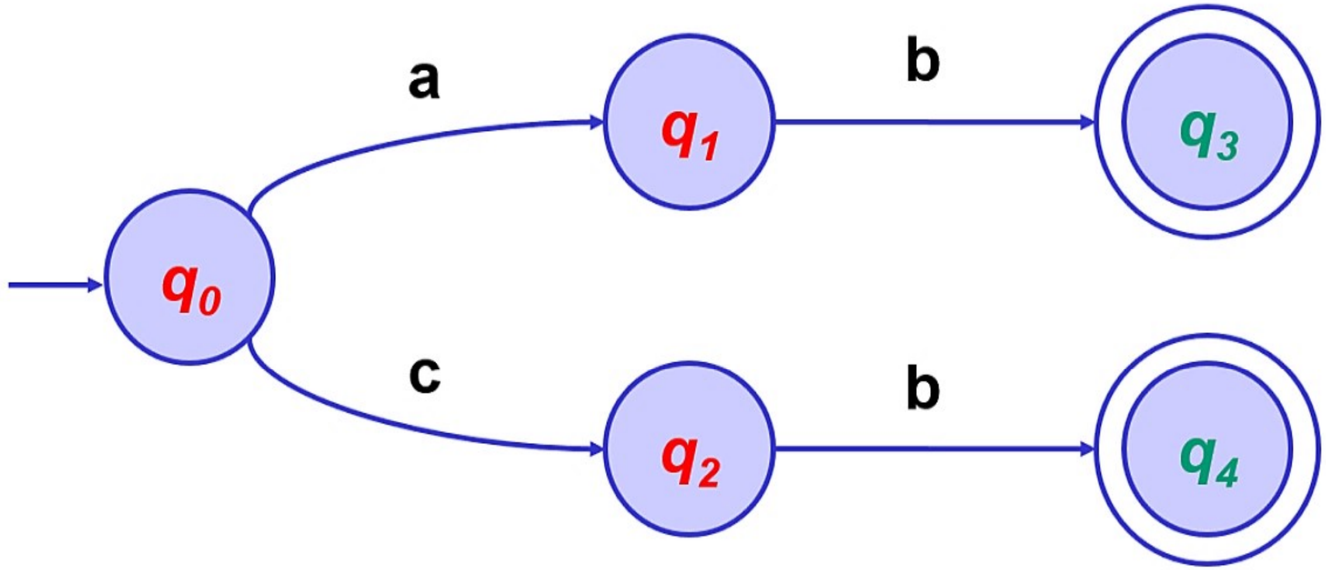
  For any string ***w*** in set Σ^* holds the following:

$$\delta(p, w) \in F \quad \wedge \quad \delta'(p', w) \in F'$$

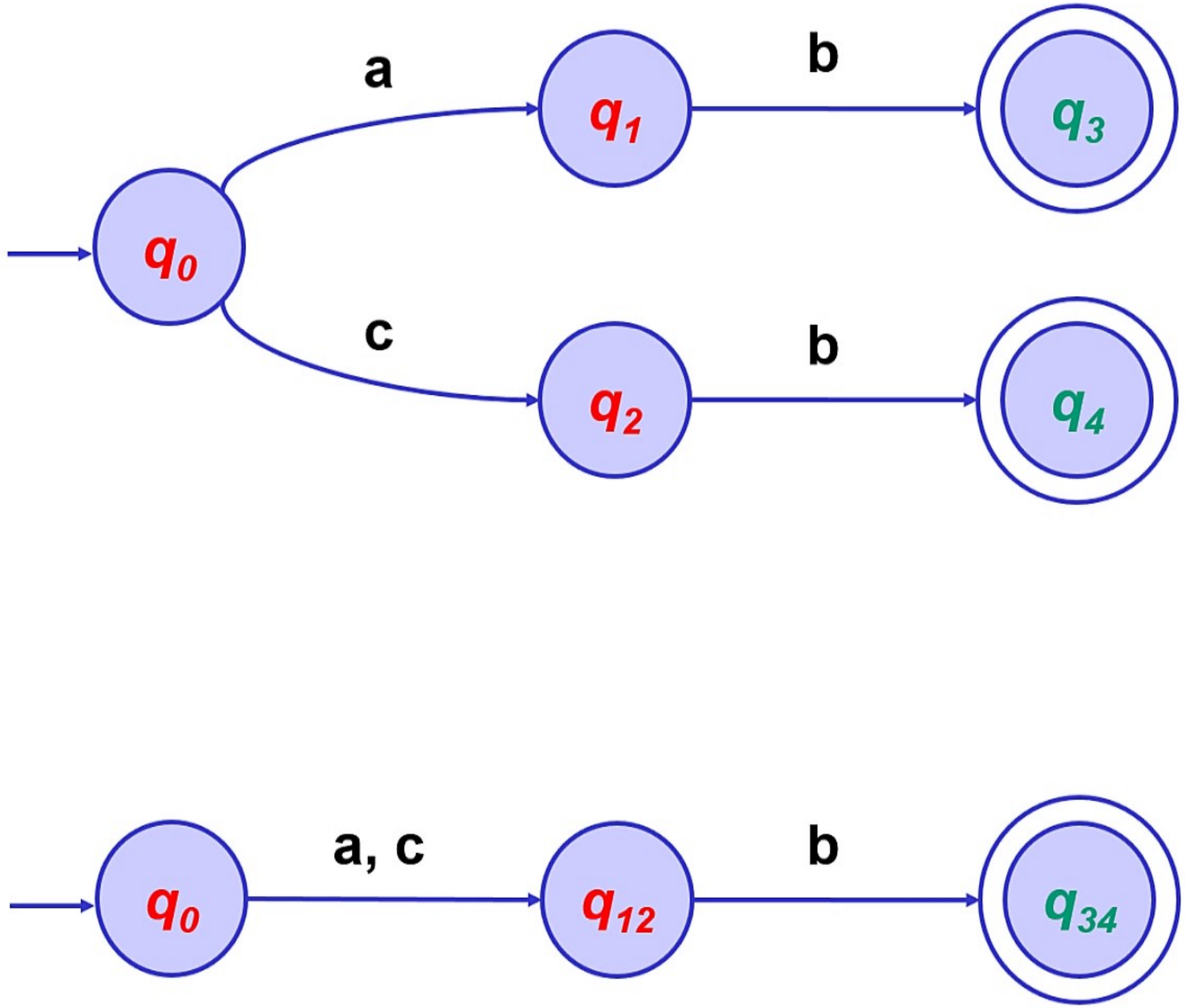
  or

$$\delta(p, w) \notin F \quad \wedge \quad \delta'(p', w) \notin F'$$

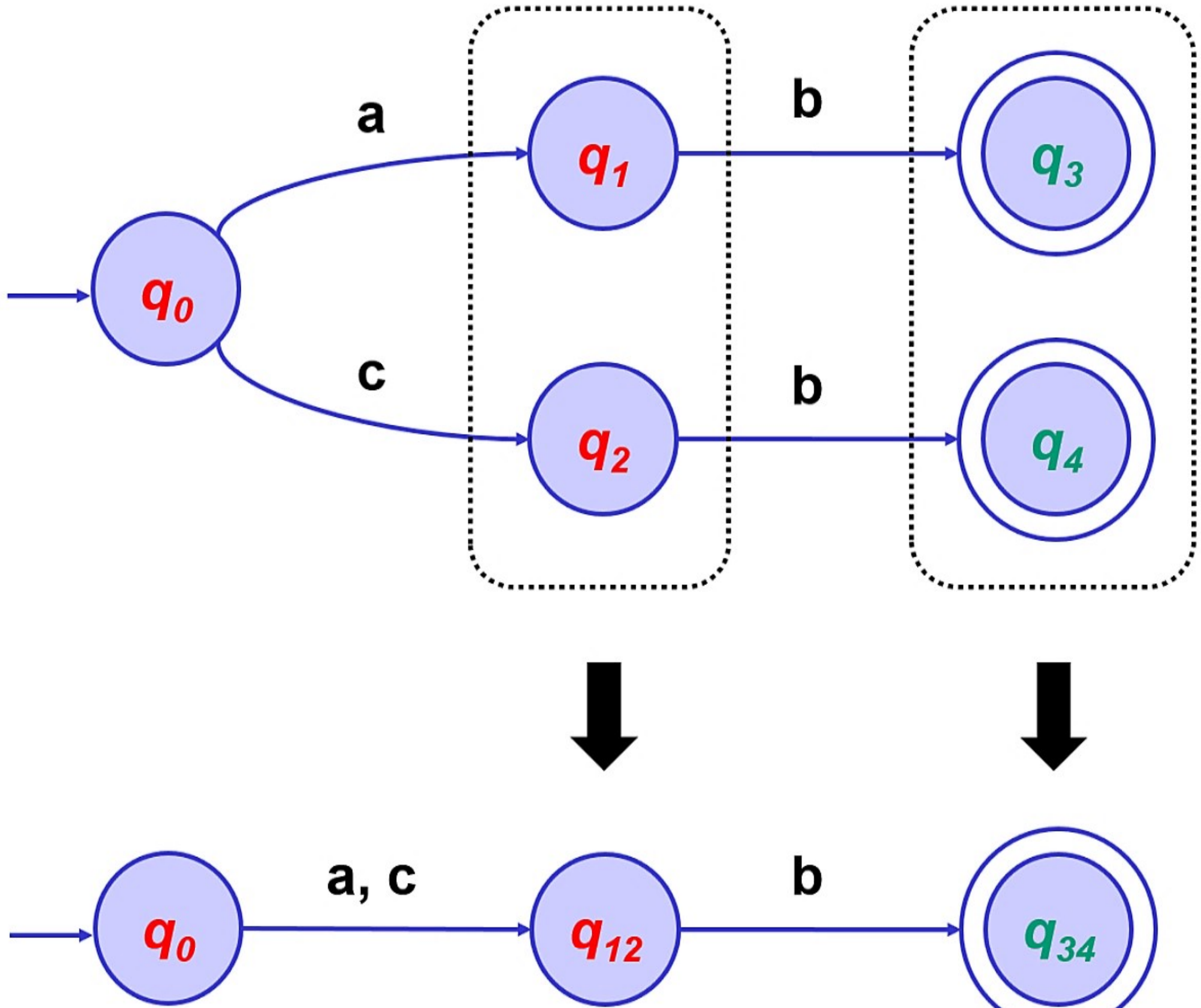
Minimization of DFA



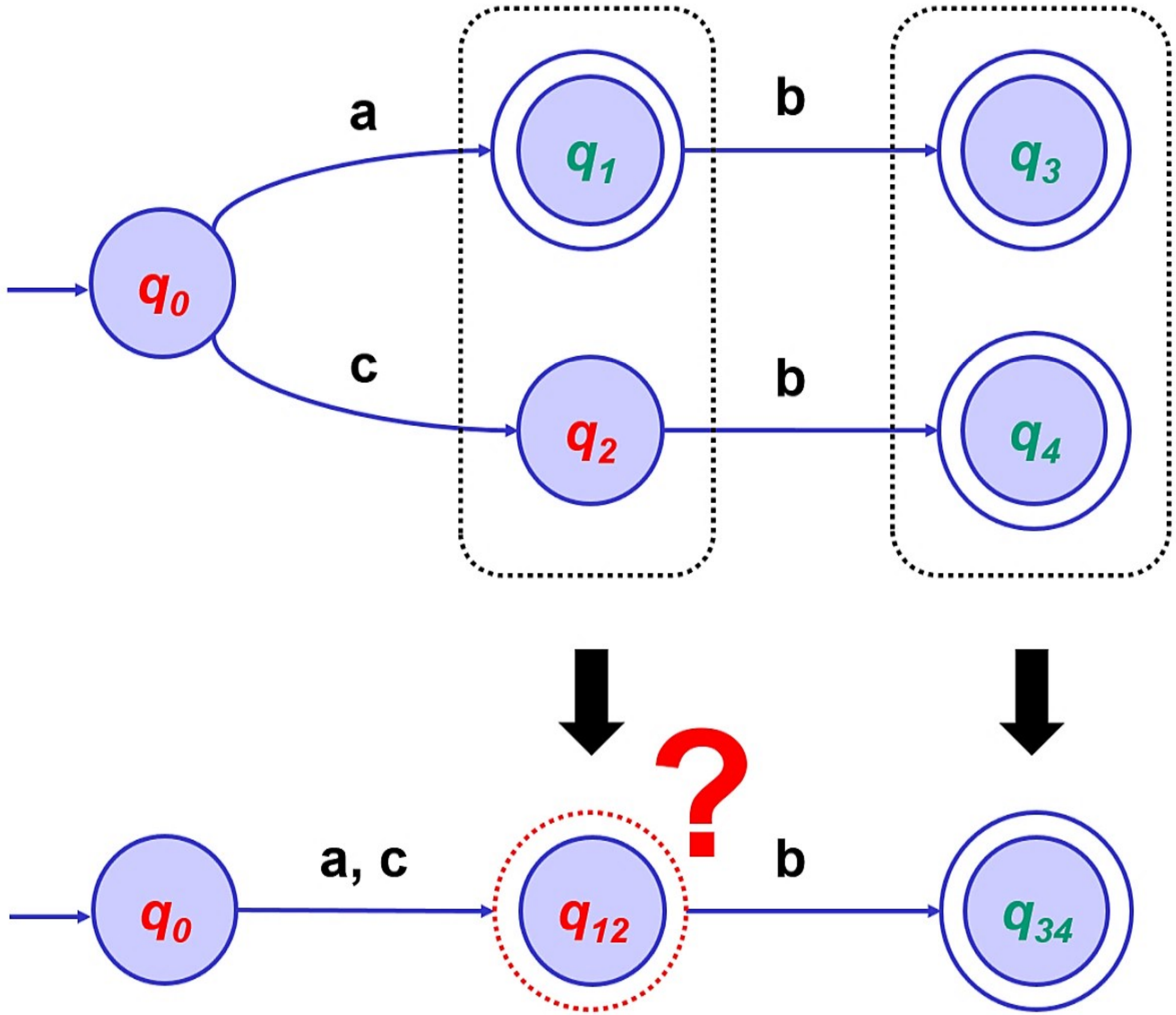
Minimization of DFA



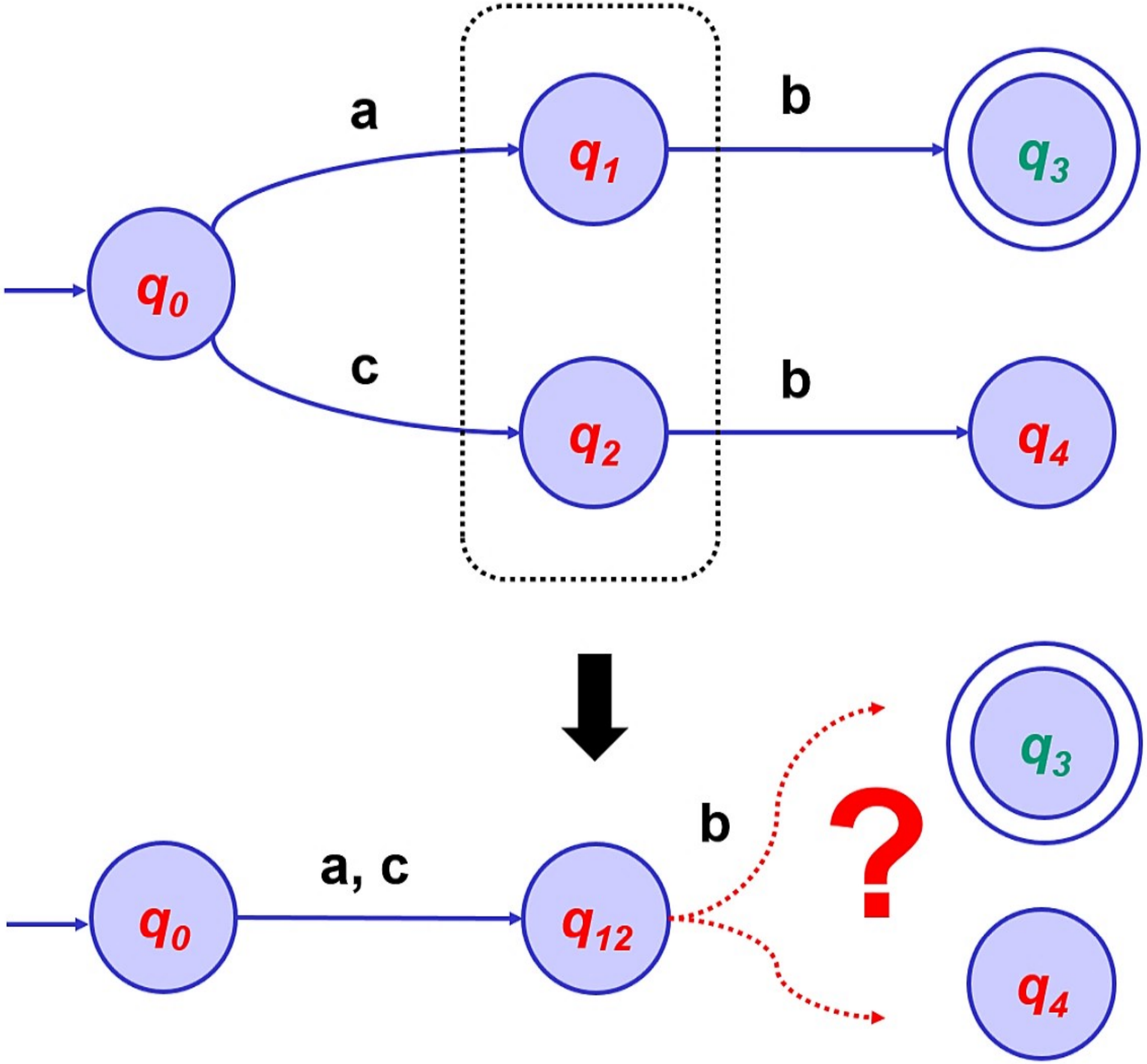
q_1 and q_2 are equivalent
q_3 and q_4 are equivalent



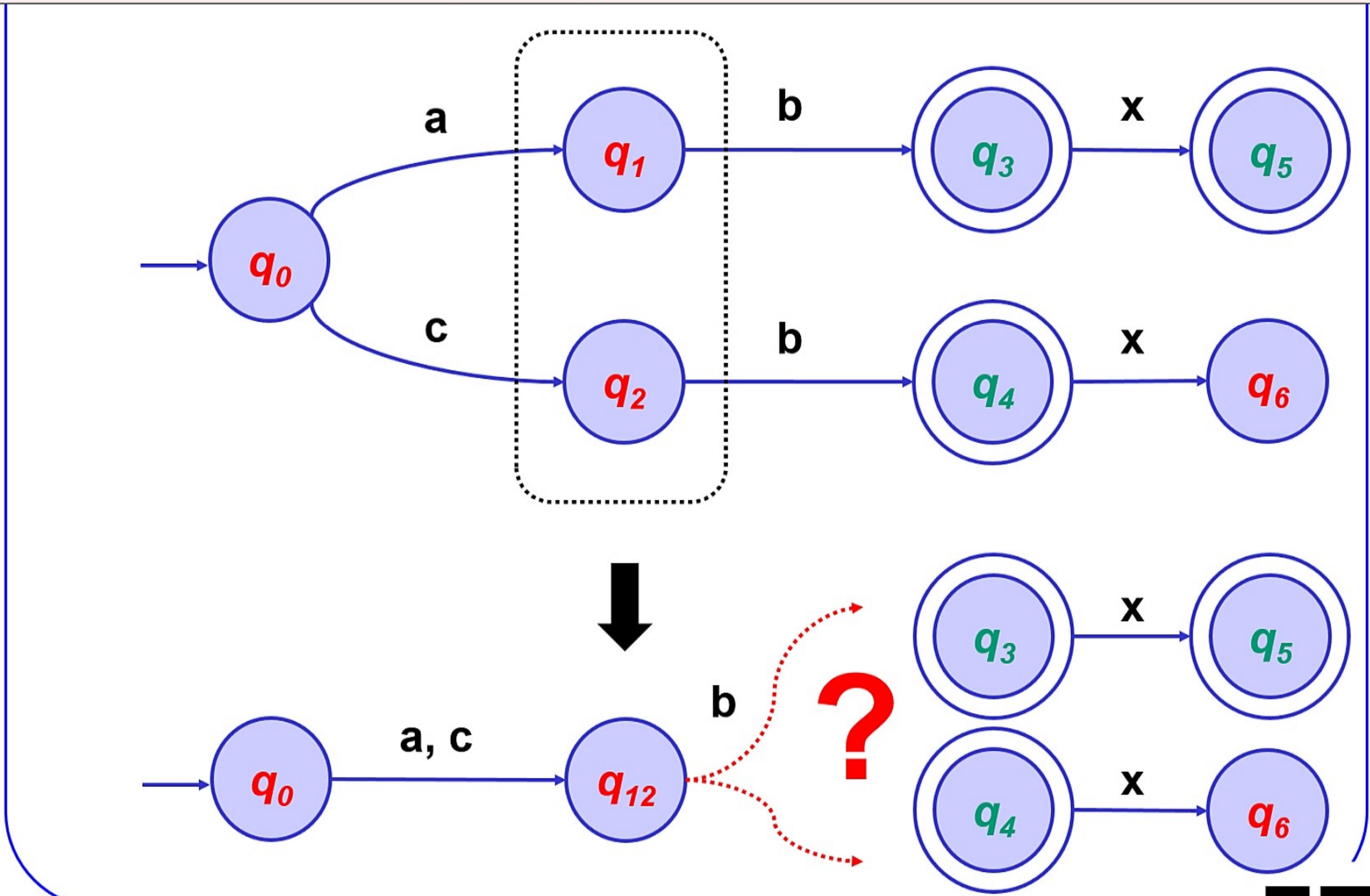
q_1 and q_2 are NOT equivalent
q_3 and q_4 are equivalent



q_3 and q_4 are NOT equivalent $\Rightarrow$ q_1 and q_2 are NOT equivalent



q_5 and q_6 are NOT equivalent $\Rightarrow$
q_3 and q_4 are NOT equivalent $\Rightarrow$
q_1 and q_2 are NOT equivalent



Minimization of DFA

- **Equivalence test for states *p* and *q***
  - **Testing of two conditions:**

Compatibility condition:

States ***p*** and ***q*** are either both accepting ($p \in F \wedge q \in F$) or both nonaccepting ($p \notin F \wedge q \notin F$).

Propagation condition:

For each input symbol ***a***, states $\delta(p, a)$ and $\delta(q, a)$ are equivalent.

Minimization of DFA – *table-filling algorithm*

	0	1	$\perp$
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

Minimization of DFA – *table-filling algorithm*

	0	1	$\perp$
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

p_2							
p_3							
p_4							
p_5							
p_6							
p_7							
p_8							
	p_1	p_2	p_3	p_4	p_5	p_6	p_7

1

Mark all pairs (p, q) for which holds that $p \in F$ and $q \notin F$

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

p_2							
p_3	X	X					
p_4			X				
p_5			X				
p_6			X				
p_7			X				
p_8			X				
	p_1	p_2	p_3	p_4	p_5	p_6	p_7

1

Mark all pairs (p, q) for which holds that $p \in F$ and $q \notin F$

Minimization of DFA – *table-filling algorithm*

	0	1	$\perp$
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2							
p_3	X	X					
p_4			X				
p_5			X				
p_6			X				
p_7			X				
p_8			X				

2

For any unmarked pair of states (p, q)

if the pair $(\delta(p,a), \delta(q,a))$ is already marked

Mark (p,q);

Recursively mark all the pairs in the list assigned to the marked pair

else

if $\delta(p,a) \;!= \delta(q,a)$

put the pair (p,q) into the list assigned to the pair $(\delta(p,a), \delta(q,a))$

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2							
p_3	X	X					
p_4			X				
p_5			X				
p_6			X				
p_7			X				
p_8			X				

2

For any unmarked pair of states (p, q)

if the pair ($\delta(p,a)$, $\delta(q,a)$) is already marked

Mark (p,q);

Recursively mark all the pairs in the list assigned to the marked pair

else

if $\delta(p,a) \; != \; \delta(q,a)$

put the pair (p,q) into the list assigned to the pair ($\delta(p,a)$, $\delta(q,a)$)

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

p_2	X						
p_3	X	X					
p_4			X				
p_5			X				
p_6			X				
p_7			X				
p_8			X				
	p_1	p_2	p_3	p_4	p_5	p_6	p_7

2

For any unmarked pair of states (*p*, *q*)

if the pair ($\delta(p,a)$, $\delta(q,a)$) is already marked

Mark (*p*,*q*);

Recursively mark all the pairs in the list assigned to the marked pair

else

if $\delta(p,a)$! = $\delta(q,a)$

put the pair (*p*,*q*) into the list assigned to the pair ($\delta(p,a)$, $\delta(q,a)$)

Minimization of DFA – *table-filling algorithm*

	0	1	$\perp$
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X		X				
p_5			X				
p_6			X				
p_7			X				
p_8			X				

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X		X				
p_5			X				
p_6			X				
p_7			X				
p_8			X				

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X		X				
p_5			X				
p_6			X				
p_7			X				
p_8			X				

List assigned to $(p_2, p_8) = \{(p_1, p_5)\}$

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X		X				
p_5			X				
p_6			X				
p_7			X				
p_8			X				

List assigned to $(p_2, p_8) = \{(p_1, p_5)\}$

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X		X				
p_5			X				
p_6	X		X				
p_7			X				
p_8	X		X				

List assigned to (p_2, p_8) = {(p_1, p_5)}

Minimization of DFA – *table-filling algorithm*

	0	1	$\perp$
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X		X				
p_5			X				
p_6	X		X				
p_7			X				
p_8	X		X				

List assigned to (p_2, p_8) = {(p_1, p_5)}

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X		X				
p_5			X				
p_6	X		X				
p_7			X				
p_8	X		X				

List assigned to $(p_2, p_8) = \{(p_1, p_5)\}$

List assigned to $(p_2, p_7) = \{(p_1, p_7)\}$

List assigned to $(p_5, p_6) = \{(p_1, p_7)\}$

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

p_2	X						
p_3	X	X					
p_4	X	X	X				
p_5		X	X				
p_6	X	X	X				
p_7		X	X				
p_8	X		X				
	p_1	p_2	p_3	p_4	p_5	p_6	p_7

List assigned to $(p_2, p_8) = \{(p_1, p_5)\}$

List assigned to $(p_2, p_7) = \{(p_1, p_7)\}$

List assigned to $(p_5, p_6) = \{(p_1, p_7)\}$

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X	X	X				
p_5		X	X				
p_6	X	X	X				
p_7		X	X				
p_8	X		X				

List assigned to $(p_2, p_8) = \{(p_1, p_5)\}$

List assigned to $(p_2, p_7) = \{(p_1, p_7)\}$

List assigned to $(p_5, p_6) = \{(p_1, p_7)\}$

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X	X	X				
p_5		X	X				
p_6	X	X	X				
p_7	X	X	X				
p_8	X		X				

List assigned to $(p_2, p_8) = \{(p_1, p_5)\}$

List assigned to $(p_2, p_7) = \{(p_1, p_7)\}$

List assigned to $(p_5, p_6) = \{(p_1, p_7)\}$

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X	X	X				
p_5		X	X	X			
p_6	X	X	X				
p_7	X	X	X	X			
p_8	X		X	X			

List assigned to $(p_2, p_8) = \{(p_1, p_5)\}$

List assigned to $(p_5, p_6) = \{(p_1, p_7)\}$

Minimization of DFA – *table-filling algorithm*

	0	1	$\perp$
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X	X	X				
p_5		X	X	X			
p_6	X	X	X		X		
p_7	X	X	X	X			
p_8	X		X	X			

List assigned to $(p_2, p_8) = \{(p_1, p_5)\}$

List assigned to $(p_5, p_6) = \{(p_1, p_7)\}$

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X	X	X				
p_5		X	X	X			
p_6	X	X	X		X		
p_7	X	X	X	X			
p_8	X		X	X			

List assigned to $(p_2, p_8) = \{(p_1, p_5)\}$

List assigned to $(p_5, p_6) = \{(p_1, p_7)\}$

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X	X	X				
p_5		X	X	X			
p_6	X	X	X		X		
p_7	X	X	X	X	X	X	
p_8	X		X	X	X	X	X

List assigned to $(p_2, p_8) = \{(p_1, p_5)\}$

Minimization of DFA – *table-filling algorithm*

	0	1	⊥
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	p_1	p_2	p_3	p_4	p_5	p_6	p_7
p_2	X						
p_3	X	X					
p_4	X	X	X				
p_5		X	X	X			
p_6	X	X	X		X		
p_7	X	X	X	X	X	X	
p_8	X		X	X	X	X	X

List assigned to $(p_2, p_8) = \{(p_1, p_5)\}$

Equivalent states:

(p_1, p_5), (p_2, p_8), (p_4, p_6)

Minimization of DFA – *table-filling algorithm*

	0	1	$\perp$
p_1	p_2	p_6	0
p_2	p_7	p_3	0
p_3	p_1	p_3	1
p_4	p_3	p_7	0
p_5	p_8	p_6	0
p_6	p_3	p_7	0
p_7	p_7	p_5	0
p_8	p_7	p_3	0

	0	1	$\perp$
$[p_1, p_5]$	$[p_2, p_8]$	$[p_4, p_6]$	0
$[p_2, p_8]$	p_7	p_3	0
p_3	$[p_1, p_5]$	p_3	1
$[p_4, p_6]$	p_3	p_7	0
p_7	p_7	$[p_1, p_5]$	0

Unreachable States in DFA

	c	d	$\perp$
q_0	q_1	q_5	0
q_1	q_2	q_7	1
q_2	q_2	q_5	1
q_3	q_5	q_7	0
q_4	q_5	q_6	0
q_5	q_3	q_1	0
q_6	q_8	q_0	1
q_7	q_0	q_1	1
q_8	q_3	q_6	0

RS $\boxed{q_0}$

1) Put the initial state q_0 into the list of reachable states *RS*

2) Extend the list *RS* with the states $\{p \mid p=\delta(q_0, a)$, za sve $a \in \Sigma\}$

3) $q_i \in RS$, $RS = RS \cup \{p \mid p=\delta(q_i, a)$, for any $a \in \Sigma\}$

Unreachable States in DFA

	c	d	$\perp$
q_0	q_1	q_5	0
q_1	q_2	q_7	1
q_2	q_2	q_5	1
q_3	q_5	q_7	0
q_4	q_5	q_6	0
q_5	q_3	q_1	0
q_6	q_8	q_0	1
q_7	q_0	q_1	1
q_8	q_3	q_6	0

RS	q_0	q_1	q_5

1) Put the initial state q_0 into the list of reachable states *RS*

2) Extend the list *RS* with the states $\{p \mid p=\delta(q_0, a)$, za sve $a \in \Sigma\}$

3) $q_i \in RS$, $RS = RS \cup \{p \mid p=\delta(q_i, a)$, for any $a \in \Sigma\}$

Unreachable States in DFA

	c	d	$\perp$
q_0	q_1	q_5	0
q_1	q_2	q_7	1
q_2	q_2	q_5	1
q_3	q_5	q_7	0
q_4	q_5	q_6	0
q_5	q_3	q_1	0
q_6	q_8	q_0	1
q_7	q_0	q_1	1
q_8	q_3	q_6	0

RS	q_0	q_1	q_5	q_2	q_7	q_3

1) Put the initial state q_0 into the list of reachable states *RS*

2) Extend the list *RS* with the states $\{p \mid p=\delta(q_0, a)$, za sve $a\in\Sigma\}$

3) $q_i\in RS$, $RS = \mathrm{RS} \cup \{p \mid p=\delta(q_i, a)$, for any $a\in\Sigma\}$

Unreachable States in DFA

	c	d	$\perp$
q_0	q_1	q_5	0
q_1	q_2	q_7	1
q_2	q_2	q_5	1
q_3	q_5	q_7	0
q_4	q_5	q_6	0
q_5	q_3	q_1	0
q_6	q_8	q_0	1
q_7	q_0	q_1	1
q_8	q_3	q_6	0

RS	q_0	q_1	q_5	q_2	q_7	q_3

1) Put the initial state q_0 into the list of reachable states *RS*

2) Extend the list *RS* with the states $\{p \mid p=\delta(q_0, a)$, za sve $a\in\Sigma\}$

3) $q_i\in RS$, $RS = \mathrm{RS} \cup \{p \mid p=\delta(q_i, a)$, for any $a\in\Sigma\}$

Unreachable States in DFA

	c	d	$\perp$
q_0	q_1	q_5	0
q_1	q_2	q_7	1
q_2	q_2	q_5	1
q_3	q_5	q_7	0
q_4	q_5	q_6	0
q_5	q_3	q_1	0
q_6	q_8	q_0	1
q_7	q_0	q_1	1
q_8	q_3	q_6	0

RS					
q_0	q_1	q_5	q_2	q_7	q_3

	c	d	$\perp$
q_0	q_1	q_5	0
q_1	q_2	q_7	1
q_2	q_2	q_5	1
q_3	q_5	q_7	0
q_5	q_3	q_1	0
q_7	q_0	q_1	1

1) Put the initial state q_0 into the list of reachable states *RS*

2) Extend the list *RS* with the states $\{p \mid p=\delta(q_0, a)$, za sve $a \in \Sigma\}$

3) $q_i \in RS$, $RS = \mathrm{RS} \cup \{p \mid p=\delta(q_i, a)$, for any $a \in \Sigma\}$

Minimal DFA

	c	d	$\perp$
p_0	p_0	p_3	0
p_1	p_2	p_5	0
p_2	p_2	p_7	0
p_3	p_6	p_7	0
p_4	p_1	p_6	1
p_5	p_6	p_5	0
p_6	p_6	p_3	1
p_7	p_6	p_3	0

	c	d	$\perp$
A	A	B	0
B	C	B	0
C	C	B	1

	c	d
p_0, A	p_0, A	p_3, B
p_3, B	p_6, C	p_7, B
p_6, C	p_6, C	p_3, B
p_7, B	p_6, C	p_3, B